

# WARNING: Physics Envy May Be Hazardous To Your Wealth!\*

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## Abstract

The quantitative aspirations of economists and financial analysts have for many years been based on the belief that it should be possible to build models of economic systems—and financial markets in particular—that are as predictive as those in physics. While this perspective has led to a number of important breakthroughs in economics, “physics envy” has also created a false sense of mathematical precision in some cases. We speculate on the origins of physics envy, and then describe an alternate perspective of economic behavior based on a new taxonomy of uncertainty. We illustrate the relevance of this taxonomy with two concrete examples: the classical harmonic oscillator with some new twists that make physics look more like economics, and a quantitative equity market-neutral strategy. We conclude by offering a new interpretation of tail events, proposing an “uncertainty checklist” with which our taxonomy can be implemented, and considering the role that quants played in the current financial crisis.

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*Imagine how much harder physics would be if electrons had feelings!*

– Richard Feynman, speaking at a Caltech graduation ceremony.

## 1 Introduction

The Financial Crisis of 2007–2009 has re-invigorated the longstanding debate regarding the effectiveness of quantitative methods in economics and finance. Are markets and investors driven primarily by fear and greed that cannot be modeled, or is there a method to the market’s madness that can be understood through mathematical means? Those who rail against the quants and blame them for the crisis believe that market behavior cannot be quantified and financial decisions are best left to individuals with experience and discretion. Those who defend quants insist that markets are efficient and the actions of arbitrageurs impose certain mathematical relationships among prices that can be modeled, measured, and managed. Is finance a science or an art?

In this paper, we attempt to reconcile the two sides of this debate by taking a somewhat circuitous path through the sociology of economics and finance to trace the intellectual origins of this conflict—which we refer to as “physics envy”—and show by way of example that “the fault lies not in our models but in ourselves”. By reflecting on the similarities and differences between economic phenomena and those of other scientific disciplines such as psychology and physics, we conclude that economic logic goes awry when we forget that human behavior is not nearly as stable and predictable as physical phenomena. However, this observation does not invalidate economic logic altogether, as some have argued.

In particular, if, like other scientific endeavors, economics is an attempt to understand, predict, and control the unknown through quantitative analysis, the kind of uncertainty affecting economic interactions is critical in determining its successes and failures. Motivated by Knight’s (1921) distinction between “risk” (randomness that can be fully captured by probability and statistics) and “uncertainty” (all other types of randomness), we propose a slightly finer taxonomy—fully reducible, partially reducible, and irreducible uncertainty—that can explain some of the key differences between finance and physics. Fully reducible uncertainty is the kind of randomness that can be reduced to pure risk given sufficient data, computing power, and other resources. Partially reducible uncertainty contains a component that can never be quantified, and irreducible uncertainty is the Knightian limit of unparametrizable randomness. While these definitions may seem like minor extensions of Knight’s clear-cut dichotomy, they underscore the fact that there is a continuum of randomness in between risk and uncertainty, and this nether region is the domain of economics

and business practice. In fact, our taxonomy is reflected in the totality of human intellectual pursuits, which can be classified along a continuous spectrum according to the type of uncertainty involved, with religion at one extreme (irreducible uncertainty), economics and psychology in the middle (partially reducible uncertainty) and mathematics and physics at the other extreme (certainty).

However, our more modest and practical goal is to provide a framework for investors, portfolio managers, regulators, and policymakers in which the efficacy and limitations of economics and finance can be more readily understood. In fact, we hope to show through a series of examples drawn from both physics and finance that the failure of quantitative models in economics is almost always the result of a mismatch between the type of uncertainty in effect and the methods used to manage it. Moreover, the process of scientific discovery may be viewed as the means by which we transition from one level of uncertainty to the next. This framework can also be used to extrapolate the future of finance, the subject of this special volume of the *Journal of Investment Management*. We propose that this future will inevitably involve refinements of the taxonomy of uncertainty and the development of more sophisticated methods for “full-spectrum” risk management.

We begin in Section 2 by examining the intellectual milieu that established physics as the exemplar for economists, inevitably leading to the “mathematization” of economics and finance. The contrast and conflicts between physics and finance can be explained by considering the kinds of uncertainty they address, and we describe this taxonomy in Section 3. We show how this taxonomy can be applied in two contexts in Sections 4 and 5, one drawn from physics (the harmonic oscillator) and the other drawn from finance (a quantitative trading strategy). These examples suggest a new interpretation of so-called “black swan” events, which we describe in Section 6. They also raise a number of practical issues that we address in Section 7, including the introduction of an “uncertainty checklist” with which our taxonomy can be applied. Finally, in Section 8 we turn to the role of quants in the current financial crisis, and consider three populist views that are either misinformed or based on incorrect claims, illustrating the benefits of a scientific approach to analysis crises. We conclude in Section 9 with some speculation regarding the finance of the future.

Before turning to these issues, we wish to specify the intended audience for this unorthodox and reflective article. While we hope the novel perspective we propose and the illustrative examples we construct will hold some interest for our academic colleagues, this paper can hardly be classified as original research. Instead, it is the engineer, research scientist, newly minted Wall Street quant, beleaguered investor, frustrated regulators and policymakers, and anyone else who cannot understand how quantitative models could have failed so spectacularly over the last few years that we intend to reach. Our focus is not on the origins of

the current financial crisis—there are now many popular and erudite accounts—but rather on developing a logical framework for understanding the role of quantitative models in theory and practice. We acknowledge at the outset that this goal is ambitious, and beg the reader’s indulgence as we attempt to reach our target audience through stylized examples and simplistic caricatures, rather than through formal theorem-and-proof.

## 2 Physics Envy

The fact that economics is still dominated by a single paradigm is a testament to the extraordinary achievements of one individual: Paul A. Samuelson. In 1947, Samuelson published his Ph.D. thesis titled *Foundations of Economics Analysis*, which might have seemed presumptuous—especially coming from a Ph.D. candidate—were it not for the fact that it did, indeed, become the foundations of modern economic analysis. In contrast to much of the extant economic literature of the time, which was often based on relatively informal discourse and diagrammatic exposition, Samuelson developed a formal mathematical framework for economic analysis that could be applied to a number of seemingly unrelated contexts. Samuelson’s (1947, p. 3) opening paragraph made his intention explicit (italics are Samuelson’s):

*The existence of analogies between central features of various theories implies the existence of a general theory which underlies the particular theories and unifies them with respect to those central features.* This fundamental principle of generalization by abstraction was enunciated by the eminent American mathematician E.H. Moore more than thirty years ago. It is the purpose of the pages that follow to work out its implications for theoretical and applied economics.

He then proceeded to build the infrastructure of what is now known as microeconomics, routinely taught as the first graduate-level course in every Ph.D. program in economics today. Along the way, Samuelson also made major contributions to welfare economics, general equilibrium theory, comparative static analysis, and business-cycle theory, all in a single doctoral dissertation!

If there is a theme to Samuelson’s thesis, it is the systematic application of scientific principles to economic analysis, much like the approach of modern physics. This was no coincidence. In Samuelson’s (1998, p. 1376) fascinating account of the intellectual origins of his dissertation, he acknowledged the following:

Perhaps most relevant of all for the genesis of *Foundations*, Edwin Bidwell Wilson (1879–1964) was at Harvard. Wilson was the great Willard Gibbs’s last (and, essentially only) protégé at Yale. He was a mathematician, a mathematical physicist, a mathematical statistician, a mathematical economist, a polymath

who had done first-class work in many fields of the natural and social sciences. I was perhaps his only disciple . . . I was vaccinated early to understand that economics and physics could share the same formal mathematical theorems (Euler’s theorem on homogeneous functions, Weierstrass’s theorems on constrained maxima, Jacobi determinant identities underlying Le Chatelier reactions, etc.), while still not resting on the same empirical foundations and certainties.

Also, in a footnote to his statement of the general principle of comparative static analysis, Samuelson (1947, p. 21) added, “It may be pointed out that this is essentially the method of thermodynamics, which can be regarded as a purely deductive science based upon certain postulates (notably the First and Second Laws of Thermodynamics)”. And much of the economics and finance literature since *Foundations* has followed Samuelson’s lead in attempting to deduce implications from certain postulates such as utility maximization, the absence of arbitrage, or the equalization of supply and demand. In fact, one of the most recent milestones in economics—rational expectations—is founded on a single postulate, around which a large and still-growing literature has developed.

## 2.1 The Mathematization of Economics and Finance

Of course, the mathematization of economics and finance was not due to Samuelson alone, but was advanced by several other intellectual giants that created a renaissance of mathematical economics during the half century following the Second World War. One of these giants, Gerard Debreu, provides an eye-witness account of this remarkably fertile period: “Before the contemporary period of the past five decades, theoretical physics had been an inaccessible ideal toward which economic theory sometimes strove. During that period, this striving became a powerful stimulus in the mathematization of economic theory” (Debreu, 1991, p. 2).

What Debreu is referring to is a series of breakthroughs that not only greatly expanded our understanding of economic theory, but also held out the tantalizing possibility of practical applications involving fiscal and monetary policy, financial stability, and central planning. These breakthroughs included:

- Game theory (von Neumann and Morganstern, 1944; Nash, 1951)
- General equilibrium theory (Debreu, 1959)
- Economics of uncertainty (Arrow, 1964)
- Long-term economic growth (Solow, 1956)
- Portfolio theory and capital-asset pricing (Markowitz, 1954; Sharpe, 1964; Tobin, 1958)
- Option-pricing theory (Black and Scholes, 1973; Merton, 1973)

- Macroeconometric models (Tinbergen, 1956; Klein, 1970)
- Computable general equilibrium models (Scarf, 1973)
- Rational expectations (Muth, 1961; Lucas, 1972)

Many of these contributions have been recognized by Nobel prizes, and they have permanently changed the field of economics from a branch of moral philosophy pursued by gentlemen scholars to a full-fledged scientific endeavor not unlike the deductive process with which Isaac Newton explained the motion of the planets from three simple laws. Moreover, the emergence of econometrics, and the over-riding importance of theory in guiding empirical analysis in economics is similar to the tight relationship between experimental and theoretical physics.

The parallels between physics and finance are even closer, due to the fact that the Black-Scholes/Merton option-pricing formula is also the solution to the heat equation. This is no accident, as Lo and Merton (2009) explain:

The origins of modern financial economics can be traced to Louis Bachelier’s magnificent dissertation, completed at the Sorbonne in 1900, on the theory of speculation. This work marks the twin births of the continuous-time mathematics of stochastic processes and the continuous-time economics of option pricing. In analyzing the problem of option pricing, Bachelier provides two different derivations of the Fourier partial differential equation as the equation for the probability density of what is now known as a Wiener process/Brownian motion. In one of the derivations, he writes down what is now commonly called the Chapman-Kolmogorov convolution probability integral, which is surely among the earlier appearances of that integral in print. In the other derivation, he takes the limit of a discrete-time binomial process to derive the continuous-time transition probabilities. Along the way, Bachelier also developed essentially the method of images (reflection) to solve for the probability function of a diffusion process with an absorbing barrier. This all took place five years before Einstein’s discovery of these same equations in his famous mathematical theory of Brownian motion.

Not surprisingly, Samuelson was also instrumental in the birth of modern financial economics (see Samuelson, 2009), and—together with his Nobel-prize-winning protege Robert C. Merton—created much of what is now known as “financial engineering”, as well as the analytical foundations of at least three multi-trillion-dollar industries (exchange-traded options markets, over-the-counter derivatives and structured products, and credit derivatives).

The mathematization of neoclassical economics is now largely complete, with dynamic stochastic general equilibrium models, rational expectations, and sophisticated econometric techniques having replaced the less rigorous arguments of the previous generation of economists. Moreover, the recent emergence of “econophysics” (Mantegna and Stanley,



2000)—a discipline that, curiously, has been defined not so much by its focus but more by the techniques (scaling arguments, power laws, and statistical mechanics) and occupations (physicists) of its practitioners—has only pushed the mathematization of economics and finance to new extremes.<sup>1</sup>

## 2.2 Samuelson’s Caveat

Even as Samuelson wrote his remarkable *Foundations*, he was well aware of the limitations of a purely deductive approach. In his introduction, he offered the following admonition (Samuelson, 1947, p. 3):

. . . [O]nly the smallest fraction of economic writings, theoretical and applied, has been concerned with the derivation of *operationally meaningful* theorems. In part at least this has been the result of the bad methodological preconceptions that economic laws deduced from *a priori* assumptions possessed rigor and validity independently of any empirical human behavior. But only a very few economists have gone so far as this. The majority would have been glad to enunciate meaningful theorems if any had occurred to them. In fact, the literature abounds with false generalization.

We do not have to dig deep to find examples. Literally hundreds of learned papers have been written on the subject of utility. Take a little bad psychology, add a dash of bad philosophy and ethics, and liberal quantities of bad logic, and any economist can prove that the demand curve for a commodity is negatively inclined.

This surprisingly wise and prescient passage is as germane today as it was over fifty years ago when it was first written, and all the more remarkable that it was penned by a twentysomething year-old graduate student. The combination of analytical rigor and practical relevance was to become a hallmark of Samuelson’s research throughout his career, and despite his theoretical bent, his command of industry practices and market dynamics was astonishing. Less gifted economists might have been able to employ similar mathematical tools and parrot his scientific perspective, but few would be able to match Samuelson’s ability to distill the economic essence of a problem and then solve it as elegantly and completely.

Unlike physics, in which pure mathematical logic can often yield useful insights and intuition about physical phenomena, Samuelson’s caveat reminds us that a purely deductive approach may not always be appropriate for economic analysis. As impressive as the achievements of modern physics are, physical systems are inherently simpler and more stable than

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<sup>1</sup>However, this field is changing rapidly as physicists with significant practical experience in financial markets push the boundaries of theoretical and empirical finance; see Bouchaud, Farmer, and Lillo (2009) for an example of this new and exciting trend.

economic systems, hence deduction based on a few fundamental postulates is likely to be more successful in the former case than in the latter. Conservation laws, symmetry, and the isotropic nature of space are powerful ideas in physics that simply do not have exact counterparts in economics because of the nature of economic interactions and the types of uncertainty involved.

And yet economics is often the envy of the other social sciences, in which there are apparently even fewer unifying principles and operationally meaningful theorems. Despite the well-known factions within economics, there is significant consensus among practicing economists surrounding the common framework of supply and demand, the principle of comparative advantage, the Law of One Price, income and substitution effects, net present value relations and the time value of money, externalities and the role of government, etc. While false generalizations certainly abound among academics of all persuasions, economics does contain many true generalizations as well, and these successes highlight important commonalities between economics and the other sciences.

Samuelson's genius was to be able to deduce operationally meaningful theorems despite the greater uncertainty of economic phenomena. In this respect, perhaps the differences between physics and economics are not fundamental, but are due, instead, to the types of uncertainty inherent in the two respective disciplines. We expand on this possibility in Sections 3–5. Before turning to that framework, it is instructive to perform a side-by-side comparison of economics and its closest intellectual sibling—psychology.

## 2.3 Economics vs. Psychology

The degree of physics envy among economists is more obvious when we compare economics with the closely related field of psychology. Both disciplines focus on human behavior, so one would expect them to have developed along very similar ideological and methodological trajectories. Instead, they have developed radically different cultures, approaching human behavior in vastly different ways. Consider, first, some of the defining characteristics of psychology:

- Psychology is based primarily on observation and experimentation.
- Field experiments are common.
- Empirical analysis leads to new theories.
- There are multiple theories of behavior.
- Mutual consistency among theories is not critical.

Contrast these with the comparable characteristics of economics:

- Economics is based primarily on theory and abstraction.
- Field experiments are not common.
- Theories lead to empirical analysis.
- There are few theories of behavior.
- Mutual consistency is highly prized.

Although there are, of course, exceptions to these generalizations, they do capture much of the spirit of the two disciplines.<sup>2</sup> For example, while psychologists certainly do propose abstract theories of human behavior from time to time, the vast majority of academic psychologists conduct experiments. Although experimental economics has made important inroads into the mainstream of economics and finance, the top journals still publish only a small fraction of experimental papers, the majority of publications consisting of more traditional theoretical and empirical studies. Despite the fact that new theories of economic behavior have been proposed from time to time, most graduate programs in economics and finance teach only one such theory: expected utility theory and rational expectations, and its corresponding extensions, e.g., portfolio optimization, the Capital Asset Pricing Model, and dynamic stochastic general equilibrium models. And it is only recently that departures from this theory are not dismissed out of hand; less than a decade ago, manuscripts containing models of financial markets with arbitrage opportunities were routinely rejected from the top economics and finance journals, in some cases without even being sent out to referees for review.

But thanks to the burgeoning literature in behavioral economics and finance, the Nobel prizes to Daniel Kahneman and Vernon Smith in 2002, advances in the cognitive neurosciences, and the recent financial crisis, psychological evidence is now taken more seriously by economists and finance practitioners. For example, going well beyond Keynes' (1936) "animal spirits", recent research in the cognitive neurosciences has identified an important link between rationality in decision-making and emotion,<sup>3</sup> implying that the two are not antithetical, but in fact complementary. In particular, emotions are the basis for a reward-and-punishment system that facilitates the selection of advantageous behavior, providing a numeraire for animals to engage in a "cost-benefit analysis" of the various actions open to them (Rolls, 1999, Chapter 10.3). Even fear and greed—the two most common culprits in the downfall of rational thinking, according to most behaviorists—are the product of

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<sup>2</sup>For example, there is a vast econometrics literature in which empirical investigations are conducted, but almost always motivated by theory and involving an hypothesis test of one sort or another. For a less impressionistic and more detailed comparison of psychology and economics, See Rabin (1998, 2002).

<sup>3</sup>See, for example, Grossberg and Gutowski (1987), Damasio (1994), Elster (1998), Lo (1999), Lo and Repin (2002), Loewenstein (2000), and Peters and Slovic (2000).

evolutionary forces, adaptive traits that increase the probability of survival. From an evolutionary perspective, emotion is a powerful tool for improving the efficiency with which animals learn from their environment and their past. When an individual’s ability to experience emotion is eliminated, an important feedback loop is severed and his decision-making process is impaired.

These new findings imply that individual preferences and behavior may not be stable through time, but are likely to be shaped by a number of factors, both internal and external to the individual, i.e., factors related to the individual’s personality, and factors related to specific environmental conditions in which the individual is currently situated. When environmental conditions shift, we should expect behavior to change in response, both through learning and, over time, through changes in preferences via the forces of natural selection. These evolutionary underpinnings are more than simple speculation in the context of financial market participants. The extraordinary degree of competitiveness of global financial markets and the outsize rewards that accrue to the “fittest” traders suggest that Darwinian selection is at work in determining the typical profile of the successful investor. After all, unsuccessful market participants are eventually eliminated from the population after suffering a certain level of losses.

This perspective suggests an alternative to the antiseptic world of rational expectations and efficient markets, one in which market forces and preferences interact to yield a much more dynamic economy driven by competition, natural selection, and the diversity of individual and institutional behavior. This approach to financial markets, which we refer to as the “Adaptive Markets Hypothesis” (Farmer and Lo, 1999; Farmer, 2002; Lo, 2004, 2005; and Brennan and Lo, 2009), is a far cry from theoretical physics, and calls for a more sophisticated view of the role that uncertainty plays in quantitative models of economics and finance. We propose such a view in Section 3.

### **3 A Taxonomy of Uncertainty**

The distinctions between the various types of uncertainty are, in fact, central to the differences between economics and physics. Economists have been aware of some of these distinctions for decades, beginning with the University of Chicago economist Frank Knight’s (1921) Ph.D. dissertation in which he distinguished between two types of randomness: one that is amenable to formal statistical analysis, which Knight called “risk”, and another that is not, which he called “uncertainty”. An example of the former is the odds of winning at the roulette table, and an example of the latter is the likelihood of peace in the Middle East

within the next five years. Although Knight’s motivation for making such a distinction is different from ours—he was attempting to explain why some businesses yield little profits (they take on risks, which easily become commoditized) while others generate extraordinary returns (they take on uncertainty)—nevertheless, it is a useful starting point for understanding why physics seems so much more successful than economics. In this section, we propose an even more refined taxonomy of uncertainty, one capable of explaining the differences across the entire spectrum of intellectual pursuits from physics to biology to economics to philosophy and religion.

### **3.1 Level 1: Complete Certainty**

This is the realm of classical physics, an idealized deterministic world governed by Newton’s laws of motion. All past and future states of the system are determined exactly if initial conditions are fixed and known—nothing is uncertain. Of course, even within physics, this perfectly predictable clockwork universe of Newton, Lagrange, LaPlace, and Hamilton was recognized to have limited validity as quantum mechanics emerged in the early twentieth century. Even within classical physics, the realization that small perturbations in initial conditions can lead to large changes in the subsequent evolution of a dynamical system underscores how idealized and limited this level of description can be in the elusive search for truth.

However, it must be acknowledged that much of the observable physical universe does, in fact, lie in this realm of certainty. Newton’s three laws explain a breathtakingly broad span of phenomena—from an apple falling from a tree to the orbits of planets and stars—and has done so in the same manner for more than 10 billion years. In this respect, physics has enjoyed a significant head start when compared to all the other sciences.

### **3.2 Level 2: Risk without Uncertainty**

This level of randomness is Knight’s (1921) definition of risk: randomness governed by a known probability distribution for a completely known set of outcomes. At this level, probability theory is a useful analytical framework for risk analysis. Indeed, the modern axiomatic foundations of probability theory—due to Kolmogorov, Wiener, and others—is given precisely in these terms, with a specified sample space and a specified probability measure. No statistical inference is needed, because we know the relevant probability distributions exactly, and while we do not know the outcome of any given wager, we know all the rules and the odds, and no other information relevant to the outcome is hidden. This is life in

a hypothetical honest casino, where the rules are transparent and always followed. This situation bears little resemblance to financial markets.

### **3.3 Level 3: Fully Reducible Uncertainty**

This is risk with a degree of uncertainty, an uncertainty due to unknown probabilities for a fully enumerated set of outcomes that we presume are still completely known. At this level, classical (frequentist) statistical inference must be added to probability theory as an appropriate tool for analysis. By “fully reducible uncertainty”, we are referring to situations in which randomness can be rendered arbitrarily close to Level-2 uncertainty with sufficiently large amounts of data using the tools of statistical analysis. Fully reducible uncertainty is very much like an honest casino, but one in which the odds are not posted and must therefore be inferred from experience. In broader terms, fully reducible uncertainty describes a world in which a single model generates all outcomes, and this model is parameterized by a finite number of unknown parameters that do not change over time and which can be estimated with an arbitrary degree of precision given enough data.

The resemblance to the “scientific method”—at least as it is taught in science classes today—is apparent at this level of uncertainty. One poses a question, develops a hypothesis, formulates a quantitative representation of the hypothesis (i.e., a model), gathers data, analyzes that data to estimate model parameters and errors, and draws a conclusion. Human interactions are often a good deal messier and more nonlinear, and we must entertain a different level of uncertainty before we encompass the domain of economics and finance.

### **3.4 Level 4: Partially Reducible Uncertainty**

Continuing our descent into the depths of the unknown, we reach a level of uncertainty that now begins to separate the physical and social sciences, both in philosophy and model-building objectives. By Level-4 or “partially reducible” uncertainty, we are referring to situations in which there is a limit to what we can deduce about the underlying phenomena generating the data. Examples include data-generating processes that exhibit: (1) stochastic or time-varying parameters that vary too frequently to be estimated accurately; (2) nonlinearities too complex to be captured by existing models, techniques, and datasets; (3) non-stationarities and non-ergodicities that render useless the Law of Large Numbers, Central Limit Theorem, and other methods of statistical inference and approximation; and (4) the dependence on relevant but unknown and unknowable conditioning information.

Although the laws of probability still operate at this level, there is a non-trivial degree of

uncertainty regarding the underlying structures generating the data that cannot be reduced to Level-2 uncertainty, even with an infinite amount of data. Under partially reducible uncertainty, we are in a casino that may or may not be honest, and the rules tend to change from time to time without notice. In this situation, classical statistics may not be as useful as a Bayesian perspective, in which probabilities are no longer tied to relative frequencies of repeated trials, but now represent degrees of belief. Using Bayesian methods, we have a framework and lexicon with which partial knowledge, prior information, and learning can be represented more formally.

Level-4 uncertainty involves “model uncertainty”, not only in the sense that multiple models may be consistent with observation, but also in the deeper sense that more than one model may very well be generating the data. One example is a regime-switching model in which the data are generated by one of two possible probability distributions, and the mechanism that determines which of the two is operative at a given point in time is also stochastic, e.g., a two-state Markov process as in Hamilton (1989, 1990). Of course, in principle, it is always possible to reduce model uncertainty to uncertainty surrounding the parameters of a single all-encompassing “meta-model”, as in the case of a regime-switching process. Whether or not such a reductionist program is useful depends entirely on the complexity of the meta-model and nature of the application.

At this level of uncertainty, modeling philosophies and objectives in economics and finance begin to deviate significantly from those of the physical sciences. Physicists believe in the existence of fundamental laws, either implicitly or explicitly, and this belief is often accompanied by a reductionist philosophy that seeks the fewest and simplest building blocks from which a single theory can be built. Even in physics, this is an over-simplification, as one era’s “fundamental laws” eventually reach the boundaries of their domains of validity, only to be supplanted and encompassed by the next era’s “fundamental laws”. The classic example is, of course, Newtonian mechanics becoming a special case of special relativity and quantum mechanics.

It is difficult to argue that economists should have the same faith in a fundamental and reductionist program for a description of financial markets (although such faith does persist in some, a manifestation of physics envy). Markets are tools developed by humans for accomplishing certain tasks—not immutable laws of Nature—and are therefore subject to all the vicissitudes and frailties of human behavior. While behavioral regularities do exist, and can be captured to some degree by quantitative methods, they do not exhibit the same level of certainty and predictability as physical laws. Accordingly, model-building in the social sciences should be much less informed by mathematical aesthetics, and much more by pragmatism in the face of partially reducible uncertainty. We must resign ourselves to



models with stochastic parameters or multiple regimes that may not embody universal truth, but are merely useful, i.e., they summarize some coarse-grained features of highly complex datasets.

While physicists make such compromises routinely, they rarely need to venture down to Level 4, given the predictive power of the vast majority of their models. In this respect, economics may have more in common with biology than physics. As the great mathematician and physicist John von Neumann observed, “If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is”.

### **3.5 Level 5: Irreducible Uncertainty**

Irreducible uncertainty is the polite term for a state of total ignorance; ignorance that cannot be remedied by collecting more data, using more sophisticated methods of statistical inference or more powerful computers, or thinking harder and smarter. Such uncertainty is beyond the reach of probabilistic reasoning, statistical inference, and any meaningful quantification. This type of uncertainty is the domain of philosophers and religious leaders, who focus on not only the unknown, but the unknowable.

Stated in such stark terms, irreducible uncertainty seems more likely to be the exception rather than the rule. After all, what kinds of phenomena are completely impervious to quantitative analysis, other than the deepest theological conundrums? The usefulness of this concept is precisely in its extremity. By defining a category of uncertainty that cannot be reduced to any quantifiable risk—essentially an admission of intellectual defeat—we force ourselves to stretch our imaginations to their absolute limits before relegating any phenomenon to this level.

### **3.6 Level $\infty$ : Zen Uncertainty**

Attempts to understand uncertainty are mere illusions; there is only suffering.

### **3.7 The Uncertainty Continuum**

As our sequential exposition of the five levels of uncertainty suggests, whether or not it is possible to model economic interactions quantitatively is not a black-and-white issue, but rather a continuum that depends on the nature of the interactions. In fact, a given phenomenon may contain several levels of uncertainty at once, with some components being completely certain and others irreducibly uncertain. Moreover, each component’s categorization can vary over time as technology advances or as our understanding of the phenomenon deepens.



For example, 3,000 years ago solar eclipses were mysterious omens that would have been considered Level-5 uncertainty, but today such events are well understood and can be predicted with complete certainty (Level 1). Therefore, a successful application of quantitative methods to modeling any phenomenon requires a clear understanding of the level of uncertainty involved.

In fact, we propose that the failure of quantitative models in economics and finance is almost always attributable to a mismatch between the level of uncertainty and the methods used to model it. In Sections 4–6, we provide concrete illustrations of this hypothesis.

## 4 The Harmonic Oscillator

To illustrate the import of the hierarchy of uncertainty proposed in Section 3, in this section we apply it to a well-known physical system: the simple harmonic oscillator. While trivial from a physicist’s perspective, and clearly meant to be an approximation to a much more complex reality, this basic model of high-school physics fits experimental data far better than even the most sophisticated models in the social sciences. Therefore, it is an ideal starting point for illustrating the differences and similarities between physics and finance. After reviewing the basic properties of the oscillator, we will inject certain types of noise into the system and explore the implications for quantitative models of its behavior as we proceed from Level 1 (Section 4.1) to Level 4 (Section 4.4). As more noise is added, physics begins to look more like economics.<sup>4</sup> We postpone a discussion of Level-5 uncertainty until Section 6, where we provide a more expansive perspective in which phenomena transition from Level 5 to Level 1 as we develop deeper understanding of their true nature.

### 4.1 The Oscillator at Level 1

Consider a mass situated at the origin  $x=0$  of a frictionless surface, and suppose it is attached to a spring. The standard model for capturing the behavior of this one-dimensional system when the mass is displaced from the origin—which dates back to 1660—is that the spring exerts a restoring force  $F$  which is proportional to the displacement and in the opposite direction (see Figure 1); that is,  $F = -kx$  where  $k$  is the spring constant and  $x$  is the position of the mass  $m$ . This hypothetical relation works so well in so many circumstances that physicists refer to it as a “law”, Hooke’s Law, in honor of its discoverer, Robert Hooke.

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<sup>4</sup>However, we must emphasize the illustrative nature of this example, and caution readers against too literal an interpretation of our use of the harmonic oscillator with noise. We are not suggesting that such a model is empirically relevant to either physics or economics. Instead, we are merely exploiting its simplicity to highlight the differences between the two fields.

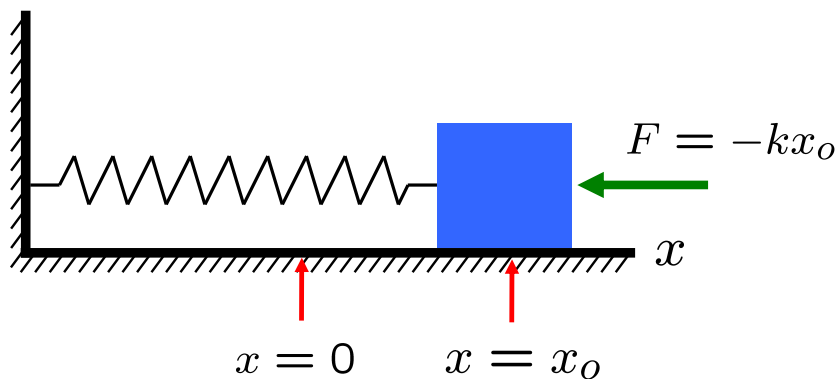


Figure 1: Frictionless one-dimensional spring system.

By applying Newton's Second Law ( $F = ma$ ) to Hooke's Law, we obtain the following second-order linear differential equation:

$$\ddot{x} + \frac{k}{m}x = 0 \quad (1)$$

where  $\ddot{x}$  denotes the second time-derivative of  $x$ . The solution to (1) is well-known and given by:

$$x(t) = A \cos(\omega_o t + \phi) \quad (2)$$

where  $\omega_o \equiv \sqrt{k/m}$  and  $A$  and  $\phi$  are constants that depend on the initial conditions of the system. This is the equation for a harmonic oscillator with amplitude  $A$ , initial phase angle  $\phi$ , period  $T = 2\pi\sqrt{m/k}$ , and frequency  $f = 1/T$ . Despite its deceptively pedestrian origins, the harmonic oscillator is ubiquitous in physics, appearing in contexts from classical mechanics to quantum mechanics to quantum field theory, and underpinning a surprisingly expansive range of theoretical and applied physical phenomena.

At this stage, the model of the block's position (2) is capturing a Level-1 phenomenon, perfect certainty. For parameters  $A = 2$ ,  $\omega_o = 1.5$ , and  $\phi = 0.5$ , Figure 2 traces out the displacement of the block without error—at time  $t = 3.5$ , we know with certainty that  $x = 1.7224$ .

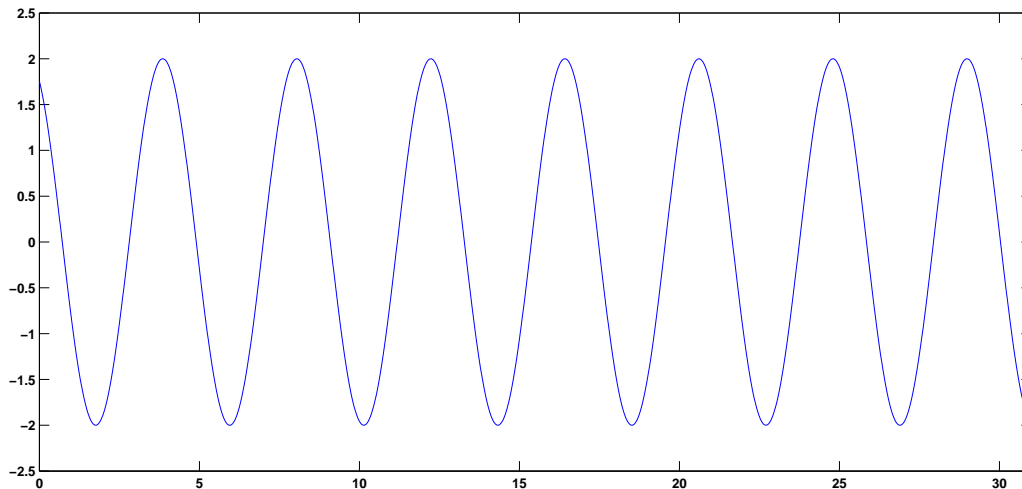


Figure 2: Time series plot of the displacement  $x(t) = A \cos(\omega_o t + \phi)$  of a harmonic oscillator with parameters  $A=2$ ,  $\omega_o=1.5$ , and  $\phi=0.5$ .

## 4.2 The Oscillator at Level 2

As satisfying as this knowledge is, physicists are the first to acknowledge that (2) is an idealization that is unlikely to be observed with real blocks and springs. For example, surface and air friction will dampen the block’s oscillations over time, and the initial displacement cannot exceed the spring’s elastic limit otherwise the spring could break or be permanently deformed. Such departures from the idealized world of (2) will cause us to leave the comforting world of Newtonian mechanics and Level-1 certainty.

For the sake of exposition, consider the simplest departure from (2), which is the introduction of an additive noise term to the block’s displacement:

$$x(t) = A \cos(\omega_o t + \phi) + \epsilon(t) \quad , \quad \epsilon(t) \text{ IID } \mathcal{N}(0, \sigma_\epsilon^2) \quad (3)$$

where ‘IID’ stands for “independently and identically distributed”,  $\mathcal{N}(0, \sigma_\epsilon^2)$  indicates a normal distribution with mean 0 and variance  $\sigma_\epsilon^2$ , and all parameters, including  $\sigma_\epsilon^2$ , are fixed and known.<sup>5</sup> Although  $x(t)$  is no longer deterministic, Level-2 uncertainty implies that the probabilistic structure of the displacement is completely understood. While we can no

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<sup>5</sup>This example of Level-2 uncertainty corresponds to a number of actual physical phenomena, such as thermal noise in elements of an electronic circuit, where the analogue of the parameter  $\sigma_\epsilon^2$  is related in a fundamental way to the temperature of the system, described by an instance of a fluctuation-dissipation theorem, Nyquist’s theorem (Reif, 1965, Chapter 15).

longer say with certainty that at time  $t=3.5$ ,  $x=1.7224$ , we do know that if  $\sigma_\epsilon=0.15$ , the probability that  $x$  falls outside the interval  $[1.4284, 2.0164]$  is precisely 5%.

### 4.3 The Oscillator at Level 3

Level 3 of our taxonomy of uncertainty is fully-reducible uncertainty, which we have defined as uncertainty that can be reduced arbitrarily closely to pure risk (Level 2) given a sufficient amount of data. Implicit in this definition is the assumption that the future is exactly like the past in terms of the stochastic properties of the system, i.e., stationarity.<sup>6</sup> In the case of the harmonic oscillator, this assumption means that the oscillator (2) is still the data-generating process with fixed parameters, but the parameters and the noise distribution are unknown:

$$x(t) = A \cos(\omega_o t + \phi) + \epsilon(t) \quad , \quad E[\epsilon(t)] = 0 \quad , \quad E[\epsilon(t_1)\epsilon(t_2)] = 0 \quad \forall t_1 \neq t_2 \quad . \quad (4)$$

However, under Level-3 uncertainty, we can estimate all the relevant parameters arbitrarily accurately with enough data, which is a trivial signal-processing exercise. In particular, although the timescale and the units in which the amplitude is measured in (4) are completely arbitrary, for concreteness let the oscillator have a period of 1 hour given a 24-hour time sample period. With data sampled every minute, this system yields a sample size of 1,440 observations.<sup>7</sup> Assuming a signal-to-noise ratio of 0.1 and applying the Fast Fourier Transform (FFT) to this time series yields an excellent estimate of the oscillator's frequency, as Figure 3 demonstrates. If this system were even a coarse approximation to business cycles and stock market fluctuations, economic forecasting would be a trivial task.

### 4.4 The Oscillator at Level 4

We now turn to Level-4 uncertainty, the taxon in which uncertainty is only partially reducible. To illustrate the characteristics of this level, suppose that the displacement  $x(t)$  is

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<sup>6</sup>In physics, the stationarity of physical laws is often taken for granted. For example, fundamental relationships such as Newton's  $F=ma$  have been valid over cosmological time scales, and appear to be accurate descriptions of many physical phenomena from very early in the history of the Universe (Weinberg, 1977). Economics boasts no such equivalents. However, other physical systems exhibit localized nonstationarities that fall outside Level-3 uncertainty (e.g., the properties of semiconductors as they are subjected to time-varying and non-stationary temperature fluctuations, vibration, and electromagnetic radiation), and entire branches of physics are devoted to such non-stationarities, e.g., non-equilibrium statistical mechanics.

<sup>7</sup>Alternatively, for those who may wish to picture a system with a longer time scale, this example can be interpreted as sampling daily observations of the oscillator with a period of 60 days over a total time period of slightly less than four years.

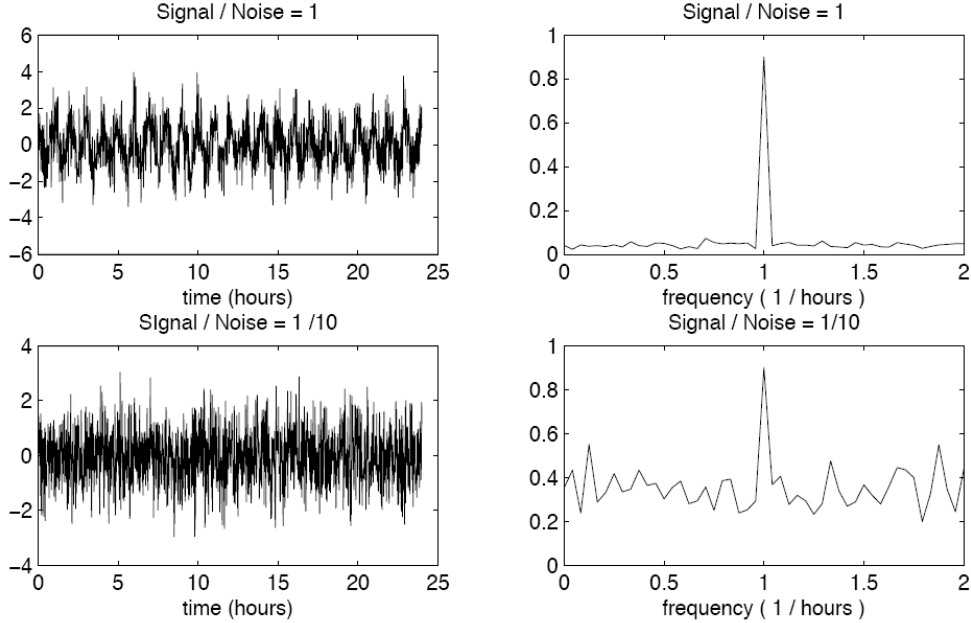


Figure 3: Simulated time series and Fourier analysis of displacements  $x_t$  of a harmonic oscillator with a period of 1 hour, sampled at 1-minute intervals over a 24-hour period, with signal-to-noise ratio of 1 (top graphs) and 0.10 (bottom graphs).

not governed by a single oscillator with additive noise, but is subject to regime shifts between two oscillators without noise (we will consider the case of an oscillator with additive noise below). Specifically, let:

$$x(t) = I(t) x_1(t) + (1 - I(t)) x_2(t) \quad (5a)$$

$$x_i(t) = A_i \cos(\omega_i t + \phi_i) \quad , \quad i = 1, 2 \quad (5b)$$

where the binary indicator  $I(t)$  determines which of the two oscillators  $x_1(t)$  or  $x_2(t)$  is generating the observed process  $x(t)$ , and let  $I(t)$  be a simple two-state Markov process with the following simple transition probability matrix  $\mathbf{P}$ :

$$\mathbf{P} \equiv \begin{matrix} & I(t) = 1 & I(t) = 0 \\ \begin{matrix} I(t-1) = 1 \\ I(t-1) = 0 \end{matrix} & \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix} \end{matrix} \quad (6)$$

As before, we will assume that we observe the system once per minute over a timespan of 24 hours, and let the two oscillators' periods be 30 and 60 minutes, respectively; their dimensionless ratio is independent of the chosen timescale, and has been selected arbitrarily. Although the most general transition matrix could allow for different probabilities of exiting each state, for simplicity we have taken these probabilities to be equal to a common value  $p$ . The value of  $p$  determines the half-life of remaining in a given state, and we will explore the effects of varying  $p$ . We also impose restrictions on the parameters of the two oscillators so that the observed displacement  $x(t)$  and its first time-derivative  $\dot{x}(t)$  are continuous across regime switches, which ensures that the observed time series do not exhibit sample-path discontinuities or "structural breaks".<sup>8</sup> This example of Level-4 uncertainty illustrates one possible form of the stochastic behavior of the oscillator frequency, namely a regime switching model. Such stochastic oscillators can be useful in building models of a variety of physical systems (see Gitterman, 2005). For example, the description of wave propagation in random media involves the behavior of certain types of stochastic oscillators, as in the propagation of sound waves through the ocean.

In keeping with our intention to illustrate Level-4 uncertainty, suppose that none of the details of the structure of  $x(t)$  are known to the observer attempting to model this system. Figure 4 shows three different examples of the observed time series  $x(t)$  generated by (5). Each panel corresponds to a different value for the switching probability  $p$ : the top panel corresponds to a half-life of 4 hours (significantly greater than the periods of either oscillator); the middle panel corresponds to a half-life of 30 minutes (the same period as oscillator 1 and comparable to oscillator 2); and the bottom panel corresponds to a half-life of 30 seconds (considerably shorter than the period of both oscillators).

In these cases, the behavior of  $x(t)$  is less transparent than in the single-oscillator case. However, in the two extreme cases where the values of  $p$  imply either a much greater or much smaller period than the two oscillators, there is hope. When the regime half-life is much

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<sup>8</sup>Specifically, we require that at any regime-switching time, the block's position and velocity are continuous. Of course the block's second time-derivative, i.e., its acceleration, will not be continuous across switching times. Therefore, in transiting from regime  $i$  with angular frequency  $\omega_i$  to regime  $j$  with angular frequency  $\omega_j$ , we require that  $A_i \cos(\omega_i \Delta t_i + \phi_i) = A_j \cos(\phi_j)$  and  $A_i \omega_i \sin(\omega_i \Delta t_i + \phi_i) = A_j \omega_j \sin(\phi_j)$ , where  $\Delta t_i$  is the time the system has just spent in regime  $i$ , which is known at the time of the transition. These equations have a unique solution for  $A_j$  and  $\phi_j$  in terms of  $A_i$  and  $\phi_i$ :

$$A_j^2 = A_i^2 [\cos^2(\omega_i \Delta t_i + \phi_i) + (\omega_i/\omega_j)^2 \sin^2(\omega_i \Delta t_i + \phi_i)] \quad , \quad \tan(\phi_j) = (\omega_i/\omega_j) \tan(\omega_i \Delta t_i + \phi_i)$$

In simulating  $x(t)$ , these equations are applied iteratively, starting from the chosen initial conditions at the beginning of the sample, and the regime shifts are governed by the Markov chain (6).

It is also worth mentioning that energy is not conserved in this system, since the switching of frequencies can be characterized as some external influence injecting or extracting energy from the oscillator. Accordingly, the amplitude of the observed time series can and does change, and the system is unstable in the long run.

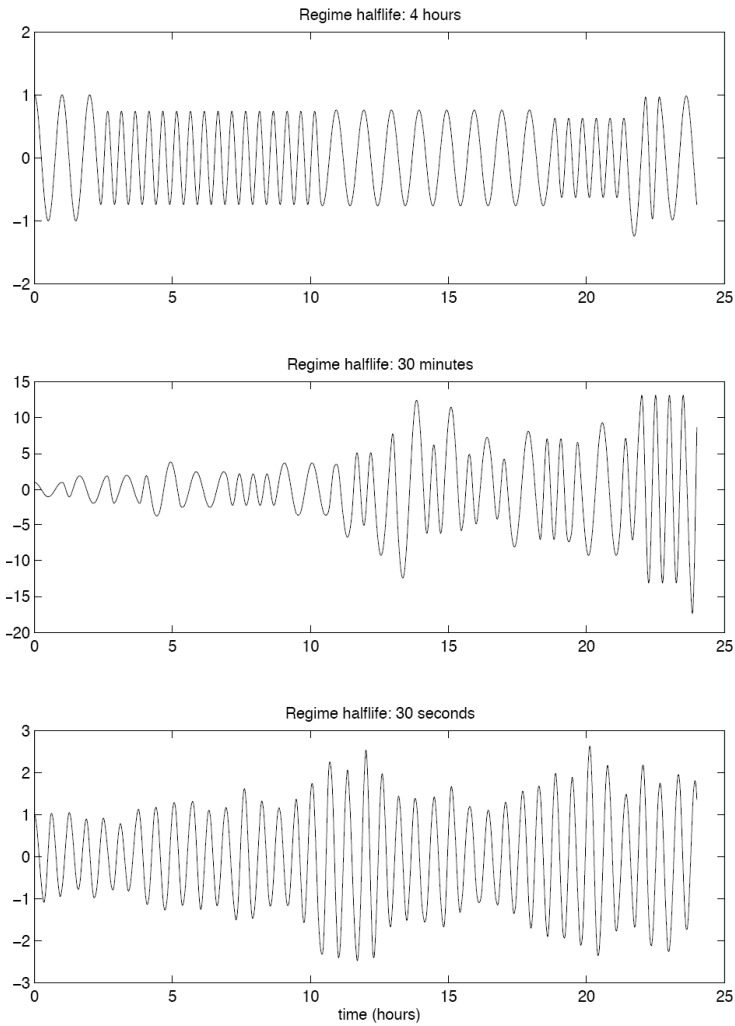


Figure 4: Simulated time series of a harmonic oscillator with regime-switching parameters in which the probability of a regime switch is calibrated to yield a regime half-life of 4 hours (top panel), 30 minutes (middle panel), and 30 seconds (bottom panel). The oscillator's parameters are completely specified by the half-life (or equivalently the transition probability) and the frequencies in each regime.

greater than both oscillators' periods, enough data can be collected during each of the two regimes to discriminate between them. In fact, visual inspection of the Fourier transform depicted in the top panel of Figure 5 confirms this intuition. At the other extreme, when the regime half-life is much shorter than the two oscillators' periods, the system behaves as if it were a single oscillator with a frequency equal to the harmonic mean of the two oscillators' frequencies, and with an amplitude that is stochastically modulated. The Fourier transform in the bottom panel of Figure 5 confirms that a single effective frequency is present. Despite the fact that two oscillators are, in fact, generating the data, for all intents and purposes, modeling this system as a single oscillator will yield an excellent approximation.

The most challenging case is depicted in the middle panel of Figure 5, which corresponds to a value of  $p$  that implies a half-life comparable to the two oscillators' periods. Neither the time series nor its Fourier transform offers any clear indication as to what is generating the data. And recall that these results do not yet reflect the effects of any additive noise terms as in (4).

If we add Gaussian white noise to (5), the signal-extraction process becomes even more challenging, as Figure 6 illustrates. In the top panel, we reproduce the Fourier transform of the middle panel of Figure 5 (that is, with no noise), and in the two panels below, we present the Fourier transforms of the same regime-switching model with additive Gaussian noise, calibrated with the same parameters as in the middle panel of Figure 5 for the oscillators, and with the additive noise component calibrated to yield a signal-to-noise ratio of 0.1. The middle panel of Figure 6 is the Fourier transform of this new simulation using the same sample size as in Figure 5, and the bottom panel contains the Fourier transform applied to a dataset 50 times larger. There is no discernible regularity in the oscillatory behavior of the middle panel, and only when we use significantly more data do the two characteristic frequencies begin to emerge.

The relevance of Level-4 uncertainty for economics is obvious when we compare the middle panel of Figure 6 to the Fourier transform of a standard economic time series such as growth rates for U.S. real gross domestic product from 1929 to 2008, displayed in the bottom panel of Figure 7. The similarities are striking, which may be why Paul Samuelson often quipped that "economists have predicted five out of the past three recessions".

Note the markedly different data requirements between this model with Level-4 uncertainty and those with Level-3 uncertainty. Considerably more data is needed just to identify the presence of distinct regimes, and this is a necessary but not sufficient condition for accurate estimation of the model's parameters, i.e., the oscillator frequencies, the transition matrix for the Markov process, and the variance of the additive noise. Moreover, even if we are able to obtain sufficient data to accurately estimate the model's parameters, it may still



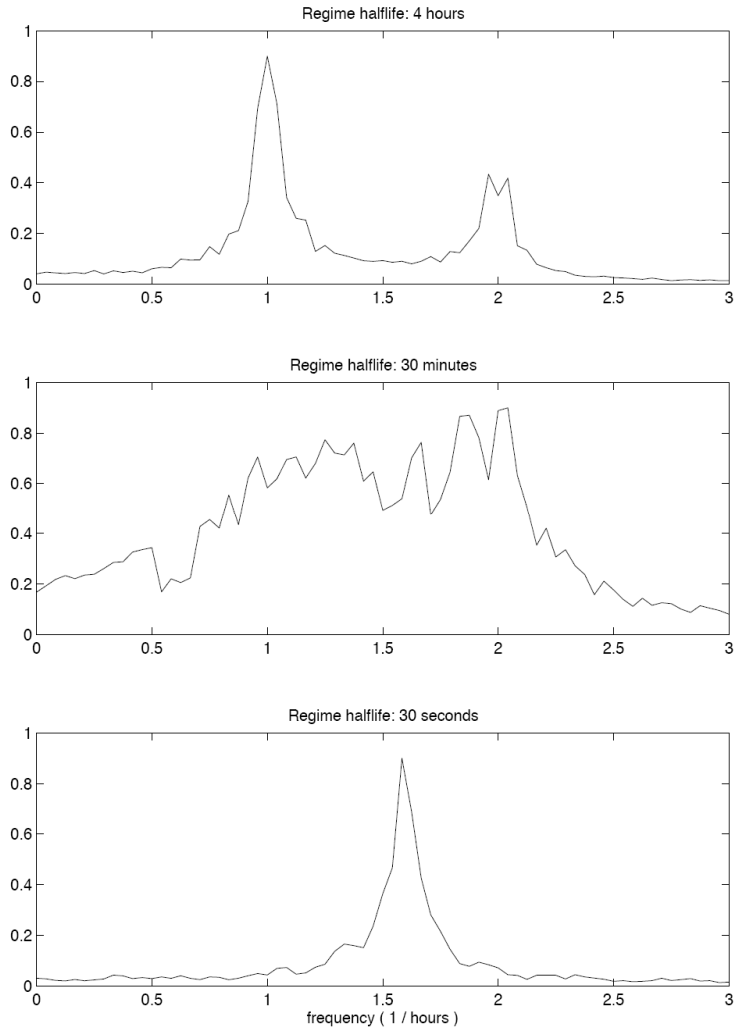


Figure 5: Fast Fourier transforms of the simulated time series of a harmonic oscillator with regime-switching parameters in which the probability of a regime switch is calibrated to yield a regime half-life of 4 hours (top panel), 30 minutes (middle panel), and 30 seconds (bottom panel). The oscillator’s parameters are completely specified by the half-life (or equivalently the transition probability) and the frequencies in each regime.

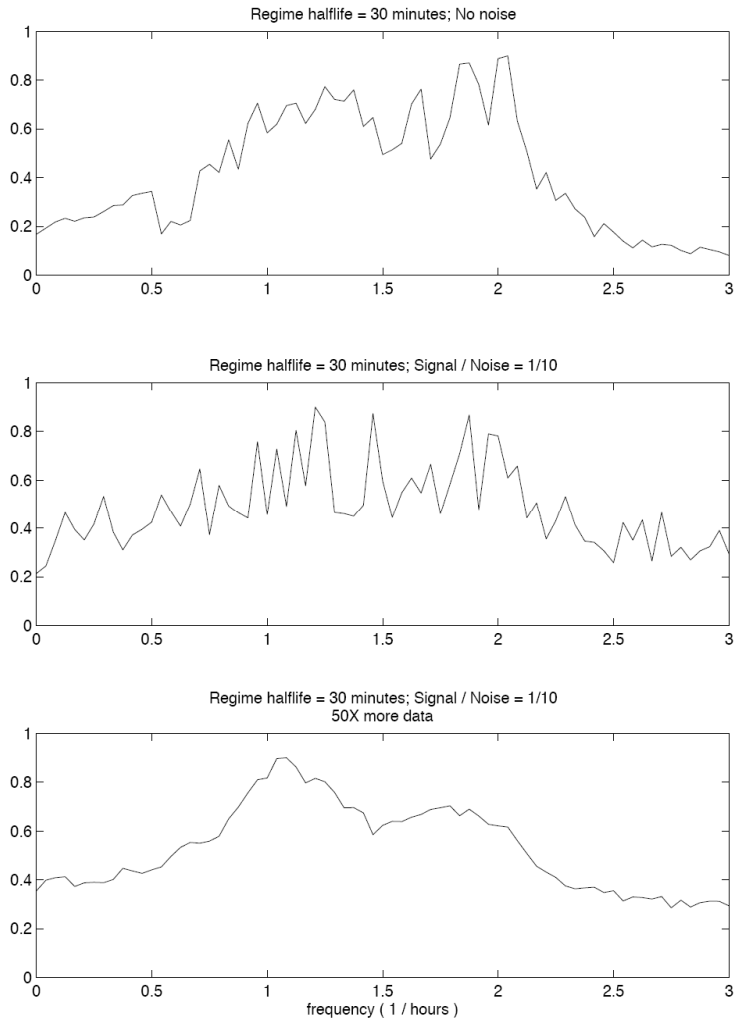


Figure 6: Fast Fourier transforms of simulated time series the two-state regime-switching harmonic oscillator with additive noise. For comparison, the case without additive noise and where the regime half-life is comparable to the oscillators' periods is given in the top panel. The middle panel corresponds to the case with additive noise and regime half-life comparable to the oscillators' periods, and the bottom panel is the same analysis applied to a dataset 50 times longer than the other two panels.

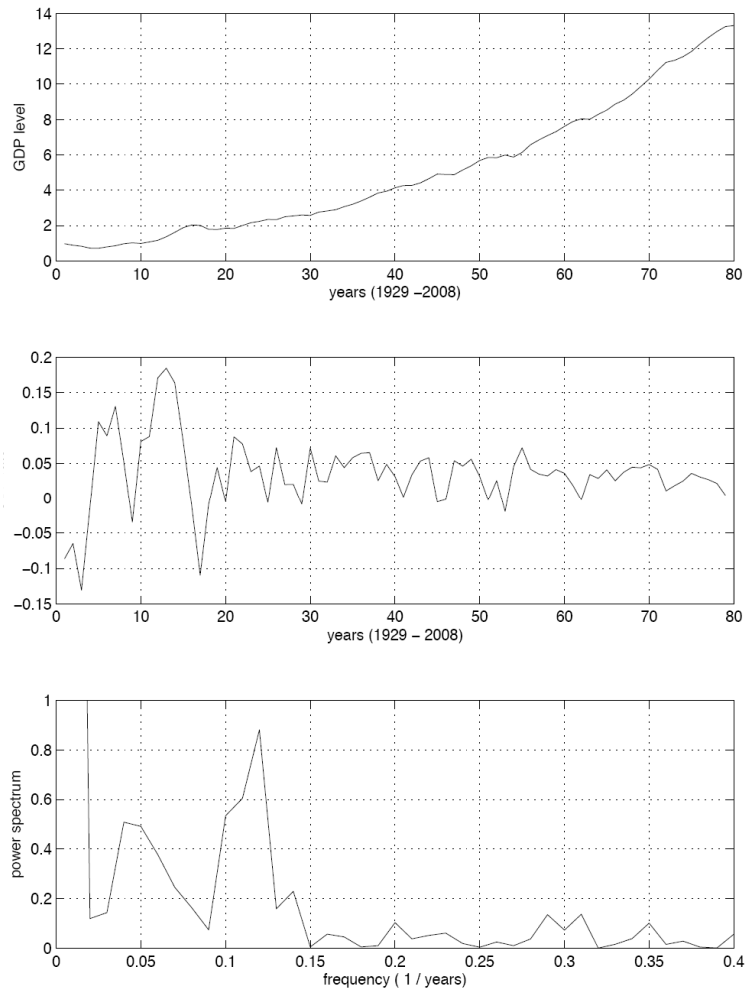


Figure 7: Annual levels (top panel), growth rates (middle panel), and Fourier transform of U.S. GDP (bottom panel) in 2005 dollars, from 1929 to 2008.

be impossible to construct reliable forecasts if we cannot identify in real time which regime we currently inhabit. In the financial markets context, such a model implies that even if markets are inefficient—in the sense that securities returns are not pure random walks—we may still be unable to profit from it, a particularly irksome state of affairs that might be called the “Malicious Markets Hypothesis”.

If Tycho, Kepler, Galileo and Newton had been confronted with such “quasi-periodic” data for the cyclical paths of the planets, it may have taken centuries, not years, of observational data for them to arrive at the same level of understanding of planetary motion that we have today. Fortunately, as challenging as physics is, much of it apparently does not operate at this level of uncertainty. Unfortunately, financial markets can sometimes operate at an even deeper level of uncertainty, uncertainty that cannot be reduced to any of the preceding levels simply with additional data. We turn to this level in Section 6, but before doing so, in the next section we provide an application of our taxonomy of uncertainty to a financial context.

## 5 A Quantitative Trading Strategy

Since the primary focus of this paper is the limitations of a physical-sciences approach to financial applications, in this section we apply our taxonomy of uncertainty to the mean-reversion strategy of Lehmann (1990) and Lo and MacKinlay (1990a), in which a portfolio of stocks is constructed by buying previous under-performing stocks, and short-selling previous outperforming stocks by the same dollar amount. This quantitative equity market-neutral strategy is simple enough to yield analytically tractable expressions for its statistical properties, and realistic enough to illustrate many of the practical challenges of this particular part of the financial industry, which has come to be known as “statistical arbitrage” or “statarb”. This strategy has been used more recently by Khandani and Lo (2007, 2008) to study the events surrounding the August 2007 “Quant Meltdown”, and the exposition in this section closely follows theirs. To parallel our sequential exposition of the oscillator in Section 4, we begin in Section 5.1 with those few aspects of statarb that may be interpreted as Level-1 certainty, and focus on progressively more uncertain aspects of the strategy in Sections 5.2–5.4. As before, we will postpone any discussion of Level-5 uncertainty to Section 6 where we address the full range of uncertainty more directly.

## 5.1 StatArb at Level 1

Perhaps the most obvious difference between physics and financial economics is the fact that almost no practically relevant financial phenomena falls into the category of Level 1, perfect certainty. The very essence of finance is the interaction between uncertainty and investment decisions, hence in a world of perfect certainty, financial economics reduces to standard microeconomics. Unlike the oscillator of Section 4, there is no simple deterministic model of financial markets that serves as a useful starting point for practical considerations, except perhaps for simple default-free bond-pricing formulas and basic interest-rate calculations. For example, consider a simple deterministic model of the price  $P_t$  of a financial security:

$$P_t = P_{t-1} + X_t \tag{7}$$

where  $X_t$  is some known increment. If  $X_t$  is known at date  $t-1$ , then a positive value will cause investors to purchase as many shares of the security as possible in anticipation of the price appreciation, and a negative value will cause investors to short-sell as many shares as possible to profit from the certain price decline. Such behavior implies that  $P_t$  will take on only one of two extreme values—0 or  $\infty$ —which is clearly unrealistic. Any information regarding future price movements will be exploited to the fullest extent possible, hence deterministic models like (7) are virtually useless in financial contexts except for the most elementary pedagogical purposes.

## 5.2 StatArb at Level 2

Consider a collection of  $N$  securities and denote by  $\mathbf{R}_t$  the  $N \times 1$ -vector of their date- $t$  returns  $[R_{1t} \cdots R_{Nt}]'$ . To be able to derive the statistical properties of any strategy based on these security returns, we require the following assumption:

- (A1)  $\mathbf{R}_t$  is a jointly covariance-stationary multivariate stochastic process with known distribution, which we take to be Gaussian with expectation and autocovariances:

$$\begin{aligned} \mathbb{E}[\mathbf{R}_t] &\equiv \boldsymbol{\mu} = [\mu_1 \ \mu_2 \ \cdots \ \mu_N]' \\ \mathbb{E}[(\mathbf{R}_{t-k} - \boldsymbol{\mu})(\mathbf{R}_t - \boldsymbol{\mu})'] &\equiv \boldsymbol{\Gamma}_k \end{aligned}$$

where, with no loss of generality, we let  $k \geq 0$  since  $\boldsymbol{\Gamma}_k = \boldsymbol{\Gamma}'_{-k}$ .

This assumption is the embodiment of Level-2 uncertainty where the probability distributions of all the relevant variables are well-defined, well-behaved, and stable over time.<sup>9</sup>

Given these  $N$  securities, consider a long/short market-neutral equity strategy consisting of an equal dollar amount of long and short positions, where at each rebalancing interval, the long positions consist of “losers” (underperforming stocks, relative to some market average) and the short positions consist of “winners” (outperforming stocks, relative to the same market average). Specifically, if  $\omega_{it}$  is the portfolio weight of security  $i$  at date  $t$ , then

$$\omega_{it}(k) = -\frac{1}{N}(R_{it-k} - R_{mt-k}) \quad , \quad R_{mt-k} \equiv \frac{1}{N} \sum_{i=1}^N R_{it-k} \quad (8)$$

for some  $k > 0$ . By buying the previous losers and selling the previous winners at each date, such a strategy actively bets on mean reversion across all  $N$  stocks, profiting from reversals that occur within the rebalancing interval.<sup>10</sup> For this reason, (8) has been called a “contrarian” trading strategy that benefits from overreaction, i.e., when underperformance is followed by positive returns and vice-versa for outperformance. Also, since the portfolio weights are proportional to the differences between the market index and the returns, securities that deviate more positively from the market at time  $t-k$  will have greater negative weight in the date- $t$  portfolio, and vice-versa. Also, observe that the portfolio weights are the negative of the degree of outperformance  $k$  periods ago, so each value of  $k$  yields a somewhat different strategy. Lo and MacKinlay (1990a) provide a detailed analysis of the unleveraged returns (9) of the contrarian trading strategy, tracing its profitability to mean reversion in individual stock returns as well as positive lead/lag effects and cross-autocorrelations across stocks and across time.

Note that the weights (8) have the property that they sum to 0, hence (8) is an example of an “arbitrage” or “market-neutral” portfolio where the long positions are exactly offset by the short positions.<sup>11</sup> As a result, the portfolio “return” cannot be computed in the

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<sup>9</sup>The assumption of multivariate normality in (A1) is not strictly necessary for the results in this section, but is often implicitly assumed, e.g., in Value-at-Risk computations of this portfolio’s return, in the exclusive focus on the first two moments of return distributions, and in assessing the statistical significance of associated regression diagnostics.

<sup>10</sup>However, Lo and MacKinlay (1990a) show that this need not be the only reason that contrarian investment strategies are profitable. In particular, if returns are positively cross-autocorrelated, they show that a return-reversal strategy will yield positive profits on average, even if individual security returns are serially independent. The presence of stock market overreaction, i.e., negatively autocorrelated individual returns, enhances the profitability of the return-reversal strategy, but is not required for such a strategy to earn positive expected returns.

<sup>11</sup>Such a strategy is more accurately described as a “dollar-neutral” portfolio since dollar-neutral does not necessarily imply that a strategy is also market-neutral. For example, if a portfolio is long \$100MM of high-

standard way because there is no net investment. In practice, however, the return of such a strategy over any finite interval is easily calculated as the profit-and-loss of that strategy's positions over the interval divided by the initial capital required to support those positions. For example, suppose that a portfolio consisting of \$100MM of long positions and \$100MM of short positions generated profits of \$2MM over a one-day interval. The return of this strategy is simply \$2MM divided by the required amount of capital to support the \$100MM long/short positions. Under Regulation T, the minimum amount of capital required is \$100MM (often stated as 2 : 1 leverage, or a 50% margin requirement), hence the return to the strategy is 2%. If, however, the portfolio manager is a broker-dealer, then Regulation T does not apply (other regulations govern the capital adequacy of broker-dealers), and higher levels of leverage may be employed. For example, under certain conditions, it is possible to support a \$100MM long/short portfolio with only \$25MM of capital—leverage ratio of 8 : 1—which implies a portfolio return of  $\$2/\$25 = 8\%$ .<sup>12</sup> Accordingly, the gross dollar investment  $V_t$  of the portfolio (8) and its unleveraged (Regulation T) portfolio return  $R_{pt}$  are given by:

$$V_t \equiv \frac{1}{2} \sum_{i=1}^N |\omega_{it}| \quad , \quad R_{pt} \equiv \frac{\sum_{i=1}^N \omega_{it} R_{it}}{V_t} . \quad (9)$$

To construct leveraged portfolio returns  $L_{pt}(\theta)$  using a regulatory leverage factor of  $\theta : 1$ , we simply multiply (9) by  $\theta/2$ :<sup>13</sup>

$$L_{pt}(\theta) \equiv \frac{(\theta/2) \sum_{i=1}^N \omega_{it} R_{it}}{V_t} . \quad (10)$$

Because of the linear nature of the strategy, and Assumption (A1), the strategy's statistical properties are particularly easy to derive. For example, Lo and MacKinlay (1990a)

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beta stocks and short \$100MM of low-beta stocks, it will be dollar-neutral but will have positive market-beta exposure. In practice, most dollar-neutral equity portfolios are also constructed to be market-neutral, hence the two terms are used almost interchangeably.

<sup>12</sup>The technical definition of leverage—and the one used by the U.S. Federal Reserve, which is responsible for setting leverage constraints for broker-dealers—is given by the sum of the absolute values of the long and short positions divided by the capital, so:

$$\frac{|\$100| + |-\$100|}{\$25} = 8 .$$

<sup>13</sup>Note that Reg-T leverage is, in fact, considered 2:1 which is exactly (9), hence  $\theta : 1$  leverage is equivalent to a multiple of  $\theta/2$ .

show that the strategy's profit-and-loss at date  $t$  is given by:

$$\pi_t(k) = \boldsymbol{\omega}'_t(k)\mathbf{R}_t \quad (11)$$

and re-arranging (11) and taking expectations yields the following:

$$\mathbb{E}[\pi_t(k)] = \frac{\boldsymbol{\iota}'\boldsymbol{\Gamma}_k\boldsymbol{\iota}}{N^2} - \frac{1}{N}\text{trace}(\boldsymbol{\Gamma}_k) - \frac{1}{N}\sum_{i=1}^N(\mu_i - \mu_m)^2 \quad (12)$$

which shows that the contrarian strategy's expected profits are an explicit function of the means, variances, and autocovariances of returns. See Lo and MacKinlay (1990, 1999) for further details of this strategy's statistical properties and a detailed empirical analysis of its historical returns.

This initial phase of a strategy's development process is characterized by the temporary fiction that Level-2 uncertainty holds, and not surprisingly, we are able to derive fairly explicit and analytically tractable results for the strategy's performance. By making further assumptions on the return-generating process  $\{\mathbf{R}_t\}$ , we can determine the profitability of this strategy explicitly. For example, suppose that the return of the  $i$ -th security is given by the following simple linear relation:

$$R_{it} = \mu_i + \beta_i\Lambda_{t-i} + \epsilon_{it} \quad , \quad \beta_i > 0 \quad , \quad i = 1, \dots, N \quad (13)$$

where  $\epsilon_{it}$  is Gaussian white noise and  $\Lambda_{t-i}$  is a factor common to all stocks, but with a lag that varies according to the index of security  $i$ , and which is also Gaussian and IID. This return-generating process is a special case of assumption (A1), and we can compute the



autocovariance matrices  $\mathbf{\Gamma}_k$  explicitly:

$$\mathbf{\Gamma}_1 = \begin{pmatrix} 0 & \beta_1\beta_2 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \beta_2\beta_3 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \beta_{N-1}\beta_N \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \sigma_\lambda^2 \quad (14a)$$

$$\mathbf{\Gamma}_2 = \begin{pmatrix} 0 & 0 & \beta_1\beta_3 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \beta_2\beta_4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \beta_{N-3}\beta_{N-1} \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \sigma_\lambda^2 \quad (14b)$$

$$\mathbf{\Gamma}_3 = \dots$$

Therefore, for  $k=1$ , the expected profit of the strategy (8) is given by:

$$\mathbb{E}[\pi_t(1)] = \frac{\sigma_\lambda^2}{N^2} \sum_{i=1}^{N-1} \beta_i\beta_{i+1} - \frac{1}{N} \sum_{i=1}^N (\mu_i - \mu_m)^2 \quad (15)$$

If the  $\beta_i$ 's are of the same sign, and the cross-sectional variability of the  $\mu_i$ 's is not too great, (15) shows that the strategy will be profitable on average. By calibrating the parameters of (15) to a specific set of values, we can evaluate the portfolio weights  $\{\omega_{it}(k)\}$  and average profitability  $\mathbb{E}[\pi_t(1)]$  of the strategy (8) explicitly.

### 5.3 StatArb at Level 3

Having analyzed the mean-reversion strategy from a theoretical perspective, a natural next step is to apply it to historical stock returns and simulate its performance. However, even the most cursory review of the data will show some obvious inconsistencies between assumption (A1) and reality, including non-normality and non-stationarities.<sup>14</sup> In fact, the

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<sup>14</sup> There is substantial evidence that financial asset returns are not normally distributed, but characterized by skewness, leptokurtosis, time-varying volatility and other non-Gaussian and non-stationarity properties

complexities of financial data and interactions make it virtually impossible to determine the precise probability laws generating the data, hence the best-case scenario we can hope for is Level-3 uncertainty when we engage in empirical analysis. Fortunately, assumption (A1) can be relaxed considerably—albeit at the expense of some notational simplicity—by using approximation theorems such as the Law of Large Numbers and the Central Limit Theorem, and using asymptotic statistical inference in place of finite-sample methods.<sup>15</sup>

With these caveats in mind, consider the empirical results reported by Khandani and Lo (2007), who apply this strategy to the daily returns of all stocks in the University of Chicago’s CRSP Database, and to stocks within 10 market-capitalization deciles, from January 3, 1995 to August 31, 2007.<sup>16</sup> Table 1 reports the year-by-year average daily return, standard deviation, and annualized Sharpe ratios (the annualized ratio of a strategy’s expected return minus a riskfree rate of return to its return standard deviation) of (8) and the results are impressive. In the first year of their sample, 1995, the strategy produced an average daily return of 1.38% per day, or approximately 345% per year assuming a 250-day year! The results for the following years are progressively less spectacular, but even in 2007, the average daily return of 0.13% translates into a still-remarkable annualized return of 33%! Moreover, the Sharpe ratios (computed under the assumption of a 0% riskfree rate) are absurdly high at 53.87 in 1995, and still extreme at 2.79 in 2007. In comparison, the S&P 500 had an annualized Sharpe ratio of 0.83 based on monthly returns from January 1995 to December 2007, and Warren Buffett’s Sharpe ratio during this same period was 0.75 based on the daily returns of Berkshire Hathaway stock. As Khandani and Lo (2008, 2009) observe, such performance is clearly too good to be true. The absurdly profitable results suggest that Level-3 uncertainty does not fully characterize the nature of this strategy’s performance, and we will return to this issue in Section 5.4.

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(see, for example, Cootner (1964), Mantegna and Stanley, 1994, and Lo and MacKinlay, 1999).

<sup>15</sup>For example, Lo and MacKinlay (1990a) show that the qualitative features of their analysis does not change under the weaker assumptions of weakly dependent heterogeneously distributed vectors  $\mathbf{R}_t$  when expectations are replaced with corresponding probability limits of suitably defined time-averages. See White for (1984) for further details.

<sup>16</sup>Specifically, they use only U.S. common stocks (CRSP share code 10 and 11), which eliminates REIT’s, ADR’s, and other types of securities, and they drop stocks with share prices below \$5 and above \$2,000. To reduce unnecessary turnover in their market-cap deciles, they form these deciles only twice a year (the first trading days of January and July). Since the CRSP data are available only through December 29, 2006, decile memberships for 2007 were based on market capitalizations as of December 29, 2006. For 2007, they constructed daily close-to-close returns for the stocks in their CRSP universe as of December 29, 2006 using adjusted closing prices from `finance.yahoo.com`. They were unable to find prices for 135 stocks in their CRSP universe, potentially due to ticker symbol changes or mismatches between CRSP and Yahoo. To avoid any conflict, they also dropped 34 other securities that are mapped to more than one CRSP PERMNO identifier as of December 29, 2006. The remaining 3,724 stocks were then placed in deciles and used for the analysis in 2007. Also, Yahoo’s adjusted prices do not incorporate dividends, hence their 2007 daily returns are price returns, not *total returns*. This difference is unlikely to have much impact on their analysis.

Year	Market Capitalization Deciles										All
	Smallest	Decile 2	Decile 3	Decile 4	Decile 5	Decile 6	Decile 7	Decile 8	Decile 9	Largest	
<b>Average Daily Returns</b>											
1995	3.57%	2.75%	1.94%	1.62%	1.07%	0.61%	0.21%	-0.01%	-0.02%	0.04%	1.38%
1996	3.58%	2.47%	1.82%	1.34%	0.84%	0.52%	0.19%	-0.11%	-0.04%	0.02%	1.17%
1997	2.83%	1.94%	1.34%	1.02%	0.62%	0.28%	0.04%	-0.12%	0.06%	0.14%	0.88%
1998	2.38%	1.45%	1.11%	0.62%	0.29%	0.03%	-0.04%	-0.12%	0.03%	0.10%	0.57%
1999	2.56%	1.41%	0.82%	0.38%	-0.01%	-0.11%	-0.21%	-0.35%	-0.01%	0.06%	0.44%
2000	2.58%	1.59%	0.92%	0.14%	0.03%	-0.02%	-0.14%	0.16%	0.00%	0.03%	0.44%
2001	2.15%	1.25%	0.57%	0.24%	-0.01%	0.06%	0.13%	-0.10%	-0.11%	-0.11%	0.31%
2002	1.67%	0.85%	0.53%	0.29%	0.28%	0.26%	0.28%	0.20%	0.11%	0.09%	0.45%
2003	1.00%	0.26%	-0.07%	0.04%	0.11%	0.20%	0.18%	0.15%	0.04%	0.05%	0.21%
2004	1.17%	0.48%	0.31%	0.38%	0.25%	0.29%	0.22%	0.15%	0.05%	-0.01%	0.37%
2005	1.05%	0.39%	0.13%	0.11%	0.09%	0.11%	0.05%	0.08%	0.01%	0.02%	0.26%
2006	0.86%	0.26%	0.11%	0.06%	0.05%	-0.02%	-0.02%	0.05%	0.06%	0.00%	0.15%
2007	0.57%	0.09%	0.08%	0.18%	0.16%	-0.08%	0.04%	-0.04%	0.00%	-0.04%	0.13%
<b>Standard Deviation of Daily Returns</b>											
1995	0.92%	0.88%	0.81%	0.82%	0.78%	0.77%	0.73%	0.67%	0.63%	0.65%	0.40%
1996	1.07%	1.00%	0.79%	0.81%	0.88%	0.84%	0.90%	0.90%	0.83%	0.73%	0.48%
1997	1.04%	0.98%	0.96%	0.96%	1.12%	1.00%	0.91%	0.99%	0.98%	0.77%	0.68%
1998	1.59%	1.67%	1.23%	1.22%	1.57%	1.25%	1.29%	1.43%	1.08%	1.00%	0.84%
1999	1.66%	1.82%	1.44%	1.44%	1.79%	1.57%	1.71%	1.70%	1.57%	1.07%	1.02%
2000	1.57%	1.69%	2.06%	1.89%	1.76%	2.15%	2.18%	2.29%	2.44%	2.56%	1.68%
2001	1.33%	1.26%	1.46%	1.62%	1.65%	1.64%	1.83%	1.91%	2.28%	2.29%	1.43%
2002	1.17%	0.89%	1.14%	1.07%	1.25%	1.11%	1.30%	1.42%	1.50%	1.50%	0.98%
2003	1.11%	0.81%	0.95%	0.89%	0.86%	0.81%	0.77%	0.76%	0.75%	0.56%	0.54%
2004	1.35%	1.01%	0.87%	0.76%	0.76%	0.78%	0.80%	0.74%	0.69%	0.57%	0.53%
2005	1.35%	0.80%	0.89%	0.70%	0.77%	0.77%	0.65%	0.73%	0.57%	0.56%	0.46%
2006	1.07%	0.90%	0.83%	0.84%	0.70%	1.07%	0.68%	0.68%	0.64%	0.61%	0.52%
2007	0.96%	1.02%	1.00%	0.99%	1.06%	1.44%	1.00%	0.87%	0.67%	0.56%	0.72%
<b>Annualized Sharpe Ratio (0% Riskfree Rate)</b>											
1995	61.27	49.20	37.79	31.26	21.49	12.68	4.62	-0.22	-0.54	0.87	53.87
1996	53.08	39.12	36.27	26.10	15.17	9.85	3.38	-1.89	-0.69	0.36	38.26
1997	43.15	31.19	22.00	16.66	8.67	4.45	0.74	-1.88	0.95	2.79	20.46
1998	23.61	13.78	14.22	8.09	2.92	0.39	-0.54	-1.32	0.43	1.58	10.62
1999	24.32	12.25	9.05	4.22	-0.11	-1.08	-1.93	-3.23	-0.09	0.82	6.81
2000	25.96	14.91	7.04	1.18	0.31	-0.18	-1.04	1.14	0.01	0.21	4.17
2001	25.56	15.68	6.15	2.30	-0.05	0.57	1.09	-0.79	-0.79	-0.73	3.46
2002	22.54	15.10	7.30	4.28	3.57	3.68	3.38	2.24	1.13	0.98	7.25
2003	14.32	5.19	-1.11	0.63	1.94	3.91	3.64	3.09	0.89	1.33	5.96
2004	13.76	7.55	5.60	7.96	5.11	5.90	4.27	3.20	1.12	-0.33	11.07
2005	12.33	7.72	2.26	2.42	1.95	2.29	1.31	1.74	0.36	0.62	8.85
2006	12.72	4.49	2.08	1.18	1.14	-0.26	-0.56	1.08	1.60	-0.03	4.47
2007	9.40	1.45	1.33	2.93	2.40	-0.84	0.69	-0.74	-0.05	-1.03	2.79

Table 1: Year-by-year average daily returns, standard deviations of daily returns, and annualized Sharpe ratios ( $\sqrt{250} \times (\text{average daily return}/\text{standard deviation})$ ) of Lo and MacKinlay's (1990a) contrarian trading strategy applied to all U.S. common stocks (CRSP share codes 10 and 11) with share prices above \$5 and less than \$2,000, and market-capitalization deciles, from January 3, 1995 to August 31, 2007.

Table 1 also confirms a pattern long recognized by long/short equity managers—the relation between profitability and market capitalization. Smaller-cap stocks generally exhibit more significant inefficiencies, hence the profitability of the contrarian strategy in the smaller deciles is considerably higher than in the larger-cap portfolios. For example, the average daily return of the strategy in the smallest decile in 1995 is 3.57% in contrast to 0.04% for the largest decile. Of course, smaller-cap stocks typically have much higher transactions costs and price impact, hence they may not be as attractive as the data might suggest.

## 5.4 StatArb at Level 4

Of course, not even the most naive quant would take the results of Table 1 at face value. There are many factors that must be weighed in evaluating these remarkable simulations, some easier to address than others. For example, the numbers in Table 1 do not reflect the impact of transaction costs, which can be quite substantial in a strategy as active as (8). The high turnover and the large number of stocks involved underscores the importance that technology plays in strategies like (8), and why funds that employ such strategies are predominantly quantitative. It is nearly impossible for human portfolio managers and traders to implement a strategy involving so many securities and trading so frequently without making substantial use of quantitative methods and technological tools such as automated trading platforms, electronic communications networks, and mathematical optimization algorithms. Indeed, part of the liquidity that such strategies seem to enjoy—the short holding periods, the rapid-fire implementation of trading signals, and the diversification of profitability across such a large number of instruments—is directly related to technological advances in trading, portfolio construction, and risk management. Not surprisingly, many of the most successful funds in this discipline have been founded by computer scientists, mathematicians, and engineers, not by economists or fundamental stock-pickers.

This reliance on technology and algorithms may give inexperienced quants the mistaken impression that the mean-reversion strategy involves only Level-3 uncertainty. For example, trading costs are relatively easy to incorporate into simulations since most costs are explicit and can therefore be readily imputed.<sup>17</sup> However, certain implicit costs are harder to estimate, such as the impact of the strategy itself on market prices, i.e., the reaction of other market participants to the strategy's purchases and sales, and the ultimate effect on market prices and dynamics. If the strategy is deployed with too much capital, financial markets

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<sup>17</sup>Known costs include brokerage commissions, bid/offer spreads (unless one is a broker-dealer, in which case the spread is a source of revenue, not an explicit cost), exchange fees, ticket charges, implicit net borrowing costs for any leverage employed by the strategy, and the associated costs of the accounting, legal, systems, and telecommunications infrastructure required to implement this strategy.

tend to equilibrate by increasing the prices of the securities the strategy seeks to buy and decreasing the prices of the securities the strategy seeks to shortsell, thereby reducing the strategy's potential profits. If, in 1995, we had implemented the mean-reversion strategy with \$1 billion of capital and leveraged it 10:1, the very act of trading this strategy would likely have generated significant losses, due solely to the market's reaction to the strategy's large and frequent trades. While such "price impact" is a fact of financial life, it is notoriously difficult to estimate accurately because it depends on many quantities that are beyond the portfolio manager's control and knowledge, including the identities of the manager's competitors on any given day, the strategies they are using, their business objectives and constraints, the likely reactions they might exhibit in response to the manager's trades, and how all of these factors may change from day to day as market conditions evolve. This is but one example of Level-4 uncertainty.

A more troubling symptom of Level-4 uncertainty is the potential non-stationarity of the strategy's returns implied by the near-monotonic decline in the average daily return of the strategy from 1995 to 2007. While certain types of non-stationarities such as time-varying volatilities can be accommodated through more sophisticated econometric techniques (see White, 1984), no statistical method can fully capture wholesale changes in financial institutions and discrete structural shifts in business conditions. Of course, the strategy's declining profitability over this period may simply be a statistical fluke, a random outcome due to observational noise of an otherwise consistently profitable strategy. Khandani and Lo (2007, 2008) propose a different hypothesis, one in which the decline is symptomatic of the growing popularity of strategies such as (8), and a direct consequence of an enormous influx of assets into such strategies during this period (see Figure 8). As more capital is deployed in similar strategies, downward pressure is created on the strategies' returns.<sup>18</sup> This secular decline in profitability has significant implications for the use of leverage, and the potential over-crowding that can occur among trading strategies of this type. We will return to this important issue in the next section.

Statistical fluke or crowded trade—which of these two hypotheses is closer to the truth? This question is not merely of academic interest, but is central to the business decision of whether or not to implement the mean-reversion strategy (8), how much capital to allocate to it if implemented, and when to shut it down and whether and when to restart it in the face of varying losses and gains. A particularly sobering episode of this simulated strategy's history is August 2007—the main focus of Khandani and Lo (2007, 2008)—when the strategy

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<sup>18</sup>Specifically, Khandani and Lo (2007, 2008) provide several related causal factors for this trend: increased competition, changes in market structure (e.g., decimalization), improvements in trading technology and electronic connectivity, and the growth in assets devoted to this type of strategy.

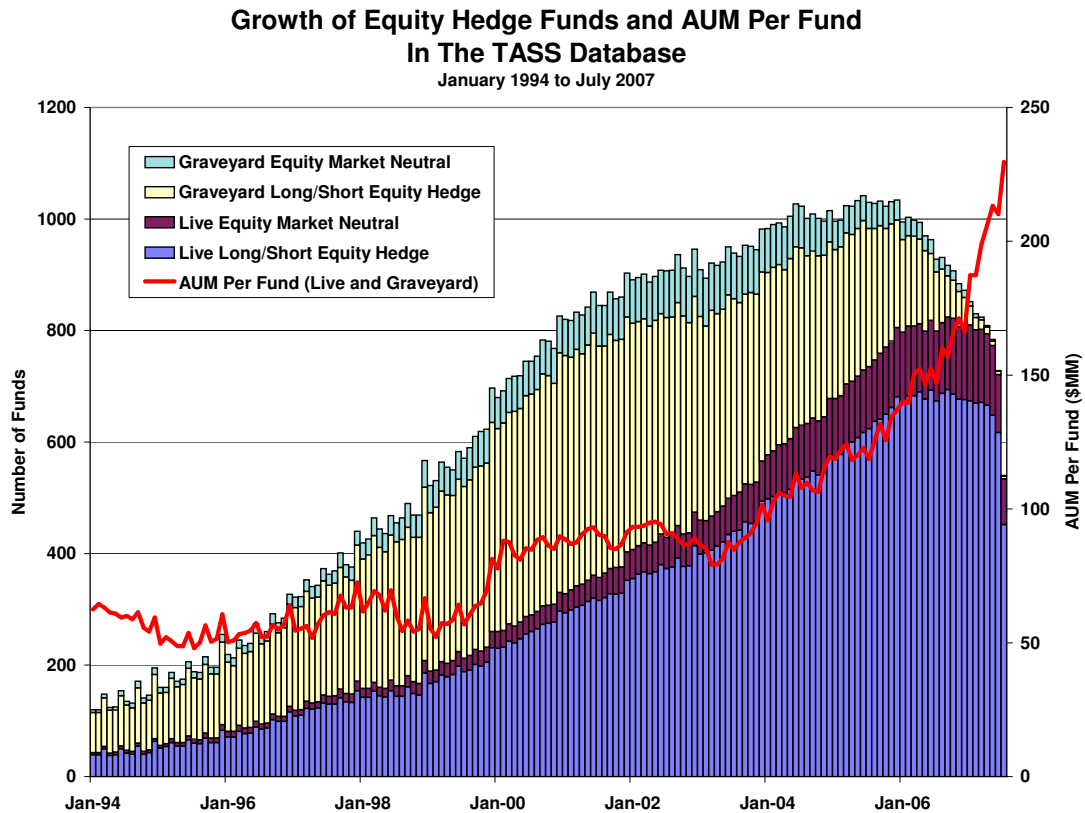


Figure 8: Number of funds in the Long/Short Equity Hedge and Equity Market Neutral categories of the TASS database, and average assets under management per fund, from January 1994 to July 2007.

produced a cumulative loss of approximately 25% over a three-day period.<sup>19</sup> Given a daily standard deviation of 0.52% for the strategy in 2006, the year prior to this event, and an 8:1 leverage ratio assumed by Khandani and Lo (2007, 2008), a 25% three-day loss represents a 7-standard-deviation event, which occurs with probability  $p=1.3\times 10^{-12}$  assuming normality. This probability implies that on average, such an event happens once every 6.4 billion years, hence the last such event would be expected to have happened approximately 1.7 billion years before the Earth was formed (see Lloyd, 2008).

Events such as these seem to happen more than rarely, which is one of the reasons that some portfolio managers use “stop-loss” policies in which they reduce the size of the bets after experiencing a certain level of cumulative losses. Such policies are often criticized as being ad hoc and “outside the model”, but implicit in this criticism is that the model is a complete description of the world, i.e., Level-3 uncertainty. If there is some chance that the model does not capture every aspect of reality, and if that omitted aspect of reality can cause significant losses to a portfolio, then stop-loss policies may have their place in the quant manager’s toolkit.<sup>20</sup>

The point of these thought experiments is not to discredit mean-reversion strategies or modern capitalism—many successful businesses and socially valuable services have emerged from this milieu—but rather to provide a concrete illustration of the impact of Level-4 uncertainty on financial decisions. Using a longer history of stock returns would do little to shed light on the strategy’s losses during August 2007; even an infinite amount of data cannot reduce Level-4 uncertainty beyond a certain level because of non-stationarities in financial markets and the economic environment. Yet financial decisions must be made even in the face of Level-3 uncertainty, and managers, shareholders, investors, and regulators must live with their consequences.

## 6 Level-5 Uncertainty: Black Swan Song?

Given the abstract nature of Level-5 uncertainty, and the fact that we did not consider this level explicitly in the examples of Sections 4 and 5, we expand on Level 5 in this section and provide some justification for its practical relevance. In the context of the harmonic

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<sup>19</sup>Khandani and Lo’s (2007, 2008) simulation is quite involved, and we refer readers to their papers for the full details.

<sup>20</sup>However, this observation does not imply that stop-loss policies should be used blindly. During the Quant Meltdown of August 2007, Khandani and Lo (2007, 2008) observe that those managers who cut risk and unwound their positions during August 7–9 effectively locked in their losses and missed out on the sharp reversal of the strategy on August 10. The moral of this story is that the only way to avoid or reduce the impact of such extreme events is to develop a deeper understanding of the strategy’s drivers, and in some cases, such understanding comes only after traumatic events like August 2007.

oscillator and statistical arbitrage, Level-5 uncertainty cannot be addressed by estimating parameters in more sophisticated ways. Instead, under this type of uncertainty, there is no plausible model or data-generating process that we can identify which systematically and accurately generates the prices we observe in financial markets. Even if there are extended periods over which some version of our model is a useful and effective description of the data, we must be prepared for times when the limitations of the model are revealed, and human nature—in the form of fear, greed, and “animal spirits”—appears in ways that few of us can anticipate.

The Financial Crisis of 2007–2009 has generated renewed interest in this kind of uncertainty, which has come to be known in the popular media and within the finance community by a variety of evocative and often euphemistic names: “tail events”, “extreme events”, or “black swans” (Taleb, 2007). These cultural icons refer to disasters that occur so infrequently that they are virtually impossible to analyze using standard statistical inference. However, we find this perspective less than helpful because it suggests a state of hopeless ignorance in which we resign ourselves to being buffeted and battered by the unknowable.

In this section, we explore this possibility by considering the entire range of uncertainty, initially from the rather unusual perspective of deterministic phenomena that are sometimes taken as random, and then in the context of financial markets and institutions. We propose that the entire taxonomy of uncertainty must be applied at once to the different features of any single phenomenon, and these features can transition from one taxon to the next as we develop deeper understandings of the phenomenon’s origins.

## 6.1 Eclipses and Coin Tosses

Consider an event that occurs with probability  $p$  in any given year and let  $I_t$  be the 0/1 indicator variable of this event, hence:

$$I_t = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases} . \quad (16)$$

Let  $\hat{p}$  denote the standard statistical estimator for  $p$  based on the historical relative frequency of the event in a sample of  $T$  observations:

$$\hat{p} = \frac{1}{T} \sum_{t=1}^T I_t . \quad (17)$$



The accuracy of  $\hat{p}$  can be measured by computing its standard error, which is given by:

$$\text{SE}[\hat{p}] = \sqrt{\text{Var}[\hat{p}]} = \sqrt{\frac{1}{T}\text{Var}[I_t]} = \sqrt{\frac{p(1-p)}{T}}. \quad (18)$$

Now consider a genuinely rare event, one that occurs once every 50 years, hence  $p = 0.02$ . According to Table 2, even with 100 years of data, the standard error of the estimator  $\hat{p}$  is 1.4%, the same order of magnitude as the estimator itself. And this result has been derived under ideal conditions; any serial correlation, heteroskedasticity, measurement error, or nonstationarities in  $\{I_t\}$  will only act to increase the standard errors of  $\hat{p}$ .

<b>p</b>	<b>5</b>	<b>10</b>	<b>20</b>	<b>50</b>	<b>100</b>	<b>500</b>	<b>1000</b>
<b>0.01%</b>	0.447%	0.316%	0.224%	0.141%	0.100%	0.045%	0.032%
<b>0.05%</b>	1.000%	0.707%	0.500%	0.316%	0.224%	0.100%	0.071%
<b>0.10%</b>	1.414%	0.999%	0.707%	0.447%	0.316%	0.141%	0.100%
<b>0.50%</b>	3.154%	2.230%	1.577%	0.997%	0.705%	0.315%	0.223%
<b>1.00%</b>	4.450%	3.146%	2.225%	1.407%	0.995%	0.445%	0.315%
<b>2.00%</b>	6.261%	4.427%	3.130%	1.980%	1.400%	0.626%	0.443%
<b>3.00%</b>	7.629%	5.394%	3.814%	2.412%	1.706%	0.763%	0.539%
<b>4.00%</b>	8.764%	6.197%	4.382%	2.771%	1.960%	0.876%	0.620%
<b>5.00%</b>	9.747%	6.892%	4.873%	3.082%	2.179%	0.975%	0.689%

Table 2: Standard errors of event probability estimator  $\hat{p}$  as a function of the actual probability  $p$  and sample size  $T$ , assuming independently and identically distributed observations.

This simple calculation seems to suggest that rare events are difficult to analyze by virtue of their rarity. However, such a conclusion is a reflection of the limitation of a pure statistical approach to analyzing rare events, not a statement about rare events and irreducible uncertainty. For example, certain types of total solar eclipses (central, with one limit) are exceedingly rare—only 49 will occur during the 10,000-year period from 4,000 BCE to 6,000 CE—yet they can be predicted with extraordinary accuracy.<sup>21</sup> The language of probability and statistics is so well-developed and ingrained in the scientific method that we often forget the fact that probabilistic mechanisms are, in fact, proxies for deterministic phenomena that are too complex to be modeled in any other fashion.<sup>22</sup>

Perhaps the clearest illustration of this probabilistic approach to deterministic complexity is the simple coin toss. The 50/50 chances of “heads or tails” is used every day in countless

<sup>21</sup>Eclipse predictions by Fred Espenak and Jean Meeus (NASA’s GSFC) are gratefully acknowledged. See <http://eclipse.gsfc.nasa.gov/SEcatmax/SEcatmax.html> for further details.

<sup>22</sup>In a simple mathematical model of atmosphere convection, Lorenz (1963) was perhaps the first to construct an example of a deterministic process that exhibited complex behavior, i.e., sensitive dependence to initial conditions. Since then, the field of nonlinear dynamical systems and “chaos theory” has identified numerous natural phenomena that are chaotic (see, for example, Strogatz, 1994).

situations requiring randomization, yet it has been demonstrated conclusively that the outcome of a coin toss is, in fact, deterministic, a product of physics not probability (see Figure 9 from Diaconis, Holmes, and Montgomery, 2007). Why, then, do we persist in treating the coin toss as random? The reason is that in all but the most sophisticated and controlled applications, we have no idea what the initial conditions are, nor does the typical coin-tosser understand enough of the physics of the toss to make use of such initial conditions even if known. Without the proper data and knowledge, the outcome is effectively random. But with sufficient data and knowledge, the outcome is certain.

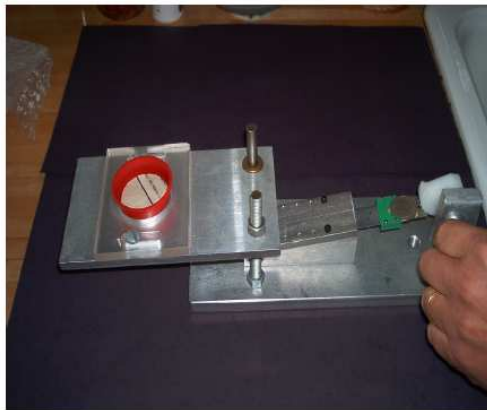


Figure 1.a

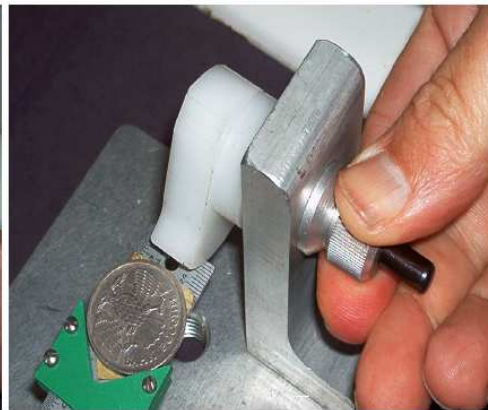


Figure 1.b



Figure 1.c

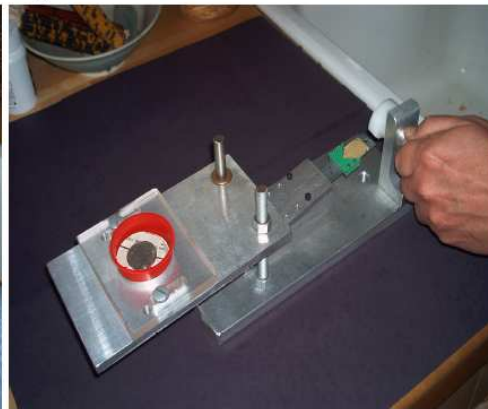


Figure 1.d

Figure 9: Coin-tossing machine of Diaconis, Holmes, and Montgomery (2007, Figure 1): “The coin is placed on a spring, the spring released by a ratchet, the coin flips up doing a natural spin and lands in the cup. With careful adjustment, the coin started heads up always lands heads up—one hundred percent of the time. We conclude that coin-tossing is ‘physics’ not ‘random’”.

Within this simple example, we can observe the full range of uncertainty from Level 5 to Level 1 just by varying the information available to the observer. Uncertainty is often in the eyes of the beholder, but we can transition from one level of uncertainty to another as we deepen our understanding of a given phenomenon.

## 6.2 Uncertainty and Econometrics

An example of irreducible uncertainty more closely related to financial economics is provided by the econometric notion of identification. If  $\boldsymbol{\theta}$  is a vector of unknown parameters that characterize the probability law  $P(\mathbf{X}; \boldsymbol{\theta})$  of a particular economic model with a vector of state variables  $\mathbf{X}$ , then the parameters are said to be “identified” if and only if  $\boldsymbol{\theta}_1 \neq \boldsymbol{\theta}_2$  implies that  $P(\mathbf{X}; \boldsymbol{\theta}_1) \neq P(\mathbf{X}; \boldsymbol{\theta}_2)$ . If this condition does not hold, then there is no way to distinguish between the two models characterized by  $\boldsymbol{\theta}_1$  and  $\boldsymbol{\theta}_2$  since they yield the same probability laws for the observable manifestation of the model,  $\mathbf{X}$ .

A well-known application of this simple but powerful concept of identification is the econometric estimation of simultaneous linear equations (see Theil, 1971). Consider the law of supply and demand which is taught in every introductory economics course: the price  $P$  and quantity  $Q$  that clear a market are determined by the intersection of the market’s supply and demand curves. Let the (linear) demand curve at time  $t$  be given by:

$$Q_t^d = d_0 + d_1 P_t + d_2 Y_t + \epsilon_t^d \quad (19)$$

where  $P_t$  is the purchase/selling price,  $Y_t$  is household income and  $\epsilon_t^d$  is a random demand shock, and let the (linear) supply curve at time  $t$  be given by:

$$Q_t^s = s_0 + s_1 P_t + s_2 C_t + \epsilon_t^s \quad (20)$$

where  $C_t$  is production costs and  $\epsilon_t^s$  is a random supply shock. An economic equilibrium or market-clearing occurs when supply equals demand, hence:

$$Q_t^s = Q_t^d = Q_t \quad (21)$$

where  $Q_t$  is the equilibrium quantity produced and consumed at time  $t$ .

Suppose that the variable to be forecasted is the quantity demanded  $Q_t^d$  as given by

(19),<sup>23</sup> and we are given an infinite history of past prices  $\{P_t\}$  and quantities  $\{Q_t\}$  observed in the marketplace, but no other information. It is easy to show that it is impossible to infer either the demand or supply curves from this history, and forecasts of  $Q_t^d$  will be hopelessly unreliable because every single data point we observe is the intersection of the two curves, hence there is no way to differentiate between the two (see Figure 10). In econometric terminology, this system is not “identified”.

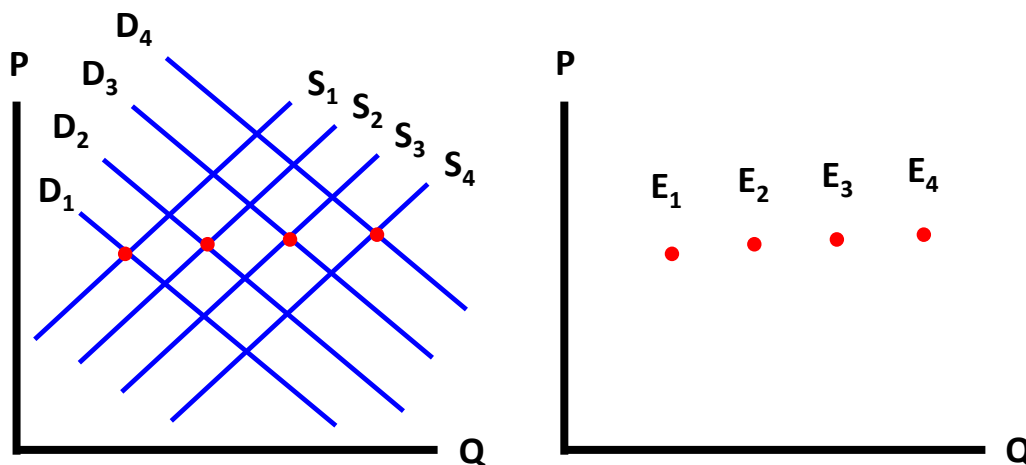


Figure 10: Illustration of the identification problem in the estimation of supply and demand curves (left). If only the intersections  $E_i$  of the two curves are observed (right), i.e., equilibrium prices and quantities, it is impossible to determine the individual supply and demand curves generating those equilibria, even with an infinite amount of data.

Under the stated assumptions, achieving identification is quite simple. Given sufficient historical data on household income  $\{Y_t\}$  and production costs  $\{C_t\}$ , a variant of linear regression—two-stage least squares or instrumental variables—can accurately estimate both supply and demand curves (Theil, 1971). These additional variables allow us to identify the two curves because they affect one curve but not the other. In particular, demand fluctuations due to variations in household income can identify the supply curve, and supply fluctuations due to variations in production costs can identify the demand curve. The framework of supply and demand, and the additional data on household income and production costs, allows us to move from Level-5 to Level-4 uncertainty.

<sup>23</sup>This is not as frivolous an example as it may seem. In particular, a significant portion of consumer marketing research is devoted to estimating demand curves. Public policies such as gasoline, alcohol, and cigarette taxes, energy tax credits, and investment incentives also often hinge on price-elasticity-of-demand estimates. Finally, economic analysis of demand is frequently the basis of damage awards in a growing number of legal disputes involving securities fraud, employment discrimination, and anti-trust violations.

As with the coin toss of Section 6.1, this example contains the full range of our taxonomy of uncertainty, and illustrates the limitations of the black swan metaphor. Rather than simply admitting defeat and ascribing a particular outcome to a “tail event”, we propose a more productive path. By developing a deeper understanding of the underlying phenomena generating rare events, we are able to progress from one level of uncertainty to the next.

### 6.3 StatArb Revisited

While the Quant Meltdown of August 2007 provides a compelling illustration of Level-4 uncertainty in a financial context, the analysis of Khandani and Lo (2007, 2008) offers some hope that additional quantitative investigation can change the proportion of irreducible uncertainty affecting mean-reversion strategies. But what happened to these strategies from July to October 2008 may be more appropriately categorized as Level-5 uncertainty: in the wake of Bear Stearns’ collapse and increasingly negative prospects for the financial industry, the U.S. Securities and Exchange Commission (SEC) temporarily prohibited the shorting of certain companies in the financial services sector, which eventually covered 799 securities (see SEC Release No. 34–58592). This unanticipated reaction by the government is an example of irreducible uncertainty that cannot be modeled quantitatively, yet has substantial impact on the risks and rewards of quantitative strategies like (8). Even the most sophisticated simulation cannot incorporate every relevant aspect of market conditions as they evolve through time. A truly thoughtful quant will understand this basic insight viscerally—having experienced losses first-hand thanks to Level-4 and Level-5 uncertainty—and will approach strategy development with humility and skepticism.

The presence of Level-5 uncertainty should not, however, imply capitulation by the quant. On the contrary, the need for analysis is even more urgent in these cases, but not necessarily of the purely quantitative type. Human intelligence is considerably broader than deductive logic, and other forms of reasoning can be effective when mathematics is not. For example, in the case of the mean-reversion strategy (8), it may be useful to observe that one possible source of its profits is the fact that the strategy provides liquidity to the marketplace. By definition, losers are stocks that have underperformed relative to some market average, implying a supply/demand imbalance, i.e., an excess supply that caused the prices of those securities to drop, and vice-versa for winners. By buying losers and selling winners, mean-reversion strategies are adding to the demand for losers and increasing the supply of winners, thereby stabilizing supply/demand imbalances. Traditionally, designated market-makers such as the NYSE/AMEX specialists and NASDAQ dealers have played this role, for which they have been compensated through the bid/offer spread. But over the last decade,

hedge funds and proprietary trading desks have begun to compete with traditional market-makers, adding enormous amounts of liquidity to U.S. stock markets and earning attractive returns for themselves and their investors in the process.

However, on occasion information affecting all stocks in the same direction arises, temporarily replacing mean reversion with momentum as the information is impounded into the prices of all securities. In such scenarios, liquidity providers will suffer large losses as this information is processed by the market, and only after such information is fully reflected in all prices will liquidity providers begin to profit again.

This explanation by Khandani and Lo (2007, 2008) is plausible, but it is not a purely quantitative hypothesis (although it certainly contains quantitative elements). Moreover, while certain components may be empirically tested and either refuted or corroborated,<sup>24</sup> it may never be possible to determine the truth or falsity of their conjectures. This is Level-5 uncertainty.

## 7 Applying the Taxonomy of Uncertainty

While the taxonomy of uncertainty may be useful from an academic perspective, we believe that a clearer understanding of the types of uncertainty at play is critical to the proper *management* of risk and uncertainty for all stakeholders in the financial system.<sup>25</sup> In this section, we describe some of these practical implications in more detail.

In Section 7.1, we consider the question asked by most investors of their quant portfolio managers: do you really believe your model? The taxonomy of uncertainty suggests that the answer is not a simple yes or no. This more nuanced perspective also applies to traditional methods of risk management, leading to the distinction between risk models and model risk discussed in Section 7.2, the impact of misaligned incentives considered in Section 7.3, and the potential hazards of mismatched timescales in Section 7.4. We illustrate the practical relevance of all of these considerations in Section 7.5 by proposing an uncertainty “checklist” that we apply to the quantitative strategy of Section 5.

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<sup>24</sup>For example, in providing liquidity to the market, mean-reversion strategies also have the effect of reducing market volatility because they attenuate the movement of prices by selling stocks for which there is excess demand and buying stocks for which there is excess supply. Therefore, an increasing amount of capital dedicated to market-making strategies is one potential explanation for the secular decline in U.S. equity-market volatility from 2003 to 2006. Once this market-making capital is withdrawn from the marketplace, volatility should pick up, as it did in the aftermath of the August 2007 Quant Meltdown.

<sup>25</sup>Although one of the central ideas of this paper is to distinguish between risk and the various levels of uncertainty, we do not expect standard terminology such as “risk management” to reflect this distinction, becoming “risk and uncertainty management” instead. Accordingly, at the risk of creating some uncertainty, we shall conform with standard terminology in this section.



## 7.1 Do You Really Believe Your Models??

Quantitative investment managers are often asked whether they ever over-ride their models on a discretionary basis. Prospective investors are often more direct: “Do you really believe your models??”. One of the key ideas in applying the taxonomy of uncertainty is the fact that a given model typically exhibits multiple levels of uncertainty, hence an important ingredient in the successful implementations of any model is recognizing its domain(s) and, in particular, the boundaries of its validity. Newtonian mechanics—while a remarkably useful description of a wide range of physical phenomena—is inadequate for many systems that require more sophisticated and encompassing theories, such as special relativity or quantum mechanics. These exceptions do not cause us to abandon Newtonian mechanics, which forms the basis for every introductory physics course. However, we acknowledge that developing new theories to extend the existing framework is a difficult, time-consuming, and worthwhile enterprise.

In fact, it is possible to “believe” a model at one level of the hierarchy but not at another. Consider, for example, a forecasting model for asset returns—often called an “alpha model” (such as the mean-reversion strategy of Section 5)—that seems genuinely effective in generating positive profits on average after deducting all transactions costs and adjusting for risk.<sup>26</sup> Specifically, suppose that the historical or “in-sample” simulations of this strategy generates a profit of 2% per day with 55% probability and  $-2\%$  per day with 45% probability, and let these estimates be highly statistically significant. Despite the fact that this strategy experiences losses 45% of the time, over the course of a typical year, its expected compound return is 65%! Moreover, suppose that in testing this strategy on an “out-of-sample” basis using real capital and live trading, the statistical properties of the strategy’s realized returns are similar to those of the backtest, after accounting for the inevitable effects of backtest bias (see, for example, Lo and MacKinlay, 1990b). In this situation, it is understandable for the strategy’s author to reply “Yes, I do believe in my model”.

However, this response is likely based on the perspectives of Level-2 and Level-3 uncertainty, which may be the only perspectives of an inexperienced quant. But the experienced quant also understands that statistical changes in regime, estimation errors, erroneous in-

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<sup>26</sup>Risk adjustments are not without controversy, especially among non-economists in the financial industry. The basic idea is that positive expected profits can be due to superior information and skill (alpha), or due to particular risk exposures for which the positive expected profits are the commensurate compensation (beta). This distinction—first espoused by Sharpe (1964) and Lintner (1965) and embodied in the “Capital Asset Pricing Model” or CAPM—is important because it shows that strategies with positive expected profits are not always based on proprietary knowledge, but can sometimes be the result of risk premia. However, there are now many versions of the CAPM, and hence many possible ways to adjust for risk, and there is no consensus as to which one is best. Moreover, all of these adjustments require the assumption of general equilibrium (the equality of supply and demand across all markets at all times), which can also be challenged on many levels.

put data, institutional rigidities such as legal and regulatory constraints, and unanticipated market shocks are possible, i.e., Level-4 and Level-5 uncertainty. In such circumstances, any given model may not only fail—it may fail spectacularly, as fixed-income arbitrage strategies did in August 1998, as internet companies did in 2001–2002, as statistical arbitrage strategies did in August 2007, and as mortgage-backed securities did in 2007–2009.

There are two responses to the recognition that, in the face of Level-4 or Level-5 uncertainty, the model is outside of its domain of validity. The first is to develop a deeper understanding of what is going on and to build a better model, which may be very challenging but well worth the effort if possible (see, for example, Lo and MacKinlay's, 1990a, analysis of the role of lead/lag effects in the profitability of Section 5's mean-reversion strategy). There are no simple panaceas; as with any complex system, careful analysis and creativity are required to understand the potential failures of a given model and the implications for a portfolio's profits and losses. By developing a more complete understanding of the model's performance in a broader set of environments, we bring aspects of the model from Level-5 uncertainty to Level-4, from Level-4 to Level-3, and so on.

The other response is to admit ignorance and protect the portfolio by limiting the damage that the model could potentially do. This brings us to the subject of risk management, which lies at the heart of investment management.

## 7.2 Risk Models vs. Model Risk

Risk management always generates considerable support in concept, but its realities often differ from its ideals. These discrepancies are largely attributable to the different levels of uncertainty. To see why, let us return to the example in the previous section of the alpha model that yields 2% each day with 55% probability and  $-2\%$  with 45% probability. Although its compound annual expected return is 65%, this strategy's annualized return standard deviation is a whopping 53% (in comparison, the annual standard deviation of the S&P 500 is currently around 25%). Such a high level of risk implies an 11% chance that this highly profitable strategy's annual return is negative in any given year assuming Gaussian returns (which may not be appropriate; see footnote 14), and a 30% chance that over a 10-year period, at least two years will show negative returns. Suppose in its first year of trading, the strategy experiences a  $-50\%$  return—what is the appropriate response?

From the perspective of Level-2 and Level-3 uncertainty, there is nothing awry with such an outcome. With an annual standard deviation of 53%, an annual return of  $-50\%$  is well within the bounds of normality, both figuratively and statistically. Therefore, a plausible response to this situation is to continue with the strategy as before, and this may well be



the correct response.

However, the perspective of Level-4 and Level-5 uncertainty suggests the possibility that the losses are due to factors that are not so innocuous and random. While a  $-50\%$  loss is not rare for a strategy with a  $53\%$  annual standard deviation, it cannot help but change, to some degree, a rational individual's prior belief that the strategy has a positive expected return. Moreover, even if a loss of  $50\%$  is statistically common for such a strategy, there are business implications that may intercede, e.g., the possibility of being shut down by senior management or investors who do not wish to risk losing all of their capital.

More generally, much of the recent debate regarding the role of quantitative models in the current financial crisis involves criticizing standard risk analytics such as Value-at-Risk measures, linear factor models, principal components analysis, and other statistical methods that rely on assumptions of linearity, stationarity, or multivariate normality, i.e., Level-2 and Level-3 uncertainty. Under such assumptions, these techniques can be readily implemented by estimating a covariance matrix from the data, and then calculating the probabilities of gain and loss based on the estimated parameters of a multivariate normal distribution that incorporate volatility and correlation information. The natural conclusion drawn by the popular press is that such a naive approach to the complexities of the real world caused the financial crisis (see the discussion in Section 8 on the Gaussian copula formula); quants failed spectacularly. In our view, this is a disingenuous caricature that oversimplifies the underlying causes of the crisis—giving too much weight to quants and too little to senior management—and misses an important gap that exists in many risk management protocols today.

There are certainly some quantitative analysts who have enough experience to understand the business end of trading desks and portfolio management. Similarly, there are certainly some senior managers who have more than a passing familiarity with the models that are used to calculate the risk exposures of their positions. But when, in a single organization, the specialization of these two perspectives becomes too great—with quants focusing on Level-1 certainty and Level-2 uncertainty, and senior managers focusing on Level-3 and Level-4 uncertainty—a potentially dangerous risk-management gap is created. This gap usually emerges gradually and without notice, perhaps through the departure of quantitatively sophisticated senior management, the rapid promotion of business managers without such training, or the rising influence of inexperienced quants who happened to have enjoyed a period of profitable trading. The appropriate response to this gap is neither to discard quantitative models entirely, nor to defend them to the death, but rather to acknowledge their limitations and develop additional methods for addressing them.

One approach is to develop a set of risk-management protocols that may lie outside of the

framework of the alpha model. In contrast to the typical linear factor models that use factors drawn from the alpha model to estimate a strategy’s risk exposures, volatility, Value-at-Risk statistics, and other traditional risk analytics, we are referring to a “meta-model” that can account for “alpha decay” (the declining profitability of a strategy) and “model risk” (the complete failure of a model due to circumstances outside the model’s set of assumptions). In other words, a complete risk management protocol must contain risk models but should also account for model risk. Specific methods for dealing with model risk include stop-loss policies that incorporate the risk of ruin or franchise risk; limits on capital commitments and positions (to prevent “outlier” signals from making unusually large or concentrated bets); statistical regime-switching models that can capture changes in the profitability of the strategy; and Bayesian decision rules to trade off “Type I and Type II errors” (making a bet when the signal is wrong vs. not making a bet when the signal is right).

These are crude but often effective responses to Level-5 uncertainty, whether they come in the form of counterparty failure, sovereign or corporate defaults, currency devaluations, accounting fraud, rapid shifts in regulatory requirements or capital controls, political and social upheaval, a change in business strategy or corporate control, or any other event that is difficult or impossible to anticipate. Such policies are closer to business decisions than trading decisions—but, of course, one important objective of trading is to stay in business—and requires managers who understand both the business environment and the limitations of models and the consequences of their failure. When senior management is missing one of these two perspectives, the consequences can be disastrous.

### 7.3 Incentives and Moral Hazard

A more prosaic example of Level-4 uncertainty in managing the risks of proprietary trading strategies such as (8) is the role of incentives. The so-called “principal-agent” problem of delegated portfolio management is well-known (see Stiglitz, 1987), but this challenge is easier to state than to solve in a real-world context. Consider the following thought experiment: you are given an investment opportunity in which you stand to earn \$10 million with 99.9% probability, but with 0.1% probability, you will be executed. Is it clear what the “right” decision is? Now suppose it is your boss that is to be executed instead of you; does that change your decision? Similar choices—perhaps with less extreme downsides—are presented everyday to portfolio managers in the financial industry, and the incentive structures of hedge funds, proprietary trading desks, and most non-financial corporations have a non-trivial impact on the attendant risks those financial institutions face.

A broader challenge regarding incentives involves the typical corporate governance struc-

ture in which risk management falls under the aegis of the CEO and CIO, both of whom are focused on maximizing corporate profits, and where risk is viewed primarily as a constraint. Moreover, most Chief Risk Officers (CROs) are compensated in much the same way that other senior management is: through stock options and annual bonuses that depend, in part, on corporate profits. One alternative is for a corporation to appoint a Chief Risk Officer (CRO) who reports directly to the board of directors and is solely responsible for managing the company's enterprise risk exposures, and whose compensation depends not on corporate revenues or earnings, but on corporate stability. Any proposed material change in a corporation's risk profile—as measured by several objective metrics that are specified in advance by senior management and the board—will require prior written authorization of the CRO, and the CRO can be terminated for cause if a corporation's risk profile deviates from its pre-specified risk mandate, as determined jointly on an annual basis by senior management and the board.

Such a proposal does invite conflict and debate among senior management and their directors, but this is precisely the point. By having open dialogue about the potential risks and rewards of new initiatives—which would provide a natural forum for discussing Level-4 and Level-5 uncertainty, even among the most quantitatively sophisticated models—senior management will have a fighting chance of avoiding the cognitive traps that can lead to disaster.

## 7.4 Timescales Matter

Another practical consideration that must figure in any application of our taxonomy of uncertainty is the natural timescale of the strategy and how it may differ from the timescale of the various decisionmakers involved. In some cases, a significant mismatch in timescales can lead to disaster. For example, if a strategy is based on quarterly earnings information and a portfolio of positions designed to exploit such information is updated only once a quarter, it cannot adjust to sharp changes in volatility that may occur within the quarter. Accordingly, such a portfolio may contain more risk than its manager or investors expect. Similarly, if financial regulations are updated only once every few years, it is easy to see how such laws can easily be outpaced and outmaneuvered by more rapid financial innovation during periods of rapid economic growth.

A simple and intuitive heuristic is to set the timescale of investment decisions and risk management to be at least as fine as the finest time interval in which information relevant to the strategy is generated. If, for example, daily signals are employed in a strategy (such as the strategy in Section 5), then daily risk analytics, stop-loss policies, and position limits are

necessary; monthly versions would be nearly useless. On the other hand, a monthly portfolio strategy may still require daily rebalancings and analytics if intra-monthly volatility spikes or large profit-and-loss swings affect the strategy’s operations or business viability. In contrast, the case of legendary investor Warren Buffett’s portfolio—which is constructed under a timescale of years over which its investments are expected to pay off—is unlikely to require monthly rebalancings (although Buffett does have the extra advantage of very deep pockets, which is perhaps the most effective risk management tool of all).

Alternatively, when the timescale of the uncertainty matches the timescale of investment decisions, the challenges are greatest, as the regime-switching harmonic oscillator of Section 4.3 illustrates. If the timescale of uncertainty is much greater than that of the investment decision (e.g., the impact of climate change on the profitability of statistical arbitrage strategies) or much less (e.g., the impact of restricting high-frequency trading on Buffett’s portfolio), such uncertainty can usually be ignored as a first approximation. But when the timescale of the uncertainty is comparable to the timescale of the investment decisions, extreme caution must be exercised in managing such uncertainty. In such circumstances, a proper understanding of the taxonomy of uncertainty and how it applies to every aspect of an investment strategy becomes critical.

## 7.5 The Uncertainty Checklist

To illustrate the practical relevance of the taxonomy of uncertainty, we propose a simple “uncertainty checklist” that can be applied to any business endeavor. In addition to providing a compact summary of the implications of the five levels of uncertainty, the checklist also serves as a useful starting point for designing a comprehensive enterprise risk management protocol that goes well beyond the standard framework of VaR, stress tests, and scenario analysis.

The idea of an uncertainty checklist is straightforward: it is organized as a table whose columns correspond to the five levels of uncertainty of Section 3, and whose rows correspond to all the business components of the activity under consideration. Each entry consists of all aspects of that business component falling into the particular level of uncertainty, and ideally, the individuals and policies responsible for addressing their proper execution and potential failings.

For concreteness, consider the quantitative trading strategy described in Section 5. The components of such a strategy might include:

- theoretical framework (alpha models and forecasts);
- empirical analysis (data and backtest platform);

- portfolio construction (optimization engine);
- trading and implementation (market frictions, telecommunications, operations);
- risk management (limits on positions, VaR, and losses);
- legal and regulatory (trading restrictions, reporting requirements); and
- business issues (funding sources, investor relations, economic conditions).

Table 3 contains an example of what an uncertainty checklist might include; the entries are by no means complete, but are meant to illustrate the type of issues to be considered at each level of uncertainty and for each component of the business. This example is also meant to convey the multi-dimensional nature of truly comprehensive enterprise-risk-management protocols, and the different skills and systems needed to address the various cells of this two-dimensional checklist. In particular, issues involving the upper left portion of the checklist require strong quantitative skills, whereas issues involving the lower right portion require deep business experience and intuition. But this table highlights the critical fact that the business involves all of these cells, which any risk management protocol must integrate to be effective.

The many facets of uncertainty can even affect an activity as mundane as cash management. At the start of a new trading strategy, the amount of capital available for deployment is usually known with certainty (Level 1). However, over time, capital inflows and outflows can occur, injecting some randomness—typically small in relation to the total amount of capital deployed—into the day-to-day sizing of the strategy (Levels 2 and 3). On occasion, extreme positive or negative performance can generate large capital flows over short time intervals, creating significant trading and implementation challenges with outcomes that cannot be completely quantified (Level 4). And because certain contractual agreements allow counterparties the flexibility to re-assign obligations (swap novation) or to re-use collateral in unspecified ways with unspecified third parties (re-hypothecation), it is sometimes impossible to determine even the qualitative nature of one’s risk exposures (Level 5). While cash management may seem trivial and pedestrian from the portfolio manager’s perspective, those who had significant cash holdings at Lehman Brothers in 2008 may now have a deeper appreciation of its subtleties and hidden dangers.

The purpose of the uncertainty checklist is not simply to catalog potential sources of disaster, but rather to develop a systematic approach to risk management that acknowledges the presence of distinct types of uncertainty, each requiring a different set of skills and systems to manage properly. For many of the cells in Table 3, there may be little to do other than to admit ignorance, but even this can be valuable from the perspective of corporate strategy. The very process of completing the uncertainty checklist may generate important questions, concerns, and insights that might otherwise not have surfaced during business as usual.

Components of a Quantitative Investment Strategy	Level 1	Level 2	Level 3	Level 4	Level 5
	Perfect Certainty	Risk	Fully Reducible Uncertainty	Partially Reducible Uncertainty	Irreducible Uncertainty
<b>Theoretical Framework</b>	Net present value relationships, law of one price	Mathematical framework of mean reversion	Statistical framework of time-series analysis	Unforeseen nonlinearities, omitted variables	Complexity
<b>Empirical Analysis</b>		Econometric estimators and methods of statistical inference	Backtest results based on historical data	Backtest bias, survivorship bias, omitted variables, etc.	Outliers, data errors, insufficient data
<b>Portfolio Construction</b>	Mathematics of optimization	Mean-variance optimization given model parameters	Statistical estimation of model parameters	Time-varying parameters, multiple regimes	Corporate actions, trading halts, shortsales restrictions
<b>Trading and Implementation</b>	Direct trading costs, required technology infrastructure	Probability distributions of trading volume, limit-order fill rates, and market-order impact	Statistical estimation of model parameters	Indirect trading costs (e.g., price impact, opportunity cost), technology failures and changes	Global flight-to-liquidity, regulatory changes (e.g., shortsales restrictions, ban on flash orders)
<b>Risk Management</b>		Probability theory of loss distributions	Statistical inference for parameters of loss distributions	Time-varying parameters, multiple regimes, and non-stationarities	Tail risk (e.g., terrorism, fraud, flu pandemic)
<b>Business Considerations</b>		Commoditized business services (e.g., market-making, liquidity provision, insurance)	Existing business practices, products, and clients	Near-term business trends, revenue and cost projections, market conditions, re-hypothecation and counterparty risk	Disruptive technologies, global economy-wide shocks, insolvency rumors, flight-to-liquidity risk
<b>Legal and Regulatory Issues</b>			Existing rules, regulations, and contract terms	Regulatory reform, new tax rules	Government intervention

Table 3: The uncertainty checklist as applied to a quantitative equity trading strategy.

## 8 Quants and the Current Financial Crisis

No discussion of the future of finance would be complete without considering the role that quantitative models may have played in creating the current financial crisis. There is, of course, little doubt that securitization, exotic mortgages, credit default swaps, credit ratings for these new instruments, and other financial innovations that depended on quantitative models were complicit. But so too were homeowners, mortgage brokers, bankers, investors, boards of directors, regulators, government sponsored enterprises, and politicians. There is plenty of blame to go around, and the process of apportioning responsibility has only just begun. But blaming quantitative models for the crisis seems particularly perverse, and akin to blaming arithmetic and the real number system for accounting fraud.

The typical scientist's reaction to failure is quite the opposite of vilification or capitulation. Faced with the complete failure of classical physics to explain the spectrum of electromagnetic radiation in thermal equilibrium, or the energies of electrons ejected from a metal by a beam of light, physicists in the opening decades of the twentieth century responded not by rejecting quantitative analysis, but by studying these failures intensely and redoubling their efforts to build better models. This process led to the remarkable insights of Planck and Einstein and, ultimately, to an entirely new model of Nature: quantum mechanics. In fact, in every science, knowledge advances through the painstaking process of successive approximation, typically with one generation of scholars testing, refining, or overturning the models of the previous generation. As Newton put it, "If I have seen further it is only by standing on the shoulders of giants". Samuelson, paraphrasing Planck's, was somewhat darker: "Progress in the sciences occurs funeral by funeral".

Among the multitude of advantages that physicists have over financial economists is one that is rarely noted: the practice of physics is largely left to physicists. When physics experiences a crisis, physicists are generally allowed to sort through the issues by themselves, without the distraction of outside interference. When a financial crisis occurs, it seems that everyone becomes a financial expert overnight, with surprisingly strong opinions on what caused the crisis and how to fix it. While financial economists may prefer to conduct more thorough analyses in the wake of market dislocation, the rush to judgment and action is virtually impossible to forestall as politicians, regulators, corporate executives, investors, and the media all react in predictably human fashion. Imagine how much more challenging it would have been to fix the Large Hadron Collider after its September 19, 2008 short circuit if, after its breakdown, Congress held hearings in which various constituents—including religious leaders, residents of neighboring towns, and unions involved in the accelerator's construction—were asked to testify about what went wrong and how best to deal with its



failure. Imagine further that after several months of such hearings, politicians—few of whom are physicists—start to draft legislation to change the way particle accelerators are to be built, managed, and staffed, and compensation limits are imposed on the most senior research scientists associated with the facility.

Although there are, of course, many differences between the financial system and a particle accelerator, the point of our analogy is to underscore the dangers of reacting too quickly to crisis in the case of highly complex systems. Today’s financial system is considerably more complex than ever before, requiring correspondingly greater concerted effort and expertise to overhaul. By comparison, the Large Hadron Collider—as complex as it is—is a *much* simpler system. Although it has been designed to probe new concepts beyond the current Standard Model of elementary particle physics, the accelerator, detectors, and computers that constitute the Collider are governed by physical laws that are either perfectly certain or fully reducibly uncertain.<sup>27</sup> Yet a collaboration of over 10,000 scientists was required to design and build it, and the September 19, 2008 short circuit took over a year to troubleshoot and repair, having come back on line on November 20, 2009.

To underscore the importance of the scientific method in analyzing the current financial crisis, in this section we present three commonly held beliefs about the crisis that, after more careful scrutiny, are demonstrably untrue or unwarranted. Our purpose is not to criticize those who hold such beliefs, but rather to remind ourselves that one of the drawbacks of human creativity is the remarkable ease with which we draw conclusions in the absence of facts. This tendency is especially common in emotionally charged situations, perhaps because the urgency of action is that much greater (see de Becker, 1997). Scientific inquiry is a potent, and perhaps the only, antidote.

## 8.1 Did the SEC Allow Too Much Leverage?

On August 8, 2008, the former director of the SEC’s Division of Market Regulation (now the “Division of Markets and Trading”), Lee Pickard, published an article in the *American Banker* with a bold claim: a rule change by the SEC in 2004 allowed broker-dealers to greatly increase their leverage, contributing to the financial crisis.<sup>28</sup> In particular, Mr. Pickard

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<sup>27</sup>When we claim that these laws are perfectly certain or have fully reducible uncertainty, we are referring to the forms of the laws and the statistical nature of the predictions they make. Quantum mechanics is intrinsically probabilistic in predicting the outcomes of individual experiments, but the form of the Schrödinger equation is not probabilistic, and repeated experiments lead to well-defined statistical outcomes. More formally, the functioning of superconducting magnets and calorimeters can be accurately represented as having a much smaller degree of partially-reducible uncertainty than statistical (fully-reducible) uncertainty, which is the essential point of this simple analogy.

<sup>28</sup>We thank Jacob Goldfield for bringing this example to our attention.



argued that before the rule change, “the broker-dealer was limited in the amount of debt it could incur, to about 12 times its net capital, though for various reason broker-dealers operated at significantly lower ratios... If, however, Bear Stearns and other large broker-dealers had been subject to the typical haircuts on their securities positions, an aggregate indebtedness restriction, and other provisions for determining required net capital under the traditional standards, they would not have been able to incur their high debt leverage without substantially increasing their capital base.” He was referring to a change in June 2004 to SEC Rule 15c3-1, the so-called “net capital rule” by which the SEC imposes net capital requirements and, thereby, limits the leverage employed by broker-dealers. This serious allegation was picked up by a number of newspapers, including the *New York Times* on October 3, 2008 (Labaton, 2008):

In loosening the capital rules, which are supposed to provide a buffer in turbulent times, the agency also decided to rely on the firms’ own computer models for determining the riskiness of investments, essentially outsourcing the job of monitoring risk to the banks themselves.

Over the following months and years, each of the firms would take advantage of the looser rules. At Bear Stearns, the leverage ratio—a measurement of how much the firm was borrowing compared to its total assets—rose sharply, to 33 to 1. In other words, for every dollar in equity, it had \$33 of debt. The ratios at the other firms also rose significantly.

These reports of sudden increases in leverage from 12-to-1 to 33-to-1 seemed to be the “smoking gun” that many had been searching for in their attempts to determine the causes of the Financial Crisis of 2007–2009. If true, it implied an easy fix according to Pickard (2008): “The SEC should reexamine its net capital rule and consider whether the traditional standards should be reapplied to all broker-dealers.”

While these “facts” seemed straightforward enough, it turns out that the 2004 SEC amendment to Rule 15c3-1 did nothing to change the leverage restrictions of these financial institutions. In a speech given by the SEC’s director of the Division of Markets and Trading on April 9, 2009 (Sirri, 2009), Dr. Erik Sirri stated clearly and unequivocally that “First, and most importantly, the Commission did not undo any leverage restrictions in 2004”.<sup>29</sup>

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<sup>29</sup>SEC Rule 15c3-1 is complex, and not simply a leverage test. The rule does contain a 15-to-1 leverage test with a 12-to-1 “early warning” obligation. However, this component of the rule only limits unsecured debt, and did not apply to large broker-dealers, who were subject to net capital requirements based on amounts owed to them by their customers, i.e., a customer-receivable or “aggregate debit item” test. This test requires a broker-dealer to maintain net capital equal to at least 2% of those receivables, which is how the five large investment banks had been able to achieve higher leverage ratios in the 1990’s than after the 2004 rule change (see Figure 11 below). Similarly, their broker-dealer subsidiaries (which were the entities subject to the net capital rule) had long achieved leverage ratios far in excess of 15-to-1. The historical leverage ratios of the investment banks were readily available in their financial reports, and the facts regarding the

He cites several documented and verifiable facts to support this surprising conclusion, and this correction was reiterated in a letter from Michael Macchiaroli, Associate Director of the Division of Markets and Trading to the General Accountability Office (GAO) on July 17, 2009, and reproduced in the GAO Report GAO-09-739 (2009, p. 117).

What about the stunning 33-to-1 leverage ratio reported by the press? According to the GAO (Report GAO-09-739, 2009, p. 40):

In our prior work on Long-Term Capital Management (a hedge fund), we analyzed the assets-to-equity ratios of four of the five broker-dealer holding companies that later became CSEs and found that three had ratios equal to or greater than 28-to-1 at fiscal year-end 1998, which was higher than their ratios at fiscal year-end 2006 before the crisis began (see fig. 6).

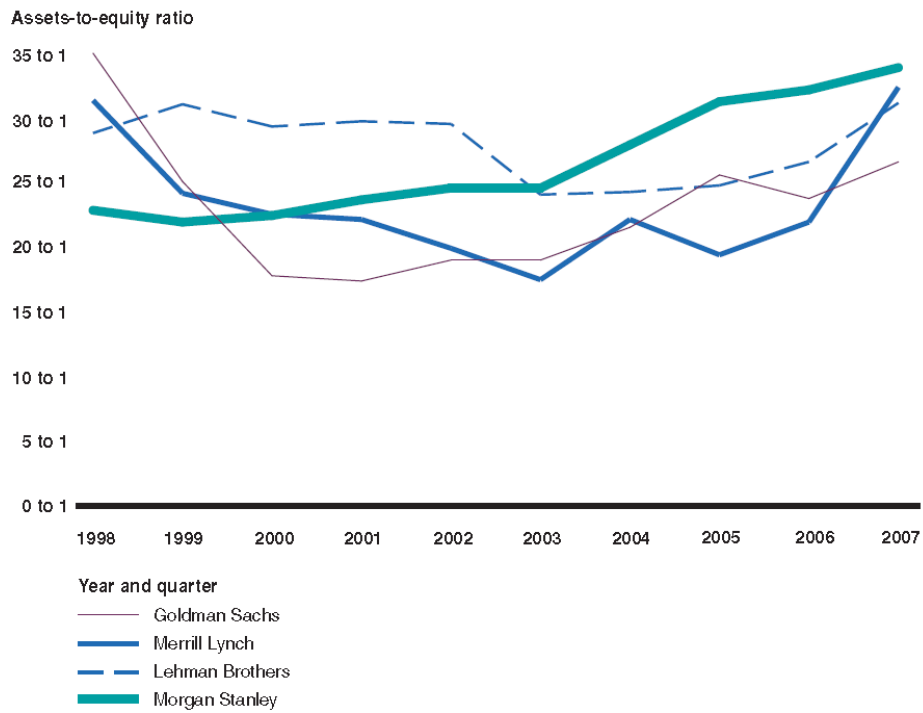
In footnote 68 of that report, the GAO observes that its 1999 report GAO/GGD-00-3 (1999) on Long-Term Capital Management "... did not present the assets-to-equity ratio for Bear Stearns, but its ratio also was above 28 to 1 in 1998". The GAO's graph of the historical leverage ratios for Goldman Sachs, Merrill Lynch, Lehman Brothers, and Morgan Stanley is reproduced in Figure 11. These leverage numbers were in the public domain and easily accessible through company annual reports and quarterly SEC filings.

Now it must be acknowledged that the arcane minutiae of SEC net capital rules may not be common knowledge, even among professional economists, accountants, and regulators. But two aspects of this particular incident are noteworthy: (1) the misunderstanding seems to have originated with the allegation by Mr. Pickard, who held the same position as Dr. Sirri at the SEC from 1973-1977 and was involved in writing the original version of Rule 15c3-1; and (2) the allegation was used by a number of prominent and highly accomplished economists, regulators, and policymakers to formulate policy recommendations without verifying the scientific merits of the claim that "changes in 2004 net capital rules caused investment banks to increase leverage".<sup>30</sup> If such sophisticated and informed individuals can be so

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true nature of the SEC net capital rule were also available in the public domain. We thank Bob Lockner for helping us with the intricacies of the SEC net capital rule.

<sup>30</sup>For example, former SEC chief economist Susan Woodward cited this "fact" in a presentation to the Allied Social Sciences Association Annual Meeting (the largest annual gathering of economists) on January 3, 2009 (Woodward, 2009). On December 5, 2008, Columbia University Law Professor John Coffee (2008) wrote in the *New York Law Journal* that after the rule change, "The result was predictable: all five of these major investment banks increased their debt-to-equity leverage ratios significantly in the period following their entry into the CSE program". In a January 2009 *Vanity Fair* article, Nobel-prize-winning economist Joseph Stiglitz (2009) listed five key "mistakes" that led to the financial crisis and "One was the decision in April 2004 by the Securities and Exchange Commission, at a meeting attended by virtually no one and largely overlooked at the time, to allow big investment banks to increase their debt-to-capital ratio (from 12:1 to 30:1, or higher) so that they could buy more mortgage-backed securities, inflating the housing bubble



Source: GAO analysis of the firms' annual report data.

Figure 11: Ratio to total assets to equity for four broker-dealer holding companies from 1998 to 2007. Source: U.S. Government Accountability Office Report GAO-09-739 (2009, Figure 6).

easily misled on a relatively simple and empirically verifiable issue, what does that imply about less-informed stakeholders in the financial system?

This unfortunate misunderstanding underscores the need for careful deliberation and analysis, particularly during periods of extreme distress when the sense of urgency may cause us to draw inferences and conclusions too quickly and inaccurately. We conjecture that the misunderstanding was generated by the apparent consistency between the extraordinary losses of Bear, Lehman, and Merrill and the misinterpretation of the 2004 SEC rule change—after all, it seems perfectly plausible that the apparent loosening of net capital rules in 2004 could have caused broker-dealers to increase their leverage. Even sophisticated individuals form mental models of reality that are not complete or entirely accurate, and when new information is encountered, our cognitive faculties are hardwired to question first those pieces that are at odds with our mental model. When information confirms our preconceptions, we usually do not ask why.

As of March 12, 2010, the *New York Times* has yet to print a correction of its original stories on the 2004 change to Rule 15c3–1, nor did the *Times* provide any coverage Dr. Sirri’s April 9, 2009 speech. Apparently, correcting mistaken views and factual errors is not always news. But it is good science.

## 8.2 If Formulas Could Kill

At the epicenter of the current financial crisis are bonds backed by large pools of geographically diversified residential mortgages and stratified into “tranches” by credit quality. Such securities are examples of “asset-backed securities” (ABSs) or, when the tranches of several ABSs are pooled, “collateralized debt obligations” (ABS-CDOs). ABS-CDOs have been used to securitize a wide variety of assets in addition to mortgages, such as credit card receivables, student loans and home equity loans. The mortgage-backed bonds reflected the aggregate risk/return characteristics of the cashflows of the underlying mortgages on which the bonds’ cashflows were based. By pooling mortgages from various parts of the country, issuers of ABS-CDOs were diversifying their homeowner default risk in precisely the same way that an equity index fund diversifies the idiosyncratic risks associated with the fortunes of each company. Of course, the diversification benefits of the pool depends critically on how correlated the component securities are to each other. Assuming that mortgages in a given pool are statistically independent leads to substantial diversification benefits, and greatly

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in the process”. And on January 24, 2009, Alan Blinder, a professor of economics at Princeton University and a former vice-chairman of the U.S. Federal Reserve, published a piece in the “Economic View” column of the *New York Times* titled “Six Errors on the Path to the Financial Crisis” in which this rule change was Error Number 2 on his list (Blinder, 2009).

simplifies the bond-pricing analytics, but may not accurately reflect reality.

Not surprisingly, aggregating the risks, expected returns, and correlations of a collection of mortgages to price a given ABS-CDO is accomplished through a mathematical formula. One of the most widely used formulas is the so-called “Gaussian copula” developed by Li (2000), which is a specific functional form that is both analytically tractable and sufficiently general to capture the joint statistical properties of a large number of correlated mortgages.<sup>31</sup> In a February 23, 2009 article in *Wired* by Salmon (2009) titled “Recipe for Disaster: The Formula That Killed Wall Street”, this contribution was described in the following manner:

One result of the collapse has been the end of financial economics as something to be celebrated rather than feared. And Li’s Gaussian copula formula will go down in history as instrumental in causing the unfathomable losses that brought the world financial system to its knees.

According to this account, the financial crisis was caused by a single mathematical expression.

However, in attempting to expose the specific shortcomings of this model, Salmon contradicts his story’s captivating title and premise, inadvertently providing a more nuanced explanation for the crisis:

The damage was foreseeable and, in fact, foreseen. In 1998, before Li had even invented his copula function, Paul Wilmott wrote that “the correlations between financial quantities are notoriously unstable.” Wilmott, a quantitative-finance consultant and lecturer, argued that no theory should be built on such unpredictable parameters. And he wasn’t alone. During the boom years, everybody could reel off reasons why the Gaussian copula function wasn’t perfect. Li’s approach made no allowance for unpredictability: It assumed that correlation was a constant rather than something mercurial. Investment banks would regularly phone Stanford’s Duffie and ask him to come in and talk to them about exactly what Li’s copula was. Every time, he would warn them that it was not suitable for use in risk management or valuation.

In hindsight, ignoring those warnings looks foolhardy. But at the time, it was easy. Banks dismissed them, partly because the managers empowered to apply the brakes didn’t understand the arguments between various arms of the quant universe. Besides, they were making too much money to stop.

Li’s copula function was used to price hundreds of billions of dollars’ worth of CDOs filled with mortgages. And because the copula function used CDS prices to calculate correlation, it was forced to confine itself to looking at the period of time when those credit default swaps had been in existence: less than a decade, a period when house prices soared. Naturally, default correlations were very low in those years. But when the mortgage boom ended abruptly and home values started falling across the country, correlations soared.

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<sup>31</sup>Despite the impact of Li’s (2000) formula on industry practice, the literature on credit models is considerably broader, hence the negative publicity surrounding Li’s contribution may be misplaced. See Duffie and Singleton (2003) and Caouette et al. (2008) for a more complete review of this literature.

Apparently, the combination of historically low correlation of residential mortgage defaults and the profitability of the CDO business caused managers to ignore the warnings issued by a number of quants. But Salmon (2009) makes an even bolder claim:

Bankers securitizing mortgages knew that their models were highly sensitive to house-price appreciation. If it ever turned negative on a national scale, a lot of bonds that had been rated triple-A, or risk-free, by copula-powered computer models would blow up. But no one was willing to stop the creation of CDOs, and the big investment banks happily kept on building more, drawing their correlation data from a period when real estate only went up.

If this claim is true, the natural follow-up question is why did no one allow for the possibility of a national-level housing market decline in their models?

One answer may be found in Figure 12, which contains a plot of the national U.S. residential nominal home price index constructed by Yale economist Robert J. Shiller from 1890 to 2009Q2.<sup>32</sup> This striking graph shows that the U.S. has not experienced a significant national housing-market downturn since 1933. And as long as housing prices do not decline, it is easy to see why the default correlations of a geographically diversified pool of mortgages will remain low. On the other hand, once housing prices do decline on a national level, it is also easy to see why default correlations might spike up abruptly. Contrary to Salmon's (2009) claim that mortgage default correlation estimates were biased by the shorter estimation window when CDS spreads were available and home prices were rising, Figure 12 suggests that using historical data from the previous seven decades would not have materially changed the outcome.

This chain of logic brings us to the main point of this example. If, over a 70-year period, the U.S. did not experience a significant national-level decline, how could the possibility of such a decline be factored into the analysis? From our perspective, there is a two-part answer.

First, it must be acknowledged that the pricing of ABS-CDOs involves a significant degree of Level-4 uncertainty, implying that assumptions of stationarity and fixed laws of motion do not apply. Therefore, no single formula will yield a complete description of the relevant risks and rewards of any security, no matter how sophisticated. Because of the presence of Level-4 uncertainty, the need for additional institutional and economic structure is considerably greater.

One example of such additional structure is provided by the post-crisis analysis of Khandani, Lo, and Merton (2009) in which they argue that the heightened default correlations

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<sup>32</sup>The vertical axis is scaled logarithmically so that a given percentage change in home prices corresponds to the same vertical increment regardless of what year it occurred and the level of the index at that time.

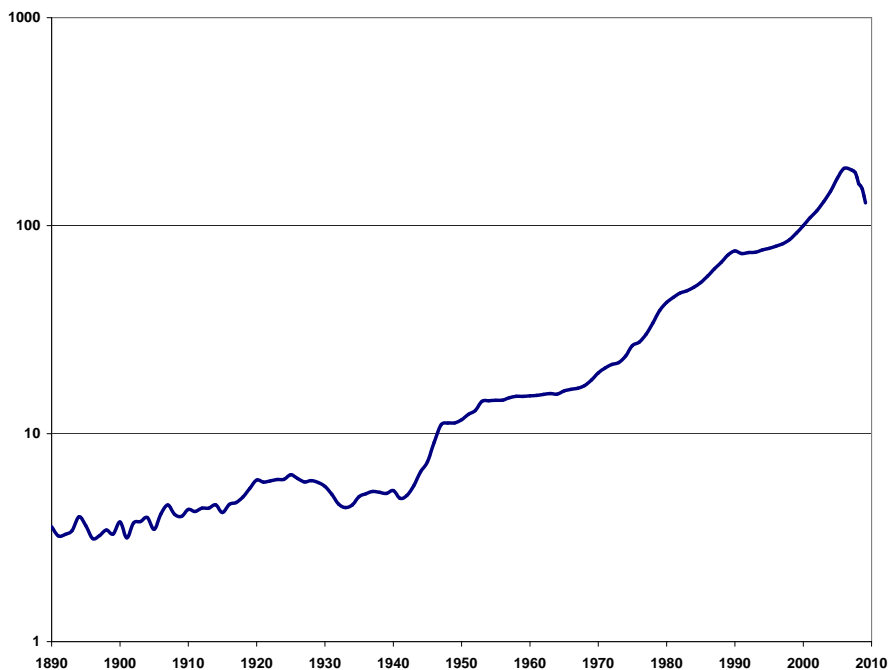


Figure 12: Nominal U.S. residential home price index from 1890 to 2009:Q1, on a logarithmic scale. Source: Robert J. Shiller.

during housing-market declines are due to the combination of low interest rates, rising home prices, and greater access to cheap refinancing opportunities that typically precede declines. Individually, each of the three conditions is benign, but when they occur simultaneously, as they did over the past decade, they impose an unintentional synchronization of homeowner leverage. This synchronization, coupled with the indivisibility of residential real estate that prevents homeowners from selling a fraction of their homes to reduce leverage when property values decline and homeowner equity deteriorates, conspire to create a “ratchet” effect in which homeowner leverage is maintained or increased during good times without the ability to decrease leverage during bad times. If refinancing-facilitated homeowner-equity extraction is sufficiently widespread—as it was during the years leading up to the peak of the U.S. residential real-estate market in June 2006—the inadvertent coordination of leverage during a market rise implies higher correlation of defaults during a market drop.

While this analysis also uses quantitative methods—in particular, numerical simulation and derivatives pricing models—the methods are simple in comparison to the institutional and economic structure on which the simulations are based. By incorporating richer and more accurate institutional features, the proportion of irreducible uncertainty in CDO pricing models can be reduced.

Second, the ultimate response to Level-4 uncertainty is human judgment, and Salmon’s



(2009) observation that senior management had neither the motives nor the required expertise to limit their risk exposures to the CDO market is, ultimately, a behavioral argument. During extended periods of prosperity, the individual and collective perception of risk declines, e.g., as losses become historically more remote, human perception of small probabilities of loss quickly converge to zero (Lichtenstein et al., 1982).<sup>33</sup> Although a national decline in home prices was certainly possible, historical experience suggested it was highly improbable. Therefore, it is not surprising that during the period from 1998 to 2006, senior managers at major financial institutions were not concerned with this risk—the last such event was too far back for anyone to care about, especially given how much financial markets had changed in the interim.

In the case of CDOs, another relevant aspect of human behavior is the possibility that a number of decisionmakers simply did not have the technical expertise to properly evaluate the risk/reward trade-offs of these securities. As Salmon (2009) puts it:

Bankers should have noted that very small changes in their underlying assumptions could result in very large changes in the correlation number. They also should have noticed that the results they were seeing were much less volatile than they should have been—which implied that the risk was being moved elsewhere. Where had the risk gone?

They didn't know, or didn't ask. One reason was that the outputs came from "black box" computer models and were hard to subject to a commonsense smell test. Another was that the quants, who should have been more aware of the copula's weaknesses, weren't the ones making the big asset-allocation decisions. Their managers, who made the actual calls, lacked the math skills to understand what the models were doing or how they worked. They could, however, understand something as simple as a single correlation number. That was the problem.

Quantitative illiteracy is not acceptable in science. Although financial economics may never answer to the same standards as physics, nevertheless, managers in positions of responsibility should no longer be allowed to take perverse anti-intellectual pride in being quantitatively illiterate in the models and methods on which their businesses depend.

### 8.3 Too Many Quants, or Not Enough?

A third example of misguided reaction to the financial crisis is the impression that there were too many quants on Wall Street and their concerted efforts created the crisis. We find this response to the crisis particularly troubling because it is inconsistent with the

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<sup>33</sup>If readers have any doubt about this human tendency, they should ask long-time residents of California how they live with the constant threat of earthquakes.



equally popular view that the crisis was caused by securities too complex for anyone to fully understand. If anything, the preliminary evidence accumulated from the aftermath of the crisis suggests that we need more financial-engineering expertise and training at the most senior levels of management at banks, broker-dealers, insurance companies, mutual funds, pension funds, and regulatory agencies, not less. The excess demand for financial expertise has only increased over time as markets have become more complex, competitive, and globally integrated.

A skeptic might respond by claiming that it may be preferable to return to a simpler time with fewer complex securities, alleviating the need for quantitatively sophisticated stakeholders. However, this view misses the fact that risk pooling and risk transfer have been essential components of capital markets for centuries, with innovations that have arisen in response to the legitimate needs of individuals and corporations.<sup>34</sup> As with most new technologies, problems arise when their limits are not fully understood or when they are purposely abused to attain short-run benefits at the expense of safety or stability. But when properly modeled and managed, securitization can play an extremely positive role in the risk-pooling and risk-transfer functions that capital markets are uniquely designed to accomplish. This potential has no doubt contributed to the growing popularity of quants in the financial industry.

Indirect evidence for an excess demand of finance expertise is documented in Philippon and Reshef's (2007) comparison of the annual incomes of U.S. engineers and finance-trained graduates from 1967 to 2005. The comparison between finance and engineering students is a useful one because both are technical disciplines, and over the past 20 years, engineers have been making significant inroads into the finance labor market. Figure 13 shows that until the mid-1980's, college graduates in engineering enjoyed higher incomes than college graduates in finance, and post-graduates in engineering had about the same compensation as post-graduates in finance. However, since the 1980's, finance-trained college graduates have caught up to their engineering counterparts, and surpassed them in 2000 and every year thereafter. But the more impressive comparison is for post-graduates-since 1982, the annual income of finance post-graduates has exceeded that of engineers every year, and the gap has widened steadily over these two decades. This pattern suggests that the demand for financial expertise has grown considerably during this time.

Table 4, which reports the number of MIT engineering and finance degrees awarded from 1999 to 2007, provides another perspective on the dearth of financial expertise. In 2007, MIT's School of Engineering graduated 337 Ph.D.'s in engineering; in contrast, the MIT Sloan School of Management produced only 4 finance Ph.D.s. These figures are not

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<sup>34</sup>Merton's (1992, 1993) functional approach to financial innovation suggests that new products and services may vary considerably over time, but the functions they serve are more stable. See also Crane et al. (1995).

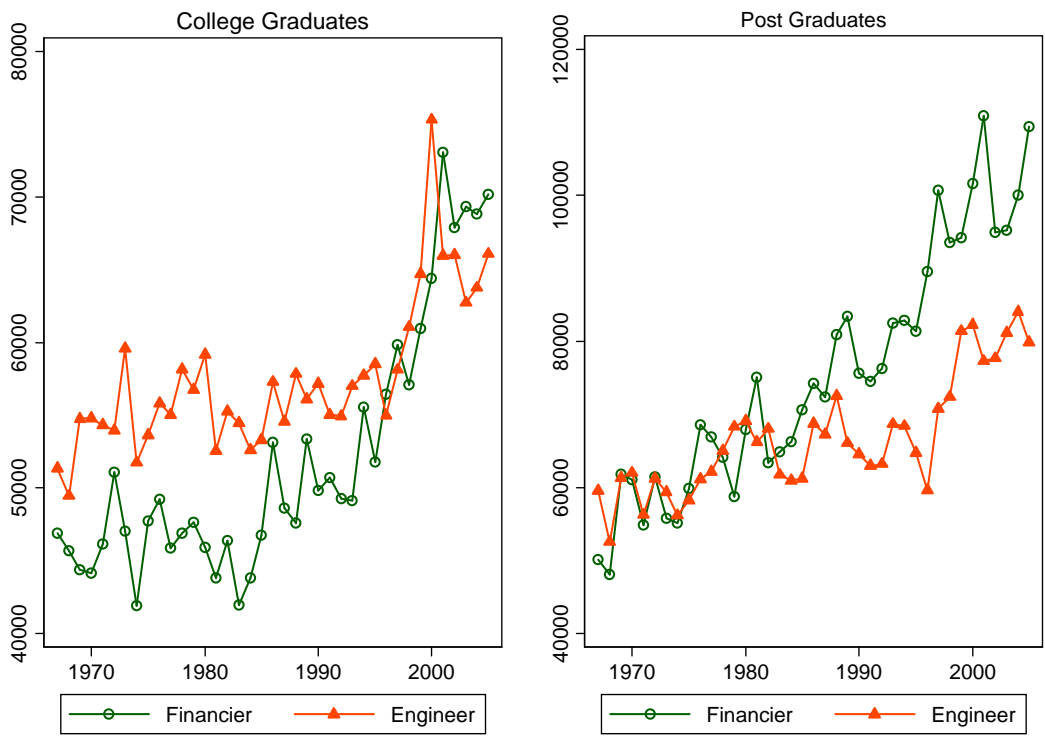


Figure 13: Comparison of finance and engineering graduates from 1967 to 2005, in 2001 U.S. dollars. Source: Philippon and Reshef (2007).

unique to MIT, but are, in fact, typical among the top engineering and business schools. Now, it can be argued that the main focus of the Sloan School is its M.B.A. program, which graduates approximately 300 students each year, but most M.B.A. students at Sloan and other top business schools do not have the technical background to implement models such as Li’s (2009) Gaussian copula formula for ABSs and CDOs, nor does the standard M.B.A. curriculum include courses that cover such models in any depth. Such material—which requires advanced training in arcane subjects such as stochastic processes, stochastic calculus, and partial differential equations—is usually geared towards Ph.D. students.<sup>35</sup>

Year	MIT School of Engineering			MIT Sloan
	Bachelor's	Master's and MEng	PhD and ScD	Finance PhD
2007	578	710	337	4
2006	578	735	298	2
2005	593	798	286	1
2004	645	876	217	5
2003	679	817	210	7
2002	667	803	239	3
2001	660	860	248	1
2000	715	739	237	2
1999	684	811	208	4

Table 4: Number of MIT degrees awarded in engineering and in finance from 1999 to 2007. Source: MIT Annual Report of the President, 1999 to 2007.

The disparity between the number of Ph.D.s awarded in engineering and finance in Table 4 raises the question of why such a difference exists. One possible explanation may be the sources of funding. MIT engineering Ph.D. students are funded largely through government grants (DARPA, DOE, NIH, and NSF), whereas MIT Sloan Ph.D. students are funded exclusively through internal MIT funds. Given the importance of finance expertise, one proposal for regulatory reform is to provide comparable levels of government funding to support finance Ph.D. students. Alternatively, funding for finance Ph.D. students might be raised by imposing a small surcharge on certain types of derivatives contracts, e.g., those that are particularly complex or illiquid and, therefore, contribute to systemic risk. This surcharge may be viewed as a means of correcting some of the externalities associated with the impact of derivatives on systemic risk. A minuscule surcharge on, say, credit default swaps, could support enough finance Ph.D. students at every major university to have a noticeable and permanent impact on the level of financial expertise in both industry and

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<sup>35</sup>However, due to the growth of the derivatives business over the past decade, a number of universities have begun to offer specialized Master’s-level degree programs in financial engineering and mathematical finance to meet the growing demand for more technically advanced students trained in finance. Whether or not such students are sufficiently prepared to fill the current knowledge gap in financial technology remains to be seen.

government.

## 9 Conclusion

Financial economics may be a long way from physics, but this state of affairs is cause for neither castigation nor celebration—it is merely a reflection of the dynamic, non-stationary, and ultimately human aspect of economic interactions. In sharp contrast to the other social sciences, economics does exhibit an enviable degree of consistency across its many models and methods. In the same way that scientific principles are compact distillations of much more complex phenomena, economic principles such as supply-and-demand, general equilibrium, the no-arbitrage condition, risk/reward trade-offs, and Black-Scholes/Merton derivatives-pricing theory capture an expansive range of economic phenomena. However, any virtue can become a vice when taken to an extreme, particularly when that extreme ignores the limitations imposed by uncertainty.

In this respect, the state of economics may be closer to disciplines such as evolutionary biology, ecology, and meteorology. In fact, weather forecasting has become noticeably better in our lifetimes, no doubt because of accumulated research breakthroughs and technological advances in that field. However, like economics, it is impossible to conduct experiments in meteorology, even though the underlying physical principles are well understood. Despite the fact that long periods of “typical” weather allow us to build simple and effective statistical models for predicting rainfall, on occasion a hurricane strikes and we are reminded of the limits of those models (see Masters, 1995). And for the truly global challenges such as climate change, the degree of subjectivity and uncertainty gives rise to spirited debate, disagreement, and what appears to be chaos to uninformed outsiders. Should we respond by discarding all forecasting models for predicting rainfall, or should we simply ignore the existence of hurricanes because they fall outside our models?

Perhaps a more productive response is to delineate the domain of validity of each model, to incorporate this information into every aspect of our activities, to attempt to limit our exposure to the catastrophic events that we know will happen but which we cannot predict, and to continue developing better models through data collection, analysis, testing, and reflection, i.e., becoming smarter.

We all use models of one sort or another—with wildly varying degrees of reliability—in making sense of the past and attempting to look into the future. Mental models are virtually synonymous with human intelligence (see, for example, Gigerenzer, 2000, and Hawkins, 2004), and some models are explicit, systematic, and quantitative while others are more

intuitive or discretionary, based on ineffable expertise. Qualitative information is difficult to incorporate in the former, while testability, repeatability, and scalability are challenges to the latter. And both approaches are, in the financial industry, routinely and intentionally hidden behind a proprietary veil, further hampering a direct assessment of their respective costs and benefits.

Faith in any person or organization claiming to have a deep and intuitive grasp of market opportunities and risks is no better or worse than putting the same faith and money behind a mysterious black-box strategy. What matters in each case is the transparency of the process, an opportunity to assess the plausibility and limitations of the ideas on which a strategy is based, clarity about expectations for risks as well as returns, an alignment of incentives between the investment manager and the investor, and proper accountability for successes and failures. Part of the recent distrust of quantitative models is that they are inanimate, and therefore inherently difficult to trust. Even when we manage to develop a certain level of comfort with a quantitative strategy, ongoing due diligence is needed to assess the consistency, integrity, and incentives of the human beings responsible for implementing the strategy. It is important to distinguish our emotional needs from the formal process of assessing the models themselves, for which a degree of quantitative literacy will always be required.

So what does this imply for the future of finance? Our hope is that the future will be even brighter because of the vulnerabilities that the recent crisis has revealed. By acknowledging that financial challenges cannot always be resolved with more sophisticated mathematics, and incorporating fear and greed into models and risk-management protocols explicitly rather than assuming them away, we believe that the financial models of the future will be considerably more successful, even if less mathematically elegant and tractable. Just as biologists and meteorologists have broken new ground thanks to computational advances that have spurred new theories, we anticipate the next financial renaissance to lie at the intersection of theory, practice, and computation.

While physicists have historically been inspired by mathematical elegance and driven by pure logic, they also rely on the ongoing dialogue between theoretical ideals and experimental evidence. This rational, incremental, and sometimes painstaking debate between idealized quantitative models and harsh empirical realities has led to many breakthroughs in physics, and provides a clear guide for the role and limitations of quantitative methods in financial markets, and the future of finance.

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