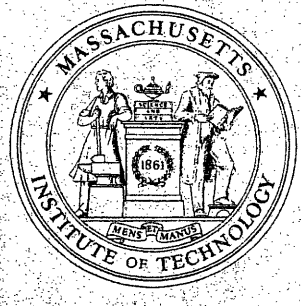
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

AN EFFICIENT MODEL FOR PLANNING BUS ROUTES IN COMMUNITIES WITH POPULATIONS BETWEEN 20,000 and 250,000

by

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ABSTRACT

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The paper discusses a flexible, inexpensive, interactive computer model specifically designed to act as an aid for planning routes for conventional bus systems in communities with populations between 20,000 and 250,000.

The characteristics of these communities, the desires of decision makers, the cost and availability of data, and the special problems inherent in route planning are examined and a useful routing model is designed. A computer implementation of the structure of its major component was developed. Usage costs are examined. (An implementation of the entire model has not yet been completed.)

The completed model will not replace community leaders or transit managers; instead, it will enable a decision maker to quickly and inexpensively test a potential route or operating decision without actually implementing it.

To test the route, the decision maker need only input the operating decision or route choice. The routing model automatically traces the route through the community and estimates whom the route serves, how well, and at what cost. It predicts how many people will use the route, their geographic location, possibly their income distribution, the travel time, wait time, the walking distance they experience, and the operating cost of the route. Based on these estimates, the decision maker can alter any decision and immediately test the new route.

In the course of model design, two new demand models are formulated: an alternative logit model for probability estimation or modal split calculations, and an extension of the intervening opportunities model for desire prediction or trip generation.

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TABLE OF CONTENTS

		Page
Part I	Problem Examination and Model Formulation	1
	1.1 Criteria	2
	1.1.1 Statement of Criteria	2
	1.1.2 Development of Criteria	2
	1.2 Features of the Routing Model	9
	1.2.1 Statement of Model Features	9
	1.2.2 Formulation of Model Features	10
	1.3 Summary	15
Part II	The Routing Model	16
	2.1 Description of the Model	16
	2.1.1 List of the Components of the Routing Model	17
	2.1.2 Initialization Components	17
	2.1.2.1 Model of the City	17
	2.1.2.2 Desire Prediction Algorithm	22
	2.1.3 Route Testing Phase	24
	2.1.3.1 Route Examination/Demand Prediction	24
	2.1.3.2 Equilibrium Model/Summary	28
	2.1.4 Output and Human Feedback Loop	29
	2.1.5 Summary of the Routing Model	29
	2.2 Usage Costs	29
	2.2.1 Usage Costs of the Trial Encodement	31
	2.2.2 Estimated Usage Costs in a Real Community	33
	2.2.3 Usage Cost Summary	35
	2.3 Conclusion	36
Footnotes		A28
Bibliography		A34

		Page
Appendix I	An Alternative Formulation of the Discriminant Function for the Two-Dimensional Logistic Model	A1
	A1.1 An Alternative Discriminant Function for the Logistic Model	A3
	A1.2 Elasticity Analysis of the Alternative Logistic Formulation	A6
	A1.2.1 Calculation of Elasticity	A8
	A1.2.2 Special Case 1: Equal Share	A9
	A1.2.3 Special Case 2: Bus Dominates	A10
	A1.2.4 Special Case 3: Auto Dominates	A11
	A1.2.5 Special Case 4: Scale Dependence of Disutilities	A13
	A1.2.6 Summary of the Special Cases	A13
	A1.2.7 Conclusion	A14
Appendix II	Extended Opportunity Model	A15
	A2.1 An Alternative Approach to Trip Generation and Trip Distribution	A15
	A2.1.1 Traditional UTP Distribution Models	A16
	A2.1.2 The Extended Opportunity Model	A17
	A2.2 Development of the Model	A17
	A2.2.1 The Simple Opportunity Model	A17
	A2.2.2 Assumptions Relaxed in the Extended Opportunity Model	A19
	A2.2.3 Derivation of the Extended Opportunity Model	A19
	A2.2.4 A Discrete Zone Approximation	A23
	A2.3 Special Cases	A23
	A2.3.1 Special Case 1: Reduction to Simple Opportunity Model	A23
	A2.3.2 Special Case 2: Gravity Form	A24
	A2.4 Further Work	A26
	A2.4.1 Transformation	A26
	A2.4.2 Probability of Termination Less Than 1	A27
	A2.4.3 Level of Service Function	A27
	A2.5 Conclusion	A27

LIST OF TABLES

Number	Title	Page
1	List of Variables in the Routing Model	18
2	Variables and Fixed Usage Costs for the Trial Encodement	34
A1.1	Comparison of Discriminant Functions for Different Travel Time Combinations	A5

LIST OF FIGURES

1	Interactions of the Components of the Routing Model	19
2	Sample of a Map Prepared to Aid the Decision Maker	21
3	Desire Matrix	23
4	Route Examination/Demand Prediction Component	25
5	Bus Stop-to-Bus Stop Origin-Destination Matrix	27
6	Sphere of Influence and the Responsibility of the Decision Maker and the Analyst	30
A1.1	Sensitivity of the Probability Function to Disutility Scaling	A11
A2.1	Representation of the Area 'dA'	A20
A2.2	Distributed Destinations	A21
A2.3	Destination Zone	A22
A2.4	Gravity Form	A24

PART I

PROBLEM EXAMINATION AND MODEL FORMULATION

According to the 1970 census there are over 70 million people in the 2245 "places"[1] in the United States which are of population greater than 10,000 but less than 250,000. Because this is a significant portion (about 34%) of the population of the United States, it is worth the while to examine the transportation needs of these communities.

This paper does not explicitly discuss the transportation problem in small communities. Instead, it examines a particular problem faced by one part of a transportation system: the planning of routes and operating policies for conventional bus systems.

The paper begins with an in-depth study of this special problem and develops a set of criteria to aid in model development. These criteria consider the necessary tradeoffs and establish a set of rules from which model formulation is to proceed.

1.1 Criteria

1.1.1 Statement of Criteria

The model should:

1. be inexpensive
2. have no extensive data requirements
3. be sensitive to characteristics that make a route unique
4. be sensitive to varying conditions
5. supply sufficient information to identify the incidence of the social benefits (costs) of route change
6. be sensitive to the needs and desires of potential bus riders
7. make marginal cost and revenue predictions
8. be acceptable to a decision maker who is not technically trained
9. be applicable to a variety of cities
10. consider competing and complementary modes of travel
11. be flexible
12. be efficient.

1.1.2. Development of Criteria

1. Inexpensive: Although some transit companies in major cities spend over $100 million[2a] annually, many small operations spend less than $500,000.[2b] This includes capital expenditures, wages, and other operating expenditures. Faced with increasing expenses and declining patronage, transit companies have already streamlined operations in an attempt to reduce costs.[3] Planning departments have been among the first to be cut back. A transit manager or public official, (hereafter referred to as decision maker) would be reluctant to spend a large sum of money for planning new routes or changing old ones if he has already cut back planning expenses. It is hard to estimate how much he would actually be willing to spend but it would certainly be much less than the amount spent by comprehensive studies such as were done for Portland, Maine ($320 thousand)[4] or Manchester, N. H. ($236 thousand)[5]. In any case, it is clear that a transit routing model for small communities should attempt to provide

necessary accuracy as inexpensively as possible, i.e. a decision maker will more likely choose a less-accurate, inexpensive model than a more accurate, expensive model.

2. No extensive data requirements: Extensive data requirements can make a model very expensive to use. For example, in the early 1960's the Southeast Wisconsin Study (Milwaukee) spent almost 1.2 million dollars[6] on data, and the Penn-Jersey Study spent almost 1.8 million dollars[7]. Data collection accounted for over 60% of the cost of these studies. True, they were comprehensive studies of major cities, but they do give an indication of how expensive data collection can be, and they do indicate that every effort should be made to minimize expensive data requirements.

Besides being expensive, massive data collection is time-consuming, often taking many months[8]. One expected use of the routing model is during crisis periods such as the potential failure of a private company. In such crises, a decision maker must have estimates quickly and cannot wait for the results of a major data collection effort. The model should be designed to use readily available inexpensive data.

3. Sensitive to characteristics that make a route unique, i.e. characteristics that make it different from other routes: this will enable the model to differentiate between alternative routes.

One important characteristic of a route is its location in the city. Because demand for transit is a derived demand, patronage will depend upon how well people and activity centers are connected. Since different routes will serve different people and different activity centers, the model should be quite sensitive to the location of the people and of activity centers with respect to bus routes.

Demand is extremely sensitive to walking distance, often dropping off dramatically if people are forced to walk more than one block.[9] The model should estimate how far people must walk, and should be sensitive to changes in walk distance of as little as one block.

Another important characteristic of a route is travel time along that route. The model should be sensitive to the street network and traffic

conditions which determine travel time. It should declare infeasible any route that includes a link such as a narrow winding road which a bus simply cannot effectively traverse. It should do this even if it requires judgmental estimates by the model user.

4. Sensitive to varying conditions: Transit demand is extremely sensitive to time of day[10], day of week[11], and season of the year[12]. Different routes may be more suitable during different hours of the day or different days of the week.

For example, a transit company may want to run different routes during peak hours, when trips are predominantly work trips than during off-peak hours, when shopping trips are relatively more common. Similarly, the company may want to periodically revise its routes because of seasonal fluctuations in demand. Thus the model should be sensitive to time of day, day of week, and season of the year.

Furthermore, a company may wish to examine special routes for special events. The model should be flexible enough to adapt to changed conditions resulting from special events such as shopping center openings or county fairs.

5. Supply sufficient information to identify the incidence of social benefits (costs) of route change: Any route change can have disruptive effects on the travel habits of many passengers, but a new route might serve the majority of the people better and increase patronage. Many people affected by a route change will be very vocal in objecting to the change. Some will object to loss of service while others will object to noise, dirt, and a perceived safety hazard[13] resulting because a new route brings a bus past their homes. A decision maker faced with unfavorable publicity will want to justify his route choice by having an indication of the incidence of benefits and costs of that new route.

The model should indicate level of service parameters, such as travel time, wait time, and walk distance, predict ridership, revenue, and costs, and should indicate both the location and some of the social characteristics[14] of the people served by the route.

6. Sensitive to the needs and desires of potential bus riders: The reason a person will use one route and not another is that the chosen route fulfills his needs and desires. For example, a poor person would be more likely to sacrifice savings in travel time to save on travel cost. To account for this, the model should be sensitive to the characteristics of the people who are served by the bus route.

A good way to make a model sensitive to these needs is to base it on probabilistic behavioral assumptions about individual choice. Of the demand models currently existing, the closest to this ideal are the behavioral demand models[15].

7. Marginal cost and revenue predictions: Whether he represents a private company which is interested in profit or a public authority that is interested in social benefits, the decision maker will want to know the net cost of a transit route. That is, he will want an estimate of the marginal cost and the marginal revenue of the additional route.

Revenue depends upon patronage (demand estimates are discussed elsewhere) but not every rider of a bus route yields a marginal addition to total system revenue equal to his fare. Some gains in patronage will result from losses to other routes. Some routes will act as feeder routes making others more popular. Finally, some passengers will pay no fare because of free transfers from other routes.

Such network effects will be significant in highly dependent networks such as exist in Bogata, Colombia, which has a network of 384 routes and, in some sections, more than 50 different bus routes passing along the same street section.[16] To consider such effects in detail, a model must be an extremely complex network model that simulates the entire network of routes simultaneously. This would add greatly to the expense of model use because such a complex model would need to be run to test every individual routing decision.[17]

If routes are more or less independent, as they might well be in small communities that require significantly fewer routes[18], then a good first-order approximation[19] is to ignore network effects and treat intersecting routes as sources of demand dependent on the level of service of the route under

consideration. The routing model being developed in this paper assumes such independence, but this assumption must be tested before the model is used in the field.

Cost of a route is the result of a variety of components. Some costs such as drivers' wages occur by the bus hour; some costs such as fuel and tire wear occur by the bus mile; and some costs are overhead costs and may or may not increase if an additional route is added. To accurately predict the marginal cost of a route, the routing model should properly consider all three types of costs.

8. Acceptable to a decision maker who is not technically trained: A model may be 100% accurate, but if it is not acceptable to the person who must make a decision based on the model then the model will never be used.

Predicting demand for transit routes is a problem in marketing research. The consumer is presented a product in the form of a transit route and the model tries to predict his reaction to it. John Little[20], through working with managers responsible for marketing decisions, has developed a set of requirements that a model must meet to be acceptable to managers. The model should be:

1. simple
2. robust
3. easy to control
4. adaptive
5. as complete as possible
6. easy to communicate with.

"By simple is meant easy to understand; by robust, hard to get absurd answers from; by easy to control, that the user knows what input data would be required to produce desired output answers; adaptive means that the model can be adjusted as new information is acquired; completeness implies that important phenomena will be included even if they require judgmental estimates of their effect; and, finally, easy to communicate, which means that the manager can quickly and easily change inputs and obtain and understand the outputs."[21]

These are important requirements that promote trust in a model and make it easy to use. The typical transit manager or public official is not technically trained and if he cannot understand and trust a model or if he finds it difficult to use, then he will probably not use it.

9. Applicable to a variety of cities: If a separate model has to be developed for each community, then each application could become very expensive and require extensive data. Thus the model should identify similarities between communities and identify the factors that influence an individual's choice of transportation. This implies that the model should not be a naive or correlative model, but instead should be a causal model[22] based on behavioral assumptions[23] about individual choice.

To account for differences among communities the model should attempt to include as many relevant phenomena as possible "even if they require judgmental estimates."[24]

One important phenomenon is network effects. This has already been discussed. Another is the equilibrium effects of supply and demand. Demand is certainly dependent upon supply, i.e. patronage depends upon the level of service and fare, but supply is also dependent upon demand. If more people ride a bus, then the crowding conditions and travel time[25] might significantly increase. Since an equilibrium component of the model can significantly increase the complexity of the overall model[26], the question naturally arises: Can equilibrium effects be ignored?

Damian Kulash, in a single route simulation analysis, found that "performance is not greatly affected by the volume of passengers using the route in most cases."[27] This result indicates that for simplicity, equilibrium effects might be ignored in the initial modelling attempt, but because there is some doubt to the validity of ignoring these effects, the model should be designed to allow the later additions of an equilibrium model. The effects of including the equilibrium model would be tested before the model is used in the field.

10. Consider competing and complementary modes of transportation: If buses were the only means of travel, then all trips would be made by bus. Since there are other modes such as automobiles or taxicabs that

account for significant numbers of trips[28], it is important that a bus routing model consider the effects of alternative modes of transportation. Besides allowing for more accurate demand predictions, these comparisons could aid a public official if he must decide how much money to allocate to a public transit system.

The model should also consider complementary modes of transportation. For example, local buses (or automobiles) can act as feeders to express buses, subway systems, or airports. Such consideration of the interaction of different modes of transportation can aid the development of a balanced transportation system.

11. Flexible: Since the model is to be used for many types of applications in a variety of communities, it must be flexible enough to adapt to changing conditions.

There is the possibility that some types of data may be available in some communities but not in others. Some existing transit companies might have kept very accurate patronage counts, some may have made comprehensive studies such as authorized under the 1962 Highway Act[29], and some may have only census data available. The model should adapt to make maximum use of any existing data.

It should also allow for a choice of specific demand models. There are many types of demand models available such as aggregate models[30], and disaggregate behavioral models[31]. A previous section indicated that disaggregate models are good because they are sensitive to the desires of individuals. Indeed they are, but other models might be better in specific circumstances depending upon the available data and the required accuracy. The routing model should allow for various demand models to be included if necessary. Besides, research in demand theory may yield models far superior to disaggregate behavioral models, and if the routing model is not to become obsolete it should be possible to easily adapt it to include newly developed demand models.

Finally, for ease of use, it should be a simple task to change any model parameter.

12. Efficient: The final criteria for a routing model is efficiency. The model should avoid consideration of irrelevant data and it should

avoid making the same calculation a number of times.

Not every part of a community is served by a single route. Tremendous savings in computation can be realized if the model tries to consider only those parts of a community that are actually significantly served by the route under consideration. For example, if the city is partitioned into 100 zones but the route only serves 20, then only approximately 400 (20 X 20) zonal interactions need be considered, instead of 10,000 (100 X 100).

1.2 Features of the Routing Model

Before a routing model could be developed, it was necessary to examine existing models and methods to avoid their weaknesses and build upon their strengths. Development of criteria and examination of existing models was fruitful in that it focused attention and indicated what type of routing model need be developed and investigated.

This section formulates model features which attempt to satisfy as many criteria as possible. The formulation of these features organizes model development and is an important step in model creation. These features set tight bounds on the type of model that can be developed and thus insure that the model fulfills many of the criteria of section 1.1.

1.2.1 Statement of Model Features

The following features are developed in section 2.3.2:

1. Simple basic prediction process.
2. Causal reasoning based upon behavioral assumptions about individual choice.
3. Sufficiently small analysis zones to enable the model to be sensitive to to route location within the city.
4. Interactive computer model on a time-sharing system.
5. Two-phase model usage
   a. initialization by experienced analyst.
   b. usage by non-technically trained decision maker.
6. Modular subprograms for greater flexibility.
7. Programs encoded in PL/I.
8. Low cost of computer package.
9. Concentrate on census data because it is easily accessible but adapt to make maximum use of other existing data.

1.2.2 Formulation of Model Features

1. Simple basic prediction process: To be applicable to a large number of cities (Criterion 9) the model should be flexible enough to adapt to changing conditions (Criterion 11). One way to gain flexibility is to have a simple underlying prediction process. A simple process based on common sense would be understandable and acceptable to a decision maker (Criterion 8).

The routing model uses the following basic procedure: it automatically traces the route through the city, determines who and what activity centers the route serves, and estimates the level of service potential riders would experience. Then, based upon characteristics of the potential riders, the level of service they experience and how well they are connected to activity centers, the model estimates for each individual[49] the probability[50] that he will ride the bus in the period under consideration. The patronage estimate is then the expected value of the number of riders choosing the route.

Note that this basic process supplies information on the incidence of benefits of a bus route (Criterion 5) and is sensitive to the desires of potential riders (Criterion 6). It implies the use of a disaggregate demand but could be adapted (Criterion 11) to aggregate models because it gives level of service estimates. To calculate the probability that transit is chosen, the model must consider competing modes. Complementary modes such as airports or rail stations can be treated as activity centers (Criterion 10).

Thus the basic process partially fulfills criteria 5, 6, 8, 9, 10 and 11.

2. Causal reasoning based upon behavioral assumptions about individual choice: discussed earlier (criteria 6, 9), to be applicable to many cities without extensive restructuring or data collection, the model must identify factors that influence an individual's choice. That is, the model can not be simply correlative; it must be as deductive as the current state of demand theory allows.

Two models necessary for the basic prediction process were (1) a model to estimate the probability that transit is chosen given a trip is made, and (2) a model to predict the spatial trip desires of potential riders. In the initial implementation of this routing model, the two-dimensional logistic

model was chosen as the primary probability model and the opportunity and extended opportunity models were chosen as the most promising predictors of spatial trip desires.

Currently MIT is engaged in basic research on demand theory, and hopefully superior models will result from this research.

3. Sufficiently small analysis zones to enable the model to be sensitive to route location within the city: Considering every individual separately could become very expensive. For example, if a community had only 10,000 people and each one had to be interviewed separately, then, based upon a minimum estimate[54] for the cost of a home interview survey, it would cost $100,000 for the interviews alone. To cut down on this expense (as per Criterion 1) it is necessary to divide the community into analysis zones and within the zones, group together people with similar characteristics.

Another reason for having analysis zones is that one possible source of data is the United States Census. The Census Bureau will not divulge information about individual households because it must protect an individual's right to privacy.

Analysis zones are necessary but they cannot be too large. In a study in Madison, Wisconsin, Fleet and Robinson[55] found that larger aggregation causes more information to be lost. In fact, in their study 80% of the variation in socio-economic data was within zones and 20% between zones. Furthermore, demand for transit is highly sensitive to walking distance and using large zones can obscure explicit calculation of

individual walking distances. Hopefully, such losses of information can be minimized by identifying homogeneous subgroups within small zones.

How small should the zones be? Analysis of small zones requires more detailed data and many more calculations[56], hence there is a tradeoff between accuracy and cost.

The routing model is designed to accept zones of any size; but an initial estimate[57] of the optimal zone size is four square blocks. A field test of the routing model is necessary to verify if this is a proper choice. Hopefully, zones this size will enable the routing model to be sensitive to the location of a route with respect to people and activity centers, and with respect to walking distance[58]. If the model proves sensitive enough, it will be able to differentiate between alternative bus routes (Criterion 3).

4. Interactive computer model on a time-sharing system: One of the fundamental criteria is that the model must be acceptable to the decision maker (Criterion 8). To be acceptable, it must be simple to use and easy to communicate with. Many repetitive calculations can overwhelm a manual procedure and obscure the basic process, but a computer can quickly do complex repetitive calculations and thus leave the model user free for more important tasks.

An interactive computer program enables a previously inexperienced user to control model calculations even if he does not completely understand them. He need only to interact with the model on a level he finds convenient. Default options enable the non-technical user to ignore any complications he finds unnecessary, but enable the technical user to completely control all aspects of the model.

An interactive computer program is also easy to communicate with. A decision maker can very quickly get estimates of the results of his decisions. Based on these estimates, he can then alter his decision and try again or proceed to the next phase in his decision process.

In summary, an interactive computer model is more acceptable to a decision maker because it is simple to understand, easy to use, and easy to communicate with.

5. Two-phase model usage: Some tasks necessary to the prediction of the impacts of routing decisions require an experienced analyst and need only be done once. Initial data acquisition and input, choice of demand model, calibration and parameter choice, and instruction of the decision maker are all tasks that require the experience and technical competence of a trained analyst. But once these tasks are completed, the model can be turned over to the decision maker who will then use it as an aid to making routing decisions.

A stratification of model usage into initialization and actual usage is more efficient (Criterion 12) and enables the model to be more accurate without becoming more difficult for the decision maker to use. It is more efficient because some calculations, such as estimation of trip desires[59], need only be made once. They can be stored on magnetic tape or disk storage and can be quickly read instead of recalculated each time they are needed. The routing model appears simpler to the decision maker because he need not concern himself with the details of the necessary, more comprehensive analysis which is done by a trained analyst.

6. Modular subprograms: Modular subprograms enable the model to be flexible (Criterion 11) and sensitive to varying conditions (Criterion 4). The interactive program presents the analyst (or even the decision maker) with a choice of a variety of subprograms for many of the components of the routing model. Or, if he wishes, he can substitute his own subprogram for any of the components.

This modularization makes possible two-phase model usage and allows for eventual incorporation of an equilibrium model or other extensions. If changes in demand theory result in superior demand models, then those models could be encoded and added as options.

Since both simple and complex models will eventually be added, an analyst can choose the model that is best suited for the community under study and the data available. Furthermore, the user will have a choice between complete output, or abridged versions. These are but an indication of the flexibility afforded through the use of modular subprograms.

7. Programs encoded in PL/I:[60a] Machine language or assembler language has the potential to be more efficient, but higher level language such as PL/I are much easier to understand and use. Ease of use is an important criterion and the model should be easy to use for the analyst (phase I) as well as for the decision maker (phase II). New models and other extensions are more likely to be tried if they do not require working with assembler or machine language.

PL/I is as easy to learn as other higher level languages (e.g. FORTRAN) but is much more flexible. "A beginner can take advantage of the many automatic features of the language to do much of his work for him. An experienced programmer can use PL/I to specify almost every detail of every step of a highly complicated program."[60b] But the primary reason for choosing PL/I is that it allows tree-structures which make data handling much more convenient. Unfortunately it is an IBM language and is not yet available on all machines, though this is gradually changing.

8. Low cost of computer package: Criterion 1 states that the routing model must be inexpensive to use. One major expense is the cost of computation; thus every effort should be made to keep that cost as low as possible. The program described in Part II costs less than ten dollars per run. This represents a single test with phase II, i.e. the portion of the program run each time a route is tested.

9. Concentrate on Census Data because it is easily accessible but adapt to make maximum use of other existing data: As discussed in section 2.1.2 (Criterion 2), data collection can become very expensive. Home interview surveys cost at least $10 per household in 1963[61] and can be expected to cost more than that now. Postcard surveys, cardon-counts, and revenue counts collect data on current users but not on potential users.

An alternative source of data is the U.S. Census[62]. Census data is very inexpensive. For example, 31 reels of magnetic tape, which cost only $60/reel, contain all the census data on the state of Kentucky; 2054 reels contain all the data on the United States. Handling costs are

reduced because the data is already on magnetic tape.

During the design of this routing model, an extensive investigation[63] was made of census data and it was found to be a potentially feasible data source if certain problems can be overcome.

Sometimes a community has data available that is more extensive, more complete, or more transportation-oriented than census data. Modular subprograms make it possible to choose demand models that increase the accuracy of the model by including such data. One likely form of "extra" data is ridership counts on an existing bus network. Such ridership counts would prove useful in the calibration phase of model usage.[64]

In short, if census data is the only data available, the model can make predictions based on it, but the model will make use of other data if it is readily available.

1.3 Summary

Section 1.1 examined the special problem of planning routes for conventional buses in small communities and developed a set of criteria. These criteria were then used by section 1.2 to formulate model features which lay a foundation for the development of the routing model.

Part II postulates the routing model and describes the usage costs incurred by the "route-testing phase" of the model.

PART II
THE ROUTING MODEL

2.1 Description of the Model

The model proposed in this section is to be used to aid a decision maker plan routes for small communities. Once the model is initialized, the decision maker can simply test a route by inputting a series of bus stops and/or certain operating characteristics. Given this information, the model will predict cost and revenue estimates, patronage estimates, and level of service estimates for the entire route or for any zone or population subgroup.

The model is a short-run planning model, that is, it predicts the impacts of bus systems given the current characteristics of the community and its people. It does not attempt to model long-term population shifts or activity system changes. It observes the activity system and its effects on the demand for bus service, but it does not model how a bus system might cause changes in the activity system.

The model includes six components. Two components, the model of the city and the desire prediction algorithm are part of the initialization phase of model use. (Two-phase model use is described in Part I.) The analyst accumulates and inputs data to those models which provide a mathematical description of the city for the second phase of the model.

The route testing phase also contains two components, route examination/demand prediction, and equilibrium model/summary. The decision maker interacts directly with this phase of the model which performs the simulation of the route.

Finally there is the output component and the human feedback loop. The loop is included simply as an indication that the decision maker will want to improve his route choice based upon the results of the route test.

Table 1 summarizes the exogenous, control, state, and "other endogenous" variables of the model. Figure 1 is a representation of how the various components interact.

2.1.1 List of the Components of the Routing Model.

1. Model of the city.	Initialization Phase
2. Desire prediction algorithm.	
3. Route examination/demand prediction.	Route Testing Phase
4. Equilibrium model/summary.	
5. Output.	
(6. Human feedback loop.)	

2.1.2 Initialization Components

2.1.2.1 Model of the City

In preparation for use of the model, the analyst must identify the potential bus network and divide the city into analysis zones.

He must identify feasible bus stops and the streets capable of supporting a bus route. (He must consider technical feasibility, i.e. is the street wide enough, but he may also consider political feasibility, i.e. is a certain bus route unacceptable to the community.)

He defines a coordinate system and assigns coordinates to the bus stops and/or measures the travel times along links connecting adjacent bus stops.

He then divides the city into analysis zones making a reasonable effort to define the zones such that no zone contains more than one feasible "bus stop". Zones chosen in this fashion are very sensitive to route location.

(One feasible bus stop means one location within the city at which a bus can stop to take on or let off passengers. In the case of

Exogenous Variables	Control Variables
location of the people	route location
characteristics of the people (income, age, sex, etc.)	fare
location and type of activity center	frequency (in each time period)
street network	driver's wage
permissible streets and bus stops (political and technological reasons)	headway distribution at dispatch point
speed on street network	
parking charges	
access time to automobile	
time of day, day of week, season of the year	
maintenance, fuel, etc. costs of bus (per mile costs)	
overhead charges for bus system	
direct cost per mile of automobile travel	
overhead cost of automobile	
tastes and preferences of the people	
predisposition to one mode or the other	

State Variables	Other Endogenous Variables
wait time for bus	desire for travel
walking distance (origin and destination)	probability of choosing bus
travel time by bus (line haul and total)	simple level of service estimates for transportation system
cost to user of bus	bus hours required by route
cost to user of automobile	bus miles required by route
demand for bus route	
cost to operator of bus route (marginal, net, and total)	
travel times and costs for other modes	

Table 1

List of Variables in the Routing Model

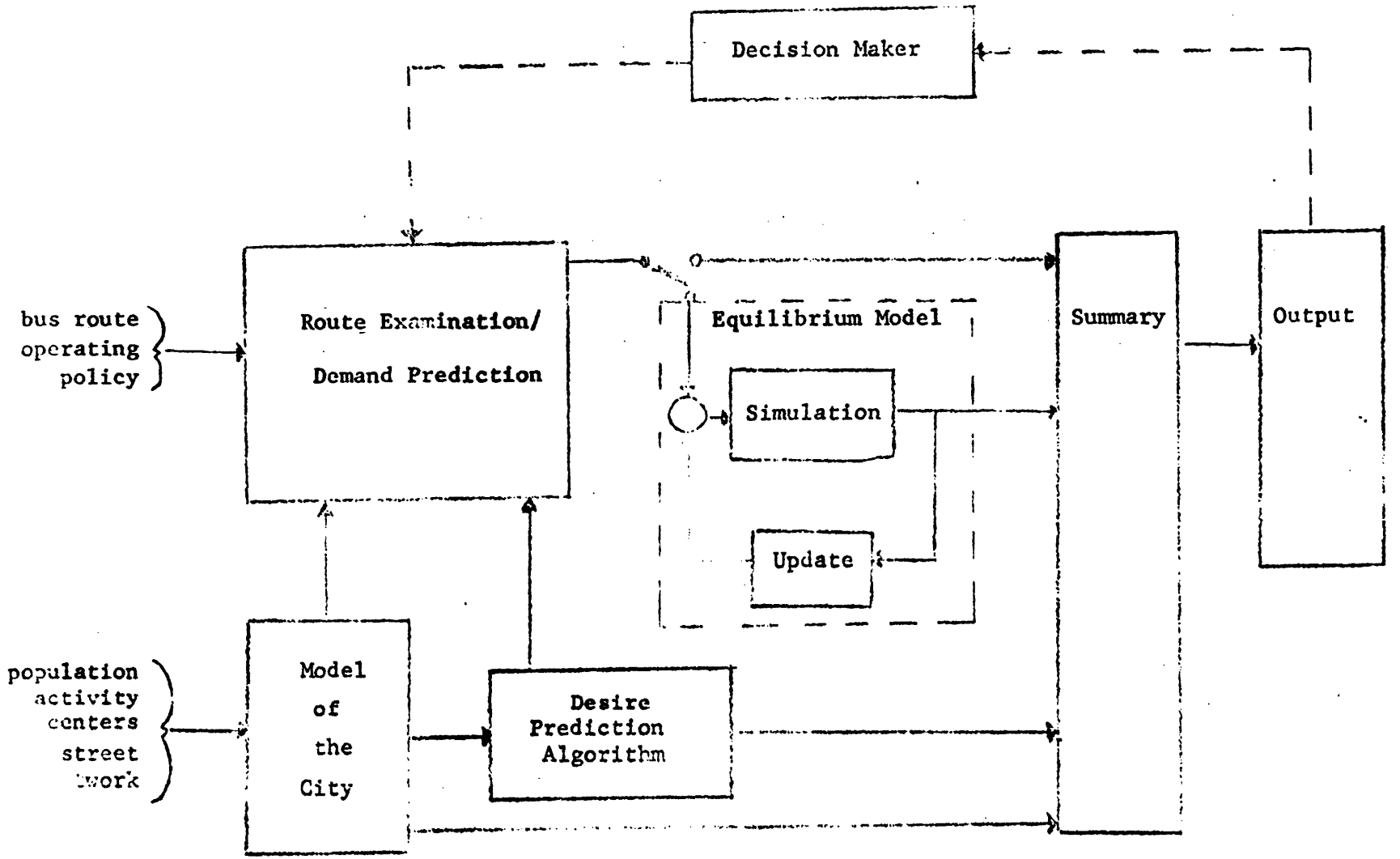


Figure 1

Representation of the Routing Model

intersecting streets, the four corners are not considered unique bus stops. In fact, one feasible "bus stop" can serve more than one route if those routes intersect at that bus stop.)

Having done this, he prepares a map, similar to Figure 2, which will help the decision maker visualize much of this information and enable him to use his pattern recognition ability and intuition to augment any predictions from the computer.

Now, the analyst must obtain the census data (or any other available relevant data) describing the location and characteristics of the population, and data, perhaps from the Chamber of Commerce, describing the location and type of trip generators. (Trip generators are activity centers which people visit, such as hospitals, shopping centers, schools, or factories.) The interactive nature of the program makes it easy for him to input this data. For example:

Computer: Input the number of zones.
User: 16
Computer: Input the coordinates of the zonal centroid and the area of the zone in the form: x(i), y(i), area(i).
Computer: Zone 1 (user:) 16, 6, 28.3
Computer: Zone 2 (user:) 17, 29, 64.5
...
Computer: Zone 16 (user:) 6, 103, 12.4
Computer: How many population subgroups are in each zone?
User: 2
Computer: Input the population of the subgroups for each zone in the form: population (i, 1), population (i, 2). (etc.)

Finally, the analyst inputs certain information, if it is available, that describes a current transit system, a taxi system, or a dial-a-ride system. The model can proceed without such information, but if available it enhances model accuracy.

Once the data is entered, the computer gives the analyst a choice of using previously encoded demand models or of using any special model he wishes to encode. Most demand models have parameters which must be set.

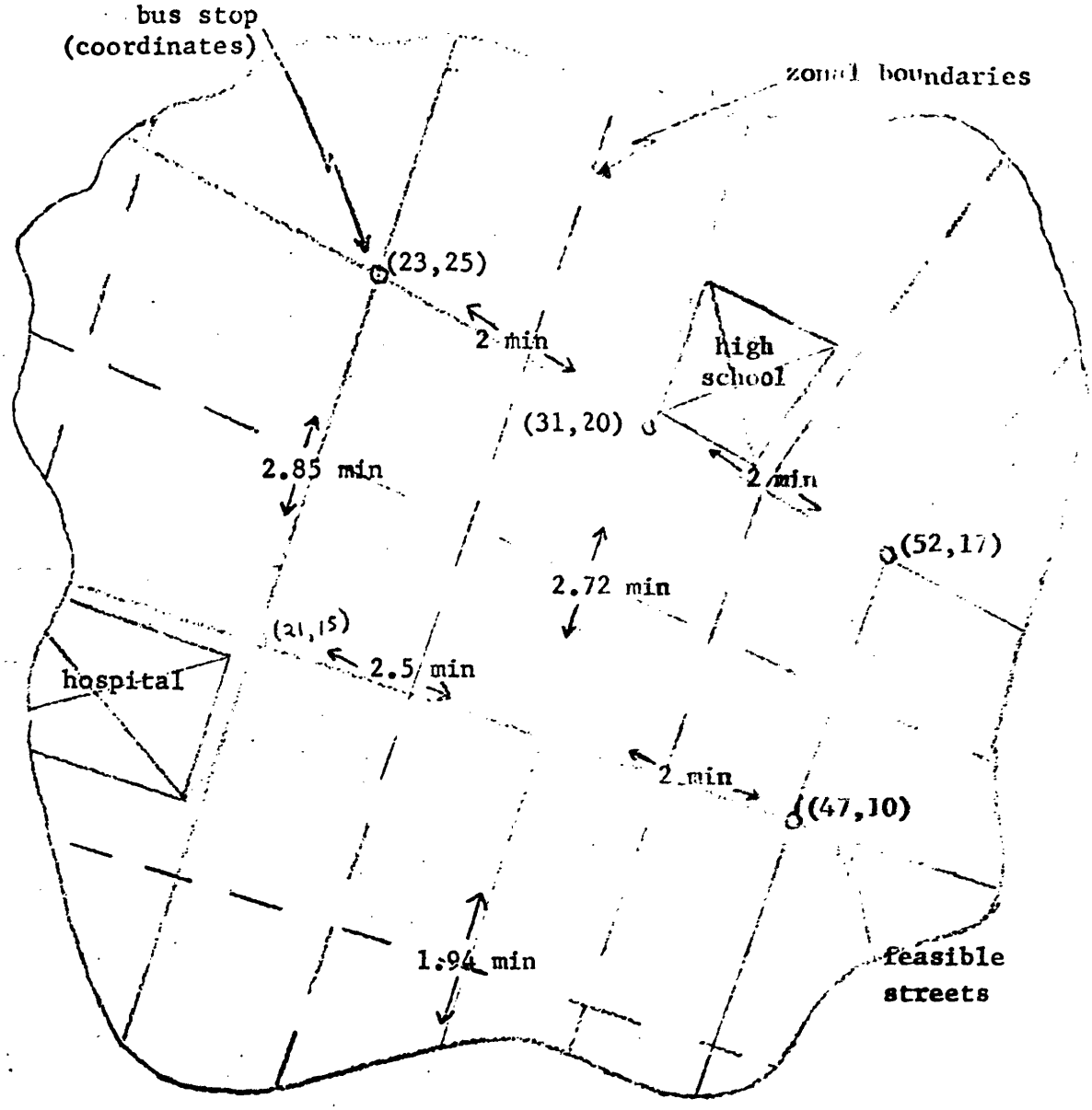


Figure 2

Sample of a Map Prepared
to Aid the Decision Maker

If the chosen model requires any, the computer asks for them or for information that will enable it to calculate them.

As described above, the main task of the model of the city is to store data describing the city. It also makes some simple heuristic interzonal level of service estimates based upon the input data. These data and estimates are stored in disk memory or magnetic tape so as to be readily available to both the desire prediction algorithm and the route testing phase.

2.1.2.2 Desire Prediction Algorithm

Given population characteristics, including their locations, the relative locations of trip generators, the street network, and relative and absolute interzonal level of service estimates, this algorithm predicts specific desires for travel and represents this information in the form of a desire matrix (see Figure 3). The elements of this matrix are proportional to the number of people in zone n, of population subgroup p, who wish to go to trip generator of class k, in zone m. (An example of a population subgroup might be people who have an income less than $4,000 per year, own no car, and are employed as laborers. An example of a class of trip generators might be department stores.) This algorithm is not exactly a trip generation model because it tries to estimate desires for trips, not actual trips. Whether the trips are actually made depends upon the level of service offered by the transportation system.

Note that this algorithm estimates where people want to go. A later component decides if and how they travel. This is not an exact modelling of human behavior; people do not usually make two-part travel decisions. The two-part model structure partially violates the criteria of behavioral assumptions, but the gains in efficiency and in ease of use for the decision maker more than offset the loss in model causality. A field test of the model will determine if this two-step process can accurately model travel demand for a bus route.

Ideally, the desire model should depend upon the relative and absolute level of service estimates from the origin zone to all destination zones; it should contain different parameters for each population subgroup,

trip generator class, time of day, day of week, and season of the year. The degree to which these goals are satisfied depends upon the choices made by the analyst.

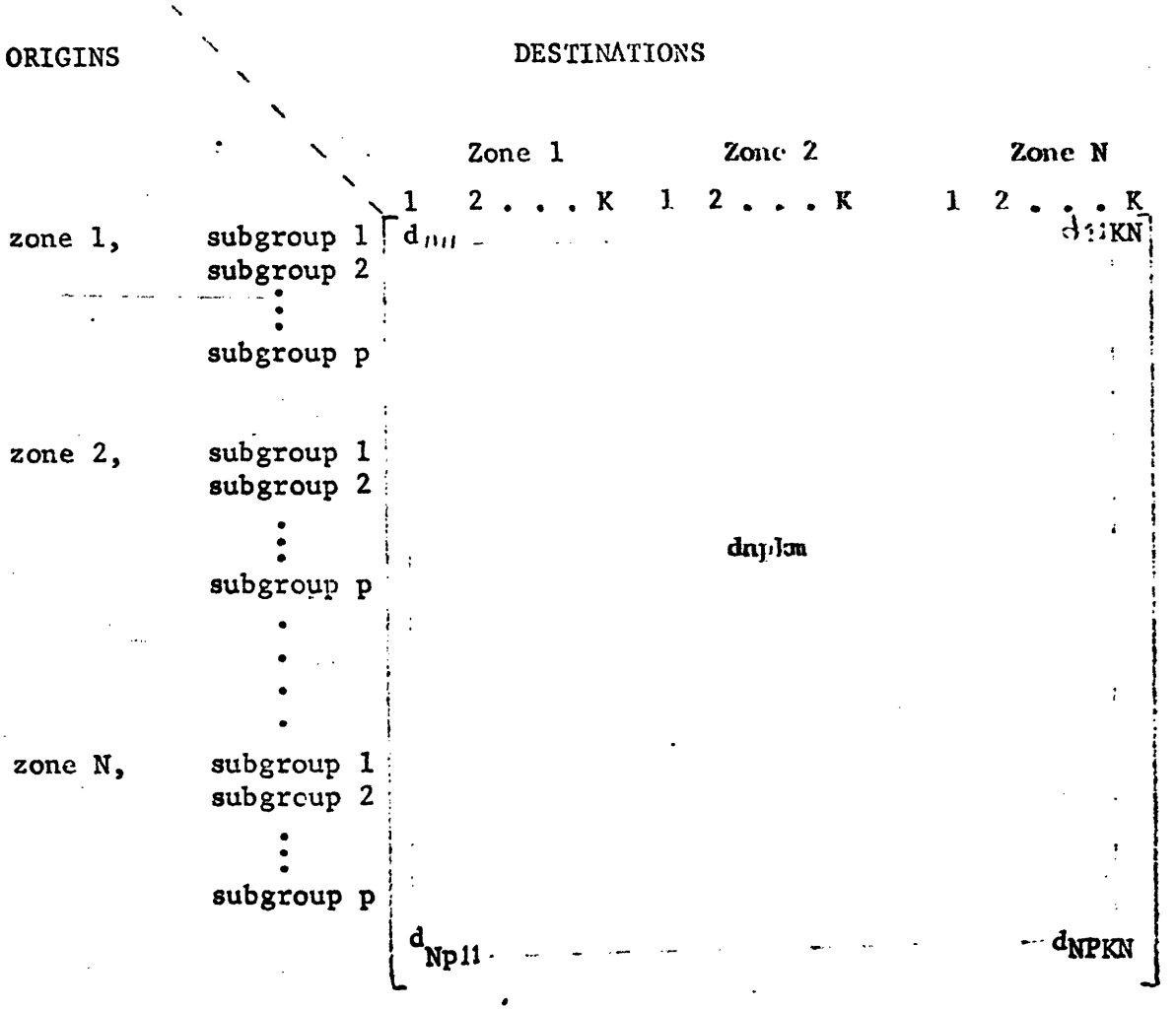


Figure 3

Desire Matrix

Eventually the analyst will be given a choice of many desire generation models, but in this research effort none has been completely developed. Preliminary research favors either the intervening opportunities model[3] or the extended opportunity model[4]. Both of these models are described in Appendix 2.

2.1.3 Route Testing Phase

Unlike phase I which is activated just once, the route testing phase is activated each time a route is tested. It asks the decision maker to input the route and operating characteristics. Then it uses the mathematical representation of the community to predict cost, revenue, patronage, and level of service for the entire route or for any zone or population subgroup.

This phase consists of two components, route examination/demand prediction and equilibrium model/summary.

2.1.3.1 Route Examination/Demand Prediction

The route examination/demand prediction component[6] is the core of the routing model because it actually predicts the impacts of specific bus routes. This prediction requires many tasks and is composed of five subcomponents. (See Figure 4.) The model user has a choice of a variety of options and a variety of demand models for many of the subcomponents.

The input subcomponent requests information in simple form from the decision maker. He is to input the specific bus route as a series of bus stops. (Express routes are input as a series of local bus stops with a no-board restriction on the local stops.) He can also set operating characteristics such as fare, frequency, and driver's wage. Eventually he will be able to choose a headway distribution at the dispatch point.

The zone-to-bus stop assignment subcomponent traces the route through the city and decides which zones are actually served, which bus stop serves those zones, and how far a person must walk to that bus stop. This step drastically reduces the number of calculations necessary for prediction.[7]

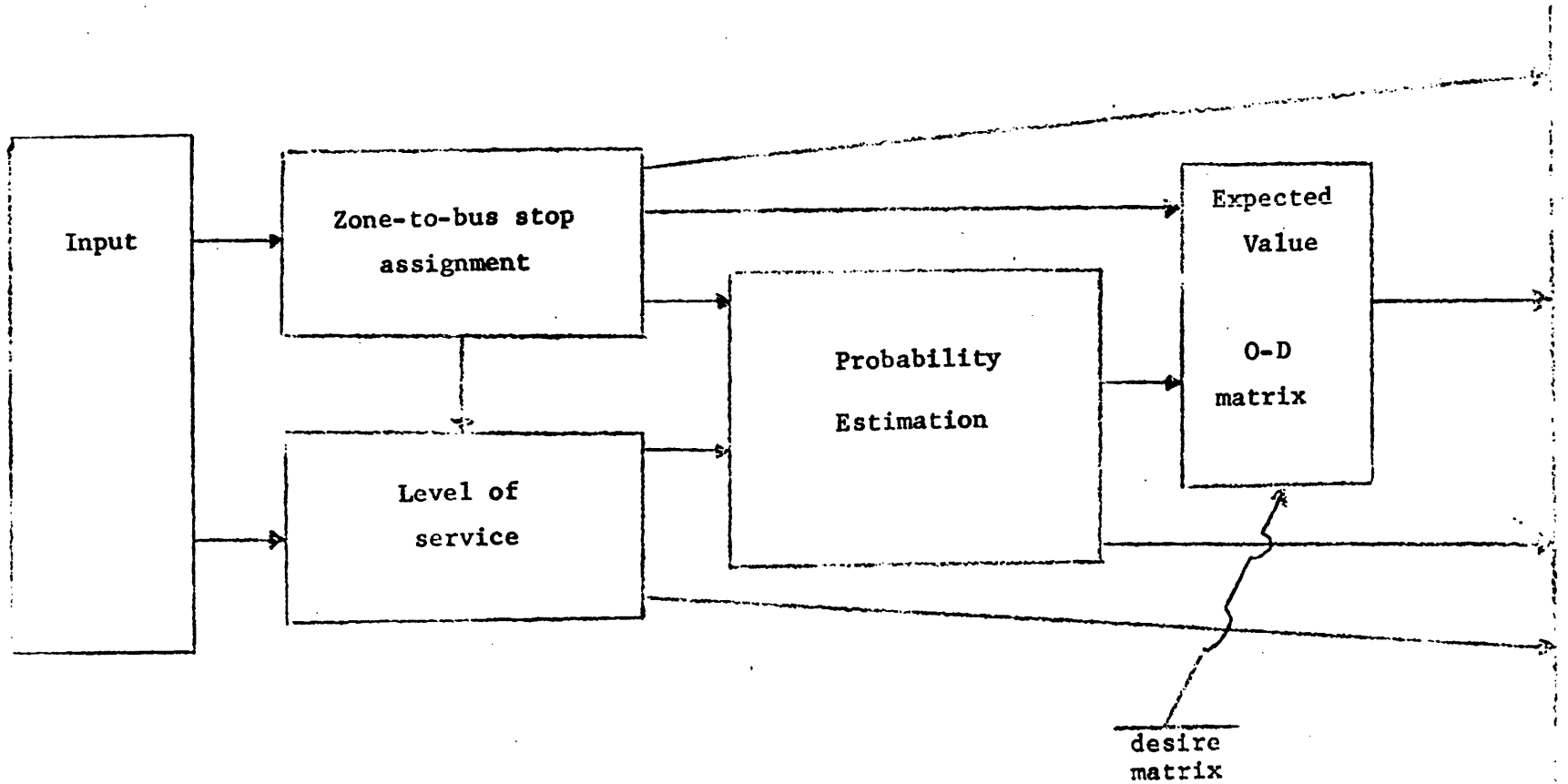


Figure 4

Route Examination/Demand Prediction Component

The level of service subcomponent predicts the wait time and travel time a person will experience if he uses the route. These estimates are made for each combination of zones served by the route.

The probability estimation subcomponent is the heart of the route examination/demand prediction component. For each combination of origin zone, population subgroup, trip generator type, and destination zone, it predicts the probability that a person of that subgroup will use the bus route under test to make that particular trip. The demand model and its parameters used in this estimation will have been chosen in the intitalization phase, but the decision maker can change them if he so desires.

(Appendix 1 describes the primary demand model used in this subcomponent.)

Finally, the expected value/O-D matrix subcomponent uses the probabilities and the desire coefficients to produce an origin-destination matrix (O-D matrix). The elements of this matrix, λ_{ij}, are the expected value of the number of people who will use the route to travel from bus stop i to bus stop j, for all i and j (see figure 5). This value is simply a sum over all origin zones near bus stop i, all destination zones near bus stop j, all population subgroups, and all trip generator classes. The elements of the sum are the product of the desire coefficient and the probability that that trip will be made by the route under test. That is:

If: S_i = the set of zones within walking distance of bus stop i.

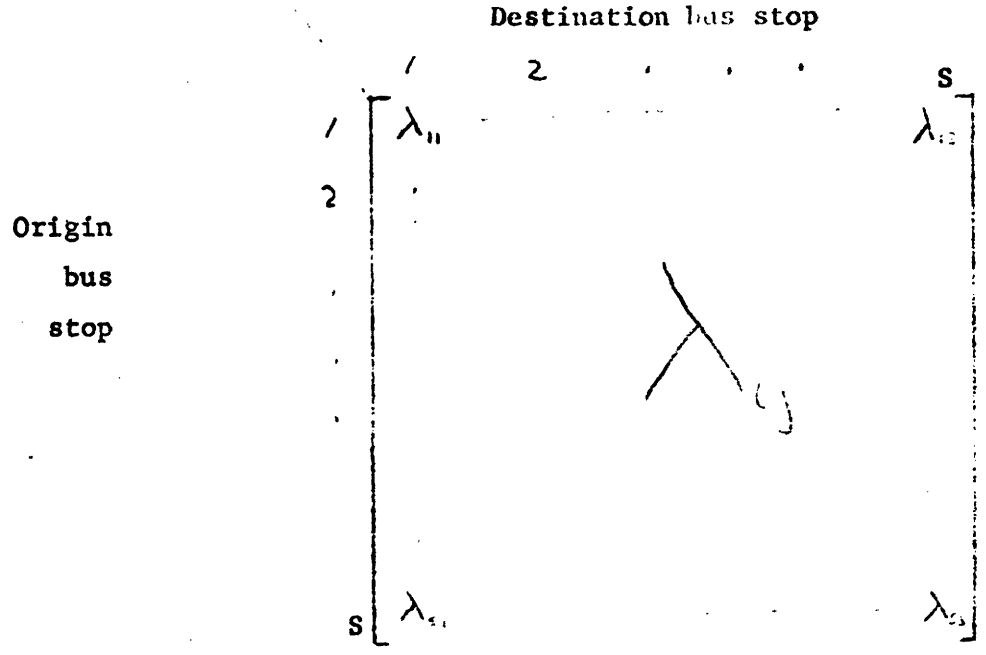
S_j = the set of zones within walking distance of bus stop j.

d_{npkn} = the desire coefficient (see Figure 3).

$P(n,p,k,m)$ = the probability that a person in zone n, of population subgroup p, will use the route under test if he is making a trip to trip generator of class k, in zone m.

Then:

$$\lambda_{ij} = \sum_{S_i} \sum_{P} \sum_{k} \sum_{S_j} d_{npkm} * P(n,p,k,m) \qquad (2.1)$$



λ_{ij} = expected value of the number of people who will use the bus route to travel from bus stop i to bus stop j.

Figure 5

Bus Stop-to-Bus Stop

Origin-Destination Matrix

2.1.3.2 Equilibrium Model/Summary

Although the equilibrium model has not been encoded, the model is structured to allow it to be easily included, and if included, to allow the user to choose whether or not to use it.

The equilibrium model considers the interaction of supply and demand. An initial O-D matrix and initial level of service estimates are produced by the route examination/demand prediction component. Given these, the equilibrium model uses Kulash's single route model[8] to simulate the route to obtain more accurate estimates of travel time, wait time, and crowding conditions. These estimates are used to update the patronage estimates. This process is continued until equilibrium or near-equilibrium[9] is reached.

The summary program is necessary because the decision maker may not want to know the detailed predictions of the model. He may feel that certain aggregated summaries are simpler to use and sufficient aids to his decision process. The summary program collates estimates from the route examination/demand prediction component to produce these summaries. For example, if the decision maker wants to know how many people of zone n used the route, then that number, N, is calculated thus:

$$N = \sum_{\forall p} \sum_{\forall k} \sum_{\forall m} d_{npkm} * P(n,p,k,m) \qquad (2.2)$$

One important summary the decision maker will want is the cost of operating the route. The summary program uses the patronage and running time estimates along with route distance estimates to predict cost and revenue for the route. Certain cost components, for example the cost of fuel per mile, are set in the initialization phase but can be reset by the decision maker in this phase. This option is useful for testing certain cost sensitivities, e.g. the sensitivity of cost to driver's wage.

2.1.4 Output and Human Feedback Loop

The output phase allows the decision maker to choose which summaries he wants and provides him with the option of having only summaries printed out on the teletypewriter and all the detailed calculations printed out on the high speed printer.

Human feedback loop: The decision maker examines the summaries and based on them he makes a judgment as to how "good" the route is. If he is not satisfied with the route he can alter it and test the new version. The routing model does not plan routes, it simply provides the decision maker with information on which to base his decisions. If he so desires, he can examine the detailed printout (from the high speed printer) at his leisure and check decisions he made based upon the summaries.

2.1.5 Summary of the Routing Model

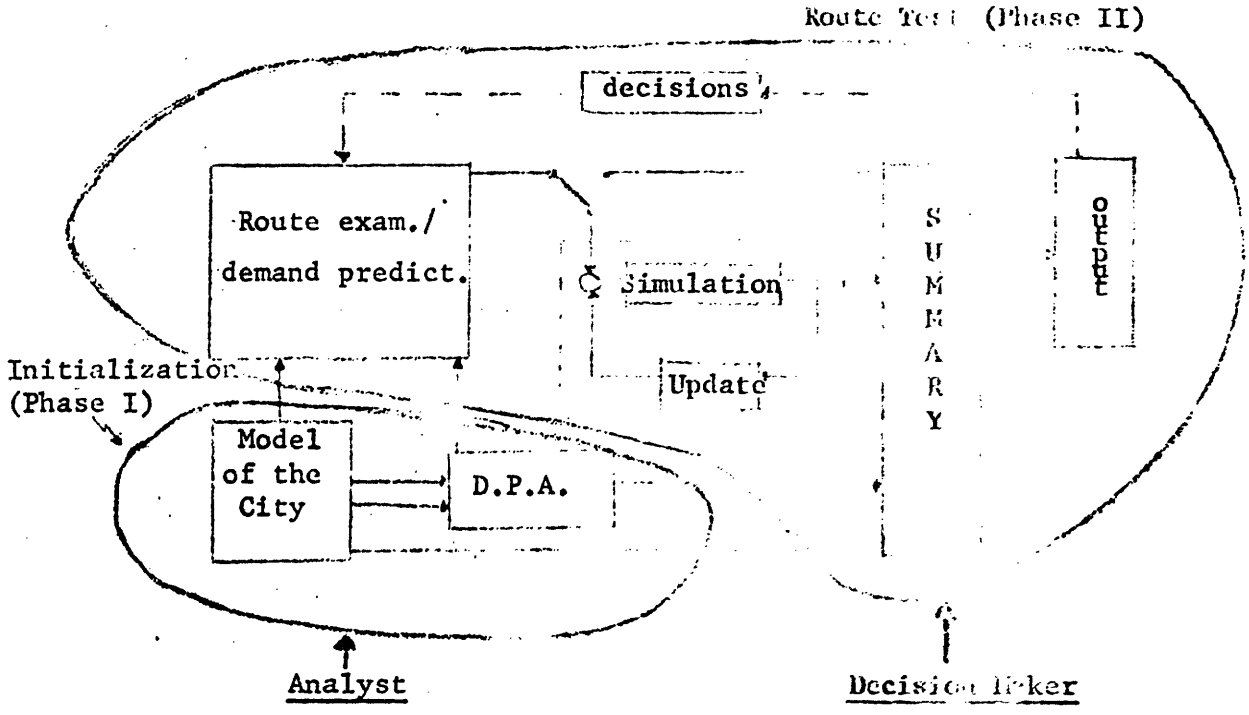
Figure 6 indicates the sphere of influence and responsibility of the decision maker and analyst.

2.2 Usage Costs

The basic structure of the route testing phase of the routing model was developed in detail and encoded as an interactive computer program. This gave an indication of how the complete development of that component and of the other components should proceed as well as indicated the feasibility, complexity, and cost of the complete model.

Only usage costs are described here. For a more complete description of the trial encodement and of data and calibration techniques, please see "An Efficient Method to Predict the Impacts of Operating Decisions for Conventional Bus Systems,"[9a] by John Hauser.

The trial encodement was developed on the Multics[10] system, a general purpose time sharing system developed at MIT's Project MAC[11] on a GE-645. The costs discussed in this section are dependent upon the system used. If further work is done on a different system, then future costs may differ.



Input census data
Obtain and input activity center data
Input "other" data
Choose models and parameters
prepare map
define coordinate system
estimate travel time information
"test" model
instruct decision maker

route location
fare
frequency
driver's wage
(headway of dispatch point)
"other" operating decisions

Figure 6

Sphere of Influence and the Responsibility

of the Decision Maker and the Analyst

2.2.1 Usage Cost for the Trial Encodement

The trial encodement is very flexible; usage costs depend upon the route and city being tested and the options chosen to test them. The total cost of one run of the program can be divided into a fixed charge and a variable charge.

The fixed charge, which is about 34 cents per run, is the result of program overhead charges such as interrogation of the user and the cost and summary programs. It varies somewhat depending upon the state of the Multics system and upon the extent of the user interaction, but these variations are usually not significant.[12]

The variable cost can be expected to be dependent upon the size of the city, the size of the bus route, and upon the particular demand models.

A single demand calculation is made for each zonal interaction. The complexity of that calculation depends upon the particular demand model chosen. Cost is proportional to the number of zonal interactions. The size of the city and the size of the bus route effect usage cost by determining the number of zones which must be considered for each route test, that is, by determining the number of zones which are within walking distance of the route.

The number of zonal interactions are calculated as follows:

If S_i = the set of all zones within walking distance of bus stop i

$||S_i||$ = number of zones in S_i

z = total number of zones within walking distance of the route

s = total number of stops

n = total number of zonal interactions

Then

$$z = \left|\left| \bigcup_{i=1}^{s} S_i \right|\right| \tag{2.3}$$

$$z = \sum_{i=1}^{s} ||S_i|| \tag{2.4}$$

(note $S_i \cap S_j = \phi$ if $i \neq j$)

$$n = \sum_{i=1}^{s} \text{(number of zones within walking distance of stop i) * (number of zones within walking distance of the rest of the route)} \quad (2.5)$$

$$n = (\,||S_i||\,)\,(z - ||S_i||\,) \quad (2.6a)$$

$$n = z^2 - \sum_{i=1}^{s} ||S_i||^2 \quad (2.6b)$$

If the number of zones within walking distance of each bus stop is approximately equal for all bus stops, then:

$$||S_i|| \sim z/s \quad (2.7)$$

$$\sum_{i=1}^{s} ||S_i||^2 \sim z^2/s \quad (2.8)$$

$$n \sim z^2 (1 - 1/s) \quad (2.9a)$$

$$n \sim z^2 \quad (s \text{ 'large'}) \quad (2.9b)$$

Equation 5.6 can also be derived by considering $||S_i||$ a random variable and assuming its variance is small. In fact, if 'var' is the sample variance for $||S_i||$ then:

$$n = z^2(1 - 1/s - (s - 1)\,\text{var}/z^2) \quad (2.10)$$

A number of tests were conducted using the trial encodement. These tests indicated that usage costs of the trial encodement could be approximated by:[13]

$$c = .34 + .0037n \quad \text{(with logit)} \quad (2.11a)$$

$$c = .34 + .0024n \quad \text{(without logit)}^{14} \quad (2.11b)$$

where c = the dollar cost of a single route test

n = total number of zonal interactions (as calculated by equation 2.6)

2.2.2 Estimated Usage Costs in a Real Community

In a real world application of the routing model there will be two costs involved: an initialization cost associated with phase I and a marginal cost associated with phase II. Because phase I is not yet developed, it is difficult at this time to make an accurate initialization cost estimate. Hopefully the initialization cost will not represent significantly more than 50% of the total model costs. The marginal cost of a single route test with phase II can be more accurately estimated. The trial encodement contains most of the structure of phase II and its costs can be used as an indicator of phase II costs.

The cost equation, equation 2.11, for the trial encodement tends to overestimate the variable cost for a real world application. Future savings will result due to elimination of inefficiencies in the substitute phase I and in the PL/I code. Although simple, the substitute phase I models do add to the variable cost of each run because the desire model and most of the level of service models must be used for each zonal interaction. A realistic phase I will determine these values before the route test, thus enabling phase II to reference rather than calculate these values for each zonal interaction. Also, the final version of phase II will be a "production" model, that is, every attempt will be made to use as efficient a PL/I code as possible.

The fixed cost might decrease because the final model will be a "production" model or it might increase because the user will be given a wider range of alternatives. In either case, it is doubtful that the change will be very significant as compared to the variable cost. Examination of table 2 reveals that in most cases the effect of the variable cost is dominant.

Number of Zones	Fixed Cost	Variable Cost	Total Cost
16	$.34	$ .95	$ 1.29
25	.34	2.31	2.65
50	.34	9.25	9.59
100	.34	37.00	37.34
200	.34	148.00	148.34
400	.34	592.00	592.34

Table 2

Variable and Fixed Usage Costs

for the Trial Encodement

Equation 2.11makes an excellent conservative indicator of phase II costs if the number of zonal interactions are known. If the number of zonal interactions is not explicitly known, then another approach can be taken. Equation 2.9 reveals that the number of zonal interactions can be approximated by the square of the number of zones within walking distance, i.e. z^2, and equation 2.6 reveals that z^2 is an upper bound to the number of zonal interactions. Thus equation 2.12 can be used as a simple conservative indicator of phase II costs.

$$\text{cost} \sim .34 + .0037\, z^2 \quad \text{(in dollars)} \tag{2.12}$$

If zones are chosen to be about the size of a census block group (population about 1000 each) or somewhat smaller,[15] then a realistic

estimate for the number of zones is about 100[16]. If all zones need be considered for each route test, then it would cost almost forty dollars (using equation 2.12) for each test, but if only one-quarter (¼) of the city is within walking distance of the route, then the model costs less than three dollars per test. (If the maximum walking distance is chosen wisely and realistically, then it is reasonable to assume that no more than one-quarter of the zones of a community will be within walking distance of any one bus stop.)

One component of phase II which has not yet been discussed is the equilibrium model. Part I indicated that the effect of demand feedback on supply is usually negligible for single bus routes because bus routes are rarely used to capacity. Still, an optional supply-demand equilibrium model should be included in the routing model so that equilibrium effects could be tested if necessary.

The best existing single route simulation model is Kulash's[17], which costs about two dollars per run. Use of that model coupled with a demand updating component similar to the trial encodement would result in a cost of about four to five dollars per iteration. Depending upon the number of iterations required, this implies that the equilibrium model might cost ten or even twenty dollars per route test.

2.2.3 Usage Cost Summary

The cost of one run of the trial encodement consists of a fixed overhead cost of about $.34 and a variable cost of about $.0037 per zonal interaction.

Based upon the results of the trial encodement, an estimate for the marginal cost of each route test using the completed model will be about $10 to $20 with the equilibrium model and about $2 to $3 if it is not necessary. The latter case, $2 to $3, is the more probable case.

2.3 Conclusion

Many previous models have been developed to aid in the complex process of planning bus routes. The routing model designed in this thesis is important because in selecting criteria and designing the model, not just the technological problems of route design were considered, but also the problems of the cost of the model, data availability, and interaction with the decision maker.

The routing model is specially designed to effectively utilize the talents of a trained analyst and an experienced decision maker and it is specially designed for small communities. It is designed to be inexpensive and to use as little data as possible and it does not assume exogenous demand.

The routing model, once completely developed, will be unprecedented in its ability to examine route location within a community and effectively communicate predictions about the impacts of routing and operating decisions to those who actually make the decisions.

Finally, in the course of model design, two new demand models have been formulated: the alternative logit model for probability estimation or modal split calculations, and the extended opportunities model for desire prediction or trip generation.

APPENDIX I

AN ALTERNATIVE FORMULATION OF THE DISCRIMINANT FUNCTION FOR THE TWO-DIMENSIONAL LOGISTIC MODEL

This appendix briefly describes the primary demand model used in the probability estimation component of the routing model. Another model, the walk distance decay model, which is simply a monotonic decreasing function of walk distance is also currently available to the user of the routing model.

The basic logistic model is a disaggregate behavioral choice model which compares two modes, in this case bus versus all other modes. First a discriminant function is computed based upon the level of service offered by both modes, the characteristics of the people, and assumptions about how the behave. Then with the help of Bayes' Theorem, the discriminant function is transformed into a probability function to represent the probability that a person will choose one of the two modes. How this is done is best illustrated by the following example.

Consider only two modes, bus and automobile, and assume that the only quantities of interest are the travel time and travel cost of the two modes.

The most common discriminant function is a simple linear function which assumes that travel time has a constant monetary value and that a consumer favors the least cost mode on the basis of the cost difference. That is:

$$Z = a_{pk}(t_a - t_b) + b_{pk}(c_a - c_b) \qquad \text{(A1.1)}$$

where

a_{pk}, b_{pk} = constants dependent upon population and trip generator class

c_b = cost of bus

c_a = cost of automobile

t_b = time by bus

t_a = time by automobile

a_{pk}/b_{pk} = value of time

The discriminant function can be transformed[1] to produce the probability that an individual will choose to ride the bus given his characteristics and given the level of service offered by both modes. That is:

if P(bus) = probability of choosing bus
P(auto) = probability of choosing automobile

$$\text{then } P(\text{bus}) = \frac{e^z}{1 + e^z} \tag{A1.2}$$

$$P(\text{auto}) = 1 - P(\text{bus}) \tag{A1.3}$$

$$= \frac{1}{1 + e^z}$$

If bus is a very superior mode, then almost everyone chooses bus; if automobiles are a far superior mode, then almost no one chooses bus, and if the two modes are equal, then people are indifferent between them. That is:

bus superior	$t_a >> t_b$ $c_a >> c_b$	$z \to \infty$	implies $P(\text{bus}) \to 1$
auto superior	$t_a << t_b$ $c_a << c_b$	$z \to -\infty$	implies $P(\text{bus}) \to 0$
equal	$t_a = t_b$ $c_a = c_b$	$z = 0$	implies $P(\text{bus}) = \frac{1}{2}$

A1.1 An Alternative Discriminant Function for the Logistic Model

The linear discriminant function assumes that the consumer favors the least cost mode on the basis of the cost (and value of time) difference. This is a reasonable assumption and it allows for a relatively simple value of time calculation.

But consider the following example:

Case 1: bus time = 1 minute, automobile time = 5 minutes

Case 2: bus time = 50 minutes, automobile time = 54 minutes

In the first case there is a 500% difference and in the second case only an 8% difference in travel time. It is doubtful that a consumer will treat these two cases identically, but the linear discriminant function does.

An alternative form is to use ratios, i.e.:

$$Z_2 = a^{(2)}_{pk} \frac{t_a}{t_b} + b^{(2)}_{pk} \frac{c_a}{c_b} \qquad \text{(A1.4)}$$

This functional form is sensitive to percentage change but does not allow an explicit value of time calculation nor is it sensitive to absolute differences. Consider the following example:

Case 3: bus time = 1 minute, automobile time = 2 minutes

Case 4: bus time = 30 minutes, automobile time = 60 minutes

Again, it is doubtful that a consumer will treat these two cases identically, but the ratio discriminant function does.

One compromise between the linear and ratio functions is to consider the absolute difference divided by the average. That is:

$$Z_3 = a^{(3)}_{pk} \frac{t_a - t_b}{(t_a + t_b) / 2} + b^{(3)}_{pk} \frac{c_a - c_b}{(c_a + c_b) / 2} \qquad \text{(A1.5)}$$

Unfortunately, this function does not allow an explicit calculation of the value of time and it suffers from the same fault as does the ratio function, i.e. it considers Cases 3 and 4 to be identical.

Another compromise is to assume that a linear additive function represents the disutility a consumer places on each mode. That is:

$$u_b = a_{pkb} t_b + b_{pkb} c_b \qquad \text{(A1.6a)}$$

$$u_a = a_{pka} t_a + b_{pka} c_a \qquad \text{(A1.6b)}$$

where a_{pkb}, b_{pkb}, a_{pka}, b_{pka} are constants.

Then assume that he favors the least "cost" mode on the basis of the relative difference in utility. That is:

$$Z_4 = k \frac{(u_a - u_b)}{(u_a + u_b) / 2} \qquad \text{(A1.7)}$$

k = constant

In the special case of all other costs being zero, this function treats Cases 3 and 4 the same way as does Z_2 or Z_3. But if all other costs are not zero, then it is sensitive to both relative and absolute differences. (See Table A1.1 for one example.) Furthermore, it allows an explicit calculation of the value of time for each mode.

Equation A1.7 might be a useful representation of how a consumer behaves, but before one can comment upon its usefulness, it is necessary to check the plausibility of its implied elasticities with an analytic sensitivity analysis and its classification powers with a field test.

	Travel Times		Linear		Ratio		Average Cost Difference		Average Utility Difference	
	ta	tb	Z	P(bus)	Z2	P(bus)	Z3	P(bus)	Z4	P(bus)
Case 1	5	1	-1.0	.269	5.0	.993	- .67	.338	-.18	.455
Case 2	54	50	-1.0	.269	1.08	.746	-1.92	..128	-.02	.495
Case 3	2	1	-4.0	.018	2.0	.881	-1.33	.209	-1.0	.269
Case 4	60	30	25.0	1.000	2.0	.881	-1.33	.209	.53	.629

Other parameters: $C_a = 0$, $C_b = 5$, $a_{pk} = b_{pk} = a^{(2)}_{pk} = b^{(2)}_{pk} = a^{(3)}_{pk} = b^{(3)}_{pk} = 1$

$a_{pkb} = b_{pkb} = a_{pka} = b_{pka} = k = 1$

Table A1.1

Comparison Of Discriminant Functions

for Different Travel Time Combinations

A1.2 Elasticity Analysis of the Alternative Logistic Formulation

An elasticity is a dimensionless quantity which represents the percentage change of a dependent variable with respect to a percentage change of an independent variable. Given the functional form of a model, it is possible to analytically compute the various elasticities and then examine their implications to determine if any counter-intuitive results are implied. Any strong counter-intuitive results could form the basis for rejecting a model.

The dependent variable in the logit model is the probability of choosing transit (or automobile), and the independent variables are the various forms in the linear disutility functions. For example, the elasticity with respect to travel time is:

$$E_{tt} = \frac{\text{\% change in probability of choosing transit}}{\text{\% change in travel time}}$$

$$= \frac{\text{\% change in } P(n, p, k, m)}{\text{\% change in } t} \qquad \text{(A1.8)}$$

$$= \frac{\partial P}{\partial t} * \frac{t}{P}$$

The form for the postulated disutility function is an additive linear form in the level of service variables (parameterized by characteristic variables). There is a symmetry which can be exploited to save calculations, i.e., once the elasticity is computed for one of the level of

service variables, say travel time, then to compute the elasticity for another level of service variable, say wait time, just interchange travel time with wait time and the travel time constant with the wait time constant. Thus the following computations will just compute the elasticities with respect to travel time, t. For simplicity of notation, all other level of service variables will be lumped into a "cost" factor, c.

Furthermore, there is a symmetry between automobile and transit which can be exploited.

$$\text{If} \quad G = \frac{u_a - u_b}{(u_a + u_b) / 2} \tag{A1.9}$$

$$\text{then } P(\text{bus}) = \frac{e^{KG}}{1 + e^{KG}} \tag{A1.10}$$

$$P(\text{auto}) = 1 - P(\text{bus})$$

$$= \frac{1}{1 + e^{KG}}$$

$$= \frac{e^{-KG}}{1 + e^{-KG}} \tag{A1.11}$$

thus

$$P(\text{auto}) = \frac{e^{KG'}}{1 + e^{KG'}} \tag{A1.12}$$

$$\text{where} \quad G' = -G = \frac{u_b - u_a}{(u_b + u_a) / 2} \tag{A1.13}$$

Thus, any results for automobile elasticities will be analogous to the results for transit elasticities; therefore, for simplicity, this appendix will compute only the elasticities for transit.

Finally, without loss of generality, the travel "cost" constant for both modes can be set equal to 1.0.

A1.2.1 Calculation of Elasticity

The following notation will be used:

- $P(\cdot)$ = probability of choosing transit
- G = discriminant function
- u_a = disutility of automobile
- u_b = disutility of bus
- t_a = travel time by automobile
- t_b = travel time by bus
- c_a = "cost" for automobile
- c_b = "cost" for bus
- v_a = "value of time" for automobile
- v_b = "value of time" for bus
- K, K_a, K_b = constants
- E = elasticity of the probability of choosing transit with respect to travel time by transit

$$P = \frac{e^{KG}}{1 + e^{KG}} \tag{A1.14}$$

$$G = \frac{u_a - u_b}{(u_a + u_b) / 2} \tag{A1.15}$$

$$u_a = K_a + v_a t_a + c_a \tag{A1.16a}$$

$$u_b = K_b + v_b t_b + c_b \tag{A1.16b}$$

$$E = \frac{\partial P}{\partial t_b} * \frac{\partial t_b}{\partial P} \tag{A1.17}$$

Doing the necessary calculations:

$$\frac{\partial P}{\partial t_b} = \frac{\partial P}{\partial G} * \frac{\partial G}{\partial u_b} * \frac{\partial u_b}{\partial t_b} + \frac{\partial P}{\partial G} * \frac{\partial G}{\partial u_a} * \frac{\partial u_a}{\partial t_b} \tag{A1.18}$$

$$\frac{\partial P}{\partial t_b} = \frac{Ke^{KG}}{(1 + e^{KG})^2} * \frac{-2u_a}{(u_a + u_b)^2} * v_b \qquad (A1.19)$$

$$E = \frac{\partial P}{\partial t_b} * \frac{t_b}{P} \qquad (A1.8)$$

$$E = -2Kv_b t_b * \frac{u_a}{(u_a + u_b)^2} * \overbrace{\frac{1}{1 + e^{KG}}}^{\text{auto share}} \qquad (A1.20)$$

Equation A1.20 is a rather complex expression, but it does imply that, as expected, an increase in travel time by bus results in a decrease in the probability that bus is chosen, (negative elasticity). To further examine the implications of equation A1.20, it is helpful to identify four special cases and interpret them. The special cases are:

case 1: equal share $(u_a = u_b)$
case 2: bus dominates $(u_a >> u_b)$
case 3: auto dominates $(u_b >> u_a)$
case 4: scale dependence of utilities $(u_a \longrightarrow ku_a,\ u_b \longrightarrow ku_b)$

A1.2.2 Special Case 1: Equal Share $(u_a = u_b)$

In this case, riders are indifferent between the two modes, thus the discriminant function is zero, $G = 0$, and the probability of choosing bus is one-half, $P(\text{bus}) = \frac{1}{2}$. Applying these values to equation A1.20 gives the following:

$$E_1 = \frac{-K}{4} * \frac{v_b t_b}{v_b t_b + c_b + K_b} \qquad (A1.21)$$

This implies that the elasticity (sensitivity) of the probability of choosing transit with respect to transit travel time is proportional to the share that the "value" of travel time, $v_b t_b$, holds in the disutility function. This is reasonable because it implies that the larger the value of time, the more sensitive passengers are to travel time.

Equation A1.21 also implies that the elasticity at the equal share point is proportional to the scaling factor, K. This means that the larger K is, the more sensitive the probability is with respect to the disutility functions. (See Figure A1.1).

In summary, the elasticities in the equal share case imply intuitive interpretations.

A1.2.3 Special Case 2: Bus Dominates ($u_a >> u_b$)

In this case, the disutility of automobile is much greater than the disutility for bus, thus people should strongly prefer bus and the probability of choosing bus should approach 1.0. The probability of choosing bus when bus dominates is:

$$u_a >> u_b \text{ implies } \quad G = \frac{u_a - u_b}{(u_a + u_b) / 2} \longrightarrow 2$$

$$P(\text{bus}) \rightarrow \frac{e^{2K}}{1 + e^{2K}} \tag{A1.22}$$

P(bus) can be made to approach 1.0 arbitrarily close by choosing a large enough value of K. For any finite value of K there is a finite residual which represents those loyalists who will drive an automobile no matter how costly or time consuming it is.

Applying the criteria $u_a >> u_b$ to equation A1.20 gives

$$E_2 = \frac{-2K v_b t_b}{u_a} * \frac{1}{1 + e^{2K}} \tag{A1.23a}$$

$$E_2 = -2K \frac{v_b t_b}{u_a} * \text{auto share} \tag{A1.23b}$$

note: $u_a >> u_b > v_b t_b$

Equation A1.23 implies that if the disutility of automobile is so bad that the transit share dominates, then it takes a major increase in travel time by bus to cause transit to lose a significant percentage of its ridership. The proportionality to automobile share indicates that because

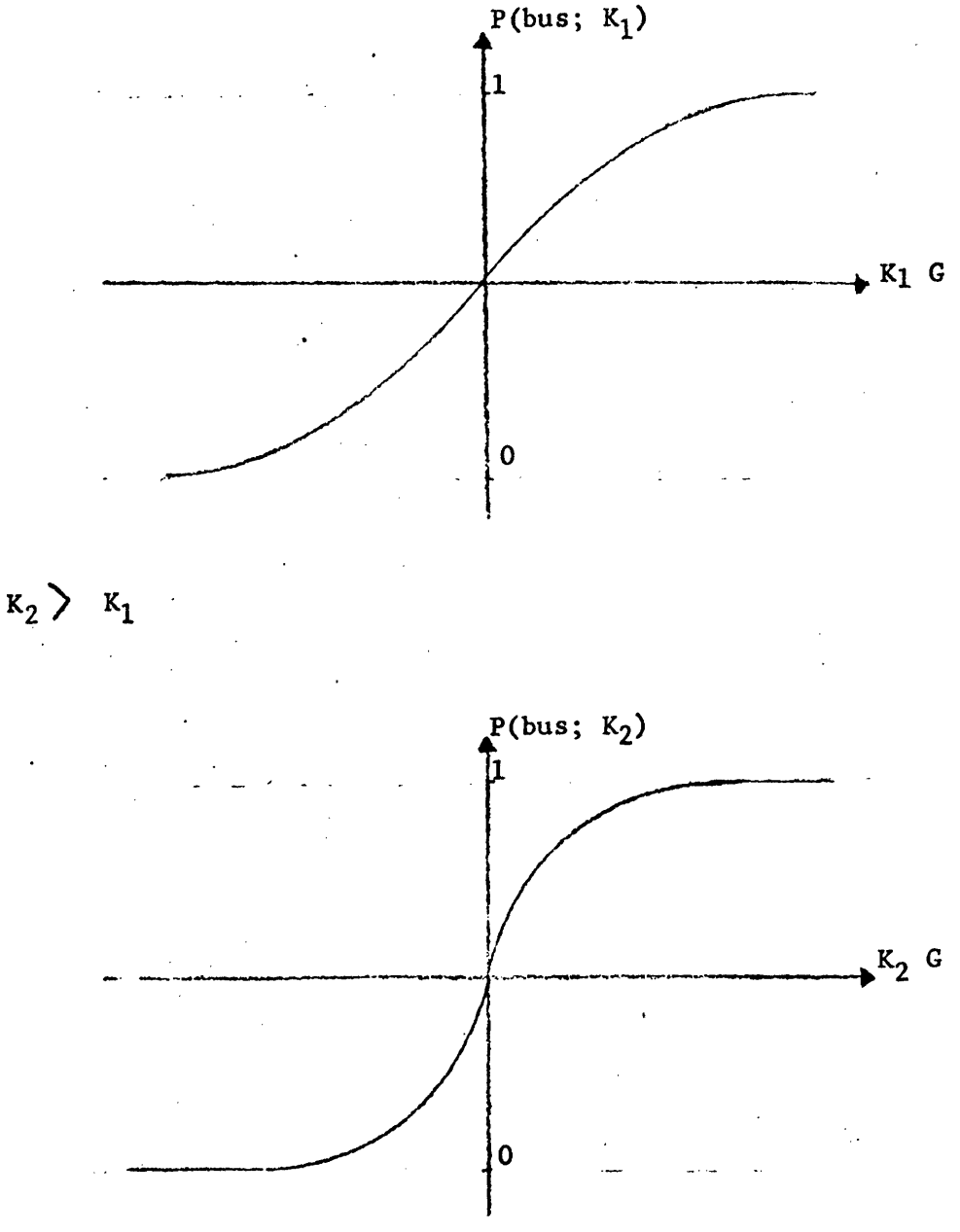


Figure A1.1
Sensitivity of the Probability Function to
Scaling Factor

auto share is small, transit share is large, and any change in transit share will be a small percentage change.

Thus, the elasticities in the case of transit dominance imply intuitive interpretations.

A1.24 Special Case 3: Auto Dominates ($u_b >> u_a$)

This case is similar to the bus dominance case, but in this case the disutility of transit is much greater than the disutility of automobile; thus people should strongly prefer automobile and the probability of choosing bus should approach 0.0. The probability of choosing bus when auto dominates is:

$$P(\text{bus}) \longrightarrow \frac{e^{-2K}}{1 + e^{-2K}} \qquad \text{(A1.24)}$$

P(bus) can be made to approach zero arbitrarily close by choosing a large value of K. For any finite K, there is a finite residual who will choose bus no matter how bad it gets. This may represent transit buffs or captive riders.

Applying the criteria $u_b >> u_a$ to Equation A1.20

$$E_3 = \frac{-2K v_b t_b}{a_b(1 + e^{2K})} \qquad \text{(A1.25a)}$$

$$E_3 = -2K \quad \frac{v_b t_b}{u_b} \quad * \quad \text{auto share} \qquad \text{(A1.25b)}$$

Once again, the elasticity is proportional to the share that the "value" of time holds in the disutility function. This means that the more significant the "value" of time in the disutility function, the more sensitive transit ridership is to travel time. If automobile dominates, then the automobile share is almost 1.0. Thus, the transit share is very small. Any change in the probability of choosing transit is more likely to be a larger percentage change.

In conclusion, the elasticities in the case of automobile dominance also imply intuitive interpretations.

A1.2.5 Special Case 4: Scale Dependence of Disutilities ($u_a \longrightarrow k u_a$, $u_b \longrightarrow k u_b$)

One feature of the alternative form of the discriminant function that was not discussed in section A1.1 is that the probability of choosing transit is not dependent upon the units that the disutility is measured in. Once the logit scaling factor, K, is chosen, the units of the disutility functions can be arbitrarily chosen as long as the relative values, i.e. the ratio of the "cost" to the "time" components of the function, are maintained. In other words, the scaling factor K, not the absolute values of the units of the disutility function, determine the sensitivity of the probability estimate to the differences in the utilities.

Analytically, this can be seen by substituting $k\,u_a$ and $k\,u_b$ in equations A1.15 and A1.20. (Of course, $v_b \longrightarrow k v_b$.) Note that both the probability estimates and the elasticities are invariant under this substitution implying the lack of dependence on disutility scaling.

A1.2.6 Summary of the Special Cases

Examination of the four special cases revealed no counter-intuitive results. Each special case yields results that could be given intuitive interpretations that meet with a priori beliefs about how people would react to changes in travel time by transit.

The proportionality of the elasticity to automobile share is interpreted to be intuitive in both the limiting case of automobile dominance and transit dominance. In all other cases it is bounded between 0.0 and 1.0 and its effect is dominated by the other terms in Equation A1.13.

In the case of equal share and of automobile dominance, the elasticity is proportional to the share that the "value" of time holds in the disutility function. This means that the more significant the effect of time in the disutility function, the greater its effect in causing changes to the probability of choosing transit. In the case of transit dominance, travel time must significantly increase relative to the disutility of automobile before it can strongly effect the probability of choosing transit.

Finally, the elasticity and the probability estimate are dependent upon the scaling factor, K, but not upon the units that disutility is measured in.

A1.2.7 Conclusion

This appendix only discusses the direct elasticity of the probability of choosing transit with respect to travel time on transit. Because of symmetry, analogous results are obtained for the dependence on other transit level of service parameters, for the cross elasticity terms (dependence upon the automobile level of service), and for the probability of choosing automobile.

Thus, the sensitivity analysis of the alternative discriminant function proposed in this appendix yields no counter-intuitive results. Although this is further evidence for the plausibility of the model, it is not grounds for the model's acceptance. It does imply that the model should not be rejected on the basis of this analysis.

APPENDIX II

EXTENDED OPPORTUNITY MODEL

This appendix proposes a desire prediction model which is an extension of the intervening opportunities model. This model is particularly adapted to phase I of the routing model.

Section A2.1 proposes an alternative approach to the traditional trip generation-trip distribution models. Section A2.2 then derives the mathematical form of the extended opportunity model and section A2.3 examines two special cases. Section A2.4 suggests further work and section A2.5 is a summary.

A2.1 An Alternative Approach to Trip Generation and Trip Distribution

Many planning or policy decisions affect the level of service that travelers perceive. In turn, level of service affects travelers' decisions on how, where, when, or whether to travel. Thus, any accurate estimate of travel volume must be sensitive to the level of service.

An often used approach to demand estimation is the urban transportation planning process[1] (UTP) consisting of trip generation, trip distribution, and modal split. In the traditional UTP process, an estimate of the total trips for an area (trip generation) is made without regard to the level of service. This fixed number of total trips is then "distributed" to various links depending upon the level of service they offer. This section proposes an alternative two-step procedure to replace the traditional trip generation-trip distribution step.

The first step of the alternative procedure is to estimate desires. A desire is a measure of the upper bound on the total trips that will originate in an area, i.e. estimate the number of trips that would be made if the level of service were "perfect". Then, for each origin zone distribute some, but not all, of the "desires" to destination zones. The number of trips actually made and where these trips terminate depend upon the level of service that

the system provides.

In symbols:

1. Estimate $d_i(\underline{A})$ = upper bound on the number of trips that can originate in zone i given the activity system of the area.
2. Distribute trips: t_{ij} = f(LOS on link i⟶ j and all other links from i) $*$ $d_i(\underline{A})$

   where $f(.) \leq 1$.

   Thus $t_{ij} \leq d_i(\underline{A})$.

Why is this process chosen? Does it have intuitive appeal? Is it based on behavioral assumptions?

Begin by examining the concept of an upper bound on total trips made in an area. Clearly, no matter how good the transportation system is, only a finite number of trips will be made. For example, a loose upper bound on daily trips might be 1000 $*$ (population of the area). If the concept of desires is to be useful, it should give a much tighter, more realistic bound. Such a bound should be based on the socio-economic characteristics of the traveler and on the activity system as a whole. For example, it should consider all possible travel attractions.

The second step, distribution, assumes that when a traveler decides to make a trip, he considers the complete transportation system. His decision depends in some way on the availability, desirability, and accessibility by all modes of all destinations. Such a calculation would be tremendously hard and time-consuming. A tradeoff between ease and accuracy must be made.

The remainder of this appendix assumes that some process for estimating desires exists, and concentrates on developing a method of distributing these desires.

A2.1.1 Traditional UTP Distribution Models

The traditional UTP trip distribution procedure assigns total generated trips among various travel destinations depending upon the relative level of service each link offers, and upon measures of attractiveness at the various destinations. Because it distributes all, not just some, of the generated trips, the total number of trips in an area is insensitive to the

level of service.

Two trip generation techniques available are the gravity model[2] and the opportunity model.[3] The very popular, traditional gravity model is analytically simple. It is dependent upon measures of zonal attractiveness and upon absolute level of service measures for each link, but its weak point is that it is not based on any causal reasoning. The only underlying behavioral assumption is that travelers choose destinations based on some inverse power of the travel time between origin and destination.

The opportunity model is intuitively more appealing than the gravity model because it is based on behavioral assumptions. Unfortunately. it is dependent only upon a rank order of interzonal travel times, not upon absolute separation of zones or upon absolute level of service. It assumes that the probability that a destination is accepted, if it is considered, is constant, independent of the level of service.

A2.1.2 The Extended Opportunity Model

The extended opportunity model is an attempt to combine the intuitive appeal of the opportunity model with the level of service dependence of the gravity model. Currently it is only a model for the probability distribution of trip ends given that a trip is made. A model sufficient for the second step in the proposed process would have the property that the probability that a trip is made is less than 1. That is:

$$P(\text{Trip from origin } i \text{ to destination } j) \leq 1.$$

A2.2 Development of the Model

A2.2.1 The Simple Opportunity Model

The opportunity model is a mathematical model of trip distribution derived from hypotheses of human behavior. It is based on the following hypotheses:[4]

1. Total travel time from a point is minimized, subject to the condition that every destination point has a stated problem of being accepted if it is considered.
2. The probability of a destination being accepted, if it is considered, is a constant, independent of the order in which destinations

are considered, and independent of the level of service.

Thus if

$F_{V\leq}(V_o)$ = cumulative density function (cdf) for the probability that a trip has terminated in subtended volume V_o.

V_o = possible destinations already considered, or subtended volume

$f_v(V_o) = dF_{V\leq}(V_o)$ = the probability that a trip will terminate in volumes between V_o and $V_o + dV_o$.

The behavioral assumptions lead to:

Probability (trip terminates in volume between V_o and $V_o + dV_o$)
= constant * Probability (trip terminates in volumes yet to be considered) * (number of volumes between V_o and $V_o + dV_o$), or:

$$dF_{V\leq}(V_o) = L * (1 - F_{V\leq}(V_o)) * dV_o \qquad \text{(A2.1)}$$

The solution of equation A2.1 which satisfies $F_{V\leq}(\infty) = 1$ is:

$$f_v(V_o)dV_o = L * \exp(-L * V_o) * dV_o \qquad \text{(A2.2)}$$

If j less than k implies that <u>all</u> destinations in zone j are closer to the origin zone than any destination in zone k, then the expected trip interchange from zone i to zone j is the volume of trip origins, O_i, in zone i, multiplied by the probability of a trip from i terminating in j:

$$T_{ij} = O_i * \int_{V_j}^{V_{j+1}} F_v(V_o)dV_o \qquad \text{(A2.3)}$$

$$\text{or } T_{ij} = O_i * (F_{V\leq}(V_{j+i}) - F_{V\leq}(V_j)) \qquad \text{(A2.4)}$$

$$\text{or } T_{ij} = Oi * (\exp(-LV_j) - \exp(-LV_{j+1})) \qquad \text{(A2.5)}$$

$$\text{where } V_j = \sum_{k=1}^{j-1} O_k. \qquad \text{(A2.6)}$$

A2.2.2 Assumptions Relaxed in the Extended Opportunity Model

The opportunity model as derived depends only upon a rank order of travel time not on absolute level of service or absolute spatial distribution of destinations.

Some of the behavioral assumptions can be relaxed. Let the probability of a destination being accepted, if it is considered, be a variable, independent of the order of consideration but dependent upon the level of service. Let this variable be monotonically decreasing in the level of service. That is, if the level of service is worse, then the value of this variable is smaller.

Let the possible destinations be spatially distributed with a density $p(r, \theta)$.

For the sake of exposition and clarity of calculations, let it be the case that for any r_o, a destination closer to the origin than r_o is preferred to a destination further from the origin than r_o. This is equivalent to saying that the probability that a trip has terminated by r_o is a monotonically increasing function of r_o.

A2.2.3 Derivation of the Extended Opportunity Model

Assume the origin under consideration is located at $r = 0$.

Let $F_{r\leq}(r_o)$ = Probability that a trip has terminated at $r = 0$.

$p(r_o, \theta)$ = density of possible destinations

$L(r, \theta)$ = a measure of the level of service from the origin to the point (r,θ). (The value of L is smaller if the level of service is worse.) For simplicity, assume $0 \leq L(r,\theta) \leq 1$.

B = probability that a possible destination is accepted given it is considered and given "perfect" level of service. $(L(r,\theta) = 1)$

$f_{r,\theta}(r_o\theta_o)$ = probability that a trip will terminate given $r \in (r_o, r_o + dr_o)$ and $\theta \in (\theta_o, \theta_o + d\theta_o)$.

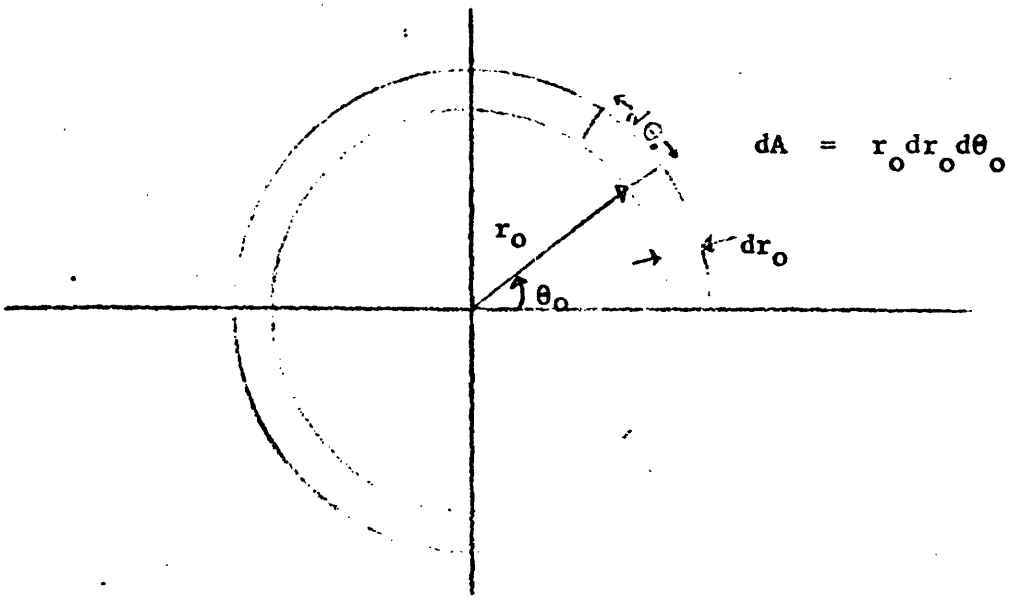


Figure A2.1

Representation of the Area 'dA'

The behavioral assumptions lead to:

Prob(trip terminates in area dA)
= B * L(r, θ) * Prob(trip has not terminated by r_o)
* (number feasible destinations in dA).

The number of possible destinations in dA is:

$$dV = p(r_o,\theta_o) r_o dr_o d\theta_o \qquad \text{(A2.8)}$$

Analytically, the behavioral assumptions can be written

$$f_{r,\theta}(r_o,\theta_o) dr\, d\theta_o = B * L(r_o,\theta_o) * (1 - Fr_{\leq}(V_o)) * p(r_o,\theta_o) r_o dr_o d\theta_o \qquad \text{(A2.9)}$$

If all trips possible are made, then $F_{r\leq}(\infty) = 1$. The solution of equation A2.9 satisfying $F_{r\leq}(\infty) \leq 1$ is:

$$F_r(r_o) = 1 - \exp(-B*z(r_o)) \text{ where } z(r_o) = \int_0^{r_o}\int_0^{2\pi} L(r,\theta)\, p(r,0)\, d\theta\, r\, dV \qquad \text{(A2.10)}$$

or substituting back into equation A2.9

$$f_{r,\theta}(r_o,\theta_o)dr_o d\theta_o = B\, L(r_o,\theta_o)e^{-Bz(r_o)} * p(r_o,\theta_o)r_o dr_o d\theta_o \qquad \text{(A2.11)}$$

Equation A2.11 implies that the origin is a point source and the destinations are chosen from a continuous medium surrounding the origin.

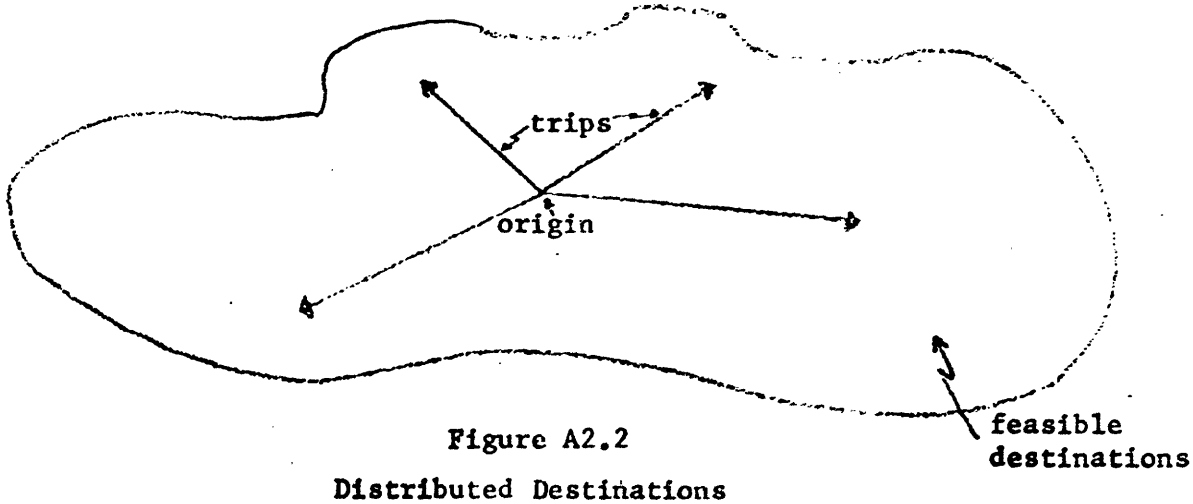


Figure A2.2
Distributed Destinations

In any real life situation, an analyst will be concerned with trips between zones of finite size. Consider now the following set of zones. The origin is still a point source but the destination zone now covers a finite area.

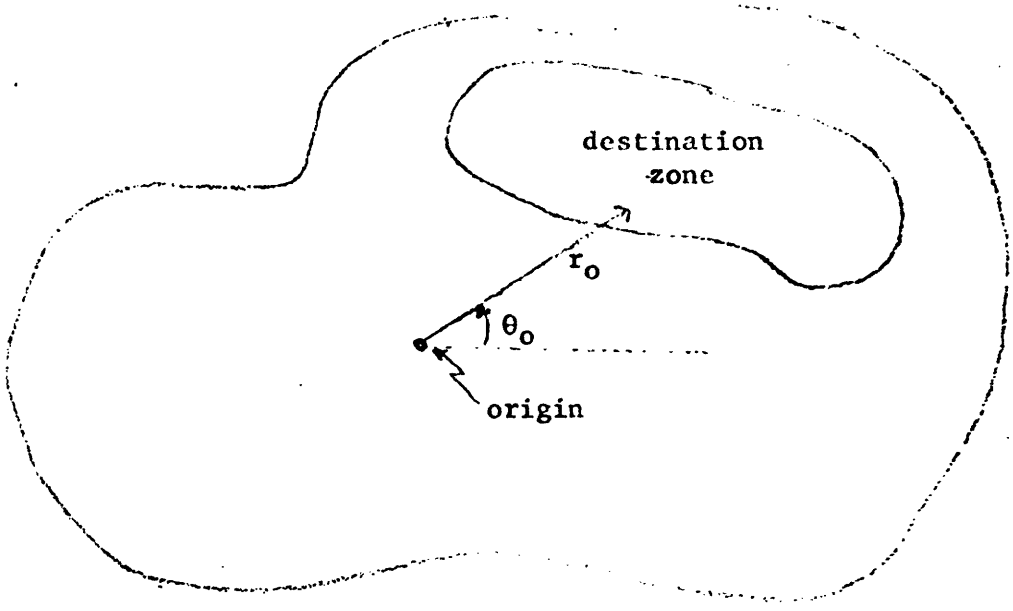


Figure A2.3
Destination Zone

If P_{ij} = Prob(trip end in zone j begins in zone i) then,

$$P_{ij} = \int_{\{r_o\theta_o\} \in \text{ zone } j} f_{r,\theta}(r_o\theta_o)dr_o d\theta_o \qquad \text{(A2.12)}$$

If there are O_i trips originating in zone i, then

$$T_{ij} = O_i * P_{ij} \qquad \text{(A2.13)}$$

or

$$T_{ij} = O_i * \int_{\{r_o,\theta_o\} \in \text{ zone } j} B\, L(r_o\theta_o)e^{-Bz(r_o)}p(r_o,\theta_o)r_o dr_o d\theta_o \qquad \text{(A2.14a)}$$

where $$z(r_o) = \int_0^{r_o}\int_0^{2\pi} L(r_o,\theta_o)p(r_o,\theta_o)r_o dr_o d\theta_o \qquad \text{(A2.14b)}$$

(There is a straightforward extension that holds if the origin is of finite area and not just a point source.)

A2.2.4 A Discrete Zone Approximation

If the area of zone j is sufficiently small, then the integral of equation A2.14a can be approximated to be a mean value times the area of integration and the integral of equation A2.14b can be approximated to be a sum. That is:

$$T_{ij} \cong O_i BL(r_j,\theta_j)e^{-Bz(rj)}p(r_j,\theta_j) * dA_j \qquad \text{(A2.15a)}$$

where $$z(r_j) = \sum_{\forall r_k < r_j} L(r_k,\theta_k)p(r_k,\theta_k) * dA_k \qquad \text{(A2.15b)}$$

If the zonal areas are not sufficiently small, but are large enough to be significant when compared to the spatial area of the urban region, then an approximation error is introduced.

A2.3 Special Cases

By keeping the derivation as general as possible, it was possible to avoid specific assumptions about the form of the level of service function, $L(r,\theta)$. To gain a more intuitive feel for what equations A2.14 really mean, it is helpful to examine certain special functional forms of $L(r,\theta)$ and $p(r,\theta)$.

A2.3.1 Special Case 1: Reduction to Simple Opportunity Model

If $L(r,\theta) \equiv 1$, then the extended opportunity model should reduce to the simple opportunity model.

Substituting $L(r,\theta) \equiv 1$ into equation A2.14:

$$T_{ij} = \theta_i \int_{\{r_o,\theta_o\} \in \text{zone } j} B\, e^{-Bz(ro)} \underbrace{p(r_o,\theta_o) r_o dr_o d\theta_o}_{dV_o}$$

$$z(r_o) = \underbrace{\int_0^{r_o}\int_0^{2\pi} p(r_o,\theta_o)\, r_o dr_o d\theta_o}_{V_o} \qquad \text{(A2.16)}$$

If all destinations in zone j are closer to zone i than those in j + 1 this reduces to:

$$T_{ij} = O_i \int_{V_j}^{V_{j+1}} B\, e^{-B\, Vo}\, dV_o \qquad \text{(A2.17)}$$

$$T_{ij} = O_i \left(e^{-B\, Vj} - e^{-B\, Vj+1}\right) \qquad \text{(A2.18)}$$

This is identical in form to the equations for the simple opportunity model, thus the extended opportunity model reduces to the simple opportunity model in the special case where $L(r,\theta) = 1$.

A2.3.2 Special Case 2: Gravity Form

Another special case to consider is one where the probability that a trip is accepted decays with distance from the origin. If this decay is in the form of a power-law, i.e. $L(r,\theta) \sim r^{-a}$, then the model is called the gravity form of the extended opportunity model. The particular function $L(r,\theta) = r^{-a}$ violates the restriction that $L(r,\theta) \leq 1$. Hence:

$$L(r,\theta) = \begin{cases} 1 & r \leq q \\ (r/q)^{-a} & r > q \end{cases} \qquad \text{for all } \theta \qquad \text{(A2.19)}$$

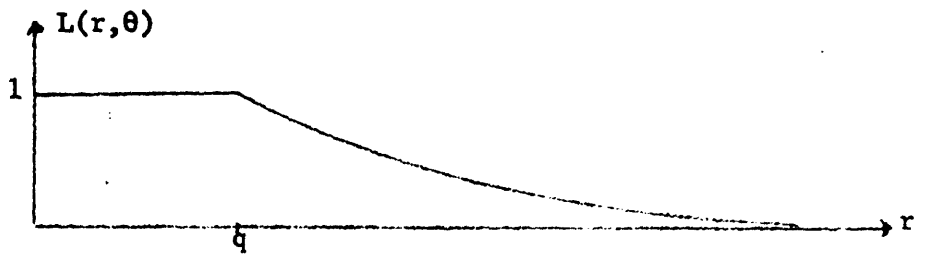


Figure A2.4: Gravity Form

If the above equation is used for $L(r,\theta)$, the algebra becomes rather complex. For clarity, only the results are stated.

If equation A2.20 is used for $L(r,\theta)$ and if $p(r,\theta)$ is constant, then:

$$f(r) = \begin{cases} 2\pi\, pBr\, e^{-Bpr^2}\, dr & r \leq q \\ 2\pi\, pB(r/q)^{-a} rk_1\, e^{-k_2 r^{2-a}} dr & r > q \quad a \neq -2 \end{cases} \qquad \text{(A2.20a)}$$

where $k_1 = \exp(B\, paq^2/(2-a))$ (A2.20b)

$$k_2 = 2\pi\, pBq^a/(2-a) \qquad \text{(A2.20c)}$$

(The case where a = 2 is similar.)

If origin zone i covers area from r = 0 to r = q then:

$$T_{ij} = O_i * e^{-BV_i} \qquad \text{(A2.21)}$$

If the destination zones are concentric rings, then:

$$T_{ij} = O_i * k_3 * (e^{-BV(r/q)^{-a}} - e^{-BV(r/q)^{-a}}) \qquad \text{(A2.22a)}$$

where $k_3 = \exp(aBV_i/(2-a))$ (A2.22b)

$$B_1 = 2 \cdot B/(2-a) \qquad a \neq 2 \qquad \text{(A2.22c)}$$

(The case where a = 2 is similar.)

Examination of equation A2.22 reveals that it is very similar in form to the simple opportunity model. Except for a few changes in the values of the constants, the only major structural change is that the subtended volume, V_j, is replaced by the subtended volume multiplied by a gravity-like decay factor, $V_j * (r/q)^{-a}$. The decay factor will cause the distribution of trips to be more concentrated near the origin. The larger a is, the more concentrated the distribution is.

This result, though analytically more complex, is more intuitively appealing than the simple opportunity model. It is based more on behavioral assumptions than the gravity model, but it does include a measure of absolute separation of the zones. The particular functional form of equations A2.19, A2.20, and A2.22 is not advocated. That form was presented only as an example of how the more general result, equation A2.14 (or equation A2.15) could be used.

A2.4 Further Work

A2.4.1 Transformation

One of the implicit assumptions in the development of the extended opportunity model was that for any r_o, a destination closer to the origin than r_o is preferred to a destination further from the origin than r_o. It might be possible to relax this assumption so that travel time or some other measure of the level of service is the dependent variable instead of distance. It might be possible to do this with some Jacobian-like transformation on equation A2.14.

Though a change of variable in the general equation is analytically complex, a change of variable in the approximation equations is analytically simple and straightforward. Such a change would give:

$$T_{ij} \cong O_i B\, L(r_j,\theta_j) e^{-Bz(r_j)} p(r_j,\theta_j)\, dAj \qquad \text{(A2.23a)}$$

$$Z(r_j) = \sum_{\forall k \text{ s.t. } L(r_k,\theta_k) \le L(r_j,\theta_j)} L(r_k,\theta_k)\, p(r_k,\theta_k) dA_k \qquad \text{(A2.23b)}$$

Note that the only change is in the order of the summation to determine $Z(r_j)$. This means that the traveler favors those destinations that have a "higher level of service", i.e. destinations that are more readily accessible.

A2.4.2 Probability of Termination Less Than 1

The extended opportunity model as it now stands is still a trip distribution model; it distributes spatially all the trips. It is quite easy to extend equation A2.14 so that the sum of the probability that a trip terminates is less than 1. To achieve this, all one has to do is change the boundary condition on $F_{r\leq}(r_o)$ to $F_{r\leq}(\infty)$ = P = probability that a trip terminates ($P \leq 1$). But to do this one must first be able to determine P as a function of the level of service.

A2.4.3 Level of Service Function

The derivation in section A2.2 was kept as general as possible so that it would not be necessary to rederive results for each $L(r,\theta)$ function. No reasons were given for prefering one analytic result to another; the gravity form was derived purely as an exposition of how the general result might be used. Before the result can be used with confidence, it is necessary to derive a $L(r,\theta)$ function which is based upon causal reasoning from behavioral assumptions. A good choice would be some function based upon a measure of a traveler's utility.

A2.5 Conclusion

The derivation of the extended opportunity model presented in this appendix is not yet complete. The model is designed explicitly for use in the desire prediction component of phase I of the routing model, but before it can actually be used, a causal derivation of P, the probability of termination, and $L(r,\theta)$, the level of service function, must be proposed. Furthermore, a means to generate desires must be derived from behavioral assumptions.

Though not yet ready for desire prediction, the model can be used as a distribution model. It combines the causal reasoning of the opportunity model with the spatial and level of service dependence of the gravity model.

FOOTNOTES

Part I

1. 70,312,824
United States Department of Commerce, Bureau of the Census, Number of Inhabitants--United States Summary, December, 1971. PC(1)-A1, page I-45, table 6.

This includes incorporated and unincorporated places. See the Census Users' Guide for complete definitions. United States Department of Commerce, Bureau of the Census, 1970 Census Users' Guide, Part I, October, 1970, page XI.

2a. Chicago spent $150 million in 1966. Friedlander, Alex E., A Method of Schedule and Route Planning in Urban Mass Transit, Ph.D. Thesis, Mass. Inst. of Tech., September, 1968, page 217.

2b. For example, net revenue for 1961 on the following bus lines was: Barre Bus Lines, $32,222; Berkshire Street Railway, $350,300; Brush Hill Transportation, $307,200; Fitchburg & Leominster St. Ry., $543,900; Johnson Bus Lines, $395,500; Lynnfield Community, Inc., $169,800; Mass. Northeastern Trans., $338,400; Plymouth & Brockton, $370,400; Saugus Transit, Inc., $136,400; Service Bus Lines, Inc., $208,600.
Systems Analysis and Research Corporation (SARC), Mass Transportation in Massachusetts: A Final Report on a Mass Transportation Demonstration Project, May, 1964, pages 77-80.

3. Voorhees, Alan M. and Associates, Inc., A Systems Analysis of Transit Routes and Schedules, prepared for the Washington Metropolitan Area Transit Commission. Mass Transportation Demonstration Project. INT-MTD-14. November, 1964.

4. Portland Area Comprehensive Transportation Study, June, 1963 to April, 1965. Abend, Norman A., and Melvin R. Levin, Bureaucrats in Collision: Case Studies in Area Transportation Planning, MIT Press, Cambridge, Mass., 1971, page 207.

5. Manchester Metropolitan Planning Study, August 1964 to August, 1967. Ibid., pages 176-178.

6. $1,193,000.
Zettel, Richard M., and Richard R Carll, Summary Review of Major Metropolitan Area Transportation Studies in the United States. The Institute of Transportation and Traffic Engineering. Univ. of California, Berkeley, November, 1962.

7. $1,803,000: Taken from the prospectus published in 1959. Later the cost of the Penn-Jersey study more than doubled. Ibid.

8. For example, the PACTS study in Portland, Maine required about nine months for data collection. Abend and Levin, op.cit.

9. 51% of transit riders walked less than one block in Boston, Mass. Commonwealth of Massachusetts; Mass Transportation Commission, Tentative Conclusions, Demonstration Project Mass-MTD-3 Progress Report #5, Boston, November 22,1969, page 98.
68% walked less than one block in Chattanooga, Tenn. Levinson, Herbert S. and F. Houston Wynn, "Some Aspects of Future Transportation in Urban Areas," Highway Research Board Bulletin 326, Urban Transportation: Demand and Coordination, 1962, page 10.
"In Springfield, Mass. 39 percent were within 200 feet of a bus route, 60 percent within 400 feet, and 83 percent within 800 feet," Levinson, Herbert S. and F. Houston Wynn, "Some Considerations in Appraising Bus Transit Potentials," Highway Research Record, No. 197, 1967, page 17.
"Data collected as part of the Chicago Area Transportation Study and other surveys in recent years strongly suggest that people actually using transit vehicles for work trips are willing to walk an average of two or three blocks but not very much more." Meyer, J.R., J.F. Kain, and M. Wohl, The Urban Transportation Problem, Harvard University Press, Cambridge, Mass. 1965, pages 188-189.

10. 40% of daily volume occurred during the four peak hours in Chicago, Ill., 53% in Washington, D.C., (1959-1962 data). Meyer, J.R., J.F. Kain, and M. Wohl, The Urban Transportation Problem, Harvard University Press. Cambridge, Mass. 1965.

11. In 1959, average Saturday riding was 61.2% of the weekday average, Sunday riding 30.2%. Although not as pronounced in small communities, similar variations occur. Meyer, Kain, and Wohl, ibid.

12. 17.6% variation as calculated from the Transportation Facts for the Boston Region, Boston Redevelopment Authority, 1968/69 Edition.

13. Wilson, Nigel, Wayne Pecknold, and Brian Kullman, Service Modification Procedures for Local Bus Operations of the Massachusetts Bay Transportation Authority. The Boston Urban Observatory, August, 1972. page 25.

14. Such as income, age, sex, etc.

15. See for example:
Lave, Charles A., "A Behavioral Approach to Modal Split Forecasting", Transportation Research, 1969, Vol. 3, No. 4, pp. 463-480.
Quarmby, D.A., "Choice of Travel Mode for the Journey to Work: Some Findings", Journal of Transportation Economics and Policy, 1967, Vol. 1, No. 3, pp. 273-314.
Stopher, P.R. and T.E. Lisco, "Modelling Travel Demand: A Disaggregate Behavioral Approach-Issues and Applications", Transportation Research Forum Proceedings, 1970, pp. 195-214.

16. Kulash, Damian, Routing and Scheduling in Public Transportation Systems, Ph.D. Thesis, Mass. Inst. of Tech. January 1971, page 208.

17. For example, if there are 20 routes under consideration, then the cost of testing a single routing decision would increase 20-fold.

18. 766 thousand people are served with 42 routes in the Providence-Pawtucket area. Simpson and Curtin, Acquisition and Public Operation of Transit Service in the Providence-Pawtucket Metropolitan Area. Transportation Engineering June 1965. 110 thousand people are served with 11 routes in Allentown, Pa. Lehigh and Northampton Transportation Authority.

19. Though not explicitly modelled, network effects can be included via judgmental estimates.

20. Little, John D.C., "Models and Managers: The Concept of a Decision Calculus", Management Science, May, 1970.

21. Ibid.

22. For a more complete discussion of naive correlative and causal models, see: de Neufville, Richard, and Joseph H. Stafford, Systems Analysis for Engineers and Managers, McGraw-Hill Book Company, New York, 1971, chapter 12.

23. The need for behavioral assumptions was discussed in criterion 6. Some references given in footnote 15.

24. Little, op.cit.

25. Travel time will increase due to additional boarding delays.

26. The design of an equilibrium component is discussed in detail in chapter 3.

27. Kulash, op.cit. page 273.

28. For example, in Lehigh Valley, Pa. only 3.3% of all trips made were made by public bus, 5.3% by school bus.
Wynn and Levinson, op.cit., page 16, table 9.

29. Highway Act of 1962, Public Law 87-866.

30. For example:
Quandt, R.E. and Baumol, W.J., "The Demand for Abstract Transport Modes; Theory and Measurement", Journal of Regional Science, Vol. 6, No. 2, 1966.
Systems Analysis and Research Corporation (SARC), Demand for Intercity Passenger Travel in the Washington-Boston Corridor, Report to U.S. Department of Commerce, 1963.
McLynn, J.M., A.J. Goldman, P.R. Myers, and E.B. Watkins, "Analysis of a Market Split Model", Journal of Research of the National Bureau of Standards B. Mathematical Science, Vol. 72B, No. 1, 1968.

31. Lave, op.cit.; Quarmby, op.cit.; Stopher and Meco, op.cit.

49. Actually, probabilities are estimated for groups of similar individuals.

50. Determination of the probability that a potential rider will ride the bus route.can be a complex calculation, but its complexity does not change the basic simplicity of the process. The complex calculation can be easily done by a computer. (See feature 4.)

54. $10 in 1961. Abend and Levin, op.cit. 10,000 people at $10,000 = $100,000.

55. Fleet, Christopher R. and Sydney R. Robertson, "Trip Generation in the Transportation Planning Process," Highway Research Record, No. 240, 1968, pages 11-31.

56. Since zonal interactions are considered, the number of zones varies roughly as the square of the number of zones within walking distance of the route. (Equation 5.7, chapter 5). Smaller zones mean a greater number of zones within walking distance.

57. Based on engineering judgment.

58. If the spatial distribution of the population within a four square block square zone is homogeneous, then the mean right-angle walking distance is two blocks.

59. The calculation of trip desires is discussed in chapter 3, section 3.1.2.2.

60a. International Business Machines (IBM) publication, A PL/I Primer, Endicott, N.Y., February, 1970.
International Business Machines (IBM) publication, IBM System/360 Operating System PL/I(F) Language Reference Manual, Endicott, N.Y., June, 1970.

60b. IBM, A PL/I Primer, Endicott, N.Y., February, 1970, page 4.

61. Abend and Levin, op.cit.

62. Chapter 4, section 4.1 contains an in-depth investigation of what is available from the Census Bureau.

"An Efficient Method to Predict the Impacts of Operating Decisions for Conventional Bus Systems," John R. Hauser, Masters Thesis, MIT, Feb. 1973, chapter 4.

63. Hauser, op. cit., chapter 4.

64. Hauser, op. cit., See chapter 4, section 4.2 for model calibration.

Part II

3. Ruiter, Earl R., "Toward a Better Understanding of the Intervening Opportunities Model", Transportation Research, pages 47-56, 1967. Martin, Brian V., Frederick W. Memmott, III, and Alexander J. Bone, Principals and Techniques of Predicting Future Demand for Urban Area Transportation, MIT Report No. 3, MIT Press, Cambridge, Mass., pages 149-151, June, 1961.

4. See Appendix 2.

5. Witheford, David K., "Comparison of Trip Generation by the Opportunity Model and the Gravity Model," Paper presented to the Origin and Destination Committee, Highway Research Board, January 9, 1961.

6. A computer implementation of the basic structure of this component has been developed and is described in more detail in section 3.2.

7. See criterion 12 in part I.

8. Kulash, Damian, Routing and Scheduling in Public Transportation Systems, Ph.D. thesis, Mass. Inst. of Tech., January, 1971.

9. The equilibrium seeking process can be set to stop after a preset number of iterations or if the last iteration produces a percentage change less than a preset value.

9a. Hauser, John R., An Efficient Method to Predict the Impacts of Operating Decision for Conventional Bus Systems, Masters Thesis, Mass. Inst. of Tech., Feb. 1973.

10. Multiplexed Information and Computing Service.

11. Machine-Aided Cognition and Multiple-Access Computer.

12. In all tests conducted, variation was less than 10%.

13. Coefficients are only stated to two significant figures because costs are dependent upon the state of the Multics system. Equations 2.11 were calibrated via simple linear regression on ten data points.

14. That is, replacing the logistic model with a model which returns a constant value for the probability of choosing transit.

15. The exact size needed depends upon the particular community.

16. A heuristic zoning attempt was made for Allentwon, Pa. (population 110,000). The result was 130 zones.

17. Kulash, op. cit.

Appendices

Appendix 1.

1. Quarmby, D. A. Choice of Travel Mode for the Journey to Work: Some Findings, Journal of Transportation Economics and Policy, 1967, Vol. 1, No. 3, pp. 273-314, appendix.

Appendix 2.

1. Martin, Brian V., Frederick W. Memmott, III, and Alexander J. Bone, Principals and Techniques of Predicting Future Demand for Urban Area Transportation, MIT Report No. 3, MIT Press, Cambridge, Mass., pages 149-151, June, 1961.

2. Martin, Memmott, and Bone, op.cit., pages 138-146.

3. Martin, Memmott, and Bone, op.cit., pages 149-151.
Ruiter, Earl R., "Toward a Better Understanding of the Intervening Opportunities Model", Transportation Research, pages 47-56, 1967.

4. Ruiter, op.cit., page 48.

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