Online Supplement for "A New Local Search Algorithm for Binary Optimization"

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The interested reader can find below the continuation of the set-packing example (2) from Section 2, which we restate for ease of exposition:

$$\begin{array}{llllll} \max & x_1 & +x_2 & +x_3 & \\ \text{s.t.} & x_1 & & +x_3 & \leq 1 \\ & & x_2 & +x_3 & \leq 1 \\ & \multicolumn{4}{c}{x_1, x_2, x_3 \in \{0,1\}} \end{array}$$

- (Steps taken in Section 2, ending with the following values for the variables:

  $$\boldsymbol{z} = [1,1,0];\ \mathcal{SL} = \{\,[1,1,0]\,\};\ \mathcal{TB}[i] = -\infty, \forall\, i. \quad)$$

- (Step 3) $\boldsymbol{x} \leftarrow [1,1,0]$. Adjacent solutions are $[0,1,0], [1,0,0], [1,1,1]$.
  - (Step 4) $\boldsymbol{y} = [0,1,0], \mathit{trace}(\boldsymbol{y}) = [0,0;1,0]$.
    - (Step 5) $\boldsymbol{y}$ feasible, but $\boldsymbol{e}'\boldsymbol{y} = 1 < \boldsymbol{e}'\boldsymbol{z}\,(=2)$.
    - (Step 9) (A1)-(A3) true, so $\boldsymbol{y}$ is interesting.
    - (Steps 10 - 11) $\mathcal{TB}[2] \leftarrow 1;\ \mathcal{SL} \leftarrow \{\,[0,1,0]\,\}$.
  - (Step 4) $\boldsymbol{y} = [1,0,0], \mathit{trace}(\boldsymbol{y}) = [0,0;0,1]$.
    - (Step 5) $\boldsymbol{y}$ feasible, but $\boldsymbol{e}'\boldsymbol{y} = 1 < \boldsymbol{e}'\boldsymbol{z}\,(=2)$.
    - (Step 9) (A1)-(A3) true, so $\boldsymbol{y}$ is interesting.
    - (Steps 10 - 11) $\mathcal{TB}[1] \leftarrow 1;\ \mathcal{SL} \leftarrow \{\,[0,1,0];[1,0,0]\,\}$.
  - (Step 4) $\boldsymbol{y} = [1,1,1], \mathit{trace}(\boldsymbol{y}) = [1,1;0,0]$.

* (Step 5) $\boldsymbol{y}$ infeasible.
* (Step 9) (A2) false, since $\|trace(\boldsymbol{y}) - trace(\boldsymbol{z})\|_1 = \|[1,1;0,0]\|_1 = 2 > Q$, so $\boldsymbol{y}$ is not interesting.

- (Step 2) $\mathcal{SL} = \{\,[0,1,0];[1,0,0]\,\} \neq \emptyset$.
- (Step 3) $\boldsymbol{x} \leftarrow [0,1,0]$. Adjacent solutions are $[1,1,0],[0,0,0],[0,1,1]$.
  - (Step 4) $\boldsymbol{y} = [1,1,0], trace(\boldsymbol{y}) = [0,0;0,0]$.
    * (Step 5) $\boldsymbol{y}$ feasible, but $\boldsymbol{e}'\boldsymbol{y} = 2 = \boldsymbol{e}'\boldsymbol{z}$.
    * (Step 9) (A1)-(A3) true, so $\boldsymbol{y}$ is interesting.
    * (Steps 10 - 11) $\mathcal{TB}[0] \leftarrow 2$; $\mathcal{SL} \leftarrow \{\,[1,0,0];[1,1,0]\,\}$.
  - (Step 4) $\boldsymbol{y} = [0,0,0], trace(\boldsymbol{y}) = [0,0;1,1]$.
    * (Step 5) $\boldsymbol{y}$ feasible, but $\boldsymbol{e}'\boldsymbol{y} = 0 < \boldsymbol{e}'\boldsymbol{z}\,(=2)$.
    * (Step 9) (A2) false, since $\|trace(\boldsymbol{y}) - trace(\boldsymbol{z})\|_1 = \|[0,0;1,1]\|_1 = 2 > Q$, so $\boldsymbol{y}$ is not interesting.
  - (Step 4) $\boldsymbol{y} = [0,1,1], trace(\boldsymbol{y}) = [0,1;0,0]$.
    * (Step 5) $\boldsymbol{y}$ infeasible.
    * (Step 9) (A1)-(A3) true, so $\boldsymbol{y}$ is interesting.
    * (Steps 10 - 11) $\mathcal{TB}[4] \leftarrow 2$; $\mathcal{SL} \leftarrow \{\,[1,0,0];[1,1,0];[0,1,1]\,\}$.
- (Step 2) $\mathcal{SL} = \{\,[1,0,0];[1,1,0];[0,1,1]\,\} \neq \emptyset$.
- (Step 3) $\boldsymbol{x} \leftarrow [1,0,0]$. Adjacent solutions are $[0,0,0],[1,1,0],[1,0,1]$.
  - (Step 4) $\boldsymbol{y} = [0,0,0], trace(\boldsymbol{y}) = [0,0;1,1]$.
    * (Step 5) $\boldsymbol{y}$ feasible, but $\boldsymbol{e}'\boldsymbol{y} = 0 < \boldsymbol{e}'\boldsymbol{z}\,(=2)$.
    * (Step 9) (A2) false, since $\|trace(\boldsymbol{y}) - trace(\boldsymbol{z})\|_1 = \|[0,0;1,1]\|_1 = 2 > Q$, so $\boldsymbol{y}$ is not interesting.
  - (Step 4) $\boldsymbol{y} = [1,1,0], trace(\boldsymbol{y}) = [0,0;0,0]$.
    * (Step 5) $\boldsymbol{y}$ feasible, but $\boldsymbol{e}'\boldsymbol{y} = \boldsymbol{e}'\boldsymbol{z} = 2$.
    * (Step 9) (A3) false, since $\boldsymbol{e}'\boldsymbol{y} = \mathcal{TB}[0] = 2$, so $\boldsymbol{y}$ is not interesting.
  - (Step 4) $\boldsymbol{y} = [1,0,1], trace(\boldsymbol{y}) = [1,0;0,0]$.

    * (Step 5) $\boldsymbol{y}$ infeasible.
    * (Step 9) (A1)-(A3) true, so $\boldsymbol{y}$ is interesting.
    * (Steps 10 - 11) $\mathcal{TB}[8] \leftarrow 2$; $\mathcal{SL} \leftarrow \{\, [1,1,0]; [0,1,1]; [1,0,1] \,\}$.

- (Step 2) $\mathcal{SL} = \{\, [1,1,0]; [0,1,1]; [1,0,1] \,\} \neq \emptyset$.
- (Step 3) $\boldsymbol{x} \leftarrow [1,1,0]$. Adjacent solutions are $[0,1,0], [1,0,0], [1,1,1]$.
  - (Step 4) $\boldsymbol{y} = [0,1,0]$, $\mathit{trace}(\boldsymbol{y}) = [0,0;1,0]$.
    * (Step 5) $\boldsymbol{y}$ feasible, but $\boldsymbol{e}'\boldsymbol{y} = 1 < \boldsymbol{e}'\boldsymbol{z}\,(=2)$.
    * (Step 9) (A3) false, since $\boldsymbol{e}'\boldsymbol{y} = 1 = \mathcal{TB}[2]$, so $\boldsymbol{y}$ is not interesting.
  - (Step 4) $\boldsymbol{y} = [1,0,0]$, $\mathit{trace}(\boldsymbol{y}) = [0,0;0,1]$.
    * (Step 5) $\boldsymbol{y}$ feasible, but $\boldsymbol{e}'\boldsymbol{y} = 1 < \boldsymbol{e}'\boldsymbol{z}\,(=2)$.
    * (Step 9) (A3) false, since $\boldsymbol{e}'\boldsymbol{y} = \mathcal{TB}[1] = 1$, so $\boldsymbol{y}$ is not interesting.
  - (Step 4) $\boldsymbol{y} = [1,1,1]$, $\mathit{trace}(\boldsymbol{y}) = [1,1;0,0]$.
    * (Step 5) $\boldsymbol{y}$ infeasible.
    * (Step 9) (A2) false, so $\boldsymbol{y}$ is not interesting.
- (Step 2) $\mathcal{SL} = \{\, [0,1,1]; [1,0,1] \,\} \neq \emptyset$.
- (Step 3) $\boldsymbol{x} \leftarrow [0,1,1]$. Adjacent solutions are $[1,1,1], [0,0,1], [0,1,0]$.
  - (Step 4) $\boldsymbol{y} = [1,1,1]$. Infeasible, and not interesting.
  - (Step 4) $\boldsymbol{y} = [0,0,1]$, $\mathit{trace}(\boldsymbol{y}) = [0,0;0,0]$.
    * (Step 5) $\boldsymbol{y}$ feasible, but $\boldsymbol{e}'\boldsymbol{y} = 1 < \boldsymbol{e}'\boldsymbol{z}\,(=2)$.
    * (Step 9) (A3) false, since $\boldsymbol{e}'\boldsymbol{y} = 1 < \mathcal{TB}[0] = 2$, so $\boldsymbol{y}$ is not interesting.
  - (Step 4) $\boldsymbol{y} = [0,1,0]$. Feasible, but not better than $\boldsymbol{z}$. Not interesting, since (A3) false.
- (Step 2) $\mathcal{SL} = \{\, [1,0,1] \,\} \neq \emptyset$.
- (Step 3) $\boldsymbol{x} \leftarrow [1,0,1]$. Adjacent solutions are $[0,0,1], [1,1,1], [1,0,0]$.
  - (Step 4) $\boldsymbol{y} = [0,0,1]$. Feasible, but not better than $\boldsymbol{z}$. Not interesting, since (A3) false.

  - (Step 4) $\boldsymbol{y} = [1, 1, 1]$. Infeasible. Not interesting, since (A2) false.
  - (Step 4) $\boldsymbol{y} = [1, 0, 0]$. Feasible, but not better than $\boldsymbol{z}$. Not interesting, since (A3) false.

- (Step 2) $\mathcal{SL} = \emptyset$.
- (Step 12) Return $z = [1, 1, 0]$.