Garicano, Luis, Claire Lelarge and John Van Reenen. 2016. "Firm Size Distortions and the Productivity Distribution: Evidence from France." American Economic Review, 106(11): 3439-79.
DOI: 10.1257/aer.20130232

ISSN 2042-2695

CENTRE *for* ECONOMIC
PERFORMANCE

CEP Discussion Paper No 1128

Revised June 2016
(Replaced February 2012 version)

Firm Size Distortions and the Productivity Distribution: Evidence from France

Luis Garicano
Claire Lelarge
John Van Reenen

LSE THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

E·S·R·C ECONOMIC & SOCIAL RESEARCH COUNCIL

Abstract
We show how size-contingent laws can be used to identify the equilibrium and welfare effects of labor regulation. Our framework incorporates such regulations into the Lucas (1978) model and applies it to France where many labor laws start to bind on firms with 50 or more employees. Using population date on firms between 1995 and 2007, we structurally estimate the key parameters of our model to construct counterfactual size, productivity and welfare distributions. We find that the cost of these regulations is equivalent to that of a 2.3% variable tax on labor. In our baseline case with French levels of partial real wage inflexibility welfare costs of the regulations are 3.4% of GDP (falling to 1.3% if real wages were perfectly flexible downwards). The main losers from the regulation are workers – and to a lesser extent, large firms – and the main winners are small firms.

Keywords: firm size, productivity, labor regulation, power law
JEL classification: L11; L51; J8; L25.6

This paper was produced as part of the Centre's Growth Programme. The Centre for Economic Performance is financed by the Economic and Social Research Council.

Acknowledgements
This is a revised version of Garicano, Lelarge and Van Reenen (2013). The first draft of this paper was produced and presented at the MIT Organizational Economics seminar in May 2011. We would like to thank the editor, Luigi Pistaferri and three anonymous referees for very helpful comments. We would also like to thank Ufuk Akcigit, Joe Altonji, Samuel Bentolila, Nick Bloom, Richard Blundell, Pierre Cahuc, Jesus Fernandez-Villaverde, Amy Finkelstein, Chang-Tai Hsieh, Eddie Lazear, Costas Meghir, Esteban Rossi-Hansberg, Jesse Shapiro and participants of seminars in Brown, Chicago, Columbia GSB, Cornell, CREI, CREST, Insead, Louvain, LSE, MIT, Nantes, NBER, Oxford, Penn, Princeton, Stanford GSB, UCL, USC and Yale. We separately thank our formal discussants Panle Jia and Ariel Pakes. Finance for this project has been kindly provided by the Economic and Social Research Council, International Growth Centre and the European Commission. The authors declare that they have no relevant or material financial interests that relate to the research described in this paper.

Luis Garicano, London School of Economics, Centre for Economic Performance and CEPR. Claire Lelarge, Centre for Economic Performance, foreign affiliate. John Van Reenen, London School of Economics, Centre for Economic Performance, CEPR and NBER.

Published by
Centre for Economic Performance
London School of Economics and Political Science
Houghton Street
London WC2A 2AE

1 Introduction

A recent literature has documented empirically how distortions can affect aggregate productivity through misallocating resources towards less productive firms. These distortions mean efficient firms produce too little output and employ too few workers (Restuccia and Rogerson, 2008). Hsieh and Klenow (2009) argue that such misallocations account for a significant proportion of the difference in aggregate productivity between the US, China and India.[1] However, the causes of the distortions remain a "black box" in these approaches. In this paper, we focus on understanding the impact of one specific distortion on the French firm size distribution: regulations that increase labor costs when firms reach 50 workers. This type of size dependent regulation is relevant to many debates around the world. For example, under the US Affordable Care Act there are penalties on firms with more than 50 employees that do not offer health care insurance to their employees.[2] Hence, critics have claimed that this will reduce the incentives for efficient firms to grow large whereas supporters are skeptical about the magnitude of any such effect.

The idea that misallocations of resources lie behind aggregate productivity gaps is attractive in understanding some differences in economic performance between the US and Europe.[3] According to the European Commission (1996) the average production unit in the EU employed 23% less workers than in the US. Consistent with this, Figure 1 shows that there is a lower proportion of relatively large firms in France compared with the US. In particular there is a large bulge in the number of firms with employment just below 50 workers in France, but not in the US. This is also illustrated in Figure 2 which shows the exact number of French manufacturing firms by number of workers in the year 2000. There is a sharp fall in the number of firms with exactly 50 employees compared to those who have 49 employees.

The burden of French legislation substantially increases when firms employ 50 or more workers. As we explain in detail below, firms of 50 workers or more must create a works council ("comité d'entreprise"), establish a health and safety committee, report detailed information on all employees to the Labor Ministry, appoint a union representative and so on. What are the implications for firm size, firm productivity and aggregate welfare from those laws? Intuitively, some more productive firms that would have been larger without the regulation choose to remain below the legal threshold to avoid these costs. In addition, the higher labor costs make firms above the threshold smaller than they would be in the unregulated economy. In this paper we show how these two sets of changes in the firm size distribution can be exploited to infer the level and distribution of the welfare (and employment) cost of such regulations.

There has been extensive discussion of the importance of labor laws for unemployment and productivity. The

---

[1] Many empirical papers have shown that deregulation (e.g. Olley and Pakes, 1996), higher competition (e.g. Syverson, 2004) and trade liberalization (e.g. Pavcnik, 2002) have tended to improve reallocation by increasing the correlation between firm size and productivity.

[2] To be precise, there are penalties for firms with more than 50 full-time employees who (i) do not offer health coverage and (ii) pay workers too little to buy coverage on their own without using federal subsidies. The penalty is $2,000 for each employee (except for the first 30 employees). If the firm has more than 50 full-time employees and offers some of them coverage but others have to apply for federal subsidies to buy coverage themselves, the firm must pay the lesser of $3,000 for each employee receiving insurance subsidies or $2,000 for each full-time employee (again excluding the first 30 employees). For more details see: http://www.washingtonpost.com/blogs/wonkblog/wp/2012/11/19/cheer-up-papa-johns-obamacare-gave-you-a-good-deal/

[3] Bartelsman, Haltiwanger and Scarpetta (2013) examine misallocation using micro-data across eight OECD countries. They argue that consumption is more than 10% below US levels in some European nations because of misalloaction. In particular, they find that the "Olley Pakes" (1996) covariance term between size and productivity is much smaller in France (0.24 in their Table 1) and other European countries compared to the US (0.51 in their Table 1). Bloom, Sadun and Van Reenen (2015) also report a more efficient allocation of employment to better managed firms in the US than in Europe and developing countries.

OECD and other agencies have developed indices of the importance of these regulations, based on detailed analysis of the laws and expert surveys. It is hard, however, to see how these can be rigorously quantified, since "adding up" the regulatory provisions has a large arbitrary component. A contribution of our paper is a methodology for quantifying the tax equivalent of a regulation, albeit in the context of a specific model. The calculation is extremely transparent, economically intuitive and can be applied in many other settings.

We start by setting up a simple model following Lucas' (1978) observation that the economy-wide observed resource distribution results from allocating productive factors over managers of different ability so as to maximize output.[4] Given limits to managerial time or attention, the better managers are allocated more workers to manage which results in a "scale-of-operations" effect whereby differences in talent are amplified by the resources allocated. We show that there are four main effects of a size-dependent labor tax with variable and fixed cost components in such a Lucas (1978) style model: (1) Equilibrium wages fall as a result of the reduction in the demand for workers (i.e. some of the tax incidence of regulation falls on workers); (2) Firm size increases for all firms below the regulatory threshold as a result of the general equilibrium effect on wages; (3) Firm size decreases to precisely the regulatory threshold for a set of firms that are not productive enough to justify incurring the new regulatory costs; and (4) Firm size decreases proportionally if the variable cost component dominates (as it does empirically) for all firms that are productive enough to incur the additional cost of regulation.

We use the model to guide our estimation of the impact of these regulatory costs. The theory tells us there is a deviation from the "correct" firm size distribution as a result of the regulation. That is, we expect to see a departure from the usual power law firm size distribution[5] as firms bunch up below the threshold of 50 workers. Given factors such as measurement error, however, the observed empirical departure from the power law is not just at 49 workers but also affects firms of slightly smaller sizes causing a "bulge". Similarly, there is not precisely zero mass to the right of the threshold, but rather a "valley" where there are significantly fewer firms than we would expect from an unbroken power law. Then, at some point the firm size distribution becomes again a power law, with a lower intercept (in log-log space) if the variable cost dominates. The break in the power law from the bulge and valley of firms around the threshold, together with the downward shift in the power law, empirically identify the magnitude of the distortion.[6] We formally allow for measurement error and also develop a dynamic extension to the model with adjustment costs to understand the presence of some firms immediately to the right of the regulatory threshold.

We find that the increase in the per worker variable cost (identified from the downward shift in the power law after

---

[4] In a model of this kind, the source of decreasing returns are on the production size, and are linked to limits to managerial time. For our purposes here, as Hsieh and Klenow (2009) show, this source of decreasing returns is equivalent to having the decreasing returns come from the demand/utility side (as is more common in the recent literature following Melitz, 2003).

[5] This means the probability density function of firm size is log-linear (e.g. Axtell, 2001). There is a large literature on the size and productivity distribution of firms in macro, trade, finance and IO. Appropriately, the first major study in this area was by Gibrat (1931) who studied French industrial firms, the main focus of the empirical part of our paper.

[6] Our descriptive findings are consistent with earlier findings documented in a publication of the French National Statistical Institute (Insee) by Ceci-Renaud and Chevalier (2011). These authors use the same datasets for employment and thus our descriptive statistics in terms of the firm size distribution are very similar. Their econometric analysis is an implementation of Schivardi and Torrini (2008) which amounts to estimating the distortive impact of the regulations on the firm size distribution by assuming it to be a fixed cost. In contrast to their analysis, we allow the regulation to take a more general form (in particular, we include variable cost components which appear to be the most important component of the cost), and we propose a general equilibrium approach which enables to assess the impact of the regulation on entry and prices (and the firm size distortions they generate).

49 workers) is large - about 2.3%. What are the welfare losses due to these costs? It is not just the labor regulation, but also the interaction between regulation and downwardly rigid real wages that causes large welfare losses. When wages are fully flexible, the aggregate welfare loss is about 1% of GDP. There are also important distributional consequences: aggregate wages and the profits of large firms drop by more than 1%, but profits of smaller firms *rise* by about 7% because they enjoy lower equilibrium wages without suffering the regulatory burden. In France, the high minimum wage and powerful unions, mean that real wages are partially inflexible. Taking this into account we find that GDP falls by 3.5% mainly through increased unemployment, but also by keeping the most productive firms below their optimal size. Too many employees work for smaller firms, and too few for large firms. In the US, where wages are more flexible than France, one could speculate that similar regulations have smaller welfare effects.

In terms of the existing literature, one closely related paper is Braguinsky, Branstetter and Regateiro (2011) who seek to explain changes in the support of the Portuguese firm size distribution in the context of the Lucas model with labor regulations.[7] Their calibrations also show substantial effects of the regulations on aggregate productivity. Portuguese laws have multiple regulatory thresholds, however, and their firm size data does not show a clear mass point like ours. Thus their approach cannot exploit the discontinuity to identify the structural parameters of their model. Our paper is also related to the more general literature using tax "kinks" to identify behavioral parameters (e.g. Saez, 2010; Chetty et al, 2011; Kleven and Waseem, 2013). Kaplow (2013) discusses issues in the optimal structure of size-related regulations. Finally, our approach of "pricing out" the cost of regulation as a tax is in the spirit of Posner (1971).

The structure of the paper is as follows. Section 2 describes our theory, Section 3 the empirical strategy, Section 4 the institutional setting and data and Section 5 the main results. We start by showing that the main empirical predictions of the model in terms of the size and productivity distribution are consistent with the data, and we then estimate the parameters of the structural model and use this to show the welfare and distributional impact of the regulation. We present various extensions and robustness tests in Section 6. Here we show our results are robust to: considering ways to evade the regulations; allowing for parameter heterogeneity across industries; introducing a second regulatory threshold; allowing for labor-capital substitution; using a Hsieh-Klenow (2009) approach and introducing dynamic considerations. We offer some concluding comments in Section 7. Our extensive Web Appendices contain more details on theoretical proofs (A); the econometric estimation (B); various robustness checks (C); capital-labor substitutability (D); dynamics (E) and the regulations themselves (F).

2 Theory

Consider the simplest possible version of Lucas (1978), where there is only one input in production, labor.[8] The primitive of the model is the pdf $\phi(\alpha)$ of "managerial ability" α, $\phi : [\underline{\alpha}; \alpha_{\max}[\rightarrow \mathbb{R}$. We assume that talent is scarce in

---

[7] On the quantitative theory side Guner, Ventura and Yi (2006, 2008) also consider a Lucas model with size-contingent regulation. They calibrate this to uncover sizeable welfare losses. Unlike our paper and Braguinsky et al (2011), however, there is no econometric application.

[8] As in Lucas, this is a one sector economy. For size-contingent regulations in a multi-sector Lucas (1978) model see Garcia-Santana and Pijoan-Mas (2014) who apply it to Indian textiles.

the sense that $\alpha\phi(\alpha)$ is decreasing in α (i.e. ϕ is decreasing in α at a faster rate than $1/\alpha$). A manager who has ability α and is allocated n workers produces $y = \alpha f(n)$, with $f' > 0$, and (given limited managerial time and attention), $f'' < 0$. We study the impact of a regulatory tax on labor which may have both fixed and variable cost components.[9] This tax is only borne by firms after they reach a given size N, that is for $n > N$ ($N = 49$ in our main empirical application, although we also study other thresholds).

2.1 Individual Optimization

Let $\pi(\alpha)$ be the profits obtained by a manager with ability α when he manages a firm at the optimal size:

$$\pi(\alpha) = \max_{n} \begin{cases} \alpha f(n) - wn & \text{if } n \leq N \\ \alpha f(n) - w\tau n - F & \text{if } n > N \end{cases} \tag{1}$$

where w is the worker's wage, n is the number of workers, F is the fixed cost, and τ is the variable tax parameter, both of which only apply to firms over a minimum threshold of N. Firm size at each side of the threshold is then implicitly determined by the first order condition:

$$\alpha f'(n^*) - \overline{\tau} w = 0, \text{ with } \begin{cases} \overline{\tau} = 1 & \text{if } n \leq N \\ \overline{\tau} = \tau & \text{if } n > N \end{cases} \tag{2}$$

so that $n^*(\alpha, \tau, w) = f'^{-1}(\frac{\overline{\tau} w}{\alpha})$. To simplify our notation we write this firm size as $n^*(\alpha)$ and omit throughout the explicit reference to the regulatory regime (N, F, τ) and to wages. Note that firm size is increasing in α, and decreasing in τ and w. Managerial ability α_c (sub-script "c" denotes "constrained") at the regulatory threshold is given by:

$$\alpha_c = \frac{w}{f'(N)}. \tag{3}$$

Firms can legally avoid being hit by the regulation by choosing to remain small. The cost of this avoidance strategy is increasing in α. The ability level α_r of the marginal manager is defined by the indifference condition between remaining small or jumping to a larger size and paying the regulatory cost:

$$\alpha_r f(N) - wN = \alpha_r f(n^*(\alpha_r)) - w\tau n^*(\alpha_r) - F \tag{4}$$

where $n^*(\alpha_r)$ is the optimal firm size for an agent of talent α_r. Subscript r refers to the marginal firm that is "regulated" in the sense that they choose to be larger than the regulatory threshold and pay the tax.

2.2 Equilibrium

The most skilled individuals choose to be manager-entrepreneurs, since they benefit from their higher ability in two ways. First, for a given firm size n, they earn more profits. Second, the most skilled individuals hire a larger team,

---

[9] Previous studies of this problem, such as particularly Kramarz and Michaud (2003) suggest that the fixed cost element is less important compared to the marginal cost component. Empirically, we also find this result.

$n^*(\alpha)$. We denote the ability threshold between managers and workers as $\alpha_{\min}$: individuals with ability below $\alpha_{\min}$ are workers. A competitive equilibrium is defined as follows:

Definition 1 *Given a distribution of managerial talent $\phi(\alpha)$ over $[\underline{\alpha}; \alpha_{\max}]$, a production function $y = \alpha f(n)$, a per worker implicit labor tax τ and a fixed cost F that binds all firms of size $n > N$, a competitive equilibrium consists of (1) a wage level w^*; (2) an allocation $n^*(\alpha)$ that assigns a firm of size n^* to a manager of skill α; and (3) a triple of cutoffs $\{\alpha_{\min} \leq \alpha_c \leq \alpha_r\}$, where $[\underline{\alpha}, \alpha_{\min}[$ is the set of workers, $[\alpha_{\min}, \alpha_c]$ is the set of unconstrained, unregulated managers, $[\alpha_c, \alpha_r[$ is the set of size constrained but unregulated managers, with firm size at $n^* = N$, and $[\alpha_r, \alpha_{\max}]$ is the set of "regulated" managers, such that: (E1) No agent wishes to change occupation (worker vs. manager); (E2) The choice of $n^*(\alpha)$ for each manager α is optimal given their skills, taxes (τ, F) and wages w^*; (E3) Supply of labor equals demand for labor.*

Condition (E1) implies continuity of earnings:

$$\alpha_{\min} f\left(n^*(\alpha_{\min})\right) - w^* n^*(\alpha_{\min}) = w^* \tag{5}$$

Equilibrium condition (E2), from the first order condition (2) implies that firm sizes are given by:

$$n^*(\alpha) = \begin{cases} 0 & \text{if } \ \alpha < \alpha_{\min} & \text{(6a)} \\ f'^{-1}\left(\dfrac{w^*}{\alpha}\right) & \text{if } \ \alpha_{\min} \leq \alpha \leq \alpha_c & \text{(6b)} \\ N & \text{if } \ \alpha_c \leq \alpha < \alpha_u & \text{(6c)} \\ f'^{-1}\left(\dfrac{\tau w^*}{\alpha}\right) & \text{if } \ \alpha \geq \alpha_u & \text{(6d)} \end{cases}$$

Thus, as Figure 3 shows, we have four categories of agents: workers, small firms, constrained firms and regulated firms. Finally, condition (E3) requires that supply equals demand of workers:

$$\int_{\underline{\alpha}}^{\alpha_{\min}} \phi(\alpha) \mathrm{d}\alpha = \int_{\alpha_{\min}}^{\alpha_{\max}} n^*(\alpha)\phi(\alpha) \mathrm{d}\alpha, \tag{7}$$

where $n^*(\alpha)$ is the continuous and piecewise differentiable function above.

Solving the model involves finding four parameters: the cutoff levels $\alpha_{\min}$, α_c, α_r, and the equilibrium wage w^*. For this we use the four equations (3), (4), (5) and (7). The equilibrium is unique and has the following relevant properties:

Proposition 1 *The introduction of a tax/variable cost τ of hiring workers only for firms of size $n > N$: (a) Reduces the equilibrium wage w as a result of the reduction in the demand for workers; (b) increases small firm entry (by reducing $\alpha_{\min}$) and increases firm size for small firms (i.e. $\alpha < \alpha_c$) because of the reduction in w; (c) reduces firm size for all firms that are taxed – i.e. those beyond the threshold α_r.*

The proof of this proposition is in Appendix A. Intuitively, the increasing equilibrium schedule that solves the occupational choice equation (5) determining w for each $\alpha_{\min}$ shifts down when the regulatory tax τ increases (for each cutoff $\alpha_{\min}$ a lower wage w yields indifference), while the decreasing schedule that yields equality between supply and demand (7) also shifts down (left), since all desired team sizes increase and wages must decrease in equilibrium. Thus the equilibrium w unambiguously decreases as τ increases (proposition 1(a)), and this immediately means that all firms which do not pay the regulatory tax are larger (proposition 1(b)). Proposition 1(c) follows since when talent is scarce, the fall in wages is never sufficient to compensate fully the direct impact of the increase in tax.

Proposition 2 *The introduction of a fixed cost F for sizes $n > N$ (a) reduces equilibrium wage w and (b) increases small firm entry (by reducing $\alpha_{\min}$) and increases firm size for small firms (i.e. $\alpha < \alpha_c$); (c) creates a bulge at N (a spike in the firm size distribution) and valley (α_r increases); (d) increases firm size n (through the general equilibrium effect) for firms beyond the threshold α_r.*

Again the proofs are in Appendix A. Intuitively, again the demand for workers is lower, so wages must fall, leading to more entry by small firms. Part (c) is immediate from equation (4) and similarly part (d) from the wage reduction.

Example. Consider a power law, $\phi(\alpha) = \frac{0.6}{\alpha^{1.6}}$ and a returns to scale parameter of $\theta = 0.5$.[10] Panel A of Figure 4 shows the firm size distribution for a firm size cut-off at 49 employees, and an employment taxes of $\tau - 1 = 0.01$ (1%) and $F/w = 0.07$ (such that $n_r = 60$). As in the distribution in the data, there is a spike at 49 employees that breaks the power law. Panel B of Figure 4 reports the productivity α as a function of firm size n. It shows that we should expect a spike in the productivity distribution at the point in which the regulation starts to bind. We find strong empirical support for both of these predictions in the data.

2.3 Empirical Implications

Our econometric work uses the theory as a guide to estimate the welfare losses that result from this regulation. As is well known, the firm size distribution generally follows a power law.[11] We follow Lucas (1978) in using the returns to scale function of $f(n) = n^{\theta}$. He shows that for it to be consistent with a power law the managerial ability or productivity distribution must also be a power law, $\phi(\alpha) = c_{\alpha}\alpha^{-\beta_{\alpha}}$, $\alpha \in [\underline{\alpha}; \alpha_{\max}]$ with the constants $c_{\alpha} > 0$ and $\beta_{\alpha} > 0$. In this case, from the first order conditions in equations (6a) to (6d), firm sizes for the equilibrium wage w^* are given by $n^*(\alpha) = \left(\frac{\theta}{w^*}\right)^{1/(1-\theta)} \alpha^{1/(1-\theta)}$ for $\alpha_{\min} \leq \alpha < \alpha_c$ and $n^*(\alpha) = \left(\frac{\theta}{w^*}\right)^{1/(1-\theta)} \tau^{-1/(1-\theta)}\alpha^{1/(1-\theta)}$ for $\alpha_r \leq \alpha \leq \alpha_{\max}$.

The distribution of firm sizes $\chi(n)$ is, by the change of variable formula, $\chi(n) = \phi(\alpha(n)) * \frac{1}{p}n^{-\theta}w\overline{\tau}\frac{(1-\theta)}{\theta}$ (omitting the threshold, and where p denotes the share of entrepreneurs among all agents in the economy). After some straightforward manipulation, relegated to Appendix B, the "broken"power law on firm size n^* is then given by:

---

[10] These values are selected simply to illustrate the graphical patterns.

[11] See Ramsden and Kiss-Haypal (2000) for OECD countries and Hernández-Pérez, Angulo-Browna and Tun (2006) for developing countries.

$$\chi^*(n) = \begin{cases} \left(\frac{1-\theta}{\theta}\right)^{1-\beta} (\beta-1)n^{-\beta} & \text{if } \theta/(1-\theta) \leq n < N \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta} \left(N^{1-\beta} - Tn_r^{1-\beta}\right) & \text{if } n = N \\ 0 & \text{if } N < n < n_r \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta} (\beta-1)Tn^{-\beta} & \text{if } n_r \leq n \end{cases} \tag{8}$$

where $\beta = \beta_\alpha(1-\theta)+\theta$ and where $T = \tau^{-\frac{\beta-1}{1-\theta}}$. The upper employment threshold, n_r, is unknown and must be estimated alongside β, the power law term, θ, the return to scale parameter, and T, which is a function of τ. Note that the shape parameter β in the power law is unaffected by the regulation; instead, in log-log space, the labor regulations generate a parallel shift in the firm size distribution driven by T (see Figure 4). Thus the key empirical implication is that the variable part of the regulatory tax can be recovered from the shift T in the power law.

2.4 Welfare Calculations

A manager with talent α and firm size $n^*(\alpha)$ produces $y(\alpha) = \alpha f(n^*(\alpha))$ and total output in this economy is found by integrating over all agents of different managerial ability:

$$Y = \int_{\alpha_{\min}}^{\alpha_c} \alpha f(n^*(\alpha))\phi(\alpha)\mathrm{d}\alpha + \int_{\alpha_c}^{\alpha_r} \alpha f(N)\phi(\alpha)\mathrm{d}\alpha + \int_{\alpha_r}^{\alpha_{\max}} \alpha f(n^*(\alpha))\phi(\alpha)\mathrm{d}\alpha$$

It follows that the welfare change between the regulated and unregulated economy is then given by:

$$\begin{aligned} \Delta Y &= Y(N,\tau,F) - Y(.,1,0) \\ &= \int_{\alpha_{\min}}^{\alpha_{\min}^{NR}} \alpha \left[f(n^*(\alpha)\right]\phi(\alpha)\mathrm{d}\alpha + \int_{\alpha_{\min}^{NR}}^{\alpha_c} \alpha\left[f(n^*(\alpha)) - f(n_{NR}^*(\alpha))\right]\phi(\alpha)\mathrm{d}\alpha \\ &+ \int_{\alpha_c}^{\alpha_r} \alpha\left[f(N) - f(n_{NR}^*(\alpha))\right]\phi(\alpha)\mathrm{d}\alpha \\ &+ \int_{\alpha_r}^{\alpha_{\max}} \alpha\left[f(n^*(\alpha)) - f(n_{NR}^*(\alpha))\right]\phi(\alpha)\mathrm{d}\alpha \end{aligned} \tag{9}$$

where n_{NR}^* is the firm size given w_{NR}^*, the equilibrium wage rate, and $\alpha_{\min}^{NR}$ the cutoff between workers and managers in the unregulated economy. Thus the welfare losses in our simple framework are the result of adding up three effects:

(1) The top row of equation (9) captures two positive effects on total output from the fall in the equilibrium wage arising from the regulation. First, there are some additional firms since marginal workers are drawn into becoming entrepreneurs by cheaper labor. Second, the firms which are below the regulatory threshold (and not paying the tax) are able to hire more workers as their wages are lower.

(2) There is a "local" output loss, that is the result of the firms that would have been larger but instead are constrained at N workers. This is the second row of equation (9).

(3) Finally, there is another output loss from the larger firms in the economy, which incur higher labor costs due to the regulatory tax (even after netting off the lower equilibrium wage), and choose "too small" a size.

Note that $\alpha_{\min}$ unambiguously decreases under regulation. In other words, the measure of firms increases and the measure of workers decreases, so the average firm size will fall in the regulated economy.

2.5 Partial Inflexibility in Real Wages

We also provide an analysis under the case when real wages are not perfectly flexible downwards after the regulation. In France there is a high minimum wage and strong unions (about 90% of all workers in France are covered by a collective bargain[12]) which restrict the ability of wages to offset the effect of regulations. More generally, there is likely to be a reservation wage below which individuals will not work, particular when welfare benefits are generous. Note that these are not the temporary nominal frictions highlighted in the business cycle macro literature, but real rigidities that prevent equilibrium real wages falling by more than a certain amount. Incorporating inflexible real wages requires a small extension of the model. We define the equilibrium with inflexible wages as:

Definition 2 *A competitive equilibrium with inflexible wages consists of: (1) a wage level* w_a^* *paid to all* <u>*employed*</u> *workers indexed by a (for "fraction of adjustment"), which corresponds to the percentage of adjustment that is feasible, with* $w_a^* = w_0^* + a(w^* - w_0^*)$*, (thus* $a = 1$ *means wages are fully flexible); (2) an allocation* $n_a^*(\alpha)$ *that assigns a firm of size n to a particular manager of skill* α; *(3) a triple of cutoffs* $\{\alpha_{\min}^a \leq \alpha_c^a \leq \alpha_r^a\}$ *like in Definition 1; and (4) an* <u>*unemployment rate*</u> u^a *defined as the number of unemployed workers as a share of the total number of potential workers, such that conditions* $(E1)$ *and* $(E2)$ *in Definition 1 above hold and* $(E3)$ *becomes:*
$(E3^a)$ *Supply of labor is equal to the sum of the demand for labor and* <u>*unemployment*</u>*.*

The model with partial inflexible wages is solved in the same way as before; the main differences relate to condition $(E1)$ and to the labor market equilibrium. The new version of equation 5 (implied from condition $(E1)$) now compares the profit a "small" (untaxed) potential entrepreneur with the *expected* wage of a worker, who earns w_a^* when he is employed, but 0 if she is unemployed:

$$\alpha_{\min}^a f\left(n_a^*(\alpha_{\min}^a)\right) - w_a^* n_a^*(\alpha_{\min}) = (1 - u^a) w_a^* \tag{10}$$

The labor market equilibrium condition $(E3)$ is also modified, since now the regulation generates unemployment:

$$(1 - u^a) \int_{\underline{\alpha}}^{\alpha_{\min}} \phi(\alpha) \mathrm{d}\alpha = \int_{\alpha_{\min}}^{\alpha_{\max}} n_a^*(\alpha) \phi(\alpha) \mathrm{d}\alpha \tag{11}$$

The remainder of the analysis is otherwise unaltered.

---

[12] http://www.eurofound.europa.eu/eiro/country/france.pdf. See also Bertola (2000) and Bertola and Rogerson (1997) for a more extensive discussion of the literature on the reasons for this inflexibility.

3 Empirical Strategy

3.1 Empirical Model

We propose a baseline empirical model in which we introduce an error term in our measure of employment so that we can take it to the data. Such empirical model must account for two departures in the data of Figure 2 from the predictions in the theory. First, the departure from the power law does not start precisely at the regulatory threshold N, but slightly earlier: there is a "bump" in the firm size distribution beginning at around 46 workers. Second, the region immediately to the right of N does not have zero density, but rather there are some firms with positive employment levels just to the right of the regulatory threshold, N.

The baseline model we propose to account for these departures features a measurement error in employment. The reasons for this specification are twofold. First, no dataset will perfectly measure the underlying theoretical concept - measurement error is a fact of empirical life. Second, several different regulations start at size 50, and as explained in greater detail in subsection 4.1 and Appendix F, they rely on slightly different concepts of employment size, defined respectively in the Code du Travail (labor laws), Code du Commerce (commercial law), Code de la Sécurité Sociale (social security) and in the Code Général des Impôts (fiscal law). The measurement of firm size that we use[13] corresponds to the fiscal definition that is a mandatory item reported in the firm's tax accounts - the arithmetic mean of the number of workers measured at the end of each of the four quarters of the fiscal year. This measure of size is available accurately for essentially all firms. However, it does not exactly correspond to the concept of size that is relevant for all of the regulations, since no (single) index of employment having this property exists. An additional reason for the existence of some firms to the immediate right of the size threshold could be labor adjustment costs. If large changes in employment generate such costs, a firm experiencing a positive shock may spend a period passing through the valley to the right of the 50 employee threshold in order to smooth these adjustment costs. In order to consider this we build an explicit dynamic model in sub-section 6.6. Although more complex, the basic intuitions all go through in this extended model.

Recall that our starting point is the pdf of n^*as given by equation (8). Employment is measured with error so we assume that rather than observing $n^*(\alpha)$ we observe $n(\alpha, \varepsilon) = n^*(\alpha)e^{\varepsilon}$ where the measurement error ε is unobservable. In the data we observe the distribution of n, and thus obtaining the likelihood function requires that we obtain the density function of n. The conditional cumulative distribution function is given by (see Appendix B):

$$\mathbb{P}(x < n|\varepsilon) = \begin{cases} 0 & \text{if } \ln(n) - \ln\left(\frac{\theta}{1-\theta}\right) < \varepsilon \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta} \left(ne^{-\varepsilon}\right)^{1-\beta} & \text{if } \ln(n) - \ln(N) < \varepsilon \leq \ln(n) - \ln\left(\frac{\theta}{1-\theta}\right) \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta} T\left(n_r\right)^{1-\beta} & \text{if } \ln(n) - \ln(n_r) < \varepsilon \leq \ln(n) - \ln(N) \\ 1 - \left(\frac{1-\theta}{\theta}\right)^{1-\beta} T\left(ne^{-\varepsilon}\right)^{1-\beta} & \text{if } \varepsilon \leq \ln(n) - \ln(n_r) \end{cases} \tag{12}$$

---

[13] Fiscal definition, Article 208-III-3 du Code Général des Impôts. We also test the robustness of our results using alternative datasets and concepts of employment; see Appendix C.4.

Let ε be normally distributed[14] with mean 0 and variance σ. Integrating over ε we can compute the unconditional CDF (convolution of the broken power law and Gaussian distributions) simply as:

$$\forall n > 0, \quad \mathbb{P}(x < n) = \int_{\mathbb{R}} \mathbb{P}(x < n|\varepsilon)\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right) \mathrm{d}\varepsilon.$$

In Appendix B we show that no further constraints on the parameters are required for this object to be a CDF:

Lemma 1 *Let* ε *be normally distributed with mean* 0 *and variance* σ *so that the measurement error is log normal. Then the function* $\mathbb{P}(x < n)$ *is a cumulative distribution function, that is strictly increasing in* n*, with* $\lim_{n\longrightarrow 0} \mathbb{P} = 0$ *and* $\lim_{n\longrightarrow\infty} \mathbb{P} = 1$ *for all feasible values of all parameters,* σ*,* θ*,* T*,* β*, and* n_r*.*

Thus taking the derivative of $\mathbb{P}$ formulated in this way we can obtain the density of the observed n. Given such a density, it is straightforward to estimate the parameters of the model by maximum likelihood. Specifically, ML yields estimates of the parameters: $\widehat{\sigma}$, $\widehat{T}, \widehat{\beta}, \widehat{n}_r$ while $\widehat{\tau}$ and $\widehat{\frac{F}{w}}$ are computed as (see full details in Appendix B):

$$\widehat{\tau} = \widehat{T}^{-\frac{1-\widehat{\theta}}{\widehat{\beta}-1}} \tag{13}$$

$$\widehat{\frac{F}{w}} = N - \frac{\widehat{\tau}}{\widehat{\theta}}.\widehat{n}_r.\left[\left(\frac{N}{\widehat{n}_r}\right)^{\widehat{\theta}} - 1 + \widehat{\theta}\right] \tag{14}$$

Figure 5 shows the difference between the pure theory model where employment was measured without error and the true model where there is measurement error. The thickest solid line shows the firm size distribution under the pure model of Section 2 (same as Figure 4A) whereas the dashed line shows the firm size distribution when we allow for measurement error. The smoothness of the bulge around 50 will depend on the degree of measurement error. If we increase the measurement error to $\sigma = 0.5$ (thin solid line) instead of $\sigma = 0.15$ it is almost impossible to visually identify the effects of the regulation.

3.2 Identification and Inference

Our ML estimation over the size distribution allows us to obtain most of the parameters of interest. Intuitively, the slope of the line in Figure 5 (which is the same before and after the cut-off) identifies β, the power law parameter. The estimates of the two 'tax' parameters can be recovered from three features of the distribution. First, the downward shift of the power law after 49 employees. Second, the "bulge" of firms just before the regulatory threshold at 50 employees and third the width of the "valley" in the size distribution between 49 employees and where the power law recovers at n_r. A variable tax τ without any fixed cost results in a parallel shift of the power law. Instead, if there were only a fixed cost F of the regulation we should only see a bulge, and a valley, but no shift down in the power law after n_r. Hence the existence of a downward shift in the firm size distribution after the regulatory threshold is powerful evidence of a variable cost component of the regulation. Last, the measurement error, σ, is identified from the size of the random deviations of the size distribution from the broken power law.

---

[14] In Appendix C.2, we examine the sensitivity of our estimates to this Gaussian assumption and show that the cost of the regulation is robustly estimated.

Given the empirical estimates of T and β, we still need an estimate of returns to scale θ in order to identify the key tax parameter, τ. There are several ways to obtain θ. Our first approach is to calibrate from existing estimates. Since θ is well recognized to be an important parameter in the macro reallocation literature there are a number of papers to draw on. Basu and Fernald (1997) show a number of estimates and suggest a value of 0.8. We use this as a baseline and consider reasonable variations around this value used in other papers.[15] Our second approach is to estimate firm level production functions and sum the coefficients on the elasticities of output with respect to factor inputs to calculate returns to scale.[16] Our third method is to use the relationship between size and TFP to back out an estimate of the returns to scale.[17]

We implement all these alternative methods to show that our results are robust to plausible values of θ. As previously explained in section 3.1, given an estimate of θ, we have an estimate of the implicit variable tax of regulation as (hats denoting estimated parameters): $\widehat{\tau} = \widehat{T}^{-\frac{1-\widehat{\theta}}{\widehat{\beta}-1}}$

We obtain standard errors for the estimates of the tax using either standard errors clustered at the four-digit industry level and the delta method, or block-bootstraping at the industry level, with 100 replications.

4 Institutional Setting and Data

4.1 Institutions: The French Labor Market and Employment Costs

France is renowned for having a highly regulated labor market (see Abowd and Kramarz, 2003; Cahuc and Kramarz, 2005, Kramarz and Michaud, 2010). What is less well known is that most of these laws only bind a firm when it reaches a particular employment size threshold. Although there are some regulations that bind when a firm (or less often, a plant) reaches a lower threshold such as 10 workers, 50 is generally agreed by labour lawyers and business people to be the critical threshold when costs rise significantly. In particular, when firms get to the 50 employee threshold they need to undertake the following duties (see Appendix F): (i) they must set up a "works council" ("comité d'entreprise") with minimum budget of 0.3% of total payroll; (ii) they must establish a committee on health, safety and working conditions (CHSCT); (iii) a union representative (i.e. not simply a local representative of the firm's workers) must be appointed if wanted by workers; (iv) they must establish a profit sharing plan; (v) they incur higher liability in case of a workplace accident; (vi) they must report monthly and in detail all of the labor contracts to the administration; and (vii) firing costs increase substantially in the case of collective dismissals of 10 or more workers (this can be regarded as an implicit tax on firm size - e.g. Bentolila and Bertola, 1990); and (viii) they must undertake to do a formal "Professional assessment" for each worker older than 45.

How important are such provisions for firms? Except in the case of the minimum regulatory budget that is to be

---

[15] Guner et al (2006) use a $\theta = 0.802$ for Japan. Atkeson and Kehoe (2005) use a version of the Lucas model with organizational capital and suggest a value of 0.85.Hsieh and Klenow (2009) use a value of $\theta = 0.5$.

[16] Appendix C.3.1 details how we do this using a variety of methods such as Levinsohn and Petrin (2003), Olley and Pakes (1996) and the more standard Solow residual approach.

[17] Note that in principle, θ can be recovered from the size distribution itself (see Appendix C.3). This method relies on rather strong assumptions over the identity of the smallest firm from the indifference condition between being a worker and a manager in equation (5). As discussed in Appendix C.3, empirically the data is not rich enough to estimate θ from the size distribution alone (although we can reject very large values of the parameter), so we consider several alternatives in order to examine the empirical robustness of our estimates of τ.

allocated to firm councils, it is extremely hard to get a handle on this. For example, what is the opportunity cost of managerial time involved in dealing with works councils, union representatives, health and safety committees, etc.? Our framework is designed to recover the costs of such regulations by examining the revealed preferences of firms. We focus on the 50 employee threshold, but it is straightforward to extend the analysis to other threshold. Indeed, we implement such an extension to cover the thresholds at both 10 and 50 employees in Section 6.

4.2 Data

Our main dataset is FICUS which is constructed from administrative (fiscal) data covering the universe of French firms between 1995 and 2007. These are based on the mandatory reporting of firms' income statements to tax authorities for all French tax schemes - the BRN[18], RSI and BNC. The BRN is the standard tax regime; the RSI is a simplified regime that small firms can opt into for a cost and the BNC covers "non commercial" professions such as legal and accounting firms. Although the BRN has been more commonly used by researchers it misses out on large numbers of small firms. For example, less than 20% of single employee firms are in the BRN and even for firms with 10 employees the BRN only covers 80% of FICUS firms. Hence, for looking at the firm size distribution one needs to use the whole FICUS data.

All firms have to report tax returns (even if they owe no tax in the fiscal year) and there are about 2.2m firms per year in our data. Our baseline results are on the approximately 200,000 firms active in the manufacturing sector (NACE2 industry classes 15 to 35) as productivity is easier to measure in these industries. But we also present results estimating the model on all the other non-manufacturing sectors and show the robustness of our results. The laws mainly apply to the administrative unit in FICUS, namely the firm ("entreprise"), so this dataset is well suited to our purpose.

The employment measure in the FICUS relates to a headcount of the number of workers in the firm averaged over the four quarters of the fiscal year in France. A headcount of employees is taken on the last day of the fiscal year (usually December 31st) and on the last day at the end of each of the previous three quarters. Employment is then the simple arithmetic average over these four days. This is close to the concept used for most of the regulations (see Appendix F), but not all of them and this is one of the motivations for allowing employment to be measured with error with respect to the regulation.[19] In addition, FICUS contains balance sheet information on capital, investment, the wage bill, materials, four digit industry affiliation, etc. that are important in estimating productivity. We also use the DADS (Déclarations Annuelles de Données Sociales) dataset which contains worker-level information on hours, occupation, gender, age, etc. This dataset provides a variety of proxies to measure firm size in terms of employment and full time equivalents (see Appendices F and C.4 for more details). Neither dataset is perfect. FICUS has the advantage that the time period for reporting is common to employment and the accounting measures used to build

---

[18] See Di Giovanni, Levchenko and Ranciere (2011) and Caliendo, Monte and Rossi-Hansberg (2012) for other work on these BRN data.

[19] We attempted to use the differences in the exact regulatory definition of "50 employees" to tease out the impact of different regulations (e.g. regulations using headcounts on a single precise date vs. those averaging over a longer period). Unfortunately, the data are not sufficiently precise to enable us to uncover such subtle distinctions. This could be an area for future work in other countries where more precise data exists.

productivity. The timing of the reporting of DADS employment differs from this accounting information[20], but it has the advantage of having hours information from which full-time equivalents can be computed, although unfortunately, full-time equivalent information in DADS misses almost a quarter of jobs.[21] Empirically, we obtain similar results whether we use FICUS or DADS for our econometric estimation (see next section and Appendix C3). Details of the TFP estimation procedure, which in the baseline specification uses the Levinsohn and Petrin (2003) version of the Olley and Pakes (1996) method,[22] are reported in Appendix C.3.1.

5 Results

5.1 Qualitative Analysis of the Data

We first examine some qualitative features of the data to see whether they are consistent with our model. Many commentators have expressed skepticism about the quantitative importance of employment regulations as it is sometimes hard to observe any clear change in the size distribution around important legal thresholds.[23] Figure 6 presents the empirical distribution of firm size around the cut-off of 50 employees for two datasets, FICUS and DADS - the left hand side panels are the frequencies immediately around 50, whereas the right panels are over a wider support in log-log space. The top left hand side figure in Panel A is the same as Figure 2. As previously discussed, there is a mass point at 49 and a sharp discontinuity in size which is strong evidence for the importance of the regulation. The top right hand side of Figure 6 shows this in log-log space clearly indicating the evidence of a “broken power law”. There is a downward shift in the “intercept” of the slope of the power law at 50 employees.

Panel B of Figure 6 uses DADS. We first aggregate employment up to the appropriate level for each FICUS firm using head counts dated on 31st December. The discrete jump at 50 also shows up here, especially in the log-log plot, but less clearly than in the FICUS data. Panel C of Figure 6 uses Full-Time Equivalents (over one calendar year) which also shows less of a jump than the straight count of employees in the previous panels, probably because of the way full time equivalents are estimated in the files. In Appendix C.4, we show that our estimates are robust across all datasets and employment definitions in Figure 6, because the main component of the estimated tax generates a shift of the distribution of the power law downwards rather than just a mass point at 49.

---

[20] In DADS many firms report employment and wage information from December to November rather than from January to December, as is (most frequently) done for value added in the fiscal files.

[21] Full time equivalents are computed from “non-annexes” jobs only, i.e. those lasting more than 30 days with more than 1.5 hours worked per day, or those that were associated with (total) gross earnings higher than 3 times the monthly minimum wage. As documented in the methodology of the DADS files, in 2002, the “non-annexes” jobs represent 76.6% of all jobs, which means that almost a quarter of the most short-end, low paid jobs were removed. This is unfortunate as such jobs are in fact covered by labor laws. Note also that the type of contract (full time vs. part time) is not observed in the DADS data. It is estimated from the hours paid (taking the 75th percentile of the distribution of hours per job as a reference), and there is a lot of variation in this due to the heterogeneous implementation of the 35 hours maximum working week.

[22] We use a control function approach to deal with unobserved productivity shocks and selection when estimating production functions. Because we have a panel of firms we can implement this and estimate the production function coefficients. There are several issues with this approach (e.g. Ackerberg et al, 2007 or 2015) to estimating production functions so we also estimate TFP using a variety of other methods (see Appendix C.3.1 for details).

[23] Hsieh and Olken (2014) note this point in their analysis of developing countries like India. See also Schivardi and Torrini (2008) and Boeri and Jimeno (2005) on Italian data, Braguinsky et al (2011) on Portuguese data, Abidoye et al (2010) on Sri Lanka data or Martin et al (2014) on India. These authors find that there is slower growth just under the threshold consistent with the regulation slowing growth (as we also show below), but they find relatively little effect on the cross-sectional distribution. This may be because of the multitude of regulations, variable enforcement or measurement error in the employment data.

In all panels of Figure 6, firm size seems to approximate a power law in the employment size distribution prior to the bulge around 50. After 50, there is a sharp fall in the number of firms and the line becomes flatter before resuming another power law with a similar slope. Broadly, outside a "distorted" region around 50 employees, one could describe this pattern a "broken power law" with the break at 50.[24] The finding of the power-law for firm size in France is similar to that for many other countries and has been noted by other authors (e.g. Di Giovanni, Levchenko and Rancière, 2011), but the finding of the break in the law precisely around the main labor market regulation is a specific feature of French data. There does appear to be some break in the power law at firm size 10 which corresponds to the size thresholds from other pieces of labor and accounting regulations (see Appendix F). In order to avoid conflating these issues we initially focus our analysis first on firms with 10 or more employees in the main analysis, and therefore on the additional costs generated by regulations at threshold 50 relative to average labor cost for firms having 10 to 49 employees. We extend later (Section 6.3) our analysis to the case with two thresholds (10 and 50) and show that our main specification does indeed pick up the main costs of the regulation.

Our basic model has the implication that more talented managers leverage their ability over a greater number of workers. Figure 7 examines this idea by plotting mean TFP levels by firm size.[25] Productivity appears to rise monotonically with size, although there is more heteroskedacity for the larger firms as we would expect because there are fewer firms in each bin. The relationship between TFP and size is broadly log-linear. What is particularly interesting for our purposes, however, is the bulge in productivity just before the 50 employee threshold. We mark these points in light shading (red). This is consistent with our model (see Panel B of Figure 4) where some of the more productive firms who would have been just over 50 employees in the counterfactual unregulated economy, choose to be below 50 employees to avoid the cost of the regulation. Firms just below the cut-off are a mixture of firms who would have had a similar employment level without the implicit tax and those firms whose size is distorted by the size-related regulation.

We also examined dynamic patterns in the data. Firms are much more likely to stay at size 49 and not grow. 12% of firms were exactly of size 49 employees for two years running compared to only 4% of firms who stayed exactly at size 47 and only 2% of firms who stayed at exactly size 52 employees (Figure A1). Only 0.32% of firms with 49 employees or less crossed over the threshold to 50 (or more) workers in an average year (about 550 firms), compared to 0.40% of firms with 47 employees or less who crossed the 48 employee threshold (about 700 firms). Another way of seeing this is that the average duration of firms at 49 employees was 0.30 of a year, compared to 0.23 of a year for firms with 50 employees (Figure A2). These dynamic patterns are consistent with firms being reluctant to grow beyond the threshold at 49 employees. We will explore this more in the context of an explicit dynamic model in Section 7.

---

[24] See Howell (2002) for examples of how to estimate these types of distributions. More generally, see Bauke (2007) for ways of consistently estimating power laws.

[25] TFP is difficult to measure accurately (see Appendix C). We do not have firm-specific information on prices so high measured TFP may reflect higher mark-ups as well as higher productivity. Additionally, adjustment costs will drive a wedge between the true and estimated production function parameters (see Section 7). Such measurement issues may explain why some of the firms to the right of the threshold have lower measured TFP.

5.2 Econometric Results

The key parameters are estimated from the size distribution of firms using the ML procedure described in Section 3. Table 1 shows a set of baseline results using different values of θ for French manufacturing firms in 2000.[26] We begin by using a calibrated value of $\theta = 0.8$ from Basu and Fernald (1997) in column (1). The slope of the power law, β, is about 1.8 and significant at conventional levels. The upper employment threshold, n_r, is estimated to be an employment level of 59 and we obtain a standard deviation of the measurement error of 0.12. Turning to the estimates of the tax equivalent costs of the regulation, we estimate the implicit variable labor tax to be $\tau - 1 = 0.023$ (2.3%) and highly significant.

One surprise in Table 1 is that the fixed cost estimate, F/w, is negative and equal to about 94% of a single worker's wages. The implications of this for the implied regulatory cost is shown in Panel A of Figure 8. The bold line is our main estimates of the total regulatory costs from column (1) of Table 1 (as a proportion of the average wage in the economy). These are zero until the firm employs the 50th worker where they jump discontinuously to 20.9% (= 50*0.023 – 0.941) before rising linearly with the estimated variable cost of 2.3%. If the fixed cost were zero then the total cost function would be given by the dashed red line. The negative fixed cost implies that there is a sudden jump in costs at the threshold, but that it is much less than what we would expect from just the variable cost.[27] So no firm ever benefits from crossing the regulatory threshold as the total variable costs exceed the fixed costs.

This is also reflected in the fact that the bulge in the firm size distribution just below the regulatory threshold is smaller than it would be without the estimated negative fixed cost. One economic explanation for the negative fixed cost is the presence of a (small) within firm positive spillover from the effort in information collection that firms must undertake to comply with the costly regulatory reporting requirements (such as monthly reports on every worker's pay and conditions to the Labor Ministry). Having to collect such information may also enable firm to make efficiency savings in organizing labor or other parts of their business.[28] Appendix C.1 shows that in general small spillovers that are concave can lead to an estimate of a negative fixed costs of the sort we find.

Column (2) of Table 1 considers an alternative calibration of $\theta = 0.85$ from Atkeson and Kehoe (2005). The obtained results are stable, although the estimate of the variable cost of regulation falls from 2.3% to 1.7%. This is because the importance of the distortions of the tax depend on returns to scale. When returns to scale are close to unity the most efficient firms produce a large share of output, so it only takes a small distortionary tax to have a large effect on the size distribution. As decreasing returns set in it takes a much larger estimate of τ to rationalize any given distorted distribution (the "downward shift", "bulge" and "valley" in firm size). To further illustrate this

---

[26] We use a sample of firms with between 10 and 1,000 employees correcting our estimates for censoring at the lower and upper known thresholds.

[27] A cross check on the plausibility of the results is to see how much extra profits would be gained from switching from 49 to 59 workers. Using the estimate in column (1) of Table 1 this is €13,863 $= [(F/w) + (\tau - 1) \times n_r] \times w = [-0.941 + 0.023 \times 59.27] \times$€$31,912$. If we take a cost of capital of 15% to reflect the greater riskiness of small firms, the difference in entrepreneurial rents for firms between the two size classes using accounting data is similar at €13,521.

[28] These could be through adopting information technologies specifically designed to keep track of inputs like labor in the firm such as Enterprise Source Planning (ERP) software. Bloom,Garicano, Sadun and Van Reenen, (2014) discuss more details of the economics and organizational consequences of ERP introduction. It could also mean adopting the kind of management practices discussed by Bloom, Sadun and Van Reenen (2015).

effect column (3) considers $\theta = 0.50$ (as in Hsieh and Klenow, 2009). This case implies a large tax of 5.9%. Column (4) goes to the other extreme with a $\theta = 0.9$ generating a smaller tax rate of 0.7%. Column (5) of Table 1 uses the TFP estimates from the production function to estimate the TFP-size relationship as in Figure 7. This generates an estimate of the variable tax of 2.4%. Column (6) uses the returns to scale parameter directly estimated from a production function, giving a value of $\theta = 0.860$. Other parameter estimates remain stable and we obtain an estimate of the variable tax of 1.6%.

Our results use the year 2000 cross section but are robust to pooling across all years between 1995 and 2007 data or choosing individual years (see Table A1). These experiments show that the upper employment threshold lies between 58 and 60 workers, the variable tax $(\tau - 1)$ between 1.7% and 2.3%.

Panel B of Figure 8 shows how the fit of the model compares with the raw data using the estimated parameters in column (1) of Table 1. Although imperfect, we seem to do a reasonable job at mimicking the size distribution even around the regulatory threshold. Another way to assess the model's performance is to recall that we are using the distribution only of firm size and employment to estimate the parameters of our model. Thus we can also assess the model's performance by using it to predict the distribution of output, a moment we are not targeting in the baseline estimation. If alternative margins of adjustment were important, then the observed distribution of output would be very different (and total output larger) than what is predicted by the model. In particular, we would expect to under-estimate the share of output produced by large firms, and to over-estimate the share of output produced by small firms, as larger firms can reduce their regulatory cost by increasing hours, skills or capital intensity (see below for evidence of this).

Panels A and B of Table 2 compare the actual and predicted distributions of firms and employment. Unsurprisingly we match these moments closely as this is the data that we are using to fit our parameters. Panel C has the output estimation. The proportion around the bulge of 49-59 workers predicted by our model is close (at 4.1%) to the actual data (3.5%). As expected, we underestimate the output of larger firms, but not by too much. Our parameters suggest that 68.6% of output should be in firms with over 58 employees whereas the number is 71.6% in the data. Symmetrically, we overestimate the output share of the small firms (27.2% vs. 24.9%). These mis-predictions are exactly as basic economics suggests as the baseline results do not account for other margins of substitution. Nevertheless, as shown by the standard errors, our estimates are within the 95% confidence intervals which is a good performance for such a simple model. In Section 6 we will explicitly model some substitution margins (e.g. over capital) and come to a similar conclusions over the robustness of our model.

5.3 Changes in the Level and Distribution of Welfare

We can fully calculate the impact of the regulation on the firm size distribution, output and welfare. As shown in Section 2, the slope of the power law does not change as a result of the implicit tax. According to the theory, the impact of the tax is a parallel move upwards of the firm size distribution at sizes $n < 49$, a spike at $n = 49$, and a parallel move down for $n > 58$. Thus the counterfactual firm size distribution is a power law with the exponent β

we calculated in our analysis, and $\tau = 1,\ F = 0$. The position of the intercept is pinned down by the labor market condition, which requires that the total number of agents in the economy is constant, and by the minimum firm size which in our specification is pinned down by the returns to scale parameter and also stays constant.[29]

As discussed in sub-section 2.5, an important factor influencing the welfare effects of the regulation is the degree of real wage flexibility, a. We begin by assuming that real wages are perfectly flexible ($a = 1$). But given that France has a high minimum wage and strong unions we calibrated the degree of inflexibility using three alternative methods (see Appendix B.5.3 for details). In our baseline model we used the average difference in the structural unemployment rates between France and the US between 2002 and 2007 from OECD (2015). The US serves as the benchmark of the least regulated labor market in the OECD. We then use our parameter estimates to fit the value consistent with the difference in unemployment rates. This generated $a = 0.70$, i.e. real wages fall by 70% of what we would expect in a world of fully flexible real wages. Second, we used the estimates in Aeberhardt et al (2012) of the impact of changes in the value of the minimum wage over the entire wage distribution. Their quantile regression estimates imply $a = 0.67$, a very similar degree of flexibility to the first method. Thirdly, we use the degree of partial adjustment estimated by the official mid-term macro-model used by the French Finance Ministry (e.g. de Loubens and Thornary, 2010). This has a higher degree of rigidity $a = 0.62$ after two years. We will take $a = 0.70$ as our baseline, but the similarity of the calibration of the wage rigidity parameter across several different methods is reassuring.

Panel A of Figure 9 presents the change in the firm size distribution in the world with regulation (bold line) and without regulation (dashed line) using the estimated parameters from our model (the lower figure zooms in around the threshold to make the changes more visible). As the theory led us to expect, in the counterfactual unregulated economy there are fewer firms under 49 employees. This is because in the regulated economy (i) there is a spike at 49 employees for those firms who are optimally avoiding the regulation and (ii) since equilibrium wages have fallen there is an expansion in the number of small firms. Compared to the unregulated economy, the regulated world has fewer large firms since although wages have fallen there is an additional regulatory tax. Panel B presents the same information in terms of the output distribution which has a similar pattern.

We illustrate the effects of different degrees of real wages inflexibility in Figure 10 for jobs (left hand side figures) and welfare (right hand side figures). Panel A has full flexibility $a = 1$, Panel B is our baseline case of $a = 0.7$. Turning first to the top left figure, we examine the changes in employment in the regulated economy compared to an unregulated economy for small firms, medium firms, large firms and the overall economy. With fully flexible wages there is basically no aggregate loss of jobs, but we can see that there is an increase in employment in small firms of nearly 200,000, and decreases in employment in medium sized and large firms. In our baseline scenario in panel B, there are overall job losses of around 140,000 as unemployment emerges. The same pattern emerges of the largest job losses coming from the biggest firms and actually some gains in the smaller firms.

The welfare losses in terms of income (wages or profits) are given in the right hand side of Figure 10 with the exact numbers in Table 3. With fully flexible wages (Panel A) there is a an overall loss of about 1.3% of GDP. Large firms

---

[29] Note that this is not the case when the regulation binds for firms of all sizes (i.e. $N = 0$ and $\tau > 1$), or when wages are rigid. See Section 2 and Appendix B for the details of the derivation.

and workers are losers whereas smaller firms are the winners. Labor costs rise by 0.5% for large firms and their size is reduced by 2.5% (rows 5 and 8 of Table 3). Small firms' costs fall by 1.8% and these firms are 9.0% larger (rows 4 and 7 of Table 3). The pure deadweight output loss is quite small (0.02% of GDP, row 9b)[30], as the wage adjustment is able to undo most of the cost, but the overall welfare loss depends on how one regards the implicit "tax revenues", which are 1.3% of GDP (row 9a). These are not actual revenue streams to the government but rather are the costs of reporting and enforcing the regulations that firms bear - diverted employee time, the costs of employing union officials, in-house lawyers, etc. One view is that these also simply represent waste so the welfare cost are in the total sum in 9c (1.326%). An alternative view is that these amenities are valued by workers. But if this was the case we would expect to see workers over the 50 threshold receiving lower wages as a compensating differential for these amenities. We can examine this by looking at wages around the regulatory threshold (see Figure 11). As expected the wage is upward sloping in size, but there does not appear to be a fall in wages after the regulatory threshold, which suggests that workers do not place much benefit on the extra regulations. A caveat to this is that there may be positive externalities from the regulations, for example if the health and safety committee reduces accidents and so saves public healthcare costs. On the other hand, there may be negative externalities, for example through more industrial disputes.

Returning to Figure 10, it is clear that welfare losses are larger if real wages are more inflexible. In Panel B welfare falls by 3.45% in our baseline scenario where wages adjust by 70%. Since equilibrium wages are not falling as much as in the fully flexible case, larger firms lose from not being able to offset the regulatory cost and low ability agents are losing because there are higher levels of unemployment.[31] Small firms increase their size by about 6%, but large firms drop in size by 5%. The welfare losses increase in the degree of downward real wage rigidity. Looking across columns (3) and (4) of Table 3, welfare losses increase (to 3.7% and 4.0%) as unemployment rises.

Table 4 examines the robustness of the welfare results to returns to scale. We use a much lower value of $\theta = 0.5$ (as in column (3) Table 1 and Hsieh and Klenow, 2009) and repeat the analysis. Welfare losses rise in the perfectly flexible wage case of column (1) to 1.93% (from 1.33%) but fall a bit to 3.29% (from 3.45%) in our baseline case of column (2) that has a degree of partial wage rigidity calibrated to US-French structural wage differences.[32] The robustness of our baseline welfare results to a large change in the scale parameter is reassuring.[33]

In summary, we have three main quantitative results. First, aggregate welfare losses from the regulation are around 3.5% of GDP with employment losses of 140,000 for realistic levels of partially inflexible real wages. Second, welfare and job losses are smaller as wages become more flexible. Third, the regulation redistributes income away from workers

---

[30] The modest magnitude of the welfare cost in the case of flexible wages is perhaps unsurprising as the regulation does not cause the rank order of firm size to change with the ability distribution. Hopenhayn (2014) shows in a general context that first order welfare losses from misallocation require some rank reversals between ability and size.

[31] In the basic model workers are homogenous so they make an *ex ante* decision to enter based on their expected wage (relative to their expected profits from being an entrepreneur) and there is a random draw *ex post* to determine who will be unemployed. In our risk neutral set up low ability individuals lose out to a similar degree regardless of the degree of wage inflexibility. We consider heterogeneous production skill amongst employees in one of the extensions below.

[32] Since the model parameters are changed by using more decreasing returns to scale paarmeter ($\theta = 0.5$ instead of 0.8) the calibrated wage inflexibility parameter that is consistent with this model also changes a but (from $a = 0.70$ to $a = 0.68$).

[33] The implied elasticity of labor demand is 2 in Table 4. This is consistent with French evidence in Kramarz and Philippon (2001) and Abowd et al (2006) who found elasticities of about 2 for men and 1.5 for women. These are more reasonable than the higher implied elasticity (of 5) in Table 3. We note, however, that a recent study by Cahuc et al (2016) on France does find elasticities in that range (4 to be precise).

and larger firms and towards small firms (i.e. those with mediocre managerial ability) across all scenarios.

6 Extensions and Robustness

In this section we consider several extensions to our framework and robustness tests of the results. We allow the parameters to be different by industry (sub-section 6.1); examine whether firms respond by splitting up their operation (sub-section 6.2) and/or substituting into other factors (sub-section 6.4); allowing for another threshold (sub-section 6.3) and in sub-section 6.5 comparing our results to Hsieh and Klenow (2009).

6.1 Industry Heterogeneity

We examine the other main sectors outside manufacturing in our baseline case where $\theta = 0.80$ see (Table A2). We first estimated the model separately for each of four other sectors: Transport, Construction, Wholesale/Distribution and Business Services industries. The tax τ is significant in all sectors, highest in Transport (3.5%) and smallest in Business Services (0.8%). We also estimated production functions separately by three digit sectors and implemented column (6) of Table 1 allowing the scale (θ) and all other parameters to be freely estimated.[34] There is substantial degree of heterogeneity with some sectors having estimates of the regulatory tax from near zero to over 3% and this is related to industry characteristics in an intuitive way. For example, when labor costs are a smaller share of total value added, the estimated taxes tend to be bigger and when the capital-labor ratio of the sector is high they tend to be smaller. The distortion associated with the regulation is less damaging[35] in sectors when labor is a less important factor.

6.2 Compliance, Misreporting and Changing Corporate Structure

A business group could respond to the regulation by outright misreporting employee numbers. The authorities and unions are well aware of this incentive and threaten hefty fines and prison sentences for employers who lie to the fiscal or social security authorities. In some countries there is certainly evidence of cheating from tax returns (e.g. Almunia and Lopez-Rodriguez, 2015), although generally researchers are surprised at how low are these rates of non-compliance given the incentives (e.g. Kleven, Kreiner and Saez, 2009). We doubt these evasion strategies can be a major factor behind our results. First, we show below that there is some adjustment on a number of other margins that is consistent with a real effect of the regulation such as investment and hours of work (see below). There is no incentive for the firm to report these other items in a systematically misleading way. Second, hiding taxable revenues is much easier than hiding workers who have a very physical presence with a legal contract, health and pension rights. Third, in 2009 alone the French state employed 2,190 "agents de contrôle" to monitor firms compliance.[36] That is about one agent per 100 firms with ten or more employees, a rather high degree of observation. Fourth, hidden workers could be considered an

---

[34] Some of the industries have insufficient number of firms to perform this estimation but we are still able to do this for a large number of sectors. The full results are available on request and are plotted in Figure A3.

[35] Note also that the standard, economy wide regulations described in Appendix F interact with industry specific regulations and agreements, which are sometimes also size dependent. This generates some industry-level variation in the strength of regulations which is reflected in our estimates.

[36] http://www.insee.fr/fr/ffc/ipweb/ip1399/ip1399.pdf

additional factor of production. As we discussed earlier our model performs reasonably well in predicting output even abstracting away from such considerations, and subsection 6.4 below shows more formally that our results are robust to this extension.

A more subtle way of avoiding the regulation for business is by splitting a company into smaller subsidiaries. For example, a firm which wished to grow to 50 employees could split itself into two 25 employee firms controlled by the group owner. There are costs to such a strategy - the firm must then file separate fiscal and legal accounts, demonstrate that the affiliates are operating autonomously and suffer from greater problems of loss of control.[37] One way to check for this issue is to split the sample into those firms that are stand-alone businesses and those that are subsidiaries/affiliates of larger groups. Panel A of Figure A4 compares the power law for these two types of firms. For both stand-alone firms and affiliates we can observe the broken power law at 50. The fact that the discontinuity exists for stand-alone firms implies that our results are not being driven solely by corporate restructuring.[38,39]

6.3 A model with Two Thresholds: Additional Threshold at 10 Employees

We have focused on the most obvious of the regulatory threshold at size 50, but there is another important threshold at 10 employees where employers must pay monthly (rather than quarterly) social security obligations, transport aid, and a higher training tax. Table A3 reports results for this analysis for manufacturing and the other main sectors. The new threshold is estimated to involve near zero variable regulatory cost in all cases (the highest is in transport which is only 0.5% compared to 3% at 50). The fixed cost at 10 is most often insignificant, except in the construction and trade industries, where the point estimate is positive but small (around 2% of the cost of one worker). Importantly, adding this new threshold does not impact materially our estimates of the regulatory costs at 50 which remains at about 2% in most sectors, including manufacturing. Table A4 uses these two threshold estimates to re-do our welfare and distributional analysis. Reassuringly, the welfare costs are concentrated on the 50 employee threshold, and both the flexible and partially rigid wages cases are virtually the same as in the main analysis with only one threshold. Our analysis thus suggests that the costs introduced after size 50 are the important regulatory costs.

6.4 Allowing for Capital-Labor Substitution and Other Margins of Adjustment

Firms may react to the regulation by substituting labor for capital or other inputs. Figure 12 shows that investment per worker increases on average with firm size, but spikes up (like employment) around the threshold of 50 employees, consistent with capital-labor substitution. To analyze the robustness of our baseline estimation to such substitutability,

---

[37] A more extreme reaction of the firm would be to engage in franchising. This has some further costs as the CEO no longer has claims over the residual profits of the franchisee and loses much control. In any case, franchising is rare in manufacturing.

[38] Panel B repeats the distribution for the standalones, but now considers affiliates aggregated up to the *group* level. Although the power law is still broken at 50 it is less pronounced than at the subsidiary level in Panel A which could indicate some degree of corporate restructuring in response to the regulation. One might ask why there should be any discontinuity at 50 employees at the group level when aggregating across multiple subsidiaries? The reason is that a small number of regulations do bind at the group level, mainly through case law.

[39] Another way to look at this issue is to examine patterns of entry and exit. If firms where splitting up we would expect to see a mass of new entry around the bump of 45-49 employees. Figure A5 shows that this is not the case: entrants tend to be small (Panel C) and the fraction of entrants in each size class is rather uniform for firms between 35 and 75 employees (Panel A). There is somewhat of an *exit* spike at 50 employees (Panel B), but this is to be expected if some firms have mistakenly crossed the threshold and been hit by a large regulatory cost shock.

we add capital as an input into a CES production function (see Appendix D). A manager who has ability α and is allocated n workers and k units of capital produces $\alpha f(n,k) = \alpha\left(\lambda n^{\rho} + (1-\lambda)k^{\rho}\right)^{\frac{\theta}{\rho}}$ with $\theta \in (0,1)$ and $\rho \in \mathbb{R}$. The production function is increasing and strictly concave in n and k. As before let $\pi(\alpha)$ be the profits obtained by a manager with ability α when he manages a firm at the optimal size. These profits are given by:

$$\pi(\alpha) = \max_{n,k} \begin{cases} \alpha f(n,k) - wn - rk & \text{if } n \leq N \\ \alpha f(n,k) - w\tau n - rk - F & \text{if } n > N \end{cases} \tag{15}$$

where w is the worker's wage, n is the number of workers, r is the marginal cost of capital, k is the number of units of capital. Firm size at each side of the threshold (but not "at" the threshold) is then determined by the first order condition:

$$[n] : \alpha\theta\left(\lambda n^{\rho} + (1-\lambda)k^{\rho}\right)^{\frac{\theta}{\rho}-1} \lambda n^{\rho-1} - w\overline{\tau} = 0 \qquad \text{with} \begin{cases} \overline{\tau} = 1 & \text{if } n < N \\ \overline{\tau} = \tau & \text{if } n > N \end{cases} \tag{16}$$

$$[k] : \alpha\theta\left(\lambda n^{\rho} + (1-\lambda)k^{\rho}\right)^{\frac{\theta}{\rho}-1} (1-\lambda)k^{\rho-1} - r = 0 \tag{17}$$

Dividing equation (16) by equation (17), obtain the labor-capital ratio:

$$\frac{n}{k} = \left[\frac{r\lambda}{w\overline{\tau}(1-\lambda)}\right]^{\eta} = \gamma^{\eta} \qquad \text{with} \begin{cases} \overline{\tau} = 1 & \text{if } n < N \\ \overline{\tau} = \tau & \text{if } n > N \end{cases} \tag{18}$$

where $\eta = \frac{1}{1-\rho}$ is the elasticity of substitution.

We re-estimate the extended model as before using ML (as in Table 1 column (1)). Figure 13 reports the sensitivity of the estimated variable and fixed costs of regulation to capital-labor substitution. As the elasticity of substitution increases, the estimated variable tax $\tau - 1$ (left hand scale) decreases from the baseline "no substitution" case ($\eta = 0$) of around 3.1% to about 2.2% in our highest substitution case (an unrealistically high $\eta = 6$).[40] Thus it is stable to reasonable assumptions over substitutability.[41]

Given these parameter estimates we can calculate in Figure 14 the sensitivity of our welfare losses to the elasticity of substitution. In Figure A for fully flexible wages, welfare losses for no substitutability ($\eta = 0$) are essentially the same (1.3%) as our baseline in Table 3 column (1). As expected, greater degrees of substitutability reduce the welfare loss, but not tremendously (about 1.1% at $\eta = 3$ for example). Figure B presents partial wage inflexibility as calibrated to the French-US unemployment difference (column (2) of Table 3). Again, welfare changes are quite stable across a wide range of values of substitutability. The level of welfare loss of 2.5% is somewhat lower than in the baseline of 3.4%. This reflects the ability of employers to avoid reducing their output so much in the face of higher labor costs by substituting into capital.[42]

Although welfare losses are somewhat smaller than in the baseline case, the ability of firms to substitute labor for

---

[40] See Appendix D for details of how our baseline model and CES model differ. This is true even at $\eta = 0$ (Leontief) which is why the estimates of τ at this boundary case are not identical. Most empirical estimates have $\eta \leq 1$ (Hamermesh, 1996). More recently Karabarbounis and Neiman (2013) argue for a higher η , using η =1.3 as their preferred estimate.

[41] See Appendix 6.4 for a comparison of the CES, two input case with the single input specification.

[42] Note that the implied value for of a is different for every value of η in Figure 14 to be consistent with (calibrated) structural French-US unemployment difference.

capital does not warrant a major change in our conclusions. The main message remains that the extent to which the wage adjustment can transfer the cost of the regulation to the workers determines the cost of the regulation. If wages were fully flexible, the welfare losses would be about half the level they are given the current level of French wage rigidity.

Apart from capital, the most obvious other way firms could adjust is by increasing hours per worker (e.g. through longer overtime) rather than expanding the number of employees. The number of annual hours does increase just before the threshold of 50 employees (Figure A6), from the expected around 1,650 to 1,710, that is by around 3.6%. Firms are limited in their ability to increase hours not just because of imperfect substitutability between overtime and regular hours but also because of France's 35 maximum hours per week regulation. There also is evidence for adjustment along other margins such as using more intermediate inputs to mitigate the costs of the regulation. Allowing for these other margins does not substantially affect our main findings, as is also suggested by the accuracy of our output predictions in Table 3.

6.5 A Comparison with the Hsieh-Klenow (2009) Approach

Hsieh and Klenow (2009) take a related approach to examining regulatory distortions by examining the distributions of size and revenue-based productivity measures in the US, India and China. Like us, they found that such distortions could have substantial macro-economic effects. Their approach, however, focuses on the variation in marginal revenue productivity (MRP) as an indicator for distortions because, when factor prices are the same, MRPs should be equalized across firms even when underlying managerial ability is heterogeneous. In our context, we can follow Hsieh and Klenow (2009) and estimate the distortion (τ) from the change in the MRP of labor. The relation can be seen in Figure A7. Naively using data around the threshold to look at the change in MRP would be incorrect, however, as this local distortion in the MRP is due to the decision of firms in the 50-58 range to optimally choose to be at 49.[43] We rather implement the Hsieh-Klenow method of value added per worker (relative to the industry average) as an index of the MRP of labor for firms having 20 to 42 or 60 to 150 employees and obtain an estimate of the tax of 2.86% (Panel A of Figure A8).[44] Although we prefer our more structural approach, we note that these estimates are similar to our own estimate of the implicit tax of 2.3%.

6.6 A Dynamic Extension to the Model

Our baseline model as well as all extensions presented in the remainder of section 6 are essentially static. In this sub-section, we consider a dynamic extension of our framework to investigate whether the existence of adjustment costs (rather than simply measurement error) can explain the positive mass of firms to the right of the threshold. The dynamic approach will also be helpful in interpreting some of the dynamic features of the data such as growth rates

---

[43] Note that the treatment effect in the Regression Discontinuity Design would reflect the local distortion and not the global distortion and would lead to misleading estimates of the implicit tax. Instead, our approach would suggest comparing the MRP of labor for firms away from the threshold.

[44] This is using the average for firms between 20 to 42 workers compared to firms with 57 to 200 workers. Reasonable changes of the exact thresholds make little difference. Another approach is to use our own estimates of the MRP of labor from the model ($\alpha\theta\,(n^*)^{\theta-1}$). When doing this we obtain a value of the implicit tax of 3.3% (Panel B of Figure A8).

around the regulatory threshold. This experiment however comes at the cost of increased complexity: in particular, we only investigate here the partial equilibrium properties of the dynamic framework.[45]

The full details of our dynamic model are in Appendix E, the main features are as follows. Upon entry firms draw their management ability from a known distribution $G(\alpha)$ that is distributed as a power law. As in e.g. Melitz (2003), there is an endogenous profit threshold such that some firms will immediately exit after observing their ability as they cannot cover their fixed costs. The dynamic nature of the model is due to the fact that TFP is allowed to change over time in this setting according to an AR(1) process: after a firm is founded it is subject each period to a shock which will usually cause it to want to change size. This is the only source of stochastic variation shifting the environment. We allow for quadratic adjustment costs in labor (and capital), such that a firm will not always adjust immediately to its long-run frictionless optimal size.[46]

Free entry implies that agents choose to become entrepreneurs and enter until the expected value of entry is equal to the sunk entry costs. Last, as in the baseline static model we assume that in the regulated economy there is a variable cost (τ) and fixed cost (F) that has to be paid when firms cross the discrete employment threshold at 50 employees.[47]

The steady-state equilibrium of this model ensures that the expected cost of entry equals the expected value of entry given optimal factor demand decisions. This equilibrium is characterized by a distribution of firms in terms of their state values α, k, n (plus their optimized choice of materials m). We simulated it numerically for 20,000 firms over 100 periods (years), and used the last 25 years to describe the obtained steady state in Figures A11 to A14. All structural parameters are either set to their estimated values or calibrated to values that have been proposed in the literature (Table A6), while we provide comparative statics for different values of the adjustment costs and of the regulatory tax parameters.

This exercise shows essentially that the intuitions underlying the static model and key empirical features of the data are preserved in such a setting. The regulatory tax distorts the steady state firm size distribution and generates a "bulge" just to the left of the regulatory threshold, a "valley" to the right of the threshold and then a continuation of the power law of the firm size distribution, which are increasing in the magnitude of the regulatory "tax". The important difference with the static approach is however that firms in the valley are fully optimizing, and not just affected by measurement error. To see this assume that current managerial ability (or productivity) would put an incumbent firm to the left of the regulatory threshold and consider what would happen if such a firm received a large

---

[45] Results in this section are based on numerical simulations of the the steady state distributions of firm size and growth in this new framework (e.g. Judd, 1998). Factor prices are treated as exogenous, common to all firms and time invariant.

[46] An alternative way to dynamically model the regulatory tax would be to consider it as a purely sunk cost (or sunk cost plus variable cost) as in Gourio and Roys (2014). This has similar predictions on the firm size distribution and simplifies the dynamics compared to the model presented here (as there are no convex adjustment costs). However, the raw data on employment dynamics do not seem compatible with this approach. Panel A of Figure A1 shows how employment adjusts around the threshold at 49 for all firms and then the sub-set of firms who had (in the past five years) already had 50 or more employees (Panel B). If the regulatory cost was sunk the latter group - who presumably have already paid the sunk cost - should not bunch much around 49 employees. However, whether we look at firms who make no change in employment in Panel A, we observe a big spike at 49 employees for both groups. This does not seem consistent with a simple sunk cost story.

[47] We complement this set of assumptions with an exogenous death rate (in addition to the endogenous exit decision) that is modeled as an iid shock that causes some (small) margin of firms to go out of business. This is not a necessary feature of the model as the TFP process is AR(1) rather than random walk, hence even a firm with a very high TFP draw will tend to revert to the average over the long run.

positive productivity shock. The firm would like to move to a larger size and earn more profits by hiring more workers. However, since there are convex adjustment costs, spending a period in the valley that is sub-optimal from a static point of view may be optimal from a dynamic point of view. Large jumps in employment are extremely costly because of convex adjustment costs and so landing for a period or two in the valley may be less costly than paying these large adjustment costs to make a large immediate expansion. When adjustment costs increase, these considerations become more important and the mass of firms in the valley rises.

To conclude, we see this dynamic extension as an important exercise, since we believe in reality a mixture of adjustment costs and measurement error helps explain the "valley" (i.e. the existence of a positive mass of firms in the dominated area). Importantly, we checked it does not delivery fundamentally new insights compared to the static model, but leave its full estimation and thorough welfare analysis[48] for future research. At this stage, we believe the considerably simpler approach of our basic model has much to recommend it.

7 Conclusions

The costliness of labor market regulation is a long-debated subject in policy circles and economics. We have tried to shed light on this issue by introducing a structural methodology that uses a simple theoretical general equilibrium approach based on the Lucas (1978) model of the firm size and productivity distribution. We introduce size-specific regulations into this model, exploiting the fact that in most countries labor regulation only bites when firms cross specific size thresholds. We show how such a model generates predictions over the equilibrium size and productivity distribution and moreover, can be used to generate an estimate of the implicit tax of the regulation. Intuitively, firms will optimally choose to remain small to avoid the regulation, so the size distribution becomes distorted with "too many" firms just below the size threshold and "too few" firms just above it. We show how the regulation creates welfare losses by (i) allocating too little employment to more productive firms who choose to be just below the regulatory threshold, (ii) allocating too little employment to more productive firms who bear the implicit labor tax (whereas small firms do not) and (iii) through reducing equilibrium wages (due to some tax incidence falling on workers) which encourages too many agents with low managerial ability to become small entrepreneurs rather than working as employees for more productive entrepreneurs.

We implement this model for France where many labor laws bite when a firm has 50 employees or more. We find that the qualitative predictions of the model fit very well. First, there is a sharp fall off in the firm size distribution precisely at 50 employees resembling a "broken power law". Second, there is a bulge in productivity just to the left of the size threshold. Third, there is a shift downwards in the power law consistent with an increase in variable labor costs after passing the regulatory threshold.

We then estimate the key parameters of the theoretical model from the firm size distribution. Our approach delivers quite a stable and robust cost of the employment regulation which seems to place an additional cost on labor of about 2% of the wage. We show that we expect this cost to translate into relatively small output losses when real wages are

---

[48] The loss of resources generated by adjustment cots might have a non-trivial impact on total welfare.

flexible (about 1% of GDP) but large losses of 3.5% of GDP in the more realistic case when real wages are (partially) downwardly rigid. Furthermore, there are large distributional effects regardless of wage flexibility with workers losing substantively and small firms benefiting from the regulation. This is unlikely to be an intended consequence of the laws. Our welfare calculations are subject to the caveat that there may be some other externalities to society from the regulations - e.g. works councils may have wider benefits than any individual worker would take into consideration.

This is just the start of our research program of opening up the "black box" of firm distortions used in many macro models. Size-contingent regulations are ubiquitous and our methodology can be used for other regulations, other parts of the size distribution, other industries[49] and other countries. We showed in the paper that all the intuitions of our baseline static model are preserved in a dynamic extension allowing for shocks to productivity (or managerial ability) and adjustment costs in terms of labor or capital. Estimating the parameters of such a dynamic model is however left for future research at this stage. Going further we could allow firms to invest in order to improve their managerial capabilities. Such investments enable small firms to grow and since size-contingent regulations "tax" this growth over the threshold, they may well discourage investment and therefore inhibit the dynamics of growth in the economy. On the other hand the regulations could conceivably encourage firms to "strike for the fence" and go for radical innovations rather than incremental innovations which would put them only marginally to the right of the threshold.

Finally, we have assumed for simplicity that workers are homogenous. One can extend our baseline model to allow for heterogeneous workers in the framework of Garicano and Rossi-Hansberg (2006). In this model managers' ability allows them to solve those problems that workers could not solve. More skilled production workers endogenously match with more talented managers, because managers can leverage their ability over a larger mass of output in this case (as in our basic model, span of control costs are in the number of workers managed, not the skill of these workers). In this set-up the regulation introduces an additional matching friction which reduces aggregate output. As before, aggregate real wages will fall, and if there is a minimum wage unskilled workers will tend to become unemployed. The qualitative implications of this extension are therefore the same as our baseline model. Structurally estimating this model is several orders of magnitude more difficult however due to the matching model of skill that would be introduced. We leave consideration of such a model for future work (see Lise, Meghir and Robin, 2015, for an attempt to structurally model heterogeneous firms and workers in a matching model).

Despite these caveats, we believe that our approach is a simple, powerful and potentially fruitful way to tackle the vexed problem of the impact of regulation on modern economies.

---

[49] For example, the retail sector has a large number of size-contingent regulations with "big boxes" being actively discouraged in many countries and US cities (e.g. Bertrand and Kramarz, 2002, or Baily and Solow, 2001).

References

Abidoye, Babatunde, Orazem, Peter and Milan Vodopivec (2009) "Firing Cost and Firm Size: A study of Sri Lanka's Severance Pay System" World Bank Social Protection Discussion Paper No. 2009.

Abowd, John M. and Francis Kramarz (2003) "The Costs of Hiring and Separations" *Labour Economics*, 10(5), 499-530.

Abowd, John M., Francis Kramarz, David N. Margolis, and Thomas Philippon (2006) "Minimum Wage and Employment in France and the United-States" CREST Working Paper.

Ackerberg, Daniel, Lanier Benkard, Stephen Berry and Ariel Pakes (2007) "Econometric Tools for Analyzing Market Outcomes" *Handbook of Econometrics 6A*, 63, 4172-4276, Amsterdam: Elsevier.

Ackerberg, Daniel, Kevin Caves and Garth Fraze (2015) "Identification Properties of Recent Production Function Estimates" *Econometrica*, 83(6), 2411-2451.

Aeberhardt, Romain, Pauline Givord and Claire Marbot (2012) "Spillover Effect of the Minimum Wage in France: An Unconditional Quantile Regression Approach" *INSEE-DESE Working Paper*, G2012/07.

Almunia, Miguel and David Lopez-Rodriguez (2015) "Staying under the Radar: Firm responses to Tax Enforcement Strategies in Spain", mimeo, Warwick.

Attali, Jacques (2008) *Rapport de la Commission pour la Libération de la Croissance Française: 300 Décisions pour Changer la France* , XO Editions: La Documentation française; Report to the French Presidency.

Atkeson, Andrew and Patrick Kehoe (2005) "Modeling and Measuring Organizational Capital" *Journal of Political Economy*, 113, 1026-1053.

Axtell, Robert (2001) "Zipf Distribution of US Firm Sizes" *Science*, 293 (5536), 1818-20.

Baily, Martin and Robert Solow (2001) "International Productivity Comparisons at the Firm Level" *Journal of Economic Perspectives*, 15, 151-172.

Bartelsman, Erik, John Haltiwanger and Stefano Scarpetta (2013) "Cross Country Differences in Productivity: The Role of Allocation and Selection" *American Economic Review*, 103(1) 305-334.

Basu, Susanto and John Fernald (1997) "Returns to Scale in US Production: Estimate and Implications" *Journal of Political Economy,* 105(2) 249-283.

Bauke, Keiko (2007) "Parameter Estimation for Power Law Distributions by Maximum Likelihood Methods" *Oxford and European Physical Journal B,* 58(2), 167-173.

Bentolila, Samuel and Bertola, Giuseppe (1990) "Firing Costs and Labour Demand: How Bad Is Eurosclerosis?" *Review of Economic Studies*, 57(3), 381-402.

Bertola, Giuseppe (2000) "Macroeconomics of Distribution and Growth" *Handbook of Income Distribution* I (eds Anthony Atkinson and François Bourguignon), Chapter 9, 477-540 Amsterdam: Elsevier.

Bertola, Giuseppe and Rogerson, Richard (1997) "Institutions and Labor Reallocation" *European Economic Review,* 41(6), 1147-1171.

Bertrand, Marianne and Francis Kramarz (2002) "Does Entry Regulation Hinder Job Creation? Evidence from The French Retail Industry" *Quarterly Journal of Economics*, 117(4), 1369-1413.

Bloom, Nicholas, Raffaella Sadun and John Van Reenen (2015) "Management as a Technology" Stanford/LSE mimeo.
http://cep.lse.ac.uk/textonly/_new/staff/vanreenen/pdf/mat_2014may.pdf

Bloom, Nicholas, Luis Garicano, Raffaella Sadun and John Van Reenen (2014) "The distinct effects of Information Technology and Communication Technology on firm organization" *Management Science* 60(12) 2259-2885.

Boeri, Tito and Juan Jimeno (2005) "The Effects of Employment Protection: Learning from Variable Enforcement" *European Economic Review*, 49, 2057-2077.

Braguinsky, Serguey, Lee Branstetter and Andre Regateiro (2011) "The Incredible Shrinking Portuguese Firm" NBER Working Paper 17265.

Cahuc, Pierre and Francis Kramarz (2005) *De la Précarité à la Mobilité: vers une Sécurité Sociale Professionnelle* La Documentation Française, collection des rapports officiels,
http://www.ladocumentationfrancaise.fr/var/storage/rapports-publics/054000092/0000.pdf

Cahuc, Pierre, Stéphane Carcilloz and Thomas Le Barbanchonx (2016) "The Effectiveness of Hiring Credits" CREST Mimeo

Caliendo, Lorenzo, Ferdinando Monte and Esteban Rossi-Hansberg (2015) “The Anatomy of French Production Hierarchies” *Journal of Political Economy*, 123(4), 809-852.

Ceci-Renaud, Nila and Paul-Antoine Chevalier (2011), “L’impact des seuils de 10, 20 et 50 salariés sur la taille des entreprises Françaises” *Economie et Statistique*, 437, 29- 45.

Chetty, Raj, John Friedman, Tore Olsen and Luigi Pistaferri (2011) “Adjustment Costs, Firm Responses, and Micro vs. Macro Labor Supply Elasticities: Evidence from Danish Tax Records” *Quarterly Journal of Economics*, 126(2), 749-804.

De Loecker, Jan (2011) “Product Differentiation, Multi-product Firms and Estimating the Impact of Trade Liberalization on Productivity” *Econometrica*, 79(5), 1407-1451.

De Loubens Aymeric and Baptiste Thornary (2010) “Modélisation de la boucle prix-salaires pour la France par une approche macrosectorielle” *Documents de Travail de la DG Trésor*, 2010/04.

Di Giovanni, Julian, Andrei Levchenko and Romain Rancière (2011) “Power Laws in Firm Size and Openness to Trade: Measurement and Implication” *Journal of International Economics*, 85(1), 42-52.

Di Giovanni, Julian, and Andrei Levchenko (2011) “Firm Entry, Trade and Welfare in Zipf’s World” *Journal of International Economics*, 89(2), 283-296.

European Commission (1996) *Enterprises in Europe (Fourth Report)* Luxembourg

García-Santana, Manuel and Josep Pijoan-Mas (2014) “The Reservation Laws in India and the Misallocation of Production Factors ” *Journal of Monetary Economics*, 66, 193-209.

Garicano, Luis and Rossi-Hansberg, Estaban (2006) “Organization and Inequality in a Knowledge Economy” *Quarterly Journal of Economics*, 121(4), 1383-1435.

Garicano, Luis, Claire Lelarge and John Van Reenen (2013) “Firm Size Distortions and the Productivity Distribution: Evidence from France” NBER Working Paper 18841.

Gibrat, Robert (1931) *Les Inégalités économiques*, Paris: Librarie du Recueil Sirey.

Gourio, Francois and Nicolas Roys (2014) “Size-Dependent Regulations, Firm Size Distribution and Reallocation” *Quantitative Economics,* 5(2) 377-416.

Guner, Nezih, Gustavo Ventura and Yi Xu (2006) “How Costly are Firm Regulations on Size?” *Japan and the World Economy*, 18, 302-320.

Guner, Nezih, Gustavo Ventura and Yi Xu (2008) “Macro-Economic Implications of Size-Dependent Policies” *Review of Economic Dynamics*, 11, 721-744.

Hamermesh, Daniel (1996) *Labor Demand,* Princeton: Princeton University Press

Hernández-Pérez, R., F. Angulo-Browna and D. Tun (2006) “Company Size Distribution for Developing Countries” *Physica A,* 359, 607-618.

Hopenhayn, Hugo (2014) “On the Measure of Distortions” NBER Working Paper 20404.

Howell, Leonard W. (2002) “Maximum Likelihood Estimation of the Broken Power Law Spectral Parameters with Detector Design Applications” *Nuclear Instruments and Methods in Physics Research A*, 489, 422-438.

Hsieh, Chang-Tai and Peter J. Klenow (2009) “Misallocation and Manufacturing TFP in China and India” *Quarterly Journal of Economics* 124(4), 1403-1448.

Hsieh, Chang-Tai and Benjamin Olken (2014) “The Missing Missing Middle” *Journal of Economic Perspectives*, 28(3), 89-108.

Judd, Kenneth (1998) *Numerical Methods in Economics*, Cambridge: MIT University Press.

Kaplow, Louis (2013) “Optimal Regulation with Exemptions and Corrective Taxation”Harvard University mimeo.

Karabarbounis, Loukas and Brent Neiman (2013) “The Global Decline of the Labor Share” *Quarterly Journal of Economics*, 129(1), 61-103.

Kramarz, Francis and Marie-Laure Michaud (2010) “The Shape of Hiring and Separation Costs in France” *Labour Economics*, 17(1), 27-37.

Kramarz, Francis and Thomas Philippon (2001) “The Impact of Differential Payroll Tax Subsidies on Minimum Wage Employment” *Journal of Public Economics*, 82, 115-146.
Kleven, Henrik and Mazhar Waseem (2013) “Using Notches to Uncover Optimization Frictions and Structural Elasticities: Theory and Evidence from Pakistan” *Quarterly Journal of Economics* 128(2), 669-723.

Kleven, Henrik, Martin Knudsen, Claus Kreiner, Søren Pedersen and Emmanuel Saez (2011) “Unwilling or Unable to Cheat? Evidence from a Randomized Tax Audit Experiment in Denmark” *Econometrica*, 79(3), 651-692.

LAMY / Informations et Solutions pour les Professionnels (2010) “Comment décompter les effectifs?” Liaisons Sociales Quotidien, 200/2010, LSQ 15710.

Lazear, Edward (1990) “Job security provisions and employment” *Quarterly Journal of Economics*, 105(3) 699-726.

Levinsohn, James and Amil Petrin (2003) “Estimating Production Functions using inputs to control for unobservable” *Review of Economic Studies*, 70(2), 317-341.

Lise, Jeremy, Costas Meghir and Jean-Marc Robin (2015) “Matching, Sorting and Wages” *Review of Economic Dynamics*, forthcoming.

Lucas, Robert E. (1978), “On the Size Distribution of Business Firms” *Bell Journal of Economics*, 9(2), 508-523.

Manne, H. G. (1965) “Mergers and the Market for Corporate Control” *Journal of Political Economy*, 73(2) 110-120.

Martin, Leslie, Ann Harrison and Shanthi Nataraj (2014) “In with the Big, Out with the Small: Removing Small-Scale Reservations in India” NBER Working Paper, 19942.

Mélitz, Marc (2003) “The Impact of Trade on Intra-industry Reallocations and Aggregate Productivity Growth” *Econometrica*, 71(6), 1695-1725.

OECD (2015) “Structural unemployment statistics” *Economic Outlook* 98, November.
http://stats.oecd.org/Index.aspx?QueryId=61365#

Olley, Stephen and Ariel Pakes (1996) “The Dynamics of Productivity in the Telecommunications Industry” *Econometrica*, 64(6), 1263-1297.

Pavcnik, Nina (2002) “Trade Liberalization, Exit, and Productivity Improvements: Evidence from Chilean Establishments”, *The Review of Economic Studies*, 69(1), 245-276.

Posner, Richard (1971) “Taxation by Regulation” *Bell Journal of Economics*, 2(1), 22-50.

Ramsden, J.J. and G. Kiss-Haypal (2000), “Company Size Distribution in Different Countries” *Physica A*, 277, 220-227.

Restuccia, Diego and Richard Rogerson (2008) “Policy Distortions and Aggregate Productivity with Heterogeneous Plants” *Review of Economic Dynamics*, 11(4), 707-720

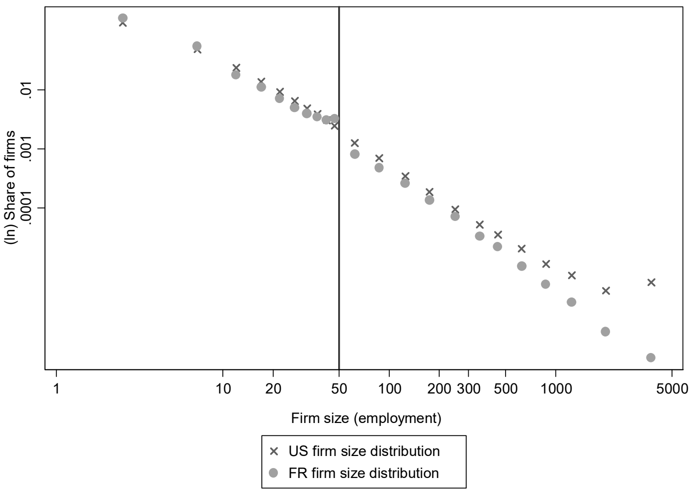
Saez, Emmanuel (2010) “Do Taxpayers Bunch at Kink Points?” *American Economic Journal: Economic Policy*, 2(3), 180-212.

Schivardi, Fabiano and Roberto Torrini (2008) “Identifying the Effects of Firing Restrictions Through Size-Contingent Differences in Regulation” *Labour Economics*, 15(3), 482-511.

Smagghue, Gabriel (2012) “Size-Dependent Regulation and Factor Income Distribution” mimeo, Sciences Po.

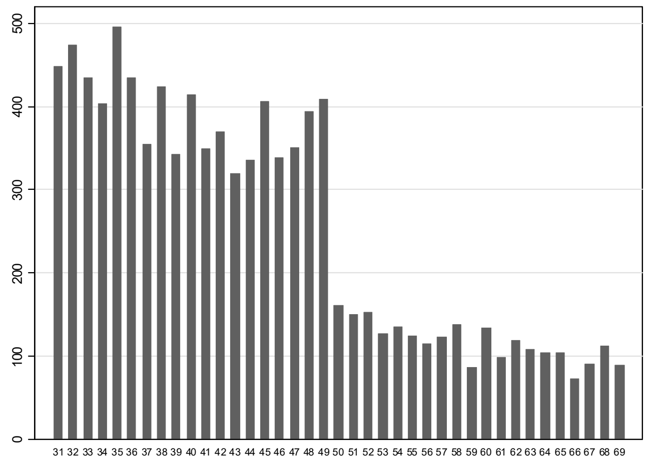
Syverson, Chad (2004) “Market Structure and Productivity: A Concrete Example” *Journal of Political Economy*, 112(6), 1181-1222.

Figure 1: Firm Size Distribution in the US and France



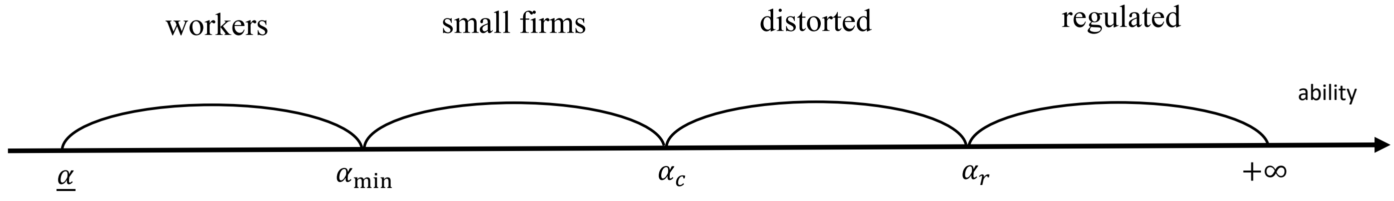
Source: FICUS 2003 for France and Census, LBD 2003 for the US. Population databases of all firms.
Notes: This is the distribution of firms (not plants). Authors' calculations

Figure 2: Number of Firms by Employment Size in France



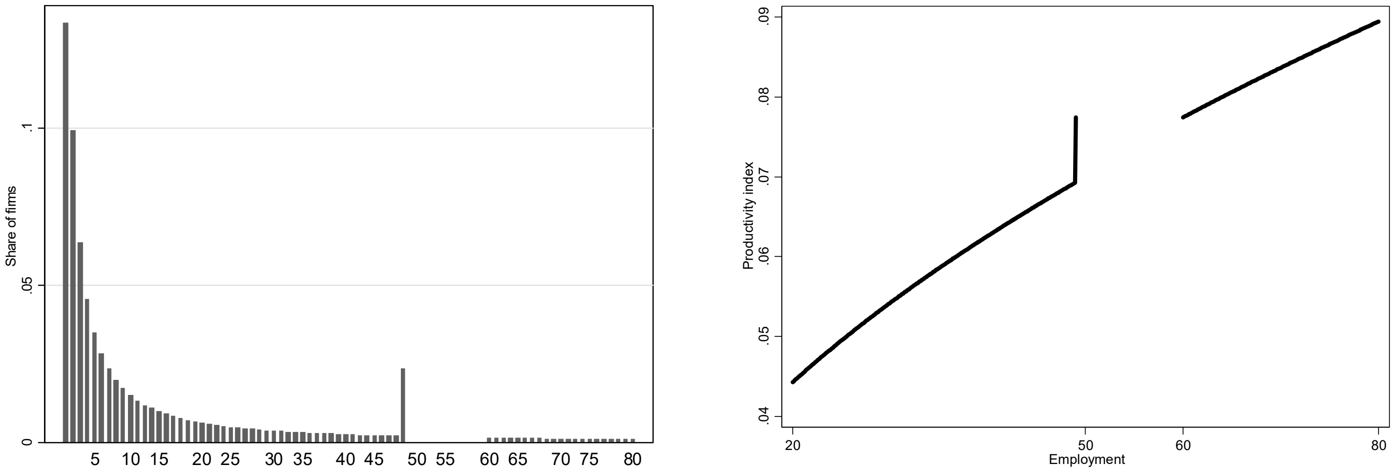
Source: FICUS.
Notes: This is the population of firms in France with between 31 and 69 employees in in manufacturing. This plots the number of firms in each exact size category (i.e. raw data, no binning) for the year 2000. There is a clear drop when regulations begin for firms with 50 or more employees.

Figure 3: Equilibrium Partition of Individuals into Workers and Firm Types by Managerial Ability, α



Note: This figure shows the definition of different regimes in our model. Individuals with managerial ability below $\alpha_{\min}$ choose to be workers rather than managers. Individuals with ability between $\alpha_{\min}$ and α_c are "small firms" who (conditional on the equilibrium wage, which is lower under regulation) do not change their optimal size. Between α_c and α_r are individuals who are affected by the regulatory constraint and choose their firm size to be smaller than they otherwise would have been – we call these individuals/ firms who are in a "distorted" regime. Individuals with ability above α_r are choosing to pay the implicit tax rather than keep themselves small.

Figure 4: Theoretical Predictions for Firm Size Distribution and TFP: Firm Size Distribution with Regulatory Constraint

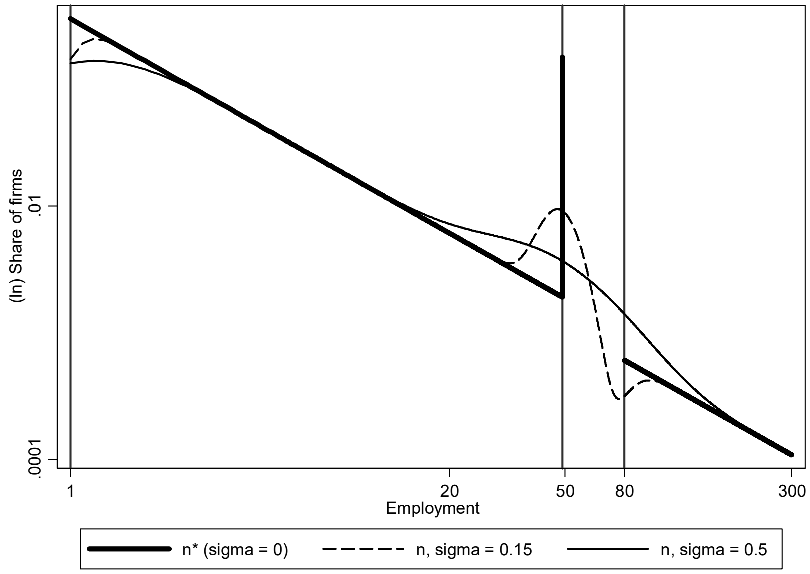


Panel A. Theoretical firm size distribution with regulatory constraint

Panel B. Theoretical Relationship between TFP (managerial talent) and firm size

Notes: Panel (A) figure shows the theoretical firm size distribution; the bar at 49 represents the point at which the size constraint bins. Panel (B) shows the theoretical relationship between TFP and firm size. There is a mass of firms at employment size = 49 where the regulatory constraint binds. Illustrative parameters (selected to amplify the graphical patterns): $\beta_\alpha = 1.6$, $\tau = 1.01$, $n_r = 60$, $\theta = 0.5$, $\beta = 1.3$.

Figure 5: Theoretical Firm Size Distribution when Employment is Measured with Error

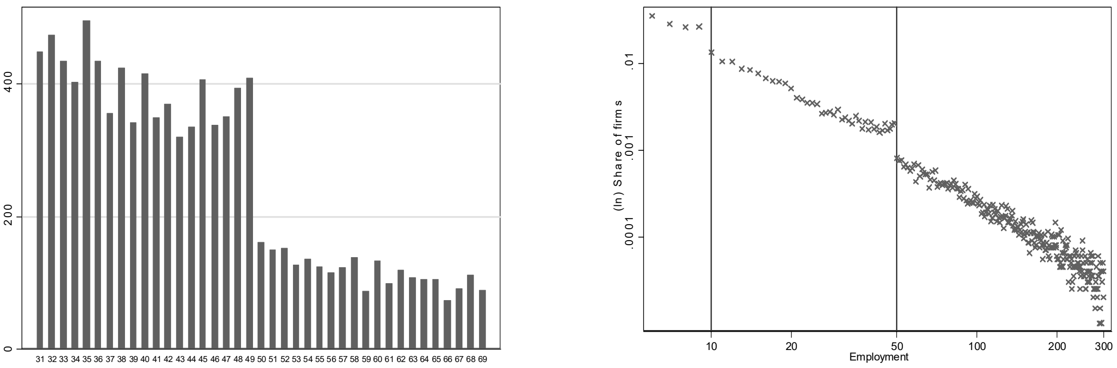


Note: The thickest solid (blue) line shows the theoretical firm size distribution (broken power law), n*. The dashed line shows the new firm size distribution when we extend the model, to allow employment size to be measured with error with $\sigma = 0.15$. The solid dark line increases the measurement error to $\sigma = 0.5$. Parameters: $\beta_\alpha = 1.6$, $\theta = 0.5$, $\beta = 1.3$, but $\tau = 1.02$ and n_r to 80.

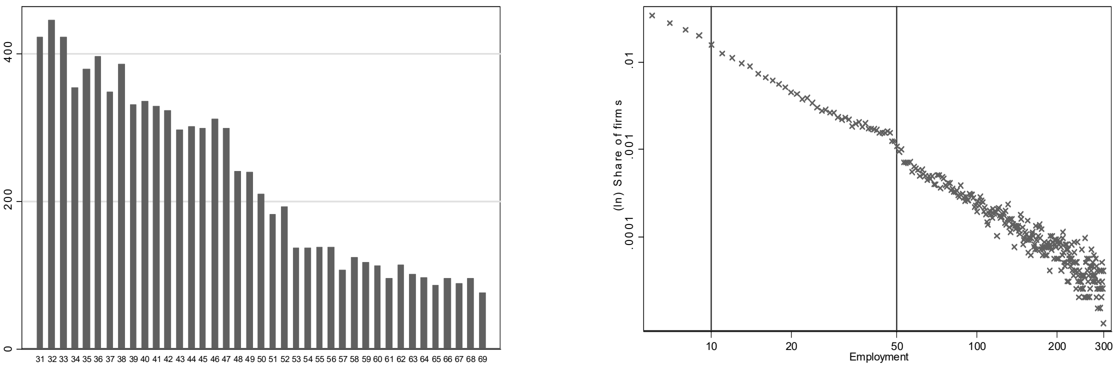
Figure 6: The Effect on the Measured Firm Size Distribution using Alternative Datasets (FICUS and DADS) and Definitions of Employment

Bar plot **Log-log plot**

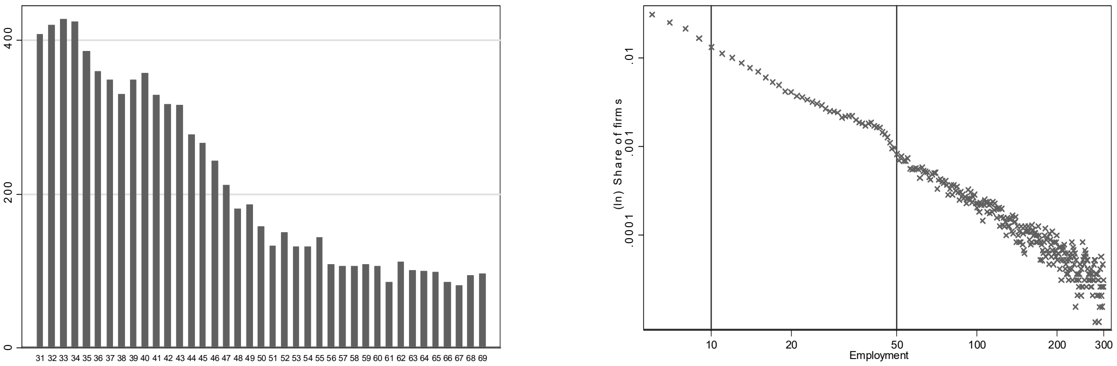
Panel A: FICUS - Arithmetic average of quarterly head counts



Panel B: DADS - Cross-sectional headcount of all workers on Dec. 31st

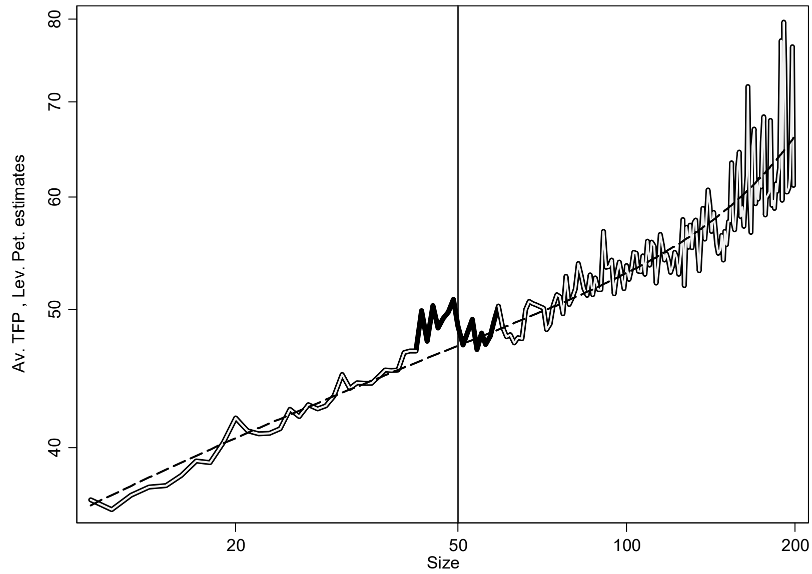


Panel C: DADS 2002 – "Full-time equivalent" (FTE) as computed by the French statistical institute



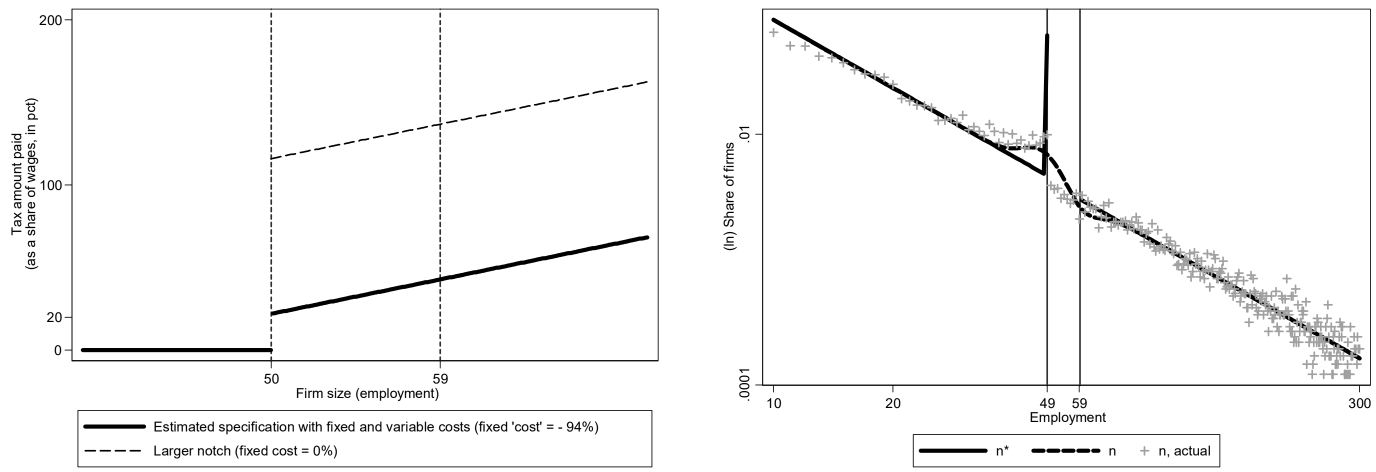
Note: Data sources are from FICUS (corporate tax collection reported to fiscal administration) and DADS (payroll tax collection reported to social administration) – see text and Appendix C3 for more details on datasets. Panels A and B relate to the year 2000, and Panel C relates to the year 2002 (as FTE calculation is unavailable in earlier years). All firms in manufacturing. FICUS employment is headcount at end of quarter averaged over four quarters (Panel A). Panel B is headcount on December 31st. FTE calculated in Panel C by the French Statistical Agency INSEE over the available sample of workers (76% of all workers). This excludes jobs lasting less than 30 days or less than 1.5 hours worked per day, or those that were associated with (total) gross earnings lower than three times the monthly minimum wage (using as a benchmark, the 75th percentile of the series of hours per workers in each two digit industries and size classes, and are bounded above by 1).

Figure 7: TFP Distribution around the Regulatory Threshold of 50 Employees



Notes: This figures plots the mean level of TFP by firm employment size using the Levinsohn-Petrin method (see Appendix B). TFP estimated using an unbalanced panel between 1995 and 2007 of firms having 10 to 1,000 workers. A fourth order polynomial is displayed using only data from the "undistorted" points (potentially “distorted” points are shown in red). Manufacturing firms only.

Figure 8: Implications of model estimates for cost of regulation and Firm size distribution



Panel A: Estimated Costs of Regulation

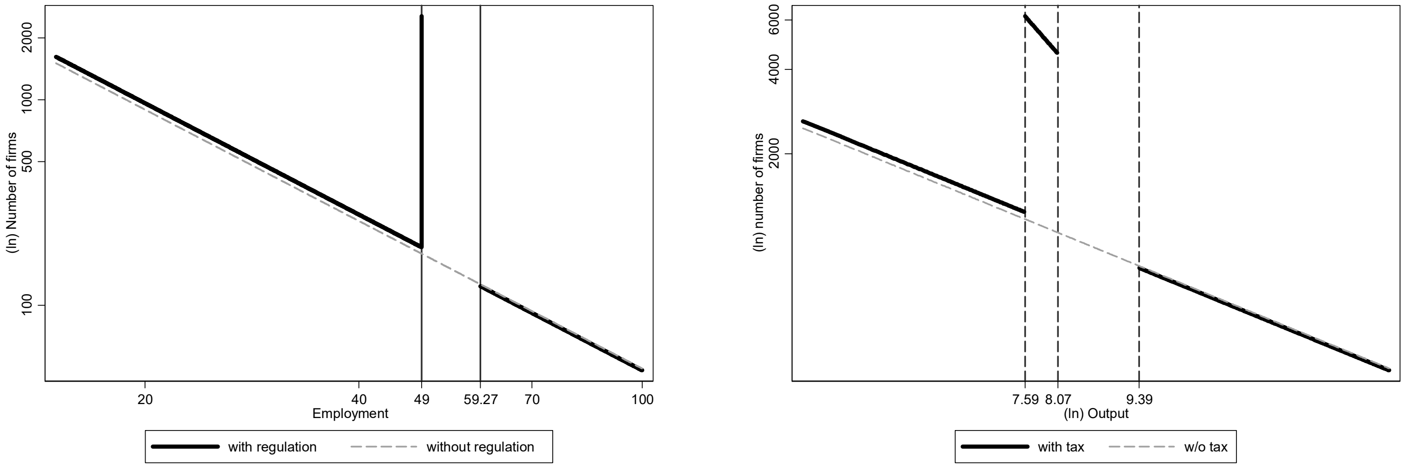
Panel B: Data and Fit of Model

Notes: Panel A plots the estimated “tax schedule” as a function of firm size. For the solid blue line, the estimates correspond to the baseline specification reported in Table 1 column (1). The dashed red line corresponds to setting the (estimated negative) fixed costs to zero. Panel B shows the difference between the fit of the model (dashed red line, *n*) which allows for measurement error with the actual data. Estimates correspond to the baseline specification reported in Table 1 column (1). We also include the “pure” theoretical predictions (in dark blue solid line, *n**). Actual data (*n*) is in crosses.

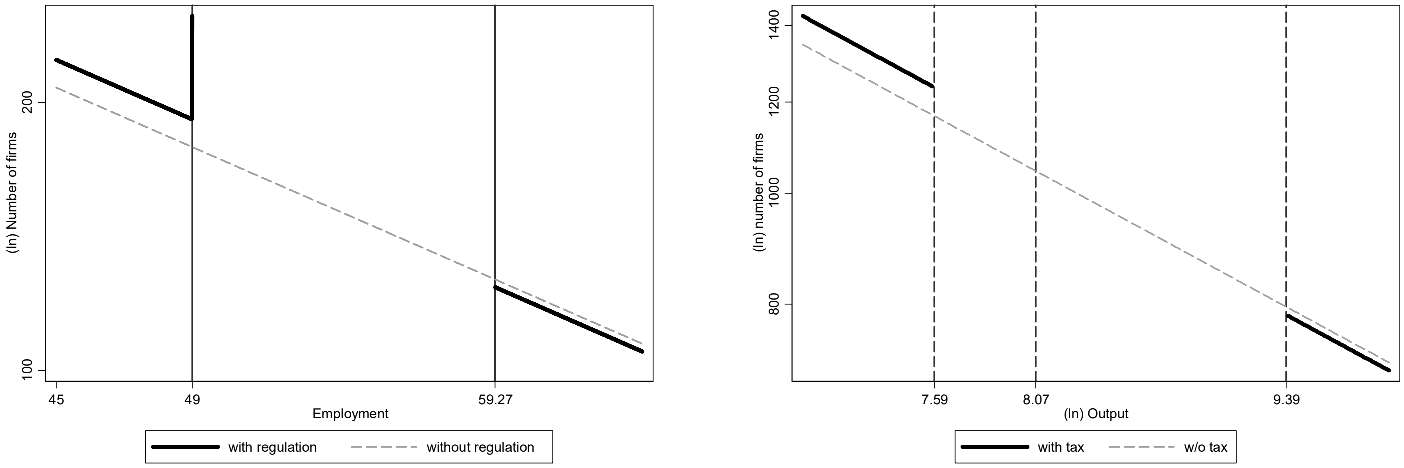
Figure 9: Firm Size Distribution With and Without Regulation

Panel A. Firm Size Measured by Employment **Panel B. Firm Size Measured by Output**

Wide Span

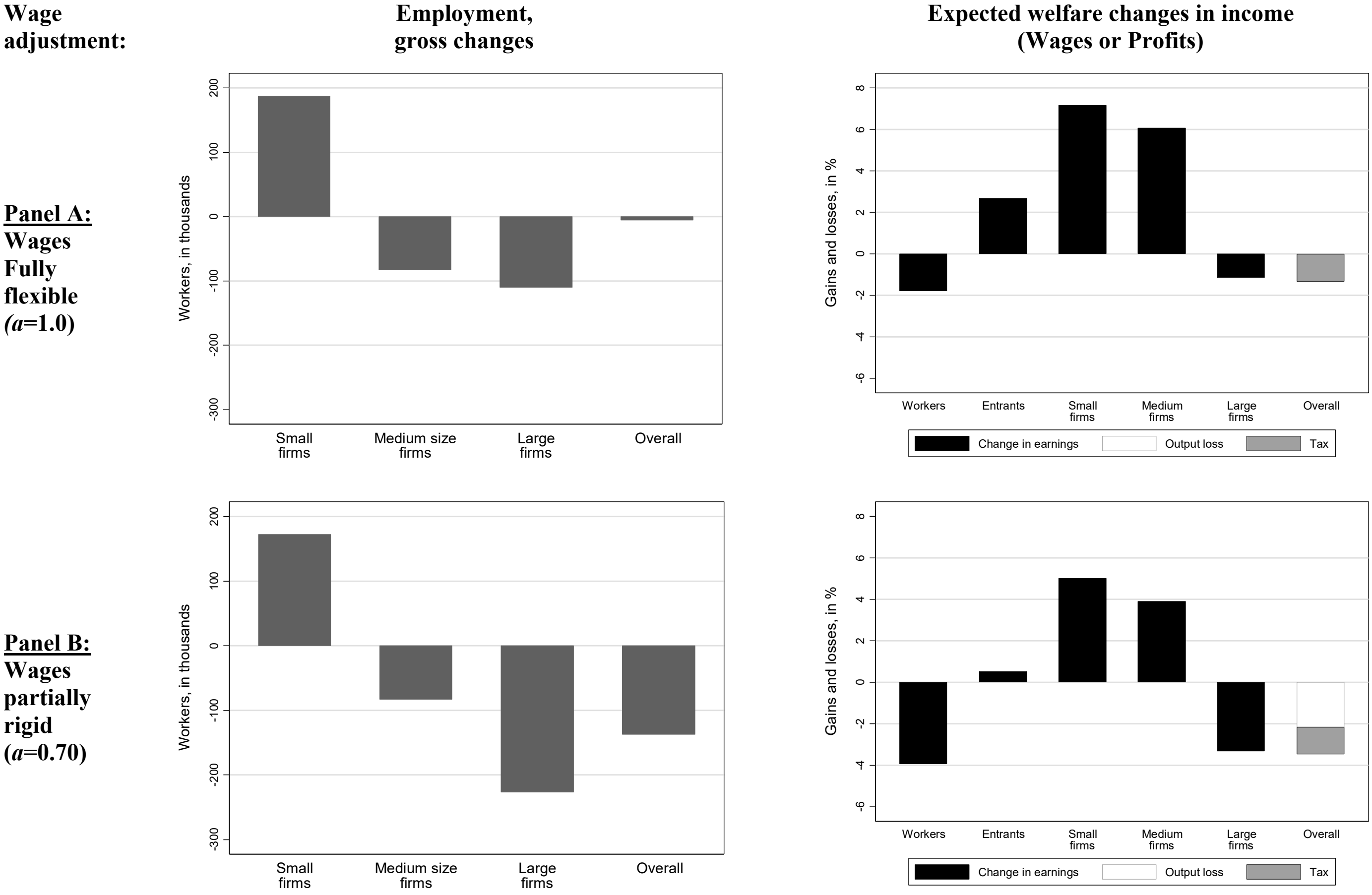


Zooming in around the Regulatory Threshold



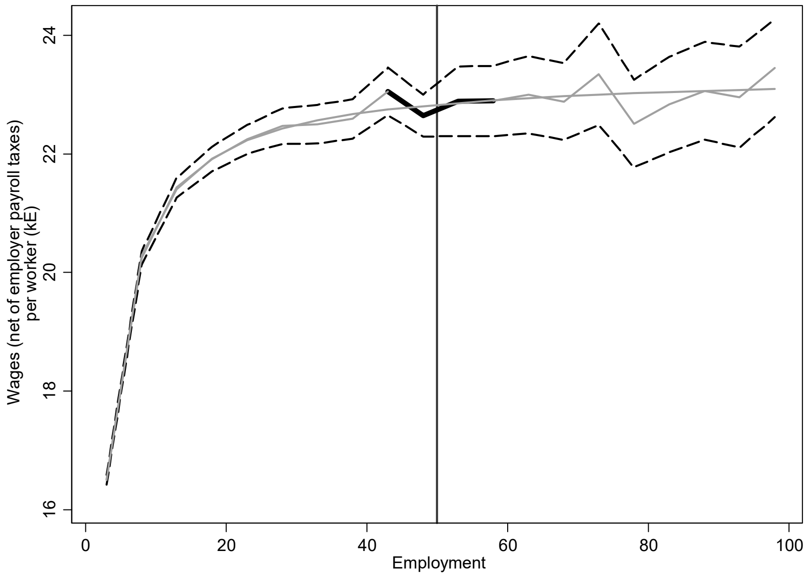
Notes: Figures in both panels compare the firm size distribution in the regulated economy (bold line) from a world without regulation (dashed line) based on the estimated parameters from our model (baseline specification reported in Table 1, column (1)). In the lower panel ("zooming around the regulatory threshold"), the "spike" has been cut.

Figure 10: Employment and Welfare Changes by Firm Size Category



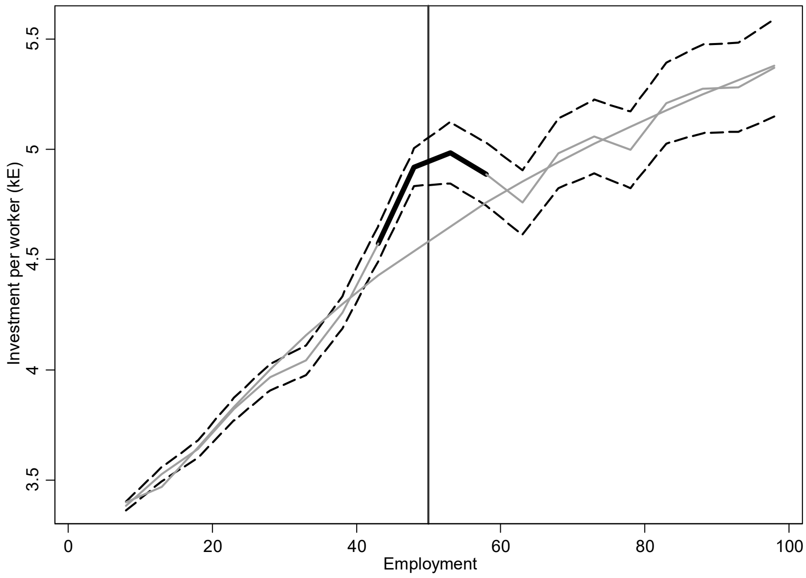
Notes: These Figures illustrate the calculations in Table 3. They are based on the parameter estimates in the baseline model of Table 1 column (1). Left hand columns present the change due to the regulation in the number of jobs in each firm size category and right hand side figures are welfare changes. "Small firms" $[\alpha_{\min}; \alpha_c\ ]$ are below the threshold (*N*=49). "Medium firms" $[\alpha_c\ ; \alpha_r\ [$ are larger than 49 but smaller than n_u (= 59.3). "Large firms" $[\alpha_r\ ; 1]$ are above n_u. Results are taken over 41,067 manufacturing firms (2.3m workers) with 10 to 1,000 workers in the year 2000 (the range of data used for estimation), employing a total of 2.29 million workers. The right side figures present percentage changes in earnings for workers and profits of firms in each category (see also table 3). "Workers" $[\underline{\alpha}; \alpha_{\min}[$ are individuals who do not choose to set-up a firm (employed or unemployed workers) in the economy with regulation. "Entrants" $[\alpha_{\min}; \alpha_{\min}^{NR}[$ are agents who are workers in the economy without regulation, but entrepreneurs of small firms in the economy with regulation. "Small firms" $[\alpha_{\min}^{NR}; \alpha_c\ ]$ are entrepreneurs of small firms in both the economy with regulation, and in the economy without regulation. "Medium firms" $[\alpha_c\ ; \alpha_r\ [$ are entrepreneurs of (mostly) medium size firms in the economy without regulation, but entrepreneurs of 49 worker firms in the economy with regulation. "Large firms" $[\alpha_r\ ; 1]$ are entrepreneurs of large firms in both the economy with regulation, and in the economy without regulation.

Figure 11: Wages around the Regulatory Threshold: Are Workers Accepting Lower Wages in Return for Amenity Value of Regulation?



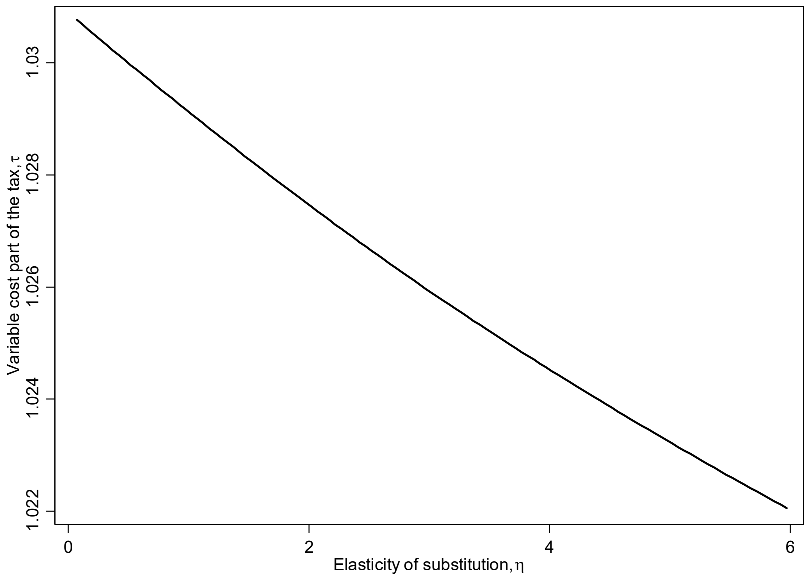
Source: FICUS, 2000.
Notes: Wages is the nominal wage (net of payroll tax) by employer size. 95% confidence intervals shown.

Figure 12: Investment per Worker



Source: FICUS, 2000.
Notes: Investment per worker, measured in thousands of euros. 95% confidence intervals shown.

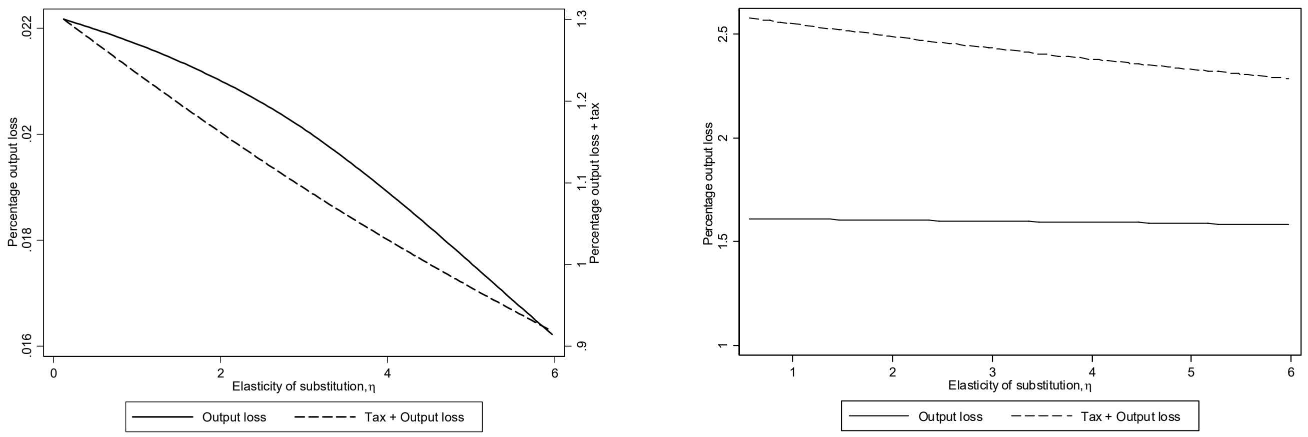
Figure 13: Allowing for Capital/Labor Substitutability (CES) –
How Regulatory Tax Estimates Vary with Calibrated Level of Elasticity of Substitution



Source: FICUS

Notes: Parameters estimated by ML. The figure shows how the estimates of the variable cost of the regulation change when we alter the assumptions over the elasticity of substitution (ranging between 0 and 6). See Appendix D for the relation between estimates of τ in the baseline, single factor model and in this CES specification.

Figure 14: Allowing for Capital/Labor Substitutability (CES) -
How Welfare Losses Change with Assumptions over Substitutability



Panel A: Fully flexible wages **Panel B: Partially Rigid wages**

Notes: Parameters estimated by ML. The figure shows how welfare changes when we alter the assumptions over the elasticity of substitution (ranging between zero and 6). Panel A looks at the case of fully flexible wages and Panel B looks at the case of partially rigid wages, where the adjustment parameter *a* is calibrated to match the difference in the structural employment rates between France and the US (*u* = 2.622 percentage points). The price of capital assumed to be completely flexible in both panels.

Table 1: Parameter Estimates (Calibrating Returns to Scale θ)

Method	(1) θ calibrated: Basu and Fernald, 1997	(2) θ calibrated: Atkeson and Kehoe, 2005	(3) θ calibrated: Hsieh-Klenow, 2009	(4) θ calibrated at 0.9	(5) TFP/size relationship	(6) Using production function estimates
θ, scale parameter	0.80	0.85	0.50	0.90	0.793	0.860
					(0.024)	(0.012)
β, power law	1.800	1.800	1.800	1.813	1.800	1.801
	(0.054)	(0.054)	(0.054)	(0.051)	(0.058)	(0.057)
n_r, upper emp.	59.271	59.265	59.271	52.985	59.271	59.200
Threshold	(2.051)	(2.026)	(2.052)	(0.317)	(2.262)	(1.715)
σ, measurement	0.121	0.121	0.121	0.041	0.121	0.120
error	(0.033)	(0.032)	(0.033)	(0.003)	(0.041)	(0.027)
τ-1, implicit tax,	0.023	0.017	0.059	0.007	0.024	0.016
variable cost	(0.008)	(0.006)	(0.021)	(0.001)	(0.008)	(0.005)
F/w, implicit tax,	-0.941	-0.704	-2.375	-0.321	-0.974	-0.655
fixed cost	(0.338)	(0.252)	(0.865)	(0.057)	(0.345)	(0.201)
Mean (Median) # of employees	55.8 (24)	55.8 (24)	55.8 (24)	55.8 (24)	55.8 (24)	55.8 (24)
Firms	41,067	41,067	41,067	41,067	41,067	41,067
Ln Likelihood	-184,128.7	-184,128.7	-184,128.7	-184,196		

Notes: Parameters estimated by ML with standard errors below in parentheses (clustered at the four digit level). Estimation is on population of French manufacturing firms with 10 to 1,000 employees, in the year 2000. These estimates of the implicit tax are based on different estimates of θ; the methods are indicated in the different columns. In columns (5) and (6), standard deviations are computed using bootstrap (100 replications). In column (5), the underlying TFP estimates are 0.109 (0.006) for capital and 0.751 (0.014) for labor.

Table 2: Comparison Estimated Results with Actual Data

	(1) Small Firms (10 to 48 workers)	**(2) Medium Firms (49 to 59 workers)**	**(3) Large Firms (60 to 10,000 workers)**
Panel A: Firm Distribution			
Distribution of firms (actual)	0.761	0.042	0.197
	(0.007)	(0.002)	(0.005)
Distribution of firms (predicted)	0.758	0.046	0.196
	(0.002)	(0.009)	(0.015)
Panel B: Employment distribution			
Distribution of employment (actual)	0.300	0.040	0.660
	(0.005)	(0.001)	(0.005)
Distribution of employment (predicted)	0.276	0.042	0.682
$n=n^*[\alpha].e^{\varepsilon}$; $\sigma_\varepsilon = 0.121$	(0.021)	(0.008)	(0.023)
Panel C: Output Distribution			
Distribution of output (actual)	0.249	0.035	0.716
	(0.003)	(0.001)	(0.003)
	[0.243;0.255]	[0.033;0.037]	[0.710;0.722]
Distribution of output (predicted)	0.272	0.041	0.686
$y = \alpha.n^{\theta}=\alpha.\ (n^*[\alpha])^{\theta}.e^{\theta\varepsilon}$, $\sigma_\varepsilon = 0.121$	(0.021)	(0.008)	(0.023)
	[0.230;0.314]	[0.025;0.057]	[0.640;0.732]

Notes: "Actual" distribution computed over our data in 2000 of population of manufacturing firms 10 to 1,000 employees. "Predicted" distribution computed using empirical model described in Section 3 (incorporating measurement error) and our baseline estimates in Table 1 column (1). Standard deviations for the distribution of actual data are computed from the time variation of empirical distributions over 1995-2007. Standard deviations for predicted values are computed using bootstrap (with 100 replications).

Table 3: Welfare and Distributional Analysis

Description of benchmark:	**(1)** **Perfectly flexible real wages**	**(2)** **Difference in US/French structural unemployment**	**(3)** **Diffusion of min. wage indexation**	**(4)** **Standard mid-term adjustment assumption**
Wage adjustment (degree of flexibility, *a*)	**1.0**	**0.7**	**0.67**	**0.62**
θ, scale parameter	**0.8**	**0.8**	**0.8**	**0.8**
1. Unemployment rate	0%	2.622%	2.909%	3.342%
2. Percentage of firms avoiding the regulation, δ	2.920%	2.859%	2.852%	2.842%
3. Percentage of firms paying tax (compliers)	10.387%	10.168%	10.144%	10.108%
4. Change in labor costs (wage reduction) for small firms (below 49)	-1.792%	-1.258%	-1.120%	-1.109%
5. Change in labor costs (wage reduction but tax increase), firms above 49	0.502%	1.0.36%	1.095%	1.185%
6. Excess entry by small firms (percent increase in number of firms)	7.184%	7.176%	7.175%	7.174%
7. Increase in size of small firms	8.958%	6.292%	5.996%	5.548%
8. Increase in size of large firms	-2.512%	-5.178%	-5.474%	-5.923%
9. Annual welfare loss (as a percentage of GDP):				
a. Implicit Tax	1.304%	1.302%	1.302%	1.302%
b. Output loss	0.022%	2.148%	2.384%	2.741%
c. Total (Implicit Tax + Output loss)	**1.326%**	**3.450%**	**3.686%**	**4.043%**
10. Winners and losers:				
a. Change in expected wage for those who remain in labor force	-1.792%	-3.915%	-4.152%	-4.509%
b. Average gain by entering entrepreneurs of small firms	2.667%	0.539%	0.303%	-0.055%
c. Average profit gain by small unconstrained firms	7.167%	5.034%	4.797%	4.438%
d. Average profit gain by firms constrained at 49	6.061%	3.928%	3.691%	3.333%
e. Change in profit for large firms	-1.159%	-3.292%	-3.529%	-3.888%

Notes: Based on the baseline estimates of Table 1 column (1), under the assumption that the maximum firm size is 10,000. Column (1) assumes real wages fully adjust. Columns (2) to (4) assume partial adjustment. "% firms avoiding the regulation" (row 2) is mass who move to the spike at 49. "% firms paying tax" (row 3) is mass of agents with productivity greater than α_u relative to agents with productivity greater than α_{min}. "Change in labor costs for small firms" (row 4) is equilibrium wage effect. "Change in labor costs for large firms" (row 5) is Row 4 plus the estimated implicit tax (τ -1). "Excess entry" (row 6) is the difference in the ln(mass of agents having productivity greater than $\alpha_{\min}$) minus ln(mass of agents having productivity greater than $\alpha_{\min}^0$), where $\alpha_{\min}^0$ is the threshold between workers and entrepreneurs in the counterfactual economy without regulation. "Increase in size of small firms" (row 7) corresponds to $\ln(n^*) - \ln(n_0^*)$ for firms having productivity smaller than α_u ; "increase in size of large firms" (row 8) corresponds to n^* -n_0^* for firms having productivity greater than α_c . "Implicit Tax" (row 9a) corresponds to the total amount of implicit tax ($\int_{\alpha \geq \alpha_c}[(\tau - 1).w^*.n^*(\alpha) + k]\phi(\alpha)\mathrm{d}\alpha$ as a share of total output, Y^*. "Output loss" (row 9b) corresponds to $\ln(Y^*) - \ln(Y_0^*)$. For "winners and losers" (row 10), we compute the average (percentage point) changes in expected wages or profits for agents in each of the following bins: 10a labor force $[\underline{\alpha}; \alpha_{\min}[$; 10b new entrepreneurs $[\alpha_{\min}; \alpha_{\min}^{NR}[$; 10c. small firms $[\alpha_{\min}^{NR}; \alpha_c]$; 10d. constrained firms $[\alpha_c ; \alpha_r[$;10e. large firms $[\alpha_r ; 1]$. In the "rigid wages" case in columns (2) to (4), expected wages are computed as $(1 - u_R^*).w_R^*$ where u denotes the unemployment rate.

Table 4: Welfare results: robustness to changing scale parameter θ

	(1)	(2)	(3)	(4)
Description of benchmark:	**Perfectly flexible real wages**	**Difference in US/French structural unemployment**	**Diffusion of min. wage indexation**	**Standard mid-term adjustment assumption**
Wage adjustment (degree of flexibility, *a*)	**1.0**	**0.68**	**0.67**	**0.62**
θ, scale parameter	**0.5**	**0.5**	**0.5**	**0.5**
1. Unemployment rate	0%	2.622%	2.731%	3.137%
2. Percentage of firms avoiding the regulation, δ	0.962%	0.941%	0.941%	0.937%
3. Percentage of firms paying tax (compliers)	3.420%	3.348%	3.345%	3.334%
4. Change in labor costs (wage reduction) for small firms (below 49)	-4.164%	-2.833%	-2.778%	-2.568%
5. Change in labor costs (wage reduction but tax increase), firms above 49	1.571%	2.902%	2.958%	3.167%
6. Excess entry by small firms (percent increase in number of firms)	6.671%	6.667%	6.666%	6.666%
7. Increase in size of small firms	8.328%	5.667%	5.554%	5.136%
8. Increase in size of large firms	-3.142%	-5.804%	-5.916%	-6.334%
9. Annual welfare loss (as a percentage of GDP):				
a. Implicit Tax	1.893%	1.891%	1.891%	1.891%
b. Output loss	0.039%	1.368%	1.424%	1.633%
c. Total (Implicit Tax + Output loss)	**1.933%**	**3.259%**	**3.332%**	**3.524%**
10. Winners and losers:				
a. Change in expected wage for those who remain in labor force	-4.464%	-5.490%	-5.546%	-5.755%
b. Average gain by entering entrepreneurs of small firms	-0.017%	-1.346%	-1402%	-1.611%
c. Average profit gain by small unconstrained firms	4.164%	2.833%	2.778%	2.568%
d. Average profit gain by firms constrained at 49	3.459%	2.128%	2.072%	1.863%
e. Change in profit for large firms	-1.052%	-2.383%	-2.439%	-2.648%

Notes: Based on the baseline estimates of Table 1 column (3), under the assumption that the maximum firm size is 10,000. Column (1) assumes real wages fully adjust. Columns (2) to (4) assume partial adjustment. All definitions are the same as Table 3.

Web Appendices:
Not Intended for Publication Unless Requested

A Proof of Propositions 1 and 2

Recall that firm optimization yields a piecewise continuous and differentiable firm size function $n^*(\alpha)$ defined by equations (6a), (6b), (6c) and (6d). The equilibrium requires finding w, $\alpha_{\min}$ and α_r such that occupational choice (equation (5)), supply equals demand (equation (7)) and managerial choice between paying or not taxes (equation (4)) holds.[50] We can rewrite these equations as:[51]

$$\alpha_{\min} f(n^*(\alpha_{\min})) - wn^*(\alpha_{\min}\ ) - w = 0 \tag{19}$$

$$\int_{\underline{\alpha}}^{\alpha_{\min}} \phi(\alpha)\mathrm{d}\alpha - \int_{\alpha_{\min}}^{\alpha_c(w,N)} n^*(\alpha)\phi(\alpha)\mathrm{d}\alpha - \int_{\alpha_c(w,N)}^{\alpha_r} N\phi(\alpha)\mathrm{d}\alpha - \int_{\alpha_r}^{\alpha_{\max}} n^*(\alpha)\phi(\alpha)\mathrm{d}\alpha = 0 \tag{20}$$

$$\alpha_r f(n^*(\alpha_r)) - w\tau n^*(\alpha_r) - F - (\alpha_r f(N) - wN) = 0 \tag{21}$$

where $\alpha_{\max}$ is finite or infinite, and where n^* has been replaced by its (constant) value between α_c and α_r. In vector form, these three equations write (ignoring N to reduce notation): $g(\alpha_{\min}, w, \alpha_r; \tau, F) = 0$.

Let (τ_0, F_0) be a specific value for the variable and fixed costs of the regulation and $\mathbf{x} = (\alpha_{\min}, w, \alpha_r)$. By the implicit function theorem there exists a function $h : A \to R$ on a neighborhood A of (τ_0, F_0) such that $(\mathbf{x}) = h(\tau, F)$ and $g(h(\tau, F), \tau, F) = 0$ for every $(\tau, F) \in A$, as long as $D_{\mathbf{X}} g(\mathbf{x}_0, \tau_0, F_0)$ is non singular (which is true as we show in the next sub-section).

Note that all functions are differentiable in the parameters in these equilibrium conditions except that n^* is not differentiable at α_r. Nevertheless all three equilibrium conditions are differentiable in α_r: the first is not a function of α_r; the second is differentiable in α_r despite the discontinuity of n^*, and the third is differentiable in α_r.

A.1 Comparative Statics with Respect to τ

To obtain the comparative statics described we compute $dh/d\tau = -\left(D_{\mathbf{X}} g(\mathbf{x}, \tau)\right)^{-1} D_\tau g(\mathbf{x}, \tau)$. First, computing derivatives and using the envelope theorem, $D_{\mathbf{X}} g(\mathbf{x}, \tau)$ simplifies to:

$$D_{\mathbf{X}} g(\mathbf{x}, \tau) = \begin{pmatrix} f(n_{\min}) & -n_{\min} - 1 & 0 \\ \phi(\alpha_{\min})(1 + n_{\min}) & -I_w & \phi(\alpha_r)\left(n_r - N\right) \\ 0 & -\left(\tau n_r - N\right) & f(n_r) - f(N) \end{pmatrix} \tag{22}$$

where:

$$\begin{aligned} I_w &= \int_{\alpha_{\min}}^{\alpha_c} \frac{\partial n^*(\alpha)}{\partial w}\phi(\alpha)\mathrm{d}\alpha + \int_{\alpha_r}^{\alpha_{\max}} \frac{\partial n^*(\alpha)}{\partial w}\phi(\alpha)\mathrm{d}\alpha \\ &= \int_{\alpha_{\min}}^{\alpha_c} \left(\frac{1}{\alpha f''(n^*)}\right)\phi(\alpha)\mathrm{d}\alpha + \int_{\alpha_r}^{\alpha_{\max}} \left(\frac{\tau}{\alpha f''(n^*)}\right)\phi(\alpha)\mathrm{d}\alpha < 0 \end{aligned}$$

The second equality follows from equation (6d), $n^*(\alpha, w, \overline{\tau})) = f'^{-1}\left(\frac{\overline{\tau} w^*}{\alpha}\right)$.

Second, we have:

$$D_\tau g(\mathbf{x}, \tau) = \begin{pmatrix} 0 \\ -I_\tau \\ -wn_r \end{pmatrix}, \tag{23}$$

---

[50] For compactness, α_c is here directly replaced with $\alpha_c(w, N) = \left(n^*\right)^{-1}(N) = \frac{w}{f'(N)}$.

[51] It is useful to also write equations (19) and (21) in terms of firm size (using $\alpha_{\min} = \frac{w}{f'(n_{\min})}$ and $\alpha_r = \frac{\tau w}{f'(n_r)}$):

$$\begin{aligned} f(n_{\min}) &= (1 + n_{\min}).f'(n_{\min}) \\ f(n_r) - f(N) &= f'(n_r).\left[n_r - \frac{N}{\tau} + \frac{F}{\tau.w}\right] \end{aligned}$$

where I_τ can be expressed as:

$$\begin{aligned} I_\tau &= \int_{\alpha_r}^{\alpha_{\max}} \frac{\partial n^*(\alpha)}{\partial \tau}\phi(\alpha)\mathrm{d}\alpha \\ &= \int_{\alpha_r}^{\alpha_{\max}} \left(\frac{w}{\alpha f''(n^*)}\right)\phi(\alpha)\mathrm{d}\alpha < 0 \end{aligned}$$

Then from $-\left(D_X g(\mathbf{x},\tau)\right)^{-1} D_\tau g(\mathbf{x}_0,\tau_0)$ and simplifying we have:

$$\begin{aligned} \frac{d\alpha_{\min}}{d\tau} &= \frac{(n_{\min}+1)\left[\underbrace{I_\tau}_{<0}.(f(n_r)-f(N)) - w.n_r.(n_r-N).\phi(\alpha_r)\right]}{D} < 0 && (24) \\ \frac{dw}{d\tau} &= \frac{f(n_{\min})\left[I_\tau.(f(n_r)-f(N)) - w.n_r.(n_r-N).\phi(\alpha_r)\right]}{D} < 0 && (25) \end{aligned}$$

where the determinant D of matrix $D_{\mathbf{X}}g(\mathbf{x},\tau)$ is:[52]

$$D = f(n_{\min}).\left[\underbrace{-I_w}_{>0}.(f(n_r)-f(N)) + (\tau n_r - N).\phi(\alpha_r).(n_r-N)\right] + \phi(\alpha_{\min}).(f(n_r)-f(N)).(1+n_{\min})^2 > 0$$

This is what we wanted to show; this result is true a fortiori starting from $\tau = 1$, the situation without taxes. Points (a) and (b) of Proposition 1 follow immediately.

Point (c) is less straightforward. The partial equilibrium effect of taxes on size is immediate as:

$$\frac{\partial n}{\partial \tau} = \frac{w}{\alpha f''(n^*)} < 0.$$

However, the general equilibrium effect is the possible reversal induced by the reduction in wages calculated above:

$$\left.\frac{dn}{d\tau}\right|_{\alpha>\alpha_r} = \frac{\partial n}{\partial \tau} + \frac{\partial n}{\partial w}\frac{dw}{d\tau} = \frac{1}{\alpha f''(n^*)}\left(w + \tau\frac{dw}{d\tau}\right) \qquad (26)$$

and thus $\frac{dn}{d\tau} < 0$ iff $w + \tau\frac{dw}{d\tau} > 0$. We get equivalently:

$$\begin{aligned} w + \tau\frac{f(n_{\min})\left[I_\tau.(f(n_r)-f(N)) - w.n_r.(n_r-N).\phi(\alpha_r)\right]}{D} &> 0 \\ f(n_{\min}).\left[\tau.I_\tau - w.I_w\right].\left[f(n_r)-f(N)\right] + w.\left[\phi(\alpha_{\min}).\left[f(n_r)-f(N)\right].(1+n_{\min})^2 - \phi(\alpha_r).f(n_{\min}).N.(n_r-N)\right] &> 0 \end{aligned}$$

This holds true because the first term is positive:

$$\begin{aligned} \tau.I_\tau - w.I_w &= \int_{\alpha_r}^{\alpha_{\max}} \left(\frac{\tau.w}{\alpha f''(n^*)}\right)\phi(\alpha)\mathrm{d}\alpha - \int_{\alpha_{\min}}^{\alpha_c} \left(\frac{w}{\alpha f''(n^*)}\right)\phi(\alpha)\mathrm{d}\alpha - \int_{\alpha_r}^{\alpha_{\max}} \left(\frac{\tau.w}{\alpha f''(n^*)}\right)\phi(\alpha)\mathrm{d}\alpha \\ &= -\int_{\alpha_{\min}}^{\alpha_c} \left(\frac{w}{\alpha f''(n^*)}\right)\phi(\alpha)\mathrm{d}\alpha > 0 \end{aligned}$$

---

[52] $D > 0$ shows that $D_X g(\mathbf{x},\tau)$ is non-singular.

The second term is also positive since we can use the conditions at $\alpha_{\min}$ and α_r to get:

$$\begin{aligned} \phi(\alpha_{\min}).\left[f(n_r) - f(N)\right].(1+n_{\min})^2 \quad &- \quad \phi(\alpha_r).f(n_{\min}).N.(n_r - N) \geq 0 \\ &\iff \quad \frac{\phi(\alpha_{\min})}{f'(\alpha_{\min})}.(1+n_{\min}) \geq \frac{\phi(\alpha_r)}{f'(\alpha_u)}.\frac{n_r - N}{n_r - \frac{N}{\tau} + \frac{F}{\tau.w}} \\ &\iff \quad \alpha_{\min}.\phi(\alpha_{\min}).\underbrace{(1+n_{\min})}_{\geq 1} \geq \alpha_r.\phi(\alpha_r).\underbrace{\frac{n_r - N}{\tau.n_r - N + \frac{F}{w}}}_{\leq 1} \end{aligned}$$

This holds true because $\alpha.\phi(\alpha)$ is assumed to be decreasing in α ("scarcity of talent").

A.2 Comparative Statics with Respect to F

Derivations are analogous with respect to F. We have:

$$D_F g(\mathbf{x}, F) = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \tag{27}$$

such that, using the same notations as before:

$$\frac{d\alpha_{\min}}{dF} = -\frac{1}{D}.(n_{\min}+1).\phi(\alpha_r).(n_r - N) < 0 \tag{28}$$

$$\frac{dw}{dF} = -\frac{1}{D}.f(n_{\min}).\phi(\alpha_r).(n_r - N) < 0 \tag{29}$$

Last, in contrast to the comparative statics with respect to τ, we get:

$$\left.\frac{dn}{dF}\right|_{\alpha > \alpha_r} = \frac{\partial n}{\partial F} + \frac{\partial n}{\partial w}\frac{dw}{dF} = 0 + \frac{\tau}{\alpha f''(n^*)}.\frac{dw}{dF} > 0$$

B Details of the Baseline Estimated Model

This Appendix describes the details of the model estimated in sections 2.3 and 3.

B.1 Firms' Objective Function and Labor Demand

In our baseline, single input model, the objective function of firms is:

$$\pi(\alpha) = \max_n \begin{cases} \alpha n^\theta - wn & \text{if } n \leq N \\ \alpha n^\theta - w\tau n - F & \text{if } n > N \end{cases} \tag{30}$$

Optimal labor demand is easily derived from the first order conditions:

$$n^*(\alpha) = \begin{cases} \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}}.\alpha^{\frac{1}{1-\theta}} & \text{if } \alpha \in [\alpha_{\min}; \alpha_c] \\ N & \text{if } \alpha \in]\alpha_c; \alpha_r] \\ \left(\frac{\theta}{\tau.w}\right)^{\frac{1}{1-\theta}}.\alpha^{\frac{1}{1-\theta}} & \text{if } \alpha \in]\alpha_r; 1] \end{cases} \tag{31}$$

The indifference condition at $\alpha_{\min}$ is:

$$\alpha_{\min}.n_{\min}^\theta - w.n_{\min} = w \tag{32}$$

Using the relation between ability and size, we obtain: $n_{\min} = \frac{\theta}{1-\theta}$.

B.2 Density of the Firm Size Distribution

Using the previous variable change, we derive the density of the theoretical firm size distribution as:

$$\chi^*(n) = \begin{cases} \underbrace{\frac{c_\alpha}{p}(1-\theta)\left(\frac{\theta}{w}\right)^{\frac{\beta-1}{1-\theta}}}_{=C} n^{-\beta} & \text{if } n_{\min} \leq n < N = n^*(\alpha_c) \\ \frac{1}{p}\int_{\alpha_c}^{\alpha_r} \phi(\alpha)\mathrm{d}\alpha = \delta & \text{if } n = N \\ 0 & \text{if } N < n < n_r \\ \underbrace{\frac{c_\alpha}{p}(1-\theta)\left(\frac{\theta}{w}\right)^{\frac{\beta-1}{1-\theta}}}_{=C} \underbrace{\tau^{-\frac{\beta-1}{1-\theta}}}_{=T} n^{-\beta} & \text{if } n^*(\alpha_r) = n_r \leq n \end{cases}$$

where p is the proportion of entrepreneurs across all agents in the economy.
The quantities δ and C can be further specified. First, δ can be expressed in terms of firm size rather than ability, using the following variable change: $n(\alpha) = \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}}.\alpha^{\frac{1}{1-\theta}}$ (see equation 31). We get:

$$\begin{aligned} \delta &= \int_{\alpha_c}^{\alpha_r} \phi(\alpha)\mathrm{d}\alpha \\ &= \int_{\left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}}.\alpha_c^{\frac{1}{1-\theta}}}^{\left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}}.\alpha_r^{\frac{1}{1-\theta}}} \frac{c_\alpha}{p}.(1-\theta).\left(\frac{\theta}{w}\right)^{\frac{\beta-1}{1-\theta}}.s^{-\beta}\mathrm{d}s = \int_N^{\tau^{\frac{1}{1-\theta}}.n_r} \frac{c_\alpha}{p}.(1-\theta).\left(\frac{\theta}{w}\right)^{\frac{\beta-1}{1-\theta}}.s^{-\beta}\mathrm{d}s \\ &= \frac{c_\alpha}{p}.\frac{1-\theta}{\beta-1}.\left(\frac{\theta}{w}\right)^{\frac{\beta-1}{1-\theta}}.\left(N^{1-\beta} - \tau^{-\frac{\beta-1}{1-\theta}}.n_r^{1-\beta}\right) = \frac{C}{\beta-1}.\left(N^{1-\beta} - T.n_r^{1-\beta}\right) \end{aligned}$$

Moreover, add-up constraints on the probability density function χ^* provides an alternative expression for δ:

$$\begin{aligned} \delta &= 1 - \frac{c_\alpha}{p}.\frac{1-\theta}{\beta-1}.\left(\frac{\theta}{w}\right)^{\frac{\beta-1}{1-\theta}}.\left((n_{\min})^{1-\beta} - N^{1-\beta} + \tau^{-\frac{\beta-1}{1-\theta}}.n_r^{1-\beta}\right) \\ &= 1 - \frac{C}{\beta-1}\left(\left(\frac{\theta}{1-\theta}\right)^{1-\beta} - N^{1-\beta} + T.n_r^{1-\beta}\right) \end{aligned}$$

Taken together, the two previous expressions imply: $\frac{C}{\beta-1} = \left(\frac{1-\theta}{\theta}\right)^{1-\beta}$ and $\delta = \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\left(N^{1-\beta} - T.n_r^{1-\beta}\right)$
We obtain a simplified expression for the theoretical firm size distribution, which corresponds to equation (8) in the main text:

$$\chi^*(n) = \begin{cases} \left(\frac{1-\theta}{\theta}\right)^{1-\beta}(\beta-1)n^{-\beta} & \text{if } n_{\min} \leq n < N \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}\left(N^{1-\beta} - Tn_r^{1-\beta}\right) & \text{if } n = N \\ 0 & \text{if } N < n < n_r \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}(\beta-1)Tn^{-\beta} & \text{if } n_r \leq n \end{cases} \tag{33}$$

B.3 Empirical Model and Proof of Lemma 1

When employment is measured with error, we can only observe the following quantity:

$$n(\alpha, \varepsilon) = n^*(\alpha).e^{\varepsilon} \tag{34}$$

We can then write the conditional CDF of this variable denoted by x below:

$$\begin{aligned}
\mathbb{P}(x<n|\varepsilon) &= \begin{cases} 0 & \text{if } n.e^{-\epsilon} \leq n_{\min} \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.(\beta-1).\int_{n_{\min}}^{n.e^{-\epsilon}} x^{-\beta}\mathrm{d}x & \text{if } n_{\min} \leq n.e^{-\epsilon} < N \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\underbrace{(\beta-1).\int_{n_{\min}}^{N} x^{-\beta}\mathrm{d}x}_{=\left(\frac{\theta}{1-\theta}\right)^{1-\beta}-N^{1-\beta}} + \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\left(N^{1-\beta}-T.n_r^{1-\beta}\right) & \text{if } N \leq n.e^{-\epsilon} \leq n_r \\ \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\left(\left(\frac{\theta}{1-\theta}\right)^{1-\beta}-T.n_r^{1-\beta}\right) + \left(\frac{1-\theta}{\theta}\right)^{1-\beta}.(\beta-1).T.\int_{n_r}^{n.e^{-\epsilon}} x^{-\beta}\mathrm{d}x & \text{if } n_r \leq n.e^{-\epsilon} \end{cases} \\
&= \begin{cases} 0 & \text{if } \ln(n)-\ln\left(n_{\min}\right) \leq \varepsilon \\ 1-\left(\frac{1-\theta}{\theta}\right)^{1-\beta}.\left(n.e^{-\varepsilon}\right)^{1-\beta} & \text{if } \ln(n)-\ln(N) < \varepsilon \leq \ln(n)-\ln\left(n_{\min}\right) \\ 1-\left(\frac{1-\theta}{\theta}\right)^{1-\beta}.T.n_r^{1-\beta} & \text{if } \ln(n)-\ln(n_r) \leq \varepsilon \leq \ln(n)-\ln(N) \\ 1-\left(\frac{1-\theta}{\theta}\right)^{1-\beta}.T.\left(n.e^{-\varepsilon}\right)^{1-\beta} & \text{if } \varepsilon \leq \ln(n)-\ln(n_r) \end{cases}
\end{aligned}$$

Assuming that ε is a Gaussian noise with mean 0 and variance σ, and denoting by φ the Gaussian pdf and by Φ the Gaussian cdf, we can compute the unconditional probability as:

$$\begin{aligned}
\forall n>0, \quad \mathbb{P}(x<n) &= \int_{\mathbb{R}} \mathbb{P}(x<n|\varepsilon)\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon \\
&= \int_{\ln(n)-\ln(N)}^{\ln(n)-\ln(n_{\min})} \left[1-\frac{C}{\beta-1}.n^{1-\beta}.e^{\varepsilon.(\beta-1)}\right]\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon + \int_{\ln(n)-\ln(n_r)}^{\ln(n)-\ln(N)} \left[1-\frac{C}{\beta-1}.T.n_r^{1-\beta}\right]\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon \\
&\quad + \int_{-\infty}^{\ln(n)-\ln(n_r)} \left[1-\frac{C}{\beta-1}.T.n^{1-\beta}.e^{\varepsilon.(\beta-1)}\right]\frac{1}{\sigma}.\varphi\left(\frac{\varepsilon}{\sigma}\right)\mathrm{d}\varepsilon \\
&= \underbrace{\Phi\left(\frac{\ln(n)-\ln(n_{\min})}{\sigma}\right)}_{=A(n)} - \underbrace{\frac{C}{\beta-1}.T.n_r^{1-\beta}.\left[\Phi\left(\frac{\ln(n)-\ln(N)}{\sigma}\right)-\Phi\left(\frac{\ln(n)-\ln(n_r)}{\sigma}\right)\right]}_{=B(n)} \\
&\quad \underbrace{-\frac{C}{\beta-1}.n^{1-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\left[\Phi\left(\frac{\ln(n)-\ln(n_{\min})}{\sigma}-\sigma.(\beta-1)\right)-\Phi\left(\frac{\ln(n)-\ln(N)}{\sigma}-\sigma.(\beta-1)\right)\right]}_{=C(n)} \\
&\quad \underbrace{-\frac{C}{\beta-1}.T.n^{1-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\Phi\left(\frac{\ln(n)-\ln(n_r)}{\sigma}-\sigma.(\beta-1)\right)}_{=D(n)}
\end{aligned} \tag{35}$$

One can easily check that this function is strictly increasing (straightforward from the way we constructed it), with limits 0 in 0 and 1 in $+\infty$:

$$\begin{aligned}
A(n) &\underset{n\to+\infty}{\longrightarrow} 1 & \qquad A(n) &\underset{n\to0}{\longrightarrow} 0 \\
B(n) &\underset{n\to+\infty}{\longrightarrow} Cst\times(1-1)=0 & \qquad B(n) &\underset{n\to0}{\longrightarrow} Cst\times(0-0)=0 \\
C(n) &\underset{n\to+\infty}{\longrightarrow} 0\times(1-1)=0 & \qquad C(n) &\underset{x\to0}{\longrightarrow} +\infty\times(0-0)=0 \text{ (*)} \\
D(n) &\underset{n\to+\infty}{\longrightarrow} 0\times1=0 & \qquad D(n) &\underset{n\to0}{\longrightarrow} +\infty\times0=0 \text{ (*)}
\end{aligned}$$

To solve the two problematic cases, marked with $(*)$, let us consider $\mathcal{F}(n)$ defined for $\mathcal{F}\in\mathbb{R}$ as:

$$\begin{aligned}
\mathcal{F}(n) \quad &= \quad n^{1-\beta}.\Phi\left(\frac{\ln(n)}{\sigma}+\mathcal{F}\right) = \frac{\Phi\left(\frac{\ln(n)}{\sigma}+\mathcal{F}\right)}{n^{\beta-1}} \\
\text{(L'Hôpital's rule)} \quad &\underset{n\to0}{\sim} \quad \frac{\frac{1}{n.\sigma}\varphi\left(\frac{\ln(n)}{\sigma}+\mathcal{F}\right)}{(\beta-1).n^{\beta-2}} \\
&\underset{n\to0}{\sim} \quad \frac{1}{\sigma.\sqrt{2\pi}.(\beta-1)}.e^{-\frac{F^2}{2}}.\underbrace{n^{1-\beta}.e^{-\frac{1}{2}.\frac{(\ln n)^2}{\sigma^2}}.e^{-\frac{1}{2}.\mathcal{F}.\frac{\ln(n)}{\sigma}}}_{=e^{\ln(n).\left(1-\beta-\frac{1}{2}.\frac{\mathcal{F}}{\sigma}-\frac{\ln(n)}{2.\sigma^2}\right)}} \\
&\underset{n\to0}{\longrightarrow} \quad 0
\end{aligned}$$

Last, the density corresponding to the previous CDF is given by:

$$\begin{aligned}
\chi(n) \quad = \quad & \frac{1}{\sigma.n}.\varphi\left(\frac{\ln(n)-\ln(n_{\min})}{\sigma}\right) - \frac{1}{\sigma.n}.\frac{C}{\beta-1}.T.n_r^{1-\beta}\left[\varphi\left(\frac{\ln(n)-\ln(N)}{\sigma}\right) - \varphi\left(\frac{\ln(n)-\ln(n_r)}{\sigma}\right)\right] \\
& -\frac{C}{\beta-1}.n^{-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.(1-\beta).\left(\Phi\left(\frac{\ln(n)-\ln(n_{\min})}{\sigma}-\sigma.(\beta-1)\right) - \Phi\left(\frac{\ln(n)-\ln(N)}{\sigma}-\sigma.(\beta-1)\right)\right) \\
& -\frac{C}{\beta-1}.n^{-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\frac{1}{\sigma}.\left(\varphi\left(\frac{\ln(n)-\ln(n_{\min})}{\sigma}-\sigma.(\beta-1)\right) - \varphi\left(\frac{\ln(n)-\ln(N)}{\sigma}-\sigma.(\beta-1)\right)\right) \\
& -\frac{C}{\beta-1}.n^{-\beta}.T.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.\left[(1-\beta).\Phi\left(\frac{\ln(n)-\ln(n_r)}{\sigma}-\sigma.(\beta-1)\right) + \frac{1}{\sigma}.\varphi\left(\frac{\ln(n)-\ln(n_r)}{\sigma}-\sigma.(\beta-1)\right)\right] \qquad (36)
\end{aligned}$$

Using the fact that $e^{\frac{\sigma^2}{2}.(\beta-1)^2}.n^{1-\beta}.\varphi\left(\frac{\ln(n)-\ln(X)}{\sigma}-\sigma.(\beta-1)\right) = X^{1-\beta}.\varphi\left(\frac{\ln(n)-\ln(X)}{\sigma}\right)$ and simplifying, we get:

$$\begin{aligned}
\chi(n) \quad = \quad & \frac{1}{\sigma.n}.\frac{C}{\beta-1}.\left(N^{1-\beta} - T.n_u^{1-\beta}\right)\varphi\left(\frac{\ln(n)-\ln(N)}{\sigma}\right) \\
& -\frac{C}{\beta-1}.n^{-\beta}.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.(1-\beta).\left(\Phi\left(\frac{\ln(n)-\ln(n_{\min})}{\sigma}-\sigma.(\beta-1)\right) - \Phi\left(\frac{\ln(n)-\ln(N)}{\sigma}-\sigma.(\beta-1)\right)\right) \\
& -\frac{C}{\beta-1}.n^{-\beta}.T.e^{\frac{\sigma^2}{2}.(\beta-1)^2}.(1-\beta).\Phi\left(\frac{\ln(n)-\ln(n_u)}{\sigma}-\sigma.(\beta-1)\right) \qquad (37)
\end{aligned}$$

We use standard ML techniques to estimate the parameters in equation (37). Note that we obtain an estimate of C from this procedure from which we can, in principle recover an estimate of the coefficient θ. This is unlikely to be a powerful way of identifying the scale parameter, however. We found empirically that the likelihood was very flat when trying to estimate θ in this way, suggesting it was not well identified: this is in particular due to the fact that we only estimate the conditional size distribution for firms having 10 (or 5) to 1,000 employees (while we expect θ to be identified from the curvature of the distribution "on the left", for the smallest firms). In the baseline specifications, we use various calibrated values of θ, with 0.8 being our preferred value. Note that the value of the ln-likelihood in column (4) of Table 1 shows that very large values of θ are rejected by the data, because the obtained ln-likelihood drops. See section C.3 for alternative strategies for the estimation of θ.

B.4 Estimation of the Fixed Cost Associated with Regulations

The fixed cost is identified and estimated from the indifference equation at α_r:

$$\alpha_r.N^\theta - w.N = \alpha_r.n_r^\theta - w.\tau.n_r - F$$

Since from equation (31) we can express α_r in terms of n_r, we get:

$$\frac{F}{w} = N - \frac{\tau}{\theta}.n_r.\left[\left(\frac{N}{n_r}\right)^\theta - 1 + \theta\right] \qquad (38)$$

B.5 Derivation of the Additional Structural Parameters Required for Counterfactual Analysis

B.5.1 Case with Fully Flexible Wages

Given our estimates of β, we have to assume that there exists an upper bound $n_{\max}$ (which is typically set to 10,000, or alternative values in Figure A10 to test robustness) in terms of firm size. Therefore, we slightly re-scale the firm size distribution in the following way:

$$\chi(n) = \begin{cases} \frac{\beta-1}{n_{\min}^{1-\beta}-T.n_{\max}^{1-\beta}}.n^{-\beta} & \text{if } n_{\min} \leq n < N \\ \frac{N^{1-\beta}-T.n_r^{1-\beta}}{n_{\min}^{1-\beta}-T.n_{\max}^{1-\beta}} & \text{if } n = N \\ 0 & \text{if } N < n < n_r \\ T.\frac{\beta-1}{n_{\min}^{1-\beta}-T.n_{\max}^{1-\beta}}.n^{-\beta} & \text{if } n_r \leq n \end{cases} \qquad (39)$$

Furthermore, we normalize the ability level $\alpha_{\max}$ corresponding to $n_{\max}$ to 1 without loss of generality, such that

$n_{\max} = \left(\frac{\theta}{\tau.w}\right)^{\frac{1}{1-\theta}}$. Applying the variable change formula, we get:

$$\frac{\beta-1}{n_{\min}^{1-\beta} - T.n_{\max}^{1-\beta}} = \frac{c_\alpha}{p}.(1-\theta).\frac{n_{\max}^{\beta-1}}{T}$$

In this expression, p is the proportion of entrepreneurs in the economy and is still unknown. It can however be estimated using the labor market equilibrium equation (version with fully flexible wages) as:

$$\begin{aligned} p &= \frac{\text{number of firms}}{\text{number of firms} + \text{number of workers in firms}} \\ &= \frac{n_{\min}^{1-\beta} - T.n_{\max}^{1-\beta}}{\left(n_{\min}^{1-\beta} - T.n_{\max}^{1-\beta}\right) + \frac{\beta-1}{\beta-2}.\left(n_{\min}^{2-\beta} - N^{2-\beta}\right) + N.\left(N^{1-\beta} - T.n_r^{1-\beta}\right) + \frac{\beta-1}{\beta-2}.T.\left(n_r^{2-\beta} - n_{\max}^{2-\beta}\right)} \end{aligned}$$

Therefore c_α can be computed as:

$$c_\alpha = \frac{T.n_{\max}^{1-\beta}}{n_{\min}^{1-\beta} - T.n_{\max}^{1-\beta}} \frac{p.(\beta-1)}{1-\theta} \tag{40}$$

and we have: $\beta_\alpha = \frac{\beta-\theta}{1-\theta}$.

B.5.2 Case with Rigid Wages

Let u be the unemployment rate on the labour market. The indifference equation at $\alpha_{\min}$ is altered as:

$$w.(1-u) = Y_{\min} - w.n_{\min} \tag{41}$$

Rearranging terms, we get a new expression for $n_{\min}$:

$$n_{\min} = \frac{\theta}{1-\theta}.(1-u) \tag{42}$$

Second, the labor market equation is:

$$\frac{\beta-1}{n_{\min}^{1-\beta} - T.n_{\max}^{1-\beta}}.\left[\frac{n_{\min}^{2-\beta} - N^{2-\beta}}{\beta-2} + \frac{N}{\beta-1}.\left(N^{1-\beta} - T.n_r^{1-\beta}\right) + T.\frac{n_r^{2-\beta} - n_{\max}^{2-\beta}}{\beta-2}\right]$$

$$= \frac{(1-p).(1-u)}{p} = \frac{1-\hat{p}}{\hat{p}} \tag{43}$$

where $p = \frac{\sharp\text{firms}}{\sharp\text{agents}}$ and $\hat{p} = \frac{\sharp\text{firms}}{\sharp\text{firms}+\sharp\text{workers in firms}} = p.\frac{\sharp\text{agents}}{\sharp\text{agents}-\sharp\text{unemployed workers}}$. This will affect how c_α is estimated.

B.5.3 Estimating the Degree of Wage Rigidity

As discussed in the text in order to calculate the welfare implications of the regulations we need to calibrate the parameter a, which indicates the degree to which real wages are flexible downwards in response to the regulation. The simplest case is complete flexibility ($a = 1$), but this is unrealistic in the French context of high minimum wages and powerful trade unions.

We therefore consider three ways of calibrating a.[53] Our preferred method is to use the average structural unemployment rate between 2002 and 2007 for the US and France as estimated by OECD (2015).[54] These were 8.308% and 5.686% leading to a difference of 2.622 percentage points. We use the US as a benchmark as it is the least regulated labor market in the industrial world so the unemployment rate could be considered as due to inescapable search and matching frictions. We then look for the value of a consistent with the model and its estimated parameters. This turns out to be an estimate of 0.70, i.e. the French real wage adjusts downwards by 70% of what it would do in the flexible wage economy (the US).

As a second alternative we exploit the institutional features of the French minimum wage. Unlike the US this is explicitly updated and indexed to past changes in the consumer price index. Aeberhardt et al (2012) estimate empirically the effect of the French minimum wage on the entire wage distribution. They look at the annual uprating

---

[53] In Garicano et al (2013) we also consider the opposite case of complete real wage rigidity which generates extremely large welfare losses).

[54] This are closest years to our sample period we could get from the OECD with a consistent time series.

of the minimum wage and what impact it has on different quantiles of the wage distribution. Using unconditional quantile regressions they estimate that the minimum wage is passed through 100% for those at or below the 10th percentile of the wage distribution; had a 40% pass through rate at for those between the 10th to 70th percentiles and had no affect for those in the top three deciles. This implies a value of $a = 0.67$, very similar to the previous method, which is reassuring. Hence we take $a = 0.70$ as our baseline case.

Finally, we consider using the degree of wage inflexibility assumed by the official macro-economic model of the French Finance Ministry (e.g. de Loubens and Thornary, 2010). This model estimates a partial adjustment of real wages of 62% after two years in response to an exogenous price shock (e.g. the oil price) which corresponds to an even greater inflexibility $a = 0.62$[55]

In the Table 3 we show all four estimates, focusing on the baseline $a = 0.70$ case.

B.6 Equilibrium Without Regulation

In the unregulated economy, the profit maximization program is simply:

$$\pi(\alpha) = \max_n \alpha.n^\theta - w.n \tag{44}$$

$$n^*(\alpha) = \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}}.\alpha^{\frac{1}{1-\theta}} \tag{45}$$

The indifference condition at $\alpha_{\min}$ is:

$$\alpha_{\min}.\left(n_{\min}\right)^\theta - w.n_{\min} = w$$

Using the relation between ability and size, we obtain as before: $n_{\min} = \frac{\theta}{1-\theta}$.

The density of the firm size distribution is given by:

$$\begin{aligned} \chi(n) &= \frac{\beta-1}{\left(n_{\min}\right)^{1-\beta} - \left(n_{\max}\right)^{1-\beta}}.n^{-\beta} && (46) \\ \frac{\beta-1}{\left(n_{\min}\right)^{1-\beta} - \left(n_{\max}\right)^{1-\beta}} &= \frac{c_\alpha}{p}.(1-\theta).\left(\frac{\theta}{w}\right)^{\frac{\beta-1}{1-\theta}} && (47) \end{aligned}$$

where maximum firm size is measured at $\alpha_{\max} = 1$ as:

$$n_{\max} = \left(\frac{\theta}{w}\right)^{\frac{1}{1-\theta}}$$

This implies:

$$p.\frac{\beta-1}{\left(n_{\min}\right)^{1-\beta} - \left(n_{\max}\right)^{1-\beta}} = c_\alpha.(1-\theta).\left(n_{\max}\right)^{\beta-1}$$

The labor market equation writes simply:

$$p.\frac{\beta-1}{\left(n_{\min}\right)^{1-\beta} - \left(n_{\max}\right)^{1-\beta}}.\int_{n_{\min}}^{n_{\max}} \left[n^{1-\beta} + n^{-\beta}\right] \mathrm{d}n = 1 \tag{48}$$

This implies that $n_{\max}$ can be expressed implicitly as a function of $n_{\min}$:

$$\frac{\left(n_{\min}\right)^{1-\beta} - \left(n_{\max}\right)^{1-\beta}}{\beta-1} + \frac{\left(n_{\min}\right)^{2-\beta} - \left(n_{\max}\right)^{2-\beta}}{\beta-2} = \frac{\left(n_{\max}\right)^{1-\beta}}{c_\alpha.(1-\theta)} \tag{49}$$

All these equations enable to solve the model in both the "regulated" and "unregulated" cases, with flexible, or rigid (indexed by a) wages. In the latter case however, we rely on the additional relation introduced in definition 2 (relating wages in the rigid economy to wages in the fully flexible economies, both regulated and unregulated), which enables to retrieve the additional parameter (the unemployment rate, u).

---

[55] Note however that this (very standard) macro-model predicts full adjustment in the *very* long run (i.e. $a = 1$), so we have a preference for the other two methods. Nevertheless it serves roughly as an "upper bound" of the welfare loss that is less extreme than assuming complete wage inflexibility.

C Extensions and Robustness Checks

C.1 Positive Spillovers Created by Regulation?

One of the empirical puzzles we found was that the fixed cost of the regulation was estimated to be negative. Such a finding can be rationalized by the existence of 'informational spillovers' within the firm from the regulation. The idea is that the reporting requirements of the regulation force a firm to keep much better track of workers' remuneration and conditions of employment. Collecting systematic information in this way may help firms reduce fixed costs by being able to better spot where there are problems - for example, with retention and recruitment, as well as potentially identify inefficiencies (e.g. through matching records on hours worked with performance of work units). Firms may develop these abilities in-house or they could purchase specialist software to help them do this, such as Enterprise Resource Planning.[56] These more systematic methods of monitoring the firm's employees and using this information to improve productivity are skin to the structured managerial practices of the type described by Bloom and Van Reenen (2007). For smaller firms, the overall costs of adopting these changes would still outweigh the benefits absent regulation - otherwise firms would already have adopted them. But being forced to collect and report information by the regulation could spill-over to other parts of a firm's operations potentially generating some small net fixed benefits.

To see this formally, assume as above that complying with the regulation involves a fixed cost $F \geq 0$ and a variable cost $\tau \geq 0$. Suppose that, by providing information to management, complying with information produces a positive internal spillover to the firm, describe by a concave function in size $G(n), G'(n) > 0$, $G(0) = 0$, $G''(n) < 0$. Assume that $F + w(1+\tau)n \geq G(n)$ for all n, so that only firms forced to comply enjoy the spillover. This ensures firms do not impose on themselves the regulatory costs voluntarily.

With these new functional forms a firm's profit becomes:

$$\pi(\alpha) = \max_n \alpha n^\theta - w\overline{\tau}n - F + 1_{\{n>N\}}G(n), \qquad \text{with } \begin{cases} \overline{\tau} = 1, F = 0 \text{ if } n \leq N \\ \overline{\tau} = \tau \geq 1; F \geq 0 \text{ if } n > N \end{cases} \tag{50}$$

To see that this type of spillovers can generate a negative intercept, approximate the concave function $G(n)$ with the function:

$$G(n) = \begin{cases} an \text{ for } n \leq N' \\ aN' \text{ for } n > N' \end{cases}$$

with $N' < N$. This functional form yields the result exactly: $\pi(\alpha) = \max_n \alpha n^\theta - w\overline{\tau}n - F + 1_{\{n>N\}}aN'$: for a sufficiently large, it will yield a negative estimate for the intercept of the fixed cost $F - aN'$ (where $1_{\{n>N\}}$ is an indicator variable that the firm is over the regulatory threshold, so $n > N$).

In general, this will be approximately true for a $G(n)$ concave and large enough.

This idea is related to an older literature which has discussed how regulations and other external cost shocks can have unexpected benefits in terms of fixed efficiency savings (e.g. the "induced innovations" of Hicks, 1932, or bad shocks that lead managers to reduce X-inefficiencies as discussed by Leibenstein, 1966). If firms are not optimizing these regulatory shocks could actually create positive welfare effects. However, in the context of our optimizing model, the net welfare impact of the regulations are still negative despite some of the offsetting fixed cost savings via such information spillovers.

C.2 Sensitivity of Estimates to the Gaussian Specification for Measurement Errors

How sensitive are our estimates to the Gaussian specification of the measurement error component? To investigate this issue, let us assume that the error term is distributed according to an unspecified pdf f, with associated cdf F. We only impose that f is sufficiently thin tailed, such that the expectation of e^ε is finite:

$$\int_{\mathbb{R}} e^{\varepsilon(\beta-1)} f(\varepsilon) \mathrm{d}\varepsilon = M < \infty \tag{51}$$

and that

$$1 - F(\ln(n)) = o\left(n^{1-\beta}\right) \text{ when } n \to +\infty \tag{52}$$

---

[56] Bloom, Garicano, Sadun and Van Reenen, (2014) discuss more details of the economics and organizational consequences of ERP introduction. Companies such as SAP and Oracle license these software systems to help companies keep better track of their human resources, procurement, customers, etc.

The unconditional cdf of n can be rewritten as[57]:

$$\begin{aligned}\forall n > 0, \quad \mathbb{P}(x < n) \quad &= \quad F\left(\ln(n) - \ln\frac{\theta}{1-\theta}\right) - \frac{C}{\beta - 1} T n_r^{1-\beta}\left[F(\ln(n) - \ln(N)) - F(\ln(n) - \ln(n_r))\right] \\ &\quad \frac{C}{\beta-1} n^{1-\beta}\left[\int_{\ln(n)-\ln(N)}^{\ln(n)-\ln\frac{\theta}{1-\theta}} e^{\varepsilon(\beta-1)} f(\varepsilon)\mathrm{d}\varepsilon + T\int_{-\infty}^{\ln(n)-\ln(n_r)} e^{\varepsilon(\beta-1)} f(\varepsilon)\mathrm{d}\varepsilon\right]\end{aligned}$$

Under the assumptions above, we can show that:

$$1 - \mathbb{P}(x < n) \underset{n\to+\infty}{\sim} \frac{C}{\beta-1} T M n^{1-\beta}$$

Therefore, for large values of n, the behavior of the optimization criterion is only affected by the distribution of ε *via* the term M. Conditional maximum likelihood estimation of the upper part of the distribution (unaffected by the "bulge" and the "valley") provides robust estimators β thanks to the slope. We can then obtain a robust estimate of the composite term $T.M$ comparing the mass of firms in the lower and upper part of the distribution, respectively. Assuming that f is centered on 0 and symmetric, one can show[58] that $M \geq 1$. In the Gaussian case, $\int_{\mathbb{R}} e^{\varepsilon(\beta-1)}\varphi(\varepsilon)\mathrm{d}\varepsilon = e^{\frac{\sigma^2}{2}(\beta-1)^2}$ which we estimate to be 1.0047 in our case. This means that the Gaussian assumption only induces a very small re-scaling of the T parameter via the M term. It is therefore very conservative with respect to τ, since the smallest value for τ would be obtained with no re-scaling at all. This lower bound for τ is estimated to be $\left(0.912 \times e^{\frac{\sigma^2}{2}(\beta-1)^2}\right)^{-\frac{1-\theta}{\beta-1}} \approx 1.022$, very close to the result presented in the main part of the text (1.023). Without any re-scaling at all, we would obtain an upper bound for T ($\widehat{T} = 0.951 > 0.948$) and a lower bound on τ ($\widehat{\tau} = 1.012 < 1.013$) which are in fact very close to the results presented in the main part of the text.

C.3 Using Information from the Productivity Distribution

As discussed in the main text, we consider three alternative routes of estimating θ. Our baseline is simply calibration, but we document here the two alternative strategies reported in Table 1: (i) estimates from the production function and (ii) using the TFP-size relationship. When we use these econometric methods to estimate θ, we take into account the variance around the estimation of θ in calculating the correct variance-covariance matrix by block-bootstrapping.

C.3.1 Estimation of TFP

There is no one settled way of best estimating TFP on firm level data and there are many approaches suggested in the literature. Fortunately, at least at the micro-level, different methods tend to produce results where the correlation of TFP estimated by different methods is usually high (see Syverson, 2011).[59]

In the baseline result we follow the method of Levinsohn and Petrin (2003) who propose extending the Olley and Pakes (1996) control function method to allow for endogeneity and selection. Olley and Pakes proposed inverting the investment rule to control for the unobserved productivity shock (observed to firm but unobserved to econometrician) that affects the firm's decision over hiring (and whether to stay in business). Because of the problem of zero investment regimes (common especially among smaller firms that we use in our dataset) Levinsohn and Petrin (2003) recommended using materials as an alternative proxy variable that (almost) always takes an observed positive value. We use this estimator to estimate firm-level production functions on French panel data 1995-2007 (using the unbalanced panel) by each of the four-digit manufacturing industries in our dataset. We also did the same for the retail sector and the business services sector. The production functions take the form (in each industry):

$$\ln y_{it} = \beta_n \ln n_{it} + \beta_k \ln k_{it} + \beta_m \ln m_{it} + \omega_{it} + \tau_t + \eta_{it} \tag{53}$$

where y = output, n = labour, k= capital, m = materials ω is the unobserved productivity shock, τ_t is a set of time dummies and η is the idiosyncratic error of firm i in year t. From estimating the parameters of the production function we can then recover our estimate of the persistent component of TFP.

There are of course many problems with these estimation techniques. For example, Ackerberg et al (2006) focus on the problem of exact multicollinearity of the variable factors conditional on the quasi-fixed factors given the assumption that input prices are assumed to be common across firms. Ackerberg et al (2007) suggest various solutions to this issue.

---

[57] The derivations below can alternatively be established in terms of the pdf.

[58] This is because in the case distribution of a distribution that is uniformly distributed over $[-x; x]$ (where $x > 0$), we have:

$$\frac{1}{2.x}\int_{-x}^{x} e^{\varepsilon(\beta-1)}\mathrm{d}\varepsilon = \frac{e^{x(\beta-1)} - e^{-x(\beta-1)}}{2x(\beta-1)} > 1$$

Therefore, if f is centered on 0 and symmetric, then it can be approximated by below by a bound that is arbitrarily close to 1.

[59] Results are all available on request.

We consider alternative ways to estimate TFP including the more standard Solow approach. Here we assume that we can estimate the factor coefficients in equation (53) by using the observed factor shares in revenues. We do this assuming constant returns to scale, so $\beta_n = \frac{wn}{py}$; $\beta_m = \frac{p^m m}{py}$and $\beta_k = 1 - \frac{wn}{py} - \frac{p^m m}{py}$ where p^m is the price of materials. We used the four digit industry factor shares averaged over our sample period for the baseline but also experimented with some firm-specific (time invariant) factor shares. As usual these alternative measures led to similar results.

A problem with both of these methods is that we do not observe firm-specific prices so the estimates of TFP as we only control for four digit industry prices. Consequently, the results we obtain could be regarded as only revenue-based TFPR instead of quantity-based TFPQ (see Hsieh and Klenow, 2009). TFPQ is closer to what we want to theoretically obtain as our estimate of managerial ability, α. In practice, there is a high correlation between these two measures as shown by Foster et al (2008) who have actual data on plant level input and output prices.[60] So it is unclear whether this would make too much of a practical difference to our results.

C.3.2 Using the Relationship Between Firm Size and TFP to Estimate θ

First, recall from sub-section 2.2 the relationship between firm size and TFP:

$$n^*(\alpha) = \begin{cases} 0 & \text{if } \alpha < \alpha_{\min} \\ \left(\frac{\theta}{w}\right)^{1/(1-\theta)} \alpha^{1/(1-\theta)} & \text{if } \alpha_{\min} \leq \alpha \leq \alpha_c \\ N & \text{if } \alpha_c \leq \alpha < \alpha_r \\ \left(\frac{\theta}{w}\right)^{1/(1-\theta)} \tau^{-1/(1-\theta)} \alpha^{1/(1-\theta)} & \text{if } \alpha_r \leq \alpha < \infty \end{cases}$$

The empirical model adds a stochastic error term to this to obtain:

$$n^*(\alpha) = \begin{cases} 0 & \text{if } \alpha < \alpha_{\min} \\ \left(\frac{\theta}{w}\right)^{1/(1-\theta)} \alpha^{1/(1-\theta)} e^{\varepsilon} & \text{if } \alpha_{\min} \leq \alpha \leq \alpha_c \\ N e^{\varepsilon} & \text{if } \alpha_c \leq \alpha < \alpha_r \\ \left(\frac{\theta}{w}\right)^{1/(1-\theta)} \tau^{-1/(1-\theta)} \alpha^{1/(1-\theta)} e^{\varepsilon} & \text{if } \alpha_r \leq \alpha < \infty \end{cases}$$

Or

$$\begin{aligned} \ln n_1 &= \frac{1}{1-\theta} \ln \alpha + \frac{1}{1-\theta} \ln \left(\frac{\theta}{w}\right) + \varepsilon \\ \ln n_2 &= \ln(N) + \varepsilon \\ \ln n_3 &= \frac{1}{1-\theta} \ln \alpha + \frac{1}{1-\theta} \ln \tau + \frac{1}{1-\theta} \ln \left(\frac{\theta}{w}\right) + \varepsilon \end{aligned}$$

Combining these together:

$$\ln n = \ln n_1 I_{\{\alpha_{\min} \leq \alpha \leq \alpha_c\}} + \ln n_2 I_{\{\alpha_c \leq \alpha < \alpha_r\}} + \ln n_3 I_{\{\alpha_r \leq \alpha\}} \tag{54}$$

where I is an indicator function for a particular "regime". We use the previous firm-level estimates of TFP (α) to estimate equation (54) and provide an alternative estimates of parameter θ. Results are reported in Table 1, column (5).

C.4 Using Alternative Datasets and Concepts of Employment

As discussed in sub-section 4.2, we also estimate our baseline model using alternative datasets and concepts of employment. In labor laws, the concept of employment in most regulations is a full-time equivalent concept (LAMY, 2010), with many special cases related to different types of workers (sick leaves, apprenticeship, etc.) and based on type of contract (full time *vs.* part time). This central information is unfortunately not available precisely in any dataset.

- The employment measure in the FICUS relates to a headcount of the number of workers in the firm averaged over the four quarters of the fiscal year in France. A headcount of employees is taken on the last day of the fiscal

---

[60] An alternative approach would be to follow de Loecker (2011) and put more structure on the product market. For example, assuming that the product market is monopolistically competitive enables the econometrician in principle to estimate the elasticity of demand and correct for the mark-up implicit in TFPR to obtain TFPQ.

year (usually December 31st) and on the last day at the end of each of the previous three quarters. Employment is then the simple arithmetic average over these four days: this "smoothing" of headcounts renders this concept closer to full time equivalent jobs.

- We also experiment with the measure of full time equivalent that is available in the DADS files (Panel B of Table A9). The main problem here is that full time equivalents are computed from "non-annexes" jobs only, i.e. those lasting more than 30 days with more than 1.5 hours worked per day, or those that were associated with (total) gross earnings higher than three times the monthly minimum wage. As reported in the methodological documentation of the DADS, in 2002, the "non-annexes" jobs represented 76.6% of all jobs, which means that almost a quarter of the most short-end, low paid jobs were removed. This is unfortunate as these jobs are in fact covered by labor laws. Note also that the type of contract (full time vs. part time) is not observed in the DADS data: it is estimated from the 75th percentile of the distribution of hours paid/worked per job (in firms having 20 to 1,000 workers). Unfortunately, this strategy only provides approximations of full time equivalents because the implementation of the mandatory 35 hour work week in 2000 was complex and differentiated across firms, which renders the relation between hours and full time equivalents heterogeneous across firms.
- We also estimate our model with a simple headcount on December 31th, that is available in the DADS (Panel C of Table A9).

The results of using alternative datasets and definitions of employment are presented in Table A5 and Figure A9. The alternative datasets differ mainly in terms of the mass point and discontinuity at the threshold at 49. The the spike as well as the discontinuity are more pronounced in FICUS than in DADS. In the DADS files, they are more pronounced in terms of headcounts rather than full time equivalents (which seems unsurprising from the way they are computed). Nevertheless a bulge and shift of the power law in clearly present in all panels in Figure A9.

Table A5 shows that these differences only affect the estimated fixed cost component of the regulation, while the variable cost part is remarkably stable across all datasets and concepts. This is not surprising, because the main component of the estimated tax generates a shift of the distribution rather than just mass points or discontinuities. Finally, Figure A9 shows that the fit of our model is broadly similar across all three concepts of employment.

C.5 Robustness of the Results to the Definition of the Size of the Largest Firm in the Economy

Although the empirical maximum firm size is 86,587 in the data, we choose for the welfare calculation to focus on costs to firms up to 10,000 since there are on average only 5 firms per year having a size greater than 10,000 (out of an average of 170,000 firms with positive employment in manufacturing industries). In Figure A10 we show that our quantitative estimates of welfare and employment are not much changed when we vary our assumption about the upper bound of firm size (using alternative values of 1,000, 5,000 and 10,000 employees) with the exception of job losses at large firms, which are halved when we only consider firms up to 1,000 employees.

D Allowing for Capital-Labor Substitution: Underlying CES Model in Section 6.4

This Appendix presents the details of the CES model underlying sub-section 6.4 and its estimation. Figures 14 and 15 correspond to the estimation results for different calibrated values of the elasticity of substitution, η.

D.1 CES Specification of Firms' Objective Function

With a CES production function, the objective function of firms is:

$$\pi(\alpha) = \max_{n,k} \begin{cases} \alpha f(n,k) - wn - rk & \text{if } n \leq N \\ \alpha f(n,k) - w\tau n - rk - F & \text{if } n > N \end{cases} \tag{55}$$

with $f(n,k) = (\lambda.n^\rho + (1-\lambda).k^\rho)^{\frac{\theta}{\rho}}$ and where w is workers' wage, n is the number of workers, r is capital (marginal) cost, and k is the number of units of capital. Moreover, $\theta, \lambda \in (0,1)$ and $\rho \in \mathbb{R}$.
Optimal input demand in each regime (but not at the threshold) is then determined by the first order conditions:

$$[n] : \alpha\theta\left(\lambda.n^\rho + (1-\lambda).k^\rho\right)^{\frac{\theta}{\rho}-1} \lambda n^{\rho-1} - w\overline{\tau} = 0 \text{ with } \begin{cases} \overline{\tau} = 1 & \text{if } n < N \\ \overline{\tau} = \tau & \text{if } n > N \end{cases} \tag{56}$$

$$[k] : \alpha\theta \left(\lambda.n^{\rho} + (1-\lambda).k^{\rho}\right)^{\frac{\theta}{\rho}-1} (1-\lambda).k^{\rho-1} - r = 0 \tag{57}$$

Solving for this system of equations, we get the optimal demand for inputs in each regime (again, these hold everywhere, except at the threshold, N):

$$k^{*} = \alpha^{\frac{1}{1-\theta}} . \left[\frac{\theta(1-\lambda)}{r}\right]^{\frac{1}{1-\theta}} . \left(\lambda. \left(\frac{\gamma}{\overline{\tau}}\right)^{\eta-1} + (1-\lambda)\right)^{\frac{\theta-\rho}{\rho.(1-\theta)}} \tag{58}$$

$$n^{*} = \alpha^{\frac{1}{1-\theta}} . \left[\frac{\theta\lambda}{w.\overline{\tau}}\right]^{\frac{1}{1-\theta}} . \left(\lambda + (1-\lambda). \left(\frac{\overline{\tau}}{\gamma}\right)^{\eta-1}\right)^{\frac{\theta-\rho}{\rho.(1-\theta)}} \tag{59}$$

where $\gamma = \frac{r}{w}.\frac{\lambda}{1-\lambda}$ and $\eta = \frac{1}{1-\rho} \in \mathbb{R}^{+}$ is the elasticity of substitution.

D.2 Normalization and Calibrated Values

First, the labor-capital ratio is obtained by taking the ratio of equations (58) and (59):

$$\frac{n^{*}}{k^{*}} = \left[\frac{r.\lambda}{w\overline{\tau}.(1-\lambda)}\right]^{\eta} = \left(\frac{\gamma}{\overline{\tau}}\right)^{\eta} \tag{60}$$

This ratio is normalized to 1 for small firms (having $\overline{\tau} = 1$) without loss of generality: this simply defines the measurement unit of capital (to the quantity of capital allocated per worker in small, untaxed firms). This calibration is attractive since capital is only reported at historical costs in firms' balance sheets, such that capital intensity is typically difficult to measure empirically. Note that this implies that γ is always 1 for $\eta > 0$[61], such that λ will simply be estimated from the ratio of prices: $\lambda = \frac{w}{w+r}$.

Second, we calibrate the labor share in small firms to 0.6 (NIPA). This restriction together with the previous choice of measurement unit for capital fully determines λ and implies that wages are three times as high as capital price:

$$\frac{w.n^{*}}{Y} = \theta.\frac{\frac{\lambda}{1-\lambda}.\gamma^{\eta-1}}{\frac{\lambda}{1-\lambda}\gamma^{\eta-1}+1} = \theta.\frac{w}{w+r} = \theta.\lambda \tag{61}$$

where the second equality holds for $\eta > 0$ and follows from the definition of γ and from equation 60. θ is calibrated to 0.8, such that $\lambda = 0.75$ and $w = 3.r$.

Notice that this also determines the value of $n_{\min}$ in case of fully flexible prices (wages). Indeed, the indifference equation at $\alpha_{\min}$ writes:

$$\begin{aligned} w &= Y_{\min} - w.n_{\min} - r.k_{\min} \\ &= Y_{\min}.\left(1 - \frac{\gamma^{\eta-1}.\theta.\lambda}{\lambda\gamma^{\rho\eta} + (1-\lambda)} - \frac{\theta.(1-\lambda)}{\lambda\gamma^{\rho\eta} + (1-\lambda)}\right) \end{aligned}$$

Rearranging terms, we get:

$$n_{\min} = \frac{\theta}{1-\theta}.\frac{\frac{\lambda}{1-\lambda}.\gamma^{\eta-1}}{\frac{\lambda}{1-\lambda}.\gamma^{\eta-1}+1} = \frac{1}{1-\theta}.\frac{w.n^{*}}{Y}. \tag{62}$$

---

[61] This normalization makes clear that the CES specification formally reduces to the baseline single factor model (w', τ') for small and large firms, with:

$$\begin{aligned} w' &= w + r \\ \tau' &= \frac{\tau.w + \tau^{\eta}.r}{w+r} \end{aligned}$$

At the threshold N however, for ability ranging from α_c to α_r, the two problems are different, since except in the Leontief case (see section D.7 for more details in this specific case), output is not linear in α.

D.3 Estimated Density for the Firm Size Distribution

The density of the firm size distribution is given by:

$$\chi\left(n\right)=\begin{cases}\frac{\beta-1}{n_{\min}^{1-\beta}}.n^{-\beta} & \text{if } n_{\min}\leq n<N\\ \frac{N^{1-\beta}-T.n_r^{1-\beta}}{n_{\min}^{1-\beta}} & \text{if } n=N\\ 0 & \text{if } N<n<n_r\\ T.\frac{\beta-1}{n_{\min}^{1-\beta}}.n^{-\beta} & \text{if } n_r\leq n\end{cases} \tag{63}$$

Note that this estimation problem is strictly similar to the baseline case, except that $n_{\min}$ is calibrated to a different value, and that the quantity T has a different interpretation:

$$T^{\frac{1}{\beta-1}}=\tau^{-\frac{1}{1-\theta}}.\left[\frac{\lambda+(1-\lambda).\tau^{\eta-1}.\gamma^{-(\eta-1)}}{\lambda+(1-\lambda).\gamma^{-(\eta-1)}}\right]^{\frac{\theta-\rho}{\rho.(1-\theta)}} \tag{64}$$

Equation (64) is solved numerically to retrieve the estimate of τ.
In practice however, and as in the baseline model, we assume that there exists an upper bound $n_{\max}$ (which is typically set to 10,000) in terms of firm size. Therefore, we slightly re-scale the firm size distribution in the following way:

$$\chi\left(n\right)=\begin{cases}\frac{\beta-1}{n_{\min}^{1-\beta}-T.n_{\max}^{1-\beta}}.n^{-\beta} & \text{if } n_{\min}\leq n<N\\ \frac{N^{1-\beta}-T.n_r^{1-\beta}}{n_{\min}^{1-\beta}-T.n_{\max}^{1-\beta}} & \text{if } n=N\\ 0 & \text{if } N<n<n_r\\ T.\frac{\beta-1}{n_{\min}^{1-\beta}-T.n_{\max}^{1-\beta}}.n^{-\beta} & \text{if } n_r\leq n\end{cases} \tag{65}$$

D.4 Estimation of the Fixed Cost Associated with Regulations

In this section, we show how to compute the fixed cost $(\frac{F}{w})$ associated with the regulation as a function of the estimated parameters. As previously, the computation is mainly based on the indifference relation at α_r:

$$\alpha_r.f(N,K)-w.N-r.K=\alpha_r.f(n_r,k_r)-w.\tau.n_r-r.k_r-F \tag{66}$$

where K is optimal capital demand for a firm of productivity α_r choosing to remain at an employment level of N, while k_r is optimal capital demand for a firm of productivity α_r choosing to grow and pay taxes (remember profits are the same in both cases).
k_r is easily computed as $n_r.\frac{\tau^\eta}{\gamma^\eta}$. Since γ^η is calibrated to unity, this is simply:

$$k_r=n_r.\tau^\eta \tag{67}$$

Notice also that α_r can be expressed as a function of n_r:

$$n_r=\alpha_r^{\frac{1}{1-\theta}}.\left[\frac{\theta\lambda}{w.\tau}\right]^{\frac{1}{1-\theta}}.\left(\lambda+(1-\lambda).\left(\frac{\tau}{\gamma}\right)^{\eta-1}\right)^{\frac{\theta-\rho}{\rho.(1-\theta)}}$$

$$\Longleftrightarrow\alpha_r=n_r^{1-\theta}.\frac{w.\tau}{\theta.\lambda}.\left(\lambda+(1-\lambda).\left(\frac{\tau}{\gamma}\right)^{\eta-1}\right)^{\frac{\rho-\theta}{\rho}} \tag{68}$$

K can be computed from the first order condition of the optimization program for “constrained” firms:

$$\max_K\left\{\alpha_r.\left(\lambda.N^\rho+(1-\lambda).K^\rho\right)^{\frac{\theta}{\rho}}-w.N-r.K\right\} \tag{69}$$

The first order condition is:

$$\alpha_r.\theta.(1-\lambda).K^{\rho-1}\left(\lambda.N^\rho+(1-\lambda).K^\rho\right)^{\frac{\theta}{\rho}-1}=r \tag{70}$$

Using the previous expression for α_r and remembering that $r = \frac{1-\lambda}{\lambda}.\gamma.w$, we get:

$$n_r^{1-\theta}.K^{\rho-1}.\left(\frac{1+\frac{1-\lambda}{\lambda}.\tau^{\eta-1}.\left(\frac{1}{\gamma}\right)^{\eta-1}}{N^\rho + \frac{1-\lambda}{\lambda}.K^\rho}\right)^{\frac{\rho-\theta}{\rho}} = \frac{\gamma}{\tau} \tag{71}$$

Equation (71) can be solved numerically to get an estimate of K.
Lastly, fixed costs can be computed as:

$$\begin{aligned} \frac{F}{w} = N \quad &+ \quad \frac{1-\lambda}{\lambda}.\gamma.K \\ &- \quad n_r.\tau.\left(1+\tau^{\eta-1}.\frac{1-\lambda}{\lambda}.\left(\frac{1}{\gamma}\right)^{\eta-1}\right).\left[\frac{\theta-1}{\theta}+\frac{1}{\theta}.\frac{\left(N^\rho+\frac{1-\lambda}{\lambda}.K^\rho\right)^{\frac{\theta}{\rho}}}{n_r^\theta.\left(1+\tau^{\eta-1}.\frac{1-\lambda}{\lambda}.\left(\frac{1}{\gamma}\right)^{\eta-1}\right)^{\frac{\theta}{\rho}}}\right] \end{aligned} \tag{72}$$

D.5 Estimation of the Additional Structural Parameters Required for Counterfactual Analysis

D.5.1 Case with Fully Flexible Prices

To compute alternative equilibria (under the fully flexible price assumption), we are still missing the scaling parameter of the ability distribution, c_α, and the amount of capital per agent in the economy, k_{TOT}. Given our calibration choices[62], this quantity will entirely capture the new margin of adjustment when we will allow for different values of η.
As in the baseline model, we normalize $\alpha_{\max}$ to 1 without loss of generality, such that, using the relation between ability and size and applying the variable change formula, we get [63]:

$$\frac{\beta-1}{n_{\min}^{1-\beta} - T.n_{\max}^{1-\beta}} = \frac{c_\alpha}{p}.(1-\theta).\frac{n_{\max}^{\beta-1}}{T}$$

In this expression, p is the proportion of entrepreneurs in the economy and is still unknown. It can however be estimated using the labor market equilibrium equation (version with fully flexible wages) as:

$$\begin{aligned} p \quad &= \quad \frac{\text{number of firms}}{\text{number of firms} + \text{number of workers in firms}} \\ &= \quad \frac{n_{\min}^{1-\beta} - T.n_{\max}^{1-\beta}}{\left(n_{\min}^{1-\beta} - T.n_{\max}^{1-\beta}\right) + \frac{\beta-1}{\beta-2}.\left(n_{\min}^{2-\beta} - N^{2-\beta}\right) + N.\left(N^{1-\beta} - T.n_r^{1-\beta}\right) + \frac{\beta-1}{\beta-2}.T.\left(n_r^{2-\beta} - n_{\max}^{2-\beta}\right)} \end{aligned}$$

Therefore c_α can be computed as:

$$c_\alpha = \frac{T.n_{\max}^{1-\beta}}{n_{\min}^{1-\beta} - T.n_{\max}^{1-\beta}} \frac{p.(\beta-1)}{1-\theta} \tag{73}$$

and we have: $\beta_\alpha = \frac{\beta-\theta}{1-\theta}$, as in the baseline specification.

Last, we can compute the total stock of capital in the economy (normalized by the total number of agents),k_{TOT}, via the stock of capital utilized by firms in each size class.
The two easy cases are the following:

---

[62] Especially the normalization $n/k = 1$ for small firms. An alternative but strictly equivalent approach would have been to calibrate the total amount of capital and allow the parameters γ, λ (or r/w) adjust. This would however make the comparison with the Leontief case less straightforward (see section D.7).

[63] This also determines prices since $r = w/3$ and:

$$w = \theta.\frac{T^{\frac{1-\theta}{\beta-1}}}{n_{\max}^{1-\theta}}.\lambda^{\frac{\theta}{\rho}}.\left(1+\frac{1-\lambda}{\lambda}.\left(\frac{1}{\gamma}\right)^{\eta-1}\right)^{\frac{\theta-\rho}{\rho}}$$

- Small firms in $[n_{\min}; N[$

$$p.\frac{\beta-1}{n_{\min}^{1-\beta}-T.n_{\max}^{1-\beta}}.\int_{n_{\min}}^{N}\left(\underbrace{\gamma^{-\eta}}_{=1}\right)n^{1-\beta}\mathrm{d}n = p.\frac{\beta-1}{\beta-2}.\frac{n_{\min}^{2-\beta}-N^{2-\beta}}{n_{\min}^{1-\beta}-T.n_{\max}^{1-\beta}} \tag{74}$$

- Large firms in $[n_r; n_{\max}]$

$$p.\tau^{\eta}.T.\frac{\beta-1}{\beta-2}.\frac{n_r^{2-\beta}-n_{\max}^{2-\beta}}{n_{\min}^{1-\beta}-T.n_{\max}^{1-\beta}}$$

The difficulty lies in the computation of the stock of capital $k(\alpha)$ used by firms at N. In this range, there is no closed form for $k(\alpha)$, but the FOC of the profit maximization program delivers a closed form expression for the inverse of this function, $\alpha(k)$:

$$\alpha(k) = \frac{r}{1-\lambda}.\frac{1}{\theta}.k^{1-\rho}.\left[\lambda.N^{\rho}+(1-\lambda).k^{\rho}\right]^{1-\frac{\theta}{\rho}}$$

We use a variable change to get an expression of the integral to compute:

$$\int_{\alpha_c}^{\alpha_r} k(\alpha).c_{\alpha}.\alpha^{-\beta_{\alpha}}\mathrm{d}\alpha = c_{\alpha}.\left(\frac{r}{1-\lambda}\frac{1}{\theta}\right)^{1-\beta_{\alpha}}\int_{N}^{K} k \quad\times\quad \left[\lambda.N^{\rho}+(1-\lambda).k^{\rho}\right]^{(\beta_{\alpha}-1).\frac{\theta}{\rho}-\beta_{\alpha}}$$
$$\left[(1-\rho).\lambda.\left(\frac{N}{k}\right)^{\rho}+(1-\lambda).(1-\theta)\right]\mathrm{d}k \tag{75}$$

This integral is then evaluated numerically.

D.5.2 Case with "Rigid" Wages but Where the Price of Capital is Fully Flexible

Let u be the unemployment rate on the labour market. The indifference equation at $\alpha_{\min}$ is altered as:

$$w.(1-u) = Y_{\min} - w.n_{\min} - r.k_{\min}$$

Rearranging terms, we get a new expression for $n_{\min}$, replacing former equation (62):

$$n_{\min} = \frac{\theta}{1-\theta}.(1-u).\frac{\frac{\lambda}{1-\lambda}.\gamma^{\eta-1}}{\frac{\lambda}{1-\lambda}.\gamma^{\eta-1}+1} = \frac{1}{1-\theta}.(1-u).\frac{w.n^{*}}{Y}$$

Second, the labor market equation is:

$$\frac{\beta-1}{n_{\min}^{1-\beta}-T.n_{\max}^{1-\beta}}.\left[\frac{n_{\min}^{2-\beta}-N^{2-\beta}}{\beta-2}+\frac{N}{\beta-1}.\left(N^{1-\beta}-T.n_r^{1-\beta}\right)+T.\frac{n_r^{2-\beta}-n_{\max}^{2-\beta}}{\beta-2}\right]$$
$$= \frac{(1-p).(1-u)}{p} = \frac{1-\hat{p}}{\hat{p}} \tag{76}$$

where $p = \frac{\sharp\text{firms}}{\sharp\text{agents}}$ and $\hat{p} = \frac{\sharp\text{firms}}{\sharp\text{firms}+\sharp\text{workers in firms}} = p.\frac{\sharp\text{agents}}{\sharp\text{agents}-\sharp\text{unemployed workers}}$. This will affect how c_{α} is estimated. The derivation of k_{TOT} is unaltered: in the previous equation (75), only the estimation of c_{α} is affected by the new labor market equation.

D.6 Equilibrium without Regulation

In the unregulated economy, the profit maximization program is simply:

$$\pi(\alpha) = \max_{n,k}\alpha.\left(\lambda.n^{\rho}+(1-\lambda).k^{\rho}\right)^{\frac{\theta}{\rho}} - w.n - r.k \tag{77}$$

First order conditions are:

$$\begin{aligned} [n] \quad &: \quad \alpha\theta\left[\lambda.n^{\rho}+(1-\lambda).k^{\rho}\right]^{\frac{\theta}{\rho}-1}.\lambda.n^{\rho-1}=w && (78)\\ [k] \quad &: \quad \alpha\theta\left[\lambda.n^{\rho}+(1-\lambda).k^{\rho}\right]^{\frac{\theta}{\rho}-1}.(1-\lambda).k^{\rho-1}=r && (79) \end{aligned}$$

We obtain optimal input demand as:

$$\begin{aligned} \frac{n}{k} &= \left[\frac{r.\lambda}{w.(1-\lambda)}\right]^{\eta}=\gamma^{\eta} && (80)\\ k &= \alpha^{\frac{1}{1-\theta}}.\left[\frac{\theta(1-\lambda)}{r}\right]^{\frac{1}{1-\theta}}.\left(\lambda.\gamma^{\eta-1}+(1-\lambda)\right)^{\frac{\theta-\rho}{\rho.(1-\theta)}} && (81)\\ n &= \alpha^{\frac{1}{1-\theta}}.\left[\frac{\theta\lambda}{w}\right]^{\frac{1}{1-\theta}}.\left(\lambda+(1-\lambda).\left(\frac{1}{\gamma}\right)^{\eta-1}\right)^{\frac{\theta-\rho}{\rho.(1-\theta)}} && (82) \end{aligned}$$

The density of the firm size distribution is given by:

$$\begin{aligned} \chi(n) &= \frac{\beta-1}{(n_{\min})^{1-\beta}-(n_{\max})^{1-\beta}}.n^{-\beta} && (83)\\ \frac{\beta-1}{(n_{\min})^{1-\beta}-(n_{\max})^{1-\beta}} &= \frac{c_{\alpha}}{p}.(1-\theta).\left(\frac{\theta.\lambda}{w}\right)^{\frac{\beta-1}{1-\theta}}.\lambda^{\frac{(\theta-\rho)(\beta-1)}{\rho(1-\theta)}}.\left[1+\frac{1-\lambda}{\lambda}.\left(\frac{1}{\gamma}\right)^{\eta-1}\right]^{\frac{(\theta-\rho)(\beta-1)}{\rho(1-\theta)}} && (84) \end{aligned}$$

The minimum firm size is obtained as previously as:

$$n_{\min}=\frac{\theta}{1-\theta}.\frac{\frac{\lambda}{1-\lambda}.\gamma^{\eta-1}}{\frac{\lambda}{1-\lambda}.\gamma^{\eta-1}+1}=\frac{\theta}{1-\theta}.\left[1-\frac{1}{\frac{\lambda}{1-\lambda}.\gamma^{\eta-1}+1}\right] \tag{85}$$

Last, maximum firm size is measured at $\alpha_{\max}=1$ as:

$$n_{\max}=\left(\frac{\theta.\lambda}{w}\right)^{\frac{1}{1-\theta}}.\lambda^{\frac{\theta-\rho}{\rho(1-\theta)}}.\left[1+\frac{1-\lambda}{\lambda}.\left(\frac{1}{\gamma}\right)^{\eta-1}\right]^{\frac{\theta-\rho}{\rho(1-\theta)}} \tag{86}$$

which implies:

$$p.\frac{\beta-1}{(n_{\min})^{1-\beta}-(n_{\max})^{1-\beta}}=c_{\alpha}.(1-\theta).(n_{\max})^{\beta-1} \tag{87}$$

The labor market equation writes simply:

$$p.\frac{\beta-1}{(n_{\min})^{1-\beta}-(n_{\max})^{1-\beta}}.\int_{n_{\min}}^{n_{\max}}\left[n^{1-\beta}+n^{-\beta}\right]\mathrm{d}n=1 \tag{88}$$

This implies that $n_{\min}$ can be expressed implicitly as a function of $n_{\max}$:

$$\frac{(n_{\min})^{1-\beta}-(n_{\max})^{1-\beta}}{\beta-1}+\frac{(n_{\min})^{2-\beta}-(n_{\max})^{2-\beta}}{\beta-2}=\frac{(n_{\max})^{1-\beta}}{c_{\alpha}.(1-\theta)} \tag{89}$$

The capital market equation is:

$$p.\frac{\beta-1}{(n_{\min})^{1-\beta}-(n_{\max})^{1-\beta}}.\gamma^{-\eta}\int_{n_{\min}}^{n_{\max}}n^{1-\beta}\mathrm{d}n=k_{TOT} \tag{90}$$

Using the previous relations, this can be rewritten as:

$$\frac{(n_{\max})^{1-\beta}}{c_{\alpha}.(1-\theta)}.\left[1-k_{TOT}.\left(\frac{\lambda}{1-\lambda}\right)^{\frac{\eta}{1-\eta}}.\left[\frac{(1-\theta).n_{\min}}{\theta-(1-\theta).n_{\min}}\right]^{\frac{\eta}{\eta-1}}\right]=\frac{(n_{\min})^{1-\beta}-(n_{\max})^{1-\beta}}{\beta-1} \tag{91}$$

All these equations enable to solve the model in both the "regulated" and "unregulated" cases, with flexible or rigid wages. In the latter case however, as in sections B.5 and B.6, we also use the additional relation introduced in definition 2 (relating wages in the rigid economy to wages in the fully flexible economies, both regulated and unregulated), which enables to retrieve the additional parameter (the unemployment rate, u).

D.7 Relation with the Baseline, Single Input Model

The baseline, single input model features $\rho \to -\infty$, $\eta \to 0$. However, it does not map directly into the Leontief limiting case of this section. To see this more clearly, let us rewrite the model with two inputs combined into a Leontief production function. As previously, the measurement unit of capital is chosen such that, in small firms (below the threshold), $n^*/k^* = 1$. In the Leontief case however, if this relation holds for small firms, then it also holds for all other firms, since the optimal capital to labor ratio is fully constrained by technology. Therefore, the profit function can be written as:

$$\pi(\alpha)=\max_{n}\begin{cases}\alpha.n^{\theta}-\underbrace{(w+r)}_{=\tilde{w}}.n & \text{if } n\leq N\\ \alpha.n^{\theta}-\underbrace{(w+r)}_{=\tilde{w}}.\underbrace{\left(\frac{\tau.w+r}{w+r}\right)}_{=\tilde{\tau}}.n-F & \text{if } n>N\end{cases} \tag{92}$$

In the case of flexible prices, the analogy with the baseline case in the main text is straightforward. There are two important differences however, which prevents to interpret the Leontief case as the limit to the baseline, single factor model:

- The indifference condition defining $\alpha_{\min}$ is altered, since equation 5 is altered in the following way:

  $$\alpha_{\min}.\left(n^*(\alpha_{\min})\right)^{\theta}-(w+r).n^*(\alpha_{\min})=w \tag{93}$$

  which implies: $n_{\min}.(1-\theta)=\theta.\frac{w}{w+r}$. In the Leontief case, we still have that in small firms $\frac{w.n^*}{Y}=\theta.\frac{w}{w+r}$, such that the calibration in section D.2 drives both the theoretical minimum firm size, which differs from the baseline case (in main text), and the split of costs between w and r.

- Most importantly, equilibrium conditions on the capital market in both the regulated and unregulated economies imply that the number of firms is the same (and therefore, the total number of workers, which equates the aggregate supply of capital in the two cases). In other words, in the Leontief case with two factors of production, there is no adjustment via new entrants.

These two features explain why the CES extension does not map perfectly with the baseline, single factor model, though the difference in the obtained results is small.

The case where we assume that wages w are rigid but capital prices r are fully flexible also departs from the baseline case with rigid wages presented in sub-section 2.5.
As previously, the indifference equation at $\alpha_{\min}$ is altered, as well as $n_{\min}$:

$$\alpha_{\min}.\left(n^*(\alpha_{\min})\right)^{\theta}-(w+r).n^*(\alpha_{\min})=(1-u).w \tag{94}$$

which implies: $n_{\min}.(1-\theta)=(1-u).\theta.\frac{w}{w+r}$.
The labor market equations in both the regulated and unregulated economies are identical to the case with only one input described in the main text but equilibrium on the capital market however generates an important difference between the Leontief case with two inputs and the single input case. In the regulated economy with rigid wages, equilibrium on the capital market requires:

$$k_{TOT}=(1-u).\int_{\underline{\alpha}}^{\alpha_{\min}}c_{\alpha}.\alpha^{-\beta_{\alpha}}\mathrm{d}\alpha \tag{95}$$

In the unregulated economy, equilibrium on the capital market requires that:

$$k_{TOT} = \int_{\underline{\alpha}}^{\alpha_{\min}^{0}} c_{\alpha}.\alpha^{-\beta_{\alpha}} \mathrm{d}\alpha \tag{96}$$

This implies that $\alpha_{\min}^0 < \alpha_{\min}$, i.e. there is a *deficit* of entry in the equilibrium with regulations. Using the indifference conditions at $\alpha_{\min}$ and $\alpha_{\min}^0$ respectively, one can show that this deficit is driven by the rise of the price of capital, $r > r_0$.

This difference with the baseline model is however not a robust feature of the Leontief framework with two inputs: for example, allowing the aggregate supply of capital to be increasing in capital price would reduce entry in the undistorted equilibrium. Alternatively, moving away from the Leontief case and allowing η to increase rises r_0, which reduces entry in the undistorted economy (and raises unemployment very quickly in the regulated economy).

E Details of the Simulated Dynamic Model in Section 6.6

Our baseline approach, presented in sections 2 to 5, is static. Although such a parsimonious approach can be interpreted as the outcome of long term adjustments, it might miss important empirical patterns that arise in a dynamic environment. In section 6.6, we therefore consider a generalization of our model to allow dynamics which is more realistic but obviously more complex. All technical details are enclosed in this appendix.[64]

We make several important extensions to the basic static framework. First, we allow managerial ability (TFP) to change over time. Managerial ability is idiosyncratic when firms startup due to heterogeneous talent, but after a firm is founded it is subject each period to a shock which will usually cause it to want to change size. Second, we allow for adjustment costs in labor (and capital).[65] Thus, a firm will not always adjust immediately to its long-run frictionless optimal size.

Given this additional complexity, we can only obtain limited analytical results so we rely on numerical methods to simulate the steady state distributions of firm size and growth (e.g. Judd, 1998). In this appendix, we focus on the partial equilibrium effects of introducing adjustment costs (on the behavior of firms). We do not attempt to include general equilibrium effects as we have done in the static model as doing so would introduce substantial additional complexity to the problem without, we feel, delivering any additional analytical insights over and above the static model. Consequently, factor prices are treated as exogenous, common to all firms and time invariant.

E.1 Production, Costs and Regulation

Firm output y is determined by productivity α, capital k, labor n and materials m with common factor prices r, w and p_m respectively. Firm-specific values are indicated by an i sub-script: $y_{it} = \alpha_{it} k_{it}^{\beta_k} n_{it}^{\beta_n} m_{it}^{\beta_m}$. Span of control problems mean that ($\beta_k + \beta_n + \beta_m = \theta < 1$). Profits are defined as revenues less costs (variable and a fixed overhead cost, O). Materials adjust at no cost, but there are adjustment costs for capital ($C_k(k_{it}, k_{it-1})$) and for labor $C_n(n_{it}, n_{it-1})$ that take the following quadratic form:

$$C_n(n_{it}, n_{it-1}) = \gamma_n n_{it-1} (\frac{n_{it} - n_{it-1}}{n_{it-1}} - \delta_n)^2 \text{ and } C_k(k_{it}, k_{it-1}) = \gamma_k k_{it-1} (\frac{k_{it} - k_{it-1}}{k_{it-1}} - \delta_k)^2$$

Labor and capital are defined by the accumulation equations $n_{it} = (1-\delta_n) n_{it-1} + H_{it}$ and $k_{it} = (1-\delta_k) k_{it-1} + I_{it}$where H_{it} is hiring, $I_{it}$is gross investment, δ_n is the separation rate and δ_k is the depreciation rate of capital. Normalizing output prices to unity, flow profits are:

$$\begin{aligned} \pi_{it} &= \alpha_{it} k_{it}^{\beta_k} n_{it}^{\beta_n} m_{it}^{\beta_m} - \gamma_n n_{it-1} (\frac{n_{it} - n_{it-1}}{n_{it-1}} - \delta_n)^2 - \gamma_k k_{it-1} (\frac{k_{it} - k_{it-1}}{k_{it-1}} - \delta_k)^2 - r k_{it} \\ &\quad - w \overline{\tau} n_{it} - p_m m_{it} - F - O \end{aligned}$$

As in the baseline static model we assume that in the regulated economy there is a variable cost (τ) and fixed cost (F) that has to be paid when firms cross the discrete employment threshold at 50 employees.

E.2 Timing, Shocks and Equilibrium

Agents choose to become entrepreneurs and enter until the expected value of entry is equal to the sunk entry costs (S). Upon entry firms draw their management ability from a known distribution $G(\alpha)$. There is an endogenous profit threshold such that some firms will immediately exit after observing their ability as they cannot cover their fixed

---

[64] The code for running the dynamic simulations is in Matlab available on request.

[65] In this extension, we include both capital and materials as additional factors of production to labor.

costs (as in e.g. Melitz, 2003). In contrast to the static model we allow managerial ability/TFP to evolve over time according to an AR(1) process: $\ln \alpha_{it} = \rho_A \ln \alpha_{it-1} + u_{it}$. We assume u_{it} is i.i.d. normal with mean zero and variance σ_u. Firms decide their choices of hiring and investment (the policy correspondences) based on this shock, adjustment costs and the economic environment they face.[66]

Given the firm's three state variables - productivity α, capital k, and labor n - and the discount factor ϕ, we can write a value function (dropping i-subscripts for brevity) as:

$$\begin{aligned} V(\alpha_t, k_t, n_t) &= \max[V^c(\alpha_t, k_t, n_t), 0] \\ V^c(\alpha_t, k_t, n_t) &= \max_{k_{t+1}, n_{t+1}} y_t - rk_t - w\overline{\tau}n_t - p_m m_t - C_n(n_{t+1}, n_t) - C_k(k_{t+1}, k_t) - F - O \\ &\quad +\phi E_t V(\alpha_{t+1}, k_{t+1}, n_{t+1}) \end{aligned}$$

where the first maximum reflects the decision to continue in operation or exit (where exit occurs when $V^c < 0$), and the second (V^c for "continuers") is the optimization of capital and labor conditional on operation. Firms begin with no factor inputs. Hence, the free entry condition is $S = \int V(\alpha, n_0, k)dG(\alpha)$. We solve for the steady-state equilibrium ensuring that the expected cost of entry equals the expected value of entry given optimal factor demand decisions. This equilibrium is characterized by a distribution of firms in terms of their state values α, k, n plus their optimized choice of materials m.

We choose a set of calibration values drawn from our data and the general literature to simulate the model, which are given in Table A6. The results seem robust to reasonable changes in the exact values of these calibration values. For example, the labor adjustment cost parameter has generally be found to be lower than the capital adjustment cost parameter (e.g. Bloom, 2009), but it is not settled on exactly how much lower.

The initial distribution of TFP (α) is a power law. Firms have zero values of all factor inputs when they enter the economy. We discretize the state space for L, K and α into exponential bins for purposes of the numerical simulation. This means for TFP that there is a transition matrix between discrete points of the grid with fixed transition probabilities. We use results in Tauchen (1986) to approximate the AR(1) TFP process by this discrete transition matrix. TFP is the only source of stochastic variation shifting the environment. Firms adjust their factor inputs in response to changes in this state variable taking into account expectations, adjustment costs and their current state. Labor and capital are the other two state variables. For TFP we use a 30 by 30 grid. For employment we use a much finer grid to see the effect of regulation (100 points in the baseline). We use a coarser grid for capital (10 points), but not much hinges upon this (except the run-times for our MatLab code). We then generate the value function for entrants and incumbents. Using the contraction mapping (e.g. Stokey and Lucas, 1986) we iterate until we obtain a fixed point of the value function. The policy correspondences for hiring workers and capital investment are formed from the optimal choices given these value functions. Materials is simply the static period by period level based on the first order condition, as we are assuming no adjustment costs for this variable factor.

We simulate data for 20,000 firms over 100 periods (years). This time span was sufficient for the data to settle down to a steady state. We drop the first 75 "training years" and use the last 25 years to form the steady state in Figures A11 to A14. In the various experiments, we observe how these change when we alter key model parameters such as the level of adjustment costs and the regulatory tax parameters. When making changes we simply re-calibrate the parameter (e.g. magnitude of the adjustment cost) and re-simulate the economy again as previously.

E.3 Results

Figure A11 shows the firm size distribution (in employment terms) in the steady state for our baseline calibration. Just as we saw in the static model there is a discontinuity at 49 employees with a large spike breaking the firm size distribution. To the right of the threshold there is then a valley before the power law continues again. Interestingly, there is a positive mass of firms in this valley, as there is in the real world data, even though there is no measurement error as we had in the baseline static model. This is essentially because of the adjustment cost structure as we will discuss below.

Figure A12 shows the impact of changing the level of the regulatory tax. In Panel A we simulate an economy where the tax is 8 times larger than in the baseline case (so 16% instead of 2%). As expected, the higher regulatory tax increases the spike at 49 and the length of the valley to the right of the threshold. There are now some employment cells with no firms to the right of the 50 employee cut-off. By contrast, Panel B reduces the regulatory tax to 25% of the baseline (i.e. to 0.5% instead of 2%). Here, the spike and the valley are almost muted as we would expect.

The intuition from the static model of changing the firm size distribution as a function of the regulatory tax are therefore robust to our dynamic extension. But the more interesting results are in Figure A13 which changes the level of the labor adjustment cost parameter. In Panel A we double the adjustment cost. In this experiment we still observe a small spike at 49, but now the valley to the right of the regulatory threshold is much more populated with firms and the sharp discontinuity is blurred. By contrast, in Panel B we reduce the labor adjustment costs to about 13% of the baseline case (i.e. 0.010 instead of 0.075) and we see that there is a much smaller mass of firms in the valley and a much larger spike.

---

[66] In addition to the endogenous exit decision, we also allow for an exogenous death rate that is modelled as an iid shock that causes some (small) margin of firms go out of business. This is not a necessary feature of the model as the TFP process is AR(1) rather than random walk, hence even a firm with a very high TFP draw will tend to revert to the average over the long run.

The intuition for this is that all else equal, firms try to avoid the valley - in the static model for example, no firm would optimally choose this point between N and n_r. But with in a dynamic equilibrium with labor adjustment costs, firms may find it optimal to transit through the valley. To see this assume that current managerial ability/productivity would put an incumbent firm to the left of the regulatory threshold and consider what would happen if such a firm received a large positive productivity shock. The firm would like to move to a larger size[67] and earn more profits by hiring more workers. However, since there are convex adjustment costs, spending a period in the valley that is sub-optimal from a static point of view may be optimal from a dynamic point of view. Large jumps in employment are extremely costly because of adjustment costs so landing for a period or two in the valley may be less costly than paying these large adjustment costs to make a large immediate expansion. Hence when we increase adjustment costs, these considerations become more important and the mass of firms in the valley rises.

We can see this even more clearly when we consider the dynamic adjustment properties of the model and compare them with the actual data. Figure A14 shows the proportion of firms who increase employment in the next year as a function of firm size in the current year. Panel A shows the raw data. We see that there is a big fall in the probability of increasing employment for firms to the left of the regulatory threshold. Panel B shows the same graph for the simulated data in our baseline calibration (of Figure A11). As with the raw data there is much less incentive to grow when a firm is just below the threshold. Panel C shows the same graph in the calibration that uses higher adjustment costs (Figure A13, Panel A). We see the same qualitative picture, but with a lower level of employment changes and less volatility as we would expect from the higher adjustment costs. The sharpness of the fall in growth rates to the left of the threshold becomes more muted.[68]

To conclude, when there are labor adjustment costs, the previous exercises show that steady state simulations reproduce the intuitions underlying the static model (and key empirical features of the data). There is a bulge of firms with employment just to the left of the regulatory threshold, a valley to the right of the threshold and then a continuation of the power law of the firm size distribution. Just as in the static model, the size of the bulge and the width of the valley are increasing in the magnitude of the regulatory "tax". An additional insight from the dynamic model is that there is a positive mass of firms in the valley, even when all firms are fully optimizing. In the static model we rationalized the positive mass in the valley in terms of measurement error, whereas in reality we believe a mixture of adjustment costs and measurement error helps explain the valley. Although a useful exercise, ultimately the dynamic model does not delivery fundamentally new insights compared to the static model, so we believe the considerably simpler approach of our basic model has much to recommend it. Nevertheless, estimating the regulatory costs through Simulated Method of Moments on the dynamic model, would be an interesting extension that we leave for future work.

F More Details of Some Size-Related Regulations in France

The size-related regulations evolve over time and are defined in four groups of laws. The Code du Travail (labor laws), Code du Commerce (commercial law), Code de la Sécurité Social (social security) and in the Code Général des Impôts (fiscal law). The main bite of the labor (and some accounting) regulations comes when the firm reaches 50 employees. But there are also some other size-related thresholds at other levels. The main other ones comes at 10-11 employees. For this reason we generally trim the analysis below 10 employees to mitigate any bias induced in estimation from these other thresholds. For more details on French regulation see inter alia Abowd and Kramarz (2003) and Kramarz and Michaud (2010), or, more administratively and exhaustively, LAMY (2010).

F.1 Main Labor Regulations

The unified and official way of counting employees has been defined since 2004[69] in the Code du Travail[70], articles L.1111-2 and 3. Exceptions to the 2004 definition are noted in parentheses in our detailed descriptions of all the regulations below. Employment is taken over a reference period which from 2004 depends on the precise date when employment is estimated and covers the 365 previous days. There are precise rules over how to fractionally count part-year workers, part-time workers, trainees, workers on sick leave, etc. For example, say a firm employs 10 full-time workers every day but in the middle of the year all 10 workers quit and are immediately replaced by a different 10 workers. Although in the year as a whole 20 workers have been employed by the firm the standard regulations would mean the firm was counted as 10 employee firm. In this case this would be identical to the concept used in our main

---

[67] Although the firm expects to eventually converge back down to average productivity, the positive productivity shock is persistent (recall $\rho_A = 0.9$ in the baseline calibration).

[68] An alternative way to dynamically model the regulatory tax would be to consider it as a purely sunk cost (or sunk cost plus variable cost) as in Gourio and Roys (2014). This has similar predictions on the firm size distribution and simplifies the dynamics compared to the model presented here (as there are no convex adjustment costs). However, the raw data on employment dynamics do not seem compatible with this approach. Panel A of Figure A1 shows how employment adjusts around the threshold at 49 for all firms and then the sub-set of firms who had (in the past five years) already had 50 or more employees (Panel B). If the regulatory cost was sunk the latter group - who presumably have already paid the sunk cost - should not bunch much around 49 employees. However, whether we look at firms who make no change in employment in Panel A, we observe a big spike at 49 employees for both groups. This does not seem consistent with a simple sunk cost story.

[69] Before that date, the concept of firm size was different across labor regulations.

[70] The text is available at:
`http://www.legifrance.gouv.fr/affichCode.do?cidTexte=LEGITEXT000006072050&dateTexte=20120822`

data (FICUS, see the discussion in section C.4 above).
Last, almost all of these regulations strictly apply to the firm level, which is where we have the FICUS data. Some case law has built up, however, which means that a few of them are also applied to the group level. We report below regulations that were stable over time, but some unreported regulations were altered, introduced or removed during our estimation period. The precise tracking of these events is very difficult in practice.

From 200 employees:

- Obligation to appoint nurses (Code du Travail, article R.4623-51)
- Provision of a place to meet for union representatives (Code du Travail, article R.2142-8)

From 50 employees:

- Monthly reporting of the detail of all labor contracts to the administration (Code du Travail, article D.1221-28)
- Obligation to establish a staff committee ("comité d'entreprise") with business meeting at least every two months and with minimum budget = 0.3% of total payroll (Code du Travail, article L.2322-1-28, threshold exceeded for 12 months during the last three years)
- Obligation to establish a committee on health, safety and working conditions (CHSC) (Code du Travail, article L.4611-1, threshold exceeded for 12 months during the last three years)
- Appointing a shop steward if demanded by workers (Code du Travail, article L.2143-3, threshold exceeded for 12 consecutive months during the last three years)
- Obligation to establish a profit sharing scheme (Code du Travail, article L.3322-2, threshold exceeded for six months during the accounting year within one year after the year end to reach an agreement)
- Obligation to do a formal "Professional assessment" for each worker older than 45 (Code du Travail, article L.6321-1)
- Higher duties in case of an accident occurring in the workplace (Code de la sécurité sociale and Code du Travail, article L.1226-10)
- Obligation to use a complex redundancy plan with oversight, approval and monitoring from Ministry of Labor in case of a collective redundancy for 9 or more employees (Code du Travail, articles L.1235-10 to L.1235-12; threshold based on total employment at the date of the redundancy)

From 25 employees:

- Duty to supply a refectory if requested by at least 25 employees (Code du Travail, article L.4228-22)
- Electoral colleges for electing representatives. Increased number of delegates from 25 employees (Code du Travail, article L.2314-9, L.2324-11)

From 20 employees:

- Formal house rules (Code du Travail, articles L.1311-2)
- Contribution to the National Fund for Housing Assistance;
- Increase in the contribution rate for continuing vocational training of 1.05% to 1.60% (Code du Travail, articles L.6331-2 and L.6331-9)
- Compensatory rest of 50% for mandatory overtime beyond 41 hours per week

From 11 employees:

- Obligation to conduct the election of staff representatives(threshold exceeded for 12 consecutive months over the last three years) (Code du Travail, articles L.2312-1)

From 10 employees:

- Monthly payment of social security contributions, instead of a quarterly payment (according to the actual last day of previous quarter);
- Obligation for payment of transport subsidies (Article R.2531-7 and 8 of the General Code local authorities, Code général des collectivités territoriales);
- Increase the contribution rate for continuing vocational training of 0.55% to 1.05% (threshold exceeded on average 12 months).

Note that, in additions to these regulations, some of the payroll taxes are related to the number of employees in the firm.

F.2 Accounting rules

The additional requirements depending on the number of employees of entreprises, but also limits on turnover and total assets are as follows (commercial laws, Code du Commerce, articles L.223-35 and fiscal regulations, Code général des Impôts, article 208-III-3):

From 50 employees:

- Loss of the possibility of a simplified presentation of Schedule 2 to the accounts (also if the balance sheet total exceeds 2 million or if the CA exceeds 4 million);
- Requirement for LLCs, the CNS, limited partnerships and legal persons of private law to designate an auditor (also if the balance sheet total exceeds 1.55 million euros or if sales are more than 3.1 million euros, applicable rules of the current year).

From 10 employees:

- Loss of the possibility of a simplified balance sheet and income statement (also if the CA exceeds 534 000 euro or if the balance sheet total exceeds 267 000 euro, applicable rule in case of exceeding the threshold for two consecutive years).

References

Ackerberg, Daniel, Lanier Benkard, Stephen Berry and Ariel Pakes (2007) in Edward Leamer and James Heckman (eds) "Econometric tools for analyzing market outcomes" *Handbook of Econometrics Vol 6A*, 63, 4172-4276, Amsterdam: Elsevier.

Armbruster, Heidi, Steffan Kinkel, Gunter Lay and Someka Maloca (2005) "Techno-organizational innovation in European manufacturing industries" *European Manufacturing Survey Bulletin*, 1, 1-16.

Bloom, Nicholas (2009) "The Impact of Uncertainty Shocks" *Econometrica,* 77(3), 623–685.

Bloom, Nicholas, Raffaella Sadun and John Van Reenen (2014) "Management as a Technology", LSE mimeo.

Bond, Stephen and John Van Reenen (2007) "Micro-econometric models of investment and employment" Chapter 65 in Heckman, J. and Leamer. E. (eds) *Handbook of Econometrics Volume 6A* (2007) 4417-4498.

Cooper, Russell and John Haltiwanger (2006) "On the nature of Capital Adjustment Costs" *Review of Economic Studies*, 73, 611-633.

Foster, Lucia, John Haltiwanger and Chad Syverson (2008) "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review,* 98(1): 394-425.

Hicks, John (1932) *The Theory of Wages.* London: Macmillan.

Leibenstein, Harvey (1966) "Allocative Efficiency vs. X-Efficiency" *American Economic Review,* 56 (3): 392–415.

Stokey, Nancy and Robert Lucas (1986) *Recursive Methods in Economic Dynamics*, Cambridge: Harvard University Press.

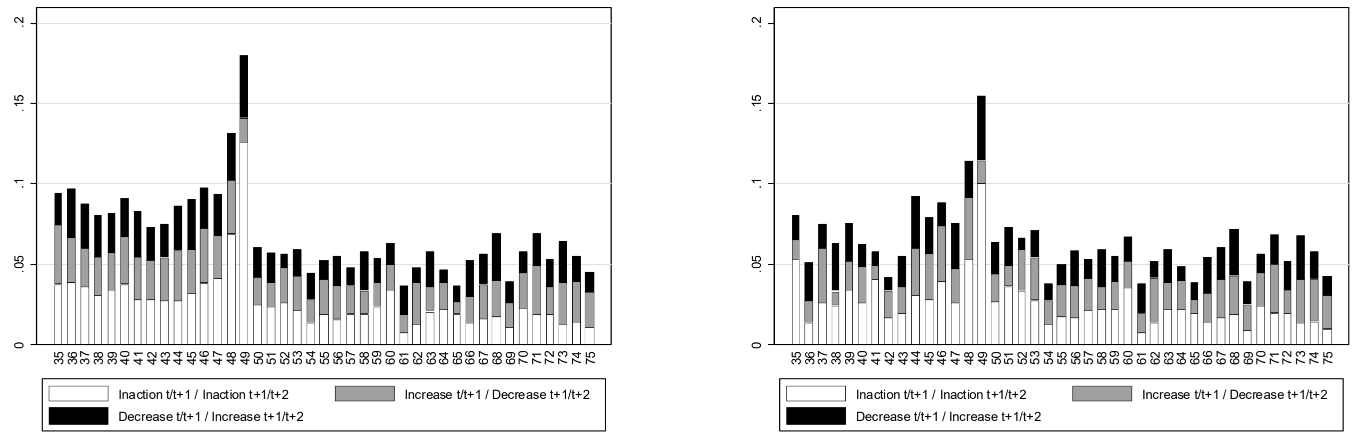
Stokey, Nancy (2009) *The Economics of Inaction: Stochastic Control Models with Fixed Costs*, Princeton: Princeton University Press.

Syverson, Chad (2011) "What determines productivity?" *Journal of Economic Literature*, 49(2) 326-65.

Tauchen, George (1986) "Finite State Markov Chain Approximation to univariate and vector autoregressions" *Economics Letters*, 20: 177-181.

FOR ONLINE PUBLICATION ONLY: APPENDICES

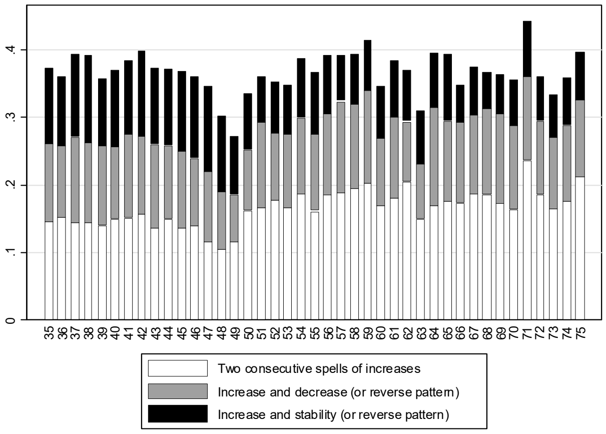
Figure A1: Inaction Rate over a Two Year Period (Between t and t+2)



Panel A: All Firms

Panel B: Conditional on Having Crossed the 49 Threshold in the Past 5 Years (Sunk vs. Fixed Costs)

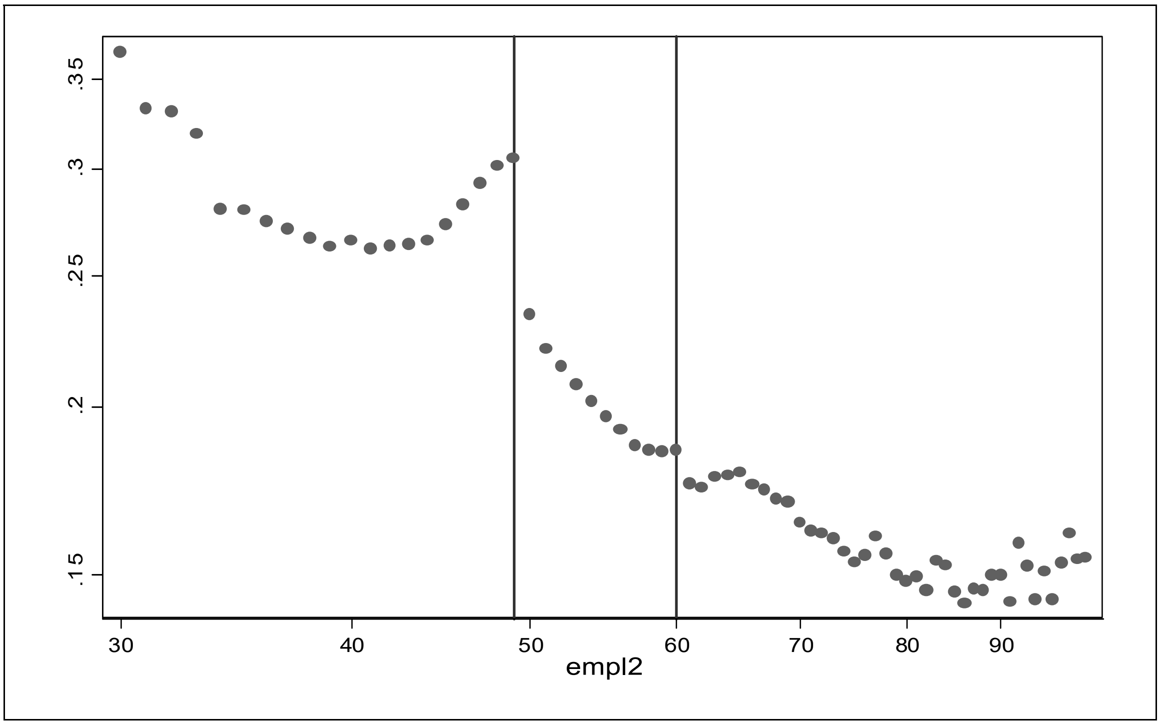
Panel C: Patterns of Firm Growth over a Two Year Period (Between *t* and *t*+2)



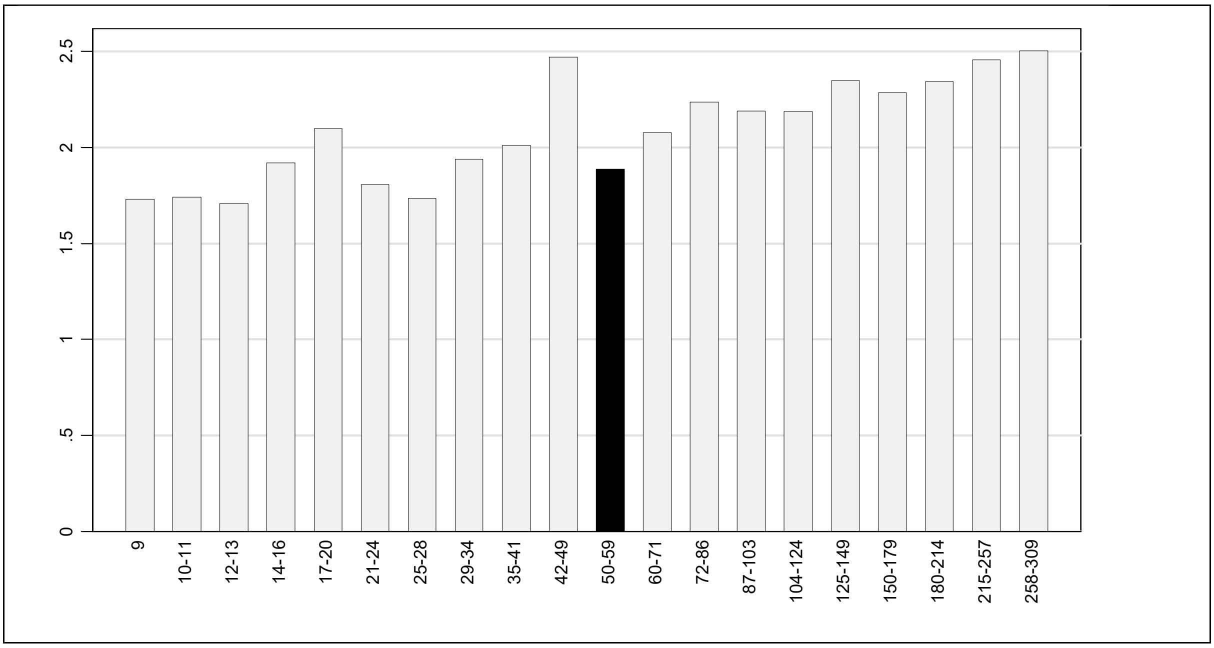
Notes: This Figure shows employment transitions over a two year period, broken down by size class. In Panels A and B the "inaction rate" is the share of firms having strictly the same size at dates *t* and *t+ 2*. We break this down into firms who have stayed at exactly the same size for two years (white bar) vs. firms who got larger than shrank back to their original size (grey bar) vs. firms who shrank and then grew back to their original size (black bar). Panel A is for the full sample and Panel B condition on the subset of firms who had 50 or more employees at least once in the previous 5 years. Panel C describes the patterns of firms growth over a two year period for firms at different employment sizes that are larger at time t+2 as compared with time t. The grey bar are for firms who increased then decreased (or decrease then increased). The white bar is the proportion of firms who grew in both periods, the grey bar is for firms who increased and then did not change (or who did not change and then increased).

Figure A2: Average Spell Length in an Employment Size Bin

Panel A: Raw Firm Size Bins

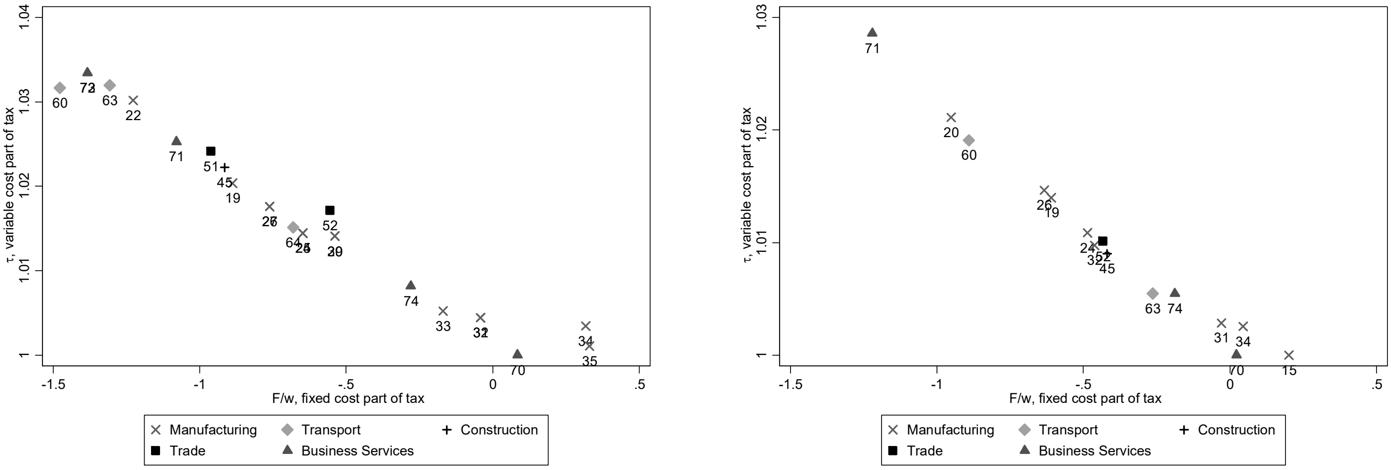


Panel B: Exponential Firm Size Bins



Notes: Manufacturing firms observed between 1995 and 2007 (censored and uncensored spells - results are robust to the removing of censored spells). To compute durations in each state, we only observe firm size at an annual frequency, so we assume that a firm which we observe at size 30 at date t, and at size 35 at date t+1 has transited through sizes 31, 32, 33, 34 during year t/t+1, and we adjust spells in each state accordingly. Panel A gives durations by employment sizes and Panel B presents the same but in exponential bins.

Figure A3: Heterogeneity of Results by Two Digit Sectors

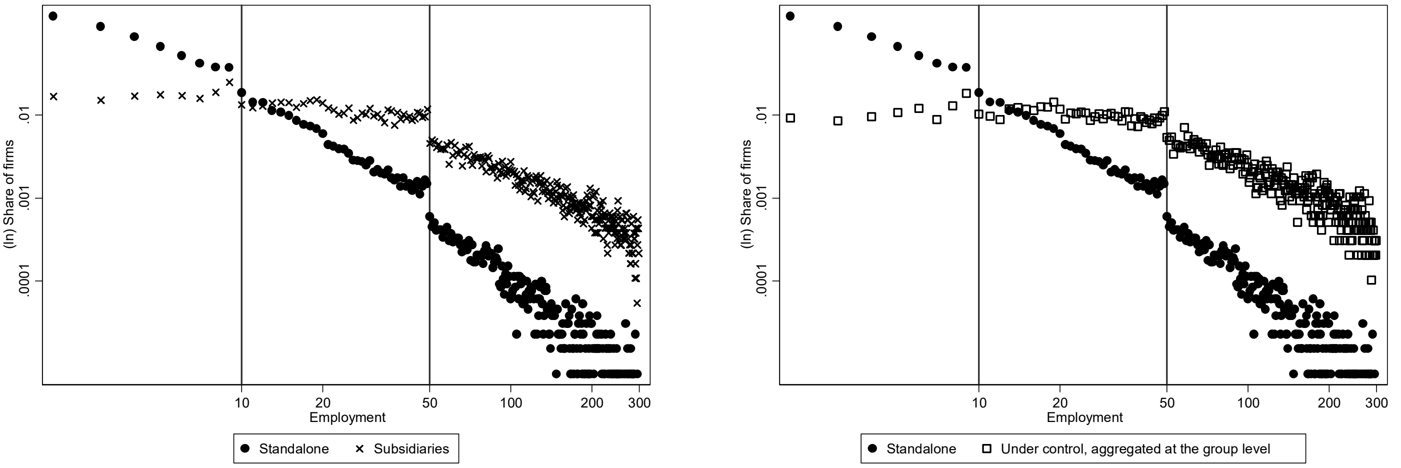


Panel A: θ Calibrated to 0.8

Panel B: Industry Specific θ's

Notes: These are the results from industry-specific estimation on the same lines as column (1) of Table 1 in panel A and column (6) of Table 1 in panel B. Industry codes correspond to the first two digits of the statistical classification of economic activities in the European Community (NACE).

Figure A4: Corporate Restructuring in Response to the Regulation? Independent Firms vs. Corporate Groups in 2000



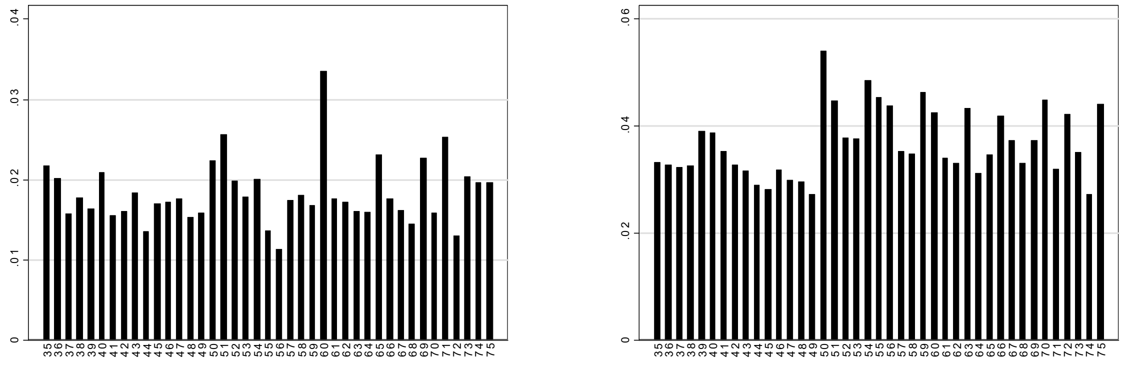
Panel A: Standalone Firms vs. Affiliates of Larger Groups

Panel B: Standalone Firms vs. Groups (i.e. All Affiliates Aggregated at the Group Level)

Sources: FICUS, 2000 and LIFI, 2000.

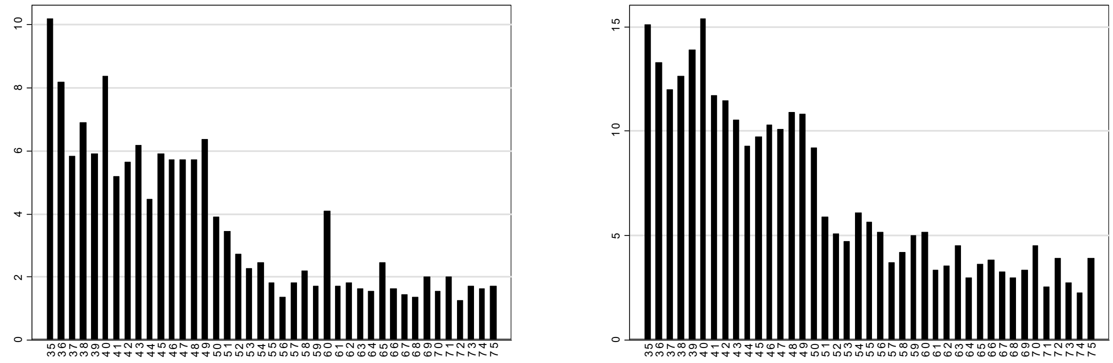
Notes: “Standalone” are independent firms that are not subsidiaries or affiliates of larger groups (blue dots). In Panel A we compare these to affiliates of larger groups with size measured at the affiliate level. A broken power law is visible in both distributions. In Panel B we repeat the standalone distribution but now compare this to affiliates aggregated to the group level (in France, we do not count overseas employees). Although there is a break in the power law for both type of firms it is stronger for the standalone firms as we would expect, i.e. the subsidiaries are not driving the results.

Figure A5: Descriptive Statistics on Entry and Exit



Panel A: Share of Entry Between *t*-1 / t in Each Size Bin

Panel B: Share of Exit t / t+1 in Each Size Bin

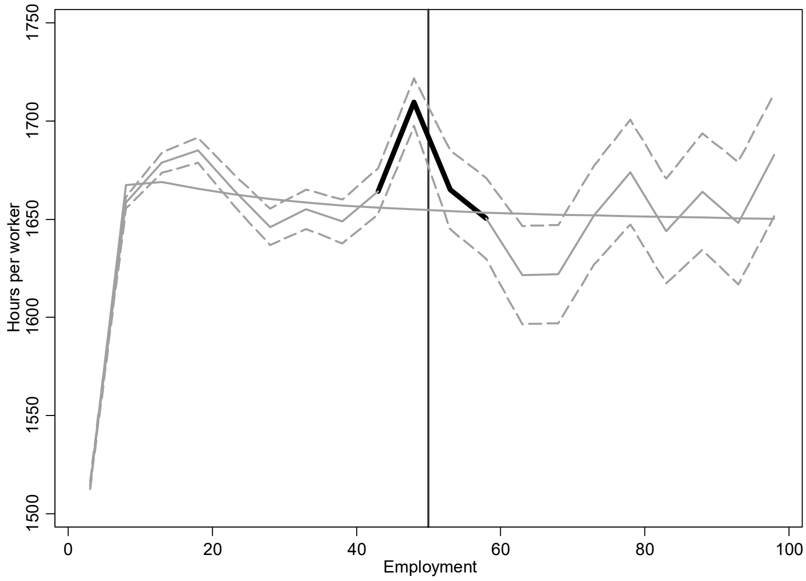


Panel C: Size Distribution at Entry (Average Number of Entrants in Each Size Bin)

Panel D: Size Distribution at Exit (Average Number of Exiters in Each Size Bin)

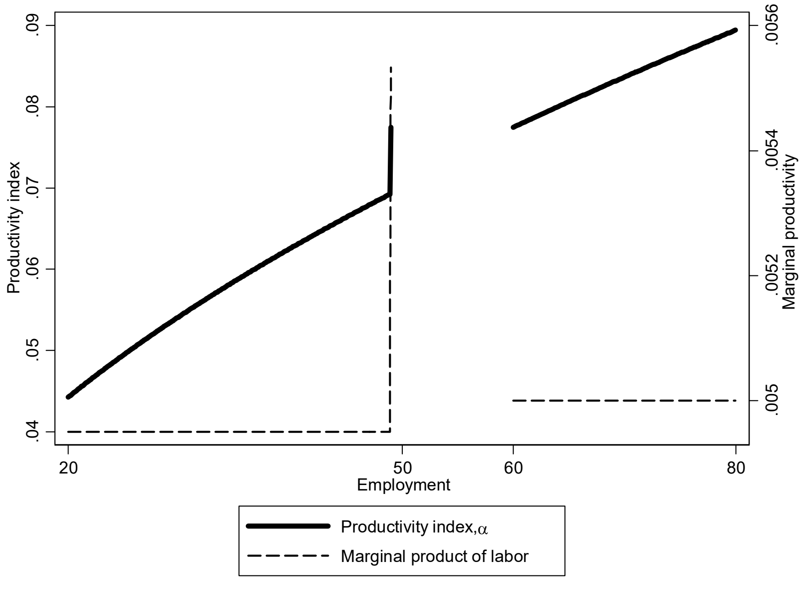
Notes: Manufacturing firms, 1995-2007, FICUS. Size is measured at time t in all panels.

Figure A6: Annual Hours per Worker



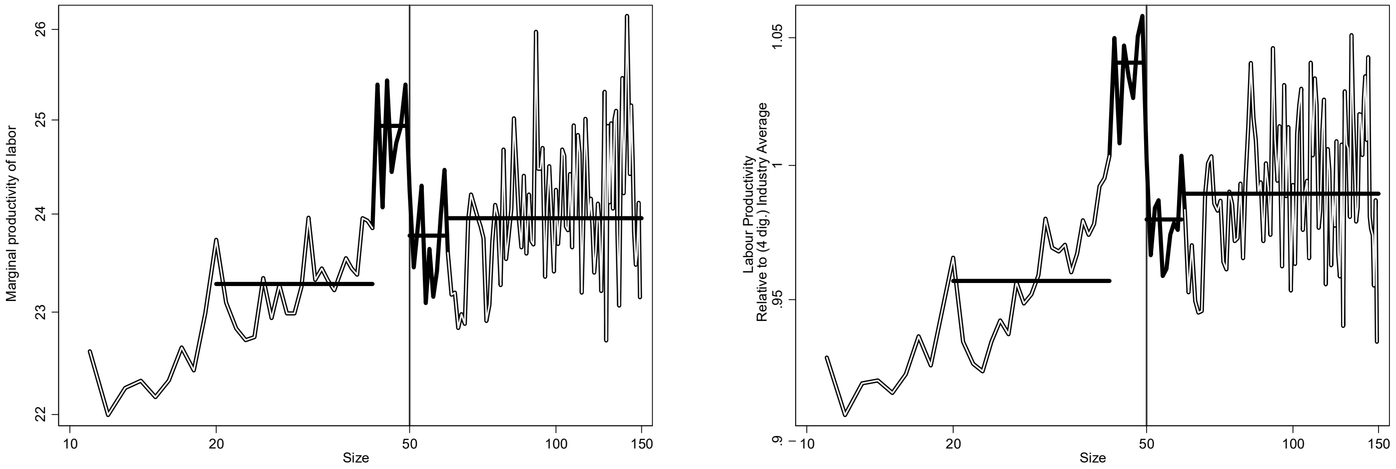
Notes: annual average hours per worker - combined FICUS and DADS data for 2002.

Figure A7: MRPL and Marginal Revenue Productivity of Labor in our Model



Notes: This figure shows the relation (correspondence) between productivity (α, left axis) or marginal product of labor ($\alpha.\theta.(n^{*}(\alpha))^{\theta-1}$, right axis) and size. The relations are defined as $\alpha.\theta.(n^{*}(\alpha))^{\theta-1} = w$ for $n < N$ and $\alpha.\theta.(n^{*}(\alpha))^{\theta-1} = \tau.w$ for $n > n_r$

Figure A8: Marginal Revenue Productivity and Firm Size (Value Added per Worker Relative to Industry Average)



Panel A: Value added per worker relative to industry average

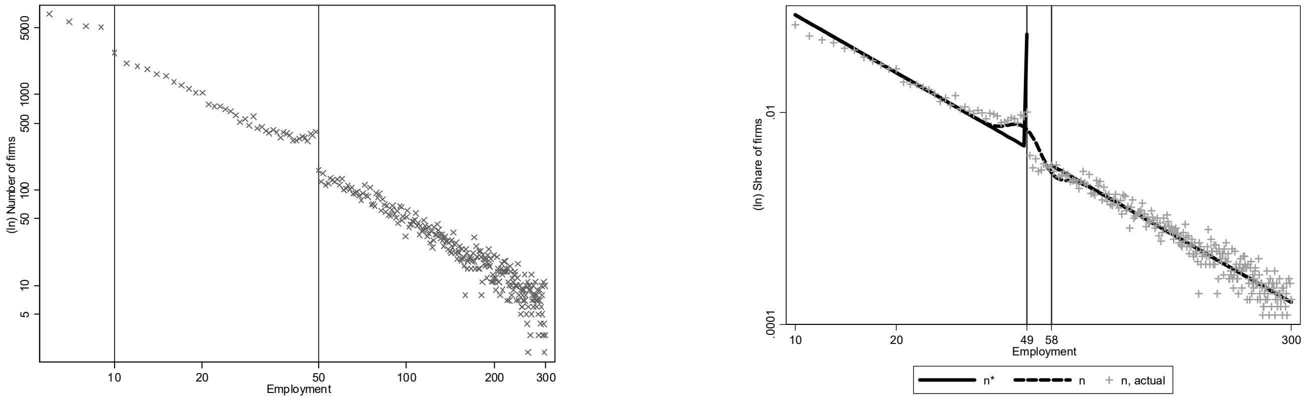
Panel B: Empirical counterpart of theory $\boldsymbol{\alpha.\theta.(n^*(\alpha))^{\theta-1}}$

Notes: Manufacturing firms in year 2000. In panel A the marginal revenue productivity of labor is measured by value added per worker (relative to the four digit industry average) by firm size. The "Hsieh-Klenow" estimate of the implicit tax distortion is the difference between average productivity for firms between 20 and 42 employees (€23,280) and average productivity for firms between 60 and 150 employees (€23,960). This implies a log difference of 0.02866 or 2.866%. In Panel B, the marginal revenue productivity of labor is measured by $\alpha.\theta.(n^*(\alpha))^{\theta-1}$ by firm size. The "Hsieh-Klenow" estimate of the implicit tax distortion is the difference between average productivity for firms between 20 and 42 employees (0.957) and average productivity for firms between 60 and 150 employees (0.989). This implies a log difference of 0.03326 or 3.326%.

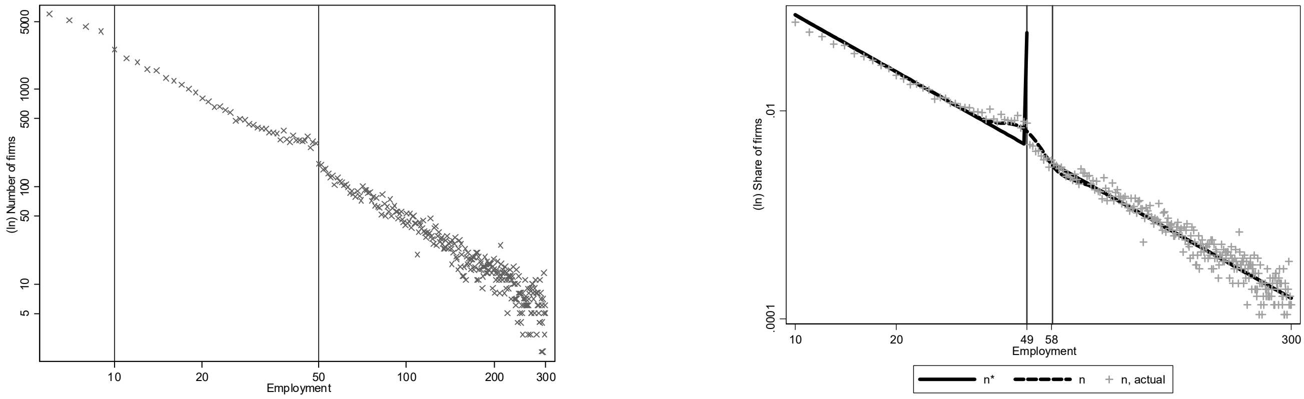
Figure A9: Model fit Using Alternative Datasets and Definitions of Employment

Log-log Plot *Model Fit*

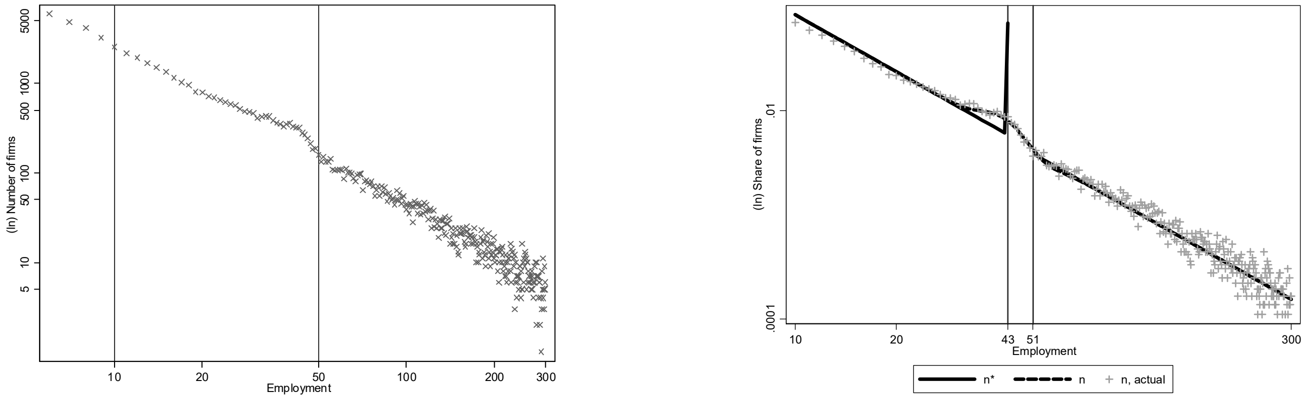
Panel A: Fiscal Source (Corporate Tax Collection to Fiscal Administration, FICUS 2002)
Arithmetic Average of Quarterly Head Counts



Panel B: Payroll Tax Reporting to Social Administration (DADS 2002)
"Declared" Workers on Dec. 31st: Cross-Sectional (and Fractional) Count



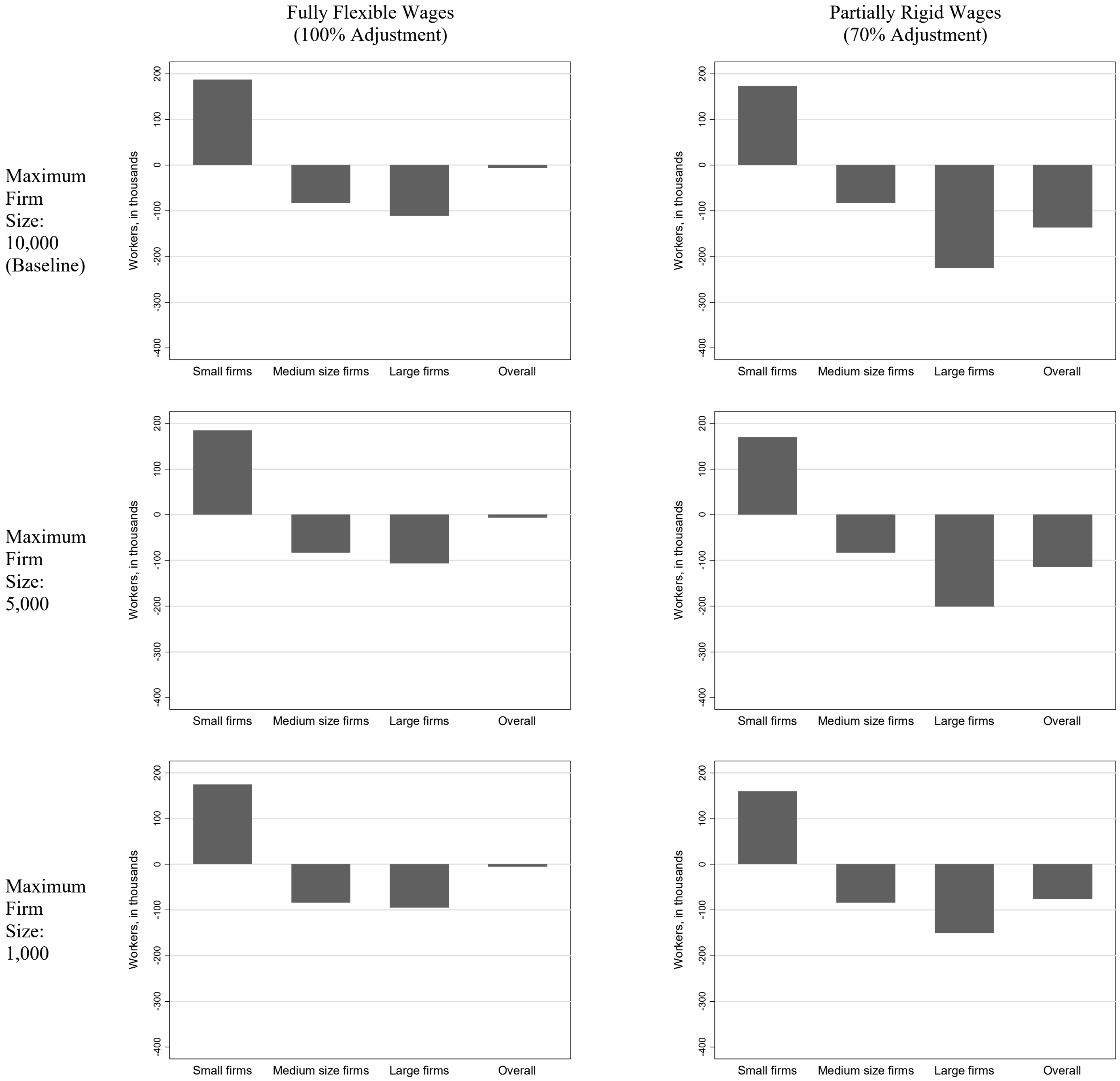
Panel C: Payroll Tax Reporting to Social Administration (DADS 2002)
"Full-Time Equivalent" (FTE), Computed by the French Statistical Institute



Notes: Data sources are indicated in the headers of the table. Full time equivalents (panel C) are only available from 2002 onwards in the files, such that all distributions have been computed in 2002. All firms in this figure had an industry affiliation in manufacturing industries at creation date. Full time equivalents in panel C are calculated by the statistical institute using as a benchmark, the 75th percentile of the series of hours per workers in each 2 digit industries and size classes, and are bounded above by 1, which produces the small shift to the left.

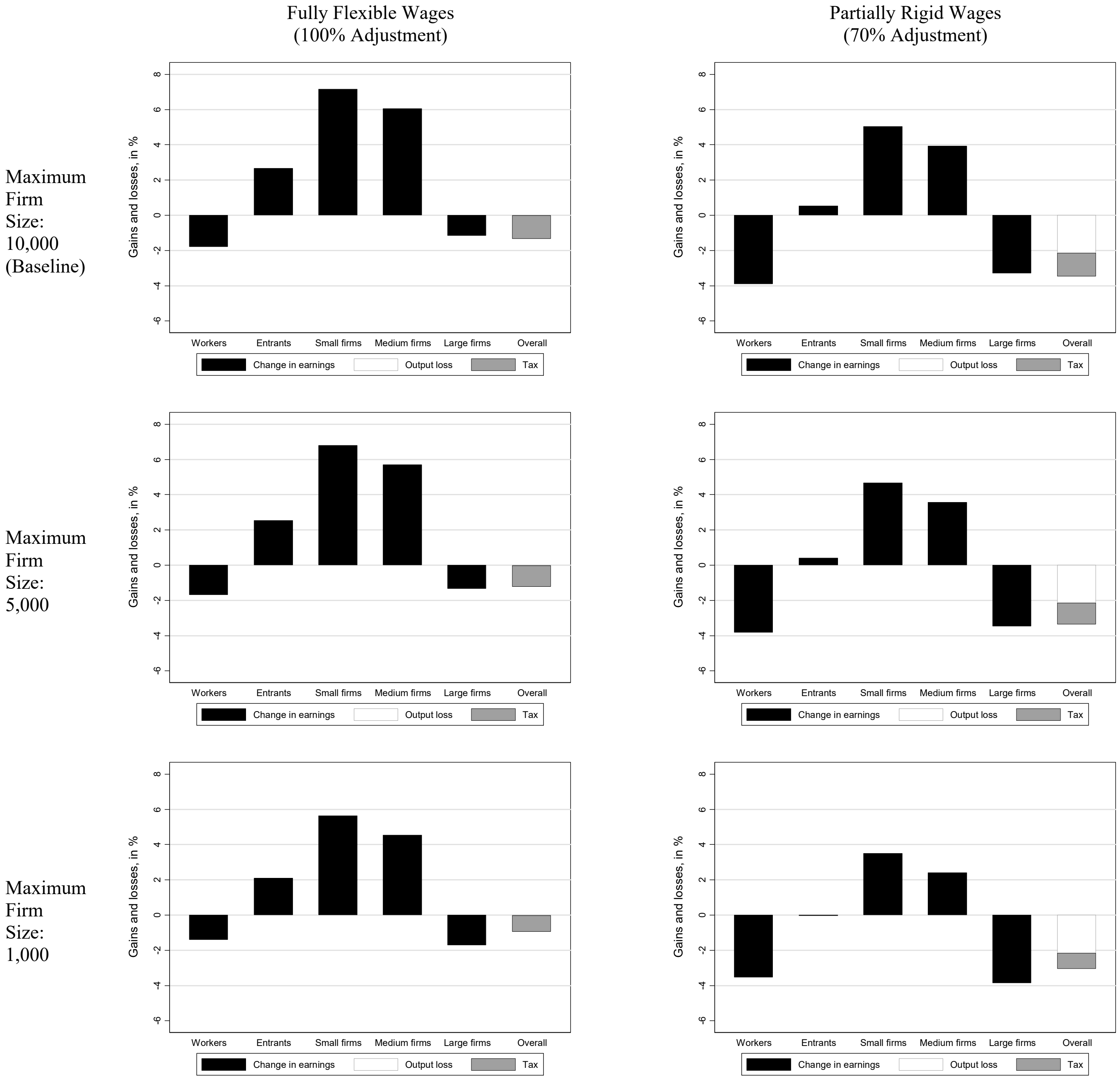
Figure A10: Welfare and Employment Reallocation Analysis under Alternative Assumptions for the Upper Bound of Firm Size

Panel A: Employment, Gross Changes



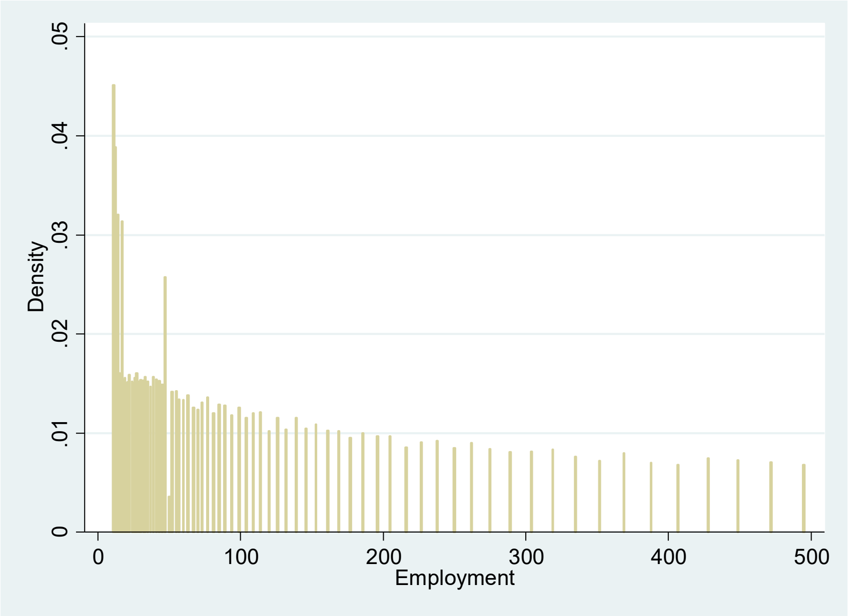
Notes: This is based on the baseline of Table 1 column (1), under various assumptions for maximum firm size. All definitions are similar to those in Figure 10.

Panel B: (Expected) Percentage Changes in Welfare (Income from Wages or Profits)



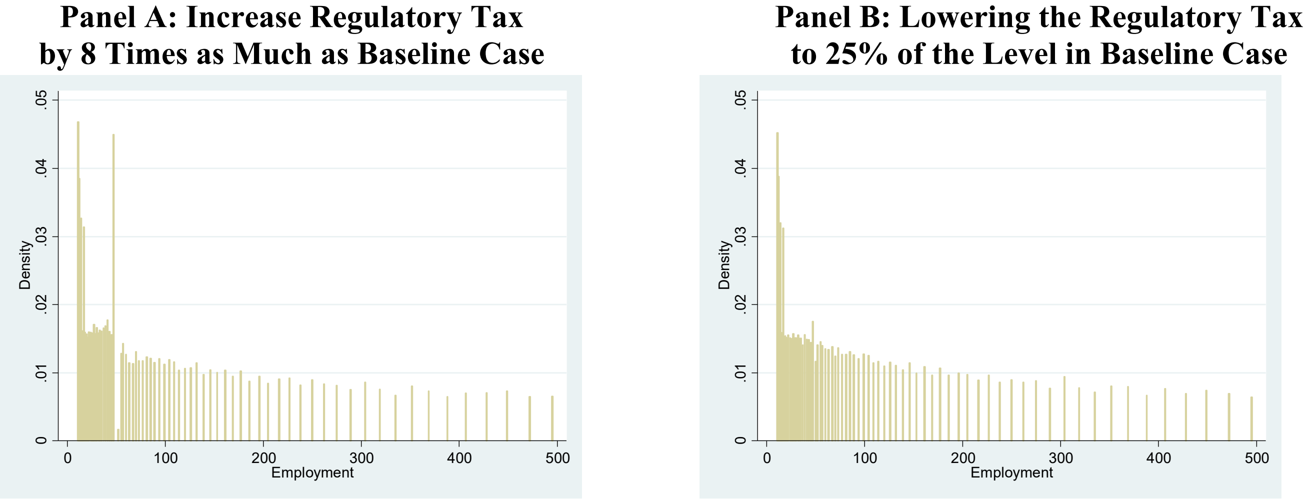
Notes: This is based on the baseline of Table 1 column (1), under various assumptions for maximum firm size. All definitions are the same as those in Figure 10.

Figure A11: Firm Size Distribution from the Simulated Dynamic Model: Baseline Specification



Notes: This is the histogram of the firm size distribution, i.e. the proportion of firms across different size classes. We run the simulation model for 100 years over 20,000 firms and keep only the last 25 years (but which time the distributions appeared to be very stable). Regulatory variable tax = 2%; regulatory fixed cost, F/w = 0.2; adjustment cost parameter for labor=0.075. Appendix E gives the full model details and other calibration values.

Figure A12: Changing the Magnitude of the Regulatory Tax

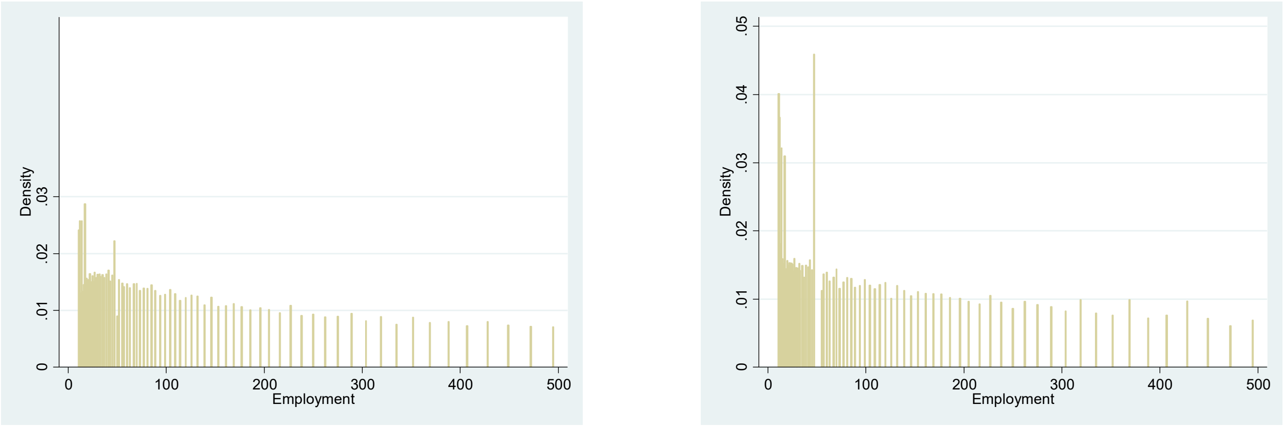


Panel A: Increase Regulatory Tax by 8 Times as Much as Baseline Case

Panel B: Lowering the Regulatory Tax to 25% of the Level in Baseline Case

Notes: This is the histogram of the firm size distribution, i.e. the proportion of firms across different size classes. The simulation model the same as baseline in Figure 15 except in Panel A the regulatory variable tax = 16% (instead of 2% as in the baseline case) and in Panel B the regulatory variable tax is lowered to 0.5%.

Figure A13: Changing the Magnitude of the Adjustment Costs



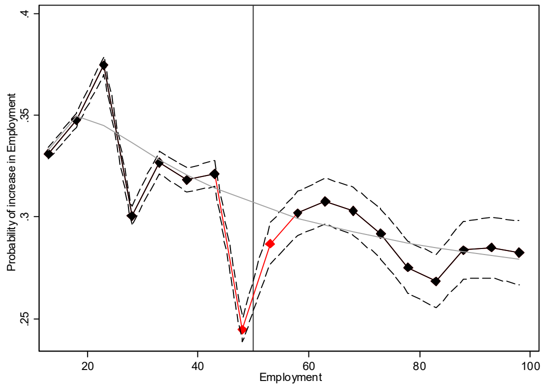
Panel A: Increasing Labor Adjustment Costs Smooths over the "Valley"

Panel B: Reducing Adjustment Costs Makes Hump Higher and Valley Wider

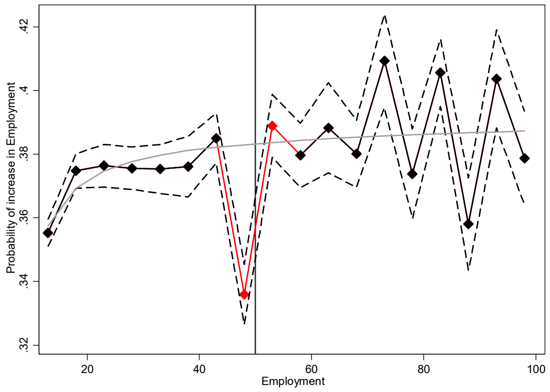
Notes: This is the histogram of the firm size distribution, i.e. the proportion of firms across different size classes. The model is the same as baseline in Figure 15 except in Panel A except the labor adjustment cost parameter is doubled to 0.150 (instead of 0.075 in baseline) and in Panel B labor adjustment costs are lowered to 0.010.

Figure A14: Percentage of Firms Increasing Employment in Actual and Simulated Data

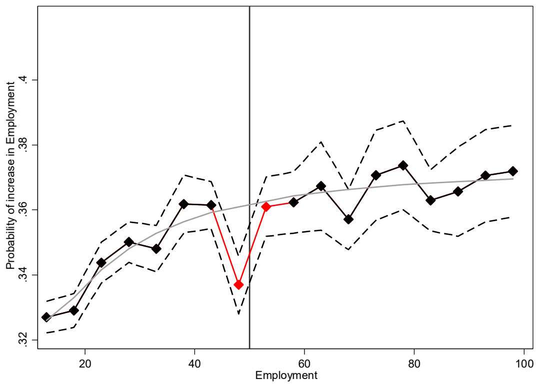
Panel A: Actual Data



Panel B: Simulated Data, Baseline



Panel C: Simulated Data, High Adjustment Costs



Notes: These graphs examine the proportion of firms whose employment in the next period as a function of employment in the current period. Panel A does this for the actual data and Panels B and C do the same for the simulated data. Panel B is the baseline calibration (as in Figure 15) and Panel C uses a higher adjustment cost (as in Figure 17, Panel A). In the empirical data we do this for all firms and for all available years. 95% confidence intervals shown.

APPENDIX TABLES

Table A1: Parameter Estimates in Different Years

Sample	(1) 1995-2007	(2) 1995	(3) 2000	(4) 2005
β, power law	1.815	1.766	1.800	1.834
	(0.055)	(0.053)	(0.054)	(0.061)
n_r, upper employment threshold	58.014	58.483	59.271	60.297
	(1.381)	(4.052)	(2.051)	(1.310)
σ, variance of measurement error	0.102	0.114	0.121	0.143
	(0.023)	(0.069)	(0.033)	(0.025)
τ-1, implicit tax, variable cost	0.017	0.022	0.023	0.017
	(0.006)	(0.013)	(0.008)	(0.008)
F/w, implicit tax, fixed cost	-0.711	-0.930	-0.941	-0.592
	(0.269)	(0.484)	(0.338)	(0.379)
Mean (Median) # of employees	55.0 (24)	57.6 (25)	55.8 (24)	53.9 (23)
Observations	517,985	39,638	41,067	37,954
Firms	74,183	39,638	41,067	37,954
Ln Likelihood	-2,314,827.4	-180,281.0	-184,128.7	-168,337.0

Notes: Parameters estimated by ML with standard errors below in parentheses (clustered at the four digit level). Estimation is on unbalanced panel 1995-2007 of population of French manufacturing firms with 10 to 1,000 employees. θ, the scale parameter, is calibrated to 0.8 in all columns. In column (4), 2005 is simply chosen for symmetry (2007 would deliver very similar results).

Table A2: Variation in Estimates Across Different Sectors

Industry	(1) Manufacturing	(2) Transport	(3) Construction	(4) Trade	(5) Business services
β, power law	1.800	1.857	2.345	2.129	1.972
	(0.054)	(0.098)	(0.122)	(0.085)	(0.079)
n_r, upper employment threshold	59.271	62.139	56.916	55.863	53.370
	(2.051)	(4.134)	(1.869)	(3.057)	(1.333)
σ, measurement error	0.121	0.150	0.083	0.072	0.045
	(0.033)	(0.053)	(0.022)	(0.035)	(0.015)
τ-1, implicit tax, variable cost	0.023	0.035	0.020	0.026	0.008
	(0.008)	(0.007)	(0.005)	(0.010)	(0.002)
F/w, implicit tax, fixed cost	-0.941	-1.397	-0.878	-1.206	-0.358
	(0.338)	(0.332)	(0.225)	(0.458)	(0.102)
Mean (Median) # of employees	55.8	48.5	30.0	36.1	47.1
	(24)	(23)	(17)	(19)	(20)
Observations	41,067	10,907	23,506	41,071	30,125

Notes: Parameters estimated by ML with standard errors below in parentheses (clustered at the four digit level). Estimation is on unbalanced the population of French firms with 10 to 1,000 employees in year 2000. These estimates of the implicit tax are based on a calibrated value for θ of 0.8.

Table A3: Parameter Estimates Across Industries in the Specification with 2 Breaks (at 9 and 49 Workers)

Industry:	(1) Manufacturing industries	(2) Transport	(3) Construction	(4) Trade	(5) Business services
β, power law	1.828	1.835	2.355	2.291	2.080
	(0.095)	(0.064)	(0.100)	(0.132)	(0.079)
σ, measurement error	0.079	0.076	0.078	0.079	0.078
	(0.002)	(0.003)	(0.002)	(0.001)	(0.002)
τ_1-1, implicit tax, variable cost,	0.001	0.005	~0	~0	0.003
threshold at 9	(0.001)	(0.001)			(0.001)
F_1/w, implicit tax, fixed cost	0.013	-0.023	0.017	0.024	-0.010
threshold at 9	(0.011)	(0.017)	(0.002)	(0.003)	(0.014)
n_{r1}	10.361	10.295	10.288	10.534	10.364
threshold at 9	(0.073)	(0.187)	(0.088)	(0.102)	(0.144)
τ_2-1, implicit tax, variable cost	0.017	0.029	0.019	0.016	0.002
threshold at 49	(0.008)	(0.009)	(0.005)	(0.009)	(0.004)
F_2/w, implicit tax, fixed cost	-0.732	-1.315	-0.845	-0.639	0.005
threshold at 49	(0.401)	(0.458)	(0.257)	(0.478)	(0.189)
n_{r2}	56.241	55.880	56.517	57.029	56.288
threshold at 49	(0.456)	(0.225)	(0.370)	(0.901)	(0.668)

Notes: Parameters estimated by ML with standard errors below in parentheses (clustered at the four digit level). Estimation is on unbalanced the population of French firms with 5 to 1,000 employees in year 2000. θ, the scale parameter, is calibrated to 0.8 in all columns. In columns (3) and (4), the variable part of tax at first threshold is set to 1 to reach convergence, indicating that the implicit tax is very close to zero.

Table A4: Welfare and Distributional Analysis in the Two Threshold Specification

Panel A: Fully Flexible Wages

(Regulated Economies - Same Unregulated Economy)		(1)	(2)	(3)
Variable		2 thresholds	Threshold at 10 only	Threshold at 50 only
1.	Unemployment rate	0%	0%	0%
	Implied wage flexibility	**100%**	**100%**	**100%**
2.	Percentage of firms avoiding the first regulation, δ_1	5.748%	5.748%	-
3.	Percentage of firms avoiding the second regulation, δ_2	2.106%	-	2.112%
4.	Percentage of firms paying the first regulation	45.277%	45.273%	
5.	Percentage of firms paying the second regulation (on top of the first)	10.296%	-	10.322%
6.	Change in labor costs (wage reduction) for small firms (below 9)	-1.341%	-0.067%	-1.273%
7.	Change in labor costs (wage reduction but first tax) for medium firms (between 10 and 49)	-1.279%	-0.005%	-1.273%
8.	Change in labor costs (wage reduction but first and second taxes) for large firms (above 50)	0.404%	-0.005%	0.410%
9.	Excess entry by small firms (percent increase in number of firms)	5.602%	0.279%	5.280%
10.	Change in size of small firms (below 9)	6.705%	0.336%	6.367%
11.	Change in size of medium firms (between 10 and 49)	6.394%	0.0250%	6.367%
12.	Change in size of large firms (above 50)	-2.022%	0.0250%	-2.048%
13.	Annual welfare loss (as a percentage of GDP):			
	a. First implicit tax (firms having more than 10 workers)	0.055%	0.055%	-
	b. Second implicit tax (firms having more than 50 workers)	0.932%	-	0.932%
	b. Output loss	0.013%	4.648e-04%	0.012%
	c. Total (Implicit Taxes + Output loss)	1.000%	0.056%	0.944%
14.	Winners and losers:			
	a. Change in expected wage for those who remain in labor force	-1.341%	-0.067%	-1.273%
	b. Average gain by entering entrepreneurs of small firms	1.999%	0.101%	1.899%
	c. Average profit gain by small unconstrained firms	5.364%	0.269%	5.094%
	d. Average profit gain by firms constrained at 9	5.103%	0.008%	
	e. Average profit gain by medium firms (10 to 48)	4.863%	-0.027%	
	f. Average profit gain by firms constrained at 49	4.415%		4.491%
	g. Change in profit for large firms (above 50)	-0.903%		-0.912%

Panel B: Partially Rigid Wages

(Regulated Economies - Same Unregulated Economy)		(1)	(2)	(3)
Variable		2 thresholds	Threshold at 10 only	Threshold at 50 only
1.	Unemployment rate	1.989%	0.101%	1.889%
	Implied wage flexibility	**70%**	**70%**	**70%**
2.	Percentage of firms avoiding the first regulation, δ_1	5.653%	5.743%	-
3.	Percentage of firms avoiding the second regulation, δ_2	2.072%	-	2.079%
4.	Percentage of firms paying the first regulation	44.529%	45.234%	
5.	Percentage of firms paying the second regulation (on top of the first)	10.126%	-	10.160%
6.	Change in labor costs (wage reduction) for small firms (below 9)	-0.938%	-0.047%	-0.891%
7.	Change in labor costs (wage reduction but first tax) for medium firms (between 10 and 49)	-0.876%	0.015%	-0.891%
8.	Change in labor costs (wage reduction but first and second taxes) for large firms (above 50)	0.807%	0.015%	0.792%
9.	Excess entry by small firms (percent increase in number of firms)	5.556%	0.279%	5.275%
10.	Change in size of small firms (below 9)	4.696%	0.235%	4.454%
11.	Change in size of medium firms (between 10 and 49)	4.379%	-0.076%	4.454%
12.	Change in size of large firms (above 50)	-4.037%	-0.076%	-3.961%
13.	Annual welfare loss (as a percentage of GDP):			
	a. First implicit tax (firms having more than 10 workers)	0.055%	0.055%	-
	b. Second implicit tax (firms having more than 50 workers)	0.931%	-	0.915%
	b. Output loss	1.620%	0.081%	1.539%
	c. Total (Implicit Taxes + Output loss)	2.606%	0.137%	2.469%
14.	Winners and losers:			
	a. Change in expected wage for those who remain in labor force	-2.947%	-0.148%	2.798%
	b. Average gain by entering entrepreneurs of small firms	0.390%	0.020%	0.372%
	c. Average profit gain by small unconstrained firms	3.752%	0.188%	3.563%
	d. Average profit gain by firms constrained at 9	3.491%	-0.073%	
	e. Average profit gain by medium firms (10 to 48)	3.251%	-0.313%	
	f. Average profit gain by firms constrained at 49	2.803%		2.961%
	g. Change in profit for large firms (above 50)	-2.515%		-2.442%

Notes: The two panels are based on the estimates of Table A3 column (1), under the additional assumption that maximum firm size is 10,000. In column (1), we use our estimates of cost parameters for the two thresholds. In column (2), we cancel the second threshold while in column (3), we cancel the first threshold. Other notations follow the same logic as those in Table 3 (the baseline case with one threshold). In particular, column 1 in panel (B) replicates the calibration exercise of column 2 in Table 3.

Table A5: Estimates Obtained Using Alternative Datasets and Definitions of Employment

Dataset	(1) FICUS, 2002	(2) DADS workers "declared" on Dec. 31st, 2002	(3) DADS, "Full-time equivalent" As estimated by Insee, 2002
θ, scale parameter	0.8	0.8	0.8
N, threshold	49	49	49
β, power law	1.808	1.820	1.822
	(0.056)	(0.065)	(0.053)
n_r, upper employment threshold	57.950	58.361	63.053
	(2.267)	(1.011)	(1.417)
σ, variance of measurement error	0.105	0.130	0.239
	(0.035)	(0.019)	(0.035)
τ-1, implicit tax, variable cost	0.020	0.021	0.022
	(0.008)	(0.008)	(0.011)
F/w, implicit tax, fixed cost	-0.806	-0.860	-0.732
	(0.311)	(0.365)	(0.492)
Mean (Median) # of employees	55.4 (24)	54.4 (24)	54.4 (24)
Observations (firms)	40,637	36,576	36,141
Ln Likelihood	-181,940	-162,970	-160,702.

Notes: Parameters estimated by ML with standard errors below in parentheses (clustered at the industry, four digit level). Estimation is on population of French manufacturing firms with 10 to 1,000 employees, in the year 2002 (rather than 2000 in baseline because the proxy for full time equivalents in the DADS files is only available from 2002 onwards).

Table A6: Values of Parameters used for Simulation in Dynamic Extension

Parameter	Mnemonic	Value	Rationale
TFP AR(1) process	ρ_A	0.9	Cooper and Haltiwanger (2006)
Capital share of gross output	β_k	1/6	FICUS
Labor share of gross output	β_n	1/3	FICUS
Material share of gross output	β_m	1/2	FICUS
Returns to scale	θ	0.8	Basu and Fernald (1997) and main text
Adjustment cost parameter for capital	γ_k	0.150	Bloom et al. (2014)
Adjustment cost parameter for labor	γ_n	0.075	Half of capital adjustment cost
Discount Factor	ϕ	0.91	Based on real interest rates
Capital depreciation rate	δ_k	10%	Bond and Van Reenen (2007)
Sunk cost of entry	S	90%	Bloom et al (2014)

Notes: Baseline values of parameters used in the numerical simulations across all model runs. Fixed costs of production normalized to 100. We use $\tau = 1.02$ and $F/w = -0.94$ as estimated in the paper.

CENTRE FOR ECONOMIC PERFORMANCE
Recent Discussion Papers

1127	Jan-Emmanuel De Neve Nicholas A. Christakis James H. Fowler Bruno S. Frey	Genes, Economics, and Happiness
1126	Robert J. B. Goudie Sach Mukherjee Jan-Emmanuel De Neve Andrew J. Oswald Stephen Wu	Happiness as a Driver of Risk-Avoiding Behavior
1125	Zack Cooper Stephen Gibbons Simon Jones Alistair McGuire	Does Competition Improve Public Hospitals' Efficiency? Evidence from a Quasi-Experiment in the English National Health Service
1124	Jörg Claussen Tobias Kretschmer Thomas Spengler	Market Leadership Through Technology - Backward Compatibility in the U.S. Handheld Video Game Industry
1123	Bernardo Guimaraes Kevin D. Sheedy	A Model of Equilibrium Institutions
1122	Francesco Caselli Tom Cunningham Massimo Morelli Inés Moreno de Barreda	Signalling, Incumbency Advantage, and Optimal Reelection Rules
1121	John Van Reenen Linda Yueh	Why Has China Grown So Fast? The Role of International Technology Transfer
1120	Francesco Giavazzi Michael McMahon	The Household Effects of Government Spending
1119	Francesco Giavazzi Michael McMahon	The Household Effects of Government Spending
1118	Luis Araujo Giordano Mion Emanuel Ornelas	Institutions and Export Dynamics
1117	Emanuel Ornelas	Preferential Trade Agreements and the Labor Market
1116	Ghazala Azmat Nagore Iriberri	The Provision of Relative Performance Feedback Information: An Experimental Analysis of Performance and Happiness
1115	Pascal Michaillat	Fiscal Multipliers over the Business Cycle

1114	Dennis Novy	Gravity Redux: Measuring International Trade Costs with Panel Data
1113	Chiara Criscuolo Ralf Martin Henry G. Overman John Van Reenen	The Causal Effects of an Industrial Policy
1112	Alex Bryson Richard Freeman Claudio Lucifora Michele Pellizzari Virginie Perotin	Paying for Performance: Incentive Pay Schemes and Employees' Financial Participation
1111	John Morrow Michael Carter	Left, Right, Left: Income and Political Dynamics in Transition Economies
1110	Javier Ortega Gregory Verdugo	Assimilation in Multilingual Cities
1109	Nicholas Bloom Christos Genakos Rafaella Sadun John Van Reenen	Management Practices Across Firms and Countries
1108	Khristian Behrens Giordano Mion Yasusada Murata Jens Südekum	Spatial Frictions
1107	Andrea Ariu Giordano Mion	Service Trade and Occupational Tasks: An Empirical Investigation
1106	Verónica Amarante Marco Manacorda Edward Miguel Andrea Vigorito	Do Cash Transfers Improve Birth Outcomes? Evidence from Matched Vital Statistics, Social Security and Program Data
1105	Thomas Sampson	Assignment Reversals: Trade, Skill Allocation and Wage Inequality
1104	Brian Bell Stephen Machin	Immigrant Enclaves and Crime
1103	Swati Dhingra	Trading Away Wide Brands for Cheap Brands
1102	Francesco Caselli Antonio Ciccone	A Note on Schooling in Development Accounting

The Centre for Economic Performance Publications Unit
Tel 020 7955 7673 Fax 020 7955 7595
Email info@cep.lse.ac.uk Web site http://cep.lse.ac.uk