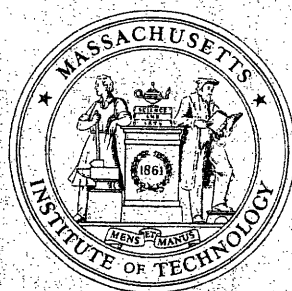# OPERATIONS RESEARCH CENTER

working paper

MASSACHUSETTS INSTITUTE
OF TECHNOLOGY

MODELING DECISIONS OF CHOICE
AMONG FINITE ALTERNATIVES: APPLICATIONS TO
MARKETING AND TO TRANSPORTATION DEMAND THEORY

by

John R. Hauser

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ABSTRACT

A NORMATIVE METHODOLOGY FOR MODELING INDIVIDUAL RESPONSE TO INNOVATION

by

John Hauser and Glen Urban

A methodology to improve the effectiveness of the design of innovation is proposed based on knowledge in the fields of psychometrics, utility theory and stochastic choice modeling. It is comprised of a consumer response and a managerial design process. The design process is one of idea generation, evaluation, and refinement while the consumer response is based on consumer measurement, an individual choice model, and aggregation of the individual choices. The consumer model interacts with the design process by providing diagnostics on consumer perceptions, preference, choice, and segmentation, as well as prediction of the share of choices.

The individual response model processes the consumer measures by "reducing" them to an underlying set of perceptual dimensions. "Abstraction" defines homogeneous groups based on perceptions and preference. "Compaction" describes how the reduced space performance measures are combined to produce a scalar measure of goodness for each consumer and for each choice alternative. This goodness measure is linked to probability of choice for the new and old alternatives. In each step, theoretical, empirical, and statistical issues are identified and various techniques are described for each phase.

The techniques are demonstrated based on survey data collected at MIT to support the design of a health maintenance organization (HMO). After discussing the issues of testing the model, the managerial design implications are shown by application to the MIT HMO case.

MODELING DECISIONS OF CHOICE AMONG FINITE ALTERNATIVES
APPLICATIONS TO MARKETING AND TO TRANSPORTATION DEMAND THEORY

by

John R. Hauser

ABSTRACT

Many of the models in marketing and in transportation demand theory ad-
dress the same questions, such as, "How many people will purchase our product?"
and "How many people will ride our bus?". This paper presents a general methodo-
logy which forces a model builder to explicitly specify his assumptions and then
allows him to easily trace out some of their implications. Most of the causal
models in both fields, particularly those which attempt to model individual
choice, are special cases of this methodology. Two examples are given: an
existing technique (disaggregate behavioral demand models) and a new technique
(direct utility assessment).

ACKNOWLEDGEMENT

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SECTION I

INTRODUCTION

Many designs require the prediction of consumer response to new alternatives or to changes in existing alternatives. Consider the following examples:

Transportation

A community considering the introduction of a new dial-a-ride system requires predictions of demand.

Health

A university medical department wishing to design its medical plans to best serve its potential users needs to know how many people favor which plans.

Finance

A brokerage house contemplating new services, wants to know how popular they will be.

A country offering tourist services, a company selling deodorant, an agency offering counseling service, all share a common need; the need to know how consumers will respond to a choice among a final number of alternatives.

As can be surmised from the variety of applications, the field of choice prediction is now new. Many researchers in diverse disciplines have developed models of choice behavior. (A brief summary of some of these efforts appears in Appendix I).

This paper endeavors to develop a general methodology which includes most of this previous work as special cases. Before describing the basic methodology, let us first consider some criteria for the development of a choice prediction theory.

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SECTION II

CRITERIA

Choice Predictions must be Theoretically Sound, have Measurable Inputs, and be Responsive to the Decision Process

The design decisions mentioned in the introduction require much more than the prediction of who will choose what. They require a choice prediction methodology which interacts well with the decision process, i.e., which reveals why consumers are reacting the way they are, indicates how to improve the consumer response, and guides creativity in the design of new alternatives.

The primary goal of this paper is to present a methodology which is as general as possible, as theoretically sound as possible and which is very responsive to design decisions. Specifically, the methodology will be:

General

1. <u>Diverse</u> - Each step in the methodology should offer a variety of techniques of varying complexity and data requirements. In this way, the methodology can respond to the diverse needs of decision makers and data availability. In addition, it can <u>adapt</u> to changes in requirements or in data.

2. <u>Encompassing</u> - The methodology should deal with choice behavior in general and as such can be used for all the applications mentioned in the introduction.

3. <u>Inclusive</u> - Almost all of the previous work on choice behavior should be covered by the methodology as special cases. This allows the incorporation of proven techniques and the exchange of subsets of techniques.

Theoretically Sound

4. <u>Reflect Behavior</u> - While not explicitly requiring microscopic models of behavior, the methodology should be consistent with what is known about be-

havior, taking account of the level of aggregation and simplification required.

5. Transparent - All models require assumptions. The methodology should make its assumptions explicit and force the submodels to make their assumptions explicit. In doing so, the methodology isolates weaknesses in existing techniques, and indicates where improvements need to be made. In addition, it prevents models from being used in applications which violate their assumptions.

6. Mathematically Consistent - Once the assumptions are isolated, the conclusions reached by the methodology must follow from these assumptions by the use of consistent mathematical logic.

Useful Predictive Powers

7. Have Measurable Inputs - The inputs must be currently available or it must be feasible to obtain them. Judgmental inputs, if required, must be feasible with the technical capabilities and experience of the design team.

8. Controllable - The methodology must predict response to changes which the decision process can control. For example, a model to be used in determining the geographic location of a transportation system may have weather as an independent variable, but a model for determining changes in frequency of service on an existing system should not.

9. Extendable - The methodology should not be simply descriptive but should be normative. It should have the capability of predicting response to new or changed alternatives rather than simply describing how consumers respond to the existing alternatives.

Elicit Creativity

10. Identify Characteristics - By identifying how the consumer perceives the alternatives, i.e., by identifying which characteristics are relevant to the choice decision, the methodology can indicate in which directions the de-

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sign should go.

11. <u>Identify Tradeoffs</u> - It is often possible to improve one characteristic at the expense of another; thus, in the design process it is necessary to know which characteristics are most important and by how much.

Acceptable to the Design Team

12. <u>Understandable Process</u> - Although some steps can be viewed as "black boxes," the overall methodology should be understandable to non-technical members of the design team.

13. <u>Understandable Output</u> - Presentation of the results of <u>each step</u> in the methodology should be clear and understandable to the design team. Every attempt should be made to enable the design team to visualize the underlying choice processes.

14. <u>Robust</u> - If possible the methodology should have some natural robustness, i.e., it should be hard to get absurd answers.

SECTION III

THE BASIC METHODOLOGY

This section presents the basic structure of the methodology. Ongoing research will develop this structure in more detail than is presented here, summarize and critique existing techniques for each step in the methodology, develop a formal theoretic structure for some steps, and introduce new techniques wherever appropriate. In addition, a testing technique will be developed which gives a measure of overall performance, as well as dividing that measure among the various steps.

3.1 Choice Model Interacts with the Design Process

As was argued in Section I, it is imperative that the choice model be intimately tied to the design process (see Figure 1).

Design Process   Analytic Process

```
┌─────────────────────┐          ┌──────────────────────┐
│  ┌───────────────┐  │          │                      │
│  │ Changes in    │  │       ┌─▶│  Predict             │
│  │ Alternatives  │  │───────┤─▶│  Consumer            │
│  └───────────────┘  │       └─▶│  Response            │
│        │    ▲       │          │                      │
│        ▼    │       │          │                      │
│  ┌───────────────┐  │          │                      │
│  │ Decisions on  │  │          │                      │
│  │ Change        │  │          │                      │
│  └───────────────┘  │          │                      │
└─────────────────────┘          └──────────────────────┘
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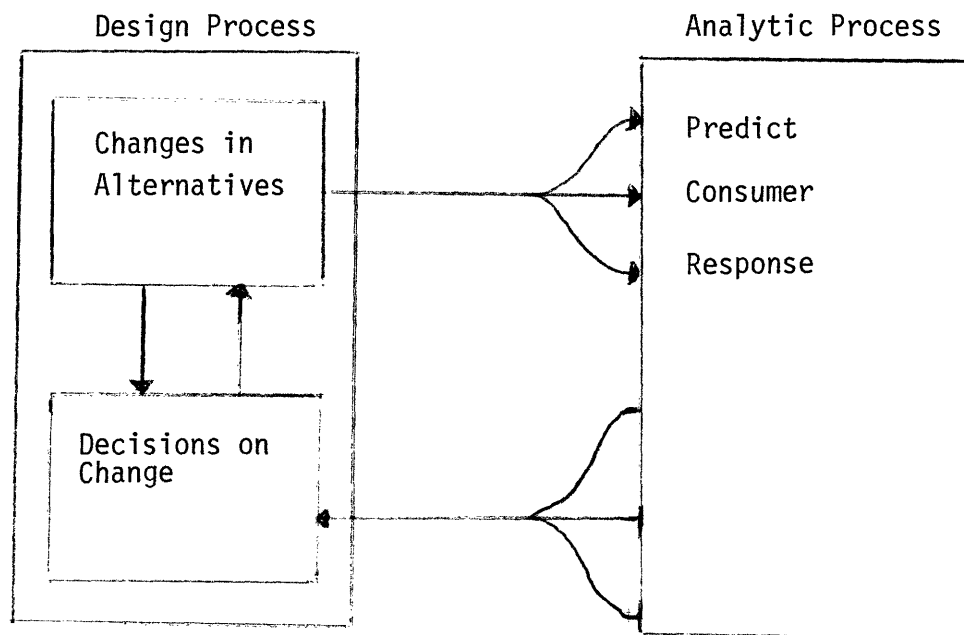
Figure 1: Analytic process interacts with design process.

In other words, the design team should be able to change alternatives on many levels, e.g., specific: waiting rooms 50% larger, or general: more comfortable waiting rooms. Conversely, the choice model should provide insight on many levels in addition to just predicting the number of people choosing the alternative. For example, it should identify relevant characteristics, their importance, and how the quality of each combine to alter choice probabilities. It should also indicate the variation of behavior across the population.

Since, ultimately, it is consumers who are choosing among alternatives, it is necessary to model individual behavior in order to gain insight on consumer response. Thus, the choice model is divided into six steps. The first two are observational, the next three model individual choice behavior in such a way to elicit creativity from the design team, and the sixth is an aggregation step which transforms individual behavior into overall market response (see Figure 2).

Design Process   Analytic Process

Changes in Alternative

Decisions on Change

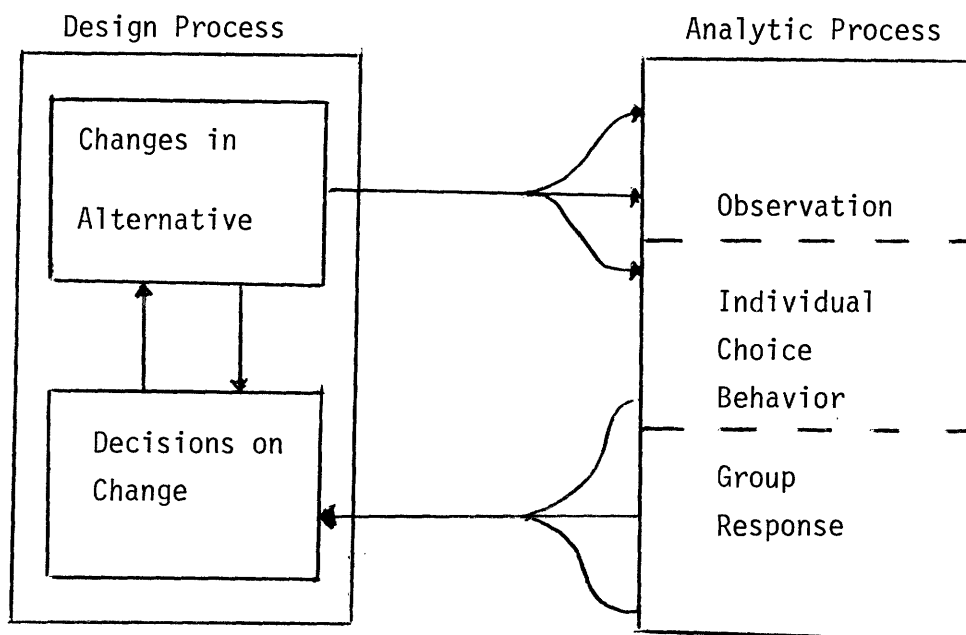Observation

Individual
Choice
Behavior

Group
Response

Figure 2: Analytic process consists of observation, individual choice behavior, and group response.

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3.2 Observation of Consumers

Observation is modeled as a two-step process to explicitly identify potential errors. The first part is <u>presenting the individual with a choice</u>. This can be a real choice or a proxy design to elicit choice behavior which approximates what it would be in a realistic choice environment. For example, individuals might be observed on which mode of transportation they choose from the existing alternatives, or they might be given descriptions of new alternatives and asked to choose among them. The second part is <u>measurement</u>. This is somehow identifying quantitative measures of consumers' perceptions of the alternatives, consumer characteristics, and choice. They can be cardinal, e.g., travel times on various modes or ordinal, e.g., a rank order on the availability of doctors in health maintenance organizations.

These measurements are input into the next section, choice behavior.

3.3 Individual Choice Behavior

The numerical goal of this section is to transform the observed measures into individual choice probabilities. That is, obtain estimates for each individual of the probabilities that that individual will choose defined alternatives. But, the more important goal is to provide insight into the individuals' choice behavior and to elicit creativity in the design of new or changed alternatives.

To provide this insight and to approximate choice behavior, the measures are transformed in three identifiable steps (see Figure 3).

<u>Reduction/Abstraction</u> - From many measures (e.g., 20), the data is reduced to a few performance measures and preference measures (e.g., 4 and 4). The reasons for this are many fold, but mostly dealing with the inability of the human mind to explicitly consider more than a few measures simultaneously.

```
┌─────────────────────┐                    ┌─────────────────────┐
│ Present the         │                    │ Measurement         │
│ individual with a   │                    │ ─────────           │
│ choice              │ ────────────────▶  │  - characteristics  │  ∿▶
│ ─────────           │                    │  - preferences      │
│                     │                    │  - choice           │
│   real vs. proxy    │                    │  - perceptions      │
└─────────────────────┘                    └─────────────────────┘
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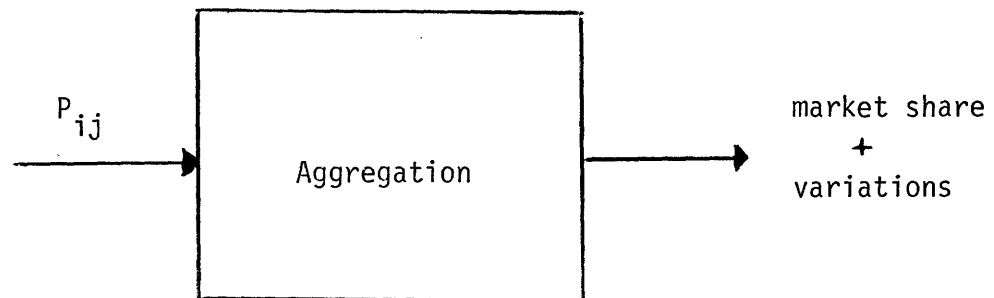
a) Observation of consumers

```
       ┌──────────────┐           ┌──────────────┐           ┌──────────────┐
       │ Reduction/   │           │  Compaction  │           │Probability of│
       │ Abstraction  │  x_j's    │   ────       │   c_i     │   Choice     │
  ∿ ▶  │ ────         │ ───────▶  │ C = C (x,λ)  │ ──────▶   │ ────         │ ──▶
       │ ∿→ X, Λ      │  λ_i      │  ij   s j i  │           │P = P (a |c ) │
       │(segmentation)│           │              │           │ ij  s  j  i  │
       └──────────────┘           └──────────────┘           └──────────────┘
```

$$\Omega \to X,\ \Lambda$$
$$C_{ij} = C_s(\underline{x}_j,\underline{\lambda}_i)$$
$$P_{ij} = P_s(a_j|\underline{c}_i)$$

b) Individual choice behavior

```
              ┌─────────────────────────┐
     P_ij     │                         │      market share
  ─────────▶  │       Aggregation       │ ──▶       +
              │                         │       variations
              └─────────────────────────┘
```
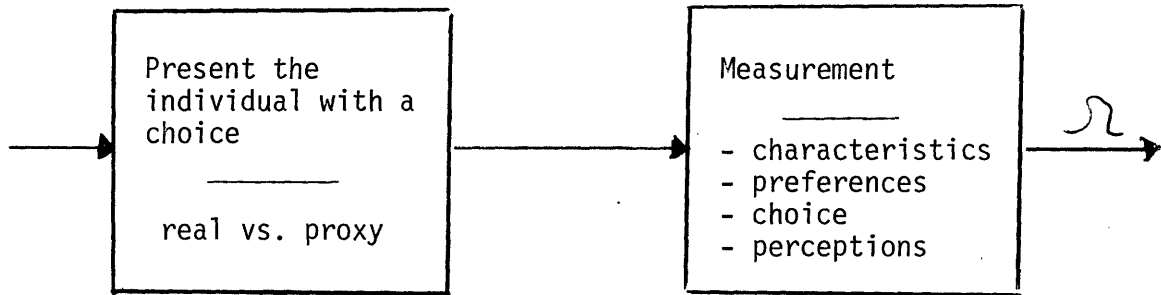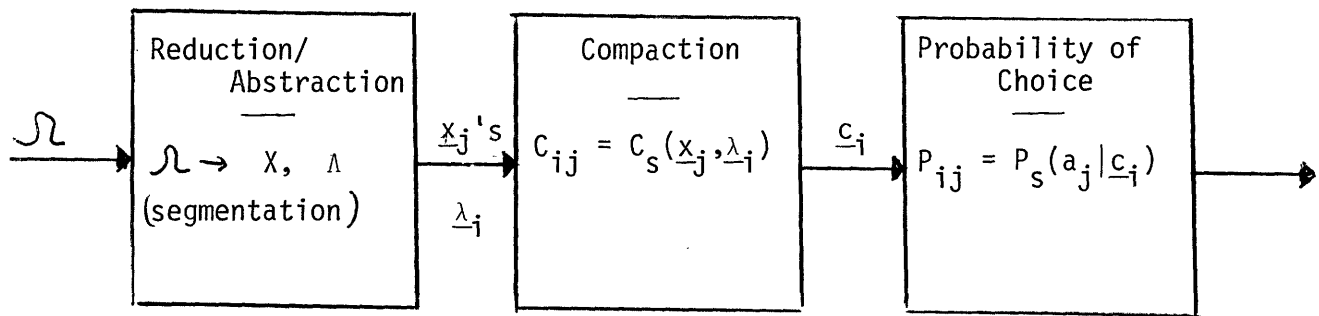
c) Group response

Figure 3:   Submodels in the methodology

Thus the reduction of dimensionality and the abstraction of the relevant performance measures allows the design team to better understand which characteristics are relevant to the choice. Furthermore, a geometric representation of the data in fewer dimensions allows visualization of the choice process.

Of course, reduction/abstraction does represent a behavioral assumption, the assumption that in order to make a choice, the individual somehow conceives of the alternatives along a relatively small number of dimensions. If the design team feels uncomfortable with this assumption, reduction/abstraction becomes a null step and the measures from the observation section are passed directly to the next step in the methodology.

In addition to reducing the data, a certain amount of segmentation occurs at this step. Because preference measures can vary significantly from individual to individual, an attempt is made to link these variations to population characteristics and to abstract appropriate relatively homogeneous segmentations. Such segmentations should be necessary (variation occurs), identifiable, and relevant to the design process.

Compaction - Even with only a few dimensions to the data, the design team needs to know the saliencies of each dimension. In other words, how do individuals trade off the performance measures, how interdependent are the performance measures, and what are the risk characteristics of the individuals' choice processes. Thus, a compaction function is determined and for each individual the performance measures for each alternative and the preference measures are compacted into a scalar measure of goodness for that alternative.

This intermediate step allows the design team to understand how the performance measures combine and thus guides the decisions they must make in the design of alternatives. For example: "increasing the safety of the train at the expense of speed" or "Increasing parking outside a medical department is

useful only if the waiting rooms are made more comfortable."

Compaction is intimately tied to the next step, probability of choice, in that every compaction function, $c(\underline{x}, \lambda)$, mapping the performance measures into a single number must have the property that with all other alternatives held fixed, the same value for the scalar measure of goodness for an alternative must yield the same probability of choice for that alternative. In other words, knowing the measures of goodness for all the alternatives is sufficient for determining the choice probabilities. Notice that a compaction function is more general than a utility function because (1) its domain includes preference measures as well as performance measures and (2) there is a randomness to behavior in that the individual does not always choose that alternative with the largest compaction value.

Again, compaction is a behavioral assumption and if found unacceptable, it becomes a null step and the performance and preference measures are transformed directly into choice probabilities.

Probability of Choice - This last step in the choice behavior section mathematically transforms the vector of compaction values into choice probabilities. Reduction/abstraction and compaction provide qualitative guides to design; this last step provides probabilities that can be used to get the numerical implications of design decisions. The output of this step can be Bernoulli probabilities, i.e., (possibly non-stationary) estimates of probabilities of selection for any choice occasion, or they can be Poisson rates, i.e., if λ is the choice rate for an alternative, then the probability that the individual will choose that alternative in (small) time Δt is $\lambda \Delta t$. Of course, inputs for more complex models are consistent with the methodology, but techniques for their determination will not be developed in this research.

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3.4 Group Response

Aggregation - The last section of the choice model is aggregation of individual choice probabilities (refer back to Figure 2). The design team is ultimately concerned with total choice behavior; thus, the individual choice probabilities must be combined to produce measures of overall consumer response. This aggregation explicitly uses the segmentations identified in the choice behavior section to extrapolate from the sample population to the target population. Some segments, e.g., elderly citizens, may be handled separately, if doing so is relevant to the design process.

The typical output of aggregation is often a single numerical estimate of market share for each alternative, but to be fully useful, measures of variation of share such as covariance should also be output. In addition, by analytically or numerically determining gradients of market share with respect to the performance measures, sensitivity analysis can be done.

A summary of the choice model portion of the methodology appears in Figure 4.

3.5 Interaction with Design Process

The choice model provides insight on four levels (1) reduction/abstraction identifies the relevant performance measures and segmentations, (2) compaction identifies tradeoffs, interactions, and risk characteristics among the performance measures, and condenses them to a scalar measure of goodness, (3) probability of choice gives numerical implications of the scalar measure of goodness, and (4) aggregation provides measures of aggregate consumer response and variation in that response.

Each level provides information, but in order for that information to provide useful insight to the design team, it must be presented to them in a
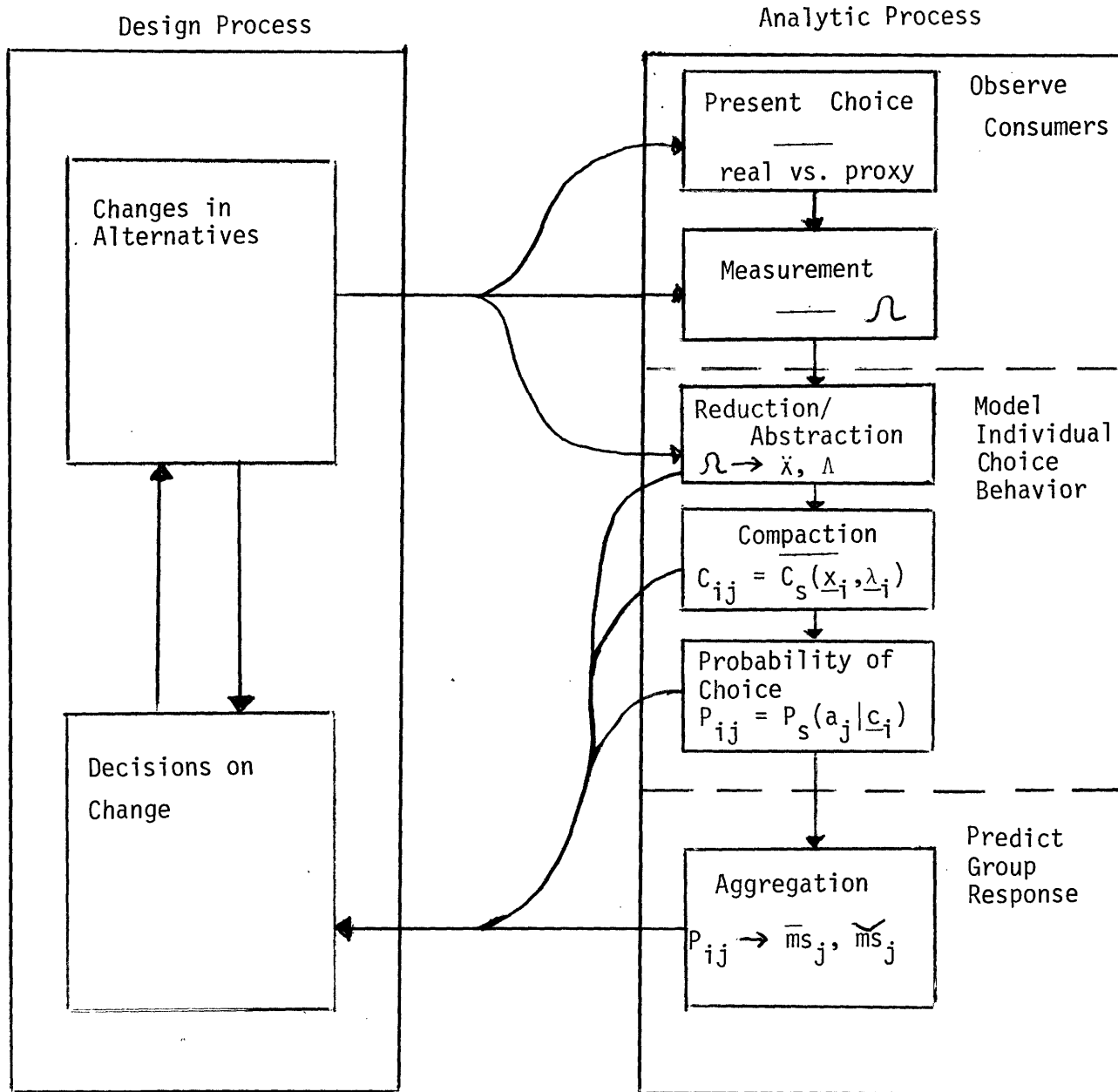
Design Process

Analytic Process

Changes in Alternatives

Decisions on Change

Present Choice
——
real vs. proxy

Measurement
—— $\int\!\!\!L$

Reduction/ Abstraction
$\int\!\!\!L \rightarrow \ddot{x}, \Lambda$

Compaction
$C_{ij} = \overline{C_s(\underline{x}_i, \underline{\lambda}_i)}$

Probability of Choice
$P_{ij} = P_s(a_j | \underline{c}_i)$

Aggregation
$P_{ij} \rightarrow \overline{ms}_j, \widetilde{ms}_j$

Observe Consumers

Model Individual Choice Behavior

Predict Group Response

Figure 4: Summary of the Choice Model

form they feel comfortable with and can mentally manipulate. For example, in the reduction/abstraction step, a map (Green and Wind [27], Urban [86]) of each alternative in "average" perceptual space might be presented, (Figure 5a), or perhaps a scatter diagram of individuals' perceptions of the alternatives in the same perceptual space, (Figure 5b). These presentations can be in terms of numbers (means and variances), maps, sensitivity curves or whatever the design team feels comfortable with.
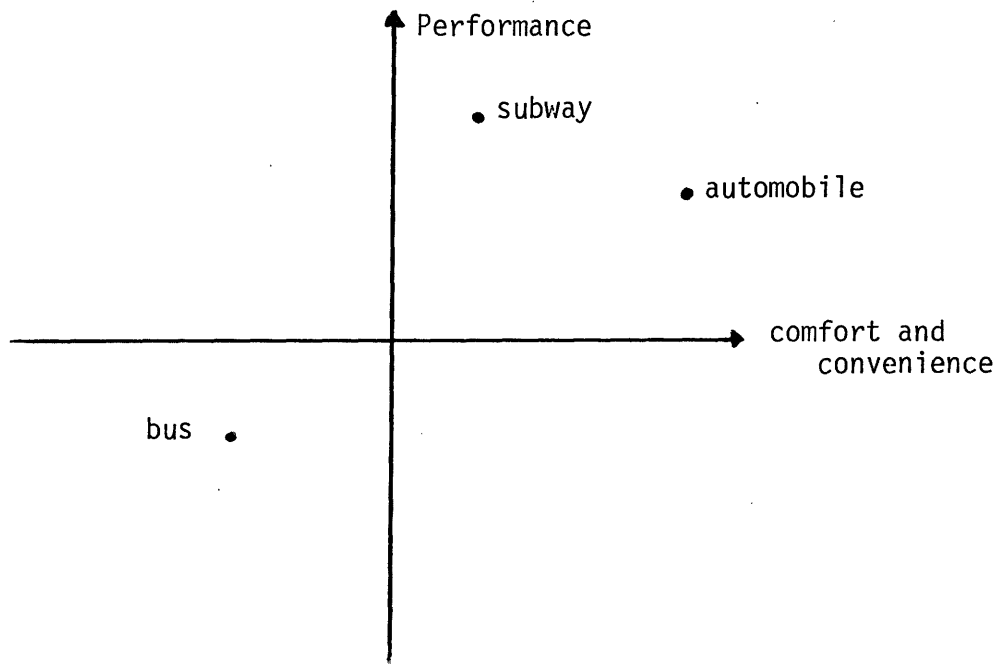
Just as presentations can occur from many steps in the methodology, changes in design decisions can be input for testing at many steps. For example, implications of a very specific change like doubling the frequency of bus service on an existing route can be tested by changing an input to the reduction/abstraction step, or a general change like improving comfort can be tested by changing an input directly into the compaction step. Similarly, an entirely new concept might require facing a sample population with a new set of alternatives, or might simply require taking a few additional measurements. Promotion and/or advertising is tested by determining their implications as changes in the preference measures.

Figure 6 summarizes the methodology.

3.6 Scope of the Methodology

By its nature, developing a general methodology is an ambitious task, but its generality does not imply a once and done procedure. Its use does not entail cycling once through the analytic process, determining the market share and then stopping. In fact, the same methodology, with different submodels, can be used during many different phases in the design process.

In the search for new alternatives or changes in existing alternatives, reduction/abstraction is particularly useful because it identifies which at-

a) "average" perceptual space
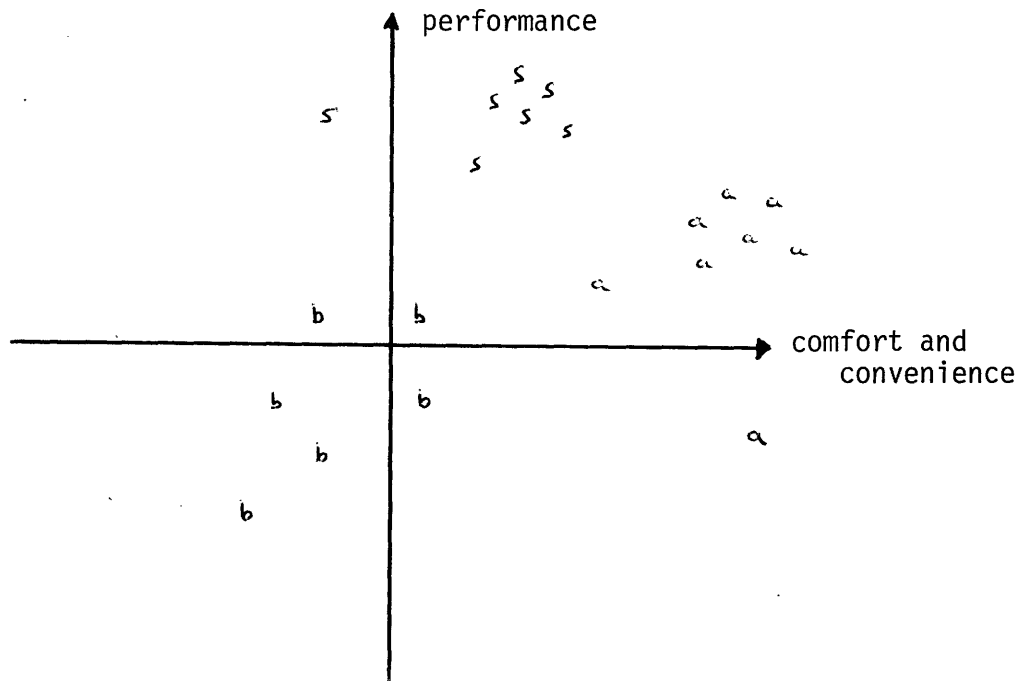
b) scatter diagram in perceptual space

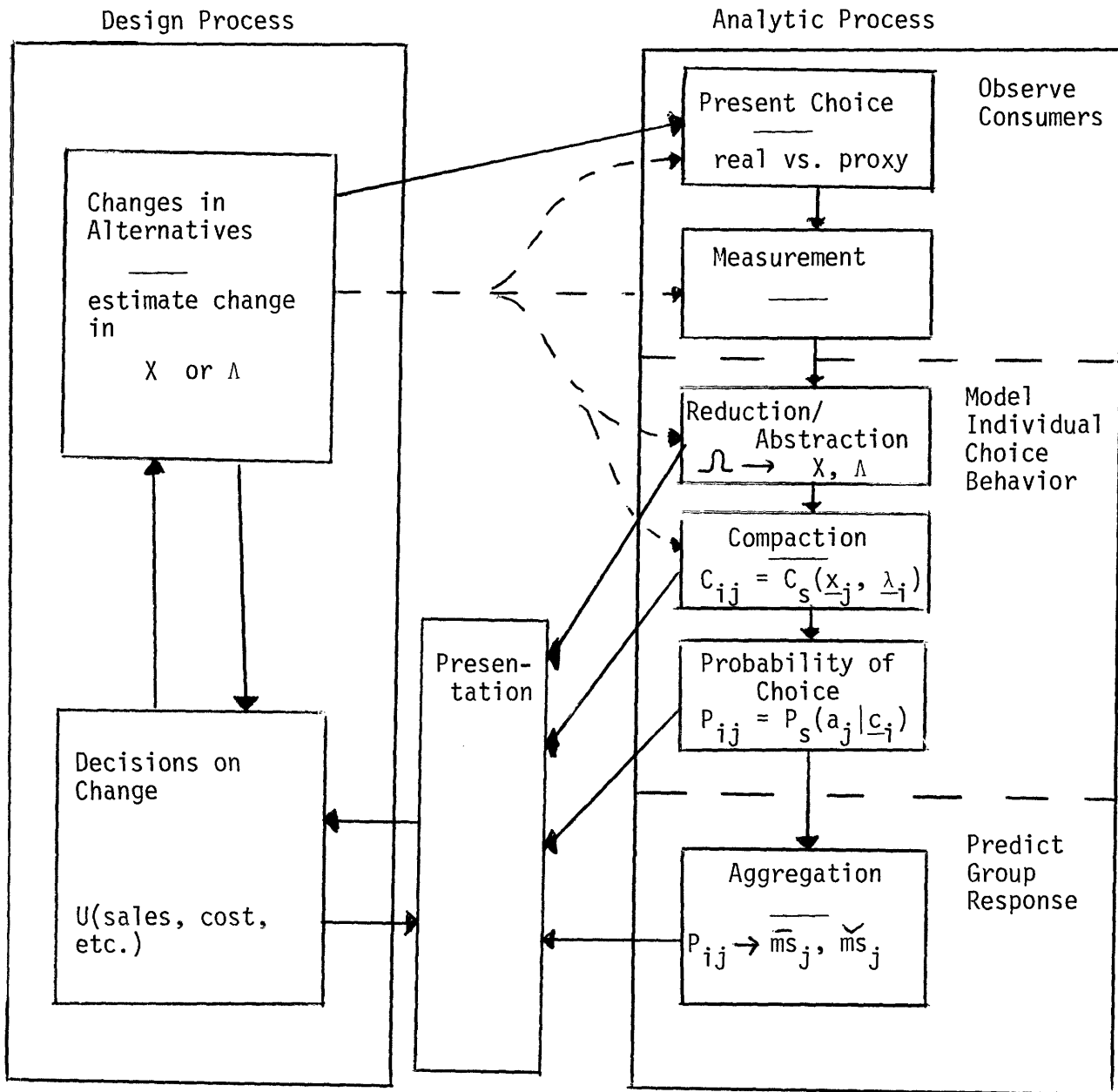Figure 5: Samples of perceptual maps

Figure 6: The complete methodology and its interactions with the design process.

tributes are most important and which ones combine to form performance measures. In addition, first cut compaction functions provide insight on tradeoffs, risk, and interaction among performance measures. All of which are useful in search.

In screening, dominance can be checked, i.e., if each of the performance measures of one alternative are dominated by another. Also, early prediction of market share and explicit modeling of the uncertainty in its estimation allows early pruning of a large set of alternatives.

In some circumstances, such as the microscopic design of a bus network, the methodology is used directly to help determine the initial design. In other circumstances, such as the design of health maintenance organizations, it does not replace test markets, but instead helps in the design of the alternative(s) which are to go to test market.

During test market, (or laboratory simulation in the case of frequently purchased consumer products) the methodology is used with more complex submodels to improve understanding of consumer behavior, and to extrapolate the entire consumer response from test market results.

Finally, during implementation, the methodology is used to monitor consumer response and to suggest improvements in the alternative, if necessary.

SECTION IV

TWO EXAMPLES

Probably the best way to explain a methodology is with examples. This section presents two methods to predict the market shares of competing transportation alternatives. The first consists entirely of existing techniques, many similar to those used by Quarmby, Ben-Akiva, and others. The second consists mostly of new techniques currently being developed at M.I.T.

4.1 Example 1: Disaggregate Behavioral Demand Models

Suppose for simplicity that total demand is known and we are only predicting market share for two alternatives, bus and automobile.

Observation

Presenting the Individual with a Choice - The observation is made in a real choice environment, i.e., the individual is questioned on the choice he most recently made when faced with the existing alternatives. This has the advantage that repeat probabilities rather than trial probabilities are determined, but the disadvantage that if entirely new options are introduced, the values of their attributes may be well outside the predictive range of the model.

Measurement - The choice behavior, the values of the attributes on existing modes, and demographic data are measured via questionnaire. Thus, some noise is introduced because of potential misunderstanding of the questionnaire. An attempt is made to relate perceived time to measured time[1] because although an individual makes a choice based on perceived time, prediction will be based on measured time.

Choice Behavior

Reduction/Abstraction - Although some is done by combinatorial experiments and some by dropping variables which are not statistically significant, this step is based primarily on the model builder's professional judgment and experience. For example, travel time, wait time, cost, access time, a dummy variable for mode, and dummy variables for income are a few of the candidates for explanatory variables. Segmentations might be by trip purpose, i.e., work trips vs. non-work trips.

Compaction - The compaction function is a linear "disutility" function, i.e., this scalar measure of goodness for each mode is calculated via a weighted sum of the variables.

Ting [81] shows that under the assumption that every pair of attributes is preferentially independent[2] of all others, an individual's cardinal utility function can be written as a function of an additive value function. In other words, there exist continuously differentiable functions $V, g^1, g^2, \ldots g^m$, such that

$$u(\underline{x}) = V[g^1(x_1) + g^2(x_2) + \ldots + g^m(x_m)].$$

Unfortunately, the theory gives no easy way to specify these functions and their form can be quite complex. The use of a linear "disutility" function assumes that these value functions can be approximated by linear transformations of the variables.

In addition to the assumptions of preferential independence and linear value functions, an assumption is made that the "disutility" function and the values of the weights are identical for all individuals within a segment. The values of these weights are determined in the next step, probability of choice.

Probability of Choice - In this step, a conditional probability law is determined which calculates the probability that an individual will choose

one mode based on the scalar measures of goodness for each mode. The following assumptions are made:

1. Individuals are independent.

2. An individual always chooses that mode which maximizes his true scalar measure of goodness (negative "disutility"), but

3. The true scalar measure of goodness is equal to our calculated scalar measure of goodness plus a random disturbance term, i.e.,

$$d_{true} = d_{calculated} + \varepsilon$$

4. The disturbance terms are independent and identically distributed with Weibull densities, i.e.,

$$P_\varepsilon(\varepsilon_0) = e^{-\varepsilon_0} \exp(-e^{-\varepsilon_0})$$

These assumptions imply that the probability of choosing mode a_1 is determined by the logit model, i.e.,

$$P(a_1 \mid d_1, d_2) = \frac{e^{-d_1}}{e^{-d_1} + e^{-d_2}}$$

The values of the weights are determined either by maximum likelihood or by linear regression, (Note that $\ln(\frac{p}{1-p}) = d_2 - d_1$ where $p = p(a_1 \mid d_1, d_2)$.) and variables which are not statistically significant are dropped.

Aggregation

Aggregation is a difficult step in transportation demand prediction since model splits (market shares) are often required for each zonal interchange [99] and because the values of the explanatory variables can vary considerably from individual to individual. Thus, the real problem is the tremendous number of calculations required. Various techniques exist for simplifying these calculations but basically grand means are calculated from individual probabilities.

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In other words, if the explanatory variables are distributed across the population segment with joint probability distribution $p(\underline{x}_1, \underline{x}_2)$ then the predicted market share of alternative 1 for that segment is:

$$\overline{ms}_1 = \int p[a_1 \mid d_1(\underline{x}_1), d_2(\underline{x}_2)] \, p(\underline{x}_1, \underline{x}_2) \, d\underline{x}_1 \, d\underline{x}_2 \qquad \text{Equation 4.1}$$

Interaction with Decision Process

The procedure just described is primarily a prediction procedure rather than a design process. Its strength is in predicting market share for already defined alternatives. It does provide some design insight because the weights for the explanatory variable are indicators of the strength of the effect that variable has. Care must be taken in interpretation because of the potential multicolinearity among the variables.

In addition, if the assumption of homogeneity of weights within a segment is valid, elasticities can be easily calculated.

4.2 Example 2: A Cardinal Utility Theoretic Approach

Again, for simplicity, assume that these are only two alternatives, bus and automobile.

Observation

Presenting the Individual with a Choice - This technique makes use of both behavioral and preference data. Thus, the individual is faced with a real choice among existing alternatives and an artificial choice among attributes of potential alternatives. This enables the model to predict choice for alternatives with values well outside the range of those available on existing modes.

Measurement - The choice behavior, the perceived values of the attributes on existing modes, demographic data, and the preferences toward the at-

-20-

tributes are all determined by questionnaire. The greatest potential for error occurs in this step because of the difficulty a consumer might have in understanding preference questions.

The three types of preference questions asked will be similar to the following:

Tradeoff Questions

1. Suppose the mode of transportation you are using costs $1.00. Suppose you can expect a waiting time of 10 minutes and a travel time of 20 minutes.

   A more reliable mode is offered which also costs $1.00. This mode guarantees only a 5 minute waiting time. What is the maximum travel time you would accept and still prefer this more reliable mode?

Risk Questions

2. Suppose the mode of transportation you are using costs $1.00. Suppose you are not sure of the waiting time, in fact, it is as if someone flipped a coin: heads meant you had to wait 5 minutes, tails meant 25 minutes. In other words, an average time of 1/2(5 + 25) = 15 minutes.

   A more reliable mode is offered which also costs $1.00. This mode can guarantee a fixed wait time. What is the maximum guaranteed wait time you would accept and still prefer this more reliable mode?

Independence Questions

3. If both the existing and the new reliable mode cost only $.50 would your answers to questions 1 and 2 change? If so, what would they now be?

The independence questions determine the mathematical form of a utility function while the risk and tradeoff questions determine the preference parameters of each individual utility function. (More on this in the discussion of compaction.)

Choice Behavior

Reduction/Abstraction - This step has two phases, exploratory and actual. The exploratory step can be performed on existing data. It makes use of statis-

tical techniques such as factor analysis and Automatic Interaction Detection [98] as guides to professional judgment in reducing the set of potential explanatory variables to a manageable number.

The actual reduction occurs via in depth interviews with decision makers, other analysts (because they have thought extensively about how consumers behave), and a relatively small number of consumers. By iteratively asking tradeoff, risk, and independence questions, it is possible to identify those attributes or combinations of attributes which can be classified as performance measures, i.e., as necessary and sufficient for the decision process.

The output of this reduction is a set of potential compaction functions. Each compaction function is an identified set of performance measures and a functional form that that individual's utility function takes. For example, when this author's utility function was assessed with the aid of an interactive computer program developed by Sicherman [97], it was found that for Dial-a-Ride trips the following form was a good approximation.

Performance Measures:

$$x_1 = \text{travel time}$$
$$x_2 = \text{wait time}$$
$$x_3 = \text{cost}$$

Preference Measures:

$$c_1, c_2, c_3 = \text{risk aversion coefficients}$$
$$k_1, k_2, k_3, k_{12}, K_{12}, K = \text{tradeoff coefficients}$$

Independence Conditions:

1. (x_1, x_2) aggregatable.

2. aggregation function multiplicative

3. after aggregation, utility function multiplicative

Thus if $u(x_1, x_2, x_3)$ = utility function

$w(x_1, x_2)$ = aggregator function

then

$$1 + Ku = [1 + Kk_3 v_3(x_3)] [1 + Kk_{12} w(x_1, x_2)]$$

$$1 + K_{12} w = [1 + K_{12} k_1 v_1(x_1)] [1 + K_{12} k_2 v_2(x_2)]$$

$$v_i(x_i) = 1 - b_i + b_i e^{-c_i x_i}$$

(b_i normalization constants)

Compaction - Cardinal utility theory (based on the von Neumann-Morgenstern axioms [90]) is normally a prescriptive technique. In other words, it is used to help people make decisions rather than predict how they will make them. Thus, we might expect that everyone will not always choose that alternative which maximizes expected utility. One of the goals of the current research effort is to develop axioms, similar to the von Neumann-Morgenstern axioms, which apply to compaction functions, i.e., which allow randomness in behavior. If these axioms are acceptable to the model builder, then the expected values of an individual's scalar measures of goodness for each alternative are sufficient to predict choice.

Based on the axiomatic theory of compaction, the compaction step in this technique consists of two phases, segmentation and distribution determination.

The final phase, segmentation, identifies segments of the population with homogeneous compaction functions. That is, it identifies which compaction functions (as determined by reduction/abstraction) apply to which popu-

lation segments. Computer packages such as AID are used to group individuals based on their answers to the risk and independence questions.

The second phase, distribution determination, uses estimates of the performance measures for the alternatives under test to determine (the joint probability distribution of) the scalar measures of goodness for each alternative, for each individual in each segment. Appropriately normalized, these are used in a Bayesian choice probability model.

Probability of Choice - In this step, a conditional probability law is determined which calculates the probability that an individual will choose one mode based on the scalar measures of goodness for each mode. The following assumptions are made:

1. individuals are independent.

2. an individual does not necessarily choose that mode which maximizes his scalar measure of goodness, but

3. two (or more) individuals from the same population segment with the same set of scalar measure of goodness have the same choice probabilities.

These assumptions allow the use of Bayes Theorem to determine choice probabilities. First, rank order the scalar measures of goodness. Then let a_1 be the event that an individual chooses the mode which maximizes his scalar measure of goodness. We can observe easily the posterior distributions $p(c_1, c_2 \mid a_1)$ and $p(c_1, c_2 \mid a_2)$ and also the total percentages, n_1 and n_2, choosing a_1 and a_2. Bayes Theorem then gives:

$$p(a_1 \mid c_1, c_2) = \frac{n_1 p(c_1, c_2 \mid a_1)}{n_1 p(c_1, c_2 \mid a_1) + n_2 p(c_1, c_2 \mid a_2)}$$

(Note that if only $c = c_1, -c_2$ matters, and if $p(c \mid a_1)$ and $p(c \mid a_2)$ are normal with common variance, then the logit model results [Quarmby (66)].)

A word about normalization. Since both utility functions and compaction functions are unique only up to positive linear transformations, the probability model can be sensitive to the choice of normalization. A number of heuristics are being developed in an attempt to guide engineering judgment in the choice of normalization.

Aggregation

The same difficulties in computation that occurred in disaggregate behavior models occur in this technique. Because the entire methodology is normative, the output of aggregation should include estimates of variation in market share as well as mean values. The question that naturally arises is: "Can this be obtained without tremendous additional computational effort?".

Since individuals are assumed independent, the Central Limit Theorem can be used to determine the entire joint distribution of aggregate market shares with little additional computation, i.e., the aggregate market shares are jointly distributed as multivariate normal random variables with means given by equation (4.1) (same as behavioral models) and covariances given by:

$$cov(ms_1, ms_2) = - \int p[a_1 \mid c_1(\underline{x}_1), c_2(\underline{x}_2)] \ p[a_2 \mid c_1(\underline{x}_1), c_2(\underline{x}_2)] \cdot$$
$$p(\underline{x}_1, \underline{x}_2)d\underline{x}_1 d\underline{x}_2$$

$$var(ms_1) = similar$$

Interaction with Decision Process

The procedure just described is primarily a normative procedure and is designed to interact will with the design process. In reduction/abstraction, the important attributes are determined and in compaction the interactions, tradeoffs, and risk qualities of these performance measures are identified. For example, in the utility function on page 23, k_1/k_2 measures the rela-

tive importance of travel time to wait time and K_{12} measures how strongly they interact. The risk aversion coefficients measure how important risk is in the choice decision. In probability of choice, the design team can examine the results of design decisions directly in terms of compaction functions or in terms of choice probabilities. Finally, in aggregation, measures of variance as well as means of market share can be determined. Notice that direct assessment of compaction functions avoids multicolinearity and that elasticities, if required, can be numerically determined.

SECTION V

RESEARCH OBJECTIVES

In addition to developing the methodology in more detail, this research will summarize and critique one or more existing techniques for each step in the methodology. New techniques and a formal structure will be developed as follows.

Measurement - An examination of utility assessment via questionnaire will be made and a few representative questions developed. If possible, these will be pretested for style on a small population.

Reduction/Abstraction - The issues involved in reduction will be identified through a series of rigorous definitions. The existing techniques of factor analysis, of automatic interaction detection, and of choosing features with maximum information content will be examined in relation to the methodology. Examples, based on a questionnaire for health maintenance organizations, will be presented. In addition, the technique of in depth utility assessment will be developed.

Compaction - A rigorous development of the theory of compaction functions based on axioms similar to the von Neumann-Morgenstern utility axioms will be developed. This will enable the utility theoretic results of Fishburn, Keeney, Ting [20,21,23,40,41,81,90] and others to be directly applied to compaction functions. Independence from irrelevant alternatives and from irrelevant partitions will be examined. The mathematics for assessing compaction functions by direct questionnaire will be developed.

Probability of Choice - Existing techniques to transform scalar measures of goodness into probabilities, such as distance in perceptual space and utility perturbation, will be summarized and critiqued. A general Bayesian

technique will be developed with examples given from deodorant and from health maintenance organization data. Also, the problem of interpersonal comparisons of utility or scalar measures of goodness will be discussed and some heuristics presented to partially circumvent these problems.

Aggregation - The general mathematical equations will be developed and the Central Limit Theorem approximation will be introduced. A discussion of numerical problems will be made, but no numerical examples are planned at this time.

Testing - A testing technique will be developed based on honest reward functions, maximum likelihood and information theory. This test tries to measure how "good" probability measures really are.

Communication with Design Process - Various techniques such as pictorial representations will be examined for presentation of the outputs to the design team.

APPENDIX

REVIEW OF SOME PREVIOUS WORK

Most (but not all) of the work in choice modeling has been done in three
fields: transportation demand modeling, marketing research, and mathematical
psychology. The purpose of this appendix is not to summarize all the work in
each field (for that could take many volumes) but to give an indication of the
types of issues involved and of various approaches taken.

A1.1 Transportation

Urban Transportation Model System - Perhaps the most classical approach to
demand modeling is the "Urban Transportation Model System" (UTMS), which has been
applied in over 200 cities in the U.S. over the last 17 years. (Manheim [55])
This process models consumer choice as sequential, i.e. the consumer first
decides to travel (trip generation), then chooses a destination (trip distri-
bution), picks a mode of travel (modal split) and finally, if multiple paths exist
from origin to destination, chooses a path of travel (assignment). Feedback
loops are often incorporated to obtain stable estimates (because the volume of
travel effects the quality of service which in turn effects volume) but still
the process is essentially sequential. Most of the submodels used in the UTMS
have been aggregate in the sense that individual choice has not been explicitly
modeled.

For example; Trip generation: Land use, automobile ownership, etc. are
extrapolated from current trends and total trips determined by multiple linear
regression (Douglas and Lewis [17], Fleet and Robertson [24], Martin, Memmott
and Bone [57]). Trip distribution: The fraction of trips to a given destination
zone is a function of the "size" of the zone, the average travel time to that

zone, and other measures of the level of service. Arbitrary functional forms, such as the gravity model (U.S. Dept. of Transportation [87]) are used as are aggregate probabilistic models such as the opportunities model (Ruiter [70], Witheford [93], Hauser [31]). Modal split: The market share each mode captures is a function of the average level of service on that mode. Techniques to do this are diversion curves (Traffic Research Corp. [82]) and other formulas (Wynn [94]). (Disaggregate approaches to modal split will be discussed later.) Assignment: this is done mostly by mathematical programming such as minimum path assignment techniques.

Direct Demand Models - The UTMS models are sequential, but many analysts feel decisions to travel are simultaneous in nature. That is, mode, destination, path, and frequency are highly interdependent decisions. The first attempts to incorporate this posulated interdependence were (aggregate) direct demand models (McLynn [53], Quandt and Baumol [64], SARC [79]). Direct demand models are essentially econometric models where the analyst postulates a function form such as:

$$d_{ijk} = \beta t_{ijk}^{a_1} c_{ijk}^{a_2} (P_i P_j)^{a_3} \ldots$$

where d_{ijk} = number of trips between city pair
$i \rightarrow j$ on mode k

c_{ijk} = cost of travel . . .

t_{ijk} = travel time . . .

P_i = population of city i

β, a_1, a_2, a_3 = constants

In this example a logarithmic transformation makes the equation linear in the parameters which are then determined by regression.

These models have the advantage of treating choice as simultaneous, but

have many disadvantages. Among these are that (1) the functional forms, although motivated by behavioral reasoning, still have a certain arbitrariness to them, (2) multicollinearity among the variable often makes interpretation difficult, (3) they are calibrated on existing alternatives and thus extrapolation to new alternatives or radically different service is difficult, and (4) because aggregate data is used, much of the detailed individual choice is not modeled.

Disaggregate Models - A more recent development in demand models have been attempts to model individual choice. For example (Lave [45], McFadden [52], Quarmby [66], Stophes [77]). Most of the applications have been in predicting modal choice although some, most notably Ben-Akiva [6], have attempted to simultaneously predict mode, destination, and frequency.

One of these models is described in more detail in example 1, section IV but basically they attempt to predict individual choice probabilities based on quality of service (and demographic) measures for each mode.

A1.2 Marketing Models

In marketing the problem is to predict how consumers will respond to new and/or mature products. Some of the important issues in these models are (1) normative vs. descriptive, (2) the degree of aggregation, (3) the degree of behavioral modeling, (4) qualitative vs. quantitative outputs, (5) type of input, and (6) extent of application.

Normative vs. Descriptive: A descriptive model strives to explain why consumers behave the way they do. Whereas a normative model endeavors to predict consumer response to management options.

Because of the complexity of consumer behavior, descriptive models tend to

have more detail and are formulated with the goals of learning about consumers. Some examples of descriptive models are Herniter's entropy model [33,35], Bass' theory of stochastic preference [5], and Butler's "Hendrodynamics" [10] which model consumer behavior as inherently stochastic and choice probabilities are determined by optimizing some measure of randomness subject to observable constraints such as market share.

Normative models also attempt to describe consumer behavior but with the goal of predicting how management decisions affect the behavior. For example Little's BRANDAID [48] relates price, advertising, promotions, and other variables to aggregate consumer response in a way that allows relatively simple evaluation of decision strategies. Calibration steps include judgment, analysis of historical data, tracking, field measurement, and adaptive control. In other words consistent use is made at all available information to uncover problems, focus managerial effort, enhance insight on market structure, and guide the decision making process.

Degree of aggregation - Ultimately most managerial decisions will be made on aggregate response, but models differ in their technique of obtaining this response. Some deal directly with the individual, modeling each consumer's decision, while others use "average" inputs.

For example Burger's new product forecasting system [9] observes individual consumers and estimates probability of purchase from a multiple linear regression model. Prediction is accomplished by first projecting the independent variables (preference, distribution, awareness, intent, and price) then predicting individual probabilities from the regression model. An example of an aggregate model is Montgomery and Silk's communication expenditures model for ethical drugs [60]. In this model, market share is considered a function of (distributed lags of) expenditures on communications mix variables such as

journal advertising, direct mail advertising, and samples and literature.

Degree of behavioral modeling - More detail in the description of consumer behavior means greater the potential for predictability, but greater detail also means greater expense in the use of the model, greater difficulty in obtaining parameter estimates, and occasionally less of an ability to isolate essential consumer dynamics. Depending upon its intended purpose different models have varying degrees of behavioral modeling.

For example, the microsimulation models of Amstutz [2] and Herniter and Cook [36] simulate the buying decisions of each and every consumer. Such models have a large number of parameters which, when set appropriately, enable the model to replicate behavior, but they are difficult and expensive to use for testing new strategies.

A model with a different degree of behavioral modeling is repeated in Claycamp and Liddy [14]. This model, which identifies causality but does not simulate individual behavior, is a multiple equation regression model which tries to predict percent of initial purchase for a product based on independent variables such as "coverage of consumer promotions adjusted for type and value of offer."

A model which is somewhere between the extremes of behavioral modeling is Farley and Ring's [19] empirical test of the Howard-Sheth model [39]. In this formulation a "conceptual model of buyers behavior" is cast in a regression format. In other words, this model identifies causalities in the consumer's buying decisions and approximates the relationships by linear equations.

Qualitative vs. quantitative outputs - The primary purpose of some models is to make a GO/NO GO decision and as such the model is required to produce

numerical estimates of market share. Other models are primarily diagnostic and as such the model is required to produce qualitative indicators.

For example in Stefflre's [76] studies of brand similarities and in Urban's PERCEPTOR [86] multi-dimensional scaling techniques are used to determine which attributes of a product are important in buying behavior. (See Green and Wind [27] or Rummel [71] for a discussion of multi-dimensional scaling techniques.) Urban goes on from there to develop quantitative estimates of market share based upon the distance in perceptual space between a product and an ideal point.

Types of measurement - The types of measurement used are highly dependent both on a model and its application. For example Ahl [1] and Massey [58] use consumer panels (diary of purchases) to forecast national demand data collected on existing products to develop dimensions of need and consumer semantics. Urban and Silk [96], and Burger [9], use a simulated store to test new products and new product concepts. Herniter [35], Butlers [10], and Bass [5] use primarily, but not exclusively, market share data. Etc.

Extent of application - Montgomery and Urban [62] categorize a new product planning system as a four-stage process; search, screen, analyze, and implement. Search is the creative generation of new ideas, screening is the narrowing down to a few promising ones. The ideas are then analyzed for their potential and the best set is implemented first in lab simulation or test market, and then in national rollout. Different models attack different phases of this process. For example, Urban's PERCEPTOR [86] and Stefflre's similarities [76] are used primarily to structure and assist in idea generation and to provide an initial screening mechanism. Urban's SPRINTER MOD III [85] and Ahl's forecasting systems [1] are designed to analyze test market results. Finally Little's BRANDAID [48] is used to aid managerial decisions dealing with mature products.

Other models and issues - The previous list of issues is not meant to be
exhaustive nor are all the important models mentioned. For example there are
sales force allocation models of Davis and Farley [15], Lodish [50], and
Montgomery, Silk, and Zaragoza [61], or the market share theorem of Little and
Bell [49], or the semi-markov descriptive model of Herniter [34], or diffusion
of innovation (Rogers and Stanfield [69], Utterbach [88]) and new products (Bass[41]).

A1.3 Mathematical Psychology

Most of the emphasis in mathematical psychology on choice behavior is on
descriptive rather than normative models. For example, much work has been done
on the measurement of attitudes.

In 1927 Thurstone [80] postulated his "law of comparative judgment" which
states that individuals respond to stimuli according to a discriminal process.
In other words, for a set of stimuli there exists some psychological continuum
(scale) and the scale value that an individual perceives for each stimuli is
normally distributed about some mean. An individual then discriminates between
two stimuli (prefers one to another) based on the maximum perceived scale value.
Thurstone then proposed a measurement technique, the method of paired comparisons,
to directly estimate the mean scale values for each stimuli. Many other scaling
techniques have since been studied such as Thurstone's method of equally
appearing intervals and his method of successive intervals, Guttman's Scalegram
analysis, and Coombs unfolding analysis. An excellent summary of these techniques
appears in Green [36].

In addition to one-dimensional scaling techniques much work has been done
on multidimensional scaling techniques. For example, factor analysis
(Rummel [71]) is a statistical technique which obtains a lower dimensional

representation of a data set of large dimensionality. In the context of choice theory this identifies the dimensions which affect choice and describe them as parsimoniously as possible. Other multi-dimensional scaling techniques are the unfolding analyses which develop perceptual maps (metric data) from nonmetric rank order input. These and other multi-dimensional techniques are described in Green and Carmone [28] and Green and Wind [27].

Another approach to the description of individual choice behavior is to begin with a fundamental set of choice axioms and derive models using deductive reasoning. Thurstone's law of comparative judgment is once such axiom and his model of behavior is representative of a class of models which have come to be known as "random utility models." (McFadden [52]) One such random utility model, the logit model, is described in section IV, example 1 of this paper.

Luce [51] begins with a choice axiom which (essentially) states that the probability of choosing some alternative, say a_j from a choice set A is equal to the probability of choosing a_j from $B \subseteq A$ times the probability of choosing B from A. I.e.

$$\text{Prob}\left\{\text{choose } a_j \text{ from A}\right\} = \text{Prob}\left\{\text{choose } a_j \text{ from B}\right\} * \text{Prob}\left\{\text{choose B from A}\right\}$$

This axiom is realistic if the choices, a_j, are distinct independent choices and Luce derives many useful results from his axiom. A generalization of Luce's axiom is the asiom of simple scalability which says that there exists a scale, u(x), which assigns to each alternative a real number and that each choice probability is a monotone function of that scale. (Tversky [83]).

These axioms, although intuitively pleasing, lead to counterintuitive results if an alternative is added to the choice set which is very similar to some but not all of the alternatives already in the choice set. For example,

suppose that given the choice between driving and walking along path 1 you will choose driving 60% of the time. Suppose further that there exists an alternative path, path 2 and that you are indifferent between path 1 and path 2. Luce's axiom then implies that given the choice between driving or walking path 1 vs. walking path 2, you would drive only 43% of the time, $[.43 \approx .6/(.6 + .4 + .4)]$ whereas a rational hierarchical model would imply driving 60% of the time.

Tversky [83] proposes an alternative formulation called "elimination-by-aspects." The motivation behind this model is that individuals first select an aspect (e.g. automatic transmission in the choice of a new car) and eliminate all alternatives that do not satisfy that criteria. This model is closely related to lexicographic rules (Fishburn [22]) but differs because the order in which aspects are selected is random. Tversky then goes on to show that Luce's formulation is a special case of elimination-by-aspects.

A1.4 Other Choice Models

There are many other choice models in the literature, for example "migration in social demography, voting behavior in political science, and mortality in bioassay."[3] A survey of these and other choice models is contained in McFadden [52]. One which is not mentioned in McFadden is Boyle's [8] use of information theoretic pattern recognition algorithms to predict whether an individual will default on a consumer loan.

Finally there is the prescriptive utility theory of decision analysis which is used to aid in decisions rather than predict decisions. See for example Raiffa [67], Keeney, [40.41,42], Ting [81], and Fishburn [20,21,23].

FOOTNOTES

1. Calculated from minimum paths along mathematical networks representing the actual transportation system.

2. Let   X = the set of attributes

    Y,Z = partitions of this set i.e. $X = Y \times Z$

    $$\underline{x}_1, \underline{x}_2 \; \varepsilon \; X \; ; \; \underline{y}_1, \underline{y}_2 \; \varepsilon \; Y; \; \underline{z}^o, \underline{z} \; \varepsilon \; Z$$

    Write

    $$\underline{x}_1 > \underline{x}_2 \quad \text{to mean } \underline{x}_1 \text{ preferred to } \underline{x}_2$$

    Then Y is preferentially independent of Z if for some $\underline{z}^o \; \varepsilon \; Z$,

    $$(\underline{y}_1, \underline{z}^o) > (\underline{y}_2, \underline{z}^o) \quad \text{implies that}$$

    $$(\underline{y}_1, \underline{z}) > (\underline{y}_2, \underline{z}) \quad \text{for all } \underline{z} \; \varepsilon \; Z.$$

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