A LINTNER MODEL OF DIVIDENDS AND MANAGERIAL RENTS *

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Abstract

We develop a model where dividend payout, investment and financing decisions are made by managers who attempt to maximize the rents they take from the firm. But the threat of intervention by outside shareholders constrains rents and forces rents and dividends to move in lockstep. Managers are risk-averse, and their utility function allows for habit formation. We show that dividends follow Lintner’s (1956) target-adjustment model. We provide closed-form, structural expressions for the payout target and the partial adjustment coefficient. Risk aversion causes managers to underinvest, but habit formation mitigates the degree of underinvestment. Changes in corporate borrowing absorb fluctuations in earnings and investment.

Keywords: payout, investment, financing policy, agency (JEL: G31, G32)

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1 Introduction

John Lintner (1956) proposed this target-adjustment formula to describe dividend payout by a mature corporation:

$$\Delta \text{Dividend}_t = \kappa + \text{PAC} (\text{Target Dividend}_t - \text{Dividend}_{t-1}) + e_t \quad (1)$$

$\Delta \text{Dividend}_t$ is the change from the previous dividend at period $t - 1$, $\text{PAC} < 1$ is a partial adjustment coefficient, $\kappa$ is a constant and $e_t$ is an error term. The target dividend is the product of a target payout ratio and net income at $t$. The target payout ratio and the coefficients $\kappa$ and $\text{PAC}$ are assumed constant over time, although they can vary across firms. Thus dividends are based on current net income, but smoothed.

Smoothing means two related things. First, if net income includes transitory shocks, dividends are smoothed relative to income. Second, a permanent shift in expected future income does not cause an immediate proportional shift in dividends; instead dividends adapt gradually.

Lintner did not derive his dividend-smoothing model. He came to it inductively, based on interviews with 28 large, public manufacturing firms. His model remains the accepted starting point for analysis of how dividends behave over time. Yet his description of dividend smoothing has, as far as we can tell, never been derived formally. We will discuss some signaling theories that suggest smoothing, but none generates Lintner’s formula.

If dividends are smoothed, something else has to absorb fluctuations in operating profitability and capital investment. Consider the budget constraint, assuming for the moment that the firm does not issue or repurchase shares:

$$\Delta \text{Debt} + \text{Net income} = \text{CAPEX} + \text{Dividends} \quad (2)$$

Lintner’s formula says that changes in dividends absorb only part of the changes in net income. The remainder must be absorbed by changes in borrowing ($\Delta \text{Debt}$) or by capital investment ($\text{CAPEX}$). If dividends follow Linter’s model and CAPEX is nailed down by the firm’s investment opportunities, then $\Delta \text{Debt}$ must soak up most of the changes in net income.

Corporate-finance theory tends to ignore the budget constraint. There are separate
theories of dividends, debt and investment. But there can be no more than two separate
theories. Given this period’s net income, a theory of dividends plus a theory of investment
must imply a theory of debt. This paper presents a combined theory of dividends, debt
and investment. We derive and interpret Lintner’s dividend-payout formula. We also
show why borrowing or lending should be the chief shock absorber in the firm’s budget
constraint.

Our model also can be interpreted more broadly as a theory of total payout, defined
as dividends plus repurchases minus stock issues. Repurchases have been important only
recently, however, and seasoned equity issues are infrequent, at least for the mature cor-
porations that Lintner interviewed and that we have in mind. Therefore we start with
dividends, but circle back later to consider implications for total payout. It turns out that
the Lintner model is a good fit to total payout by large, blue-chip U.S. corporations that
pay regular dividends.

We assume that financial decisions are made by a coalition of managers, who maximize
the present value of their (utility from) future rents that they will take from the firm.
The managers in our model are entirely self-interested and have no loyalty to outside
shareholders. The shareholders have only the most basic and primitive property right,
which is the ability to take over the firm and throw out the managers if sufficiently provoked.
The managers therefore have to observe a capital-market constraint: they have to deliver
an adequate return to investors in each period by paying a sufficient cash dividend. In
equilibrium the managers do deliver adequate returns, and shareholders do not intervene.

Our model follows Myers (2000), Jin and Myers (2006) and Lambrecht and Myers (2007,
2008). But those papers assumed risk-neutral managers and did not analyze dividends, debt
and investment jointly. Here we assume a more realistic utility function, with risk aversion
and habit formation. We do adopt those papers’ view of managerial rents, however. We
are not defining rents as psychological private benefits, such as the CEO’s warm glow from
leading a big public firm. We define rents as real resources appropriated by a broad coalition
of managers and staff, including above-market salaries, job security, generous pensions and
perks. Rich labor contracts can generate a flow of rents to blue-collar employees.

Rents follow naturally from agency issues and imperfect corporate governance. Rents
can be efficient, however. They are necessary to reward managers’ investment in firm-specific human capital. Myers (2000) and Lambrecht and Myers (2008) also show how rents can align managers’ and shareholders’ interests if the managers maximize the present value of rents subject to a capital-market constraint.

We assume perfect, frictionless financial markets. Investors in our model do not care whether dividends are stable or erratic. (There are clienteles of investors who want smooth dividends – see Baker, Nagel, and Wurgler (2007), for example – but we do not invoke them to explain smoothing.) Debt and dividend policy turn out to be irrelevant for shareholders, as in Modigliani-Miller (1958, 1961).

Our main results include the following:

1. Dividends are smoothed because managers want to smooth their flow of rents. Rents and dividends move in lockstep. An attempt to smooth rents without smoothing dividends would violate the capital-market constraint.

2. Risk aversion means that rents depend on managers’ permanent income, which is proportional to the present value of the firm’s future net income. The response of rents and dividends to transitory changes in net income is an order of magnitude less than the response to persistent changes in net income. Thus dividends smooth out transitory shocks to income.

3. Habit formation means that rents and dividends respond gradually to permanent shifts in net income. The managers’ risk aversion and habit formation together lead to dividend smoothing according to Lintner’s target-adjustment model.

4. The Lintner constant $\kappa$ increases with managers’ subjective discount factor (impatience) and habit formation, but decreases with risk aversion and earnings volatility. The partial adjustment coefficient $PAC$ decreases with habit persistence and with the market discount factor. Dividend payout increases with better investor protection.

5. We explain the ”information content of dividends,” that is, the good (bad) news conveyed by dividend increases (cuts). Managers take rents based on their forecast of the firm’s permanent income. Thus dividend changes signal managers’ view of permanent income. The smaller the firm’s $PAC$, the greater the stock-price response to a given unanticipated
dividend change.


7. Given investment, changes in debt absorb all changes in income that are not soaked up by changes in dividends or rents. Once managers smooth rents and dividends, the change in debt is the only free variable in the budget constraint. Thus we arrive at a theory of debt dynamics, similar to the pecking order, but not by relying on asymmetric information and adverse selection, as in Myers and Majluf (1984) and Myers (1984). Equity issues can be used to finance part of CAPEX, however.

We are not attempting a Theory of Everything. Our model is designed for mature, profitable, creditworthy public corporations that have access to debt and the ability to use borrowing and lending as the balancing items in their budget constraints. Our model would not apply to zero-dividend growth firms or to firms in financial distress. It would not apply to declining firms that should disinvest, as in Lambrecht and Myers (2007). Our goal is to understand dividend policy and how dividend payout interacts with borrowing and investment. Therefore we focus on mature companies that can make regular payouts to shareholders.

Section 2 of the paper solves for the optimal payout and debt policy for a given (sunk) level of investment. We analyze dividend payout policy, how dividend policy affects the firm’s stock price, and how dividend policy interacts with debt policy. We prove that rent smoothing necessarily implies dividend smoothing. We interpret the ”information content of dividends.” We also derive the managers’ optimal investment policy and its implications for payout and debt policy. The concluding Section 3 reviews empirical implications, including predictions about total payout.

The rest of this introduction does two things. First, it explains why rent smoothing necessarily implies dividend smoothing. The explanation will introduce the assumptions, setup and economic intuition of our model. Second, it reviews relevant literature in more detail.
1.1 Rent smoothing and dividend smoothing

The following example illustrates how dividend smoothing follows from rent smoothing. Start with the following market-value balance sheet:

<table>
<thead>
<tr>
<th>$V(K)$</th>
<th>$D_t$</th>
<th>Interest on debt = $\rho D_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_t$</td>
<td>Annual rents = $r_t$</td>
</tr>
<tr>
<td></td>
<td>$S_t$</td>
<td>Annual Dividends = $d_t$</td>
</tr>
<tr>
<td>$V_t$</td>
<td>$V_t$</td>
<td></td>
</tr>
</tbody>
</table>

The firm holds a capital stock $K$, which generates income with present value $V(K)$. There are three claims on this value: debt ($D_t$), the present value of managerial rents ($R_t$) and outside equity ($S_t$), with $R_t + S_t = V(K) - D_t$. The flow of rents is $r_t$. (In practice the rents will often be received as job security or perks, but here we model rents as just a flow of cash to managers.) The dividend is $d_t$. The interest rate is $\rho$. Debt service is senior to both rents and dividends. We assume that lenders and equity investors are risk-neutral.

Managers maximize the present value of their lifetime utility from all future rents, subject to a capital-market constraint. They are constrained by the shareholders’ property right to intervene and take over the company. If they do so, the managers get nothing ($R_t = 0$). But the shareholders face a cost of collective action. Their net payoff from intervening is $\alpha(V(K) - D_t)$, with $\alpha < 1$. (Think of $\alpha$ as a governance parameter capturing the shareholders’ practical property rights and the effectiveness of corporate governance.) In equilibrium the shareholders do not intervene, because it is in the managers’ interest to deliver an adequate return of $\rho \alpha (V(K) - D_t)$. The conditions for this equilibrium are described in Myers (2000) and set out more formally in Section 2.

The gross profits generated over the period $(t-1, t]$ are realized at time $t$ and given by

\footnote{The parameter $\alpha$ could also reflect portable human capital that contributes to the firm’s earnings. Shareholders could take over the firm, but still have to give up $(1 - \alpha)(\pi(K_{t-1}) - \rho D_{t-1})$ to pay for the human capital or replace it.}
\( \pi_t(K) \). Net income (after interest but before rents) is \( \pi_t(K) - \rho D_{t-1} \). Suppose that capital is sunk and constant at \( K \). With no CAPEX, the budget constraint for period \( t \) is:

\[
d_t + r_t = \pi_t(K) - \rho D_{t-1} + (D_t - D_{t-1})
\]  

If debt is constant (\( \Delta D = D_t - D_{t-1} = 0 \)), the equilibrium payout policy simply splits net income, \( \alpha(\pi(K) - \rho D_{t-1}) \) to dividends and \( (1 - \alpha)(\pi(K_{t-1}) - \rho D_{t-1}) \) to rents. With this payout policy, rents and dividends follow net income, always in the ratio \( \alpha/(1 - \alpha) \). Because all future income will also be split in this ratio, values are \( S_t = \alpha(V(K) - D_t) \) and \( R_t = (1 - \alpha)(V(K) - D_t) \). Managers would of course like to reduce dividends and take more rents, but cannot do so without violating the capital market constraint. The managers pay no more dividends than necessary, so the capital market constraint pins down dividends, rents and values exactly.

Thus value is split between managers and shareholders. From the shareholders’ viewpoint, the managers own the fraction \( (1 - \alpha) \) of the equity. But the managers, unlike the investors, are assumed to be wealth-constrained. Their claims to future rents are not tradeable, for the usual reasons of moral hazard and non-verifiability. If the managers could trade their claims, their risk aversion and habit formation would not matter.

Now suppose that managers want to smooth rents, for example by taking more than \( (1 - \alpha)(\pi(K) - \rho D_{t-1}) \) when profitability declines. They cannot raise rents by cutting dividends. But they can increase both rents and dividends by taking on more corporate debt. If the firm borrows \( \Delta D \), they can keep \( (1 - \alpha)\Delta D \) as additional rents, provided that they simultaneously pay out \( \alpha \Delta D \) in additional dividends. Thus rents can be smoothed by changes in debt, but dividends must be smoothed along the same time pattern.

Suppose that \( \alpha = .8 \). For the managers to take $1 in additional rents, the firm has to borrow \( \Delta D = 5 \), with $4 paid out as an additional dividend. The shareholders’ claim is reduced by 80% of \( \Delta D \), so they have to be given $4 extra. The managers’ claim is reduced by 20% of \( \Delta D \), but they get $1 in extra rents.

Smoothing also means gradual adjustment of rents when profitability increases. If growth in rents is held back, then dividend growth has to be held back to exactly the same extent. Otherwise shareholders would get a free gift from the managers. The cash released by holding back rents and dividends has to be used to pay down debt, however.
For example, reducing growth in rents by $1 requires reducing growth in dividends by $4 and paying off $5 of debt. (If paying down debt is inconvenient in the short run, the firm can invest the $5 in money-market or other debt securities. Net debt is still reduced by $5. Net debt is what matters in our model.)

When a corporation borrows, the debt is partly a claim against equity and partly a claim against the present value of managers’ future rents (Lambrecht and Myers (2008)). If not invested, the proceeds of additional borrowing must be distributed to managers and shareholders in the ratio $\alpha/(1 - \alpha)$. If cash flow is used to pay down debt, rents and dividends must be reduced in the same proportions. Thus rents and dividends have to move in lockstep. The shock absorber is corporate debt.

We believe the idea that dividend smoothing follows from rent smoothing is new. Therefore we develop this idea and its implications. We are not claiming that rent smoothing is the only reason for dividend smoothing.

1.2 Research on Dividends and Dividend Smoothing

The starting point of any theory of dividend payout is the Miller and Modigliani (1961) proof of dividend irrelevance with frictionless financial markets and complete information. Subsequent research has focused on the roles of taxes, information, agency costs and other imperfections. A full review of this literature is impossible here. We refer instead to excellent literature surveys by Allen and Michaely (2003), Kalay and Lemmon (2008), DeAngelo, DeAngelo, and Skinner (2008), and also the survey evidence in Brav, Graham, Harvey, and Michaely (2005). These surveys cite no derivations of the Lintner (1956) model.

Lintner’s paper was a breakthrough contribution to empirical corporate finance, but we do not claim that his target-adjustment specification fully explains dividend policy today. Brav, Graham, Harvey, and Michaely (2005) find that target payout ratios are less important now than in Lintner’s day, and the speed of adjustment has declined. Also the volume of repurchases has grown enormously. Skinner (2008, p. 584) concludes that repurchases have substituted for cash dividends and ”are now the dominant form of payout.”
Some mature, blue-chip firms – Exxon Mobil, for example – pay steady cash dividends and also repurchase shares year in and year out. Many smaller firms do not pay dividends and repurchase irregularly. We focus on companies that pay regular cash dividends, however. We will build and discuss our model assuming that cash dividends are the only form of payout. We turn later to our model’s implications for total payout, including repurchases net of stock issues.

Casual explanations of dividend smoothing sometimes start with the “information content of dividends.” One might overhear the following: ”Dividends have information content because investors expect managers to smooth dividends and to increase dividends only when they are confident about future income. Managers smooth dividends because they don’t want to send a false positive signal to investors.” Statements like this either assume some kind of smoothing or are close to circular.

The causes of dividend smoothing are not clear in prior theory. The dividend signaling models of Bhattacharya (1979), Miller and Rock (1985), and John and Williams (1985) explain why dividends can convey information, but do not explain smoothing. They are one-period exercises that explain dividend levels but not dividend changes. Some other papers suggest smoothing but not the Lintner model specifically. For example, Kumar (1988) derives a coarse signaling equilibrium in which a firm’s dividends are more stable than its performance and prospects. Allen, Bernardo, and Welch (2000) argue that well-managed firms pay dividends to attract institutional investors and to weed out tax-paying retail investors. The less well-managed firms turn to retail investors. This theory could accommodate smoothing if the dividing line between high- and low-dividend payers is stable.

The surveys by Allen and Michaely (2003) and Leary and Michaely (2008) conclude that dividend policy is better explained by agency problems than by signalling. Roberts and Michaely (2007) show that private firms smooth dividends less than their public counterparts, suggesting that the scrutiny of public capital markets leads firms to pay and smooth

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2 Miller (1987) reviews conditions for a dividend signaling equilibrium, but finds no satisfactory explanation of Linter-style dividend smoothing or the information content of dividends.

3 Fudenberg and Tirole (1995) develop a model in which managers smooth income in order to protect their jobs and private benefits. All reported income is paid out as dividends, which are thus also smoothed. But the Lintner model, backed up by ample facts, says that dividends are smoothed relative to income.
dividends. Ours is an agency model, but with a capital market constraint that forces managers to smooth dividends if they decide to smooth rents. La Porta et al. (2000) survey dividend policies worldwide and conclude that companies pay dividends because investors have (more or less imperfect) governance mechanisms that force payout.

Our paper uses insights and methods from theories of household consumption, starting with the permanent income hypothesis (PIH) of Friedman (1957). The PIH states that consumers’ consumption choices are determined not by their current income but by their longer-term income expectations. Therefore transitory, short-term changes in income have little effect on consumer spending behavior. Hall (1978) formalizes the PIH by deriving a relation between income and consumption in an intertemporal stochastic optimization framework. The assumption of quadratic utility in Hall (1978) (and in many subsequent models) switches off consumers’ motives for precautionary savings, however. Caballero (1990) shows that when marginal utility is convex, agents have an incentive to accumulate savings as a precautionary measure against income shocks.

Research on asset pricing has stressed the importance of habit formation and the links between today’s consumption and the marginal utility of future consumption. Our paper is closest to “internal habit” models, such as Muellbauer (1988), Sundaresan (1989), Constantinides (1990) and Alessie and Lusardi (1997).

Of course we are not modeling an individual manager’s utility function, but the combined utility of a coalition of managers. One can think of a ”representative manager,” like a representative agent in asset-pricing theory, or simply accept the idea of a coalition as a reduced-form description of how managers behave. (Acharya, Myers, and Rajan (2009) show how a coalition of managers can form to invest and operate the firm, even with weak or no outside governance.) But it is clearly reasonable to assume that managers as a group are risk averse. Habit formation also comes naturally. Many forms of rents, including above-market wages, job security and pension benefits, are not normally changed on short notice. The assumption of a rent-seeking coalition of managers has proved fruitful in prior work, including Myers (2000), Jin and Myers (2006) and Lambrecht and Myers (2007, 2008).
2 How managers set rents and dividends

Managers undertake financing and payout decisions in order to maximize the present value of their life-time utility. Shareholders are risk neutral, but managers are risk averse with a concave utility function. The managers are also subject to habit formation. We assume their utility of current rents is \( u(r_t - hr_{t-1}) \). The reference point \( hr_{t-1} \) is determined by last period’s rents \( r_{t-1} \) and the habit persistence coefficient \( h \in [0, 1) \). Habit formation means that utility is no longer time-separable.

At each time \( t \) the infinitely-lived managers choose a payout \((d_t, r_t)\) policy that maximizes the objective function:

\[
\max E_t \left[ \sum_{j=0}^{\infty} \omega^j u(r_{t+j} - hr_{t+j-1}) \right]
\]

where \( \omega \) is the managers’ subjective discount factor and \( \frac{1}{\omega} \) measures “impatience.” The market discount factor is \( \beta \equiv \frac{1}{1+\rho} \) where \( \rho \) is the risk-free rate of return. We assume \( \omega \leq \beta \), so that managers can be more impatient than investors.\(^4\)

Managers maximize their life-time utility subject to the following constraints that need to be satisfied at all times:

\[
S_t \equiv \sum_{j=1}^{\infty} E_t[d_{t+j}]\beta^j \geq \alpha \left[ \sum_{j=1}^{\infty} \beta^j K^\phi E_t[\pi_{t+j}(\eta_{t+j})] - D_t \right] \equiv \alpha [V_t - D_t] \quad (5)
\]

\[
D_t = D_{t-1}(1 + \rho) + d_t + r_t - K^\phi \pi_t(\eta_t) \quad (6)
\]

\[
\lim_{j \to \infty} \frac{D_{t+j}}{(1+\rho)^j} = 0 \quad (7)
\]

\( K^\phi \pi_t \) is the operating profit at time \( t \). For now we take \( K \), the amount of capital that has been invested in the firm, as fixed and constant. The amount of output produced each period is \( K^\phi \), with decreasing returns to scale \( (\phi < 1) \). \( \pi_t(\eta_t) \) is the operating profit per unit of output, which depends on the realization of a demand shock \( \eta_t \). The demand shock is exogenous and not affected by rents and dividends at time \( t \). Dividends and managerial rents are declared and paid at the end of each period, after operating profit is realized and interest is paid on start-of-period debt.

\(^4\)Managers’ will also be more impatient if they face a probability of termination in each future period. In that case \( \omega = \beta \zeta \), where \( \zeta \) is managers’ constant survival probability.
$D_t$ is net debt. If $D_t > 0$, the firm is a net borrower. If $D_t < 0$, the firm holds a surplus in liquid assets and is a net lender. The rate of interest $\rho$ is the same for financial assets and liabilities. For simplicity we ignore default risk.\(^5\) The firm can borrow and lend at the risk-free rate $\rho$.

Eq. (5) is the capital market constraint, which requires that dividend policy always supports an equity value $S_t$ that at least equals what shareholders can get from taking over. The net payoff to shareholders from taking over is $\alpha (V_t - D_t)$, with $0 < \alpha < 1$.\(^6\)

Eq. (6) is the firm’s budget constraint. The operating profit $K^\phi \pi_t$ is used to pay interest $(\rho D_{t-1})$, dividends $(d_t)$ and managerial rents $(r_t)$. Any surplus or deficit leads to a reduction or increase in debt. Debt is therefore a balancing variable that follows from the payout policy $(r_t, d_t)$ (and from investment policy, as will become clear later). The optimal debt policy allows the managers to take their optimal rents. The accounting equality between sources and uses of cash pins down debt policy once payout policy have been chosen.

Eq. (7) is a constraint that prevents the managers from running a Ponzi scheme in which they borrow to achieve an immediate increase in rents and then borrow forever after to pay the interest on the debt. The constraint prevents debt from growing faster than the interest rate $\rho$, so that claim values are bounded.

Since the budget constraint needs to be satisfied for all future times $t$, repeated forward substitution of the budget constraint Eq. (6), combined with the no-Ponzi constraint (7) gives the following intertemporal budget constraint (IBC):

$$
\sum_{j=0}^{\infty} \beta^j \left[ K^\phi \pi_{t+j} - d_{t+j} - r_{t+j} \right] = (1 + \rho)D_{t-1}
$$

The IBC gives a condition that a feasible payout plan $\{r_{t+j}, d_{t+j}\}$ ($j = 0, 1, 2, ...$) must

\(^5\)Default risk should be second-order for mature corporations that make regular payouts and have ample debt capacity. Modeling shareholders’ default put would add a heavy layer of complication. See Lambrecht and Myers (2008), who analyze the effect of default risk on debt, payout and investment policy in a continuous-time model with managerial rents and a capital market constraint.

\(^6\)For $\alpha = 0$ shareholders have no stake in the firm and the capital market constraint disappears. For $\alpha = 1$ managers can no longer capture rents and their objective function is no longer defined. Therefore $\alpha \in (0, 1)$.
satisfy. The condition essentially states that the sum of the managers’, bondholders’ and shareholders’ claims must add up to the present value of all future operating profits. Since $K$ is fixed and the profit process $\pi_{t+j}$ is exogenous (and not affected by payout policy), the IBC requires that any increases in rents and dividends at time $t$ must be compensated for by future decreases. After taking expectations and simplifying, the IBC becomes:

$$R_t \equiv \sum_{j=1}^{\infty} \beta^j E_t(r_{t+j}) = V_t - S_t - D_t$$

Note that $R_t$ equals the market value of the managers’ future, not the private value of rents that is being optimized. The difference arises because the managers are risk-averse and cannot sell or borrow against their future rents in financial markets.

The managers’ decision problem is sequential. At time $t$ the managers decide on the optimal level for $r_t$ and $d_t$ given the values for $r_{t-1}, d_{t-1}, D_{t-1}$ and $\eta_t$ (and given their expectations about the future realizations for $\eta_{t+j}$ ($j > t$)). To solve the optimization problem explicitly, we need to make assumptions about the managers’ utility function $u(.)$ and the stochastic process $\pi_t(\eta_t)$. We assume that managers have exponential utility $u(x) = 1 - \frac{1}{\theta} e^{-\theta x}$. This utility function has been used extensively in the household consumption literature because of its tractability. We assume that $\pi_t$ follows the autoregressive process $\pi_t = \mu \pi_{t-1} + \eta_t$ with $0 < \mu < 1$ (the process for $\pi_t$ is therefore stationary).

The shocks $\eta_{t+j}$ ($j = 0, 1, \ldots$) are independently and identically normally distributed with zero mean and volatility $\sigma_\eta$. Thus $E_t(\eta_{t+j}) = 0$, $E_t(\eta_{t+j}^2) = \sigma_\eta^2$ and $E_t(\eta_{t+j} \eta_{t+j+1}) = 0$ for all $j$.

The following proposition describes the linkage between dividends and managers’ rents.

**Proposition 1** Dividend payout $d_t$ is proportional to managers’ rents $r_t$, with $d_t = \left(\frac{\alpha_1}{1-\alpha}\right) r_t \equiv \gamma r_t$.

Thus dividends and rents are locked together in the ratio $\frac{d_t}{r_t} = \frac{\alpha_1}{1-\alpha} \equiv \gamma$. We show in the appendix that this result is a direct consequence of the collective action constraint

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7Our assumption of exponential utility and normally distributed shocks can lead to negative rents and dividends, which we would interpret as equity issues and managerial sweat equity. These assumptions could also lead to negative stock prices, which are impossible with limited liability. But default risk is remote for the mature and stable firms that our model is designed for. Therefore we ignore default risk for simplicity.
and does not depend on managers’ utility function. The dividend \( d_t \) is set by managers so that shareholders are indifferent between taking collective action at time \( t \) and letting managers carry on for another period. As managers raise the rent level \( r_t \), this reduces the payoffs shareholders can expect from taking collective action at time \( t + 1 \), so shareholders require a higher dividend \( d_t \) up-front. Dividends therefore move in lockstep with rents.

The following proposition gives the solution to the managers’ dynamic optimization problem.

**Proposition 2** The managers’ optimal rent policy \( r_t \) at time \( t \) is given by:

\[
 r_t = \beta h r_{t-1} + (1 - h \beta)(1 - \alpha) Y_t + c
\]

where \( c \equiv \left( \frac{\beta}{(1 - \beta) \theta} \right) \ln \left( \frac{\beta}{\omega} \right) - \frac{(1 - \alpha)^2 \beta (1 - \beta)(1 - h \beta)^2}{(1 - \mu)^2} \frac{\theta}{2} K^{2\phi} \sigma_n^2 \)

\( Y_t \) is the firm’s permanent income:

\[
 Y_t = \rho \beta \sum_{j=0}^{\infty} \beta^j E_t \left[ K^{\phi} \pi_t + j (\eta_{t+j}) \right] - \rho D_{t-1}
\]

The proposition contains the paper’s core results, which allow us to analyze (1) optimal dividend policy, (2) how it influences stock prices and (3) how dividend policy interacts with debt policy.

### 2.1 Optimal Dividend Policy

Eq. (10) implies that in the presence of habit formation \( (h > 0) \) dividends follow Lintner’s target-adjustment model. Subtracting \( r_{t-1} \) from both sides of Eq. (10) and expressing rents \( r_t \) in terms of dividends (using \( d_t = \gamma r_t \)) gives the following corollary:

**Corollary 1** The firm’s dividend policy is given by the following target-adjustment model:

\[
 d_t - d_{t-1} = (1 - \beta h) (\alpha Y_t - d_{t-1}) + \kappa
\]

where \( \kappa \equiv \frac{\alpha c}{1 - \alpha} = \left[ \left( \frac{\alpha \beta}{(1 - \alpha)(1 - \beta) \theta} \right) \ln \left( \frac{\beta}{\omega} \right) - \alpha (1 - \alpha) \left( \frac{\beta (1 - \beta)(1 - h \beta)^2}{(1 - \mu)^2} \right) \frac{\theta}{2} K^{2\phi} \sigma_n^2 \right]

13
Permanent income $Y_t$ is the rate of return on the sum of current and the present value of all future net income, net of debt service, but before rents. It is an annuity payment that, given expectations at time $t$, could be sustained forever. The partial adjustment coefficient $PAC \equiv (1 - \beta h)$ depends on the managers’s subjective discount factor $\beta$ and their habit persistence parameter $h$. Absent habit formation ($h = 0$), the previous dividend’s deviation from the current target is fully adjusted for in each period, that is, $d_t - d_{t-1} = (\alpha Y_t - d_{t-1}) + \kappa$. The target dividend is $\alpha Y_t$, so higher level of investor protection $\alpha$ increases target payout.

The constant $\kappa$ in the partial adjustment model can be expressed as the difference between managers’ dissavings due to impatience and their precautionary savings due to risk-aversion. The first dissavings term is positive (zero) for $\omega < \beta$ ($\omega = \beta$). Increased impatience raises current dividends at the expense of future dividends. This property follows directly from the first order condition (see appendix), which requires that the expected marginal utility from rents grows by a factor $\frac{\beta}{\omega}$ along the optimal path. Increased investor protection (higher $\alpha$) raises the dissavings term.

The second, negative term in the formula for the constant $\kappa$ corresponds to the standard precautionary savings term from the household consumption literature (see e.g. Caballero (1990)). A higher risk aversion coefficient $\theta$ and an autoregressive coefficient $\mu$ each increase the amount of precautionary savings and therefore reduce dividends. The higher the earnings volatility $K^\phi \sigma_\eta$, the more managers cut rents in order to save for a rainy day. More uncertainty therefore leads to higher planned payout growth. Precautionary savings increase with the autoregressive coefficient ($\mu$) and decrease with habit formation ($h$). Since habit formation by itself induces higher savings, it reduces the need for additional precautionary savings, which explains why the precautionary savings term decreases with $h$.

Whether the constant term $\kappa$ in the Lintner model is positive or negative depends on the relative importance of dissavings due to managerial impatience and precautionary savings due to earnings volatility and risk aversion. Stronger impatience (high $\frac{\beta}{\omega}$) increases the

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\[8\] If $\pi_t$ follows an autoregressive process with non-zero drift $\mu_0$ (i.e. $\pi_t = \mu_0 + \mu \pi_{t-1} + \eta_t$) then the constant $c$ (and therefore $\kappa$) would include an additional term in $\mu_0$. In particular, a higher (lower) drift $\mu_0$ would increase (reduce) the constant $\kappa$ in the partial adjustment model.
dissavings term. Higher risk aversion \((\theta)\) reduces dissavings and increases precautionary savings, and higher earnings volatility increases precautionary savings. The constant term is a U-shaped function of \(\alpha\).\(^9\) These results can be summarized in the following corollary.

**Corollary 2** If managers have exponential utility, then the constant term in the Lintner model increases with managers’ impatience and habit formation but decreases with risk aversion and earnings volatility. The constant is a U-shaped function of investor protection.

We have some empirical evidence on the Lintner constant. Lintner (1956) said that “The constant will be zero for some companies but will generally be positive to reflect the greater reluctance to reduce than to raise dividends ... as well as the influence of the specific desire for a gradual growth in dividend payments found in about a third of the companies visited.” Fama and Babiak (1968) found that the constant in the Lintner model was usually positive but insignificant, which suggests that dissavings due to impatience marginally outweighed managers’ precautionary savings. This might come as a surprise if one believes that the subjective discount rate approximately equals the market discount rate, and therefore that dissavings due to impatience are small. Habit formation dramatically reduces the amount of precautionary savings, however. Note that the precautionary savings term includes the squared partial adjustment coefficient \((1 - \beta h)^2\). For a typical Lintner \(PAC\) of 0.3, this implies that habit formation reduces precautionary savings by over 90%! Therefore, even if dissavings due to managerial impatience are small, they could still outweigh precautionary savings and generate a positive constant.

The partial adjustment coefficient \(PAC\) is determined by the risk-free rate \(\rho\) and the managers’ habit persistence coefficient \(h\). As \(h\) increases, managers’ cost of adjusting towards a new target rent level goes up, so dividends become “stickier” and less responsive to changes in permanent income.

The market discount factor \(\beta\) does not enter directly into the managers’ utility function, but \(\beta\) matters because of the IBC. The discount rate \(\rho\) sets the terms at which managers can move cash through time by changes in corporate borrowing. When \(\rho\) is high and \(\beta\) is low, it costs more to borrow against future cash flows, so managers smooth less and adjust

\(^9\)The constant term approaches zero from below as \(\alpha \to 0\) and goes towards positive infinity as \(\alpha \to 1\).
more quickly to shocks ($\frac{\partial P_{AC}}{\partial \beta} < 0$). The overall effect of the market discount rate $\rho$ on dividends is more complicated, however, because $\rho$ affects not only the partial adjustment coefficient $P_{AC}$, but also permanent income $Y_t$.

A dollar increase in the firm’s permanent income leads to an immediate increase in dividends of only $P_{AC}\alpha$. The lagged incremental effects in subsequent periods are given by $P_{AC}\alpha\beta h$, $P_{AC}\alpha (\beta h)^2$, $P_{AC}\alpha (\beta h)^3$, ... . The total, cumulative long run effect of a dollar increase in permanent income on dividends equals $P_{AC}\alpha \sum^{\infty}_{j=0}(\beta h)^j = \alpha$. Our derivation of the Lintner model can therefore be expressed as a distributed lag model in which current dividends are a function of current and past permanent income. Repeated backward substitution of (13) gives:

$$d_t = (1 - h\beta) \alpha \sum^{\infty}_{j=0}(\beta h)^j Y_{t-j} + \frac{\kappa}{1 - \beta h}$$  \hspace{1cm} (14)

Now we turn to permanent income $Y_t$ and its effect on dividends. Using the IBC it is straightforward to prove the following property.

**Property 1** The following property results directly from the IBC and is valid for all utility functions:

$$E_t[Y_{t+1}] = Y_t + \rho(Y_t - (1 + \gamma)r_t)$$  \hspace{1cm} (15)

Therefore permanent income follows a martingale process if and only if the total (shareholders’ plus managers’) payout is $(1 + \gamma)r_t = Y_t$. Proposition 2 gives this result if and only if both $\omega = \beta$ and $h = \sigma_\eta = 0$. The martingale property no longer holds in all other cases. For example, if current dividends are above (below) the target dividends, then permanent income is expected to go down (up) next period.

The IBC implies that the long run total payout target equals permanent income $Y_t$. Proposition 2 implies that a lower level of expected permanent income $E_t[Y_{t+1}]$ reduces expected payout:

$$E_t[d_{t+1}] - d_t = \beta h(d_t - d_{t-1}) + (1 - \beta h)\alpha [E_t[Y_{t+1}] - Y_t]$$  \hspace{1cm} (16)

Substituting property 1 of permanent income for expected dividend changes gives:

$$E_t[d_{t+1}] - d_t = \beta h(d_t - d_{t-1}) + (1 - \beta h)\rho [\alpha Y_t - d_t]$$  \hspace{1cm} (17)
Expected dividend changes are therefore a weighted average of lagged dividend changes and the deviation of the long run dividend target $\alpha Y_t$ from current dividends.

If there is no habit formation and uncertainty ($h = \sigma = 0$), and if $\omega = \beta$, then dividends are always on target with $E_t[d_{t+1}] = d_t$. Dividends are still smoothed, however, because the firm gears payout to permanent income ($Y_t$) rather than current net income $\pi_t - \rho D_{t-1}$.

2.2 Estimation

Next we consider how one could estimate the dividend model in proposition 2. Eq. (10) implies the following econometric specification for a firm’s dividends:

$$d_t = a_0 + a_1 d_{t-1} + a_2 Y_t + e_t$$  \hspace{1cm} (18)$$

where $a_0$, $a_1$ and $a_2$ are the regression coefficients and $e_t$ the error term. Permanent income $Y_t$ is not observable, but it could be estimated from current operating profit and the market’s expectation of future profits. If the profit margin $\pi_t$ follows the autoregressive process $\pi_t = \mu \pi_{t-1} + \eta_t$, then permanent income is $Y_t = \frac{\rho}{1+\rho-\mu} \left( K^\phi \pi_t - (1 + \rho - \mu) D_{t-1} \right)$. For example, in the limiting case where $\pi_t$ follows a random walk ($\mu = 1$) permanent income equals $K^\phi \pi_t - \rho D_{t-1}$, which is net income after interest repayments.

Lintner found a $PAC$ of about 0.3 using aggregate data on corporate earnings and dividends. Fama and Babiak (1968) tested Lintner’s model for individual firms over a 20-year period and reported a mean $PAC$ of 0.32 and a mean target payout ratio of 0.52. Their estimate of the constant term was positive but small, with a mean and median of 0.109 and 0.028. For most firms the constant was not significantly different from zero.

These estimates for the Lintner model allow some inferences about the underlying parameters. Assume a market discount factor of $\beta = 0.95$ and $PAC = 0.32$ from Fama and Babiak (1968). Then $h = (1 - 0.32)/0.95 = 0.72$. We can also infer from the insignificant, positive Lintner constant that managers are less patient than the market ($\omega < \beta$), but that the wedge between $\omega$ and $\beta$ must be small.
We can also write the target-adjustment model (10) in first differences:

\[ \Delta d_t = \beta h \Delta d_{t-1} + (1 - \beta h) \alpha \Delta Y_t \] (19)

Here we need an expression for \( \Delta Y_t \), the change in permanent income:

Property 2 The following property results directly from the IBC and is valid for all utility functions:

\[ Y_t - Y_{t-1} = \rho (Y_{t-1} - (1 + \gamma) r_{t-1}) + \nu_t \] (20)

where \( \nu_t \) is white noise defined by: \( \nu_t \equiv \rho \sum_{i=0}^{\infty} \beta^i K \sigma_\eta^2 (E_t(\pi_{t+i}) - E_{t-1}(\pi_{t+i})) = \frac{\rho K \sigma_\eta}{1 - \beta \mu} \)

Using Eq. (10) one can write \( Y_{t-1} \) as a function of \( r_{t-1} \) and substitute for \( Y_{t-1} \) into property 2, which gives \( \Delta Y_t \) as a function of \( r_{t-1} \) and \( r_{t-2} \). Substituting the resulting expression for \( \Delta Y_t \) into (19) gives \( \Delta r_t \) as a function of the exogenous variables \( r_{t-1} \) and \( r_{t-2} \) only, as stated in the following corollary:

Corollary 3 If managers have negative exponential utility, then the changes in permanent income, rents and dividends are:

\[ \Delta Y_t = \frac{\rho (\beta h \Delta r_{t-1} - c)}{(1 - \beta h)(1 - \alpha)} + \nu_t \] (21)

\[ \Delta r_t = h \Delta r_{t-1} - \rho c + (1 - \alpha)(1 - \beta h) \nu_t \] (22)

\[ \Delta d_t = h \Delta d_{t-1} - \frac{\alpha \rho c}{1 - \alpha} + \alpha (1 - \beta h) \nu_t \] (23)

\[ \text{var}(\Delta r_t) = \Lambda^2 (1 - \alpha)^2 K^{2\phi} \sigma_\eta^2 \quad \text{and} \quad \text{var}(\Delta d_t) = \Lambda^2 \alpha^2 K^{2\phi} \sigma_\eta^2 \] (24)

\[ \text{var}(\Delta d_t + \Delta r_t) = \text{var}(\Delta d_t) + 2 \text{cov}(\gamma \Delta r_t, \Delta r_t) + \text{var}(\Delta r_t) = \Lambda^2 K^{2\phi} \sigma_\eta^2 \] (25)

where \( \Lambda = \frac{(1 - \beta h)(1 - \beta)}{1 - \beta \mu} < 1 \) and \( \nu_t \) is white noise as defined in property 2.

Since \( \nu_t \) is white noise, \( h \) can be estimated from the vector autoregression (23), which allows us to calculate the partial adjustment coefficient \( 1 - \beta h \). The advantage of the regression model (23) is that we do not need to know permanent income. In fact, our estimate for \( h \) allows us to calculate the permanent income that is implied by a given dividend series as:

\[ Y_t = \frac{(d_t - \beta h d_{t-1}) - \alpha c}{\alpha(1 - \beta h)} \] (26)
The variance of dividend changes are a fraction $\Lambda^2\alpha^2$ of the variance of operating income $(K^{2\phi}\sigma^2_\eta)$.\textsuperscript{10}

Finally we consider the effects of transitory and persistent earnings shocks on dividends. A transitory shock $\tau_t$ only affects current (gross) earnings and therefore has a “one-off” effect. A persistent shock $\eta_t$ affects not only current earnings but also the expected value of all future earnings. In other words:

$$\frac{\partial K^{\phi\pi_t}}{\partial \tau_t} > 0 \quad \text{and} \quad \frac{\partial K^{\phi\pi_{t+j}}}{\partial \tau_t} = 0 \quad \text{for all } j > 0$$

$$\frac{\partial K^{\phi\pi_t}}{\partial \eta_t} > 0 \quad \text{and} \quad \frac{\partial K^{\phi\pi_{t+j}}}{\partial \eta_t} > 0 \quad \text{for all } j > 0$$ \hspace{1cm} (27)

Suppose we allow for transitory shocks by assuming that $\pi_t = p_t + \tau_t$ with $p_t = \mu p_{t-1} + \eta_t$, where $\eta_t$ and $\tau_t$ are iid shocks. Set $K = 1$, so that a $1$ transitory shock $\tau_t$ increases operating profits at time $t$ by exactly $1$. Therefore:

$$\frac{\partial \pi_t}{\partial \tau_t} = 1 \quad \text{and} \quad \frac{\partial \pi_{t+j}}{\partial \eta_t} = \mu^j \text{ with } \mu > 0 \text{ for } j = 0, 1, 2...$$

The marginal effects of a transitory and a persistent shock on permanent income are given by:

$$\frac{\partial Y_t}{\partial \tau_t} = \rho \beta \frac{\partial \pi_t}{\partial \tau_t} = \rho \beta$$

$$\frac{\partial Y_t}{\partial \eta_t} = \rho \beta \left[ \frac{\partial \pi_t}{\partial \eta_t} + \beta \frac{\partial \pi_{t+1}}{\partial \eta_t} + \beta^2 \frac{\partial \pi_{t+2}}{\partial \eta_t} + \ldots \right] = \frac{\rho \beta}{1 - \rho \beta}$$

Thus transitory shocks have a smaller effect on permanent income than persistent shocks. The transitory effect is of order $\rho \beta$ and the permanent effect is of order $\frac{\rho \beta}{1 - \rho \beta}$. In the limiting case where operating income follows a random walk (i.e. $\mu = 1$), a permanent shock is fully absorbed in permanent income (i.e. $\frac{\partial Y_t}{\partial \eta_t} = 1$). The marginal effect of a transitory shock is, however, only $\rho \beta$, which is likely to be less than 0.05 for reasonable discount rates.

The effect of an earnings shock on dividends is smaller than its effect on permanent income because of habit formation. Proposition 2 gives:

$$\frac{\partial d_t}{\partial \tau_t} = (1 - \beta h) \alpha \rho \beta \quad \text{and} \quad \frac{\partial d_t}{\partial \eta_t} = (1 - \beta h) \alpha \left( \frac{\rho \beta}{1 - \beta \mu} \right)$$ \hspace{1cm} (28)

\textsuperscript{10}Note that the sum of the variance of dividend changes and the variance of rent changes does not add up to the variance of total payout, because the latter also includes a covariance term.
Thus dividends are smoothed in two ways. First, dividends are linked to permanent income, not to contemporaneous income. Dividends do not respond much to transitory earnings. This type of smoothing is a result of managers’ risk aversion. Second, dividends adjust gradually to changes in permanent income. This type of smoothing or “stickiness” results from habit formation.\textsuperscript{11}

These results highlight the perils of an econometric model for dividends that lumps earnings all in one basket. Ideally, the econometrician should distinguish transitory and persistent earnings. The accounting literature may provide further guidance on these topics.\textsuperscript{12}

\section*{2.3 Dividends and stock prices}

The managers’ optimal rent and dividend policies give the following valuation for the firm’s stock:

\textbf{Corollary 4} \textit{Ex-dividend market capitalization is independent of dividend policy and given by:}

\begin{align*}
S_t &= \sum_{j=1}^{\infty} E_t[d_{t+j}] \beta^j = \alpha \left[ \sum_{j=1}^{\infty} \beta^j K^\phi E_t[\pi_{t+j}] - D_t \right] = \alpha [V_t - D_t] \quad (29) \\
&= \frac{\alpha Y_t}{\rho \beta^3} - d_t \quad (30)
\end{align*}

The corollary shows that the firm’s share price and overall market capitalization depend on the firm’s permanent income $Y_t$ but not on dividend policy. A dollar of extra dividends reduces the equity value by the same amount, which is the standard Modigliani and Miller (1961) result.

The corollary also shows that an extra dollar of debt reduces the firm’s market capitalization by $\alpha$, which is equity’s share of income after interest. The rest of debt is covered

\textsuperscript{11} We have explored an alternative model in which managers absorb adjustment costs when they change rents from $t - 1$ to $t$. But we had to assume implausibly high adjustment costs in order to get plausible dividend smoothing. The current model, which starts with habit formation in managers’ utility function, gives simpler and more interesting results.

\textsuperscript{12} For example, see Ohlson (1999) and Barth et al. (1999).
by managers, who pay the fraction $1 - \alpha$ of debt service by reducing rents. As Lambrecht and Myers (2008) explain, managers cannot borrow personally against their future rents, but the corporation can in effect borrow on their behalf.

### 2.4 Dividends and debt policy

Proposition 2 shows how dividend policy drives debt policy once investment is fixed. Substituting the optimal policy for $r_t$ and $d_t$ gives the following corollary.

**Corollary 5** The dynamics of the firm’s debt (assuming the capital stock $K$ is fixed) are given by:

$$D_t - D_{t-1} = \rho D_{t-1} + (1 + \gamma) r_t - K^\phi \pi_t$$

$$= \left[ Y_t - \left( K^\phi \pi_t - \rho D_{t-1} \right) \right] + \beta h [(1 + \gamma) r_{t-1} - Y_t] + \frac{\kappa}{\alpha}$$  \hspace{1cm} (31)

The corollary has interesting implications. First, the change in debt includes a fixed component $\kappa/\alpha$, where $\kappa$ is defined in corollary 1. Whether this change is positive or negative depends on whether dissavings from managerial impatience or precautionary savings dominates. For example, more impatient managers derive more utility from today’s rents and are therefore prepared to incur additional borrowing. Of course, extra borrowing raises rents and dividends now, but reduces expected future permanent income, which in turn tightens the IBC constraint on future rents and dividends. The IBC (8) means that borrowing cannot sustain rents that exceed managers’ share of permanent income. Borrowing by the firm smooths rents and tailors them to managers’ preferences. The strict enforcement of the IBC also means that debt policy can never spiral out of control.  

Second, changes in the firm’s debt level depend on permanent income $Y_t$, the firm’s operating profits $K^\phi \pi_t$ and previous period’s payout $(1 + \gamma) r_{t-1}$. If there is no habit formation ($h = 0$) then the debt level goes up (down) if permanent income exceeds (is

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13The IBC does not rule out financial distress if the firm’s operating earnings fall so far that managers and shareholders decide to default. See Lambrecht and Myers (2008) for an analysis of default with risk-neutral, rent-seeking managers. In this paper we have assumed mature, blue-chip corporations and ignored default for simplicity.
below) current income. With habit formation \((h > 0)\), a second term is added, which reflects the difference between the firm’s total payout in last period \(((1 + \gamma)r_{t-1})\) and permanent income \(Y_t\). If last period’s payout is above (below) the current target, debt increases (decreases). The increase or decrease is not completed immediately, however, because \(\beta h < 1\).

The evolution of debt will clearly be path dependent. The path dependence is caused by earnings shocks, which lead to revisions in expectations about future income. A shock in earnings is also propagated over time through its effect on dividends and rents. If the firm’s dividend is off target, then adjustment to the target must occur over time because of the IBC. Debt policy acts as a temporary shock absorber that allows the firm gradually to adjust towards the new targets for rents and dividends.

We can distinguish the effects on debt of a transitory shock \(\tau_t\) and a persistent shock \(\eta_t\), where \(\tau_t\) and \(\eta_t\) are as defined in section 2.1. Using our earlier expressions for \(\frac{\partial Y_t}{\partial \tau_t}\) and \(\frac{\partial Y_t}{\partial \eta_t}\) (and assuming again that \(K = 1\)), the marginal effects are:

\[
\frac{\partial [D_t - D_{t-1}]}{\partial \tau_t} = (1 - \beta h)\rho \beta - 1 < 0 \quad (32)
\]

\[
\frac{\partial [D_t - D_{t-1}]}{\partial \eta_t} = \frac{(1 - \beta h)\rho \beta}{1 - \beta \mu} - 1 < 0 \quad (33)
\]

A dollar of transitory earnings decreases debt by almost a full dollar, because only a small fraction \(\rho \beta\) of the windfall cash flows is paid out as dividends. An extra dollar of persistent earnings decreases debt by a much smaller amount.

### 2.5 Information Content of Dividends

The "information content of dividends" refers to the good news conveyed to investors by dividend increases and the bad news conveyed by dividend cuts. The good and bad news is obvious in our model, because dividends are proportional to rents, and managers set rents depending on their view of the firm’s permanent income. Thus an unanticipated increase in dividends signals an unanticipated increase in permanent income. The information content is clear in Eq. (18), where \(d_t\) depends on permanent income \(Y_t\) and the previous dividend \(d_{t-1}\), and in Eq. (30), which states that cum-dividend stock-market value is proportional to permanent income. If investors only observe dividends, then Eqs. (26) and
(30) say that an unanticipated dividend increase of $\Delta d_t$ should increase stock-market value by $\Delta S_t = \Delta d_t/ [(1 - \beta h)\rho \beta]$. The higher the habit parameter $h$, the more good news is conveyed by a given dividend change $\Delta d_t$. A given dollar change in dividends means more to investors when they know that managers are averse to changes in rents and dividends.

Of course shareholders can’t protect their property rights if they observe only dividends. They must also observe or infer rents – otherwise they would have no clue about when to exercise their property right and the capital market constraint would not work. Hiring accountants to report net income is a partial remedy, but net income is calculated after rents, and rents are mixed in with other business expenses. So accounting (and other monitoring and governance mechanisms) also have to provide sufficient detail about expenses so that shareholders can estimate rents with tolerable accuracy. The mechanisms have to prevent the managers from actively concealing rents or from suddenly tunneling out massive rents and leaving shareholders with an empty shell.

The information about rents does not have to be verifiable or contractible. The shareholders are not relying on legal enforcement, but on their ability to take over (at a cost) and toss out the incumbent managers.

### 2.6 Investment

Now we study the managers’ optimal investment policy. At time $t$ the firm makes a (one-off) irreversible investment $K$ that generates a future stream of operating profits of $K^\phi \pi_{t+j}$, where $\pi_{t+j}$ is the autoregressive process $\pi_{t+1+j} = \mu \pi_{t+j} + \eta_{t+1+j}$. Managers first decide on $K$ and then set rents and dividends $r_t$ and $d_t$. Of course managers take into account how investment will affect current and future rents.

Since managers are wealth constrained (and cannot borrow or save on their own behalf) the investment has to be paid for by issuing debt and equity, so $K = \Delta D + \Delta S$. The capital market constraint implies that the proceeds from an equity issue are $\Delta S = \alpha (\Delta V - \Delta D)$. The additional debt required to finance the investment $K$ is $\Delta D(K) \equiv (K - \alpha \Delta V)/(1 - \alpha) = \left[ K - \alpha K^\phi \sum_{i=1}^{\infty} \beta^i E_t(\pi_{t+j}) \right]/(1 - \alpha)$.\(^ {14}\)

\(^ {14}\)If the investment has a high NPV then $\Delta D$ could be negative. The firm could use the proceeds of the
The budget constraints at the time of investment \( t \) and later times \( t + j \) are:

\[
D_t = (1 + \gamma) r_t + (1 + \rho) D_{t-1} + \Delta D(K) - \Pi_t
\]

\[
D_{t+j} = (1 + \gamma) r_{t+j} + (1 + \rho) D_{t+j-1} - K^\phi \pi_{t+j} \quad j = 1, 2, ...
\]

where \( D_{t-1} = \Pi_t = 0 \) if the firm has no history prior to time \( t \).

Investing the amount \( K \) has two effects. First, it increases the outstanding debt at \( t \) by an amount \( \Delta D(K) \). Second, it scales all future operating profits by a factor \( K^\phi \).

Repeated substitution of the budget constraint leads to the following intertemporal budget constraint:

\[
(1 + \gamma) \sum_{j=0}^{\infty} \beta^j r_{t+j} = \Pi_t + K^\phi \sum_{j=1}^{\infty} \beta^j \pi_{t+j} - (1 + \rho) (D_{t-1} + \beta \Delta D(K))
\]

If the risk-neutral shareholders were in charge, they would simply maximize the present value of expected payout over the firm’s infinite life. Optimizing the right hand of this equality with respect to \( K \) and taking expectations, we get the shareholders’ first-best investment policy:

**Proposition 3** The investment policy \( K^* \) that maximizes shareholder value is the solution to:

\[
\phi K^\phi \sum_{j=1}^{\infty} \beta^j E_t[\pi_{t+j}] - 1 = 0
\]

The efficient investment policy \( K^* \) is given by:

\[
K^* = \left[ \frac{\beta \phi E_t[\pi_{t+1}]}{1 - \beta \mu} \right]^{\frac{1}{1-\phi}}
\]

Consider next the managers’ investment decision. We have derived managers’ optimal payout policy \( r_t \) for any given constant level of investment and for any level of debt. Once the investment \( K \) is sunk, the managers’ optimal rent and payout policy is described in proposition 2, that is:\(^{15}\)

\[
Y_t[K] \equiv \rho \left[ \Pi_t + \sum_{j=1}^{\infty} K^\phi \beta^j E_t[\pi_{t+j}] \right] - \rho (D_{t-1} + \beta \Delta D(K))
\]

\[
r_t = \beta h r_{t-1} + (1 - \beta h) (1 - \alpha) Y_t[K] + c
\]

If there is no payout history prior to time \( t \), then a benchmark value for \( r_{t-1} \) will have to be picked to have an initial starting value.
Because wealth-constrained managers do not invest directly,\textsuperscript{16} they choose $K$ in order to maximize:

$$\max_K \sum_{j=0}^{\infty} \omega^j E_t[u(\hat{r}_{t+j})] \quad \text{where } \hat{r}_{t+j} \equiv r_{t+j} - hr_{t+j-1}$$

(41)

The first-order condition is:

$$\sum_{j=0}^{\infty} \omega^j E_t \left[ u'(\hat{r}_{t+j}) \frac{\partial \hat{r}_{t+j}}{\partial K} \right] = 0$$

(42)

After lengthy calculations (see appendix) this first-order condition simplifies to:

$$\sum_{j=0}^{\infty} \omega^j E_t \left[ u'(\hat{r}_{t+j}) \frac{\partial \hat{r}_{t+j}}{\partial K} \right] = e^{-\theta \hat{r}_t} \sum_{j=0}^{\infty} \omega^j \left( \frac{\beta}{\omega} \right)^j = \frac{e^{-\theta \hat{r}_t}}{1 - \beta} = 0$$

which ultimately leads to the following proposition.\textsuperscript{17}

**Proposition 4** The managers’ optimal investment policy $K$ is the solution to:

$$\phi K^{\theta - 1} \sum_{j=1}^{\infty} \beta^j E_t[\pi_{t+j}] - 1 = \frac{\theta \sigma_\eta^2 (1 - \alpha)^2 \beta (1 - h \beta) \phi K^{2 \theta - 1}}{(1 - \beta \mu)^2}$$

(43)

Managers underinvest if they are risk-averse ($\theta > 0$) and if profits are uncertain ($\sigma_\eta > 0$). Managers adopt the efficient investment level $K^*$ if they are risk-neutral ($\theta = 0$) or if profits are deterministic ($\sigma_\eta = 0$).

The proposition has several interesting implications. First, investment is efficient only if the right side of Eq. (43) is zero. But this expression is positive, so risk-averse managers underinvest. Comparing Eqs. (43) and (10) reveals that the term on the right is proportional to the precautionary savings term. Risk aversion causes managers to save for a rainy day, which leads to underinvestment.

Consider next the role of risk-aversion and profit volatility. All outcomes in the first-order condition Eq. (42) are weighed by $u'(r_{t+j})$, the managers’ marginal utility of rents

\textsuperscript{16}Even though managers are wealth constrained, they can still co-invest by keeping current rents $r_t$ as low as possible. The budget constraint (34) shows that a dollar cutback in $r_t$ reduces debt $D_t$ by $1 + \gamma$ dollars. When managers set $K$, its effect on optimal contemporaneous and future rents is taken into account, as is clear from the first order condition (42).

\textsuperscript{17}Given the optimal payout policy, which can also be expressed as $\hat{r}_{t+j} = \hat{r}_t + j \Gamma + K^\delta \sum_{i=1}^{j} \delta \eta_{t+i}$ (where $\Gamma$ and $\delta$ are constants defined in the proof of proposition 2), the managers’ optimal investment policy essentially boils down to maximizing current (and therefore also future) habit adjusted rents $\hat{r}_t$. 

25
at $t + j$. Since rents increase with the realization of the economic shock $\eta$, and since marginal utility is declining exponentially in rents, the managers’ first order condition puts relatively more weight on bad outcomes than on good outcomes. Therefore the degree of under-investment increases with managers’ risk-aversion coefficient ($\theta$) and with profit volatility ($\sigma_\eta$).

Habit formation mitigates underinvestment. The managers’ optimal investment decreases with the partial adjustment coefficient ($1 - \beta_h$), because habit formation reduces the managers’ need for precautionary savings. Habit formation and the resulting partial adjustment of rents smooths the rent stream and dampens the effect of volatility.

A higher level of investor protection makes investment policy more efficient. Notice how the squared factor $(1 - \alpha)^2$ pushes the right side of (43) rapidly towards zero as $\alpha$ increases. As investor protection approaches perfection ($\alpha \to 1$), the risks borne by managers approach zero, and their investment policy approaches the shareholders’ first best. This result is fragile and misleading near the limit of $\alpha = 1$, however, because the managers’ rents also go to zero at this limit. ”Perfect” investor protection gives managers no hope of future rents and no reason to invest in firm-specific human capital.

Our prediction of underinvestment is opposite to the ”free cash flow” theory, which proposes that managers of mature firms always want to invest if there is cash lying around. Of course the managers in our model would be happy to increase $K$ if they could invest only the shareholders’ money, for example by cutting dividends while maintaining rents. The capital market constraint prevents this, however. Managers might be tempted to overinvest and finance the investment by borrowing, but this strategy would reduce the present value of their rents. Also cash (negative debt) ”lying around” that is invested in negative NPV projects reduces managers’ claim. Therefore, the free cash flow problem

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18 If managers were for some reason unable to issue any new equity and had to rely exclusively on debt ($\Delta D(K) = K$), then managers’ optimal investment is the same as (43) except that $(1 - \alpha)^2$ is replaced by $1 - \alpha$. Financing 100% by debt therefore increases the degree of underinvestment and reduces rents and payout.

19 Myers (2000) argues that firms that depend on firm-specific human capital go public in order to reduce investors’ bargaining power and to create space for managerial rents.

20 The free cash flow theory starts with Jensen (1986). Note also Shleifer and Vishny’s (1989) theory of entrenching investment, which we would interpret as an attempt by managers to reduce $\alpha$. 

26
does not arise for as long as managers cannot steal or divert cash for direct personal gain.

Our model does not rely on psychological private benefits, but this is one place where such benefits may enhance efficiency by offsetting risk-aversion and mitigating underinvestment. Suppose that managers get private benefits $bK$ from investment ($b > 0$), and that these benefits do not impose a financial drain on the firm. Then there is a level for $b$ that is just right and leads to value-maximizing investment. Of course a $b$ that is too high would lead to overinvestment. We see no good way of gauging the actual or optimal magnitude of private benefits for the managers of large, public corporations. This is a problem for theories of investment and financing that rely on private benefits to motivate managers.

3 Conclusion

This paper has presented a theory of payout policy. Our original goal was to explain dividend smoothing and to see whether Linter’s (1956) target-adjustment model could be derived from deeper principles. It was quickly clear, however, that the dynamics of payout policy had to be modeled jointly with debt and investment policy. The three policies are tied together by the firm’s budget constraint. A dynamic theory of payout and investment defines a dynamic theory of capital structure.

We assume a coalition of risk-averse managers who maximize their life-time utility of the rents they extract from the firm. Managers are subject to a threat of intervention by outside shareholders, however. The managers pay out just enough in each period to leave shareholders indifferent between intervention and keeping managers in place for another period. This ties dividends to managers’ rents. For example, if managers want to maintain rents during a recession, they must also maintain dividends. Rents turn out to be a constant proportion of dividends.

We assume that managers are risk averse and subject to habit formation. The managers’ risk aversion and habit formation lead them to smooth rents (and therefore dividends), but in different ways. Risk aversion ties the rent and payout targets to permanent income, not to transitory income shocks. Habit formation means that rents and payouts adjust gradually to changes in permanent income. Because of smoothing, the response of
payout to transitory changes in earnings is an order of magnitude smaller than the response to persistent changes in earnings.

We show how investment policy affects debt policy and payout policy. Manager’s risk aversion leads to underinvestment – the managers do not maximize market value. Once dividend and investment policy are set, changes in (net) debt must serve as the shock absorber. The residual flow is not dividends, because managers smooth dividends and rents, but borrowing (or lending, for cash-rich firms). For example, a positive transitory earnings shock is used primarily to reduce net debt. Only a small fraction (less than 5% for realistic parameter values) of the transitory earnings are paid out as dividends. A positive persistent shock in earnings leads to a much smaller decrease in net debt, however, and may even increase debt if the shock leads to expanded investment.

Our results have empirical implications. Over the past 50-plus years Lintner’s model has been tested for in a wide variety of settings. These tests typically estimate the partial adjustment coefficient, the target payout ratio and the constant term. We link these estimates to deeper economic fundamentals. We show that the constant term increases with managers’ subjective discount factor (impatience), but decreases with risk aversion and earnings volatility. The speed of adjustment decreases with habit persistence, managers’ impatience and with the market interest rate. Dividend payout increases with the degree of investor protection.

DeAngelo, DeAngelo, and Skinner (2008) review the history of tests of Lintner’s target-adjustment model. The tests generally report lower PACs than Lintner (1956) or Fama and Babiak (1968). (See Choe (1990) and Brav, Graham, Harvey, and Michaely (2005), for example.) One explanation is that transitory payouts were almost always packaged as specially designated dividends (SDDs) during 1950s and 1960s. Lintner and Fama and Babiak included SDDs in the dividends used to fit the target-adjustment model. But now stock repurchases account for most transitory payouts and SDDs are rare. Thus most transitory payouts are now excluded from dividends, pushing estimated PACs downward.

Our model does not distinguish between cash dividends, stock repurchases and SDDs. We have talked about dividend payout, but strictly speaking our model applies to total payout, defined as dividends plus repurchases minus issues. Thus our model should be
tested (on recent payout data) using total payouts by large, blue-chip public corporations. Skinner (2008, Table 6) has done such a test, using dividends and total payout from 1980 to 2005 for a sample of 345 firms that paid regular dividends in at least 16 years and repurchased shares in at least 11 years. The Lintner target-adjustment model seems to work better for total payout than for dividends, at least in the last decade of the Skinner sample (1995-2005). In this period, the average $PAC$ for the dividend regression was .29 and insignificant ($t = 1.48$). The average $PAC$ for the total-payout regression was .55 and highly significant ($t = 8.93$). The Lintner constant was significantly negative for the dividend regression but positive and insignificant in the total-payout regression. These results match our theory predictions.

Skinner also finds that the coefficients in the total-payout regressions are larger and more significant when a period is defined as two years rather than one. It appears that firms time repurchases based on stock prices and other tactical considerations, and that the timing shifts total payout from one year to another. That result is OK in our model, which does not specify the length of period $t$. But our model does not explain why repurchases are timed tactically and dividends are not. Nor can we distinguish between the information content of changes in dividends and changes in repurchases. Here we are in good company with much of the literature on payout policy, however.

Skinner (2008) also fits the Lintner target-adjustment model for 351 firms that repurchase regularly but do not pay cash dividends. The estimated coefficients of the Lintner target-adjustment model are extremely high and significant for two-year periods from 1995-2005. For example, the average $PAC$ was .92 ($t = 5.79$) and the implied target payout ratio was 81% of reported earnings. These coefficients seem unusually high, although the firms in this sample are younger and probably differ in other respects from the confirmed dividend-payers. Skinner did not run the Lintner model for the much larger sample of firms that paid no dividends and did not make regular repurchases. We would not expect the Lintner model to work for many of these firms, however. Our derivations of the Lintner model assume a mature firm that generates free cash flow and can use borrowing or lending as the shock absorber for changes in earnings and investment. Many smaller, younger firms will not fit this description. Payout policy for this type of firm is a leading topic for further research.
Several other important issues are not addressed in this paper and are also left for future research. We assume that corporate debt is risk-free. Therefore our model does not apply to distressed firms or firms in declining markets that sooner or later must disinvest. We ignore the forces that drive conventional theories of capital structure, including taxes, costs of financial distress and information. Adding these forces to our model may lead to further insights, but will probably pose serious technical challenges.

Given permanent income, Lintner’s target-adjustment regression would fit in our model with an R-squared of 1.0. But we have made bright-line assumptions that cannot be so crisp and bright in practice. For example, we assumed that managers know the precise specification of the income generating process, and can therefore determine permanent income exactly. In reality managers’ estimates of permanent income will be noisy. We have also assumed that shareholders know rents exactly and that they and the managers know the tipping point for shareholder intervention exactly. A more "realistic" version of the model would have the managers estimating an increasing probability of shareholder intervention as the managers’ take of rents relative to dividends and repurchases increases. The shareholders’ decision to intervene would depend on their estimate of rents and their trust in accounting and governance to stop runaway rents. Also the shareholders would not rely exclusively on the threat of all-or-nothing intervention. For example, they would encourage compensation schemes to help align top managers’ economic interests with their own. The schemes would reward top managers based on income after rents and on stock price performance. (Recall that we distinguish total rents, which go to a broad cohort of managers and staff, from compensation to the CEO and his or her inner circle.) Thus one can think of more complex models in which rents and payouts do not move in exact lockstep, as in our model, but nevertheless move together on average and in the longer run. Rents and payouts would still be smoothed in such models, and Lintner’s target-adjustment specification should still work when fitted to blue-chip firms that make regular cash payouts to investors.

See Myers (2000, pp. 1017-1018) for a discussion of equilibrium when managers do not know shareholders’ cost of intervention.
4 Appendix

Proof of proposition 1

The payoff to shareholders from taking collective action at time \( t \) (instead of accepting the proposed dividend) is by assumption given by:

\[
S_t = \alpha \left[ \pi_t - (1 + \rho)D_{t-1} + \sum_{j=1}^{\infty} \beta^j E_t[\pi_{t+j}] \right] \tag{44}
\]

The payoff to shareholders from accepting the dividend \( d_t \) and not taking collective action at \( t \) is given by:

\[
S_t = d_t + \beta E_t[S_{t+1}] = d_t + \alpha \beta \left[ E_t[\pi_{t+1}] - (1 + \rho)D_t + \sum_{j=1}^{\infty} \beta^j E_t[\pi_{t+1+j}] \right] \tag{45}
\]

The dividend set by managers is such that shareholders are indifferent between taking collective action and keeping managers in place for another period. Substituting the budget condition at time \( t \) into \( D_t \) and solving for \( d_t \) gives:

\[
d_t = \left( \frac{\alpha}{1 - \alpha} \right) r_t \equiv \gamma r_t.
\]

Hence the total payout to managers and shareholders \((r_t + d_t)\) at time \( t \) can be expressed as \((1 + \gamma)r_t\).

Proof of proposition 2

The managers decision problem is to solve for an optimal payout plan \( P^o = \{r_t^o, r_{t+1}^o, r_{t+2}^o, \ldots\} \).

The first order condition for the decision variable \( r_t \) can be found by applying a variational argument as, for example, in Hall (1978). Define a variation \( P_t^1(e) \) on the optimal plan that varies rents at \( t \).

\[
P_t^1(e) = \{(r_t^o + e; r_{t-1}), (r_{t+1}^o - (1 + \rho)e; r_t^o + e), (r_{t+2}^o; r_{t+1}^o - (1 + \rho)e), (r_{t+3}^o; r_{t+2}^o), \ldots\}
\]

For clarity’s sake we have added a second argument representing the habit stock. If the optimal plan \( P_t^o \) satisfies the budget constraint at \( t \) (as it must, by definition) then by construction the variation \( P_t^1(e) \) will also. Let \( M_t^{(1)}(e) \) denote the managers’ expected utility as of time \( t \) associated with the plan \( P_t^{(1)}(e) \). Since the variation equals the optimal plan when \( e = 0 \) and since the optimal plan maximizes expected utility, it follows that

\[
\frac{dM_t^{(1)}(e)}{de} \bigg|_{e=0} = 0,
\]

or equivalently \( r_t^o \) must satisfy:

\[
E_t[u'(r_t - hr_{t-1}) - \omega hu'(r_{t+1} - hr_t)] = \frac{\omega}{\beta} E_t[u'(r_{t+1} - hr_t) - \omega hu'(r_{t+2} - hr_{t+1})] \tag{46}
\]
The condition says that along the optimal path the managers must receive the same present utility from an extra dollar of rents today as from \((1 + \rho)\) dollars tomorrow. In the absence of habit formation \((h = 0)\) this condition simplifies to the traditional Euler equation \(u'(r_t) = \frac{\beta}{\omega} E_t [u'(r_{t+1})]\).

Following Mullerbauer (1988) we define the transformed variable: 
\[
\hat{r}_{t+j} = r_{t+j} - hr_{t+j-1}
\]
(for \(j = 0, 1, 2, \ldots\)). Substituting into the above condition and replacing \(u(.)\) by the exponential utility function \(u(r) = 1 - \frac{1}{\theta}e^{-\theta r}\) gives:

\[
\left(\frac{\beta}{\omega}\right) e^{-\theta \hat{r}_t} = E_t \left[e^{-\theta \hat{r}_{t+1}} (1 + \beta h) - \omega h e^{-\theta \hat{r}_{t+2}}\right] \quad (47)
\]
The transformed condition encapsulates the Caballero (1990) model as a special case. The proof therefore follows closely Caballero (1990) and conjectures the following solution for \(\hat{r}_t\):

\[
\hat{r}_{t+j} = \phi_{t+j-1} \hat{r}_{t+j-1} + \Gamma_{t+j-1} + v_{t+j} \quad j = 0, 1, 2, \ldots
\]

where \(v_{t+j}, \phi_{t+j-1}\) and \(\Gamma_{t+j-1}\) remain to be determined and where \(v_{t+j}\) is the shock to rents that results from the shock \(\eta_t\) in earnings.

Substituting the conjecture for \(\hat{r}_{t+1}\) and \(\hat{r}_{t+2}\) into (47) gives:

\[
\left(\frac{\beta}{\omega}\right) e^{-\theta \hat{r}_t} = e^{-\theta \phi \hat{r}_t} e^{-\theta \Gamma_t} (1 + \beta h) E_t \left[e^{-\theta v_{t+1}} - \omega h E_t \left[e^{-\theta \Gamma_{t+1}} e^{-\theta v_{t+2}} e^{-\theta \phi \hat{r}_t + \Gamma_t + v_{t+1}}\right]\right] \quad (49)
\]

Rearranging gives:

\[
\left(\frac{\beta}{\omega}\right) e^{-\theta \phi t (1 - \phi_t) + \theta \Gamma_t} = (1 + \beta h) E_t \left[e^{-\theta v_{t+1}} - \omega h E_t \left[e^{-\theta \Gamma_{t+1} + v_{t+2} + \phi \hat{r}_t (\phi_t - 1)} + v_{t+1} \phi_{t} + \Gamma_t (\phi_{t} - 1)\right]\right] \quad (50)
\]

Analogous to Caballero (1990) it must be the case that \(\phi_t = \phi_{t+1} = 1\) as otherwise rents would be determined by the Euler equation regardless of the budget constraint. Therefore:

\[
\left(\frac{\beta}{\omega}\right) e^{\theta \Gamma_t} = (1 + \beta h) E_t \left[e^{-\theta v_{t+1}} - \omega h E_t \left[e^{-\theta \Gamma_{t+1} + v_{t+2} + v_{t+1}}\right]\right] = E_t \left[e^{-\theta v_{t+1}} \left\{(1 + \beta h) - \omega h e^{-\theta \Gamma_{t+1} + v_{t+2}}\right\}\right] \quad (51)
\]

Since the shocks \(\eta_{t+j}\) in earnings are i.i.d., it follows that the shocks in rents \(v_{t+j}\) are also i.i.d.; Using a property of the moment generating function of the normal distribution, it follows that:

\[
\left(\frac{\beta}{\omega}\right) e^{\theta \Gamma_t} = \left[e^{-\theta E_t[v_{t+1}] + \frac{\beta^2}{2} \sigma^2} (1 + \beta h) - \omega h e^{-\theta \Gamma_{t+1} + \theta E_t[v_{t+2}] + \frac{\beta^2}{2} \sigma^2}\right] \quad (52)
\]
\[
\left( \frac{\beta}{\omega} \right) e^{\theta [\Gamma_t + E_t[v_{t+1}] - \frac{\theta}{2} \sigma_v^2]} = 1 + \beta h - \omega h e^{-\theta [\Gamma_{t+1} + E_t[v_{t+2}] - \frac{\theta}{2} \sigma_v^2]} 
\] (52)

For the equality to hold at all times, it must be the case that:

\[
\Gamma_t - \frac{\theta}{2} \sigma_v^2 + E_t[v_{t+1}] = \Gamma_{t+1} - \frac{\theta}{2} \sigma_v^2 + E_t[v_{t+2}] \equiv g
\] (53)

The constant \( g \) can be found by solving for the positive root of the following quadratic equation:

\[
\left( \frac{\beta}{\omega} \right) (e^{\theta g})^2 - (1 + \beta h) e^{\theta g} + \omega h = 0
\] (54)

Solving gives:

\[
e^{\theta g} = \frac{\omega}{\beta} \quad \text{or} \quad g = \frac{1}{\theta} \ln \left( \frac{\omega}{\beta} \right).
\]

Consequently:

\[
\Gamma_t = \frac{\theta}{2} \sigma_v^2 - E_t[v_{t+1}] + \frac{1}{\theta} \ln \left( \frac{\omega}{\beta} \right)
\]

\[
\Gamma_{t+1} = \frac{\theta}{2} \sigma_v^2 - E_t[v_{t+2}] + \frac{1}{\theta} \ln \left( \frac{\omega}{\beta} \right)
\] (55)

We now substitute the solution for \( \hat{r}_t \) into the Intertemporal Budget Constraint (IBC). To derive this intertemporal constraint first note that the budget constraint at times \( t, t+1, t+2, \ldots \) can be written as:

\[
(1 + \gamma)r_t = D_t - (1 + \rho)D_{t-1} + K^\phi \pi_t
\]

\[
(1 + \gamma)\beta r_{t+1} = D_{t+1} \beta - D_t + \beta K^\phi \pi_{t+1}
\]

\[
(1 + \gamma)\beta^2 r_{t+2} = D_{t+2} \beta^2 - \beta D_{t+1} + \beta^2 K^\phi \pi_{t+2}
\]

\[
(1 + \gamma)\beta^3 r_{t+3} = \ldots
\] (56)

Summing the budget conditions over time and enforcing the no-Ponzi condition (7) gives the Intertemporal Budget Constraint (IBC):

\[
(1 + \gamma) \sum_{j=0}^{\infty} \beta^j r_{t+j} = \sum_{j=0}^{\infty} \beta^j K^\phi \pi_{t+j} - (1 + \rho)D_{t-1}
\] (57)

Therefore,

\[
h \sum_{j=0}^{\infty} (1 + \gamma) \beta^j r_{t+j-1} = h \sum_{j=0}^{\infty} \beta^j K^\phi \pi_{t+j-1} - h(1 + \rho)D_{t-2}
\]

\[
= h \beta \sum_{j=0}^{\infty} \beta^j K^\phi \pi_{t+j} - h(1 + \rho)D_{t-2} + h K^\phi \pi_{t-1}
\] (58)

Using the definition of \( \hat{r}_{t+j} \) and the budget constraint \(-K^\phi \pi_{t-1} + (1 + \rho)D_{t-2} = D_{t-1} - (1 + \gamma)r_{t-1}\) gives, after simplifying, the transformed IBC:

\[
\sum_{j=0}^{\infty} (1 + \gamma) \beta^j \hat{r}_{t+j} = (1 - h\beta) \left[ \sum_{j=0}^{\infty} \beta^j K^\phi \pi_{t+j} - (1 + \rho)D_{t-1} \right] - h(1 + \gamma)r_{t-1}
\] (59)
Since $\hat{r}_{t+1} = \hat{r}_t + \Gamma_t + v_{t+1}$, repeated substitution means that our conjectured solution for $\hat{r}_{t+j} = \hat{r}_t + \sum_{i=1}^{j} (\Gamma_{t+i-1} + v_{t+i})$. Substituting $\hat{r}_{t+j}$ into the IBC gives:

$$\frac{(1+\gamma}\hat{r}_t + (1+\gamma)\sum_{j=1}^{\infty} \beta^j \sum_{i=1}^{j} (\Gamma_{t+i-1} + v_{t+i}) + h(1+\gamma)r_{t-1} = (1-h\beta) \sum_{j=0}^{\infty} \beta^j K^\phi \pi_{t+j} - (1+\rho)D_{t-1}$$

(60)

Furthermore, $\sum_{j=1}^{\infty} \beta^j \sum_{i=1}^{j} (\Gamma_{t+i-1} + v_{t+i}) = \frac{1}{1-\beta} \sum_{j=1}^{\infty} \beta^j (v_{t+j} + \Gamma_{t+j-1})$. Taking expectations conditional on the information available at $t$ and solving for $\hat{r}_t$ gives:

$$(1+\gamma)\hat{r}_t = \rho \beta(1-h\beta) \sum_{j=1}^{\infty} \beta^j E_t[K^\phi \pi_{t+j}] - \rho(1-h\beta)D_{t-1} - (1+\gamma)\rho \beta h r_{t-1} - (1+\gamma) \sum_{j=1}^{\infty} \beta^j \Gamma_{t+j-1}$$

(61)

Simplifying gives

$$(1+\gamma) r_t = (1-h\beta) Y_t + h\beta r_{t-1}(1+\gamma) - (1+\gamma) \sum_{j=1}^{\infty} \beta^j \Gamma_{t+j-1}$$

(62)

where $Y_t$ is permanent income as defined by equation (12). The stochastic sequence for $v_{t+j}$ can be derived by substituting the solution for $\hat{r}_t$ back into the (ex-post) IBC (60). Using the assumption that $\pi_t = \mu \pi_{t-1} + \eta_t$, the IBC simplifies to the following condition:

$$\frac{1+\gamma}{1-\beta} \sum_{j=1}^{\infty} \beta^j v_{t+j} = \frac{(1-h\beta)}{(1-\beta \mu)} \sum_{j=1}^{\infty} \beta^j K^\phi \eta_{t+j}$$

(63)

Consequently, this pins down the stochastic sequence $\{v_{t+j}\}$ as a function of the earning shocks sequence $\{\eta_{t+j}\}$

$$v_{t+j} = \frac{K^\phi (1-\alpha)(1-\beta)(1-h\beta)\eta_{t+j}}{(1-\beta \mu)} = \delta K^\phi \eta_{t+j}$$

(64)

It follows that $\sigma_v = \delta K^\phi \sigma_\eta$. Since $\eta_{t+j}$ are iid shocks with zero mean, it follows that $\Gamma_t = \Gamma_{t+1} = ... \equiv \Gamma$ and therefore:

$$\sum_{j=1}^{\infty} \beta^j \Gamma_{t+j-1} = \frac{\beta}{(1-\beta)} \left[ \frac{\theta}{2} \sigma_v^2 + \frac{1}{\theta} \ln \left( \frac{\omega}{\beta} \right) \right] = \frac{\beta(1-\beta)(1-\alpha)^2(1-h\beta)^2}{(1-\beta \mu)^2} \theta 2 K^{2\alpha} \sigma_\eta^2 + \frac{\beta}{1-\beta} \frac{1}{\theta} \ln \left( \frac{\omega}{\beta} \right)$$

Substituting into the solution for $r_t$ gives:

$$r_t = (1-h\beta) \frac{Y_t}{1+\gamma} + h\beta r_{t-1} - \frac{(1-\alpha)^2(1-h\beta)^2}{(1-\beta \mu)^2} \theta 2 K^{2\alpha} \sigma_\eta^2 + \frac{\beta}{(1-\beta \theta)} \ln \left( \frac{\beta}{\omega} \right)$$

Proof of proposition 4
The proof will require the evaluation of the following integral:

\[ I \equiv \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{+\infty} xe^{tx} e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma^2} \right)^2} \, dx \quad (65) \]

\[ = e^{\left( \mu t + \frac{\sigma^2 t^2}{2} \right)} \left( \mu + t \sigma^2 \right) \quad (66) \]

The first equality is obtained by completing the square. The final step follows from the fact that the second integral calculates the expected value of a normally distributed value with mean \( \mu + t \sigma^2 \).

The proof requires us to solve the following first order condition (42):

\[ \sum_{j=0}^{\infty} \omega^j E_t \left[ u'(\hat{r}_{t+j}) \frac{\partial \hat{r}_{t+j}}{\partial K} \right] = 0 \quad (68) \]

We know from the proof to proposition 2 that:

\[ \hat{r}_{t+j} = \hat{r}_t + j \left( \frac{\theta}{2} \sigma_v^2 + \frac{1}{\theta} \ln(\frac{\omega}{\beta}) \right) + \sum_{i=1}^{j} v_{t+i} \quad (69) \]

\[ = \hat{r}_t + j \frac{\theta}{2} \delta^2 K^{2\phi} \sigma_{\eta}^2 - \frac{j}{\theta} \ln(\frac{\beta}{\omega}) + \delta K^{\phi} \sum_{i=1}^{j} \eta_{t+i} \quad (70) \]

where \( \delta \equiv \frac{(1-\alpha)(1-\beta)(1-h\beta)}{(1-\beta\mu)} \). It follows that:

\[ \frac{\partial \hat{r}_{t+j}}{\partial K} = \frac{\partial \hat{r}_t}{\partial K} + j \theta \delta^2 K^{2\phi-1} \phi \sigma_{\eta}^2 + \phi \delta K^{\phi-1} \sum_{i=1}^{j} \eta_{t+i} \quad (71) \]

Before we substitute this into the first order condition note the following results:

\[ E_t \left[ e^{-\delta K^{\phi} \sum_{i=1}^{j} \eta_{t+i}} \right] = e^{\frac{1}{2} \theta \delta^2 K^{2\phi} \sigma_{\eta}^2} \quad (72) \]

\[ E_t \left[ \eta_{t+j} e^{-\delta K^{\phi} \eta_{t+j}} \right] = -\theta \delta K^{\phi} \sigma_{\eta}^2 e^{\frac{1}{2} \theta \delta^2 K^{2\phi} \sigma_{\eta}^2} \quad (73) \]

where the second result is a special case of the above integral \( I \) (for \( \mu = 0, \sigma = \sigma_{\eta} \) and \( t = -\theta \delta K^{\phi} \)).

We can now calculate:

\[ E_t \left[ \sum_{i=1}^{j} \eta_{t+i} e^{-\delta K^{\phi} \sum_{i=1}^{j} \eta_{t+i}} \right] = E_t \left[ \eta_{t+1} e^{-\delta K^{\phi} \eta_{t+1}} \right] e^{\frac{1}{2} \theta \delta^2 K^{2\phi} \sigma_{\eta}^2(j-1)} + E_t \left[ \eta_{t+2} e^{-\delta K^{\phi} \eta_{t+2}} \right] e^{\frac{1}{2} \theta \delta^2 K^{2\phi} \sigma_{\eta}^2(j-1)} \]

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where we made use of the fact that \( \eta_{t+j} \) are iid shocks. Define now the following auxiliary variables:

\[
a_t \equiv \hat{r}_t + \frac{j}{2} \theta \delta^2 K^{2\phi} \sigma_\eta^2 - \frac{j}{\theta} \ln(\frac{\beta}{\omega})
\]

(75)

\[
b_t \equiv \frac{\partial \hat{r}_t}{\partial K} + \frac{j}{\phi} \theta^2 K K^{2\phi} \sigma_\eta^2
\]

(76)

Using these results allows us to calculate:

\[
E_t \left[ u' \left( \hat{r}_{t+j} \right) \frac{\partial \hat{r}_{t+j}}{\partial K} \right] = E_t \left[ e^{-\theta \left( a_t + \delta K^{\phi} \sum_{i=1}^{j} \eta_{t+i} \right)} \left( b_t + \phi \delta K^{\phi-1} \sum_{i=1}^{j} \eta_{t+i} \right) \right]
\]

(77)

Substituting this into the first order condition gives:

\[
\sum_{j=0}^{\infty} \omega^j E_t \left[ u' \left( \hat{r}_{t+j} \right) \frac{\partial \hat{r}_{t+j}}{\partial K} \right] = e^{-\theta \hat{r}_t} \frac{\partial \hat{r}_t}{\partial K} \left( 1 + \omega \left( \frac{\beta}{\omega} \right) + \omega^2 \left( \frac{\beta}{\omega} \right)^2 + \ldots \right) = e^{-\theta \hat{r}_t} \frac{\partial \hat{r}_t}{\partial K} = 0 \iff \frac{\partial \hat{r}_t}{\partial K} = 0
\]

(78)

We know from the proof of proposition 2 that:

\[
\hat{r}_t = (1 - \alpha)(1 - h) Y_t[K] - h \rho \beta r_{t-1} - \frac{\theta \sigma_\eta^2 \delta^2 K^{2\phi}}{2\rho} + \frac{1}{\rho^2} \ln(\frac{\beta}{\omega})
\]

(79)

Using equation (39) and differentiating \( \hat{r}_t \) with respect to \( K \) gives:

\[
\frac{\partial \hat{r}_t}{\partial K} = \rho \beta (1 - \beta h) \left[ \frac{\phi K^{\phi-1} \beta \mu_t}{1 - \beta \mu} - 1 \right] - \frac{\phi \theta \sigma_\eta^2 \delta^2 K^{2\phi-1} = 0}
\]

(80)

which gives the condition in proposition 4.
Proof of property 1

\[ Y_{t+1} = \rho \left[ \beta \sum_{j=0}^{\infty} E_{t+1}[\pi_{t+j}] K^{j} \beta^{j} - D_{t} \right] \quad (81) \]

\[ = \rho \left[ \beta \sum_{j=1}^{\infty} K^{j} E_{t+1}[\pi_{t+j}] \beta^{j} - D_{t-1}(1 + \rho) - (1 + \gamma) r_{t} + K^{\phi} \pi_{t} \right] \quad (82) \]

\[ E_{t}[Y_{t+1}] = \rho \left[ \beta \sum_{j=0}^{\infty} K^{j} E_{t}[\pi_{t+j}] \beta^{j} - \frac{D_{t-1}}{\beta} \right] - \rho(1 + \gamma) r_{t} \quad (83) \]

\[ = \rho \left[ \beta \sum_{j=0}^{\infty} K^{j} E_{t}[\pi_{t+j}] \beta^{j} - D_{t-1} \right] - \rho(1 + \gamma) r_{t} \quad (84) \]

\[ = Y_{t}(1 + \rho) - \rho(1 + \gamma) r_{t} \quad (85) \]

Proof of property 2

Using the definition of permanent income it follows:

\[ Y_{t} - Y_{t-1} = \rho \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} [E_{t}(\pi_{t+j}) - E_{t-1}(\pi_{t+j-1})] - \rho (D_{t-1} - D_{t-2}) \quad (86) \]

The budget constraint requires that \( D_{t-1} - D_{t-2} = \rho D_{t-2} + (1 + \gamma) r_{t-1} - K^{\phi} \pi_{t-1} \). Substituting into the above expression gives:

\[ Y_{t} - Y_{t-1} = \rho \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} [E_{t}(\pi_{t+j}) - E_{t-1}(\pi_{t+j-1})] + \rho \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} E_{t-1}(\pi_{t+j}) \]

\[ - \rho \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} E_{t-1}(\pi_{t+j-1}) - \rho \left[ \rho D_{t-2} + (1 + \gamma) r_{t-1} - K^{\phi} \pi_{t-1} \right] \]

\[ = \eta_{t} + \rho \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} E_{t-1}(\pi_{t+j}) - \rho \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} E_{t-1}(\pi_{t+j-1}) \]

\[ - \rho \left[ \rho D_{t-2} + (1 + \gamma) r_{t-1} - K^{\phi} \pi_{t-1} \right] \]

\[ = \eta_{t} - \rho \left[ \rho \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} E_{t-1}(\pi_{t+j}) - y_{t-1} + (1 + \gamma) r_{t-1} - K^{\phi} \pi_{t-1} \right] \]

\[ + \rho \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} E_{t-1}(\pi_{t+j}) - \rho \sum_{j=0}^{\infty} \beta^{j} K^{j} E_{t-1}(\pi_{t+j-1}) \]

\[ = \eta_{t} - (1 + \rho) \rho \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} E_{t-1}(\pi_{t+j}) + \rho \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} E_{t-1}(\pi_{t+j}) \]

\[ + \rho \left[ Y_{t-1} - (1 + \gamma) r_{t-1} + K^{\phi} \pi_{t-1} \right] \quad (87) \]

where \( \eta_{t} \equiv \rho \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} [E_{t}(\pi_{t+j}) - E_{t-1}(\pi_{t+j})] \). Using the fact that \( \beta \sum_{j=0}^{\infty} \beta^{j} K^{j} E_{t-1}(\pi_{t+j}) - \)}
\[ \beta(1 + \rho) \sum_{j=0}^{\infty} \beta^j K^\phi E_{t-1}(\pi_{t+j-1}) = -K^\phi \pi_{t-1}, \] it follows:

\[ Y_t - Y_{t-1} = \nu_t - K^\phi \rho \pi_{t-1} + \rho Y_{t-1} - \rho(1 + \gamma) r_{t-1} + K^\phi \rho \pi_{t-1} \]

\[ = \nu_t + \rho Y_{t-1} - \rho(1 + \gamma) r_{t-1} \]

Finally, from \( \pi_t = \mu \pi_{t-1} + \eta_t \), it follows that \( E_t(\pi_{t+j}) - E_{t-1}(\pi_{t+j}) = \mu j \eta_t \). Consequently, \( \nu_t = \frac{\rho \beta K^\phi \eta_t}{1 - \beta \mu} \).

References


