Optimal Design of Internal Capital Markets

Andrey Malenko

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Abstract

This paper studies optimal design of a capital allocation process in a dynamic setting in a firm in which the division manager with empire-building preferences privately observes the arrival and properties of investment projects, and headquarters are able to audit each project at a cost. Under certain conditions, the optimal system takes the form of a budgeting mechanism with threshold division of authority. Specifically, headquarters: (i) allocate a spending account to the manager at the initial date and accumulate it over time; (ii) set a threshold on the size of individual projects, such that all projects below the threshold are delegated to the manager and financed out of her spending account, while all projects above the threshold are audited and financed fully by headquarters. Extensions of the model generate a set of implications for the design of capital allocation processes in organizations.
Internal capital allocation is fundamental to any organization. A fundamental issue in it is that investment projects are often conceived on lower levels of the organization, whose managers are likely to have different incentives from upper-level managers, let alone shareholders.\(^1\) In particular, an important misalignment is that division managers want to spend too much. When investment projects are relatively short-term and have observable and contractible performance signals, division managers can be provided incentives to invest efficiently with performance-based compensation. However, in many cases this solution is not feasible: for example, if projects have externalities between divisions such as marketing campaigns, or if projects have long-term horizons such as R&D, or if organizations have non-monetary goals. In such settings, to ensure that investment decisions are made efficiently, headquarters need to rely on the internal capital allocation process - a collection of rules specifying how managers at various levels share information about potential investments and how investment decisions are made. The question of fundamental importance in corporate finance and organizational economics is how this process should be designed.

While there is a large literature on internal capital allocation, with rare exceptions existing theories focus on one-shot settings\(^2\). However, the internal capital allocation problem in real-world organizations is dynamic by its nature: headquarters and division managers interact repeatedly and over multiple investment projects. The dynamic nature makes the internal capital allocation problem more flexible, because headquarters have an additional tool of incentive provision - they can provide incentives dynamically by making future decisions contingent on what happened in the past; for example, by punishing the division manager in the future for spending a lot today. This leads to a number of important questions. How does the optimal capital allocation process look like? In particular, can observed organizational practices be rationalized? What are the implications for investment? What is the role of commitment?

To study these questions, I extend the static capital allocation problem of Harris and Raviv (1996) to a dynamic environment, and study the general mechanism design problem. I consider a continuous-time principal-agent setting in which a risk-neutral principal (headquarters) employs a risk-neutral agent (the division manager). The firm has access to a sequence of heterogeneous investment opportunities that arrive stochastically over time and whose arrival and type, driving the size of investment, is observed only by the division man-

\(^1\)According to survey evidence in Petty et al. (1975), in a typical Fortune 500 firm less than 20% of investment projects are originated at the central office level. Akalu (2003) provides similar evidence for European firms based on more recent data.

\(^2\)See the end of this section for a discussion of related literature.
The agency problem stems from the fact that headquarters and the division manager have different preferences with respect to investment. In particular, headquarters operate in the interest of shareholders; the division manager enjoys utility from monetary compensation and gets an “empire-building” private benefit from each dollar of investment. Thus, from the position of headquarters there is a concern that if the division manager wants to invest a lot in a project, it can be a result of private benefits rather than project fundamentals. At any time headquarters can use two tools to incentivize the division manager. First, they can punish the division manager for high spending today by being tougher in the future. Second, they can audit the division manager at a cost and find out the quality of the current project with certainty. The goal is to find a mechanism that maximizes firm value subject to delivering the division manager the expected utility of at least a given amount.

I show that under certain conditions a simple mechanism, called a budgeting mechanism with threshold division of authority, is optimal in this setting. In this mechanism, headquarters allocate a dynamic spending account to the division manager at the initial date and accumulate it over time at a certain rate. At any time, the division manager is allowed to finance investment projects out of her account at her own discretion. In addition, headquarters specify a threshold on the size of individual projects, such that at any time the division manager has an option to pass the project to headquarters claiming that it deserves investment above the threshold. If the division manager passes the project, it gets audited and if audit confirms that the project indeed deserves an investment above the threshold, it gets financed fully by headquarters. Otherwise, the division manager is punished. In equilibrium, the division manager passes a project to headquarters if and only if it indeed deserves investment above the threshold. Thus, the mechanism completely separates decision making between the parties: all small investment decisions are made at the division level and are financed out of the division manager’s spending account; by contrast, all large projects are passed to headquarters and are financed fully by headquarters.

The intuition for optimality of this arrangement is as follows. To provide incentives not to overspend, headquarters must either audit the project or punish the division manager in the future for high spending today. Under certain conditions, the second tool can be implemented via a spending account. Intuitively, when the division manager draws on the spending account to invest in a project, the account balance goes down, which reduces the division manager’s ability to invest in the future. As a result, the spending account punishes the division manager in the future for spending today and aligns incentives of the division manager and headquarters.
However, the incentive role of a spending account comes at a cost. Higher investment out of the spending account decreases the remaining allocation for future projects and thus constrains future investment activity. If the size of the current project is large enough, the increase in financing constraints becomes costly enough, so that headquarters find it optimal to switch to the other tool of incentive provision, i.e., audit the project. Because additional fluctuations in the spending account balance are costly for headquarters, it is optimal to finance all audited projects without changing the spending account of the division manager. This can be implemented by giving the division manager an option to pass the project to headquarters claiming that it deserves an investment above the threshold and financing the project fully upon successful audit. Because the division manager obtains free financing from headquarters, she has incentives to pass the project to headquarters if the optimal investment indeed exceeds the threshold. However, because of expected punishment, she has no incentives to pass the project to headquarters if the optimal investment is below the threshold.

This mechanism captures two features of spending activity that are common in real-world organizations: a dynamic spending account, when the upper management provides an account to a subordinate and gives considerable discretion to allocate this account over time between various spending needs, and threshold division of authority, when a lower-level manager passes a project to the upper level of the organizational hierarchy, if its size exceeds a formally specified project size threshold. These features lead to path-dependent distortions of investment that are different and sometimes the opposite of distortions from prior literature on agency and investment dynamics (e.g., DeMarzo and Fishman (2007a) and DeMarzo et al. (forthcoming)). In particular, with uncorrelated investment opportunities, the model implies negative and zero serial correlation of investment at the division and headquarters levels, respectively, in contrast to positive serial correlation in the prior literature. In addition, investment decisions made at the division level are more constrained than investment decisions made at headquarters level, consistent with evidence in Ross (1986) that divisions use higher discount rates than corporate investment committees.

Optimality of the budgeting mechanism with threshold division of authority hinges on two important assumptions: (i) private benefits that the division manager gets from spending

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3 Examples of spending accounts include R&D accounts of research groups, investment budgets in corporations, budgets of loan officers, and research accounts in academic institutions. Examples of project size thresholds are provided in the study of Ross (1986). In his sample, a typical plant manager makes decisions on projects whose size is below $100,000, but passes larger projects to one of the upper levels of the organizational hierarchy.
an additional dollar are the same across all projects; (ii) performance-based compensation of the division manager is not feasible. Thus, the model suggests that the budgeting mechanism with threshold division of authority is not “one solution for all.” Instead, it is optimal only in settings in which managers deal with relatively homogeneous projects and the use of performance-based compensation is limited, e.g., because of externalities, long horizons, or non-monetary goals. I analyze these restrictions in more detail in extensions. One extension deals with a setting with two categories of investment projects with different levels of private benefits. I argue that this feature may give rise to the use of multiple spending accounts, one for each category of investment projects. This might explain why organizations often use separate budgets for different kinds of activities: for example, a separate budget for R&D and a separate budget for capital investment. I also argue that although the budgeting mechanism with threshold division of authority is not optimal in presence of performance-based compensation, it is still a natural mechanism in settings when headquarters are uncertain over preferences of the division manager. In additional extensions, I argue that limited commitment can lead to a lower spending account balance but faster replenishment and that a richer set of audit technologies can lead to multiple project size thresholds and co-financing of projects by different layers of the organizational hierarchy. Taken together, these extensions provide a set of testable implications about the design of capital allocation processes in organizations.

The rest of the paper is organized as follows. The remainder of this section discusses related literature. Section 1 describes setup of the model and formalizes the problem. Section 2 applies the revelation principle and solves for the optimal direct mechanism. Section 3 establishes optimality of the budgeting mechanism with threshold division of authority. Section 4 relates this result to the observed organizational practices and discusses implications for investment dynamics. Section 5 discusses the important assumptions behind the result and relaxes them one by one. Finally, Section 6 concludes.

Related Literature

The paper merges literature on internal capital allocation in corporate finance and organizational economics with literature on optimal dynamic contracting and mechanism design. With rare exceptions, literature on internal capital allocation approaches the problem as a one-shot interaction of headquarters with the division manager. This literature was started by [Harris et al.] (1982) and [Antle and Eppen] (1985). [Harris and Raviv] (1996, 1998) are
perhaps the most closely related papers as they consider a similar agency setting. [Harris and Raviv (1996)] study a one-project one-shot setting and show that the optimal procedure takes the form of an initial spending allocation that the division manager can increase under the threat of getting audited by headquarters. [Harris and Raviv (1998)] extend their earlier paper to the case of two projects and show that the same solution applies, with the difference that the initial spending limit is allocated for both projects. Unlike in a one-shot setting, where audit is the only way to provide incentives not to overspend, in a repeated setting, analyzed in this paper, headquarters can also provide incentives dynamically by punishing (rewarding) the manager for high (low) past spending. This has several implications. First, one could conjecture that a dynamic extension of [Harris and Raviv (1996)] leads to a sequence of short-term initial allocations, where the size of a subsequent allocation depends on all previous spendings and audit results in a complicated way. I show that under certain conditions this sequence is implemented using a simple long-term spending account that accumulates over time. Interestingly, optimality of the long-term spending account requires more severe constraints than those in [Harris and Raviv (1996, 1998)] - the direct extension of their setting will not lead its optimality. Identifying the conditions for optimality is important for understanding optimal design of capital allocation in real-world organizations. Second, the optimal mechanism in this paper implies full separation of financing, whereby headquarters finance large projects separately from the division manager’s spending account - a result not present in the prior literature. Finally, the model features different implications for distortions in investment. Section 6 of [Harris and Raviv (1998)] and [Roper and Ruckes (2011)] consider two-period settings based on [Harris and Raviv (1996)]. These models feature no audit and costless audit, respectively, so there is no trade-off between audit and dynamic punishment.

[Holmstrom and Ricart i Costa (1986), Bernardo et al. (2001), Bernardo et al. (2004), and Garcia (2008)] study the interplay between capital allocation and performance-based compensation - an important aspect that is missing in my model. Another related strand of literature studies delegation of a decision to an informed but biased expert. Optimal delegation sometimes takes the form of a threshold. To focus deeper on issues of delegation, this literature restricts the set of admissible mechanisms. In contrast, a project size threshold is part of a globally optimal mechanism in my paper. [Frankel (2011)] obtains a quota (or

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4See Section 3 for this discussion.  
budget) contract as a max-min optimal mechanism. His setting features uncertainty over the agent’s preferences, but does not include audit and monetary transfers.

The paper also builds on optimal dynamic contracting literature that uses recursive techniques to characterize the optimal contract. Within this literature, the paper is most closely related to models with risk-neutral parties based on repeated hidden information. The memory device role of the division manager’s spending account is similar to that of a credit line in DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007b) or cash reserves in Biais et al. (2007). My model contributes to this literature in several ways. First, I study a different agency setting - organization of internal capital allocation as opposed to the problem of how to split cash flows from the firm. Second, my model features investment decisions. Third, my model extends this literature by incorporating the costly state verification (CSV) framework of Townsend (1979). Related, Piskorski and Westerfield (2011) introduce non-CSV monitoring into this literature. Finally, path-dependent investment decisions make my paper related to the literature on agency and investment dynamics. This literature usually assumes that the investment opportunity set is publicly known, and investment is used to alleviate the agency problem. By contrast, in my paper, the investment opportunity set is privately observed by the agent, so investment is the source of the agency problem. This leads to very different (sometimes opposite) implications for investment dynamics, discussed in Section 4.

1 The Model

The model studies an organization consisting of risk-neutral headquarters (the principal) and a single risk-neutral division manager (the agent). The organization is characterized by a sequence of spending opportunities that arrive randomly over time. I refer to these spending opportunities as “investment projects,” but they can be any spending activities with the potential to generate monetary or non-monetary value to headquarters. Since the focus is

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on internal capital markets, I assume that headquarters are the only source of capital.\footnote{The paper does not address the question why division does not operate as a stand-alone entity. For studies that examine this issue, see Gertner et al. (1994), Stein (1997), Rajan et al. (2000), Scharfstein and Stein (2000), and others. Stein (2003) surveys this literature.}

Time is continuous, indexed by \( t \geq 0 \), and the horizon is infinite. The discount rates of headquarters and the division manager are \( r > 0 \) and \( \rho > r \), respectively. This assumption means that the division manager is more impatient than headquarters and rules out postponing payments to the former forever.\footnote{Case \( \rho = r \) can be studied in a finite-horizon version of the model with mostly similar results.} Over each infinitesimal period of time \([t, t + dt]\), the firm gets a project with probability \( \lambda dt \). Each project has quality \( \theta \), which is an i.i.d. draw from a distribution with c.d.f. \( F(\theta) \) and positive density \( f(\theta) \) defined over \( \Theta = [\theta, \bar{\theta}] \), \( \bar{\theta} > \theta > 0 \). Formally, the arrival of projects is an independently marked homogeneous point process \((T_n, \theta_n))_{n \geq 1}\), where \( T_n \) and \( \theta_n \) denote the arrival time and the quality of the \( n^{th} \) investment project. Each project is a take-it-or-leave-it opportunity that generates the net value of \( V(k, \theta) - k \) to headquarters, where \( k \geq 0 \) is the amount of capital spent on the project. Function \( V(k, \theta) \) is assumed to satisfy the following restrictions:

**Assumption 1.** \( V(k, \theta) \) has the following properties:

(a) \( V(0, \theta) = 0 \);

(b) \( V_{kk}(k, \theta) < 0 \), \( \lim_{k \to 0} V_k(k, \theta) = \infty \), and \( \lim_{k \to \infty} V_k(k, \theta) = 0 \);

(c) For any \( k > 0 \) and \( \theta_1, \theta_2 \in \Theta \) such that \( \theta_1 > \theta_2 \), \( V_k(k, \theta_1) > V_k(k, \theta_2) \).

These restrictions are natural. Part (a) of Assumption 1 means that the project generates zero value if there is no investment. Part (b) means that projects exhibit decreasing marginal returns, ranging from infinity for the first dollar invested to zero for the infinite dollar invested. Finally, part (c) means that for the same investment the marginal return is higher if the quality of the project is higher. Taken together, these restrictions will ensure that investment in each project is positive, finite, and other things equal increasing in the quality of the project. For convenience, I define \( V(k, 0) = 0 \) to be the value when no project is available.

Let \( (dX_t)_{t \geq 0} \) denote the stochastic process describing evolution of investment opportunities. Specifically, \( dX_t = 0 \) if no project arrives at time \( t \) and \( dX_t = \theta \) if a project of quality \( \theta \) arrives at time \( t \). The division manager has informational advantage over headquarters in that the arrival of projects and their qualities (i.e., \( dX_t \)) are privately observed by the division manager. Headquarters can learn about projects from two sources. First, they can rely on reports of the division manager. Second, at any time \( t \) headquarters can indepen-
dentify (audit) the division manager and learn $dX_t$ with certainty. The fact that audit at time $t$ reveals $dX_t$ allows to interpret audit as investigation of prospects of a single investment project. Following classical models of costly state verification (Townsend (1979), Gale and Hellwig (1985)), I assume that when headquarters audit the division manager, they incur a cost $c > 0$, such as the time and effort necessary to find out the true prospects of the project.

Let $(dA_t)_{t \geq 0}$ be the stochastic process describing audit decisions of headquarters: $dA_t = 1$ if headquarters audits at time $t$ and $dA_t = 0$ otherwise.

In addition to investment and audit decisions, I allow for monetary compensation of the division manager by headquarters. Then, the utility of headquarters from audit decisions $(dA_t)_{t \geq 0}$ and non-negative streams of investment $(dK_t)_{t \geq 0}$ and monetary compensation of the division manager $(dC_t)_{t \geq 0}$ is

\[ 
\int_0^\infty e^{-rt} \left( V(dK_t, dX_t) dN_t - dK_t - dC_t - cdA_t \right),
\]

where $dN_t = 1$ if $t = T_n$ and $dN_t = 0$ otherwise. When there is no conflict of interest between the parties, the problem is trivial: headquarters can simply ask the division manager to invest the optimal amount when a project arrives. It is therefore worthwhile to focus on the case in which the division manager and headquarters have conflicting preferences with respect to spending decisions. Following prior research on internal resource allocation, I assume that the division manager derives utility both from monetary compensation and from spending capital on projects. Specifically, the utility of the division manager from streams $(dK_t)_{t \geq 0}$ and $(dC_t)_{t \geq 0}$ is

\[ 
\int_0^\infty e^{-\rho t} \left( \gamma dK_t + dC_t \right),
\]

where $\gamma \in (0, 1)$ captures the relative importance of “empire-building” to the division manager. This utility is a dynamic extension of the division-manager’s utility in Berkovitch and Israel (2004). The preference for higher spending may reflect perquisite consumption associated with running larger projects as well as an intrinsic preference for empire-building.

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11 In practice, a subordinate is often able to hide a project from her supervisor, in case she wants (see Berkovitch and Israel (2004) for a discussion). While this ability is not captured in the model, it would not affect my results, because under the optimal mechanism in the model the manager does not want to hide projects.

12 The assumption that utility from spending accrues at the time of spending is without loss of generality. Equivalently, $\gamma dK_t$ can denote the present value of all future private benefits from spending $dK_t$ at time $t$. For example, if spending $dK_t$ gives the manager a flow of private benefits of $\tilde{\gamma} dK_t$, $s \geq t$, then the time-$t$ present value of private benefits is $(\tilde{\gamma}/\rho) dK_t$, which is equivalent to \( (\tilde{\gamma}/\rho) dK_t \) with $\gamma = \tilde{\gamma}/\rho$.  

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Because my goal is to examine optimal capital allocation, for the most of the paper I assume that the parties are able to commit to any long-term mechanism. Renegotiation-proofness is studied in Section 5.3. Headquarters have all bargaining power subject to delivering the division manager the time-0 utility of at least $R$. Varying $R$ allows to see how the solution is affected by the division of bargaining power between the parties.

1.1 Formulation of the Mechanism Design Problem

To consider the widest possible set of mechanisms, I start with a general communication game with arbitrary message spaces and complete history-dependence. The sequence of events over any infinitesimal time interval $[t, t + dt]$ is depicted on Figure 1. At the beginning of each period the division manager learns $dX_t$: whether the project arrives and if it does, its quality. Then, she sends a message $m_t$ from message space $M_t$. Given $m_t$ and the history of prior messages and audits, the mechanism reacts by prescribing headquarters to audit the division manager or not. Finally, given $m_t$, the audit result (if there was audit), and the prior messages and audits, the mechanism prescribes investment $dK_t$ and compensation $dC_t$.

By the revelation principle, any outcome that can be implemented with a general mechanism can also be implemented with a truth-telling direct mechanism. Thus, I can restrict attention to truth-telling direct mechanisms. In other words, it is sufficient to focus only on mechanisms in which at any time $t$ the division manager sends a report $d\tilde{X}_t \in \{0\} \cup \Theta$, i.e., whether the project is available and, if it is available, what its quality is, and in which the division manager finds it optimal to always send truthful reports $d\tilde{X}_t = dX_t$. Given this, my analysis proceeds in the following way. In Section 2, I optimize over the set of truth-telling direct mechanisms. Then, in Section 3, I show that the budgeting mechanism with threshold 

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13Myerson (1986) provides a general revelation principle for dynamic communication games.
division of authority implements the same audit, investment, and compensation policies as the optimal truth-telling direct mechanism. This allows me to conclude that the budgeting mechanism with threshold division of authority is optimal.

The problem of optimal design of a truth-telling direct mechanism is formalized in the following way. The reporting strategy $\hat{X} = \{d\hat{X}_t \in \{0\} \cup \Theta \}_{t \geq 0}$ is an $\mathcal{F}$-adapted stochastic process, where $\mathcal{F} = \{\mathcal{F}_t\}_{t \geq 0}$ is the filtration generated by $((T_n, \theta_n))_{n \geq 1}$. The direct mechanism $\Gamma$ is described by a triple $(A, K, C)$ of stochastic processes such that the audit process $A = \{dA_t \in \{0, 1\}\}_{t \geq 0}$ is measurable with respect to $\{d\hat{X}_s, s \leq t, dX_s, s < t : dA_s = 1\}_{t \geq 0}$, and the investment and compensation processes, $K = \{dK_t \geq 0\}_{t \geq 0}$ and $C = \{dC_t \geq 0\}_{t \geq 0}$, are measurable with respect to $\{d\hat{X}_s, s \leq t, dX_s, s \leq t : dA_s = 1\}_{t \geq 0}$. Given mechanism $\Gamma$ and reporting strategy $\hat{X}$, the expected discounted utilities of the division manager and headquarters are

$$E^{\hat{X}} \left[ \int_0^{\infty} e^{-\rho t} (\gamma dK_t + dC_t) \right],$$

(3)

$$E^{\hat{X}} \left[ \int_0^{\infty} e^{-rt} (V (dK_t, dX_t) dN_t - dK_t - dC_t - cdA_t) \right].$$

(4)

The reporting strategy $\hat{X}$ of the division manager is incentive compatible if and only if it maximizes her expected discounted utility (3) given mechanism $\Gamma$. A direct mechanism $\Gamma$ is truth-telling if the truth-telling reporting strategy $\hat{X} = X$ is incentive compatible. The goal is to find a truth-telling direct mechanism $\Gamma = (A, K, C)$ that maximizes the expected discounted utility of headquarters (4) subject to delivering the division manager the initial expected discounted utility of at least $R$.

2 Derivation of an Optimal Mechanism

This section solves for the optimal truth-telling direct mechanism using martingale techniques similar to those in [DeMarzo and Sannikov (2006)]. The idea is to summarize all relevant prior history at time $t$ using a single state variable and show that its evolution represents the division-manager’s incentives.

2.1 Incentive Compatibility

By the standard argument, it is optimal to punish the division manager as much as possible if the audit reveals that her report is not truthful. Maximum punishment implies that
$dK_t = 0$ and $dC_t = 0$ for any $t$ following a non-truthful report revealed by the audit. Intuitively, because lying never occurs in equilibrium, there is no cost of imposing the maximum punishment for lying. Given this, in what follows I focus only on histories in which audit always confirms reports of the division manager. Then, the past history can be summarized using only the report process $\left(d\hat{X}_t\right)_{t \geq 0}$. When deciding what report $d\hat{X}_t$ to send, the division manager evaluates how the report will affect her expected utility. Let $W_t\left(\hat{X}\right)$ denote the expected future utility of the division manager at time $t$ after a history of reports $\{d\hat{X}_s, s \leq t\}$, conditional on reporting truthfully in the future:

$$W_t\left(\hat{X}\right) = \mathbb{E}_t \left[ \int_t^\infty e^{-\rho(s-t)} \left(\gamma dK_s + dC_s\right) \right]. \tag{5}$$

In other words, $W_t\left(\hat{X}\right)$ is the expected future utility that the mechanism “promises” to the division manager at time $t$ following history $\hat{X}$, i.e., the promised utility. It is convenient to denote the left-hand limit of (5) at time $t$ by $W_{t-} \equiv \lim_{s \uparrow t} W_s$. The following lemma uses the martingale representation theorem to represent the evolution of $(W_t)_{t \geq 0}$:

**Lemma 1.** At any time $t \geq 0$, the evolution of the division manager’s promised utility $W_t$ following report $d\hat{X}_t \in \{0\} \cup \Theta$ is

$$dW_t = \rho W_{t-} dt - \gamma dK_t - dC_t + H_t\left(d\hat{X}_t\right) - \left(\lambda \int_0^{\tilde{\theta}} H_t\left(\theta\right) f\left(\theta\right) d\theta\right) dt, \tag{6}$$

where $H_t\left(d\hat{X}_t\right)$ is the sensitivity of the division manager’s utility to her report satisfying:

(i) $H_t\left(0\right) = 0$; (ii) for any fixed $\theta \in \{0\} \cup \Theta$, $H_t\left(\theta\right)$ is $\mathcal{F}$-predictable.

Equation (6) reflects the martingale property of the expected lifetime utility of the division manager discounted to the initial date. In (6), $\gamma dK_t + dC_t$ is the utility that the division manager obtains from investment and compensation at time $t$, and $dW_t$ is the change in the expected future utility. The martingale condition states that the sum of these terms less the discounting adjustment ($\rho W_{t-} dt$) is zero in expectation. Function $H_t\left(d\hat{X}_t\right)$ represents the sensitivity of the division manager’s utility to her report. Condition $H_t\left(0\right) = 0$ states that if the division manager reports that no project is available, then the evolution of the division manager’s lifetime expected utility is continuous. By contrast, if the division manager reports that a project of quality $d\hat{X}_t$ is available, then her lifetime expected utility changes by $H_t\left(d\hat{X}_t\right)$. As mentioned above, (6) describes the evolution of $W_t$ only along histories
in which audit confirms reports of the division manager. If the audit at time $t$ reveals that the division manager’s report is not truthful, then her expected future utility jumps down to zero.

In the optimal mechanism, the division manager finds it optimal to send a truthful report: $d\hat{X}_t = dX_t$. Depending on report $d\hat{X}_t$, headquarters either audit it or not. Let $D_t^A = \{d\hat{X}_t : dA_t = 1\}$ and $D_t^N = \{d\hat{X}_t : dA_t = 0\}$ be the “audit” and “no audit” regions of reports at time $t$, respectively. Because audit is costly, it is never optimal to audit if the division manager reports that there is no project. Therefore, $\{0\} \in D_t^N$. Consider any $dX_t$ from the “no audit” region $D_t^N$. The division manager has no incentive to send a report from the “audit” region, since it would be audited and lead to punishment. To have incentives not to send a non-truthful report from the “no audit” region, any report from this region must have the same effect on the division manager’s utility. Otherwise, the division manager would report $d\hat{X}_t$ with the best effect on her utility whenever any $dX_t \in D_t^N$ is realized. Hence, $H_t\left(d\hat{X}_t\right) = 0$ for any report from the “no audit” region. Next, consider any $dX_t$ from the “audit” region $D_t^A$. Again, there is no incentive to send a non-truthful report from the “audit” region, since lying would be revealed. To have incentives not to send a report from the “no audit” region, $H_t\left(d\hat{X}_t\right)$ must be greater or equal than the effect of sending the report from the “no audit” region, i.e., zero. These conclusions are summarized in the following lemma:

**Lemma 2.** At any time $t \geq 0$, truth-telling is incentive compatible if and only if:

1. $\forall d\hat{X}_t \in D_t^N : H_t\left(d\hat{X}_t\right) = 0$;

2. $\forall d\hat{X}_t \in D_t^A : H_t\left(d\hat{X}_t\right) \geq 0$.

### 2.2 Solution of the Optimization Problem

Having derived the incentive compatibility conditions, I can use the dynamic programming approach to solve for optimal investment, compensation, and audit policies subject to delivering the division manager any expected utility $W$ and insuring that truth-telling is incentive compatible. The problem is solved in two steps. First, I fix the audit region and solve for optimal investment and compensation. Then, I optimize over all possible audit strategies. I present a heuristic argument here and verify it in the proof of Proposition 1 in the appendix.

Let $P(W)$ denote the value that headquarters obtain in the optimal mechanism that delivers expected utility $W$ to the division manager. Because a mechanism can specify
randomization between any two levels of the division manager’s promised utility, \( P(W) \) must be concave. For simplicity, my analysis is based on the assumption that the optimal mechanism does not involve randomization. However, I argue in the appendix that if the optimal mechanism involves randomization, then the budgeting mechanism with threshold division of authority is also optimal, if in addition the division manager is allowed to use her spending account for any fair lottery.

Let \( W^c \) be the lowest \( W \) at which \( P'(W) = -1 \). Because headquarters can compensate the division manager with immediate payments, it must be the case that \( P'(W) \geq -1 \) for any \( W \). By concavity of \( P(W) \), optimal compensation policy follows the threshold rule:

**Property 1.** Headquarters make payments to the division manager if and only if her promised utility is at least \( W^c \). The optimal payments satisfy:

\[
dC_t = \max\{W_t - W^c, 0\}.
\] (7)

Property 1 is standard to other optimal long-term contracting models with risk-neutral parties.\(^\text{15}\) It states that it is cheaper to compensate the division manager with promises when her promised utility is low and with direct cash payments when her promised utility is high. In particular, (7) implies that on any sample path \( W_t \) never exceeds \( W^c \), except for the starting point if \( W_0 > W^c \).

Consider region \( W < W^c \). The expected instantaneous change in headquarters’ value function is

\[
rP(W_{t-})\,dt.
\] (8)

This expression must be equal to the sum of the expected flow of value over the next instant and the expected change in \( P(W) \) due to the evolution of \( W \). Clearly, zero investment is optimal if the division manager reports that no project arrives. Thus, the expected flow of value over the next instant is

\[
\lambda dt \int_{d}^{\theta} \left( V(dK_t, \theta) - dK_t - cdA_t \right) f(\theta) \,d\theta.
\] (9)

To evaluate the expected instantaneous change in \( P(W) \), I apply Itô’s lemma (e.g., Shreve\(^\text{14}\)).

\(^{14}\) If \( P'(W) > -1 \) for all \( W \). If the value function is not differentiable, then at any kink in it, I interpret \( P'(W) \) as a supergradient of \( P \) at \( W \).

and use (6):

$$
\mathbb{E}[dP(W_t)] = \left[ \rho W_t dt - \left( \lambda \int_\theta^\theta H_t(\theta) f(\theta) d\theta \right) dt \right] P'(W_t) 
+ \lambda dt \int_\theta^\theta \left[ P(W_t + H_t(\theta) - \gamma dK_t) - P(W_t) \right] f(\theta) d\theta.
$$

The first term in (10) corresponds to the drift of $W$ and the second term corresponds to the jump due to a potential arrival of an investment project. Combining (10) with (9) and equating their sum to (8) yields the following HJB equation on headquarters’ value function $P(W)$:

$$
rP(W) = \max_{\{k_\theta, a_\theta, h_\theta\} \in \Theta} \left\{ \lambda \int_\theta^\theta (V(k_\theta, \theta) - k_\theta - ca_\theta) f(\theta) d\theta 
+ \rho W - \left( \lambda \int_\theta^\theta h_\theta f(\theta) d\theta \right) \right\} P'(W) 
+ \lambda \int_\theta^\theta \left[ P(W + h_\theta - \gamma k_\theta) - P(W) \right] f(\theta) d\theta,
$$

where the maximization is subject to the constraints $k_\theta \geq 0$, $a_\theta \in \{0, 1\}$, and the incentive compatibility constraints

$$
h_\theta \geq 0, \text{ if } a_\theta = 1 \quad (12)
h_\theta = 0, \text{ if } a_\theta = 0. \quad (13)
$$

Let $k^*(\theta, W)$ and $a^*(\theta, W)$ denote the investment and audit strategies that solve this problem. Given (11), I can derive the properties of $k^*(\theta, W)$, $a^*(\theta, W)$, and the evolution of $W$ under the optimal mechanism. Consider a report $\theta$ that is audited. The derivative of (11) with respect to $h_\theta$ is proportional to

$$
-P'(W) + P'(W + h_\theta - \gamma k_\theta).
$$

This reflects two effects of a marginally higher $h_\theta$. First, it increases the promised utility of the division manager after the investment in the project of type $\theta$, which is reflected in the second term of (14). Second, it decreases the drift of $W$, which is reflected in the
first term of (14). From (14) it follows that (11) is maximized by
\[ h_\theta = \gamma k_\theta \]
whenever \( a_\theta = 1 \). Intuitively, if the project is audited, then the incentive compatibility condition is lax because the information of the division manager is verified by headquarters. As a result, a positive \( h_\theta \) simply redistributes the division manager’s promised utility across states. Other things equal, fluctuations in the promised utility of the division manager are harmful for headquarters. As a consequence, in order to minimize the fluctuations, it is optimal to keep the division manager’s promised utility constant at \( W \). This contrasts the evolution of the division manager’s promised utility after reporting a project that does not get audited. In this region, the incentive compatibility condition is binding because headquarters must rely on the division manager’s report. Indeed, if the project does not get audited, punishing the division manager in the future is the only way to provide incentives not to overstate the prospects of an investment project. As a consequence, headquarters must reduce the division manager’s promised future utility by the amount \( \gamma k_\theta \) of private benefits acquired from the current project. Combining these two cases yields the following property:

**Property 2.** Under the optimal mechanism, the evolution of the division manager’s promised utility \( W_t \) following a report of project \( \theta \) is:

1. If the report is not audited, then \( W_t \) is reduced by the amount \( \gamma k_\theta \) of private benefits from the project.

2. If the report is audited and is truthful, then \( W_t \) is unaffected.

3. If the report is audited and turns out to be non-truthful, then \( W_t \) is reduced to zero.

Property 2 allows to derive optimal investment given type \( \theta \) of the project and the division manager’s promised utility \( W \) in the “audit” and “no audit” regions. Differentiating (11) with respect to \( k_\theta \) yields

\[
\frac{\partial V (k_\theta, \theta)}{\partial k} - 1 - \gamma P' (W - \gamma k_\theta) = 0, \text{ if } a_\theta = 0, \\
\frac{\partial V (k_\theta, \theta)}{\partial k} - 1 - \gamma P' (W) = 0, \text{ if } a_\theta = 1.
\]

(15) - (16)

Concavity of \( V (k, \theta) \) and \( P (W) \) ensures that each equation has a unique solution. According to (15) - (16), the optimal investment policy maximizes firm value subject to the financing constraint implied by the agency problem. The first two terms in (15) - (16) correspond to
the net present value of a marginal dollar invested in the project.\textsuperscript{16} The last term reflects the additional effect of a financing constraint implied by the agency problem. Thus, the optimal investment policy satisfies the following property:

**Property 3.** Under the optimal mechanism, the investment policy is

\[
    k^* (\theta, W) = \begin{cases} 
    k^n (\theta, W), & \text{if } a^* (\theta, W) = 0, \\
    k^a (\theta, W), & \text{if } a^* (\theta, W) = 1, 
\end{cases}
\]

(17)

where \( k^n (\theta, W) \) and \( k^a (\theta, W) \) solve (15) and (16), respectively.

Optimal investment satisfies three natural properties. First, \( k^n (\theta, W) \) and \( k^a (\theta, W) \) are increasing in \( \theta \): investment is higher if the project is better. Second, \( k^n (\theta, W) \) and \( k^a (\theta, W) \) are increasing in \( W \): investment is higher if the promised utility of the division manager is higher. The effect of \( W \) on optimal investment is uniquely determined by the slope of the value function at the post-investment promised utility. Finally, \( k^a (\theta, W) > k^n (\theta, W) \): other things equal, investment is higher if the investment project is audited. This property holds because investment lowers the promised utility of the division manager only if the project is not audited.

Finally, it remains to solve for the optimal audit policy. From (11), \( a^* (\theta, W) = 1 \) if and only if

\[
    V(k^a(\theta,W),\theta) - k^a(\theta,W) - \gamma k^a(\theta,W) P'(W) \\
    - [V(k^n(\theta,W),\theta) - k^n(\theta,W) + P(W - \gamma k^n(\theta,W)) - P(W)] \geq c.
\]

(18)

The left-hand side of (18) is the marginal benefit of audit in that the firm’s agency constraint is reduced. In the appendix, I show that it is a strictly increasing function of \( \theta \in \Theta \). Intuitively, if the quality of the project is higher, investment in it is higher. In turn, higher investment leads to a higher increase in the agency constraint if the project is not audited. At some point \( \theta^* (W) \), the left-hand side becomes high enough so that audit is optimal if and only if the reported quality of the project is above \( \theta^* (W) \). This result is summarized in the following property:

**Property 4.** There exists at most one point \( \theta^* (W) \in \Theta \) at which (18) holds as equality. If

\textsuperscript{16}Here I use the narrow definition of the NPV that only accounts for direct value that the project generates to headquarters. Alternatively, one can define the NPV broader taking, the private benefits of the division manager into account. Both definitions are discussed in more detail in Section 4.
\[(18)\) holds strictly for all \( \theta \in \Theta \), let \( \theta^*(W) = \theta \). If \((18)\) does not hold for any \( \theta \in \Theta \), let \( \theta^*(W) \) be any value above \( \bar{\theta} \). Then, the optimal audit strategy is

\[
a^*(\theta, W) = \begin{cases} 
0, & \text{if } \theta < \theta^*(W), \\
1, & \text{if } \theta \geq \theta^*(W).
\end{cases}
\]

(19)

Combining Properties 1 - 4 leads to the following investment, audit, and evolution of the division manager’s promised utility. If the division manager reports that no project arrives, then her promised future utility accumulates at a certain rate such that her lifetime expected utility is a martingale. Once her promised utility reaches threshold \( W^c \), it does not accumulate anymore and the division manager gets paid a flow of constant bonus payments such that her promised utility is reflected at \( W^c \). If a project with quality \( \theta < \theta^*(W) \) is reported, then the report is not audited, the firm invests \( k_n(\theta, W) \), and the promised utility of the division manager falls by the amount of empire-building private benefits consumed from the investment. Finally, if a project with quality \( \theta \geq \theta^*(W) \) is reported, then the report is audited and, provided that the audit confirms the report, the firm invests \( k_a(\theta, W) \), and does not change the promised utility of the division manager. The following proposition summarizes these findings:

**Proposition 1.** Suppose that an optimal mechanism exists. Then, the following mechanism is optimal. If \( R \leq W^c \), then the initial value \( W_0 \) is \( \max \{ R, W^* \} \), where \( W^* \) is the point at which \( P(W) \) is maximized. If \( R > W^c \), then an immediate payment of \( R - W^c \) is made to the division manager and \( W_0 = W^c \). At any \( t \), the division manager sends a report \( \hat{d}X_t \) from message space \( \{0\} \cup \Theta \).

1. If \( d\hat{X}_t = 0 \), then \( dK_t = 0 \) and \( dA_t = 0 \). If \( W_t < W^c \), then

\[
dW_t = g(W_t-) W_t^- dt,
\]

where

\[
g(W) = \rho - \lambda \int_{\theta^*(W)}^{\bar{\theta}} \frac{\gamma k^a(\theta, W)}{W} f(\theta) d\theta,
\]

and \( dC_t = 0 \). If \( W_t = W^c \), then \( dW_t = 0 \) and \( dC_t = g(W^c) W^c dt \).

2. If \( d\hat{X}_t \in [\bar{\theta}, \theta^*(W_t^-)) \), then \( dK_t = k_n(d\hat{X}_t, W_t^-) \), \( dA_t = 0 \), and \( dW_t = -\gamma dK_t \).

3. If \( d\hat{X}_t \in [\theta^*(W_t^-), \bar{\theta}] \), then \( dA_t = 1 \). If the audit confirms the report, then \( dK_t = \)
Figure 2: Headquarters’ value as a function of the division manager’s promised utility in the optimal mechanism.

\[ k^a \left( d\hat{X}_t, W_t^- \right) \text{ and } dW_t = 0. \text{ If the audit does not confirm the report, then } dK_t = 0 \text{ and } dW_t = -W_t^- . \]

An example of headquarters’ value function \( P(W) \) is shown on Figure 2. It has an inverted U-shaped form. When the division manager’s promised utility \( W \) is low, little investment occurs to keep the expected private benefits of the manager low. In the extreme case of \( W = 0 \), headquarters’ value is zero, because \( W = 0 \) implies no investment. When the division manager’s promised utility is low enough, a marginal increase in it increases headquarters’ value. When the promised utility is high enough, a marginal increase in it decreases headquarters’ value. Point \( W^* \) denotes the promised utility at which headquarters’ value is maximized. When \( W > W^c \), it is optimal to compensate the division manager with payments. Hence, the slope of headquarters’ value function at these points is \(-1\).

## 3 Implementation

Although the direct mechanism from Proposition 1 is optimal, it has little resemblance to real-world capital allocation processes. This is not surprising, because of its complexity.
In this section, I show that a much simpler mechanism, the budgeting mechanism with threshold division of authority, is equivalent to the mechanism from Proposition 1 in the sense of implementing the same investment, audit, and compensation policies. Hence, the budgeting mechanism with threshold division of authority is optimal.

I begin by defining a simple budgeting mechanism:

**Definition (simple budgeting mechanism).** Headquarters allocate a spending account $B_0$ to the division manager at the initial date. All projects are financed out of the allocated account and are at the discretion of the division manager. At any time $t \geq 0$ the spending account is accumulated at the rate $g_t$: $dB_t = g_t B_t dt$.

The simple budgeting mechanism has two features. First, all investment decisions are delegated to the division manager: she has full discretion to invest any amount in any project provided that she stays within the limit of her account (“budget”). Any investment reduces the remaining balance by the amount of investment. Second, the spending account is rigid meaning that the division manager cannot get extra financing even if it leads to passing by profitable investment opportunities. The simple budgeting mechanism has two parameters: initial size $B_0$ and accumulation rate $g_t$.

The budgeting mechanism with threshold division of authority augments the simple budgeting mechanism with an additional feature. It gives the division manager an option to pass the project to headquarters claiming that it deserves investment above some pre-specified threshold. Upon the receipt of the project, headquarters audit it and, provided that audit confirms that the project deserves investment above the threshold, finance the project separately, i.e., without changing the manager’s account balance.

**Definition (budgeting mechanism with threshold division of authority).** Headquarters allocate a spending account $B_0$ to the division manager at the initial date. The division manager can use the account at her discretion to invest in projects. At time $t \geq 0$ the spending account is accumulated at rate $g_t : dB_t = g_t B_t dt$. In addition, there is a threshold on the size of individual investment projects, $k^*_t$, such that at any time $t$ the division manager can pass the project to headquarters claiming $k^a (\theta, \gamma B_t) \geq k^*_t$, where $\theta$ is the quality of the current project. If the division manager passes the project, it gets audited. If the audit confirms that $k^a (\theta, \gamma B_t) \geq k^*_t$, then headquarters invest $k^a (\theta, \gamma B_t)$ and does not alter the account balance. If the audit reveals that $k^a (\theta, \gamma B_t) < k^*_t$, then headquarters
punish the division manager by reducing her spending account balance to zero.

Thus, in addition to giving the division manager discretion to spend resources on projects within the limits of her dynamic spending account, this mechanism separates investment and financing decisions by a threshold on the size of individual projects. If the division manager exercises the option to pass the project to headquarters if and only if it deserves investment above the threshold, then under this mechanism, investment and financing of all projects below the threshold is delegated to the division manager. The division manager uses her spending account to finance small investment projects. By contrast, investment and financing of all projects above the threshold is centralized at the level of headquarters. Whenever the division manager conceives a project that deserves a large enough investment, she passes it to headquarters, and headquarters finance it independently, i.e., without using the division manager’s spending account. Thus, the budgeting mechanism with threshold division of authority has three parameters: initial size $B_0$, accumulation rate $g_t$, and project size threshold $k_t^*$. The following proposition establishes optimality of the budgeting mechanism with threshold division of authority:

**Proposition 2.** Consider a budgeting mechanism with threshold division of authority with the following parameters:

- the project size threshold of
  \[ k_t^* = k^a (\theta^* (\gamma B_t), \gamma B_t) ; \tag{22} \]
- the accumulation rate of $g_t = g (\gamma B_t)$, if $B_t < B^c$, and $g_t = 0$, if $B_t = B^c$.

Suppose that the monetary compensation of the division manager is $dC_t = 0$, if $B_t < B^c$, and $dC_t = g (\gamma B^c) \gamma B^c dt$, if $B_t = B^c$. Then, the division manager finds it optimal to (i) allocate the spending account between current and future investment opportunities in the way that maximizes headquarters’ value, $V (dK_t, \theta) + P (\gamma (B_t - dK_t))$, where $\theta$ is the quality of the project at time $t$; (ii) pass a project to headquarters if and only if $k^a (\theta, \gamma B_t) \geq k_t^*$. If, in addition, the size of the initial spending account is $B_0 = W_0/\gamma$, then this mechanism is equivalent to the one in Proposition 1.

The intuition behind Proposition 2 is as follows. To provide incentives to invest appropriately, headquarters must either audit the report of the division manager or punish her by
reducing her promised utility by the amount of private benefits that she obtains from current investment. If the project’s quality is low, the latter tool is optimal and can be implemented using a dynamic spending account. Because investing from the account reduces its balance by the amount of investment, the spending account punishes the division manager in the future for high investment today. Moreover, because the division manager’s private benefits are proportional to the amount of investment, the decrease in the division manager’s promised utility is exactly equal to the amount of private benefits consumed from the current investment. As a consequence, the division manager is indifferent between all ways of allocating her account between the current and future investment projects. In particular, she has incentives to invest in the way that maximizes headquarters’ value. The key assumption that makes the spending account a valid incentive tool is that private benefits from investing a dollar, $\gamma$, are identical for all projects. This is a more severe constraint than in a one-shot setting of Harris and Raviv (1996), who allow for $\gamma$ to depend on $\theta$. The direct extension of their setting will not lead to alignment of incentives under the spending account.

The incentive role of a spending account comes at a cost. Specifically, higher current investment decreases the remaining budget for the future. This constrains future investment of the division, which results in the loss of value of the relationship. If the current investment is high enough, the increase in the financing constraint leads to a cost above the cost of audit. Hence, any project whose size exceeds a certain threshold is audited, even the account balance exceeds investment. By concavity of the value function, ex-ante distortions in the spending account balance are costly. Hence, if the project is audited, the spending account of the division manager remains unaffected – the project is financed without the use of the division manager’s account at all. This outcome is implemented through giving the division manager an option to pass the project to headquarters claiming that the optimal investment exceeds the threshold. Because the division manager keeps the same spending account and obtains additional financing, she finds it optimal to pass the project to headquarters if it deserves investment above the threshold. At the same time, because all projects passed to headquarters are audited, the division manager has no incentives to pass the project to headquarters if the optimal investment is below the threshold. The optimal threshold is such the audit policy implied by this mechanism coincides with the audit policy in Proposition 1.

An example of headquarters’ value function as a function of the division manager’s spending account balance is shown on Figure 3. Comparison of Figures 2 and 3 illustrates the one-to-one correspondence between the division manager’s spending account balance $B$ and
her promised utility $W$. Under the mechanism in this section, headquarters gives the initial spending account to the division manager. If the division manager’s initial required payoff $R$ is below $W^*$, the size of the initial spending account is $B^* = W^*/\gamma$ - the level at which headquarters’ value is maximized. If $R$ is above $W^*$ but below $W^c$, the size of the initial spending account is $R/\gamma$. Finally, if $R$ is above $W^c$, the initial spending account is $B^c = W^c/\gamma$ and the division manager gets an upfront monetary payment of $R - W^c$. As time goes by, the spending account accumulates at the rate of $g(\gamma B_t)$. As investment projects below the threshold arrive, the account is deplenished. If the spending account balance is at the accumulation limit $B^c$, it is no longer accumulated, and the division manager receives a flow of constant monetary payments instead.

4 Discussion

The implementation in Proposition 2 captures two features of spending activity that are common in real-world organizations. The first feature is a dynamic spending account, i.e., an arrangement in which upper management provides an account to a subordinate (e.g., a di-
vision manager) and gives considerable discretion to allocate this account over time between various spending needs. Spending accounts are common in the real-world organizations: examples include R&D accounts of research groups, investment budgets in corporations, budgets of loan officers, and research accounts in academic institutions. The spending account in this model is rolled over and gets replenished over time. While the structure of replenishment in real-world spending accounts is usually quite different from the mechanism in Proposition 2, in Section 5 I argue that this difference can arise due to limited commitment of parties in the real-world organizations.

The second feature is threshold division of authority between the layers of organizational hierarchy, under which the organizational design includes formally specified thresholds on the size of individual investment projects, such that a manager passes the project to a higher level if the size of the project exceeds the threshold. In the study of Ross (1986), a typical manufacturing firm sets boundaries on the size of investment that specify the level of the corporate hierarchy at which the investment decision is made. In a typical firm in the sample of Ross (1986), a plant manager has authority to make decisions on projects whose size is below $100,000, but passes larger projects to one of the upper levels of the organizational hierarchy. Similar evidence is presented in other surveys. In the model, this feature arises because the incentive role of a spending account comes at a cost of constraining future investment activity. If the amount of investment in the current project is high enough, headquarters find it optimal to provide incentives by auditing the project. In this case, full financing of the project by headquarters is optimal, because it minimizes ex-ante distortions in the spending account. Unlike in the mechanism in Proposition 2, project size thresholds in real-world organizations are usually time-independent. This discrepancy might arise due to the assumption of the model that investment in each project cannot be delayed. If investment can be delayed, the division manager can manipulate her decision when to pass the project to headquarters. A time-independent project size threshold coupled with time-independent investment in a large project might be a way to solve this manipulation problem.

The mechanism in Proposition 2 has equivalent variations. First, although headquarters reduce the division manager’s account balance to zero if the audit reveals that the project does not deserve investment above the threshold, a similar mechanism with any positive punishment is equivalent. Intuitively, because the project passed to headquarters is always audited and the auditing technology is perfect, any positive punishment would suffice to

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prevent the division manager from passing small projects. At the same time, punishment never occurs in equilibrium, so all positive punishments are equivalent. The size of the punishment would be important if audit were either probabilistic or imperfect. Second, while the mechanism in Proposition 2 gives the division manager an option to pass a project to headquarters claiming that it deserves investment above the threshold, a similar mechanism that requires the same from the division manager is equivalent. This is because the division manager has no incentive to hide large projects under the mechanism in Proposition 2.

Finally, the mechanism has an equivalent implementation in which it is the division manager who makes investment decisions on large projects too. Under this implementation, at any time the division manager can file a capital request to headquarters for any amount of capital above the threshold. If the division manager files a capital request, headquarters audit the project and allocate the capital if and only if the audit confirms that the capital requested by the division manager to be invested in the project maximizes headquarters’ value. Both implementations lead to the same policies, and so they are equivalent in this model. In practice, however, each implementation has pros and cons. The implementation in Proposition 2 involves less communication but requires the ability of headquarters to commit to the level of investment that maximizes firm value ex ante. The alternative implementation does not require this ability but involves more communication between the parties: the division manager not only passes the project to headquarters but also specifies the exact amount of investment in it.

The budgeting mechanism with threshold division of authority implies path-dependent distortions for investment. Although these distortions are not optimal ex post, they are optimal ex ante due to the agency problem. First, compared to the level of investment \( k^0(\theta) \) that maximizes NPV, \( V(\theta, k) - k \), there can be both overinvestment and underinvestment. If the post-investment account balance \( B \) is above \( B^* \), then \( P'(B) < 0 \), so the firm overinvests compared to \( k^0(\theta) \). Otherwise, the firm underinvests compared to \( k^0(\theta) \). At the same time, there is always underinvestment relative to the size of investment that maximizes the sum of the project NPV and the division manager’s empire-building benefits, unlike in Harris and Raviv (1996). Second, because the division manager’s spending account balance decreases

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18To see this, suppose that headquarters approve a capital request for \( \Delta \) and allocate it to the division manager. Because the division manager allocates her spending account between the current and future investment projects in the way that maximizes firm value, the size of investment will be the one that maximizes \( V(\theta, k) - k + P(\gamma (B + \Delta - k)) \), where \( B \) is the spending account balance prior to filing the capital request. Knowing this, headquarters approve the capital request if and only if the requested amount \( \Delta \) is exactly equal to the size of investment \( k \). The resulting level of investment is equal to \( k^a(\theta, \gamma B) \).
only if the project is not passed to headquarters, investment by the division is negatively serially correlated over time, while investment by headquarters is serially uncorrelated over time. The negative correlation of investment is the opposite result from models of agency with a constant investment opportunity set (DeMarzo and Fishman (2007a) and DeMarzo et al. (forthcoming)). This difference arises because investment plays different roles in these models. In DeMarzo and Fishman (2007a) and DeMarzo et al. (forthcoming), investment is a response of the principal to the agency problem. In contrast, in my paper, investment is a source of the agency problem. Finally, investment decisions made on the division level are more financially constrained than investment decisions made on headquarters level. It is easy to see this from (15) and (16): by the concavity of the value function, $P' \left( \gamma (B - k) \right)$ is always higher than $P' \left( \gamma B \right)$. Intuitively, unlike investment decisions made at the division level, investment decisions made at headquarters level are not financed using the division’s spending account. As a consequence, while the former decisions increase future financing constraints of the division, the latter decisions do not. Because of this difference, it is optimal to treat projects financed by the division more “harshly” than projects financed by headquarters. Empirically, this feature seems common: for example, divisions are known to use higher discount rates than corporate investment committees (Ross (1986)).

5 Extensions

Optimality of the budgeting mechanism with threshold division of authority hinges on a number of important assumptions. The most important assumptions are: (i) investment of $1$ in any project generates the same private benefit to the division manager; (ii) utility of the division manager does not depend on project values; and (iii) headquarters have unlimited commitment power. These assumptions are reasonable approximations of reality in some settings, but not the others. In this section, I relax each of these assumptions one by one and discuss the possible consequences. I also consider an extension that allows for multiple audit technologies and show that it leads to multiple project thresholds and co-financing of some projects.

5.1 Multiple Project Categories

The base model assumes that the private benefits that the division manager gets from investment of $dK_t$ are the same for all projects. If this assumption is violated, the division
manager no longer has incentives to use her spending account in the way that maximizes headquarters' value. Heterogeneity of projects along the private benefit dimension is a practical concern: for example, a division manager probably gets higher private benefits from spending resources renovating her office than on a marketing campaign.

The base model can be extended to handle multiple project categories that differ in private benefits. Specifically, unlike in the base model, suppose that there are two groups of projects, \(L\) and \(H\), with private benefit parameters \(\gamma_L > 0\) and \(\gamma_H \in (\gamma_L, 1)\) from each dollar of investment. Letting \((dK_{L,t})_{t \geq 0}\) and \((dK_{H,t})_{t \geq 0}\) denote the streams of investment in project categories \(L\) and \(H\), respectively, the utility of the division manager is

\[
\hat{\infty} \int_0^\infty e^{-\rho t} (\gamma_L dK_{L,t} + \gamma_H dK_{H,t} + dC_t).
\]

(23)

Analogously to the main model, assume that an investment opportunity in category \(i \in \{L, H\}\) arrives independently with intensity \(\lambda_i\) and is characterized by quality \(\theta\), which is an i.i.d. draw from a distribution with c.d.f. \(F_i(\theta)\) and density \(f_i(\theta) > 0\) defined over \(\Theta = [\underline{\theta}, \bar{\theta}]\).

The optimal mechanism in this case can be implemented with a combination of two spending accounts with each account being devoted to a particular project category.\(^{19}\) The division manager is allowed to transfer funds between the two accounts at the rate that equals the ratio of private benefits from two categories of projects. For each project category, there is a time-dependent project size threshold that gives the division manager an option to pass the project to headquarters. As in the base model, the thresholds separate decision making, with all projects below the thresholds being financed out of the division manager’s accounts and all projects above the thresholds being passed to headquarters, audited, and financed separately. Intuitively, the use of a category-specific spending account does not allow the division manager to strategically invest more in projects with higher private benefits. This makes investment policy that maximizes headquarters’ value incentive compatible. The ability of the manager to transfer funds between the accounts relaxes the financing constraint, and because the rate equals the ratio of private benefits, the division manager cannot manipulate the transfer to increase her private benefits.

This extension can explain why organizations often use separate spending accounts (or budgets) for different kinds of activities: for example, an account for R&D, an account for capital investment, an account for office renovation, etc. It also implies that the optimal

\(^{19}\)The detailed solution of this extension is available from the author upon request.
number of accounts equals not the number of different project categories, but the number of project categories with different levels of private benefits: if two project categories share same private benefits, then it is optimal to have one account for both categories of projects. A feature of the optimal mechanism that is not observed in practice is conversion of funds from one account to the other. Practical implementation of conversion is, however, limited, because headquarters are unlikely to know the exact ratio of private benefits of the division manager. This might explain why the use of conversion is limited in practice.

5.2 Value-Sensitive Utility of the Division Manager

Another important assumption in the base model is that utility of the division manager does not depend on payoffs from projects. Clearly, this is unlikely to hold in many settings, such as if the division manager owns stock in the firm or if project payoffs are observed later by headquarters and used to reward or punish the division manager. If this assumption is violated, the budgeting mechanism with threshold division of authority is no longer optimal. Intuitively, if the division manager has a preference to invest in higher-quality projects, then punishing the manager for the full amount of private benefits from investment, $\gamma d K_t$, is too expensive for headquarters: it is enough to decrease the manager’s promised utility by less than that to ensure incentive compatibility of the second-best investment strategy. In the context of the spending account implementation, one can interpret the decrease of the division manager’s promised utility by less than $\gamma d K_t$ as co-financing of the project by the division manager and headquarters: the division manager finances part of the project out of her spending account, and headquarters finance the rest.

While the budgeting mechanism with threshold division of authority is no longer optimal, it is still a natural mechanism in settings when headquarters are uncertain over preferences of the division manager. Specifically, suppose that the division manager’s utility is a weighted combination of her utility in the base model (2) and the utility of headquarters (1). Suppose that headquarters give the budgeting mechanism with threshold division of authority to the division manager. Even though utility of the division manager is different, she will not mismanage the spending account because she cares about headquarters’ payoff. In this respect, the mechanism from Proposition 2 is robust to preferences of the division manager with respect to headquarters’ value. While characterization of the optimal mechanism for the case of the division manager caring about headquarters’ value is beyond the scope of

\[20\] This conjecture can be formally studied in a model in which the principal is uncertain about the agent’s preferences, as in \cite{Frankel2011}.
this paper, it is likely to rely on the details of the manager’s preferences. Therefore, the policies implied by the optimal mechanism are unlikely to be robust to misspecifications in the division manager’s preferences. This makes a budgeting mechanism with threshold division of authority a reasonable (though, not optimal) mechanism when headquarters are uncertain over preferences of the division manager.

5.3 Renegotiation-Proofness

The optimal mechanism in the model is not renegotiation-proof for two reasons. First, if the spending account balance $B$ is such that $P'(\gamma B) > 0$, both parties benefit from increasing the division manager’s spending account balance. Second, because the division manager always passes only high quality projects to headquarters, audit need not be optimal ex-post. To partially address the effects of limited commitment, suppose that headquarters are able to commit to any audit strategy ex ante, but cannot commit not to increase the division manager’s promised utility if both parties benefit from the increase. This addresses the first commitment problem but not the second.

Renegotiation-proofness in this partial sense implies that headquarters’ value function does not have positive slope. Thus, possibility of renegotiation places the lower boundary on the division manager’s promised utility to $W^*$ at which $P'(W^*) = 0$. The optimal investment in the “no audit” region is then $\hat{k}^n(\theta, W) = \min\{k^n(\theta, W), (W - W^*)/\gamma\}$, where $k^n(\theta, W)$ solves (15). Thus, possibility of renegotiation restricts the ability to invest in projects without audit when $W$ is sufficiently low. By analogy with Property 4, the optimal audit strategy is given by threshold $\theta^*(W)$ that is determined as in the base model but with $\hat{k}^n(\theta, W)$ in place of $k^n(\theta, W)$. If audit reveals that the division manager lies, then headquarters are able to lower the payoff only to $W^*$.

It turns out that the optimal renegotiation-proof mechanism can be equivalently implemented with a budgeting mechanism with threshold division of authority, similar to the one in Proposition 2.21 There are three important differences. First, at any time $t$, the spending account balance equals $B_t = (W_t - W^*)/\gamma$, not $B_t = W_t/\gamma$ as in the base model. Hence, compared to the full-commitment solution, the spending account has a lower initial balance but a higher accumulation rate. A lower account balance and faster replenishment are necessary to prevent the division manager from spending a very high amount and asking for more. To the extent that commitment power can be measured, this result provides a

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21 The detailed solution of this extension is available from the author upon request.
testable implication for the design of spending activity in real-world organizations. Second, accumulation of the spending account takes a different form. In addition to the growth of the account balance with rate \( g_t \), similar to the base model, the account balance is replenished by a constant flow of \( W^*/\gamma \), independently of the account balance. This constant replenishment that does not depend on the account balance makes replenishment in this model more similar to replenishment of spending accounts in real-world organizations. Finally, the optimal audit strategy is quite different. Specifically, when the spending account approaches zero, headquarters audit all projects with positive investment.

5.4 Multiple Audit Technologies

The model can be naturally extended to the case of multiple audit technologies, keeping the assumption that random audit is not allowed. Suppose that there are two audit technologies, 1 and 2. Technology 2 is similar to the audit technology in the base model: it reveals the type of the project with certainty but costs \( c_2 > 0 \). By contrast, technology 1 costs \( c_1 \in (0, c_2) \) but is less efficient: with probability \( p \), it reveals the type of the project with certainty (the audit is successful); with probability \( 1 - p \), it does not reveal anything (the audit is unsuccessful). One can extend the model to more than two audit technologies. If the number of technologies approaches infinity and the audit costs of all technologies are proportional to the probability of the success of the audit, the model approaches the analog of the base model that allows for random audit.

It turns out that the mechanism similar to the one in Proposition 2 is optimal in this case. Headquarters allocate a spending account to the division manager and allow to use this account at her discretion. However, headquarters specify two thresholds on the size of individual projects that separate the “no audit” region (small projects) from the “audit using technology 1” region (medium projects) from the “audit using technology 2” region (large projects). One can interpret two audit technologies as audit by the CEO (more efficient but also more expensive) and audit by the corporate investment committee. In light of this interpretation, the findings of these sections are consistent with the evidence in [Ross (1986)] of multiple project size thresholds that separate authority between levels of the organizational hierarchy.

Figure 4 illustrates the solution. If at time \( t \) the division manager obtains a project with optimal investment above \( k^*_t \), she passes it to headquarters that audit it using technology 2 and finance the project independently. If at time \( t \) the division manager obtains a project...
with optimal investment above $k_t^*$, she passes it to the headquarters that audit it using technology 1. If the audit is successful, headquarters finance the project independently. Interestingly, headquarters provide financing for the project even if the audit is not successful. In this case, the project is co-financed by the two parties or even financed by headquarters only.

6 Conclusion

The paper studies optimal design of investment activity in an organization in a dynamic framework when the key problem is the agent’s desire to overspend money. For this purpose, I develop a continuous-time principal-agent (headquarters - division manager) framework with three key properties. First, the arrival and quality of investment projects are privately observed by the division manager. Second, the division manager obtains a private benefit from each dollar spent on projects. Finally, at any time headquarters can learn the quality of the current investment project of the division manager at a cost. I show that under certain conditions, out of all possible mechanisms, a very simple one is optimal. It involves a combination of a dynamic spending account and a threshold on the size of individual projects. Headquarters allocate a spending account to the division manager, accumulate it over time, and give the division manager discretion to invest this account on projects. In addition, headquarters specify a threshold on the size of individual projects, such that the division manager at any time can pass the project to headquarters claiming that it deserves investment above the threshold. If the division manager passes the project, headquarters audit it and finance it independently. Thus, the optimal mechanism features threshold separation: all small projects are financed out of the division manager’s dynamic account, while all large projects are passed to headquarters and financed independently.

Because the result is based on several important assumptions, the model sheds light on
The pros and cons of using dynamic spending accounts and project size thresholds in capital budgeting in organizations. Section 5 discusses these assumptions and describes how optimal capital budgeting policies are affected if each of them is violated one by one. These results provide a number of empirical predictions about how spending activity in hierarchical organizations is structured. In addition, the model generates dynamic distortions for investment that are novel compared to distortions in prior literature on agency and investment dynamics.

While my main focus is on corporate investment, the results of the paper can be applicable to any principal-agent setting, in which the agent privately receives various spending needs over time and has incentives to overspend.
Appendix

Proof of Lemma 1. Note that $W_t \left( \hat{X} \right)$ is also the division manager’s expected future utility if $\{d\hat{X}_s, 0 \leq s \leq t\}$ were the realizations that the division manager reported truthfully. Hence, without loss of generality, it is sufficient to prove the lemma for the case of truthful reporting by the division manager. Let $U_t(X)$ denote the lifetime expected utility of the division manager, evaluated conditionally on information available at time $t$:

$$U_t(X) = \int_0^t e^{-\rho s} \left( \gamma dK_s + dC_s \right) + e^{-\rho t} W_t(X).$$  \hfill (24)

By definition, process $U(X) = \{U_t(X)\}_{t \geq 0}$ is a right-continuous $\mathcal{F}$-martingale. By the martingale representation theorem for marked point processes (e.g., see Theorem 1.13.2 on pages 25 - 26 in Last and Brandt (1995)), for any $t$ there exists a function $h_t(\cdot)$, where $h_t(\theta)$ is $\mathcal{F}$-predictable for any fixed $\theta \in \Theta$, such that

$$dU_t = \begin{cases} 
- \left( \lambda \int_\Theta h_t(\theta) f(\theta) d\theta \right) dt, & \text{if } t \neq T_n \text{ for any } n \geq 1, \\
h_t(\theta_n) - \left( \lambda \int_\Theta h_t(\theta) f(\theta) d\theta \right) dt, & \text{if } t = T_n \text{ for some } n \geq 1.
\end{cases}$$  \hfill (25)

For convenience, rescale $h_t(\cdot)$ by factor $e^{\rho t}$ and write as a function of $dX_t \in \{0\} \cup \Theta$, defining it to be zero if $dX_t = 0$:

$$H_t(dX_t) = \begin{cases} 
0, & \text{if } dX_t = 0, \\
e^{\rho t} h_t(dX_t), & \text{if } dX_t \in \Theta.
\end{cases}$$  \hfill (26)

Notice that $H_t(dX_t)$ is $\mathcal{F}$-predictable for any fixed $dX_t \in \{0\} \cup \Theta$, because $h_t(\theta)$ is $\mathcal{F}$-predictable for any fixed $\theta \in \Theta$ and $H_t(0) = 0$. Then,

$$dU_t = e^{-\rho t} \left( H_t(dX_t) - \left( \lambda \int_\Theta H_t(\theta) f(\theta) d\theta \right) dt \right).$$  \hfill (27)

From (24):

$$dU_t = e^{-\rho t} (\gamma dK_t + dC_t) - \rho e^{-\rho t} W_t(X) + e^{-\rho t} dW_t(X).$$  \hfill (28)

Equating (28) with (27) and rearranging the terms yields (6).

Proof of Lemma 2. Consider any $dX_t \in D^N_t$. Report $dX_t$ dominates any $d\hat{X}_t \in D^A_t$, because any report $d\hat{X}_t \in D^A_t$ leads to zero utility due to maximum punishment. Any report
$d\hat{X}_t \in D^N_t$ instead of $dX_t$ leads to the net gain of $H_t\left(d\hat{X}_t\right) - H_t(dX_t)$. Therefore, truthful reporting is optimal for all $dX_t \in D^N_t$ if and only if $H_t\left(d\hat{X}_t\right) - H_t(dX_t) \leq 0 \forall dX_t, d\hat{X}_t \in D^N_t$. Hence, $H_t(dX_t)$ must be constant $\forall dX_t \in D^N_t$. Because $\{0\} \in D^N_t$ and $H_t(0) = 0$, $H_t(dX_t) = 0 \forall dX_t \in D^N_t$. Next, consider any $dX_t \in D^A_t$. Again, report $dX_t$ dominates any $d\hat{X}_t \in D^N_t$ due to punishment. Report $d\hat{X}_t \in D^N_t$ instead of $dX_t$ leads to the net gain of $H_t\left(d\hat{X}_t\right) - H_t(dX_t)$. Therefore, truthful reporting is optimal for all $dX_t \in D^A_t$ if and only if $H_t\left(d\hat{X}_t\right) - H_t(dX_t) \leq 0 \forall dX_t \in D^A_t, d\hat{X}_t \in D^N_t$. Because $H_t\left(d\hat{X}_t\right) = 0 \forall d\hat{X}_t \in D^N_t$, this condition is equivalent to $H_t(dX_t) \geq 0 \forall dX_t \in D^A_t$.

**Proof of Property 4.** First, I show that the left-hand side of (18) is a strictly increasing function of $\theta$. It equals $F^a(\theta, W) - F^n(\theta, W)$, where

$$
F^a(\theta, W) \equiv \max_{k \in \mathbb{R}^+} \left\{ V(k, \theta) - k + P(W) - \gamma kP'(W) \right\}, \quad (29)
$$

$$
F^n(\theta, W) \equiv \max_{k \in \mathbb{R}^+} \left\{ V(k, \theta) - k + P(W - \gamma k) - P(W) \right\}. \quad (30)
$$

By the envelope theorem,

$$
\frac{d[F^a(\theta, W) - F^n(\theta, W)]}{d\theta} = \frac{\partial V(k^a(\theta, W), \theta)}{\partial \theta} - \frac{\partial V(k^n(\theta, W), \theta)}{\partial \theta}
$$

$$
= \int_{k^n(\theta, W)}^{k^a(\theta, W)} \frac{\partial^2 V(k, \theta)}{\partial k \partial \theta} dk > 0,
$$

because $k^a(\theta, W) > k^n(\theta, W)$, as follows from (15) and (16), and $\partial^2 V(k, \theta) / \partial k \partial \theta > 0$ by Assumption 1. Therefore, the left-hand side of (18) is an increasing function of $\theta$.

Second, I use this result to conclude that Property 4 holds. There are three cases. First, if the left-hand side of (18) is above $c$ for all $\theta \in \Theta$, then it is optimal to audit all investment projects. Hence, $a^\ast(\theta, W) = 1$ for any $\theta \geq \bar{\theta} = \theta^\ast(W)$. Second, if the left-hand side of (18) is below $c$ for all $\theta \in \Theta$, then audit is not optimal for any investment project $\theta$. Hence, $a^\ast(\theta, W) = 0$ for any $\theta \leq \bar{\theta} < \theta^\ast(W)$. Finally, if the left-hand side of (18) is neither above nor below $c$ for all $\theta \in \Theta$, then the result that the left-hand side of (18) is a strictly increasing function of $\theta$ implies that there is a unique point $\theta^\ast(W) \in \Theta$ at which (18) holds as equality. In this case, the left-hand side of (18) is below $c$ (hence, $a^\ast(\theta, W) = 0$) for all $\theta < \theta^\ast(W)$ and above $c$ (hence, $a^\ast(\theta, W) = 1$) for $\theta \geq \theta^\ast(W)$.

**Proof of Proposition 1.** The goal is to verify that the direct mechanism conjectured in the proposition indeed maximizes headquarters’ value. The proof follows the logic of standard
problems in optimal control theory. First, I show headquarters’ value from any incentive compatible mechanism that delivers the initial expected value of $W_0$ to the manager is at most $P(W_0)$. Second, I argue that headquarters’ value from the mechanism that satisfies the conditions of the proposition and delivers the initial expected value of $W_0$ to the division manager is $P(W_0)$.

Let $G_t$ be defined as

$$ G_t \equiv \int_0^t e^{-rs} (V(dK_s, dX_s) dN_s - dK_s - dC_s - cdA_s) + e^{-rt} P(W_t) . $$ (32)

Consider an arbitrary direct mechanism satisfying incentive compatibility of truth-telling. Because any mechanism that wastes resources when there is no investment opportunity cannot be optimal, it is enough to restrict attention to mechanisms with $dK_t = 0$ if $d\hat{X}_t = 0$. The evolution of the division manager’s expected future utility implied by the mechanism is given by (6). Applying Itô’s lemma, multiplying by $e^{rt}$, and rearranging the terms,

$$ e^{rt} dG_t = V(dK_t, dX_t) dN_t - dK_t - cdA_t $$

$$ - \left( \lambda \int_\theta^\theta (V(dK_t, \theta) - dK_t - cdA_t) f(\theta) d\theta \right) dt $$

$$ + \left( \lambda \int_\theta^\theta (V(dK_t, \theta) - dK_t - cdA_t) f(\theta) d\theta \right) $$

$$ + \left[ \rho W_{t-} - \left( \lambda \int_\theta^\theta H_t(\theta) f(\theta) d\theta \right) \right] P'(W_{t-}) $$

$$ + \lambda \int_\theta^\theta [P(W_{t-} + H_t(\theta) - \gamma dK_t) - P(W_{t-})] f(\theta) d\theta - r P(W_{t-}) dt $$

$$ + (P'(W_{t-}) - 1) dC_t . $$ (33)

The expectation of the sum of the terms on the first two lines is zero. From (11), the sum of the terms on lines 3 - 5 is less than or equal to zero. Finally, because $P'(W_{t-}) \geq -1$, the term on line 6 is less than or equal to zero. Therefore, $(dG_t)_{t \geq 0}$ is a supermartingale.
Consider headquarters’ value at time 0. For any \( t < \infty \),

\[
\mathbb{E} \left[ \int_0^\infty e^{-r s} (V(dK_s, dX_s) dN_s - dK_s - dC_s - cdA_s) \right]
= \mathbb{E} [G_t] + e^{-rt} \mathbb{E} \left[ \int_t^\infty e^{-r(s-t)} (V(dK_s, dX_s) dN_s - dK_s - dC_s - cdA_s) \right] - P(W_t)
\leq P(W_0) + e^{-rt} \mathbb{E} \left[ P^0 - W_t - P(W_t) \right],
\]

where \( P^0 \) is the “first-best” value of operations such that investment that maximizes \( V(k, \theta) - (1 - \gamma) k \) is always made and audit never occurs. The last inequality holds, because \( \mathbb{E} [G_t] \leq P(W_0) \) by the supermartingale property of \( G \) and the value of operations can never exceed \( P^0 \). Letting \( t \to \infty \),

\[
\mathbb{E} \left[ \int_0^\infty e^{-r s} (V(dK_s, dX_s) - dK_s - dC_s - cdA_s) \right] \leq P(W_0).
\]

Therefore, headquarters’ expected utility from any incentive compatible mechanism that delivers the initial expected value of \( W_0 \) to the manager is at most \( P(W_0) \).

Suppose that the mechanism satisfies the conditions of the proposition. Then, \( G_t \) is a martingale. Therefore, headquarters’ initial expected payoff from the mechanism is \( G_0 = P(W_0) \). Consequently, this mechanism is optimal, since no other direct incentive compatible mechanism can achieve the initial expected payoff above \( P(W_0) \).

**Proof of Proposition 2.** The mechanism is optimal if and only if at any time \( t \) it leads to the same investment, audit, compensation policies, and evolution of the division manager’s expected future utility as the mechanism in Proposition 1.

First, I show that the evolution of \( \gamma B_t \) is the same as the evolution of \( W_t \) in Proposition 1. The starting point is \( \gamma B_0 = W_0 \) and the evolution of \( \gamma B_t \) if \( B_t < B^c \) and the division manager does not pass the project to headquarters is

\[
d (\gamma B_t) = (g(\gamma B_t) B_t dt - dK_t) \gamma.
\]

Hence, the evolutions of \( \gamma B_t \) and \( W_t \) are the same if the investment policies are the same. Because the change in the division manager’s utility, \( dW_t + \gamma dK_t = g(W_t) W_t dt \), does not depend on \( dK_t \), allocating the spending account between the current and future investment opportunities in the way that maximizes headquarters’ value is incentive compatible. The implied amount of investment solves
\[
\max_{k \in \mathbb{R}_+} \{ V(\theta, k) + P(\gamma (B_t - k)) \}.
\] (37)

Investment that solves this problem is exactly \( k^d(\theta, \gamma B_t) = k^d(\theta, W_t) \).

Consider the division manager’s decision to pass the project to headquarters. If the division manager believes that the audit would confirm that \( k^a(\theta, \gamma B_t) \geq k^*_t \), then the division manager finds it optimal to pass the project to headquarters, because her account balance would be unaffected and she would get additional utility from private benefits. By contrast, if the division manager believes that the audit would not confirm that \( k^a(\theta, \gamma B_t) \geq k^*_t \), then the division manager does not find it optimal to pass the project to headquarters, because she would get punished.

Next, the audit decisions implied by this mechanism are the same as the audit decisions in the mechanism in Proposition 1. Conditional on getting financed by headquarters, the optimal level of investment in a project is \( k^a(\theta, \gamma B_t) \). Because \( k^a(\theta, \gamma B_t) \) is an increasing function of \( \theta \) and \( k^*_t = k^a(\theta^*(\gamma B_t), \gamma B_t) \), the division manager will pass the project to headquarters if and only if \( \theta \geq \theta^*(\gamma B_t) = \theta^*(W_t) \). Therefore, this mechanism implies the same audit decisions as the mechanism in Proposition 1. Finally, the mechanism implies the same compensation of the division manager as the mechanism in Proposition 1.

**Randomization of the division manager’s promised utility.** Suppose that the optimal direct mechanism involves randomization of \( W \). Consider the budgeting mechanism with threshold division of authority from Proposition 2 and suppose in addition that headquarters allow the division manager to use her spending account to do any fair lottery. I argue that this mechanism is equivalent to the optimal direct mechanism. To see why, let \( W \) be any level of the division manager’s expected payoff that involves optimal randomization between some levels \( W_L \) and \( W_H \), with probabilities \( \pi \) and \( 1 - \pi \). By construction, \( W = \pi W_L + (1 - \pi) W_H \). Consider the division manager’s problem in the budgeting mechanism with threshold division of authority. Because the division manager is risk-neutral and any lottery is fair, she is indifferent between all lotteries at any point. Hence, she has incentives to run a lottery between \( B_L = W_L / \gamma \) and \( B_H = W_H / \gamma \) with probabilities \( \pi \) and \( 1 - \pi \), when her account balance is \( B = W / \gamma \). Hence, this mechanism implements optimal randomization. Combining this with the argument in Proposition 2 yields equivalence.
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