The Strategic Implications of Scale in Choice-Based Conjoint Analysis

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Abstract

Choice-based conjoint (CBC) studies have begun to rely on simulators to forecast equilibrium prices for patent/copyright valuations and for strategic product positioning. While CBC research has long focused on the accuracy of estimated relative tradeoffs among attribute levels, predicted equilibrium prices and strategic positioning are surprisingly and dramatically dependent on the magnitude of the partworths relative to the magnitude of the error term (scale). Although the impact of scale on the ability to estimate heterogeneous partworths is well-known, neither the literature nor current practice has addressed the sensitivity of pricing and positioning to scale. This sensitivity is important because (estimated) scale depends on seemingly innocuous market-research decisions such as whether attributes are described by text or by pictures. We demonstrate the strategic implications of scale using a formal model in which heterogeneity is modeled explicitly. If a firm shirks on the quality of a CBC study and acts on incorrectly observed scale, a follower, but not an innovator, can make costly strategic errors. We demonstrate empirically that image quality and incentive alignment affect scale sufficiently to change strategic decisions and can affect patent/copyright valuations by hundreds of millions of dollars.

Keywords: Conjoint analysis, market research, choice models
1. Scale Affects Patent/Copyright Valuations and Strategic Decisions

With an estimated 18,000 applications per year, conjoint analysis is one of the most-used quantitative market research methods (Orme 2014; Sawtooth Software 2015). Over 80% of these conjoint applications are choice-based. Firms routinely use choice-based conjoint analysis (CBC) to identify preferred product attributes in the hopes of maximizing profit—for example, General Motors alone spends 10s of millions of dollars each year (Urban and Hauser 2004). CBC is increasingly used in litigation and courts have awarded billion-dollar judgments for patent or copyright infringement based on CBC studies (Cameron, Cragg, and McFadden 2013; McFadden 2014; Mintz 2012).

Research in CBC has long focused on the ability to estimate accurate tradeoffs among product attribute levels. Improved question selection, improved estimation, and techniques such as incentive alignment all enhance accuracy and lead to better managerial decisions. However, with the advancement of CBC simulators and faster computers, researchers, especially in litigation, have begun to use CBC studies to estimate price equilibria and the resulting equilibrium profits (e.g., Allenby et al. 2014). This use of CBC raises a new concern because, as shown in this paper, the calculated price equilibria depend critically on the relative error term, i.e., the magnitude of the partworths relative to the magnitude of the error—called “scale” in the CBC literature. (We define scale as inversely proportional to the standard deviation of the error as in Louviere [1993], recognizing that some authors define scale as proportional to the standard deviation of the error as in Train [2009, p. 40]).

The implications are significant. We show that patent/copyright valuations can differ by hundreds of millions of dollars depending upon scale. We also show that scale affects strategic positioning decisions even when holding heterogeneity and unobserved attributes constant—a different phenomenon than that commonly analyzed in the strategic positioning literature. These implications of scale are important to practice because (1) patent/copyright valuations and strategic decisions are based on the scale observed in a market-research study, (2) observed scale depends upon market research decisions
such as the realism of images or incentive alignment, and (3) observed scale can be adjusted, although relatively rarely done so in practice, to reflect how consumers will actually behave in the marketplace (vs. holdout choices or the choice-experimental setting).

We combine formal analysis with empirical research. By varying incentive alignment and the realism of the stimuli, we demonstrate that the quality of market research affects the parameters of the formal model sufficiently to have a large impact. With a subsequent market-validation study, we also show that patent/copyright valuations can be substantially different if the court bases valuations on observed (“internal”) scale versus (“external”) scale as adjusted to market validation. Furthermore, firms may make different strategic decisions depending upon the quality of the CBC study and whether or not scale is adjusted to the market.

It is obvious that, if the scale parameter upon which the court or firm bases its decisions is different from the scale parameter upon which market-based equilibrium price is based, then a court or firm might make errors in its decisions. However, neither the magnitude and direction of the errors, nor the large effect of seemingly minor market-research craft such as realistic images and incentive alignment are obvious. We reviewed the conjoint-analysis papers in Marketing Science from the last 15 years (2003-2017). Forty-six (46) papers addressed new estimation methods, new adaptive questioning methods, methods to motivate respondents, more efficient designs, non-compensatory methods, and other improvements. Mostly, papers focused on the improved estimation of relative partworths or managerial interpretations. Five of the papers address scale (or a related concept for non-CBC papers), and of those five, three focus on more accurate estimation, one on brand credibility, and one on peer influence. None discuss the strategic (price or positioning) implications of scale.

Most papers do not report whether stimuli are text-only, pictorial, or animated, but of those that do, the vast majority are text-only. Interest in incentive alignment is growing. No papers discuss the impact of either stimuli presentation or incentive alignment on scale observed for the estimation data.
2. Deriving Implications from CBC Studies

We begin with a short review of current CBC practice. We then describe the implications when CBC studies are used to calculate price equilibria.

2.1. Typical Current Practice

In CBC, products (or services) are summarized by a set of levels of the attributes. For example, a smartwatch might have a watch face (attribute) that is either round or rectangular (levels), be silver or gold colored, and have a black or brown leather band. By varying the smartwatch attribute levels systematically within an experimental design, CBC estimates preferences for attribute levels, called “part-worths,” which describe the differential value of the attribute levels. For example, one partworth might represent the differential value of a rectangular watch face relative to a round watch face.

Applied practice focuses on estimating accurately the relative partworths. For example, if rectangular and round watch faces are equally costly, but the partworth of a rectangular watch face is greater than the partworth of a round watch face for most consumers, then a typical recommendation would be to launch a product with a rectangular watch face. The relative partworths can also be used to calculate willingness-to-pay by comparing them to the CBC price coefficient. For example, if a consumer’s differential value between a rectangular and a round watch face is higher than a $100 reduction in the purchase price, firms typically infer that the consumer is willing-to-pay more than $100 for a rectangular rather than a round watch face. (There are subtleties in this calculation due to the Bayesian nature of most estimates, but this is the basic concept.)

These implications depend only on the (distribution of) relative partworths, not scale. If we double all partworths, including the price coefficient, for every respondent, the relative partworths are unchanged; product development practice would still recommend a rectangular watch face, and any patent/copyright valuation relying on willingness-to-pay would be unchanged. Nonetheless, the accuracy with which the relative partworths can be estimated (and their interpretation) depends upon ac-
counting for heterogeneity in scale during estimation (e.g., Beck, Rose, and Hensher 1993; Fiebig, et al. 2010; Pancreas, Wang, and Dey 2016; Swait and Louviere 1993). Scale heterogeneity affects partworth estimation and comparisons and/or aggregation of respondents’ willingness to pay (WTP), but, once those are accounted for, WTP does not depend upon a common (across respondent) increase or decrease in scale (e.g., Ofek and Srinivasan 2002, Eq. 15). The phenomenon we investigate is different from scale heterogeneity; we focus on the strategic implications of scale assuming accurate relative partworths and assuming the estimation accounts for scale heterogeneity.

In practice, there are two methods to adjust scale in CBC simulations. One method adjusts scale and partworths to match market shares and uses the adjusted scale and partworths in simulations (e.g., Gilbride, Lenk, and Brazell 2008). The adjustments are motivated by predictive ability rather than strategic implications. The second method adjusts scale directly or in a procedure known as randomized first choice (RFC) in which an additive error is included in the simulations. RFC automatically determines the random perturbations to yield "approximately the same scale factor as the [logit] model" (Huber, Orme, Miller 1999). Scale adjustments are easy to implement in the dominant simulation packages, but usage is reported as rare—users almost always stick with the scale observed in the CBC estimation (Orme 2017). While some use the estimation scale by default, many users report that market data, as a benchmark to adjust scale, are not available or not relevant to the simulated markets. Our formal model and empirical illustration suggest that adjustments are critical and should be used much more often than they are currently. We also provide an alternative adjustment that does not require market data.

2.2. Current Practice is Changing: The Implications of Price Equilibria

One method to calculate damages in patent/copyright cases is based on the additional profits obtained due to the infringing level of an attribute. Because CBC studies have influenced billion-dollar judgments (Mintz 2012), the state-of-the-art in CBC simulators is advancing rapidly. Due to the influence of game theory in marketing science, CBC simulators are beginning to consider competitive response.
For example, if an innovator introduces a rectangular watch face and a follower responds with a round watch face (and all other attributes are held constant), then CBC simulations can be used to calculate the Nash price equilibrium. Based on equilibrium prices, simulators can calculate the follower’s most-profitable response (rectangular vs. round) to the innovator’s new product. Allenby, et al. (2014) propose that damages be calculated as the equilibrium profits obtained with the patent/copyright infringement minus the equilibrium profits that would have been obtained without the infringement. Courts recognized the issue as early as 2005 for class-action cases (e.g., Whyte 2005, albeit not CBC) and the use of CBC studies to calculate market-price equilibria are now debated in many high-profile patent/copyright case (and in many other types of litigation cases).

Although Sawtooth Software estimates 80% of managerial CBC applications consider competition in market simulations, less than 5% of CBC applications now use “game theory stuff” (Orme 2017). Considering competitive reactions can lead to different implications than static simulations. We show that scale plays a central role when calculating price equilibria and predicting optimal competitive reactions.

2.3. Interpretation and Implications of the Error Term

The error term in CBC has many interpretations and implications. It has been interpreted as inherent stochasticity in consumer choice behavior and/or sources that are unobservable to the researcher, such as unobserved heterogeneity, unobserved attributes, functional misspecifications, or consumer stochasticity that is introduced by the CBC experiment (e.g., due to fatigue; e.g., Manski 1977; Thurstone 1927). We are most interested in what happens to the (observed) relative magnitude of the error term, and consequently scale, when the quality of the CBC study changes, say by the addition of more-realistic images or incentive alignment. To address this issue, we assume that the firm acts strategically on a CBC study anticipating the implied price equilibria. However, after the firm selects its positioning strategy (say a silver vs. gold smartwatch) and launches its product, the prices are set by market forces. Because
actual sales depend on how consumers react to the product’s attributes and price in the marketplace, we need the concept of a “true” scale that represents how the market reacts. We purposefully do not define “true” scale as a philosophical construct—it is defined as the scale that best represents how consumers actually react in the marketplace. Practically, we expect the “true” scale to be finite because of inherent stochasticity (e.g., Bass 1974), but the theory allows “true” scale to be finite or to approach infinity.

Our formal model differs from prior models in the positioning literature because we explicitly model heterogeneity in all partworths that are relevant to positioning decisions. In other words, in our formal model we assume that the error term (in estimation) captures imperfections in the CBC study (if any) plus any residual uncertainty. We explicitly assume the firms do not act on attributes that are unobserved or on heterogeneity in consumer preferences that will be revealed when consumers purchase the products. For “true” scale, the error term is limited to residual uncertainty (if any). With explicit heterogeneity we seek to rule out explanations that the firms act strategically upon heterogeneity as in de Palma, et al. (1985, p. 780) who state: “the world is pervasively heterogeneous, and we have made it clear how, in a particular model, this restores smoothness [that leads to differentiation].” Explicit heterogeneity greatly complicates the proofs, but, we hope, provides new practical insight for the use of CBC studies to calculate and act on price equilibria.

2.4. Research Philosophy

We use complementary formal models and empirical CBC studies to explore the impact of scale. Each helps the other. The formal models illustrate that the observed empirical effects are based on more-general concepts. The empirical studies show that the parameters of the formal model can be estimated and that the predicted phenomena have real-world implications. Empirically, we vary two aspects of the quality of CBC studies—realism of images and incentive alignment. Each of the aspects is important by itself, but the two aspects are chosen to be illustrative of the larger issue. The method we
use to adjust scale illustrates that adjustments affect strategic decisions. We do not claim these are the only feasible adjustments, although they are practical even when market data is unavailable.

3. Related Literature

3.1. Asymmetric Competition: Minimum versus Maximum Differentiation

The study of minimum versus maximum differentiation has a rich history in both economics and marketing. Hotelling (1929) proposed a model of minimum differentiation in which consumers are uniformly distributed along a line and two firms compete by first choosing a position (attribute level) and then a price. After demonstrating that the price equilibrium did not exist in Hotelling’s model, d’Aspremont, Gabszewicz, and Thisse (1979) proposed quadratic transport costs and obtained an equilibrium of maximum differentiation—firms choose strategic positions at opposite ends of the line. de Palma, et al. (1985) and Rhee, et al. (1992) restored minimum differentiation to Hotelling’s line with uncertainty about heterogeneity in preferences (partworth heterogeneity in our model). In their model, each consumer’s utility depends on the transport cost from the firm to the consumer’s position on the line and on an error-like term reflecting the firm’s uncertainty about the heterogeneity in consumer’s tastes (and other attributes the consumer may value). Aggregate demand is given by a logit-like function. When the unknown heterogeneity is sufficiently large, the firm’s predictions are imprecise leading to reduced price competition and minimum differentiation. We note that their “error” term captures unobserved heterogeneity in consumer preferences rather than any stochasticity that remains after heterogeneity and missing attributes are fully modeled. We extend their model to model heterogeneity explicitly in order to study the practical implications of market research quality. In our formal model, we are careful to rule out heterogeneity and strategic firm behavior with respect to unobserved attributes.

Many other researchers explore Hotelling-like models to derive conditions when differentiation is likely and when it is not (e.g., Eaton and Lipsey 1975; Eaton and Wooders 1985; Economides 1984; Graitson 1982; Johnson and Myatt 2006; Novshek 1980; Sajeesh and Raju 2010; Shaked and Sutton
In these formal models, differentiation is driven by the heterogeneity of consumer preferences—something we hold constant.

In marketing, Thomadsen (2007) shows how asymmetries in attribute levels lead one firm to favor maximum differentiation in physical location while another favors minimum differentiation. Gal-or and Dukes (2003) show that a two-sided market (commercial media serving consumers and advertisers) reverses the differentiation found in d’Aspremont, Gabszewicz, and Thissé (1979). Guo (2006) extends attribute-based analysis to forward looking consumers who observe one of two product attributes. Consumers anticipate probabilistically future valuations for the other product attribute. In these models heterogeneity in preferences (partworths) drives strategic behavior with respect to prices and profits.

In the language of CBC, all of these papers focus on the distribution of relative partworths or on the partworths of unobserved or uncertain attributes. Although many models include error terms, none analyze the effect of imperfect market research or inherent stochasticity. We show that these phenomena alone can drive firm’s decisions on differentiation. The scale-based strategic analysis is a means to an end. It gives us the machinery to understand the impact of the quality of CBC studies on forecast equilibrium prices and on strategic decisions.

3.2. Price Equilibria in Heterogeneous Logit Models (as in CBC)

When there is no heterogeneity in partworths, Choi, DeSarbo and Harker (1990) demonstrate that price equilibria exist if consumers are not overly price-sensitive. Their condition (p. 179) suggests that price equilibria are more likely to exist if there is greater uncertainty in consumer preferences—a result consistent with our model which, in addition, accounts for heterogeneity. Choi and DeSarbo (1994) use similar concepts to solve a positioning problem with exhaustive enumeration. Luo, Kannan and Ratchford (2007) extend the analysis to include heterogeneous partworths and equilibria at the retail level. They use numeric methods to find Stackelberg equilibria if and when they exist.

State-of-the-art applications of CBC explicitly model consumer heterogeneity. Hierarchical Bayes
(HB) methods are most common, but latent structure, empirical Bayes, machine learning, and polyhedral methods are also used (Andrews, Ansari, and Currim 2002; Evgeniou, Pontil, and Toubia 2007; Rossi and Allenby 2003; Toubia, Hauser, and Garcia 2007). If we are to examine price equilibria using CBC, we need to know that price equilibria exist and are unique. Unfortunately, Aksoy-Pierson, Allon, and Federgruen (APAF, 2013) warn that price equilibria in heterogeneous logit models may not exist. APAF generalize the analyses of Caplin and Nalebuff (1991) to establish sufficient conditions for price equilibria to exist, to be unique, and to be given by the first-order conditions. The APAF conditions apply to typical HB CBC studies (APAF, §6). Because of this warning, we address whether the equilibria exist and are unique in our formal model and we check existence in our empirical analyses.

3.3. Improvements in CBC Methods are Common in the Literature

Many of the forty-six Marketing Science articles that we reviewed (see §1) focused on improved estimation with a common focus on the accuracy with which partworths are estimated. Sawtooth Software (2016) reports that many of these methods have been adopted by industry. Our paper explores why the accurate estimation of scale matters for patent/copyright valuations and for managerial decisions and, hence, provides an additional dimension and justification for improved methods. Our paper on price/profit equilibrium considerations complements the academic and industry focus on the appropriate modeling of scale to improve relative partworth estimation and accurate predictive ability.

4. General Formulation and Basic Notation

We begin with the more-general model of consumer preference that we use for the empirical studies in §9. Appendix 1 summarizes notation for both the general and stylized formal models. We focus on a single attribute with two levels. Other attribute levels are common among products in the market and do not affect strategic decisions. This focus is sufficient to illustrate the impact of scale and is consistent with Irmen and Thisse (1998, p. 78), who analyze markets in which firms compete on multiple dimensions and conclude that “differentiation in a single dimension is sufficient to relax price com-
petition and to permit firms to enjoy the advantages of a central location in all other characteristics.”

Our analysis also applies to simultaneous differentiation of a composite of multiple dimensions, say a silver smartwatch with a rectangular face and a black leather band vs. a gold smartwatch with a round watch face and a metal band.

To match typical applications of CBC, we focus on discrete (horizontal) levels of an attribute that we label \( r \) and \( s \). A product can have either \( r \) or \( s \), but not both. If mnemonics help, think of \( r \) as round, regular, routine, ruby, or rust-colored and \( s \) as square, small, special, sapphire, or scarlet. We focus on two firms, each of which sells one product. We allow an “outside option” to capture other firms and products that are exogenous to the strategic decisions of the two-firm duopoly.

Let \( u_{ij} \) be consumer \( i \)’s utility for Firm \( j \)’s product, let \( u_{i0} \) be \( i \)’s utility for the outside option, and let \( p_j \) be product \( j \)’s price. Let \( \beta_{ri} \) and \( \beta_{si} \) be \( i \)’s partworths for attribute levels \( r \) and \( s \), respectively, and let \( \delta_{rj} \) and \( \delta_{sj} \) be indicator functions for whether or not Firm \( j \)’s product has \( r \) or \( s \), respectively. Let \( \eta_i \) indicate \( i \)’s preference for price, let \( \epsilon_{ij} \) be an extreme value error term with variance \( \pi^2 / 6 \gamma_i^2 \). If the error terms are independent and identically distributed, we have the standard logit model for the probability, \( P_{ij} \), that consumer \( i \) purchases Firm \( j \)’s product (relative to Firm \( k \)’s product and the outside option):

\[
u_{ij} = \beta_{ri} \delta_{rj} + \beta_{si} \delta_{sj} - \eta_i p_j + \epsilon_{ij}
\]

\[
P_{ij} = \frac{e^{\gamma_i (\beta_{ri} \delta_{rj} + \beta_{si} \delta_{sj} - \eta_i p_j)}}{e^{\gamma_i (\beta_{ri} \delta_{rj} + \beta_{si} \delta_{sj} - \eta_i p_j)} + e^{\gamma_i (\beta_{rk} \delta_{rk} + \beta_{sk} \delta_{sk} - \eta_k p_k)} + e^{\gamma_i u_{i0}}}
\]

To identify the model in estimation, we must set \( \beta_{ri}, \beta_{si}, \eta_i \), or \( \gamma_i \). For the remainder of the theory development, we follow McFadden (2014), Sonnier, Ainslie, and Otter (2007), and Train (2003, p. 44) and parameterize the model so that the coefficient of price relative to \( r \) or \( s \) is unity. In particular, we parameterize the logit model such that \( \eta_i = 1 \). In this parameterization, the \( \beta \)’s are now relative partworths and \( \gamma_i \) is scale. (We obtain the same theoretical and empirical implications with any parame-
terization in which scale is inversely proportional to the magnitude of the error term.) We focus on scale in the theory development recognizing that, empirically, the quality of CBC studies can also affect the accuracy with which the relative partworths are estimated. See §9.9. With our parameterization, the CBC logit model for the formal analysis becomes:

\[ P_{ij} = \frac{e^{\gamma l(\beta_{rijk} + \beta_{sik} \delta_{ik} - p_{ik})}}{e^{\gamma l(\beta_{rijk} + \beta_{sik} \delta_{ik} - p_{ik})} + e^{\gamma l(\beta_{rijk} + \beta_{sik} \delta_{ik} - p_{ik})} + e^{\gamma l(\beta_{rijk} + \beta_{sik} \delta_{ik} - p_{ik})}} \]

If \( V \) is the market volume (including volume due to the outside option), \( c_j \) is the marginal cost for product \( j \), \( C_j \) is Firm \( j \)'s fixed cost, and \( f(\beta_{ri}, \beta_{si}, y_i) \) is the probability distribution over the relative partworths and scale (posterior if Bayesian), then the profit, \( \pi_j \), for Firm \( j \) is given by:

\[ \pi_j = V (p_j - c_j) \int P_{ij} f(\beta_{ri}, \beta_{si}, y_i) d\beta_{ri} d\beta_{si} dy_i - C_j \]

(Empirically, if all estimates are Bayesian, we use the posterior distribution in the standard way.)

For the purposes of this paper, we assume that \( c_j \) does not depend on the quantity sold nor the choice of \( r \) or \( s \). These assumptions can be relaxed and do not reverse the basic intuition in this paper. (The effect of the relative cost of \( r \) or \( s \) is well-studied.)

5. Stylized Formal Model with Two-Segment Heterogeneity

To examine the strategic implications of scale while explicitly modeling heterogeneity, we focus on two mutually exclusive and collectively exhaustive consumer segments. We label the segments R and S, with segment sizes \( R \) and \( S \), respectively. Partworths vary between segments, but are homogeneous within segment (\( \beta_{ri} = \beta_{rR} \) and \( \beta_{si} = \beta_{sR} \) for all \( i \) in segment \( R \); \( \beta_{ri} = \beta_{rS} \) and \( \beta_{si} = \beta_{sS} \) for all \( i \) in segment \( S \)). Scale is constant across consumers such that \( y_i = y \) for all \( i \). The relative influence of a segment is captured by its size, \( R \) or \( S \). This focus enables us to set \( \beta_{rR} = \beta_{sS} = \beta^h \) and \( \beta_{rS} = \beta_{sR} = \beta^e \).

We set \( \beta^h \geq \beta^e \) and \( R \geq S \) without loss of generality so that consumers in Segment R prefer \( r \) and consumers in Segment S prefer \( s \) and so that Segment R is at least as large as Segment S. Heterogeneity of
partworths and scale is important empirically and modeled in our empirical studies (Fiebig, et al. 2010; Salisbury and Feinberg 2010). Heterogeneity in partworths between segments (homogeneous within) enables us to focus on scale and is sufficient for the formal model. We show in §9 that the same insights apply for heterogeneous relative partworths and scales obtained from standard HB CBC.

The costs, $c_j$ and $C_j$, affect strategic decisions in the obvious ways and need not be addressed in this paper. For example, a firm might require a minimum price such that $p_j \geq c_j$ or choose not to enter if $C_j$ is too large. Such effects are well-studied and affect firm decisions above and beyond the strategic effect of scale. For focus, we normalize $V$ to a unit market volume, set $C_j = 0$, and roll marginal costs into price by setting $c_j = 0$.

We label the potential strategic positions for Firms 1 and 2, respectively, as either $rr$, $rs$, $sr$, or $ss$. For example, $rs$ means that Firm 1 positions at $r$ and Firm 2 positions at $s$. Because prices, market shares, and profits depend on these strategic positioning decisions, we subscript prices, shares, and profits accordingly. For example, $p_{1rr}$ is Firm 1’s price in a market in which Firm 1’s position is $r$ and Firm 2’s position is $r$. With this notation, Equations 1 and 2 simplify as illustrated in Equation 3 for $rr$.

$$
P_{R1rr} = \frac{e^{\gamma^{true}(p_{R1rr})}}{e^{\gamma^{true}(p_{R1rr})} + e^{\gamma^{true}(p_{2rr})} + e^{\gamma^{true}u_o}}
$$

(3)

$$
\pi_{1rr} = Rp_{1rr}P_{R1rr} + Sp_{1rr}P_{S1rr}
$$

Equations for Firm 2, for Segment S, and for other positioning strategies are derived similarly. In Equation 3, $\gamma^{true}$ is the true scale—the scale that describes how consumers react in the marketplace. We have chosen to make scale homogeneous so that we can focus on a common increase or decrease in scale. Empirically, when scale heterogeneity is explicitly modeled, we consider effects where scale increases or decreases proportionally for every consumer. Scale homogeneity enables us to abstract from detailed estimation issues to focus on the effect of market-research quality on firms’ decisions.
5.1 Basic Game to Demonstrate the Impact of True Scale (Inherent Stochasticity)

The price-positioning game is consistent with key references in §3.1. Temporarily, we assume the firms know $y^{true}$, which may approach infinity. Based on this knowledge, the firms first choose their product positions ($r$ or $s$) sequentially, and then the market sets prices. The positioning decisions, once made, are not easily reversible, perhaps due to production capabilities or ephemeral advertising investments. Without loss of generality, Firm 1 is the innovator and Firm 2 is the follower. The innovator enters assuming that the follower will choose its positions optimally. (We abstracted away from entry decisions by setting $c_j = C_j = 0$.) After the firms have entered, Nash equilibrium prices, if they exist, are realized. If the firms know the true scale, they can anticipate these prices. (This two-stage game will be embedded in another game in §7 in which firms do not observe the true scale.) We use * to indicate Nash equilibrium prices, shares, and profits.

The equilibria we obtain, and strategies that are best for the innovator and follower, have the flavor of models in the asymmetric competition literature (minimum vs. maximum differentiation), but with two important differences. (1) Our results are not driven by unobserved heterogeneity or strategically-relevant unobserved attributes. (2) Our results are focused on providing a structure to understand and evaluate the impact of improvements in CBC methods. The formal structure can be used as a practical tool to evaluate whether improvements, such as more-realistic images or incentive alignment, affect strategic decisions.

Although we believe that, ex post, the qualitative effects are intuitive, they have not been discussed in the CBC literature. It is well known that scale affects logit-model-based market shares because the logit curve is steeper in both attributes and price when scale increases, but we could find no discussion of the implications of scale for price equilibria or strategic positioning. Papers which advocate the use of estimated equilibrium prices do not discuss sensitivity to scale.
5.2 Equilibria in the Price Subgame

We begin with implicit first-order conditions for the realized prices recognizing that the market will set prices based on the true scale.

\[
\frac{\partial P_{1rr}}{\partial p_{1rr}} = -y^{true}e^{y^{true}(\beta^h_p-\beta^r_p)}[e^{y^{true}(\beta^h_p-\beta^r_p)} + e^{y^{true}u_0}]^2 = -y^{true}P_{1rr}(1 - P_{1rr})
\]

\[
\frac{\partial \pi_{1rr}}{\partial p_{1rr}} = RP_{1rr} + SP_{S1rr} - y^{true}P_{1rr}[RP_{1rr}(1 - P_{1rr}) + SP_{S1rr}(1 - P_{S1rr})]
\]

Using these relationships, we obtain implicit equations for the equilibrium prices and the corresponding equilibrium profits. Similar equations apply for \(rs, sr,\) and \(ss\) and Firm 2:

\[
(4a) \quad p^*_{1rr} = \frac{1}{y^{true}} \frac{RP^*_{1rr} + SP^*_{S1rr}}{RP^*_{1rr}(1 - P^*_{1rr}) + SP^*_{S1rr}(1 - P^*_{S1rr})}
\]

\[
(4b) \quad \pi^*_{1rr} = \frac{1}{y^{true}} \frac{(RP^*_{1rr} + SP^*_{S1rr})^2}{RP^*_{1rr}(1 - P^*_{1rr}) + SP^*_{S1rr}(1 - P^*_{S1rr})}
\]

Differentiating further, we obtain implicit second-order and cross-partial conditions (given in Appendix 2, existence and uniqueness section). Using these conditions, we establish that interior equilibria exist and are unique given (mild) sufficient conditions. We test these conditions for our illustrative examples and for our empirical analyses. In all cases, the equilibria in the illustrative example exist and are unique. In our data, the empirical equilibrium exists for most posterior draws and, when they exist, they are unique.

6. Sensitivity of Valuations and Strategic Decisions

In this section, we assume the firm knows the true scale and explore how scale affects patent/copyright valuations and strategic positioning decisions. In §7, we use these results to explore what happens when decisions are based on observed scale rather than true scale.

6.1. Scale Affects the Price Equilibrium that are Calculated—Illustration

Consider the probability that a consumer in Segment R chooses the innovator’s product given
positions rs. By assumption, the relative partworths for r vs. s do not change, nor does the relative preference for r (or s) vs. price. However, the impact of these preference differences depends upon $\gamma^{true}$. A larger $\gamma^{true}$ makes $P_{RS}$ more sensitive to both attribute differences and price differences; a smaller $\gamma^{true}$ makes $P_{RS}$ less sensitive. As firms react to one another, a larger $\gamma^{true}$ will drive the equilibrium price downward.

As an illustration, we plot the equilibrium price of Firm 1 as a function of $\gamma^{true}$ using the relative partworths we obtained in our empirical study about smartwatches (details in §9). Figure 1 is based on a CBC simulation with two firms whose products differ on watch-face color. We calculate the (counterfactual) price equilibria for each level of $\gamma^{true}$. In Figure 1, the range of the scales is in the ranges reported in the literature and in our empirical studies. The predicted equilibrium prices vary substantially.

**Figure 1. Predicted Equilibrium Price Depends Upon the Scale of the CBC Study Used to Calculate the Equilibrium Prices**

Assume for this illustration that the smartwatch price swing in Figure 1 applies to smartphones. (Smartphone prices are higher so that this will likely under-estimate the effect.) We can use publicly available data to get an idea of the impact that scale would have had if equilibrium prices been used to motivate damages in the first Apple v. Samsung trial about smartphone patents (Mintz 2012; willing-
ness-to-pay, which is scale independent, was used in the 2012 trial). Using estimates of over 20 million infringing Samsung smartphones (The Verge 2012), the calculated price swing of $100 implies a swing of $2 billion in revenue. Patent/copyright valuations are based on profit differences, not revenue. Unfortunately, margins are subject to “protective orders.” If the predicted multi-year profit due to the infringement were only a small fraction of the revenue swing implied by Figure 1, damages could easily vary by tens of millions or even hundreds of millions of dollars depending upon the scale of the CBC analyses used to calculate those damages. This swing is in the order of magnitude of the jury awards in the Apple v. Samsung patent infringement cases (Mintz 2012).

6.2. True Scale Affects the Relative Profits of the Firms’ Positioning Strategies

To understand the effect of true scale on firms’ positioning strategies (choice of attribute levels in equilibrium), we examine how profit-maximizing attribute levels change as true scale increases from small to large. Because the functions are continuous, we need only show the extremes. Result 1 shows that for sufficiently low true scale, price moderation through differentiation does not offset the advantage of targeting the larger segment. The proof is driven by the fact that the logit curve becomes flatter as $\gamma^{true} \to 0$. When price is endogenous, common intuition is not correct. All shares, including the outside option, do not tend toward equality as $\gamma^{true} \to 0$. The endogenous increase in equilibrium prices offsets this effect. Instead, while the innovator and follower shares move closer to one another, the equilibrium prices increase and reduce shares relative to the outside option. The proof demonstrates that all of the countervailing forces balance to favor $rs$ for the innovator and $rr$ for the follower. We provide details in Appendix 2.

**Result 1.** For sufficiently low true scale ($\gamma^{true} \to 0$), the follower prefers not to differentiate whenever the innovator positions for the larger segment ($\pi^*_{2rr} > \pi^*_{2rs}$). However, the innovator would prefer that the follower differentiate ($\pi^*_{1rs} > \pi^*_{1rr}$) and, if the follower were to differentiate, the innovator would earn more profits than the follower ($\pi^*_{1rs} > \pi^*_{2rs}$).
We now show that the firms prefer different strategic positions if true scale is sufficiently high. It
is sufficient that (1) the relative partworth of \( r \) is larger than the relative partworth of the outside option
and (2) the relative partworth of the outside option is at least as large as the relative partworth of \( s \).
With these conditions, market shares are sufficiently sensitive to price for large \( y^{true} \). Shares for differ-
entiated markets become more extreme, the equilibrium price is driven down, and shares increase rela-
tive to the outside option. The countervailing forces balance to favor \( rs \) for the follower.

**Result 2.** Suppose \( \beta^h \) is sufficiently larger than \( u_o \) and \( u_o \geq \beta^l \). Then, there exists a sufficiently
large \( y^{true} \) such that the follower prefers to differentiate whenever the innovator positions for
the larger segment \((\pi_{2rs}^* > \pi_{2rr}^*)\). Differentiation earns more profits for the innovator than no
differentiation \((\pi_{1rs}^* > \pi_{1rr}^*)\), and those profits are larger than the profits earned by the follower
\((\pi_{1rs}^* > \pi_{2rs}^*)\).

Together Results 1 and 2 establish that, if the innovator targets the larger segment, then the fol-
lower will choose to differentiate (\( s \)) when true scale is sufficiently high and will choose not to differen-
tiate (\( r \)) when true scale is sufficiently low. All that remains is to show is that, in equilibrium, the innova-
tor will target the larger segment. While this may seem intuitive from Results 1 and 2, we need Results 3
and 4 to establish the formal result.

**Result 3.** Among the undifferentiated strategies, both the innovator and the follower prefer to
target the larger segment \((\pi_{1rr}^* = \pi_{2rr}^* > \pi_{1ss}^* = \pi_{2ss}^*)\).

**Result 4.** Suppose \( \beta^h \) is sufficiently larger than \( u_o \) and \( u_o > \beta^l \). Then, there exists a sufficiently
large \( y^{true} \) such that the innovator prefers to differentiate by targeting the larger segment ra-
ther than the smaller segment \((\pi_{1rs}^* > \pi_{1sr}^*)\).

### 6.3. Equilibrium in Product Positions

Results 1 to 4 establish necessary and sufficient conditions to prove the following propositions.
Proposition 1. For low true scale ($y^{true} \rightarrow 0$), the innovator (Firm 1) targets the larger segment ($r$) and the follower chooses not to differentiate. It also targets the larger segment ($r$).

Proposition 2. If $\beta^h$ is sufficiently larger than $u_a$ and if $u_a \geq \beta^f$, then there exists a sufficiently large $y^{true}$ such that the innovator targets the larger segment ($r$) and the follower chooses to differentiate by targeting the smaller segment ($s$).

Because the profit functions are continuous (see also APAF), Propositions 1 and 2 and the Mean Value Theorem imply that there exists a $y^{cutoff}$ such the follower is indifferent between $rr$ and $rs$.

Numerically, for a wide variety of parameter values, the profit functions are smooth, the cutoff value is unique, and $\pi^*_2rs - \pi^*_2rr$ is monotonically increasing in $y^{true}$. We have not found a counterexample.

We now have the machinery to address the issue of why scale is an important consideration when CBC simulators are used for patent/copyright valuations and/or strategic decisions.

7. Implications for Investing in the Quality of CBC Studies

Higher “quality” in CBC can be expensive. Some firms, such as Procter & Gamble, Chrysler, or General Motors are sophisticated and spend substantially on CBC. For example, some CBC studies invest 10s of thousands of dollars to create realistic animated descriptions of products and attributes complete with training videos. Incentive alignment can also be expensive: one CBC study gave 1 in 20 respondents $300 toward a smartphone and another gave every respondent $30 toward a streaming-music subscription (Koh 2014; McFadden 2014). Firms routinely use high-quality Internet panels, often paying as much as $5-10 for each respondent and up to $50-60 for hard-to-reach respondents. Our review of the literature (§1) suggests that firms believe that each of these investments increases the accuracy with which relative partworths are estimated and eliminates sources that are not due to inherent stochasticity in consumers’ choices. On the other hand, many firms reduce market research costs by using text-only attribute descriptions, less-sophisticated methods, convenience samples, and small sample sizes. Many “quality” decisions are driven by software defaults. We show that managerial implications are not trivial.
In §6, we temporarily assumed the firm knew the scale (defined as $\gamma^{true}$) that described how consumers would react to $r$, $s$, and price in the market. We are interested in what happens if the testifying experts or firms shirk on their investments in the quality of CBC studies.

We embed the game from §6 into a larger game. We assume that if the firm invests more in the CBC study, its estimate of scale becomes better, that is, $|\gamma^{market\ research} - \gamma^{true}|$ becomes smaller. To focus on scale, we assume all (reasonable) CBC studies estimate the relative partworths correctly so that the firm knows that $r > s$ in R, $s > r$ in S, and $R > S$. It is sufficient to illustrate the phenomenon if we consider a lower-quality and a higher-quality CBC studies such that $\gamma^{higher} = \gamma^{true}$ in the higher-quality study and $\gamma^{lower} \neq \gamma^{true}$ in the lower-quality study. (In §9, we show that investments in more-realistic images and incentive alignment lead to scale estimates that more accurately reflect how consumers in a marketplace behave.) We seek to understand the implications of the firm’s decisions on market-research quality. Thus,

1. The innovator decides whether to invest in the lower-quality or the higher-quality study. (It needs at least the lower-quality study to determine $r > s$ in R, $s > r$ in S, and $R > S$.)

2. The innovator completes its CBC study and observes $\gamma^{market\ research}$.

3. Based on its observed $\gamma^{market\ research}$, the innovator announces and commits to either $r$ or $s$.

4. The follower decides whether to invest in the lower-quality or the higher-quality study. (It needs at least the lower-quality study to determine $r > s$ in R, $s > r$ in S, and $R > S$.)

5. The follower completes its CBC study and observes $\gamma^{market\ research}$. (The innovator’s CBC study is private knowledge limited to the innovator.)

6. Based on its observed $\gamma^{market\ research}$, the follower announces and commits to either $r$ or $s$.

7. Both firms launch their products. The market determines sales and price based on
It will be obvious in §7.1 that the follower could have made its market research decision before learning of the innovator’s positioning—either game ordering gives the same results. Commitment to \( r \) or \( s \) implicitly assumes that positioning decisions are “sticky,” expensive, or based on know-how, patents, or copyrights. Once made, the firm cannot change its positioning even when the market price, market shares, and profits are not as forecast. Propositions 1 and 2 give us sufficient insight to understand the innovator’s and the follower’s market-research-quality decisions as they affect observed scale.

7.1 Innovator’s Strategic Positioning Decision Does Not Depend Upon Observed Scale

The innovator chooses to target the larger segment \( r \) in both Propositions 1 and 2, thus the innovator makes the same decision whether \( \gamma^{\text{market research}} = \gamma^{\text{true}} \) or \( \gamma^{\text{market research}} \neq \gamma^{\text{true}} \). Because the innovator’s strategic positioning decision is independent of the observed scale, investing in a higher-quality CBC study has no effect on the innovator’s positioning strategy. (We state and prove the result formally in Appendix 2.) The insight is consistent with recommendations in product development (e.g., Urban and Hauser 1993, Ulrich and Eppinger 2004). These texts advise innovators to use market research to identify the best attributes, but also advise that the accuracy need only be sufficient for a GO/NO-GO decision.

7.2. Follower’s Strategic Positioning Decision Depends Upon Observed Scale

If a naïve follower underinvests in CBC studies, and if either \( \gamma^{\text{lower}} < \gamma^{\text{cutoff}} < \gamma^{\text{true}} \) or \( \gamma^{\text{lower}} > \gamma^{\text{cutoff}} > \gamma^{\text{true}} \), then the follower makes a strategic error by choosing the wrong strategic position (the wrong attribute level). (We state and prove the result formally in Appendix 2.) For example, if \( \gamma^{\text{cutoff}} < \gamma^{\text{true}} \), then Proposition 1 implies that the most profitable attribute level for the follower is \( r \). However, if the follower acts on \( \gamma^{\text{market research}} = \gamma^{\text{lower}} \), and if \( \gamma^{\text{lower}} < \gamma^{\text{cutoff}} \), then, by Proposition 2, the follower will choose the less profitable attribute level, \( s \). In some cases, the naïve follower may underinvest in CBC studies, but get lucky, say if \( \gamma^{\text{true}} < \gamma^{\text{cutoff}} \) and \( \gamma^{\text{lower}} < \gamma^{\text{cutoff}} \).
The first inequality implies $s$ is the follower’s most profitable attribute level and the second inequality implies the follower chooses $s$. The important insight is that, if the naïve follower underinvests in the quality of a CBC study, then it is relying on luck to make the right decision. While it is often true that getting a parameter wrong affects managerial decisions, it is somewhat surprising that simple design decisions, such as whether or not to use text-only attributes, can have such a major effect.

Empirically, shirking on the quality of CBC market research can either increase or decrease observed scale relative to true scale. For example, all else equal, we might expect that a text-based CBC study would predict marketplace choices less precisely (lower scale) than a CBC study based on realistic stimuli—the firm might underestimate (validation) scale with a text-based study. However, consumers might answer text-based questions more consistently than realistic-stimuli-based questions. Scale as observed in the estimation data might be high with text-based stimuli. Thus, if the firm calculates scale using estimation data without adjusting scale for marketplace validation, it might overestimate scale. It cannot know *a priori* whether the increase in observed scale due to the easier task counteracts the decrease in observed scale because the task represents the market less well. The amount by which observed scale differs from true scale is an empirical question. (We provide empirical examples in §9.)

From a practical standpoint, if the cost of higher quality is small compared to the expected opportunity loss from making the wrong positioning decision, then the follower should invest in higher quality CBC studies. §7.3 provides a formal method to make this decision.

**7.3. Sophisticated Bayesian Follower’s Decision on Investments in CBC Studies**

Sophisticated followers might anticipate that higher-quality CBC studies resolve their uncertainty about true scale. Such sophisticated followers would make optimal decisions on whether or not to invest in higher-quality CBC studies. Suppose that the follower has prior beliefs, $g(y^{true})$, about the true scale and can pay $M$ dollars to resolve that uncertainty ($y^{higher} = y^{true}$). (For simplicity of exposition, we normalize the cost of the lower-quality CBC study to zero.) Suppose further that the estimates
of the relative partworths are the same for both the higher- and lower-quality CBC studies, but only the higher-quality study resolves $y_{true}$. Because the follower knows the relative partworths and is sophisticated, we assume the follower can calculate anticipated $\pi_{2rs}^{*}(y_{true})$ and $\pi_{2rr}^{*}(y_{true})$ for all values of $y_{true}$. The sophisticated follower must decide whether or not to invest $M$ dollars for higher quality.

If a sophisticated follower invests only in the lower-quality CBC study, it does not resolve $g(y_{true})$ and its expected profits are given by an expectation over $g(y_{true})$. The risk-neutral follower’s expected profits with lower-quality research are:

$$E[\pi_{2}^{*}(\text{lower quality research})] = \max \left\{ \int \pi_{2rs}^{*}(y_{true})g(y_{true})dy_{true}, \int \pi_{2rs}^{*}(y_{true})g(y_{true})dy_{true} \right\}$$

On the other hand, if the follower invests in a higher-quality CBC study, it resolves its estimate of scale such that $y_{higher} = y_{true}$. For each observed $y_{true}$, the follower anticipates that it will choose $r$ if $\pi_{2rs}^{*}(y_{true}) < \pi_{2rr}^{*}(y_{true})$, $s$ if $\pi_{2rs}^{*}(y_{true}) > \pi_{2rr}^{*}(y_{true})$, and choose randomly if $\pi_{2rs}^{*}(y_{true}) = \pi_{2rr}^{*}(y_{true})$. Let $\Delta_2(y_{true}) = 1$ indicate that it is optimal for the follower to choose $r$ for an observed $y_{true}$ and $\Delta_2(y_{true}) = 0$ indicate that it is optimal to choose $s$ after $y_{true}$ is revealed by higher-quality market research. Then the risk neutral follower will use the maximum-profit-indicator function ($\Delta_2$) to integrate over $g(y_{true})$. The expected profits when the sophisticated risk-neutral follower invests in higher-quality market research are:

$$E[\pi_{2}^{*}(\text{higher quality research})] = \int \{\pi_{2rs}^{*}(y_{true})\Delta_2(y_{true}) + \pi_{2rr}^{*}(y_{true})[1 - \Delta_2(y_{true})]\}f(y_{true})dy_{true} - M$$

To decide on whether or not to invest in the higher-quality CBC study, the sophisticated follower need only compare the profits given by Equations 6 and 7. Because the notation in Equations 6 and 7 is cumbersome, we illustrate the comparison more simply and intuitively in §8 with an illustrative example.
In summary, the solution to the larger game is that the innovator never invests to resolve scale \((y^{true})\). Lower-quality market research is sufficient as long as the CBC study reveals sufficiently accurate \textbf{relative} partworths \((r > s \text{ in } R, s > r \text{ in } S, \text{ and } R > S)\). The sophisticated follower chooses whether to invest by choosing the market research that maximizes profits comparing Equations 6 and 7). Naively relying on the scale observed in a lower-quality CBC study might lead to substantial opportunity losses. Unfortunately, our experience suggests that many firms make “gut” decisions on CBC investments and choose aspects of CBC studies that we suspect are lower-quality. To the extend such firms are unaware of the implications of scale on strategic decisions, such “gut” decisions may or may not be optimal.

The solution to the game cautions us about the use of price and profit equilibria in patent/copyright valuations. If the quality of the CBC is such that the testifying expert cannot assure the court that \(y^{market\ research} \approx y^{true}\), then the CBC simulation may not even model the correct positioning decision in a world without infringement (the “but-for” world). This caution is in addition to the caution from Figure 1 that an inaccurate estimate of scale can lead to a substantial error in the magnitude of damages.

8. Illustrative Example, with Comments on Patent/Copyright Valuations

We illustrate the formal insights with a numerical example in which \(h^h = 2, h^l = 1, u_o = 1, \text{ and } R = 0.55\). We obtain the fixed-point price equilibria by simple iteration combined with grid search. In all cases, we check that the second-order and cross-partial conditions are satisfied. (R program available from the authors.)

8.1. Representative Equilibria for Various Levels of Scale

Table 1 reports price equilibria for differentiated strategies \((r,s)\) for different values of scale \((y^{true})\). In practice we expect \(y^{true}\) to be the order of magnitude of the partworths, but to illustrate key issues we vary \(y^{true}\) over a wider range. For very small values of true scale \((y^{true} = 0.05)\), the market is less sensitive to price allowing firms to price highly and earn substantial profits. Profits and prices de-
crease with $\gamma^{true}$. For very large values of $\gamma^{true}$, the innovator’s share in segment S ($P^*_S$) approaches zero as does the follower’s share in Segment R ($P^*_R$). The market becomes more segmented when true scale increases.

Table 1. Prices, Shares, Profits, and Second-order Conditions: Differentiated Market

<table>
<thead>
<tr>
<th>Scale</th>
<th>Prices</th>
<th>Shares in Segment R</th>
<th>Shares in Segment S</th>
<th>Profits</th>
<th>Second Order Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p^r_1r$</td>
<td>$p^r_2r$</td>
<td>$P^r_1r$</td>
<td>$P^r_2r$</td>
<td>$P^r_S$</td>
</tr>
<tr>
<td>0.05</td>
<td>24.625</td>
<td>24.603</td>
<td>0.192</td>
<td>0.183</td>
<td>0.183</td>
</tr>
<tr>
<td>0.50</td>
<td>2.588</td>
<td>2.564</td>
<td>0.261</td>
<td>0.160</td>
<td>0.158</td>
</tr>
<tr>
<td>0.60</td>
<td>2.190</td>
<td>2.166</td>
<td>0.278</td>
<td>0.155</td>
<td>0.146</td>
</tr>
<tr>
<td>0.70</td>
<td>1.909</td>
<td>1.885</td>
<td>0.295</td>
<td>0.149</td>
<td>0.139</td>
</tr>
<tr>
<td>0.80</td>
<td>1.701</td>
<td>1.677</td>
<td>0.311</td>
<td>0.143</td>
<td>0.133</td>
</tr>
<tr>
<td>0.90</td>
<td>1.543</td>
<td>1.519</td>
<td>0.328</td>
<td>0.136</td>
<td>0.195</td>
</tr>
<tr>
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<td>0.126</td>
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<td>0.501</td>
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<td>0.067</td>
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<tr>
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<td>0.807</td>
<td>0.614</td>
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<td>0.030</td>
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<tr>
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<td>0.005</td>
</tr>
<tr>
<td>10</td>
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<td>0.805</td>
<td>0.876</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>20</td>
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<td>0.861</td>
<td>0.942</td>
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<td>0.000</td>
</tr>
<tr>
<td>200</td>
<td>0.974</td>
<td>0.974</td>
<td>0.995</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 2 reports equilibria for undifferentiated strategies ($rr$) using the same values of true scale as in Table 1. The second-order conditions are always satisfied for undifferentiated strategies. Low true scale implies high prices and profits. When true scale is large, the market is very sensitive to price and the shares in Segment R approach 50%. If the firms do not differentiate, the high price sensitivity due to large scale drives profits to zero. The last two columns of Table 2 compare profits between a differentiated ($rs$) strategy and an undifferentiated strategy ($rr$). For low scale (below 1.0), strategy $rr$ is more profitable than $rs$ for the follower. This is shown in a red bold font.
Table 2. Prices, Shares, Profits, and Relative Profits: Undifferentiated Market

<table>
<thead>
<tr>
<th>Scale</th>
<th>Prices</th>
<th>Shares in Segment R</th>
<th>Shares in Segment S</th>
<th>Profits</th>
<th>Relative Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y_{true})</td>
<td>(p_{1rr}^*)</td>
<td>(p_{2rr}^*)</td>
<td>(P_{R1rr}^*)</td>
<td>(P_{R2rr}^*)</td>
<td>(P_{S1rr}^*)</td>
</tr>
<tr>
<td>0.05</td>
<td>24.619</td>
<td>24.619</td>
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<td>0.190</td>
<td>0.184</td>
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<td>2.553</td>
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<td>0.240</td>
<td>0.179</td>
</tr>
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<td>2.147</td>
<td>0.251</td>
<td>0.251</td>
<td>0.178</td>
</tr>
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<td>1.858</td>
<td>0.262</td>
<td>0.262</td>
<td>0.176</td>
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<td>1.642</td>
<td>0.272</td>
<td>0.272</td>
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<td>0.283</td>
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</tr>
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<td>1.341</td>
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<td>0.294</td>
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<td>0.744</td>
<td>0.385</td>
<td>0.385</td>
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<td>0.539</td>
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<td>0.444</td>
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<tr>
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<td>0.425</td>
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<td>0.476</td>
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<td>0.349</td>
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<td>0.491</td>
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<tr>
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<td>0.500</td>
<td>0.500</td>
<td>0.127</td>
</tr>
<tr>
<td>20</td>
<td>0.089</td>
<td>0.089</td>
<td>0.500</td>
<td>0.500</td>
<td>0.127</td>
</tr>
<tr>
<td>200</td>
<td>0.009</td>
<td>0.009</td>
<td>0.500</td>
<td>0.500</td>
<td>0.127</td>
</tr>
</tbody>
</table>

8.2. Plot of Relative Profits as a Function of Scale

Figure 2 plots differentiated minus undifferentiated profit for values of true scale \(y_{true}\) in the range of \([0, 2]\). In Figure 2, the range of true scale is approximately equal to the values of the partworths \((\beta^e \text{ and } \beta^h)\). The innovator (black dashed line) always hopes that the follower will differentiate, but the follower (red solid line) only chooses to differentiate when \(y_{true}\) is approximately 1.0 or greater. In Figure 2, profit differences are smooth and monotonic; the critical value of \(y_{true}\) is unique \((y_{cutoff} \cong 1)\).

In §9 we provide a similar figure, but for empirical data.
8.3. Illustration of an Optimal Decision on CBC-Based Market Research

Decisions on CBC-based market research spending depend upon Equations 6 and 7. Suppose, for the sake of illustration, that the market potential is 10 million units and that prices are scaled in dollars. Suppose further that the follower anticipates that the higher-quality CBC study reveals the true scale, \( y_{\text{higher}} = y_{\text{true}} \). It will act on the \( y_{\text{true}} \) that is revealed. It uses its prior to anticipate the \( y_{\text{true}} \) that will be revealed. The lower-quality CBC study does not reveal \( y_{\text{true}} \), therefore the follower must act based on its prior. If the follower chooses the lower-quality CBC study, the follower bases its positioning strategy based on expected profits, integrating over \( g(y_{\text{true}}) \). The calculations are given in Table 3.

Based on Table 3, an undifferentiated strategy has a higher expected value than a differentiated strategy, hence the follower using a lower-quality CBC study would choose \( r \) as per Equation 6. If the follower invests in the higher-quality CBC study, the follower can choose its strategy (\( r \) or \( s \)) depending upon the \( y_{\text{true}} \) it observes. The follower’s decision after observing \( y_{\text{true}} \) is indicated by the “Best Strategy” column. Choosing the best strategy for each realized \( y_{\text{true}} \) yields higher expected profits ($5,034,722) compared to the best strategy based only on the lower-quality study ($4,981,407). The difference, $53,315, is the most that a sophisticated follower would pay for a higher-quality CBC study.
Table 3. Illustration of the Follower’s Decisions and Outcomes Based on Either a Lower-Quality CBC Study (Columns 3&4) or a Higher-Quality CBC Study (Column 6)

<table>
<thead>
<tr>
<th>Prior, $g(y^{true})$</th>
<th>True Scale, $y^{true}$</th>
<th>Follower Chooses s Based on Lower-Quality CBC Study</th>
<th>Follower Chooses r Based on Lower-Quality CBC Study</th>
<th>Best Strategy After $y^{true}$ Revealed</th>
<th>Follower Chooses r or s after Higher-Quality CBC Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.1</td>
<td>$23,337,834$</td>
<td>$23,509,998$</td>
<td>r</td>
<td>$23,509,998$</td>
</tr>
<tr>
<td>0.03</td>
<td>0.2</td>
<td>$12,027,032$</td>
<td>$12,186,344$</td>
<td>r</td>
<td>$12,186,344$</td>
</tr>
<tr>
<td>0.08</td>
<td>0.3</td>
<td>$8,275,610$</td>
<td>$8,420,431$</td>
<td>r</td>
<td>$8,420,431$</td>
</tr>
<tr>
<td>0.08</td>
<td>0.4</td>
<td>$6,414,787$</td>
<td>$6,543,437$</td>
<td>r</td>
<td>$6,543,437$</td>
</tr>
<tr>
<td>0.08</td>
<td>0.5</td>
<td>$5,310,777$</td>
<td>$5,421,558$</td>
<td>r</td>
<td>$5,421,558$</td>
</tr>
<tr>
<td>0.08</td>
<td>0.6</td>
<td>$4,585,625$</td>
<td>$4,676,841$</td>
<td>r</td>
<td>$4,676,841$</td>
</tr>
<tr>
<td>0.08</td>
<td>0.7</td>
<td>$4,077,318$</td>
<td>$4,147,275$</td>
<td>r</td>
<td>$4,147,275$</td>
</tr>
<tr>
<td>0.08</td>
<td>0.8</td>
<td>$3,704,817$</td>
<td>$3,751,862$</td>
<td>r</td>
<td>$3,751,862$</td>
</tr>
<tr>
<td>0.08</td>
<td>0.9</td>
<td>$3,423,066$</td>
<td>$3,445,561$</td>
<td>r</td>
<td>$3,445,561$</td>
</tr>
<tr>
<td>0.08</td>
<td>1.0</td>
<td>$3,204,993$</td>
<td>$3,201,369$</td>
<td>s</td>
<td>$3,204,993$</td>
</tr>
<tr>
<td>0.03</td>
<td>1.1</td>
<td>$3,033,356$</td>
<td>$3,002,089$</td>
<td>s</td>
<td>$3,033,356$</td>
</tr>
<tr>
<td>0.03</td>
<td>1.2</td>
<td>$2,896,596$</td>
<td>$2,836,255$</td>
<td>s</td>
<td>$2,896,596$</td>
</tr>
<tr>
<td>0.03</td>
<td>1.3</td>
<td>$2,786,715$</td>
<td>$2,695,922$</td>
<td>s</td>
<td>$2,786,715$</td>
</tr>
<tr>
<td>0.03</td>
<td>1.4</td>
<td>$2,697,959$</td>
<td>$2,575,425$</td>
<td>s</td>
<td>$2,697,959$</td>
</tr>
<tr>
<td>0.03</td>
<td>1.5</td>
<td>$2,626,085$</td>
<td>$2,470,611$</td>
<td>s</td>
<td>$2,626,085$</td>
</tr>
<tr>
<td>0.03</td>
<td>1.6</td>
<td>$2,567,891$</td>
<td>$2,378,368$</td>
<td>s</td>
<td>$2,567,891$</td>
</tr>
<tr>
<td>0.03</td>
<td>1.7</td>
<td>$2,520,907$</td>
<td>$2,296,323$</td>
<td>s</td>
<td>$2,520,907$</td>
</tr>
<tr>
<td>0.03</td>
<td>1.8</td>
<td>$2,483,216$</td>
<td>$2,222,635$</td>
<td>s</td>
<td>$2,483,216$</td>
</tr>
<tr>
<td>0.03</td>
<td>1.9</td>
<td>$2,453,264$</td>
<td>$2,155,856$</td>
<td>s</td>
<td>$2,453,264$</td>
</tr>
<tr>
<td>0.03</td>
<td>2.0</td>
<td>$2,429,844$</td>
<td>$2,094,834$</td>
<td>s</td>
<td>$2,429,844$</td>
</tr>
</tbody>
</table>

| Expected Profits      | $4,975,580$             | $4,981,407$                                        | $5,034,722$                                        |

Table 3 also illustrates that a naïve follower can make strategic errors. Suppose the follower invests in a lower-quality CBC study that tells the firm (incorrectly) that $y^{true} = 0.1$. Believing and acting on the lower-quality CBC study, the follower would choose not to differentiate ($r$) and forecast a profit of over $23.5M. If true scale were really $y^{true} = 2.0$, then the firm would (1) position the product incorrectly ($r$ rather than $s$), (2) bear an opportunity cost of $335,010, and (3) not realize anywhere near its anticipated profit ($2.1M vs. $23.5M).

8.4. Implications for Patent and Copyright Valuations

Suppose the scale of a CBC study is $y^{market research}$ and suppose that $y^{market research} \neq y^{true}$,
then calculated prices and profits may either be too low or too high. Tables 1 and 2 suggest the differences can be quite large. Consider a patent valuation scenario in which $r$ is enabled by a patent and $s$ represents the non-infringing alternative. Suppose the infringing firm competes in the market with a firm that owns the patent and chooses a position, $r$.

In this case, the proposed equilibrium-priced-based patent valuation would be the difference in the infringing firm’s profits (the follower in our model) in an $rs$ market versus the same firm in an $rr$ market. We use $-(\pi^*_rs - \pi^*_rr)$ from Table 2. For $\gamma^{market\ research} = 0.5$, the valuation would be 0.011, but for $\gamma^{market\ research} = 0.9$, the valuation would be 0.002. Such differences in $\gamma^{market\ research}$ are not unreasonable—we get at least a 2-to-1 swing in §9. But this difference implies an over five-fold difference in damages. (If $\gamma^{market\ research} = 0.05$, damages would increase nine-fold.) Given that patent valuations can be in the hundreds of millions of dollars or more, this is a huge difference. While our example is based on a stylized formal model and the scales are purely illustrative, the example cautions that scale and, by implication, the quality of the CBC study, is an important consideration for patent/copyright valuation. Interestingly, if $\gamma^{market\ research} \geq 1$, then the equilibrium profit calculations imply that the infringing firm would have been better off by not infringing—an interesting interpretation for the courts to consider. The effect is real—CBC studies used in litigation vary widely in the quality of images, incentive alignment, and other aspects that might affect the (estimated) scale that is used in the damages calculation.

9. Empirical Test: Smartwatches

We seek to influence CBC practice when price equilibria are used for patent/copyright valuations and/or strategic positioning. In particular, we seek to determine whether the insights from the stylized formal model and illustrative example translate to a real CBC study. To address practical relevance, we address three empirical questions:

- Can we manipulate observed scale ($\gamma^{market\ research}$) with more-realistic images and incentive
alignment (an illustration with two aspects of quality)?

- Can we obtain results analogous to those derived with the stylized formal model, but with heterogeneous partworths (and scales) from an HB CBC study using a representative panel of respondents?
- Does it matter whether observed scale is based on estimated partworths or whether scale is adjusted based on a validation task that mimics the marketplace (an illustration of scale adjustment)?

9.1. Higher- and Lower-Quality CBC Studies

We designed two CBC studies to match typical empirical practice. One study uses software defects and mimics a lower-quality study, which is typical of many, but not all, CBC studies used for patent/copyright valuation. The other study used what is normally considered higher (and more costly) quality by manipulating two aspects of quality, i.e., more-realistic images and incentive alignment. We hold all other design variables constant between the two studies.\(^1\)

The product category was smartwatches. We abstracted from the large number of attributes in smartwatches to focus on case color (silver or gold), watch face (round or rectangular), watch band (black leather, brown leather, or matching metal color), and price ($299 to $449). Following industry practice, we held all other attributes constant, including brand and operating system, so that we could estimate the relative tradeoffs among the attribute levels that we varied. Our focus on three smartwatch attributes and price is sufficient to test the generalizability of the stylized model; an industry study might vary more attributes. Empirically, any unobserved attributes do not vary between higher-vs.-lower quality studies. By assumption for the worlds we simulate, unobserved attributes are not used strategically for positioning decisions.

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\(^1\) The two studies were part of a larger experimental design that also manipulated other aspects of quality in a fractional factorial design—for the purposes of this comparison those aspects were randomized. For brevity, we focus on the two most impactful aspects of quality. Details on the less impactful aspects are available from the authors.
We used sixteen choice sets for estimation (and two for internal validation) with three profiles per choice set. We included the outside option via a dual-response procedure. These settings are typical for current industry applications (Meissner, Oppewal, and Huber 2016; Wlömert and Eggers 2016). We followed standard survey design principles including extensive pretesting (28 respondents in the higher-quality study and 38 in the lower-quality study) to assure that (1) the questions, attributes, and tasks were easy to understand and (2) that the manipulation of research quality between respondents was not subject to demand artifacts.

9.2. Higher-Quality Study: Animations, Realistic Pictures, and Incentive Alignment

After the screening questions, respondents entered the CBC section. Respondents completed a training task (not used in estimation), then saw an animated video to induce incentive alignment2 (e.g., Ding 2007; Ding, Grewal, and Liechty 2005; Ding, et al. 2011). Specifically, respondents were told that some respondents (1 in 500) would receive a smartwatch and/or cash with a combined value of $500—based on their answers to the survey. See Figure 3. Each respondent chose among realistic images of three smartwatches and then indicated whether or not he or she would purchase the smartwatch (Figure 4). To make the images more realistic, the respondent could toggle among a detailed view, a top view, and an app view (not shown in Figure 4). Dahan and Srinivasan (2000) suggest visual depictions and animations provide nearly the same results as physical prototypes.

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2 The incentive-alignment video is available at https://www.youtube.com/watch?v=DBLPfRJo2Ho.
9.3. Lower-Quality CBC Study: Text-based with No Incentive Alignment

In the lower-quality study, respondents also completed a training task but did not see the incentive-alignment video, did not receive an incentive-alignment promise, saw only text-based stimuli (with
simple images), and could not toggle among views. See Figure 5. Respondents took part in a lottery for a 1 in 500 chance to win $500 in cash; however, the reward was not tied to the answers given in the lower-quality survey as it was in the higher-quality study.

**Figure 5. Lower-Quality Study: Choice-Based Dual Response Task (no ability to toggle)**

9.4. Validation Task to Estimate Scale Adjustments

We created a marketplace with twelve smartwatches and an outside option. (Twelve smartwatches represents all possible design combinations.) Smartwatches varied on case color, watch face, watch band, and price. Starting three weeks after the two CBC studies, respondents, in both the higher- and lower-quality studies, were given an incentive-aligned opportunity to choose either one of the twelve smartwatches or the outside option. We used the validation data to adjust the scale factor (§9.7). Marketplace market shares were not available for these studies, but, in practice, researchers might consider other validation adjustments such as those proposed by Gilbride, Lenk, and Brazell (2008).

9.5. Sample

Our sample was drawn from a professional panel.\(^3\) We screened the sample so that respondents expressed interest in the category, were based in the US, aged 20-69, and agreed to informed consent as required by our internal review boards. We also screened out respondents who already owned a

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\(^3\) Peanut Labs is an international panel with 15 million pre-screened panelists from 36 countries. Their many corporate clients cumulatively gather data from approximately 450,000 completed surveys per month. Peanut Labs is a member of the ARF, CASRO, ESOMAR, and the MRA and has won many awards: web.peanutlabs.com.
smartwatch. Such screening is reasonable for our research purpose. Respondents in both studies re-
ceived standard panel incentives for participating in the study.

Overall, 858 respondents completed the first wave of studies: 427 in the higher-quality study,
431 in the lower-quality study. The rate of consumers who completed the validation study was equally
distributed (both 0.69). We only considered respondents in the analyses who completed the validation
study and removed respondents who always chose the outside option. There were no significant differ-
ences between the studies and the exclusion of respondents ($p = 0.74$). The final sample size was 545
(270 in the higher-quality study, 275 in the lower-quality study).

9.6. Estimation

We estimate a joint HB CBC model in which the relative partworths are drawn from the same
hyper-distribution, but scale ($\gamma$) varies between higher- and lower-quality and between estimation and
validation. We normalize a scale-adjustment to 1.0 for the lower-quality estimation sample so that
scale-adjustment for the other conditions can be identified. We assume a normal prior for the design
partworths and constrain the price coefficient by assuming a log-normal distribution (Allenby, et al.
2014). To avoid misspecification errors, we tested for interaction effects but could not detect significant
improvements in model fit; our model is based on main effects. The remaining settings followed stand-
ard procedures (Sawtooth Software 2015).

In the estimation, we used 10,000 burn-in iterations and a subsequent 10,000 iterations to draw
partworths, from which we kept every 10th draw. Based on these data, HB CBC provides a posterior dis-
tribution of relative partworths and estimated scales. All subsequent summaries, profits, and other re-
ported quantities are based on the posterior distributions.

9.7. CBC Quality Affects Observed Scale

Our first research question is whether the improvements in quality affects observed scale. The
posterior means and standard deviations of the scale-adjustment posterior distributions are given in
Table 4.

<table>
<thead>
<tr>
<th>Table 4. Posterior Means of Scale Adjustment</th>
<th>Lower-Quality Study</th>
<th>Higher-Quality Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale is based on estimation data</td>
<td>1.00&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>(n.a.)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Scale is adjusted to validation task</td>
<td>0.38</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.06)</td>
</tr>
</tbody>
</table>

<sup>a</sup> Normalized to 1.00 for identification.

First, we notice that if scale is based on the estimation data only, the lower-quality study appears to be more precise (higher scale). In the majority of posterior draws (87%), respondents were more consistent in answering text-based questions without incentive alignment than they were in answering questions based on more-realistic images with incentive alignment. If these were the only data available, the firm might conclude that investments in higher quality were counterproductive. However, when we examine the scale adjustment for the validation task, we see that higher-quality greatly enhances validation-based scale, which is a consistent finding across all posterior draws.

From our data, we do not know $y^{true}$, but we can examine (validation) predictive ability as a surrogate. Both hit rates and uncertainty explained ($U^2$, Hauser 1978) are substantially improved for the higher quality study—hit rates increase from 23% to 40% (chance is 7.7%) and $U^2$ increases from 0.15 to 0.33. There was no draw in which the lower-quality study performed better.

Our focus in this paper is to illustrate that market-research quality can drive observed scale. However, it is an interesting question, beyond the scope of this paper, as to which aspects of market-research quality have the greatest impact. In a companion paper, we demonstrate that both realistic images and incentive alignment drive quality individually, with realistic images having the greater impact. (Note to reviewers. Upon request, we can greatly expand this discussion to describe the data, the model, and statistical tests.)
9.8. The Empirical Data Produce Strategic Effects Analogous to the Stylized Model

We now address the second question: whether the phenomena predicted by the stylized model can be reproduced using partworths from a typical HB CBC study. We use the CBC simulator to compare markets with lower “true” scale and markets with higher “true” scale. We create counterfactuals for “true” scale by holding the distribution of relative partworths from the empirical studies constant and varying the scale adjustment (as in Table 4). We use the CBC simulator to examine strategic positioning decisions for smartwatch color (silver vs. gold). Our counterfactual simulations assume that smartwatch-color decisions are difficult to reverse. We use the root-finding method described in Allenby et al. (2014) to find the price equilibria. In order to avoid extrapolation beyond the price range used in the CBC experiment, we cap prices at an upper limit of $449. For illustration, we chose scale-adjustment values consistent with Table 4 and near the strategic cutoff point. Empirically, $\gamma^{cutoff} \approx 0.6$. See Figure 6.

![Figure 6. Relative Profits Based on the Smartwatch Data](image)

Table 5, based on the posterior means, summarizes the positioning equilibria with unit demand and zero costs. The equilibria exist in the majority of draws and appear to be unique. Because more respondents preferred silver to gold (66.1%) than vice versa, the analogy to the stylized model is $r = \text{silver}$, even though “$r$” is mnemonically cumbersome for silver.

As the scale-adjustment decreased from $\gamma^{true} = 0.8$ to $\gamma^{true} = 0.4$, the positioning equilib-
rium shifted from differentiated positions (silver, gold) to undifferentiated positions (silver, silver). If we assume that the market is 10 million units, then positioning based on misestimating the true scale would result in a $95 million opportunity loss for the follower. For comparison, the Apple Watch sold 11.9 million units in 2016 (Reisinger 2017). Likewise, differences in calculated patent/copyright valuations vary substantially based on observed scale (not shown in Table 5) in the same order of magnitude.

Table 5. “True” Scale Affects Strategic Positioning with HB CBC Partworths
(Relative partworths are heterogeneous, but the same in higher- and lower-scale markets. In this table, \( y_{true} \) is the scale-adjustment factor which is proportional to scale.)

<table>
<thead>
<tr>
<th>Higher-Scale ( (y_{true} = 0.8) )</th>
<th>Follower’s Position</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Innovator’s Position</strong></td>
<td><strong>Silver</strong></td>
</tr>
<tr>
<td>Silver</td>
<td>( \pi_{1rr} = 61.6 )</td>
</tr>
<tr>
<td></td>
<td>( \pi_{2rr} = 61.6 )</td>
</tr>
<tr>
<td><strong>Gold</strong></td>
<td>( \pi_{1sr} = 71.1 )</td>
</tr>
<tr>
<td></td>
<td>( \pi_{2sr} = 98.8 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lower-Scale ( (y_{true} = 0.4) )</th>
<th>Follower’s Position</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Innovator’s Position</strong></td>
<td><strong>Silver</strong></td>
</tr>
<tr>
<td>Silver</td>
<td>( \pi_{1rr} = 102.6 )</td>
</tr>
<tr>
<td></td>
<td>( \pi_{2rr} = 102.6 )</td>
</tr>
<tr>
<td><strong>Gold</strong></td>
<td>( \pi_{1sr} = 94.7 )</td>
</tr>
<tr>
<td></td>
<td>( \pi_{2sr} = 118.2 )</td>
</tr>
</tbody>
</table>

We obtained similar results when we used CBC simulators for watch face (rectangular vs. round) and watch strap (black vs. brown or other combinations). In all counterfactual tests using empirical HB CBC partworths, the market always shifted from differentiated to undifferentiated as “true” scale de-
increased through a critical value, $y_{\text{cutoff}}$. We conclude that there are examples where the stylized theory applies to empirical data with heterogeneous relative partworths and scales.

Our third question asked whether or not it mattered strategically whether scale was based on estimation or was adjusted to a validation task. For the lower-quality study, estimation-based scale implies differentiation while validation-adjusted scale does not. Thus, it can matter empirically whether or not scale is adjusted. Market-research quality also matters. Using validation-adjusted scale, the higher-quality study implies differentiation, but the lower-quality study does not. Thus, empirically, the quality of the market research can lead to different strategic outcomes.

If we assume that the higher-quality-validation-adjusted scale is closest to the true scale, then the lower-quality-estimation-based scale gets the right strategic decision for the wrong reasons—the two effects offset. Few firms would choose to rely on luck (just the right offsetting effects) for the correct strategic outcome. The key point is that strategic errors can result from relying on lower-quality studies and/or from not adjusting scale with validation tasks. The firm does not know a priori what, if any, strategic errors it will make if it shirks on quality and validation adjustment.

Our empirical study (and the stylized model) abstracted from product line decisions in order to focus on the strategic impact of the scale on price and positioning. We focused respondents so that they held constant both operating system and brand—two attributes that are used to differentiate smartwatches. Smartwatch manufacturers also differentiate on the shape of the watch face (round for Motorola; square for Apple). Both Apple and Motorola offer watches in at least three different colors—a product line (not modeled and beyond the scope of this paper).

9.9. CBC Quality Also Affects the Relative Partworths

Our stylized model focused on scale, but empirically the quality of the CBC study might also affect the relative partworths. We drew the relative partworths from the same distribution, but we can still examine 1,000 draws and compare the relative partworths between studies. Partworth differences
(relative to price differences) suggest that more-realistic images and incentive alignment led respondents to value differences in watch color and band type more than did for text-only images without incentive alignment. When we relax the specification to allow relative partworths to vary between the two studies (equivalent to independent estimation for each study), the insights and interpretations are unchanged. Our interpretation is that the higher-quality study encouraged respondents to evaluate attribute-level differences more carefully in our study (see also Vriens, et al. 1998). However, we cannot rule out other situations where greater respondent motivation and better attribute descriptions cause respondents to decrease valuations of attribute-level differences.

10. Discussion and Summary

Many previous papers establish (1) that quality improvements enhance the accuracy with which the relative partworths can be estimated and (2) that accounting for heterogeneity in scale enhances accurate estimation of the relative partworths. This paper demonstrates that market research quality affects observed scale and that observed scale affects strategic positioning decisions and predicted equilibrium prices. Patent/copyright valuations based on equilibrium prices from CBC studies are extremely sensitive to the quality of the CBC study on which those valuations are based. Shirking on image realism and incentive alignment, or not adjusting for validation tasks, can impact “but-for” damages by hundreds of millions of dollars. (We provide three illustrative examples all suggesting potentially large impacts.)

In our analyses, we abstracted from marginal and fixed costs because these effects are well-studied and do not change the insights relative to scale. Likewise, it is clear that, if a lower-quality study misidentifies the relative partworths, then (1) copyright/patent valuations will be inaccurate and (2) strategically both the innovator and the follower may make tactical errors in product design. Such errors are well-studied. It might surprise many that, even if the relative partworths are unaffected, decisions on market-research quality can affect observed scale substantially and can dramatically affect pa-
tent/copyright valuations and change strategic positioning.

Finally, a firm need not always invest in the highest-quality research to make correct strategic decisions. For every market there is a critical scale, $\gamma^{cutoff}$, above which the follower should differentiate. The follower must only assure that it invests sufficiently in the quality of a CBC study to know whether the true scale is above or below the cutoff. These critical values can be calculated empirically and can help firms decide which of the many published improvements in CBC are worth the investment. We believe many will pass the test.
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Appendix 1: Summary of Notation

\( i \) indexes consumers.

\( j \) indexes firms. Firm 1 is the innovator; Firm 2 is the follower.

\( c_j \) Firm \( j \)'s marginal cost.

\( C_j \) Firm \( j \)'s fixed costs.

\( r \) a product attribute. We can think of \( r \) as red (or rose, regular, round, or routine).

\( s \) a product attribute. We can think of \( s \) as silver (or sapphire, small, square, or special).

A firm’s product can have either \( r \) or \( s \). It cannot have both or neither.

\( p_j \) Firm \( j \)'s price.

\( p_{jrr}^* \) Nash equilibrium price for Firm \( j \) given that Firm 1 chooses \( r \) and Firm 2 chooses \( r \). Define \( p_{jrs}^* \), \( p_{jsr}^* \), and \( p_{jss}^* \) analogously.

\( P_{jrr} \) probability that consumer \( i \) purchases product from Firm \( j \) given that Firm 1 chooses \( r \) and Firm 2 chooses \( r \). Define \( P_{jrs} \), \( P_{jsr} \), and \( P_{jss} \) analogously.

\( P_{Rjrr} \) probability that a consumer in segment \( R \) purchases product from Firm \( j \) given that Firm 1 chooses \( r \) and Firm 2 chooses \( r \). Define \( P_{RsR} \), \( P_{RsS} \), \( P_{SjR} \), \( P_{SjS} \), and \( P_{Sjss} \) analogously.

\( R \) size of Segment \( R \) (We use italics for the size of the segment; non-italics to name the segment.)

\( S \) size of Segment \( S \).

\( u_{ij} \) utility that consumer \( i \) perceives for Firm \( j \)'s product.

\( u_{io} \) utility that consumer \( i \) perceives for the outside option

\( u_o \) utility of outside option for Segments \( R \) and \( S \).

\( u_{Rj} \) utility of Firm \( j \)'s product among consumers in segment \( R \).

\( u_{Sj} \) utility of Firm \( j \)'s product among consumers in Segment \( S \).

\( V \) Number (measure) of consumers.

\( \beta_{ri} \) relative partworth for \( r \) for consumer \( i \).

\( \beta_{si} \) relative partworth for \( s \) for consumer \( i \).

\( \beta_{R} \) relative partworth of \( r \) for all \( i \in R \). Define \( \beta_{S} \), \( \beta_{RS} \), and \( \beta_{SS} \) analogously.

\( \beta_h \) higher partworth, \( \beta_{R} = \beta_{S} = \beta_h \).

\( \beta^e \) lower partworth, \( \beta_{RS} = \beta_{SR} = \beta^e \). Theory holds if \( \beta^e \) normalized to zero, but is less intuitive.

\( \delta_{rij} \) indicator function for whether Firm \( j \)'s product has attribute \( r \). Define \( \delta_{sj} \) analogously.

\( \epsilon_{ij} \) error term for consumer \( i \) for Firm \( j \)'s product. Errors are independent and identically distributed and drawn from an extreme-value distribution.

\( \gamma_i \) scale. Larger values imply smaller relative magnitude of the error term.

\( \gamma \) when scale is homogeneous.

\( \gamma_{true} \) the true scale (sometimes \( \gamma \) for notational simplicity in proofs if not confused with \( \gamma_i \)).

\( \gamma_{higher} \) scale estimated with higher-quality CBC study.

\( \gamma_{lower} \) scale estimated with lower-quality CBC study.

\( \gamma_{cutoff} \) cutoff value for scale. \( \gamma > \gamma_{cutoff} \) implies differentiation. \( \gamma < \gamma_{cutoff} \) no differentiation.

\( \eta_i \) partworth (coefficient) for price, before parameterization (equals 1.0 after reparameterization).

\( \pi_j \) profits for Firm \( j \).

\( \pi_{jrr}^* \) profits for Firm \( j \) at the Nash equilibrium prices of \( p_{jrr}^* \) and \( p_{jss}^* \). Define \( \pi_{jrs}^* \), \( \pi_{jsr}^* \), and \( \pi_{jss}^* \) analogously.

\( \Delta_2 (\gamma) \) indicator of whether, for a given \( \gamma \), it is more profitable for Firm 2 to differentiate.

\( \Delta_{RSRR} \) defined in the proof to Result 2. \( \Delta_{RSRR} \) and other terms for \( rs, sr, \) and \( ss \) defined analogously.
Appendix 2. Proofs to Results and Propositions (provided for review)

Throughout this appendix, for notational simplicity, we drop the superscript on \( y^{\text{true}} \) and write it simply as \( y \). Results in this appendix are stated in notational shorthand, but are the same as those in the text.

**Result 1.** For \( y \to 0 \), \( \pi_{2rr}^* > \pi_{2rs}^*, \pi_{1rs}^* > \pi_{1rr}^* \), and \( \pi_{1rs}^* > \pi_{2rs}^* \).

**Proof.** This proof addresses first-order conditions. We address second-order and cross-partial conditions when we examine existence and uniqueness later in this appendix. As \( y \to 0 \), the logit curve becomes extremely flat, which motivates a Taylor’s Series expansion of market share around \( \beta^h = \beta^e \). When \( \beta^h = \beta^e \), the logit equations for the market shares are identical for Firm 1 and 2, identical for all strategies, \( rr, rs, sr, ss \), and symmetric with respect to Firm 1 and Firm 2. Thus, at \( \beta^h = \beta^e \) we have:

\[
p_{1rr}^* = p_{2rr}^* = p_{1rs}^* = p_{2rs}^* = p \text{ at } \beta^h = \beta^e
\]

\[
p_{R1rr}^* = p_{R2rr}^* = p_{R1rs}^* = p_{R2rs}^* = p \text{ at } \beta^h = \beta^e
\]

\[
p_{S1rr}^* = p_{S2rr}^* = p_{S1rs}^* = p_{S2rs}^* = p \text{ at } \beta^h = \beta^e
\]

Because the prices and shares are identical, we have:

\[
\pi_{1rr}^* = \pi_{2rr}^* = \pi_{1rs}^* = \pi_{2rs}^* = \frac{1}{y} \frac{P}{1 - P} \text{ at } \beta^h = \beta^e
\]

Where the last step comes from substituting the equalities for \( P \) in Equation 4b from the text, and simplifying using \( R + S = 1 \). We obtain the optimal price by solving the following fixed-point problem in \( \pi \):

\[
\gamma P = \frac{1}{1 - P} = \frac{2e^{-\gamma P} + e^{\gamma u_o}}{2e^{-\gamma P} + e^{\gamma u_o}} \text{ using } P = \frac{e^{-\gamma P}}{2e^{-\gamma P} + e^{\gamma u_o}}
\]

Because the right-hand side is decreasing in \( p \) on the range \([1, 1.5]\) there will be exactly one solution in the range of \( \gamma P \in [1, 1.5] \) for small \( \gamma \). We compute the partial derivatives of the \( P \)'s at \( \beta^h = \beta^e \):

\[
\frac{\partial p_{R2rr}}{\partial \beta^h} = \gamma P(1 - 2P) \equiv \gamma \Delta_{R2rr}, \quad \frac{\partial p_{S2rs}}{\partial \beta^h} = 0 \equiv \gamma \Delta_{S2rr}
\]

\[
\frac{\partial p_{R2rs}}{\partial \beta^h} = -\gamma P^2 \equiv \gamma \Delta_{R2rs}, \quad \frac{\partial p_{S2rs}}{\partial \beta^h} = \gamma P(1 - P) \equiv \gamma \Delta_{S2rs}, \quad \Delta = \beta^h - \beta^e
\]

We now use a Taylor’s series expansion with respect to \( \beta^h \) recognizing that higher order terms are \( o(y^2) \) or higher and, hence, vanish as \( y \to 0 \). Substituting the expressions for the partial derivatives into the first-order conditions (Equation 4b), multiplying by \( \gamma \), and using the above notation, we obtain:

\[
\gamma \pi_{2rr}^* = \frac{[R(P + \gamma \Delta_{R2rr}) + S(P + \gamma \Delta_{S2rr})]^2 + o(y^2)}{R(P + \gamma \Delta_{R2rr})(1 - P) - \gamma \Delta_{R2rr} + S(P + \gamma \Delta_{S2rr})(1 - P) - \gamma \Delta_{S2rr}} + o(y^2)
\]

\[
\gamma \pi_{2rr}^* = \frac{p^2 + 2\gamma \Delta(R\Delta_{R2rr} + S\Delta_{S2rr}) + o(y^2)}{p(1 - P) + \gamma \Delta(1 - 2P)(R\Delta_{R2rr} + S\Delta_{S2rr}) + o(y^2)}
\]
Similarly,
\[
\gamma \pi_{2rs}^* = \frac{p^2 + 2\gamma \Delta (R \Delta_{R2rs} + S \Delta_{S2rs}) + o(\gamma^2)}{P(1 - P) + \gamma \Delta (1 - 2P)(R \Delta_{R2rs} + S \Delta_{S2rs}) + o(\gamma^2)}
\]

Because all terms in the numerators and denominators of \(\gamma \pi_{2rr}^*\) and \(\gamma \pi_{2rs}^*\) are clearly positive, we show that \(\gamma \pi_{2rr}^* > \gamma \pi_{2rs}^*\) for \(\gamma \to 0\) if:

\[
p^2 + 2\gamma \Delta (R \Delta_{R2rr} + S \Delta_{S2rr})[P(1 - P) + \gamma \Delta (1 - 2P)(R \Delta_{R2rs} + S \Delta_{S2rs})] > p^2 + 2\gamma \Delta (R \Delta_{R2rs} + S \Delta_{S2rs})[P(1 - P) + \gamma \Delta (1 - 2P)(R \Delta_{R2rs} + S \Delta_{S2rs})]
\]

After simplification and ignoring terms that are \(o(\gamma^2)\), this expression reduces to:

\[
\gamma \Delta P[(R \Delta_{R2rr} + S \Delta_{S2rr}) - (R \Delta_{R2rs} + S \Delta_{S2rs})][2 - 3P + 2P^2] > 0
\]

We need only show that both terms in brackets are positive. We show the first term in brackets is positive because:

\[
(R \Delta_{R1rr} + S \Delta_{S1rr}) - (R \Delta_{R1rs} + S \Delta_{S1rs}) = (RP(1 - P) - RP^2) - (SP(1 - P) - RP^2) > 0
\]

The last step follows from \(R > S\). We show the second term is positive because its minimum occurs at \(P = \frac{3}{4}\) and its value at this minimum is \(2 - 3P + 2P^2 = \frac{7}{8}\). Thus, \(2 - 3P + 2P^2\) is positive for all \(P \in [0,1]\).

To prove that \(\pi_{1rs}^* > \pi_{1rr}^*\) for \(\gamma \to 0\) we use another Taylor’s series expansion and simplify by the same procedures. Most of the algebra is the same until we come down to the following term in brackets (now reversed because \(r\) is more profitable for Firm 1 than \(r\) as \(\gamma \to 0\)). Taking derivatives gives:

\[
\frac{\partial P_{r1r}}{\partial \beta^h} = \gamma P(1 - 2P) \equiv \gamma \Delta_{R1rr} \quad \frac{\partial P_{s1r}}{\partial \beta^h} = 0 \equiv \gamma \Delta_{S1rr}
\]

\[
\frac{\partial P_{r1r}}{\partial \beta^h} = \gamma P(1 - P) \equiv \gamma \Delta_{R1rs} \quad \frac{\partial P_{s1r}}{\partial \beta^h} = -\gamma P^2 \equiv \gamma \Delta_{S1rs}
\]

The corresponding expression in brackets becomes (for \(\gamma \pi_{1rs}^* - \gamma \pi_{1rr}^*\):

\[
(R \Delta_{R1rs} + S \Delta_{S1rs}) - (R \Delta_{R1rr} + S \Delta_{S1rr}) = (RP(1 - P) - SP^2) - (RP(1 - P) - RP^2) > 0
\]

where the last step is true because \(R > S\).

By exploiting symmetry, we have \(\pi_{1rr}^* = \pi_{2rr}^*\), yielding the result that \(\pi_{1rs}^* > \pi_{1rr}^* = \pi_{2rr}^* > \pi_{2rs}^*\).

**Lemma 1.** \(\gamma p_{1rs}^* < (1 - P_{R1rs})^{-1}\) and \(\gamma p_{2rs}^* < (1 - P_{S2rs})^{-1}\). Related conditions hold for \(rr, ss, and sr\).

**Proof.** We use the first-order conditions (for \(rs\)) as illustrated in Equation 4a. All terms are positive, so we cross multiply. The first expression holds if \(RP_{R1rs}(1 - P_{R1rs}) + SP_{S1rs}(1 - P_{S1rs}) > RP_{R1rs}(1 - P_{R1rs}) + SP_{S1rs}(1 - P_{S1rs})\), which is true if \(P_{R1rs}^* > P_{S1rs}^*\). The latter holds whenever \(\beta^h > \beta^s\) for all \(\gamma\) by substituting directly into the logit equation. We prove the second expression by using the first-order conditions for \(p_{2rs}^*\). Related expressions hold for other positionings. For example, for the \(rr\) positions, \(\gamma p_{1rr}^* < (1 - P_{R1rr})^{-1}\) and \(\gamma p_{2rr}^* < (1 - P_{R2rr})^{-1}\).
Result 2. Suppose $\beta^h$ is sufficiently larger than $u_0$ and $u_o \geq \beta^e$. Then, there exists a sufficiently large $\gamma$ such that $\pi^*_2rs > \pi^*_2rr$, $\pi^*_1rs > \pi^*_1rr$, and $\pi^*_1rs > \pi^*_2rs$.

**Proof.** In this proof we examine the first-order conditions. Second-order and cross-partial conditions are addressed when we consider existence and uniqueness later in this appendix. We first recognize that:

\[
\begin{align*}
\psi_{11} &= \psi_{11}^* + e^{y}\left(\psi_{11}^* + e^{y}\psi_{22}^*\right), \\
\psi_{22} &= \psi_{22}^* + e^{y}\left(\psi_{22}^* + e^{y}\psi_{11}^*\right), \\
\psi_{21} &= \psi_{21}^* + e^{y}\left(\psi_{21}^* + e^{y}\psi_{12}^*\right), \\
\psi_{12} &= \psi_{12}^* + e^{y}\left(\psi_{12}^* + e^{y}\psi_{21}^*\right),
\end{align*}
\]

When $\gamma$ is large relative to $\beta$ and $u_o$, $P_{11} \approx 0$, $P_{22} \approx 0$, and $P_{12} \approx 0$. Substituting and simplifying the first-order conditions gives us:

\[
\begin{align*}
P_{22}^* &= \frac{R_{22}}{R_{22}^* (1 - P_{22}^*) + S_{22}^* (1 - P_{22}^*)} \approx \frac{S_{22}^*}{S_{22}^* (1 - P_{22}^*)} = 1 - P_{22}^*
\end{align*}
\]

We substitute for $P_{22}^*$ and simplify to obtain:

\[
\begin{align*}
P_{22}^* &= \frac{R_{22}^* + S_{22}^*}{R_{22}^* (1 - P_{22}^*) + S_{22}^* (1 - P_{22}^*)} \approx \frac{S_{22}^*}{S_{22}^* (1 - P_{22}^*)} = 1 - P_{22}^*
\end{align*}
\]

As $\gamma$ gets large, the effect of $\gamma$ as an exponent is much larger than the effect of $\gamma$ as a multiplier, thus the expression in parentheses in the exponent must converge toward zero for the equality to hold. As the expression approaches zero, the solution to this fixed point problem approaches $P_{22}^* = \beta^h - u_0 - \epsilon$ where $\epsilon > 0$, $\epsilon$ is but a fraction of $\beta^h - u_0$, and $\epsilon \to 0$ as $\gamma \to \infty$. Thus, $\pi^*_2rs = P_{22}^* S_{22}^* (\beta^h - u_0 - \epsilon)$. Substituting $P_{22}^*$ into the expression for $S_{22}^*$, we get:

\[
\begin{align*}
P_{22}^* &= \frac{R_{22}^* + S_{22}^*}{R_{22}^* (1 - P_{22}^*) + S_{22}^* (1 - P_{22}^*)} \approx \frac{S_{22}^*}{S_{22}^* (1 - P_{22}^*)} = 1 - P_{22}^*
\end{align*}
\]

Thus, $\pi^*_2rs$ is greater than $S(\beta^h - u_0)/3$ as $\gamma \to \infty$. ($P_{22}^*$ actually gets close to 1 and $\pi^*_2rs$ gets close to $S(\beta^h - u_0)$ as $\gamma \to \infty$, but we only need the weaker upper bound.)

Thus, for sufficiently large $\gamma$ (relative to $\beta^h$ and $u_o$), the solution of $\pi^*_2rs$ is greater than $S(\beta^h - u_0)/3$. Similar arguments establish that $\pi^*_1rs \approx R(\beta^h - u_0 - \epsilon)P_{11}^*$ and that $\pi^*_1rs$ is greater than $R(\beta^h - u_0)/3$. (Recall that $\epsilon \to 0$ as $\gamma \to \infty$.)

We examine the price equilibrium when both Firm 1 and Firm 2 choose $r$. We first recognize that, by symmetry, $P_{11}^* = P_{22}^*$. Hence,

\[
\begin{align*}
P_{22rr} &= \frac{e^{y}(\beta^e - P_{22rr})}{2e^{y}(\beta^e - P_{22rr}) + e^{y}u_0} \\
P_{22rr} &= \frac{e^{y}(\beta^e - P_{22rr})}{2e^{y}(\beta^e - P_{22rr}) + e^{y}u_0}
\end{align*}
\]

A4
We seek to show that there is a \( p_{2rr}^* \), with the properties that \( p_{2rr}^* \bowtie \beta^h - u_o \) and \( p_{2rr}^* < u_o \), which satisfies the first-order conditions. In this case, as \( \gamma \to \infty \), \( p_{2rr}^* \equiv \frac{1}{2} \). The first-order conditions become:

\[
\gamma p_{2rr}^* = \frac{R P_{R2rr}^* + S P_{S2rr}^*}{R P_{R2rr}^* (1 - P_{R2rr}^*) + S P_{S2rr}^*(1 - P_{S2rr}^*)} \approx \frac{1}{2} R + S P_{S2rr}^* \leq \frac{1}{4} R + S P_{S2rr}^*(1 - P_{S2rr}^*) = \frac{1}{4} R < 6
\]

The third to last step, setting \( P_{S2rr}^* = 1 \) for the inequality, is possible because the fraction increases in \( P_{S2rr}^* \) to obtain its maximum at \( P_{S2rr}^* = 1 \), as shown with simple calculus. Thus, if \( p_{2rr}^* \) satisfies the first-order conditions, then \( p_{2rr}^* < 6/\gamma \). By implication, \( \pi_{2rr}^* < \frac{6}{\gamma} (\frac{1}{2} R + S) < \frac{3}{2} S (\beta^h - u_o) < \pi_{2rs}^* \) for \( \gamma \) sufficiently large. (We use the condition that \( \beta^h > u_o \) by a sufficient amount.) We establish \( \pi_{1rr}^* < \pi_{1rs}^* \) by similar arguments recognizing that, by symmetry, \( \pi_{1rr}^* = \pi_{2rr}^* \), and using the proven result that \( \pi_{1rs}^* > \pi_{1rs}^* \) is greater than \( \frac{1}{3} R (\beta^h - u_o) \) for sufficiently large \( \gamma \).

**Result 3.** \( \pi_{1rr}^* = \pi_{2rr}^* > \pi_{1ss}^* = \pi_{2ss}^* \).

**Proof.** We examine the equations for the segment-based market shares to recognize that

\[
P_{R1rr}(p_{1ss}^*, p_{2ss}^*) = P_{S1ss}(p_{1ss}^*, p_{2ss}^*) and P_{S1rr}(p_{1ss}^*, p_{2ss}^*) = P_{R1ss}(p_{1ss}^*, p_{2ss}^*), and P_{S1ss}(p_{1ss}^*, p_{2ss}^*) > P_{R1ss}(p_{1ss}^*, p_{2ss}^*).
\]

Thus, \( \pi_{1rr}^* = \pi_{1rr}^*(p_{1ss}^*, p_{2ss}^*) \geq \pi_{1rr}^*(p_{1ss}^*, p_{2ss}^*) = p_{1ss}^*[RP_{R1rr}(p_{1ss}^*, p_{2ss}^*) + S P_{S1rr}(p_{1ss}^*, p_{2ss}^*)] = p_{1ss}^*[S P_{S1ss} + R P_{R1ss}] = p_{1ss}^* \). The second inequality is by the principle of optimality. The last inequality uses \( R > S \) and \( P_{S1ss}^* > P_{R1ss}^* \). The equalities, \( \pi_{1rr}^* = \pi_{2rr}^* \) and \( \pi_{1ss}^* = \pi_{2ss}^* \), are by symmetry.

**Result 4.** Suppose \( \beta^h \) is sufficiently larger than \( u_o \) and \( u_o \geq \beta^s \). Then, there exists a sufficiently large \( \gamma \) such that \( \pi_{1rs}^* > \pi_{1sr}^* \).

**Proof.** By symmetry, we recognize that \( \pi_{1sr}^* = \pi_{2rs}^* \). In the proof to Result 2 we established that

\[\pi_{2rs}^* \equiv S(\beta^h - u_o) P_{S2rs}^* and \pi_{1rs}^* \equiv R(\beta^h - u_o) P_{R1rs}^*\]

because \( \epsilon \to 0 \) as \( \gamma \to \infty \). We also see that the fixed-point problems are identical for \( p_{1rs}^* \) and \( p_{2rs}^* \), thus, as \( \gamma \to \infty \), \( p_{1rs}^* \approx p_{2rs}^* \), which implies that \( P_{R1rs}^* \equiv P_{S2rs}^* \). Putting these relationships together implies that \( \pi_{1rs}^* \equiv R(\beta^h - u_o) P_{R1rs}^* > S(\beta^h - u_o) P_{R1rs}^* \equiv S(\beta^h - u_o) P_{S2rs}^* \equiv \pi_{2rs}^* = \pi_{1sr}^* \).

**Proposition 1.** For \( \gamma \to 0 \), the innovator targets \( r \) and the follower targets \( s \).

**Proposition 2.** If \( \beta^h \) is sufficiently larger than \( u_o \) and \( u_o \geq \beta^s \), then there exists a sufficiently large \( \gamma \) such that the innovator targets \( r \) and the follower targets \( s \).

**Proof.** We prove the two propositions together. Result 1 establishes that \( \pi_{2rr}^* > \pi_{2rs}^* \) as \( \gamma \to 0 \). Result 2 establishes that \( \pi_{2rs}^* > \pi_{2rr}^* \) when \( \gamma \) is sufficiently large. Thus, if Firm 1 chooses \( r \), Firm 2 chooses \( r \) as \( \gamma \to 0 \) and chooses \( s \) when \( \gamma \) gets sufficiently large.

To prove that Firm 1 always chooses \( r \), we first consider the case where \( \gamma \to 0 \). If Firm 1 chooses \( r \), then Firm 2 chooses \( r \) by Proposition 1. Suppose instead that Firm 1 chooses \( s \), then Firm 2 will choose \( r \).
Firm 2 will choose $r$ in this case because, by Result 1, $\pi^{rs}_1 > \pi^{rs}_2$ and, by symmetry, $\pi^{s2}_r = \pi^{st}_r$, hence $\pi^{s2}_r > \pi^{st}_r = \pi^{sr}_2$. If Firm 2 would choose $r$ whenever Firm 1 chooses $s$, Firm 1 would earn $\pi^{rs}_1$. But $\pi^{st}_r = \pi^{sr}_2$ by symmetry and $\pi^{s2rs}_1 < \pi^{s2sr}_1 = \pi^{s1sr}_1$ by Result 1. Thus, Firm 1 earns more profits ($\pi^{s1sr}_1$) by choosing $r$ than the profits it would obtain ($\pi^{st}_r$) by choosing $s$.

We now consider the case where $y$ is sufficiently large. Suppose Firm 1 chooses $r$, then Firm 2 will choose $s$ by Result 2. Firm 1 receives $\pi^{rs}_1$. Suppose instead that Firm 1 chooses $s$, then Firm 2 will choose $r$ because $\pi^{s2sr}_1 = \pi^{s1rs}_1$ by symmetry and $\pi^{s2rs}_1 > \pi^{s1ss}_1 = \pi^{s2ss}_1$ under the conditions of Result 2. Thus, if Firm 1 chooses $s$ it receives $\pi^{s1sr}_1$. Because $\pi^{s1sr}_1 > \pi^{st}_r$ by Result 4, Firm 1 will choose $r$. □

Existence and Uniqueness. The existence and uniqueness arguments require substantial algebra. To avoid an excessively long appendix, we provide the basic insight. Detailed calculations are available from the authors. The proofs to Results 1-4 rely on the first-order conditions, thus we must show that a solution to the first-order conditions, if it exists, satisfies the second-order conditions. The second-order conditions for the $rs$ positions are.

$$\frac{\partial^2 \pi^{rs}_1}{\partial p^{rs}_{1rs}^2} = -y R P^{rs}_{1rs}(1 - P^{rs}_{1rs})(2 - \gamma \pi^{rs}_1(1 - 2P^{rs}_{1rs}))(2 - \gamma \pi^{rs}_1(1 - 2P^{rs}_{1rs}))$$

We use Lemma 1 to substitute $(1 - P^{rs}_{1rs})^{-1}$ for $\gamma \pi^{rs}_1$. The former is a larger value, so if the conditions hold for the larger value, they hold for $\gamma \pi^{rs}_1$. Algebra simplifies the right-hand side of the second-order condition to $-\gamma R P^{rs}_{1rs}(1 - P^{rs}_{1rs}) + SP^{rs}_{1rs}(1 - 2P^{rs}_{1rs})$. With direct substitution in the logit model, recognizing $P^{rs}_{1rs} \geq P^{rs}_{2rs}$, we show $P^{rs}_{S2rs} \geq P^{rs}_{R1rs} \geq P^{rs}_{R2rs} \geq P^{rs}_{S1rs}$. (We show $P^{rs}_{1rs} \geq P^{rs}_{2rs}$ with implicit differentiation of the first-order conditions with respect to $R$.) These inequalities imply that $P^{rs}_{S1rs} < \min(P^{rs}_{R1rs}, 1 - P^{rs}_{R1rs})$. Hence, $R P^{rs}_{R1rs}(1 - P^{rs}_{R1rs}) \geq SP^{rs}_{1rs}(1 - P^{rs}_{1rs})$ whenever $R > S$. Thus, the second-order condition is more negative than $-\gamma SP^{rs}_{1rs}(1 - P^{rs}_{1rs})(2 - 2P^{rs}_{1r} + 2P^{rs}_{1s}) < 0$. We repeat the analysis for $p^{rs}_{2rs}$ using a sufficient technical condition that either $P^{rs}_{S2rs} \leq \frac{1}{2}$ or that the ratio of $S/R$ is above a minimum value. (The condition, not shown here, requires only $S > 0$ as $y \to \infty$.) Although our proof formally imposes the technical sufficient condition, we have not found any violation of the second-order conditions at equilibrium, even with small $S$. Thus, with a (possible) mild restriction on $S$, the second-order conditions are satisfied whenever the first-order conditions hold.

We now establish that the second-order conditions are satisfied on a compact set. We begin by showing algebraically that $(1 - P^{rs}_{1rs})^{-1}$ is decreasing in $p^{rs}_{1rs}$ and that it decreases from a finite positive value, which we call $F_{R1rs}(p^{rs}_{1rs} = 0) > 1$. As $p^{rs}_{1rs} \to \infty$, $(1 - P^{rs}_{1rs})^{-1}$ decreases to 1. But $\gamma \pi^{rs}_1$ increases from 0 to $\infty$, thus there must be a solution to $\gamma \pi^{rs}_1 = (1 - P^{rs}_{1rs})^{-1}$ for every $p^{rs}_{2rs}$. Call this solution $p^{ms}_{rs}(p^{rs}_{2rs})$. Because $(1 - P^{rs}_{1rs})^{-1}$ is decreasing in $\gamma \pi^{rs}_1$, it must be true that $\gamma \pi^{rs}_1 \leq (1 - P^{rs}_{1rs})^{-1}$ for all $p^{rs}_{1rs} \in [0, p^{ms}_{rs}(p^{rs}_{2rs})]$. Using similar arguments we show there exists a $p^{ms}_{rs}(p^{rs}_{1rs})$ such that

$$\gamma \pi^{rs}_1 \leq (1 - P^{rs}_{2rs})^{-1} \text{for all } p^{rs}_{2rs} \in [0, p^{ms}_{rs}(p^{rs}_{1rs})].$$

Together $p^{rs}_{1rs} \in [0, p^{ms}_{rs}(p^{rs}_{2rs})]$ and $p^{rs}_{2rs} \in [0, p^{ms}_{rs}(p^{rs}_{1rs})]$ define a compact set that is a subset of $p^{rs}_{1rs} \in [0, p^{ms}_{rs}(0)]$ and $p^{rs}_{2rs} \in [0, p^{ms}_{rs}(0)]$. ($p^{ms}_{rs}(p^{rs}_{2rs})$ is continuous and decreasing in $p^{rs}_{2rs}$ and $p^{ms}_{rs}(p^{rs}_{1rs})$ is continuous and decreasing in $p^{rs}_{1rs}$; $p^{ms}_{rs}(p^{rs}_{2rs}), p^{ms}_{rs}(p^{rs}_{1rs}) > 0$.) We have already established that $P^{rs}_{S2rs} \geq P^{rs}_{R1rs} \geq P^{rs}_{R2rs} \geq P^{rs}_{S1rs}$ when $p^{rs}_{1rs} \geq p^{rs}_{2rs}$. If we restrict the compact set to $p^{rs}_{1rs} \geq p^{rs}_{2rs}$ and the price difference is not too large, we
have \( P_{S2rS} \geq P_{R1rS} \geq P_{R2rS} \geq P_{S1rS} \) on the set. This simplifies the proof, but is not necessary. Thus, we can choose a compact set such that \( \gamma p_{1rS} = (1 - P_{R1rS})^{-1} \), \( \gamma p_{2rS} \leq (1 - P_{S2rS})^{-1} \), and \( P_{S2rS} \geq P_{R1rS} \geq P_{R2rS} \geq P_{S1rS} \) on the set. This set contains the interior solution to the first-order conditions. Using arguments similar to those we used for the equilibrium prices, we establish that the second-order conditions hold on this compact set. If necessary, we impose a weak technical condition on \( S/R \). This implies that both profit functions are concave on the compact set. Concavity on a compact set guarantees that the solution exists and, by the arguments in the previous paragraph, that the solution is an interior solution. Numerical calculations, for a wide variety of parameter values, suggest that the second-order conditions hold on the compact set, that the second-order conditions hold outside the set (the restrictions are sufficient but not necessary), that the second-order conditions hold for prices satisfying \( p_{2rS} > p_{1rS} \), and that, at equilibrium, the second-order conditions hold for all \( S \).

The proof for the \( rr \) positions follows arguments that are similar to those for the \( rs \) positions. We do not need the technical condition on \( S \) because \( P_{S2rr} \leq \frac{1}{2} \) implies that \( S > 0 \) is sufficient. The compact set is simpler because \( p_{1rr}^* = p_{2rr}^* \) by symmetry. The proofs for the \( sr \) and \( ss \) positions use related conditions and follow the logic of the proofs for the \( rs \) and \( rr \) positions. \( \square \)

Uniqueness requires that we examine the cross-partial derivatives, illustrated here for \( rs \):

\[
\frac{\partial^2 \pi_{1rS}}{\partial p_{1rS} \partial p_{2rS}} = \gamma p_{1S} p_{2S} [1 - \gamma p_{1S} (1 - 2p_{1rS})] + \gamma p_{R1S} p_{S2S} [1 - \gamma p_{1S} (1 - 2p_{S1rS})]
\]

Restricting ourselves to the a compact set as in the existence arguments, we can use \( \gamma p_{1rS} \leq 1/(1 - P_{R1rS}) \), \( \gamma p_{2rS} \leq 1/(1 - P_{S2rS}) \), and \( p_{S2rS} \geq P_{R1rS} \geq P_{R2rS} \geq P_{S1rS} \). We substitute to show that, when the cross-partial derivative is positive (similar conditions and a similar proof applies when it is negative):

\[
\frac{\partial^2 \pi_{1rS}}{\partial p_{1rS} \partial p_{2rS}} \geq \frac{\gamma}{1 - P_{R1rS}} \left[ (R p_{R1S} (1 - P_{R1rS})^2 + R (1 - P_{R1rS} - P_{R2rS}) p_{R1rS}^2 + S p_{S1rS} (1 - P_{S1rS}) (1 - P_{R1rS}) + S p_{S1rS} (1 - P_{S1rS} - P_{S2rS}) (2P_{S1rS} - P_{R1rS}) \right]
\]

We substitute further to show the third term on the right-hand side is larger than the, possibly negative, fourth term. Hence, the cross-partial condition is positive for \( \pi_{1rS} \) on the compact set. The cross-partial condition for \( \pi_{2rS} \) is satisfied with a technical condition on \( S \). Numerical calculations, for a wide variety of parameter values, suggest that the cross-partial conditions hold on the compact set, that the cross-partial conditions hold outside the set (the restrictions are sufficient but not necessary), that the cross-partial conditions hold for prices satisfying \( p_{2rS} > p_{1rS} \), and that, at equilibrium, the cross-partial conditions hold for all \( S \).

In summary, subject to (possible) technical conditions on the magnitude of \( S \), we have proven that interior-solution price equilibria exist and are unique. At minimum, we have shown that this is true for many, if not most, markets—markets satisfying the technical conditions on \( S/R \). \( \square \)
Corollary 1. Firm 1 selects r for both $\gamma^\text{lower}$ and $\gamma^\text{higher}$.

Proof. The result follows directly from Proposition 1 and Proposition 2. Firm chooses r if $\gamma^\text{lower} < \gamma^\text{cutoff}$ by Proposition 1 and chooses r if $\gamma^\text{lower} \geq \gamma^\text{cutoff}$ by Proposition 2. Thus, Firm 1 chooses r independently of $\gamma^\text{lower}$. We use the same arguments to show that Firm 1 chooses r independently of $\gamma^\text{higher}$. (The result also requires continuity of the profit functions, proven elsewhere.) □

Corollary 2. If Firm 2 acts on $\gamma^\text{lower}$ and $\gamma^\text{lower} \neq \gamma^\text{true}$, then Firm 2 might choose the strategy that does not maximize profits.

Proof. We provide two examples. If $\gamma^\text{lower} < \gamma^\text{cutoff}$ and $\gamma^\text{true} > \gamma^\text{cutoff}$, then, if Firm 2 acts on $\gamma^\text{lower}$ it will choose s by Proposition 1, but the profit-maximizing decision is r by Proposition 2. If $\gamma^\text{true} < \gamma^\text{cutoff} < \gamma^\text{lower}$, then Firm 2 will choose r when it’s profit-maximizing decision is to choose s. The word “might” is important. Firm 2 will choose the correct strategy, even if $\gamma^\text{lower} \neq \gamma^\text{true}$ when both $\gamma^\text{lower}$ and $\gamma^\text{true}$ are on the same side of $\gamma^\text{cutoff}$. □