Online Addendum

Appendix O.1: Equilibria in the Stackelberg Setting

Here we discuss the equilibrium analysis in a Stackelberg setting where the large firm is targeted by the NGO and hence has to determine whether to replace or to defer *before* the small firm. The extensive-form game is characterized in Figure 3.

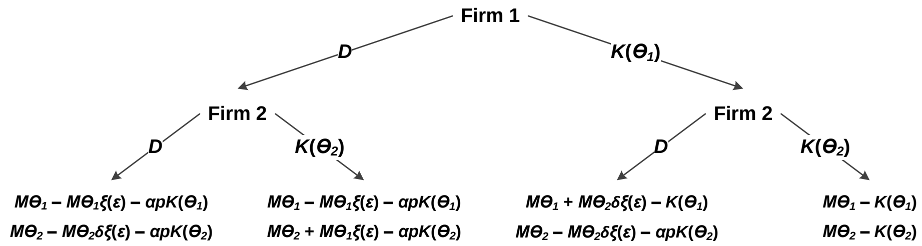

Figure 3 **Dynamics of the Stackelberg Game**

The parameter $\delta \in [0,1]$ captures the potentially asymmetric market loss due to deferring between the large and the small firm. Since the small firm is not targeted by the NGO, it may suffer from a less severe market loss than the large firm if it decides to defer replacement. The two bounds of δ represents two extreme cases: $\delta = 0$ means that the small firm does not incur a market loss at all if it defers; whereas $\delta = 1$ means that the small firm, if it defers, incurs the same level of market loss (measured by the fraction of its original market who switches or leaves the market) as the large firm. We next derive the firms' replacement equilibria under this new setting using backward induction.

First given that firm 1 replaces, firm 2's best response is to replace if and only if $\epsilon \geq [K_2(1-\alpha p) - M\delta\theta_2 q]/[M\delta\theta_2(1-q)]$; otherwise, firm 2's best response is to defer. Similarly, given that firm 1 defers, firm 2's best response is to replace if and only if $\epsilon \geq [K_2(1-\alpha p) - M(\theta_1+\delta\theta_2)q]/[M(\theta_1+\delta\theta_2)(1-q)]$; otherwise, firm 2's best response is to defer. Since $\theta_2 < \theta_1 + \delta\theta_2$, the first inequality above is more stringent than the second one. We consider 3 cases.

Case (i): $\epsilon \geq [K_2(1-\alpha p) - M\delta\theta_2 q]/[M\delta\theta_2(1-q)]$. In this case, firm 2's best response is to replace regardless of firm 1's action. Anticipating this, firm 1's best response is to replace if and only if $\epsilon \geq [K_1(1-\alpha p) - M\theta_1 q]/[M\theta_1(1-q)]$. Since $K_1/\theta_1 < K_2/\theta_2 \leq K_2/(\delta\theta_2)$, the above inequality always holds under the condition of Case (i). Hence, the firm equilibrium in this case is (K_1, K_2).

Case (ii): $[K_2(1-\alpha p) - M(\theta_1+\delta\theta_2)q]/[M(\theta_1+\delta\theta_2)(1-q)] \leq \epsilon < [K_2(1-\alpha p) - M\delta\theta_2 q]/[M\delta\theta_2(1-q)]$. In this case, firm 1's best response is determined by whether its payoff under (K_1, D) is larger or smaller than that under (D, K_2). Comparing these two payoffs, we find that (K_1, D) is the firm equilibrium if and only if $[K_1(1-\alpha p) - M(\theta_1+\delta\theta_2)q]/[M(\theta_1+\delta\theta_2)(1-q)] \leq \epsilon < [K_2(1-\alpha p) - M\delta\theta_2 q]/[M\delta\theta_2(1-q)]$, and (D, K_2) is the equilibrium if and only if $[K_2(1-\alpha p) - M(\theta_1+\delta\theta_2)q]/[M(\theta_1+\delta\theta_2)(1-q)] \leq \epsilon < [K_1(1-\alpha p) - M(\theta_1+\delta\theta_2)q]/[M(\theta_1+\delta\theta_2)(1-q)]$.

Case (iii): $\epsilon < [K_2(1-\alpha p) - M(\theta_1+\delta\theta_2)q]/[M(\theta_1+\delta\theta_2)(1-q)]$. In this case, firm 2's best response is to defer regardless of firm 1's action. Anticipating this, firm 1's best response is to defer if and only if

$\epsilon < [K_1(1-\alpha p) - M(\theta_1+\delta\theta_2)q]/[M(\theta_1+\delta\theta_2)(1-q)]$. This inequality always holds under the condition of Case (iii). Thus, the firm equilibrium is (D, D) in this case.

In summary, the firms' equilibrium replacement strategies given NGO effort ϵ are as follows:

$$(s_1^*(\epsilon), s_2^*(\epsilon)) = \begin{cases} (D,D) & \text{if } \epsilon \in \left[0, \frac{K_2(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)}\right), \\ (D,K_2) & \text{if } \epsilon \in \left[\frac{K_2(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)}, \frac{K_1(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)}\right), \\ (K_1,D) & \text{if } \epsilon \in \left[\frac{K_1(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)}, \frac{K_2(1-\alpha p)-M\theta_2 q\delta}{M\theta_2(1-q)\delta}\right), \\ (K_1,K_2) & \text{if } \epsilon \in \left[\frac{K_2(1-\alpha p)-M\delta\theta_2 q}{M\delta\theta_2(1-q)}, 1\right]. \end{cases} \tag{O.1}$$

We next derive the NGO's optimal effort level in each of the four equilibrium regions.

(D, D): The NGO's payoff is $\pi^S_{D,D} = bp[(1-\xi(\epsilon))\theta_1 + (1-\delta\xi(\epsilon))\theta_2] + \gamma[M(1-\xi(\epsilon))\theta_1 + M(1-\delta\xi(\epsilon))\theta_2 - \alpha p(K_1+K_2)] - c\epsilon^2$, where $\xi(\epsilon) = q + (1-q)\epsilon$ and the superscript S denotes the Stackelberg setting. Note that $\pi^S_{D,D}$ is decreasing in ϵ. Thus, the optimal effort to induce (D, D) is $\epsilon^S_{D,D} = 0$.

(D, K_2): The NGO's payoff is $\pi^S_{D,K_2} = b[p + (1-p)(\theta_2 + \xi(\epsilon)\theta_1)] + \gamma[M - \alpha pK_1 - K_2] - c\epsilon^2$. The first-order condition gives the interior optimal effort as $\epsilon^S_{D,K_2} = [b(1-p)(1-q)\theta_1]/(2c)$. Thus, the optimal effort to induce (D, K_2) is

$$\epsilon^*_{D,K_2} = \begin{cases} \frac{K_2(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)} & \text{if } \frac{b(1-p)(1-q)\theta_1}{2c} < \frac{K_2(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)}, \\ \frac{b(1-p)(1-q)\theta_1}{2c} & \text{if } \frac{K_2(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)} \leq \frac{b(1-p)(1-q)\theta_1}{2c} < \frac{K_1(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)}, \\ \frac{K_1(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)} & \text{if } \frac{b(1-p)(1-q)\theta_1}{2c} \geq \frac{K_1(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)}. \end{cases} \tag{O.2}$$

(K_1, D): The NGO's payoff is $\pi^S_{K_1,D} = b[p + (1-p)(\theta_1 + \delta\xi(\epsilon)\theta_2)] + \gamma(M - K_1 - \alpha pK_2) - c\epsilon^2$. The first-order condition gives the interior optimal effort as $\epsilon^S_{K_1,D} = [b(1-p)(1-q)\delta\theta_2]/(2c)$. Thus, the optimal effort to induce (K_1, D) is

$$\epsilon^*_{K_1,D} = \begin{cases} \frac{K_1(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)} & \text{if } \frac{b(1-p)(1-q)\delta\theta_2}{2c} < \frac{K_1(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)}, \\ \frac{b(1-p)(1-q)\delta\theta_2}{2c} & \text{if } \frac{K_1(1-\alpha p)-M(\theta_1+\delta\theta_2)q}{M(\theta_1+\delta\theta_2)(1-q)} \leq \frac{b(1-p)(1-q)\delta\theta_2}{2c} < \frac{K_2(1-\alpha p)-M\delta\theta_2 q}{M\delta\theta_2(1-q)}, \\ \frac{K_2(1-\alpha p)-M\delta\theta_2 q}{M\delta\theta_2(1-q)} & \text{if } \frac{b(1-p)(1-q)\delta\theta_2}{2c} \geq \frac{K_2(1-\alpha p)-M\delta\theta_2 q}{M\delta\theta_2(1-q)}. \end{cases} \tag{O.3}$$

(K_1, K_2): The NGO's payoff is $\pi^S_{K_1,K_2} = b + \gamma(M - K_1 - K_2) - c\epsilon^2$. Note that $\pi^S_{K_1,K_2}$ is decreasing in ϵ. Thus, the optimal effort to induce (K_1, K_2) is $\epsilon^S_{K_1,K_2} = [K_2(1-\alpha p) - M\delta\theta_2 q]/[M\delta\theta_2(1-q)]$.

Following our earlier notation, we denote the lower-bound solutions for ϵ^*_{D,K_2} and $\epsilon^*_{K_1,D}$ as $\epsilon^{S_B}_{D,K_2}$ and $\epsilon^{S_B}_{K_1,D}$, respectively. Comparing Equations (O.1), (O.2), (O.3) to (5), (6), (7), we obtain Lemma O.1.

Lemma O.1. *(a)* $\epsilon^{S_B}_{D,K_2} \geq \epsilon^{I_B}_{D,K_2}$ *with equality if and only if* $\delta = 1$.

(b) $\epsilon^{S_B}_{K_1,D} \geq \epsilon^{I_B}_{K_1,D}$ *if and only if* $\delta \leq (K_2\theta_1 - K_1\theta_2)/(K_1\theta_1 - K_2\theta_2)$. *Specifically,* $\epsilon^{S_B}_{K_1,D} > \epsilon^{I_B}_{K_1,D}$ *when* $\delta = 0$*, whereas* $\epsilon^{S_B}_{K_1,D} < \epsilon^{I_B}_{K_1,D}$ *when* $\delta = 1$.

(c) When $\delta = 0$*,* (K_1, D) *is always achieved at the lower-bound solution in the Stackelberg game.*

(d) $\epsilon^S_{K_1,K_2} \geq \epsilon^I_{K_1,K_2}$ *with equality if and only if* $\delta = 1$. *When* $\delta \to 0$*,* $\epsilon^S_{K_1,K_2} \to +\infty$.

Appendix O.2: Equilibria When the Replacement Cost Is Decreasing in Market Share

Here we discuss the firm replacement equilibria when the firm replacement cost is decreasing in market share; i.e., when $K(\theta)$ is decreasing in θ. Since the analysis follows the exact same procedure as in Appendix A, we simplify the discussion and only highlight the differences.

Scenario (I): The NGO targets the industry. In this scenario, the conditions on ϵ such that each of the four replacement equilibria is induced are summarized below.

$I(D,D)$	$I(D,K_2)$	$I(K_1,D)$	$I(K_1,K_2)$
$\epsilon \in \left[0, \frac{K_1(1-\alpha p)-Mq}{M(1-q)}\right)$	$\epsilon \in \left[\frac{K_2(1-\alpha p)-Mq}{M(1-q)}, \frac{K_1(1-\alpha p)-M\theta_1 q}{M\theta_1(1-q)}\right)$	$\epsilon \in \left[\frac{K_1(1-\alpha p)-Mq}{M(1-q)}, \frac{K_2(1-\alpha p)-M\theta_2 q}{M\theta_2(1-q)}\right)$	$\epsilon \in \left[\frac{K_2(1-\alpha p)-M\theta_2 q}{M\theta_2(1-q)}, 1\right]$

Since $K(\theta)$ is decreasing in θ, we have $K_2 \geq K_1$ and $K_2/\theta_2 \geq K_1/\theta_1$. Therefore, the range of ϵ inducing $I(D,K_2)$ is contained by that inducing $I(K_1,D)$. That is, we have multiple equilibria in that range. As in Appendix A.1, we follow the refinement concept of risk dominance to resolve this issue. Based on our earlier analysis, we know that $I(K_1,D)$ risk dominates $I(D,K_2)$ if $\epsilon \geq [(K_1\theta_1 - K_2\theta_2)(1-\alpha p) - M(\theta_1-\theta_2)q]/[M(\theta_1-\theta_2)(1-q)]$. However, note that $K_1 \geq (K_1\theta_1 - K_2\theta_2)/(\theta_1-\theta_2)$ since $K_2 \geq K_1$. Thus, $I(K_1,D)$ risk dominates $I(D,K_2)$ for $\epsilon \geq [K_1(1-\alpha p) - Mq]/[M(1-q)]$, where the right hand side is the lower bound of ϵ that induces $I(K_1,D)$. Thus, $I(D,K_2)$ does not occur in equilibrium, and the firm replacement equilibrium can be characterized as follows.

$I(D,D)$	$I(K_1,D)$	$I(K_1,K_2)$
$\epsilon \in \left[0, \frac{K_1(1-\alpha p)-Mq}{M(1-q)}\right)$	$\epsilon \in \left[\frac{K_1(1-\alpha p)-Mq}{M(1-q)}, \frac{K_2(1-\alpha p)-M\theta_2 q}{M\theta_2(1-q)}\right)$	$\epsilon \in \left[\frac{K_2(1-\alpha p)-M\theta_2 q}{M\theta_2(1-q)}, 1\right]$

Scenario (II): The NGO targets the regulatory body. Following a similar approach as above, we obtain the firm replacement equilibrium as follows.

$R(D,D)$	$R(K_1,D)$	$R(K_1,K_2)$
$\epsilon \in \left[0, \frac{K_1(1-\alpha p)-Mq}{\alpha K_1(1-p)}\right)$	$\epsilon \in \left[\frac{K_1(1-\alpha p)-Mq}{\alpha K_1(1-p)}, \frac{K_2(1-\alpha p)-M\theta_2 q}{\alpha K_2(1-p)}\right)$	$\epsilon \in \left[\frac{K_2(1-\alpha p)-M\theta_2 q}{\alpha K_2(1-p)}, 1\right]$

The analysis of the NGO's optimal effort level in each equilibrium region is similar to that in Appendix A and thus is omitted.

O.2.1. Numerical Results

Table O.1 summarizes the NGO's and the firms' equilibrium strategies when $K(\theta)$ is decreasing in θ. We test when $K(\theta)$ is concave (i.e., $K(\theta) = k\sqrt{1-\theta}$) or convex in θ (i.e., $K(\theta) = k(1-\theta)^2$). Note that the placement of the equilibrium regions are consistent with our findings in the base model.

Table O.2 highlights the frequently occurring cases for how the NGO's and the firms' equilibrium strategies change when $K(\theta)$ is decreasing in θ. As before, we test cases when $K(\theta)$ is convex (65,530,255 cases tested; 82.3% of all cases shown) or concave (65,530,072 cases tested; 88.8% of all cases shown) in θ. For both cases, the biggest shift in equilibrium occurs when $I(D,K_2)$ in the base model is replaced

Table O.1 Equilibria When $K(\theta)$ is Decreasing in θ

	K Convex			K Concave		
EQ	% of Cases	θ_1	b	% of Cases	θ_1	b
$R(K_1, K_2)$	25.9%	0.74	434.4	24.0%	0.72	431.3
$I(K_1, K_2)$	7.2%	0.61	424.0	0.8%	0.60	451.3
$I(K_1, D)$	62.4%	0.80	310.7	69.0%	0.77	334.0
$R(K_1, D)$	4.4%	0.85	241.1	3.4%	0.84	247.3
$R(D, D)$	0.1%	0.61	16.5	2.8%	0.69	57.4

Note: Parameter values shown are averages. For both cases, the total number of cases tested was over 62 million.

by $I(K_1, D)$ in the new setting, since $I(D, K_2)$ no longer occurs in equilibrium as shown above. The next largest shift occurs when comparing $I(K_1, D)$ and $R(K_1, K_2)$. When there exists a dominant firm in the market, regardless of the convexity or concavity of $K(\theta)$, we observe that $R(K_1, K_2)$ in the base model can change to $I(K_1, D)$ in the new setting, because it is now more costly and difficult to induce the small firm to proactively replace. Conversely, if $K(\theta)$ is convex in θ, we find that $I(K_1, D)$ in the base model may be replaced by $R(K_1, K_2)$ when the firms are more homogeneous in size. However, if $K(\theta)$ is concave in θ, the replacement cost for the small firm remains substantial even when the market is homogeneous. Hence, the NGO continues to prefer inducing $I(K_1, D)$ as in the base model.

Table O.2 Equilibrium Changes When $K(\theta)$ is Decreasing in θ

EQ	EQ	$K(\theta)$ Convex Dec				$K(\theta)$ Concave Dec			
Base Model	$K(\theta)$ Dec	No. of Cases	% of Cases	θ_1	b	No. of Cases	% of Cases	θ_1	b
$I(K_1, D)$	$I(K_1, D)$	18,207,515	27.8%	0.82	370.1	23,337,410	35.6%	0.78	369.4
$R(K_1, K_2)$	$R(K_1, K_2)$	15,682,000	23.9%	0.72	433.5	13,963,000	21.3%	0.71	431.7
$I(D, K_2)$	$I(K_1, D)$	13,823,352	21.1%	0.77	250.0	14,767,083	22.5%	0.76	248.9
$I(K_1, D)$	$R(K_1, K_2)$	3,308,570	5.0%	0.61	356.8	–	–	–	–
$R(K_1, K_2)$	$I(K_1, D)$	2,948,763	4.5%	0.86	461.5	6,182,400	9.4%	0.78	452.9

Note: Parameter values shown are averages.

Appendix O.3: When The Large Firm Can Lobby

Here we discuss the analytical results when the large firm can lobby either consumers or the regulatory body to counteract the NGO's effort. We solve for the subgame perfect Nash equilibrium $(\epsilon^*, l^*(\epsilon), s_1^*(\epsilon, l), s_2^*(\epsilon, l))$ defined as follows.

Definition 2. *The strategy profile $(\epsilon^*, l^*(\epsilon), s_1^*(\epsilon, l), s_2^*(\epsilon, l))$ constitutes a subgame perfect Nash equilibrium (SPNE) if it satisfies: (i) For all $\epsilon \in [0,1]$, $l \in [0,1]$, and given $s_1^*(\epsilon, l)$, $s_2^*(\epsilon, l) \in \arg\max_{s_2 \in \{K(\theta_2), D\}} \Pi_2(s_1^*(\epsilon, l), s_2)$, where $\Pi_2(\cdot,\cdot)$ is firm 2's payoff function given the firms' replacement strategies in Table 2; (ii) For all $\epsilon \in [0,1]$, $l \in [0,1]$, and given $s_2^*(\epsilon, l)$, $s_1^*(\epsilon, l) \in \arg\max_{s_1 \in \{K(\theta_1), D\}} \{\Pi_1(s_1, s_2^*(\epsilon, l)) - c_L l^2\}$, where $\Pi_1(\cdot,\cdot)$ is firm 1's payoff function given the firms' replacement strategies in Table 2; (iii) For all $\epsilon \in [0,1]$, $l^*(\epsilon) \in \arg\max_{l \in [0,1]} \{\Pi_1(s_1^*(\epsilon, l), s_2^*(\epsilon, l)) - c_L l^2\}$; and (iv) $\epsilon^* \in \arg\max_{\epsilon \in [0,1]} \pi_{NGO}(\epsilon, l^*(\epsilon), s_1^*(\epsilon, l^*(\epsilon)), s_2^*(\epsilon, l^*(\epsilon)))$, where $\pi_{NGO}(\cdot,\cdot,\cdot,\cdot)$ is the NGO's payoff function given its effort level, the resulting lobbying level of the large firm, and the resulting firm replacement strategies.*

In the subsequent analysis, we restrict our attention to scenarios where the NGO targets only one party and the large firm lobbies the same party as the NGO.

We first analyze the firm replacement equilibrium given ϵ and l when the NGO targets the industry. Following the same approach as in Appendix A.1, we characterize the replacement equilibrium with respect to the large firm's lobbying effort l in Table O.3.

Table O.3 NGO Targets Industry: Firm Replacement Equilibria Given the Large Firm's Lobbying Effort

$I(K_1,K_2)$	$I(K_1,D)$	$I(D,K_2)$	$I(D,D)$
if $l \in \left[0, l^{I_A}_{K_1,D}\right]$	if $l \in \left(l^{I_A}_{K_1,D}, l^{I_B}_{K_1,D}\right]$	if $l \in \left(l^{I_B}_{K_1,D}, l^{I_B}_{D,K_2}\right]$	if $l \in \left(l^{I_B}_{D,K_2}, 1\right]$

where

$$l^{I_A}_{K_1,D} \equiv 1 - \frac{K_2(1-\alpha p) - M\theta_2 q}{M\theta_2(1-q)\epsilon}; \tag{O.4}$$

$$l^{I_B}_{K_1,D} \equiv 1 - \frac{(K_1\theta_1 - K_2\theta_2)(1-\alpha p) - M(\theta_1-\theta_2)q}{M(\theta_1-\theta_2)(1-q)\epsilon}; \tag{O.5}$$

$$l^{I_B}_{D,K_2} \equiv 1 - \frac{K_2(1-\alpha p) - Mq}{M(1-q)\epsilon}. \tag{O.6}$$

To analyze the large firm's equilibrium lobbying effort, we discuss four cases corresponding to the four replacement equilibria. In case $I(K_1,K_2)$, the large firm's payoff is $\Pi^L_1(l) = M\theta_1 - K_1 - c_L l^2$, which is decreasing in l. Thus, the optimal lobbying effort in this case is $l^I_{K_1,K_2} = 0$. This result directly implies that the NGO's payoff under (K_1,K_2) is identical in the base model and the lobbying scenario. Hence, the corresponding optimal effort levels are also identical.

In case $I(K_1,D)$, the large firm's payoff is $\Pi^L_1(l) = M[\theta_1 + (q + (1-q)\epsilon(1-l))\theta_2] - K_1 - c_L l^2$, which is again decreasing in l. Hence, the optimal lobbying effort in this case is

$$l^I_{K_1,D} = \begin{cases} l^{I_A}_{K_1,D}, & \text{if } l^{I_A}_{K_1,D} > 0, \\ 0, & \text{if } l^{I_A}_{K_1,D} \le 0. \end{cases} \tag{O.7}$$

This result implies that (K_1,D) is never achieved at the lower-bound solution in the lobbying scenario (Corollary O.1), as summarized in the following result.

Corollary O.1. *When the large firm can lobby, the* (K_1,D) *equilibria are never achieved at the lower-boundary solution. This result particularly implies that the region of potential contention* $I(D,K_2)_A$ *no longer exists when the large firm can lobby.*

In addition, we claim that $l^I_{K_1,D} = l^{I_A}_{K_1,D} > 0$ never occurs in equilibrium. Note that when the firm replacement equilibrium is (K_1,D), the NGO's payoff is equal to $\pi^L_{K_1,D} = b[p + (1-p)(\theta_1 + (q + (1-q)\epsilon(1-l))\theta_2)] + \gamma(M - K_1 - c_L l^2 - \alpha p K_2) - c\epsilon^2$. If the large firm's lobbying effort were positive, then Equation (O.4) implies that the first component in $\pi^L_{K_1,D}$ is a constant. Thus, the NGO's payoff is decreasing in both l and ϵ. Also note from Equation (O.4) that when $l^{I_A}_{K_1,D} > 0$, $l^{I_A}_{K_1,D}$ is increasing in ϵ. Hence, the NGO's best response to the large firm exerting a lobbying effort of $l^{I_A}_{K_1,D}$ is to reduce ϵ such that $l^{I_A}_{K_1,D} \le 0$; i.e., $l^{I_A}_{K_1,D} > 0$ never occurs in equilibrium, proving our claim.

In case $I(D,K_2)$, the large firm's payoff is $\Pi^L_1(l) = M\theta_1(1-q)[1-(1-l)\epsilon] - \alpha p K_1 - c_L l^2$. Note that $\mathrm{d}\Pi^L_1/\mathrm{d}l = M\theta_1\epsilon - 2c_L l$ and $\mathrm{d}^2\Pi^L_1/\mathrm{d}l^2 = -2c_L < 0$. Thus, Π^L_1 is concave in l, and the optimal lobbying effort in this case is either at the boundary or at the interior solution to the first-order condition. Thus

$$l^{I}_{D,K_2} = \begin{cases} l^{I_B}_{K_1,D}, & \text{if } \frac{M\theta_1(1-q)\epsilon}{2c_L} \leq l^{I_B}_{K_1,D}, \\ \frac{M\theta_1(1-q)\epsilon}{2c_L}, & \text{if } \frac{M\theta_1(1-q)\epsilon}{2c_L} \in \left(l^{I_B}_{K_1,D}, l^{I_B}_{D,K_2}\right], \\ l^{I_B}_{D,K_2}, & \text{if } \frac{M\theta_1(1-q)\epsilon}{2c_L} > l^{I_B}_{D,K_2}. \end{cases} \tag{O.8}$$

Finally in case $I(D,D)$, the large firm's payoff is the same as in case $I(D,K_2)$. Thus

$$l^{I}_{D,D} = \begin{cases} l^{I_B}_{D,K_2}, & \text{if } \frac{M\theta_1(1-q)\epsilon}{2c_L} \leq l^{I_B}_{D,K_2}, \\ \frac{M\theta_1(1-q)\epsilon}{2c_L}, & \text{if } \frac{M\theta_1(1-q)\epsilon}{2c_L} \in \left(l^{I_B}_{D,K_2}, 1\right], \\ 1, & \text{if } \frac{M\theta_1(1-q)\epsilon}{2c_L} > 1. \end{cases} \tag{O.9}$$

Comparing Equations (O.8) and (O.9), and noting that the large firm's payoff function is concave and identical in $I(D,K_2)$ and $I(D,D)$, we find that the second case in Equation (O.8) always dominates the first case in Equation (O.9). Similarly, the second case in Equation (O.9) always dominates the third case in Equation (O.8). Given the above analysis, the large firm then has to compare its payoff under the four cases and determines its optimal lobbying effort. Finally, anticipating the large firm's lobbying effort in response to the NGO's effort, the NGO finds the optimal effort level with a similar approach as found in Appendix A.1. We omit the details here.

We next repeat the same analysis for the case where the NGO targets the regulatory body. Table O.4 shows the firm replacement equilibria given the large firm's lobbying effort.

Table O.4 NGO Targets Regulation: Firm Replacement Equilibria Given the Large Firm's Lobbying Effort

$R(K_1,K_2)$	$R(K_1,D)$	$R(D,K_2)$	$R(D,D)$
if $l \in \left[0, l^{R_A}_{K_1,D}\right]$	if $l \in \left(l^{R_A}_{K_1,D}, l^{R_B}_{K_1,D}\right]$	if $l \in \left(l^{R_B}_{K_1,D}, l^{R_B}_{D,K_2}\right]$	if $l \in \left(l^{R_B}_{D,K_2}, 1\right]$

where

$$\begin{aligned} l^{R_A}_{K_1,D} &\equiv 1 - \frac{K_2(1-\alpha p) - M\theta_2 q}{\alpha K_2(1-p)\epsilon}; \\ l^{R_B}_{K_1,D} &\equiv 1 - \frac{(K_1\theta_1 - K_2\theta_2)(1-\alpha p) - M(\theta_1-\theta_2)q}{\alpha(K_1\theta_1 - K_2\theta_2)(1-p)\epsilon}; \\ l^{R_B}_{D,K_2} &\equiv 1 - \frac{K_2(1-\alpha p) - Mq}{\alpha K_2(1-p)\epsilon}. \end{aligned}$$

We derive the large firm's equilibrium lobbying effort for the four replacement equilibria as follows.

(i) Case $R(K_1,K_2)$: $l^{R}_{K_1,K_2} = 0$.

(ii) Case $R(K_1,D)$:

$$l^{R}_{K_1,D} = \begin{cases} l^{R_A}_{K_1,D}, & \text{if } l^{R_A}_{K_1,D} > 0, \\ 0, & \text{if } l^{R_A}_{K_1,D} \leq 0. \end{cases} \tag{O.10}$$

(iii) Case $R(D,K_2)$:

$$l^{R}_{D,K_2} = \begin{cases} l^{R_B}_{K_1,D}, & \text{if } \frac{\alpha K_1(1-p)\epsilon}{2c_L} \leq l^{R_B}_{K_1,D}, \\ \frac{\alpha K_1(1-p)\epsilon}{2c_L}, & \text{if } \frac{\alpha K_1(1-p)\epsilon}{2c_L} \in \left(l^{R_B}_{K_1,D}, l^{R_B}_{D,K_2}\right], \\ l^{R_B}_{D,K_2}, & \text{if } \frac{\alpha K_1(1-p)\epsilon}{2c_L} > l^{R_B}_{D,K_2}. \end{cases} \tag{O.11}$$

(iv) Case $R(D,D)$:

$$l^R_{D,D} = \begin{cases} l^{R_B}_{D,K_2}, & \text{if } \frac{\alpha K_1(1-p)\epsilon}{2c_L} \le l^{R_B}_{D,K_2}, \\ \frac{\alpha K_1(1-p)\epsilon}{2c_L}, & \text{if } \frac{\alpha K_1(1-p)\epsilon}{2c_L} \in \left(l^{R_B}_{D,K_2}, 1\right], \\ 1, & \text{if } \frac{\alpha K_1(1-p)\epsilon}{2c_L} > 1. \end{cases} \tag{O.12}$$

As before, we remark that the NGO's optimal effort to induce (K_1, K_2) in the lobbying scenario is identical to that in the base model. Thus, we obtain the following result.

COROLLARY O.2. *When the large firm can lobby, the NGO exerts the same level of effort as in the base model to induce both firms to replace in equilibrium; i.e., (K_1, K_2).*

One can also show that $l^R_{K_1,D} = l^{R_A}_{K_1,D} > 0$ never occurs in equilibrium. Also, the second case in Equation (O.11) always dominates the first case in Equation (O.12), and the second case in Equation (O.12) always dominates the third case in Equation (O.11). As before, we omit the detailed algebra for calculating the large firm's equilibrium lobbying effort and the NGO's equilibrium effort. Instead, we show the following result regarding the NGO's optimal effort to induce (K_1, D) under the lobbying scenario as compared to that in the base model.

PROPOSITION O.1. *When the large firm can lobby, the NGO exerts equal or lower effort to achieve the equilibrium where only the large firm replaces. In particular, in parameter regions where the firm replacement equilibrium is $(K_1, D)_B$ in the base model and remains (K_1, D) in the lobbying model, the NGO's optimal effort is strictly lower.*

Proof: We prove this result for the case of the NGO targeting the industry; the proof for the targeting the regulation case is similar and thus omitted. We first characterize the NGO's optimal effort in the lobbying model when the resulting firm replacement equilibrium is (K_1, D). We consider two cases.

Case (a): $l^{I_A}_{K_1,D} < 0$. This condition is equivalent to $\epsilon < \epsilon^I_{K_1,K_2}$, where $\epsilon^I_{K_1,K_2}$ is the NGO's optimal effort in the $I(K_1, K_2)$ equilibrium as defined in Appendix A.1. In this case, the large firm does not lobby, so the NGO's payoff remains the same as in the base model. By the analysis in Appendix A.1, we know that the NGO's optimal effort in this case can take one of two values: (i) the interior solution $\epsilon^I_{K_1,D}$ (defined in Appendix A.1) when $\epsilon^I_{K_1,D} < \epsilon^I_{K_1,K_2}$, and (ii) the upper-bound solution $\epsilon^I_{K_1,K_2}$ when $\epsilon^I_{K_1,D} \ge \epsilon^I_{K_1,K_2}$. In addition, given the concavity of the NGO's payoff function, we know that in case (i) the NGO's payoff at the optimal effort level is strictly higher than that at the boundary $\epsilon^I_{K_1,K_2}$.

Case (b): $l^{I_A}_{K_1,D} \ge 0$. As shown in Appendix O.3, in this case, it is the NGO's best response to exert an effort such that $l^{I_A}_{K_1,D} = 0$; i.e., $\epsilon = \epsilon^I_{K_1,K_2}$.

Combining both cases, the NGO's optimal effort under equilibrium $I(K_1, D)$ is as follows:

$$\epsilon^{I_L}_{K_1,D} = \begin{cases} \frac{b(1-p)(1-q)\theta_2}{2c}, & \text{if } \frac{b(1-p)(1-q)\theta_2}{2c} < \frac{K_2(1-\alpha p) - M\theta_2 q}{M\theta_2(1-q)}, \\ \frac{K_2(1-\alpha p) - M\theta_2 q}{M\theta_2(1-q)}, & \text{otherwise.} \end{cases}$$

Comparing this solution with Equation (7), we observe that $\epsilon^{I_L}_{K_1,D} \le \epsilon^I_{K_1,D}$, and the inequality is strict when $\epsilon^I_{K_1,D}$ takes the lower-boundary solution. This completes the proof. ■

O.3.1. Proof of Proposition 4

Finally, we prove our main theoretical result for the lobbying scenario, i.e., Proposition 4. First, (a) follows directly from the earlier analysis for cases $I(K_1, K_2)$ and $R(K_1, K_2)$. To show (b), we first show that $R(K_1, D)_A$ is dominated by $R(K_1, K_2)$ as in Lemma A.3(c). Note that in case $R(K_1, K_2)$, the large firm does not lobby. Hence, the NGO's payoff function is exactly the same as in the base model. When $R(K_1, D)$ is achieved at the upper-boundary solution, by Table O.4 and Equation (O.10) we have $(1 - l_{K_1}^{R_A})\epsilon = \epsilon_{K_1,K_2}^R$ defined in Appendix A.2. Thus, the NGO's payoff function is equal to that in the base model minus a positive lobbying cost incurred by the large firm. Therefore, following the proof of Lemma A.3(c), we obtain $R(K_1, D)_A$ is dominated. Now note from Equations (O.7) and (O.10) that the large firm exerting positive lobbying effort is equivalent to $I(K_1, D)$ or $R(K_1, D)$ being achieved at the upper-boundary solution. Since the latter is dominated, the large firm lobbies if and only if (K_1, D) is achieved within the region of potential contention $I(K_1, D)_A$, proving (b).

We next prove (c). First observe from Tables O.3 and O.4 that when $\epsilon \to 0$, all listed constants become negative. Thus, when the NGO does not exert effort, the only possible equilibrium is for both firms to defer. The structure of l_{D,K_2}^I and l_{D,K_2}^R (Equations (O.8) and (O.11)) then implies that both of these values must be positive; i.e., the large firm always lobbies. The proof of (d) follows the same argument. ■

O.3.2. Numerical Results

To test how the NGO's and the firms' strategies change when the large firm can lobby, we compare our results in §4 with the case in which the large firm can lobby either consumers or the regulatory body to offset the NGO's activism. Due to computational constraints, we use the smaller parameter set stated at the start of Appendix C.2. We define c_L as the large firm's lobbying cost factor with $c_L \in \{25, 50, 75, 100\}$. Table O.5 shows the most frequently occurring equilibrium changes when comparing the lobbying scenario to the base model. For 88.5% of all cases tested, the NGO's optimal strategy and the firm replacement equilibrium do not change. The biggest changes occur within the $I(D, K_2)$ region when either the NGO substantially lowers its effort level or the large firm is forced to replace due to an increased NGO effort. Of the 1,998,348 $I(K_1, D)_A$ cases in our base model, we find that 1,483,700 (74.2%) change to $I(K_1, D)$ and 487,750 (24.4%) remain $I(K_1, D)_A$ under the lobbying scenario.

Table O.5 Equilibrium Comparison Between Base Model and Lobbying Model

EQ (Base)	EQ (Lobbying)	Cases	ϵ (Base)	ϵ (Lobbying)	Difference	l
$I(D, K_2)$	$I(D, K_2)$	8,468,827	0.28	0.10	**-0.18**	0.11
$\boldsymbol{I(D, K_2)}$	$\boldsymbol{I(K_1, D)}$	**3,914,382**	0.54	0.56	0.02	0.00
$I(K_1, D)$	$I(K_1, D)$	20,007,665	0.27	0.27	0.00	0.00
$R(K_1, K_2)$	$R(K_1, K_2)$	16,210,000	0.22	0.23	0.01	0.00
$R(D, D)$	$I(D, D)$	1,479,900	0.00	0.01	0.01	0.03

Note: Parameter values shown are averages. The total number of cases tested was 54,554,554; 50,080,774 cases are shown.