The usefulness of control theory techniques in economic simulations is examined. A model of the capital investment accelerator is used for illustration. It is shown that behavior approaching optimality can be achieved in a highly nonlinear system with simple heuristics based on linearization and constant state feedback with an appropriate choice of desired closed-loop poles. Applications to real-life situations are discussed, and suggestions are made for approaching potential problems using similar control theory techniques.

One can approach a system dynamics problem from either of two points of view: mathematical and intuitive. Mathematical analysis provides global knowledge of the solutions but is only feasible in simple models; it is very hard to analyze large realistic models mathematically. On the other hand, using the intuitive approach, one can investigate more comprehensive and realistic models, but it is all too easy to become lost in simulation and trial and error in designing policies to improve performance.

We seek to answer several questions:

1. How useful are the techniques of modern control theory in simulation models of social systems?
2. Can approximately optimal policies be designed that improve performance significantly, relative to the original policies of the decision makers? By "approximately optimal policies" we mean a set of policies based on heuristic methods rather than on the closed-form solution of the problem, since such analytic solutions are generally impossible to determine in the high-order, nonlinear systems typically of concern in system dynamics modeling.
3. How different are the improved and original policies? That is, are the improved rules consistent with the likely availability of information and bounded rationality of real managers?

The goal of this article is to motivate the heuristic use of some mathematical tools to aid the systems analyst. The approach can be outlined as follows:

1. Start from the full simulation model portraying the problem of interest.
2. Analyze the model formally, using decoupling, model reduction, linearization, and so on, where appropriate, to decrease the burden of mathematical analysis.
3. Design an appropriate controller (policies for improved performance).
4. Project the results back to the original model, and test the robustness of the new policies in the full system.

We assume that step 1 is given and concentrate on steps 2, 3, and 4. This article complements previous applications of control theory to policy design in social and corporate systems (e.g., Mohapatra and Sharma 1985; Coyle 1985; Kivijärvi and Törmänen 1986) by applying formal methods to a nonlinear model with complex dynamics. Further, the existence of experimental results showing how people actually manage the system provides a means for directly assessing the performance improvements that may be obtained through the use of control techniques.

We next describe some of the basic mathematical tools. The outlined approach is applied to Sterman's (1985) model of the Kondratiev cycle, or long wave. The particular example illustrates general techniques applicable to a variety of situations, such as business cycles (Mass 1975; Lyneis 1980), commodity price stabilization (Meadows 1969), and market growth (Forrester 1968). Although this article is mostly heuristic, we hope to motivate the need for rigorous analysis based on the outlined method.
The last section summarizes our results and makes recommendations for further analysis.

**Mathematical tools**

Linearization is the most common tool for formal analysis of nonlinear models. A model of the form

\[ \dot{x}(t) = f(x(t)) + g(u(t)) \]  

where \( x \) is the vector of states, \( u \) is the vector of inputs in the model, and \( f(\cdot), g(\cdot) \) are state and input functions, respectively, could be linearized around a nominal trajectory \( x_n(t) \) using the Jacobian matrices

\[ F = [f_{ij}], \quad f_{ij} = \frac{\partial f_i(x)}{\partial x_j} \]  

\[ G = [g_{ij}], \quad g_{ij} = \frac{\partial g_i(u)}{\partial u_j} \]  

Then the linearized model

\[ \delta \dot{x}(t) = F \delta x(t) + G \delta u(t) \]  

describes deviations from the nominal trajectory. The actual trajectory \( x(t) = x_n(t) + \delta x(t) \) is driven by the input \( u(t) = u_n(t) + \delta u(t) \) where

\[ f(x_n(t)) + g(u_n(t)) = 0 \]

This approximation is good for slowly varying nominal trajectories. The model can be analyzed using modern control theory (Guckenheimer and Holmes 1983; for "slowly varying," see Vidyasagar 1978).

It should always be remembered that linearization only provides information about the local region of phase space and is not a reliable guide to global dynamics. A special class of nonlinearities is the class of piecewise linear functions, such as saturation, bang-bang, and roundoff. Their derivatives are discontinuous, and therefore the Jacobian matrix is undefined. In these cases, the analysis can be performed for the different regions where the derivatives of these nonlinearities are defined.

Once the linearized model (Eq. 3) is constructed, various methods of control can be applied. The method used here is pole placement using full state feedback (Kailath 1980). In essence, a weighted sum of each state is fed back to the system as the input. The method is to calculate these weights, \( H \), in order to achieve a given set of closed-loop poles (i.e., the eigenvalues of \( F + GH \)). If the system (Eq. 3) is controllable, then
A matrix $H$ can be chosen such that the eigenvalues of $F + GH$ are as desired. The input $\delta u(t)$ is picked as

$$\delta u(t) = H\delta x(t)$$

(4)

The closed-loop system then becomes

$$\delta \dot{x}(t) = (F - GH)\delta x(t)$$

(5)

and has the desired closed-loop poles. The nominal input is picked to sustain the nominal trajectory. For example, the nominal trajectory might be given by the equilibrium of the system, which may be changing with exogenous conditions.

To achieve an approximately optimal response for a system with two poles, one rule of thumb is to select a pair of poles for the closed-loop system such that these poles are complex conjugates with equal magnitudes of real and imaginary parts and negative real part. The response to a step input exhibits a slight overshoot and is optimal in the sense of minimum overshoot and settling time (Roberge 1975). Varying the distance from the poles to the origin trades off overshoot and settling time.

An important aspect of any control design is robustness. We consider robustness by perturbation of the control parameters and by introducing uncertainty in the states.

**A model of the Kondratiev cycle**

Various models of the Kondratiev cycle, or long wave, have been developed to analyze recent economic difficulties (Rasmussen, Mosekilde, and Sterman 1985; Sterman 1985; 1986; Vasko 1987). This article adopts the model of Sterman (1985). The Kondratiev model is chosen since it is a nonlinear system with complex dynamics, yet its structure is simple enough to allow for a considerable amount of mathematical analysis to be done by hand to illustrate the proposed techniques. It is also well known in the system dynamics literature. The model has also been converted into a simulation game, Strategem-2 (Sterman and Meadows 1985). Strategem-2 provides a simulated economy in which human decision makers replace the decision rule of the original model. Thus, the game constitutes a laboratory for experimental testing of the model. Strategem-2 motivates the design of an “optimal” controller to play the game, so that we can compare the behavior of actual managers to the “optimal” response, thus providing a rough measure of the value of improved performance through formal analysis.

The long-wave model portrays the process governing capital investment in the aggregate economy. The capital-producing sector strives to satisfy the demand for capital of the goods-producing sector as well as its own capital needs. The model represents the capital self-ordering feedbacks created by the fact that capital is an input to its own production. This dependence implies that the total demand for capital can only be filled when there is sufficient capacity but also that the capacity of the capital-producing sector can only be increased by first ordering additional capacity and adding to the demand. Intuitively, such a positive feedback must de-
stabilize the adjustment of capital producers to shocks, and in fact, the model typically exhibits large-amplitude limit cycles. The model can also generate chaotic behavior, where no periodicity can be attributed to the model (Sterman 1988). In the experimental version, subjects play the role of managers for the capital-producing sector and strive to balance the supply and demand for capital. Subjects seek to minimize their costs, or score, during each trial. The score is the average absolute deviation between supply (production capacity PC) and demand (desired production DP) over the T periods of the game:

\[ S = \frac{1}{T} \sum |DP_t - PC_t| \]

When presented with an unanticipated step input in demand, a sample of about 50 subjects produced an average score of more than 500 (Sterman 1987; 1989). The optimal score for the same situation is 19. The poor performance of the subjects shows that even an approximately optimal controller may offer substantial improvements in performance.

The states in the model are capital K, the backlog of orders placed by the capital sector BKS, and the backlog of orders placed by the goods sector BGS. The inputs are new orders from the capital sector NKS (specified by the player), and new orders from the goods sector NGS (exogenous). The model can be summarized as follows (Figure 1):

\[
\begin{bmatrix}
\dot{K} \\
\dot{BKS} \\
\dot{BGS}
\end{bmatrix} = \begin{bmatrix}
\frac{BKS}{NCAT} & FDS & -\frac{K}{ALC} \\
-\frac{BKS}{NCAT} & FDS & 0 \\
-\frac{BGS}{NCAT} & FDS & 0
\end{bmatrix} + \begin{bmatrix}
0 \\
NKS \\
NGS
\end{bmatrix}
\] (6a)

\[ FDS = \text{Fraction of demand satisfied} \]
\[ = \min(\text{Desired production, Capacity}) / \text{Desired production} \] (6b)

\[ \text{Desired production} = P^* = \frac{BKS + BGS}{NCAT} \] (6c)

\[ \text{Capacity} = \frac{K}{COR} \] (6d)

where the normal capital acquisition time NCAT, average lifetime of capital ALC, and capital-output ratio COR are constants. For the experiment in this article, the values NCAT = 2, ALC = 20, and COR = 2 are used, the same values as in the Strategem-2 game. Also, the time step (2 years), simulation length (70 years), and score function are the same as in the game in order to make the present results easily comparable to the Strategem-2 results. The acquisition lag and capital-output ratio were deliberately set equal to the time step to facilitate play in the Strategem-2 game. In the original model, the parameters were based on econometric evidence, and the time step was reduced so that integration error was not significant. The difference and
New orders for capital placed by the consumer goods and services sector (NGS) are exogenous. In the experiment, the new orders of the capital sector (NKS) are determined by the player. In the analysis presented here, NKS is determined by the controller.

**Capital Sector:**

- **FDS = PR/DP**  
  Fraction of Demand Satisfied (dimensionless)
- **DP = (BKS+BGS)/NCAT**  
  Desired Production (units/year)
- **NCAT = 2**  
  Normal Capital Acquisition Time (years)
- **PR = MIN(DP,PC)**  
  Production Rate (units/year)
- **PC = K/COR**  
  Production Capacity (units/year)
- **COR = 2**  
  Capital Output Ratio (years)
- **K = K + dt * (CA - CD)**  
  Capital Stock (units)
- **CD = K/ALC**  
  Capital Discard Rate (units/year)
- **ALC = 20**  
  Average Life of Capital (years)
- **CA = FDS*BKS/NCAT**  
  Capital Acquisition Rate (units/year)
- **BKS = BKS + dt * (NKS - CA)**  
  Backlog of orders of the capital sector (units)
- **NKS = Determined by subject**  
  New Orders of the Capital Sector (units/year)

**Goods and Services Sector:**

- **BGS = BGS + dt * (NGS - GCA)**  
  Backlog of orders of the goods sector (units)
- **GCA = FDS*BGS/NCAT**  
  Goods Sector: Capital Acquisition Rate (units/year)
- **NGS = Exogenous**  
  New Orders of the Goods Sector (units/year)
- **dt = 2**  
  Simulation time step (years)
differential equation versions of the model produce the same types of behavior and respond to parameter changes in the same fashion.

**Linearization**

The only nonlinearity requiring piecewise analysis arises from the fraction of demand satisfied FDS. The capital-producing sector can only fully satisfy the demand when that demand is less than or equal to capacity. FDS is equal to 100 percent when capacity exceeds desired production. As desired production rises above capacity, FDS falls, constraining the growth of capital itself. The model operates in two distinct regimes: FDS = 1 (capacity exceeds demand), and FDS < 1 (insufficient capacity).

Considering each regime in turn, linearization of Eq. 6 yields the following:

1. For FDS = 1,

\[
\begin{bmatrix}
\delta K \\
\delta BKS \\
\delta BGS
\end{bmatrix}
= \begin{bmatrix}
\delta K \\
\delta BKS \\
\delta BGS
\end{bmatrix} + G_2 \begin{bmatrix}
\delta NKS \\
\delta NGS
\end{bmatrix}
\]

\[
F_2 = \begin{bmatrix}
-1 & 0 & 0 \\
\frac{ALC}{NCAT} & \frac{1}{NCAT} & 0 \\
0 & 0 & -1
\end{bmatrix}
\]

and

\[
G_2 = \begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}
\]

2. For FDS < 1,

\[
\begin{bmatrix}
\delta K \\
\delta BKS \\
\delta BGS
\end{bmatrix}
= \begin{bmatrix}
\delta K \\
\delta BKS \\
\delta BGS
\end{bmatrix} + G_1 \begin{bmatrix}
\delta NKS \\
\delta NGS
\end{bmatrix}
\]

where \( F_1 \) is

\[
\begin{bmatrix}
\frac{BKS}{\text{COR}(BKS + BGS)} & \frac{1}{ALC} & \frac{-K \cdot BKS}{\text{COR}(BKS + BGS)^2} \\
\frac{-BKS}{\text{COR}(BKS + BGS)} & \frac{K \cdot BGS}{\text{COR}(BKS + BGS)^2} & \frac{K \cdot BKS}{\text{COR}(BKS + BGS)^2} \\
\frac{-BGS}{\text{COR}(BKS + BGS)} & \frac{-K \cdot BKS}{\text{COR}(BKS + BGS)^2} & \frac{-K \cdot BKS}{\text{COR}(BKS + BGS)^2}
\end{bmatrix}
\]
and $\bar{X}$ denotes the value of state variable $X$ at the chosen operating point.

Let $\varphi = (\overline{K}/\text{COR})/(\overline{BKS} + \overline{BGS})$. Note that $\varphi = \text{FDS}/\text{NCAT}$, where $\text{FDS}$ is the fraction of demand satisfied at the operating point. Also, let $\pi = \overline{BKS} + \overline{BGS}$ and note that $\pi = \text{NCAT} \cdot \overline{P}^*$, where $\overline{P}^*$ is the desired production at the operating point. Then, we have

$$
G_1 = \begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 1
\end{bmatrix}
$$

For $\text{FDS} = 1$, the entire model is linear, and for $\text{FDS} < 1$, the model is highly nonlinear. Note the following observations from the analysis of the preceding models:

1. $\text{FDS} = 1$.
   a. The system is stable with time constants $\text{ALC}$ (average lifetime of capital) and $\text{NCAT}$ (normal capital acquisition time). In this regime, excess capacity permits all orders to be delivered within the normal capital acquisition time, and capital stock depreciates with time constant $\text{ALC}$.
   b. $K$ and $\overline{BKS}$ are controllable via $\text{NKS}$. $\overline{BGS}$ is uncontrollable but stable. In other words, $K$ can be controlled by $\text{NKS}$, but as long as $\text{FDS} = 1$, the goods sector gets all it desires and thus is not controllable. $\overline{BGS}$ is also decoupled from $K$ and $\overline{BKS}$.

2. $\text{FDS} < 1$.
   The eigenvalues are $0$, $-\varphi$, and $\overline{BKS}/(\text{COR} \cdot \pi) - 1/\text{ALC}$.
   a. The Jacobian matrix has a zero eigenvalue, which implies that higher-order terms ignored by the linearization are important. Thus, no stability conditions can be inferred from the Jacobian matrix. Because of its highly nonlinear nature, it seems to be safer to keep the system away from this region whenever possible.
   b. The linearized system is controllable. Therefore, the goal is to design a controller that stabilizes the system in this region or, better still, gets the system out of this region into the stable region where $\text{FDS} = 1$.

An interesting characteristic of this model is that the equilibrium point, which lets us calculate the nominal trajectory for a given $\text{NGS}$, is exactly at the boundary of these two regions, making the system harder to control. Specifically, at the equilibrium,

$$\text{FDS} = 1$$
\[ X_n = \frac{\text{ALC} \cdot \text{COR}}{\text{ALC} - \text{COR}} \cdot \text{NGS} \]  
\[ \text{BKS}_n = \frac{\text{NCAT} \cdot \text{COR}}{\text{ALC} - \text{COR}} \cdot \text{NGS} \]  
\[ \text{NKS}_n = \frac{\text{COR}}{\text{ALC} - \text{COR}} \cdot \text{NGS} \]  
\[ \text{BGS}_n = \text{NCAT} \cdot \text{NGS} \]

where \( X_n \) denotes the nominal trajectory for \( X \).

Therefore, to sustain the nominal trajectory given \( \text{NGS} \), \( \text{NKS} \) should be chosen as \( \text{COR}/(\text{ALC} - \text{COR}) \cdot \text{NGS} + \delta \text{NKS} \), where \( \delta \text{NKS} \) is chosen in order to control the deviations from the nominal trajectory.

**Controller design**

The model, implemented in STELLA, was extended to support two controllers (one for each region) to perform pole placement using full state feedback. The controller determines the new orders of the capital sector \( \text{NKS} \) and is designed to calculate \( H_i \) at every instant of time such that the eigenvalues of \( F_i + G_i H_i \) are as desired, for each region, \( i = 1, 2 \). The nominal trajectory is determined by the equilibrium rate of capital sector orders, given the current orders of the consumer sector. Then, the resulting input is

\[ \text{NKS} = \frac{\text{COR}}{\text{ALC} - \text{COR}} \cdot \text{NGS} + H_i \begin{bmatrix} \delta K \\ \delta \text{BKS} \\ \delta \text{BGS} \end{bmatrix} = \text{NKS}_n + \delta \text{NKS}_i \]  

Note that as long as the desired poles are chosen to be nonzero and stable, the trajectories will be stable around the slowly varying nominal trajectories. In the control literature, the system is usually linearized around the equilibrium point and the poles are set accordingly. Here we place the poles for all operating points (except perhaps at a few points where controllability is lost). Although this is not backed up by rigorous results, it merits further research in view of the ensuing successful results.

Figure 2 shows a policy structure diagram of the overall controller (equations for the controller are supplied in the Appendix). First, \( H_1 \) and \( H_2 \) are calculated on the basis of the current state of the system and the desired closed-loop poles. The entries of \( H_i \) were found such that the characteristic polynomial of the closed-loop matrix (determinant of \( \lambda I - (F + G_i H_i) \)) is the desired polynomial ((\( \lambda - \) desired pole 1) (\( \lambda - \) desired pole 2) . . . ). Then, \( \delta \text{NKS}_1 \) and \( \delta \text{NKS}_2 \) are calculated as the weighted sum, by multiplication via \( H_1 \) and \( H_2 \), respectively, of the deviations from the nominal trajectory, which are determined by the current value of \( \text{NGS} \) and Eq. 9. This stabilizes the dynamics of the error modes of the closed-loop system and thus drives the states to the steady-state values. Thus, the system is more robust, since the error will go to
zero and the response will not be very sensitive to perturbations in $H_1$ and $H_2$. Finally, $\delta NKS_1$ or $\delta NKS_2$ is chosen depending on the value of FDS, and NKS, is added to calculate NKS.

Three poles in the region $FDS < 1$ were placed in the left half-plane on the circumference of a circle centered at the origin. One pole was placed on the real axis with a complex conjugate pair with equal real and imaginary parts. The radius of the circle determines a trade-off between overshoot and settling time (the larger the radius, the more overshoot and faster settling). In the region $FDS = 1$, two poles were placed on the real axis in the left half-plane, both at the same location. Their distance from the origin determines the settling time in this region.

Positive shocks to NGS from equilibrium put the system in the unstable, nonlinear region $FDS < 1$. The controller is designed to move the system into the stable region in an optimal fashion—quickly and with minimum overshoot. In the region $FDS = 1$, the system is expected to settle at the equilibrium in an overdamped way, thus providing the overall desired response. Negative jumps in NGS put the system in the region $FDS = 1$. The response will be overdamped in that case.

In intuitive terms, increasing the speed of adjustment in the region $FDS < 1$ may result in building the capital stock too high and thus degrading performance by increasing the game score. Decreasing the speed when $FDS < 1$ may cause the system to linger in this region with inadequate investment to escape, also degrading performance by raising the score. Increasing the speed in the region $FDS = 1$ may cause the system to re-enter the region $FDS < 1$ because of the discrete time intervals used.
in the Strategem-2 game. This is not expected to cause any problems, since capital will be bounced back up again, but it may cause small amplitude (probably damped) oscillations. Decreasing the speed in this region may cause slow settling to the equilibrium (overconservative response) and thus increase the score.

**Experimental results**

Various speeds were tried, and simulation results for pole locations $-0.38, -0.27 \pm 0.27j$ for $\text{FDS} < 1$, and $-1, -1$ for $\text{FDS} = 1$, are illustrated in Figure 3. In the figure, desired capital $\text{DesiredK} = (\text{BKS} + \text{BGS})/\text{NCAT}$. The capital sector input supports the intuition that a positive jump in the goods sector demand is followed by an amplified positive jump in the control input $\text{NKS}$ such that the share of the capital sector demand in the desired production is increased enough to increase the capital
Fig. 4. Typical experimental results. Note the large amplitude and long period of the cycles generated by the subjects. N.B.: vertical scales differ.
and to supply the desired production (bounced to the region $FDS = 1$). Later, the input drops close to zero for a while, to compensate for the initial over-demand, and finally settles to its equilibrium value. Capital shows the desired response, while compensating for the jump in the backlog of the goods sector. $FDS$ falls to 80 percent, but recovers in about 10 years. The score for the simulation is 15.² (Compare the behavior in Figure 3 to Figure 4, which shows the cycles typically produced by subjects of the experiment.)

To investigate robustness, the speed inputs were perturbed by 10 percent in both directions independently. This resulted in negligible changes in the response, and the increase in the average score per period was less than 1 for all perturbations.

Another robustness test was performed by introducing uncertainty in choosing the appropriate controller around the equilibrium. Specifically, this boundary was chosen as the region $0.95 \leq \frac{\text{desired production}}{\text{production capacity}} \leq 1.05$. Figure 5 shows the simulation result when the controller for $FDS < 1$ was chosen for this boundary. The response exhibits some underdamped fluctuations due to the complex

---

Fig. 5. Controller for $FDS < 1$ used in the boundary whenever $0.95 \leq \frac{\text{DP}}{\text{PC}} \leq 1.05$. Score = 18. K, DesiredK, NKS in units; FDS in %.
conjugate pair, but they seem to be unimportant. Figure 6 shows the simulation result when the controller for FDS = 1 was chosen for this boundary. Since the controller for FDS = 1 was overdamped, when used in the boundary it never bounces the system into the region FDS = 1. Thus, FDS never makes it to 1 and gets into a cycle where it slides down, jumps up (but below 1), slides back down, and so on. Recall that there was an extra degree of freedom in the controller for FDS = 1, which was not used, since BGS is uncontrollable. Specifically, BGS was not included in the controller. To correct for this deficiency, a multiple of BGS was added to the controller for FDS = 1. This has no effect in the region FDS = 1, since the capital sector is decoupled from the goods sector in this region. In the boundary, this could be used to correct the undesired effect of the uncertainty. Figure 7 illustrates the simulation result when 12 percent of BGS was added to the controller of the region FDS = 1. The response is greatly improved, and FDS settles at 1 in this case.

Intuitively, when faced with an unstable region and uncertainty in the states, a
robust controller will err on the side of caution by moving the system rapidly to the stable zone even at the cost of possibly moving farther into the stable zone than necessary.

The results for other simulations, where different waveforms for the goods sector demand were chosen, are summarized in Table 1.

The controller seems to be robust with respect to perturbations in the desired pole locations, which also correspond to perturbations in the feedback gains. The discrepancy due to the uncertainty in choosing the appropriate controller around equilibrium was corrected by using the additional degree of freedom in the region FDS = 1. Thus, the results are not contingent on precise knowledge of the system parameters.

The controller designed in this article could be modified to follow ramp inputs better, but it is likely to give worse results in response to other inputs. More advanced control techniques, such as linear quadratic regulation, could be applied to this problem but require the use of packaged programs to solve the necessary equations.
### Table 1. Summary of simulations with various waveforms for NKS

<table>
<thead>
<tr>
<th>Waveform</th>
<th>Description</th>
<th>Average score per period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ramp $225 + 2.5(t - 2)$</td>
<td>Ramp with a lag; no equilibrium</td>
<td>Above 90%</td>
</tr>
<tr>
<td>Sinusoidal $225 + 50 \sin \left( \frac{2\pi t}{72} \right)$</td>
<td>Distorted sinusoidal</td>
<td>Periodic between 100% and 70%</td>
</tr>
<tr>
<td>Gaussian noise $225 + 20 \times \text{NORMAL}$</td>
<td>Noisy but stable</td>
<td>Fluctuates between 100% and 60%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Settles around 100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Settles around 125</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Below 200</td>
</tr>
</tbody>
</table>

### Conclusions and further analysis

This article presents a heuristic application of control theory to an economic model. Although we considered one case, the same approach can be applied to a wide class of problems that exhibit similar structures, that is, any system with a region in which resources are fully utilized and a region of slack. In general, the results show that a highly nonlinear problem can be controlled quite satisfactorily with the aid of simple mathematical analysis and basic control concepts. We considered two key techniques for approaching such nonlinear problems:

1. Linearization around the nominal trajectory. This is a commonly used technique. The results of Vidyasagar (1978), although complicated, are particularly useful in finding the class of slowly varying trajectories around which the system would be stable. We verified this for step inputs by simulation. Ramp inputs and sinusoids seem to present some problems.

2. Analyzing piecewise-linear nonlinearities in linear regions separately. We considered a common example of this case, saturation. However, since the general applications of control theory require the system to operate in the unsaturated region, there is not much rigorous analysis to be found in the literature for the case when the operating point is required to be on the boundary between the two regions, as in many system dynamics problems. This perhaps merits a rigorous analysis.

Many economists argue that dysfunctional oscillations such as the long wave cannot exist, since economic agents with rational expectations would behave in an optimal fashion. The analysis here shows that there are in fact optimal strategies for investment that can avoid instability. But it is very unlikely that the results presented here would be achieved in practice through conventional decision-making processes. This assertion is supported by the experimental results of Sterman (1987; 1989). The decision rule developed here requires information a manager in real life is unlikely to have. It then processes that information in a highly sophisticated fashion. In particular, the optimal rule utilizes knowledge of the equilibrium structure of the system to compute targets for capital stock and new orders. The equilibrium structure of the full economy is not known to real firms. The rule also requires very different strategies depending on which regime one is operating in, requiring a firm to be able to decide whether there is excess or insufficient capacity compared to demand. But,
in reality, an individual firm is unable to tell whether customers’ orders represent long-run needs, transient stock adjustments, or self-ordering effects. The optimal rule is not fooled, as players of the Strategem-2 game frequently are, into ordering still more in response to the rise in demand induced by their own attempt to raise capacity. It is unlikely that actual firms use dramatically different decision rules in each regime, particularly since a firm cannot distinguish with certainty in which regime the economy is operating. Rather, as experiments with managers, economists, and students confirm, people tend to use a single decision process that is locally rational, but leads to poor performance in the full system.

We stress that in problems of this type, to overcome the large gap between the requirements of optimal control and the reality of bounded rationality, the trend of the inputs is much more important in practice than precise values. The tests of controller robustness also suggest that small variations due to modeling errors and the like still produce very satisfactory results. The main finding of our analysis is the critical role of nonlinearity in determining the best strategy. Performance can be dramatically improved by determining in which regime one is operating and reacting accordingly, even if the decision rules for each regime are only approximately correct. As in any policy analysis, implementation will depend on the modeler’s ability to explicate the rationale for the policy. The use of control concepts reinforces rather than replaces the need for managerially oriented justification of the proposed policies.

**Appendix: Equations for the controller**

**FDS < 1:**
Given desired polynomial $\lambda^3 + a\lambda^2 + b\lambda + c$, the feedback vector is

$$H_1 = [h_1 \ h_2 \ h_3]$$

where

$$h_1 = \left(\frac{\pi}{\varphi \cdot BGS}\right) \left(r \cdot \varphi - r^2 + a \cdot r - b - \frac{\text{COR} \cdot c}{\varphi \cdot BGS (\pi - \text{COR})}\right), \quad r = \frac{-BKS}{\text{COR} \cdot \pi} + \frac{1}{\text{ALC}}$$

$$h_2 = \varphi + r - a$$

$$h_3 = \frac{\text{COR} \cdot c}{\varphi \cdot BGS \cdot (\pi - \text{COR})} - \frac{BKS}{BGS} (\varphi + r - a)$$

**FDS = 1:**
Only two poles can be placed. Thus, given a desired second-order polynomial $\lambda^2 + a\lambda + b$, the feedback vector is

$$H_2 = [h_1 \ h_2 \ h_3]$$

where

$$h_1 = \frac{a \cdot \text{NCAT}}{\text{ALC}} - \frac{\text{NCAT}}{\text{ALC}^2} - b \cdot \text{NCAT}$$
\[ h_2 = a - \frac{1}{\text{NCAT}} - \frac{1}{\text{ALC}} \]

The value of \( h_3 \) does not affect the characteristic polynomial. Recall that we used this degree of freedom to improve robustness.

**Notes**

1. The optimal score was determined by grid search of the score space as a function of successive decisions. The optimal strategy presumes that the step in demand is unanticipated, consistent with the information available to the subjects.
2. In the Strategem-2 game, all quantities are rounded to the nearest 10 units to simplify the decision-making task of the subjects. The simulation analyzed here permits the states to take continuous values. As a result, the score in the simulation can be less than the optimal score of 19 in the experiment.

**References**


