Supplementary Appendix

to

OPTIMAL TIME-INCONSISTENT BELIEFS: MISPLANNING, PROCRASTINATION, AND COMMITMENT

Markus K. Brunnermeier
Princeton University
CEPR, and NBER

Filippos Papakonstantinou
Imperial College London

Jonathan A. Parker
MIT Sloan School of Management and NBER

B.1 A model with time-consistent preferences

In our model from Section 1 in the paper, preferences are time inconsistent, since with objective beliefs, the optimal work w_1 from the time 0 perspective is $w_1^{*,0}(\mathbb{E}) = \min\{1, (D_0 + D_1)(1 + E)\}$ (see Equation 6 in the paper), which is different from the optimal work from the time 1 perspective, which is $w_1^*(\mathbb{E}) = \min\{1, B_1(1+E)\}$ (see Equation 5 in the paper). This time inconsistency in preferences does not alter the message of our paper, since $D_0 + D_1 < B_1$, hence $w_1^{*,0}(\mathbb{E}) < w_1^*(\mathbb{E})$, and therefore it is not this time inconsistency that causes a preference for commitment in our model, rather it is the time inconsistency in beliefs. Still, for completeness, we extend our baseline model here by introducing the possibility that the past, not just the future, matters, and therefore time-consistent preferences are possible, and then we show that all our results continue to hold.

We extend our model from Section 1 in the paper by altering Equations 2 and 3 to include past utility, i.e.:

$$\begin{aligned} V_t &:= \hat{\mathbb{E}}_t \left[\sum_{\tau \geq 0} \beta^{\tau - t} U_\tau \right] \\ U_t &:= \hat{\mathbb{E}}_t \left[\sum_{\tau \geq 0} \phi^{\tau - t} u(w_\tau) \right]. \end{aligned}$$

Now Equation 5 from the paper becomes

$$w_1^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = \min\left\{1, B_0\left(1 + \hat{\mathbb{E}}_0[\eta]\right) + B_1\left(1 + \hat{\mathbb{E}}_1[\eta]\right)\right\}, \tag{B.1}$$

where $B_0 := \phi^2 \frac{\phi}{(1+\phi)(\beta^2+\beta\phi+\phi^2)}$, and now $B_1 := \beta(\beta + \phi) \frac{\phi}{(1+\phi)(\beta^2+\beta\phi+\phi^2)}$. In addition, Equation 6 from the paper remains the same, i.e.,

$$w_1^{*,0}\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = \min\left\{1, D_0\left(1 + \hat{\mathbb{E}}_0[\eta]\right) + D_1\left(1 + \hat{\mathbb{E}}_1[\eta]\right)\right\},$$

with the difference that now $D_0 := \frac{\phi(\beta^2+\phi^2)}{(1+\phi)(\beta^2+\beta\phi+\phi^2)}$ and $D_1 := \frac{\beta\phi^2}{(1+\phi)(\beta^2+\beta\phi+\phi^2)}$. Indeed then, with objective beliefs, we have $w_1^*(\mathbb{E}) = \min\{1, (B_0 + B_1)(1 + E)\} = \min\{1, (D_0 + D_1)(1 + E)\} = w_1^{*,0}(\mathbb{E})$, so preferences are time consistent.

All the propositions (and their proofs) in the paper remain essentially unchanged. In terms of results, a slight exception is Proposition 4, whose results in this setting are essentially a more extreme version of the results in the original setting from Section 1. In terms of proofs, the exceptions are Propositions 2 and 4; their proofs in the new setting are very similar in spirit, but the details are somewhat different. As a result, in what follows we focus on restating and proving Propositions 2 and 4; the exact statements and proofs of all the remaining propositions, for the setting we introduce here, are available from the authors upon request.

B.1.1 Optimism and the planning fallacy

Proposition B.1. *(Optimism and the planning fallacy are optimal)*
(i) $\hat{\mathbb{E}}_0^{\text{ND}}[\eta]$ *and* $\hat{\mathbb{E}}_1^{\text{ND}}[\eta]$ *are piece-wise linear, weakly increasing functions of* $\mathbb{E}[\eta]$.
(ii) Optimal beliefs are optimistic, i.e., $\hat{\mathbb{E}}_0^{\text{ND}}[\eta] \leq \mathbb{E}[\eta]$ *and* $\hat{\mathbb{E}}_1^{\text{ND}}[\eta] \leq \mathbb{E}[\eta]$.
(iii) Over time, beliefs become less optimistic, i.e., $\hat{\mathbb{E}}_0^{\text{ND}}[\eta] \leq \hat{\mathbb{E}}_1^{\text{ND}}[\eta]$.
(iv) The planning fallacy (under-estimation of task duration) is optimal, i.e.,
$\hat{\mathbb{E}}_0^{\text{ND}}\left[w_1^* + w_2^*\right] < \mathbb{E}\left[w_1^* + w_2^*\right]$ *and* $\hat{\mathbb{E}}_1^{\text{ND}}\left[w_1^* + w_2^*\right] < \mathbb{E}\left[w_1^* + w_2^*\right]$.
(v) Optimal work is $w_1^{\text{ND}} := B_0\left(1+\hat{\mathbb{E}}_0^{\text{ND}}[\eta]\right)+B_1\left(1+\hat{\mathbb{E}}_1^{\text{ND}}[\eta]\right) \leq \min\{1, (B_0+B_1)(1+\mathbb{E}[\eta])\} =: w_1^{\text{RE}}$.

B.1.1.1 Proof of Proposition B.1

(i - iii) First, we argue that optimal beliefs satisfy $B_0\left(1+\hat{E}_0\right)+B_1\left(1+\hat{E}_1\right) \leq 1$: If not, from Equation B.1, the optimal work at $t = 1$ would be 1, and optimal beliefs could become more optimistic, yielding anticipatory benefits, without altering behavior, so without cost. Thus, we substitute $w_1 = B_0\left(1+\hat{E}_0\right)+B_1\left(1+\hat{E}_1\right)$ and $w_2 = 1+\eta-w_1$ into $\mathcal{W}$.

Define the following constants:

$$\begin{aligned} F &:= \phi(1+\phi)+(1+\phi)\beta+\left(1+\phi^{-1}\right)\beta^2 & M_{B_0} &:= FB_0^2+\phi^2(1-2B_0) \\ G_B &:= FB_0B_1-\phi^2B_1-\beta\phi B_0 & M_{B_1} &:= FB_1^2+\beta\phi(1-2B_1), \end{aligned}$$

where $F > 0$, and it is easy to show that $G_B < 0$, $M_{B_0} > 0$, and $M_{B_1} > 0$. Then:

$$\begin{aligned} \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_0} &= -M_{B_0}\left(1+\hat{E}_0\right)-G_B\left(1+\hat{E}_1\right)+\beta^2B_0(1+E) \\ \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_1} &= -G_B\left(1+\hat{E}_0\right)-M_{B_1}\left(1+\hat{E}_1\right)+\beta^2B_1(1+E). \end{aligned}$$

Setting the derivatives to 0, we find

$$\hat{E}_0^{\dagger} = \frac{\phi^2 B_1^2 + \beta\phi B_0 (1-B_1)}{M_{B_0} M_{B_1} - G_B^2} \beta^2 (1+E) - 1$$
$$\hat{E}_1^{\dagger} = \frac{\phi^2 B_1 (1-B_0) + \beta\phi B_0^2}{M_{B_0} M_{B_1} - G_B^2} \beta^2 (1+E) - 1,$$

and simple algebra verifies that $M_{B_0} M_{B_1} > G_B^2$ and $\hat{E}_1^{\dagger} \geq \hat{E}_0^{\dagger}$.

We also note that $B_0 \left(1+\hat{E}_0\right) + B_1 \left(1+\hat{E}_1\right) \leq 1$ binds only if $\hat{E}_0$, $\hat{E}_1$ are both positive. Otherwise, we would have $\hat{E}_0 = 0$ and $\hat{E}_1 = \frac{1-B_0-B_1}{B_1}$ since $\hat{E}_0^{\dagger} \leq \hat{E}_1^{\dagger}$, but then $\mathcal{W}$ could be increased by raising $\hat{E}_0$ by some $\varepsilon > 0$ and lowering $\hat{E}_1$ by $\frac{B_0}{B_1}\varepsilon$, since at $\hat{E}_0 = 0$, $\hat{E}_1 = \frac{1-B_0-B_1}{B_1}$ we have $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_0} - \frac{B_0}{B_1}\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_1} = \frac{B_0}{B_1^2}\beta\phi\,(1 - B_0 - B_1) > 0$.

So the possible solutions are:

- $\hat{E}_0^{\mathrm{ND}} = \hat{E}_1^{\mathrm{ND}} = 0$. In the interior we have $\hat{E}_1^{\dagger} > \hat{E}_0^{\dagger}$, so to find the Es for which $\hat{E}_0^{\mathrm{ND}} = 0$ we use $\hat{E}_1^{\dagger}$ in $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_0}$ and check when the resulting $\hat{E}_0$ is 0; we find $\hat{E}_0^{\mathrm{ND}} = 0$ if $E \leq \frac{1}{\bar{s}_0^{\mathrm{ND}}} - 1$, where $\bar{s}_0^{\mathrm{ND}} := \frac{\phi^2 B_1^2 + \beta\phi B_0(1-B_1)}{M_{B_0} M_{B_1} - G_B^2}\beta^2$. To find the Es for which $\hat{E}_1^{\mathrm{ND}} = 0$ we use $\hat{E}_0 = 0$ in $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_1}$ and check when the resulting $\hat{E}_1$ is 0; we find $\hat{E}_1^{\mathrm{ND}} = 0$ if $E \leq \frac{-\underline{c}_1^{\mathrm{ND}}}{\underline{s}_1^{\mathrm{ND}}}$, where $\underline{s}_1^{\mathrm{ND}} := \frac{\beta^2 B_1}{M_{B_1}}$, $\underline{c}_1^{\mathrm{ND}} := \underline{s}_1^{\mathrm{ND}} - \frac{G_B}{M_{B_1}} - 1$. So $\hat{E}_0^{\mathrm{ND}} = \hat{E}_1^{\mathrm{ND}} = 0$ if $E \leq \frac{-\underline{c}_1^{\mathrm{ND}}}{\underline{s}_1^{\mathrm{ND}}}$.

- $\hat{E}_0^{\mathrm{ND}} = 0 < \hat{E}_1^{\mathrm{ND}}$. We already know that $\hat{E}_0^{\mathrm{ND}} = 0$ and $\hat{E}_1^{\mathrm{ND}} = \underline{s}_1^{\mathrm{ND}} E + \underline{c}_1^{\mathrm{ND}}$ for $\frac{-\underline{c}_1^{\mathrm{ND}}}{\underline{s}_1^{\mathrm{ND}}} < E \leq \frac{1}{\bar{s}_0^{\mathrm{ND}}} - 1$. We also know that for $\hat{E}_0 = 0 < \hat{E}_1$, $B_0\left(1+\hat{E}_0\right) + B_1\left(1+\hat{E}_1\right) \leq 1$ does not bind, so can be ignored.

- $\hat{E}_0^{\mathrm{ND}} > 0$ and $\hat{E}_1^{\mathrm{ND}} > 0$ and no constraints bind. We know in this case optimal beliefs are $\hat{E}_0^{\mathrm{ND}} = \bar{s}_0^{\mathrm{ND}}(1+E) - 1 < \bar{s}_1^{\mathrm{ND}}(1+E) - 1 = \hat{E}_1^{\mathrm{ND}}$, where $\bar{s}_1^{\mathrm{ND}} := \frac{\phi^2 B_1(1-B_0) + \beta\phi B_0^2}{M_{B_0} M_{B_1} - G_B^2}\beta^2$. For these values, $B_0\left(1+\hat{E}_0\right) + B_1\left(1+\hat{E}_1\right) \leq 1$ does not bind for $E \leq \frac{1}{B_0\bar{s}_0^{\mathrm{ND}} + B_1\bar{s}_1^{\mathrm{ND}}} - 1$, so we combine this with the condition for $\hat{E}_0 > 0$, i.e., $E > \frac{1}{\bar{s}_0^{\mathrm{ND}}} - 1$.

- $\hat{E}_0^{\mathrm{ND}} > 0$, $\hat{E}_1^{\mathrm{ND}} > 0$ and $B_0\left(1+\hat{E}_0\right) + B_1\left(1+\hat{E}_1\right) \leq 1$ binds, which happens when $E > \frac{1}{B_0\bar{s}_0^{\mathrm{ND}} + B_1\bar{s}_1^{\mathrm{ND}}} - 1$. We use $E = \frac{1}{B_0\bar{s}_0^{\mathrm{ND}} + B_1\bar{s}_1^{\mathrm{ND}}} - 1$ in the unconstrained optimal beliefs, to get $\hat{E}_0^{\mathrm{ND}} = \frac{\bar{s}_0^{\mathrm{ND}}}{B_0\bar{s}_0^{\mathrm{ND}} + B_1\bar{s}_1^{\mathrm{ND}}} - 1 < \frac{\bar{s}_1^{\mathrm{ND}}}{B_0\bar{s}_0^{\mathrm{ND}} + B_1\bar{s}_1^{\mathrm{ND}}} - 1 = \hat{E}_1^{\mathrm{ND}}$.

To summarize, we have

$$\begin{cases} \hat{\mathbb{E}}_0^{\mathrm{ND}}[\eta]=0 & = & \hat{\mathbb{E}}_1^{\mathrm{ND}}[\eta]=0 & \text{if} & & \mathbb{E}[\eta] & \leq \mu_L^{\mathrm{ND}} \\ \hat{\mathbb{E}}_0^{\mathrm{ND}}[\eta]=0 & < & \hat{\mathbb{E}}_1^{\mathrm{ND}}[\eta]=\underline{s}_1^{\mathrm{ND}}\mathbb{E}[\eta]+\underline{c}_1^{\mathrm{ND}} & \text{if} & \mu_L^{\mathrm{ND}}< & \mathbb{E}[\eta] & \leq \mu_I^{\mathrm{ND}} \\ \hat{\mathbb{E}}_0^{\mathrm{ND}}[\eta]=\overline{s}_0^{\mathrm{ND}}(1+\mathbb{E}[\eta])-1 & < & \hat{\mathbb{E}}_1^{\mathrm{ND}}[\eta]=\overline{s}_1^{\mathrm{ND}}(1+\mathbb{E}[\eta])-1 & \text{if} & \mu_I^{\mathrm{ND}}< & \mathbb{E}[\eta] & \leq \mu_U^{\mathrm{ND}} \\ \hat{\mathbb{E}}_0^{\mathrm{ND}}[\eta]=\frac{\overline{s}_0^{\mathrm{ND}}}{B_0\overline{s}_0^{\mathrm{ND}}+B_1\overline{s}_1^{\mathrm{ND}}}-1 & < & \hat{\mathbb{E}}_1^{\mathrm{ND}}[\eta]=\frac{\overline{s}_1^{\mathrm{ND}}}{B_0\overline{s}_0^{\mathrm{ND}}+B_1\overline{s}_1^{\mathrm{ND}}}-1 & \text{if} & \mu_U^{\mathrm{ND}}< & \mathbb{E}[\eta] & \end{cases} \tag{B.2}$$

where the critical values are $\mu_L^{\mathrm{ND}} := \frac{-\underline{c}_1^{\mathrm{ND}}}{\underline{s}_1^{\mathrm{ND}}}$, $\mu_I^{\mathrm{ND}} := \frac{1}{\overline{s}_0^{\mathrm{ND}}}-1$, $\mu_U^{\mathrm{ND}} := \frac{1}{B_0\overline{s}_0^{\mathrm{ND}}+B_1\overline{s}_1^{\mathrm{ND}}}-1$.

We show optimal beliefs are optimistic; $\hat{E}_0^{\mathrm{ND}} \leq \hat{E}_1^{\mathrm{ND}}$ so we just show for $\hat{E}_1^{\mathrm{ND}}$:

- For $E \leq \mu_L^{\mathrm{ND}}$, we have $\hat{E}_1^{\mathrm{ND}} = 0 < E$.
- For $\mu_I^{\mathrm{ND}} < E \leq \mu_U^{\mathrm{ND}}$, we have $\hat{E}_1^{\mathrm{ND}} = \overline{s}_1^{\mathrm{ND}}(1+E)-1$, so it is optimistic if $\overline{s}_1^{\mathrm{ND}} < 1$, which after some algebra can be shown to be true.
- For $\mu_L^{\mathrm{ND}} < E \leq \mu_I^{\mathrm{ND}}$, we have $\hat{E}_1^{\mathrm{ND}} = \underline{s}_1^{\mathrm{ND}}E + \underline{c}_1^{\mathrm{ND}}$. Optimal beliefs in the ranges $\left(\mu_L^{\mathrm{ND}}, \mu_I^{\mathrm{ND}}\right]$ and $\left(\mu_I^{\mathrm{ND}}, \mu_U^{\mathrm{ND}}\right]$ must be equal at $E = \mu_I^{\mathrm{ND}}$, so $\underline{s}_1^{\mathrm{ND}}E + \underline{c}_1^{\mathrm{ND}} = \overline{s}_1^{\mathrm{ND}}(1+E)-1 < E$ at $E = \mu_I^{\mathrm{ND}}$. Also, algebra shows that $\underline{c}_1^{\mathrm{ND}} < 0$ is true, so $\underline{s}_1^{\mathrm{ND}}E + \underline{c}_1^{\mathrm{ND}} < E$ at $E = 0$. Since $\underline{s}_1^{\mathrm{ND}}E + \underline{c}_1^{\mathrm{ND}}$ is a straight line, we conclude that $\hat{E}_1^{\mathrm{ND}} < E$, i.e., optimistic, for $\mu_L^{\mathrm{ND}} < E \leq \mu_I^{\mathrm{ND}}$.
- For $\mu_U^{\mathrm{ND}} < E$, $\hat{E}_1^{\mathrm{ND}}$ is the same as for $E = \mu_U^{\mathrm{ND}}$, so it is optimistic.

(iv) The planning fallacy is that $\hat{\mathbb{E}}_t\left[w_1^* + w_2^*\right] < \mathbb{E}\left[w_1^* + w_2^*\right]$, for $t = 0$ and $t = 1$. Using Equation 1, this simply becomes $\hat{\mathbb{E}}_t[\eta] < \mathbb{E}[\eta]$, which we have shown above to be true, since optimal beliefs are optimistic.

(v) Substituting, respectively, optimal beliefs from Equation B.2 and objective beliefs into Equation B.1, we have $w_1^{\mathrm{ND}} = B_0\left(1+\hat{E}_0^{\mathrm{ND}}\right) + B_1\left(1+\hat{E}_1^{\mathrm{ND}}\right)$ and $w_1^{\mathrm{RE}} = \min\{1, (B_0+B_1)(1+E)\}$. We know that $\hat{E}_0^{\mathrm{ND}} < E$, $\hat{E}_1^{\mathrm{ND}} < E$, so $w_1^{\mathrm{ND}} \leq w_1^{\mathrm{RE}}$.

B.1.2 Preference for commitment

Proposition B.2. *(Self-imposed deadline)*

(i) If $\beta \leq \phi$, optimal beliefs are identical to those absent a commitment device and the agent does not impose a binding deadline.

(ii) If $\beta > \phi$, then:

- *Optimal expectations* $\hat{\mathbb{E}}_0^{\text{D}}[\eta]$, $\hat{\mathbb{E}}_1^{\text{D}}[\eta]$ *are weakly increasing functions of* $\mathbb{E}[\eta]$.

- *Optimal beliefs are optimistic (*$\hat{\mathbb{E}}_0^{\text{D}}[\eta] < \mathbb{E}[\eta]$ *and* $\hat{\mathbb{E}}_1^{\text{D}}[\eta] < \mathbb{E}[\eta]$*).*

- *Optimal beliefs become more optimistic over time (*$\hat{\mathbb{E}}_0^{\text{D}}[\eta] \geq \hat{\mathbb{E}}_1^{\text{D}}[\eta]$*).*

- *Time* 0 *optimal beliefs are more pessimistic (*$\hat{\mathbb{E}}_0^{\text{D}}[\eta] \geq \hat{\mathbb{E}}_0^{\text{ND}}[\eta]$*) and time* 1 *optimal beliefs more optimistic (*$\hat{\mathbb{E}}_1^{\text{D}}[\eta] \leq \hat{\mathbb{E}}_1^{\text{ND}}[\eta]$*) than absent a commitment device.*

- *The optimal deadline* $\psi^{\text{D}} := D_0\left(1+\hat{\mathbb{E}}_0^{\text{D}}[\eta]\right)+D_1\left(1+\hat{\mathbb{E}}_1^{\text{D}}[\eta]\right)$ *binds (*$w_1^*\left(\left\{\hat{\mathbb{E}}_t^{\text{D}}\right\}\right) \leq \psi^{\text{D}}$*), but is smaller than* w_1^{RE}.

- *Complete overconfidence is optimal (*$\hat{\Sigma}_0^{\text{D}} = \hat{\Sigma}_1^{\text{D}} = 0 < \Sigma$*).*

B.1.2.1 Proof of Proposition B.2

Step 1 – Optimal work given an arbitrary deadline and arbitrary beliefs

Combining the deadline $w_1 \geq \psi$ with Equation B.1, the optimal work is $w_1^*\left(\left\{\hat{\mathbb{E}}_t\right\}, \psi\right) = \min\left\{1, \max\left\{\psi, B_0\left(1+\hat{E}_0\right)+B_1\left(1+\hat{E}_1\right)\right\}\right\}$.

Step 2 – Optimal deadline given optimal work and arbitrary beliefs Deadline $\psi \notin \left[w_1^*, 1\right]$ is ignored at $t=1$, so at $t=0$ the agent chooses $\psi \in \left[w_1^*, 1\right]$ to maximize V_0. So at $t=1$ the agent chooses $w_1^*\left(\left\{\hat{\mathbb{E}}_t\right\}, \psi\right)=\psi$. Substituting in V_0, we have

$$V_0 \propto -\left\{\psi^2+D_0\left[\left(1+\hat{E}_0\right)^2+\hat{\Sigma}_0-2\left(1+\hat{E}_0\right)\psi\right]+D_1\left[\left(1+\hat{E}_1\right)^2+\hat{\Sigma}_1-2\left(1+\hat{E}_1\right)\psi\right]\right\}$$

$$\frac{\mathrm{d}V_0}{\mathrm{d}\psi} \propto -\left\{\psi - D_0\left(1+\hat{E}_0\right)-D_1\left(1+\hat{E}_1\right)\right\},$$

so imposing $\psi \in \left[w_1^*, 1\right]$ and using $B_1 - D_1 = D_0 - B_0$, we have

$$\begin{cases} \psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = B_0\left(1+\hat{E}_0\right)+B_1\left(1+\hat{E}_1\right) & \text{if } \hat{E}_0 \leq \hat{E}_1 \text{ and } B_0\left(1+\hat{E}_0\right)+B_1\left(1+\hat{E}_1\right) \leq 1 \\ \psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right) & \text{if } \hat{E}_1 \leq \hat{E}_0 \text{ and } D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right) \leq 1 \\ \psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = 1 & \text{otherwise.} \end{cases}$$

We show below that optimal beliefs are optimistic; combined with $D_0 + D_1 = B_0 + B_1$, this proves the optimal deadline is smaller than $w_1^{\text{RE}} = (B_0 + B_1)(1+E)$.

Step 3 – Optimal beliefs given optimal work and optimal deadline The optimality of complete overconfidence trivially follows from the quadratic utility assumption, so we turn our attention to optimal expectations. They satisfy $B_0\left(1+\hat{E}_0\right)+B_1\left(1+\hat{E}_1\right)\leq 1$, $D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right)\leq 1$. If not, our expressions for $w_1^*\left(\left\{\hat{\mathbb{E}}_t\right\},\psi\right)$, $\psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right)$, show that optimal work at $t=1$ would be 1, and optimal beliefs could become more optimistic, yielding anticipatory benefits without altering behavior, so without cost. So we need only consider two cases.

Step 3 – Case A: If $\psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = B_0\left(1+\hat{E}_0\right)+B_1\left(1+\hat{E}_1\right)$, then working as in Section B.1.1.1 we find the same optimal beliefs, which indeed satisfy $\hat{E}_0 \leq \hat{E}_1$.

Step 3 – Case B: If $\psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right)=D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right)$ we work as follows.

Substituting for $w_1^*\left(\left\{\hat{\mathbb{E}}_t\right\},\psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right)\right)$ in $\mathcal{W}$ and differentiating w.r.t. $\hat{E}_0$ and $\hat{E}_1$:

$$\begin{aligned}\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_0} &= -M_{D_0}\left(1+\hat{E}_0\right)-G_D\left(1+\hat{E}_1\right)+\beta^2 D_0\left(1+E\right)\\ \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_1} &= -G_D\left(1+\hat{E}_0\right)-M_{D_1}\left(1+\hat{E}_1\right)+\beta^2 D_1\left(1+E\right),\end{aligned}$$

where definitions of the constants mirror the corresponding ones in Section B.1.1.1 and where algebra shows that $G_D < 0$, $M_{D_0} > 0$ and $M_{D_1} > 0$.

Ignoring the constraints and setting the derivatives to 0 we find

$$\begin{aligned}\hat{E}_0^\dagger &= \overline{s}_0^{\mathrm{D}}\left(1+E\right)-1\\ \hat{E}_1^\dagger &= \overline{s}_1^{\mathrm{D}}\left(1+E\right)-1,\end{aligned}$$

where $\overline{s}_0^{\mathrm{D}} := \frac{\phi^2 D_1^2+\beta\phi D_0(1-D_1)}{M_{D_0}M_{D_1}-G_D^2}\beta^2$ and $\overline{s}_1^{\mathrm{D}} := \frac{\phi^2 D_1(1-D_0)+\beta\phi D_0^2}{M_{D_0}M_{D_1}-G_D^2}\beta^2$, and where simple algebra shows that $M_{D_0}M_{D_1} > G_D^2$ and $\hat{E}_1^\dagger \leq \hat{E}_0^\dagger$. In addition, we note that $D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right) \leq 1$ binds only if $\hat{E}_0$, $\hat{E}_1$ are both positive. If not, we would have $\hat{E}_0 = \frac{1-D_0-D_1}{D_0}$ and $\hat{E}_1 = 0$ since $\hat{E}_1^\dagger \leq \hat{E}_0^\dagger$, but then $\mathcal{W}$ could be increased by raising $\hat{E}_1$ by some $\varepsilon > 0$ and lowering $\hat{E}_0$ by $\frac{D_1}{D_0}\varepsilon$, since at $\hat{E}_0 = \frac{1-D_0-D_1}{D_0}$ and $\hat{E}_1 = 0$ we have $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_1} - \frac{D_1}{D_0}\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_0} = \frac{D_1}{D_0^2}\phi^2\left(1-D_0-D_1\right) > 0$.

So the possible solutions are:

- $\hat{E}_0^{\mathrm{D}} = \hat{E}_1^{\mathrm{D}} = 0$. In the interior we have $\hat{E}_0 > \hat{E}_1$, so to find the Es for which $\hat{E}_1^{\mathrm{D}} = 0$ we use the interior $\hat{E}_0$ in $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_1}$ and check when the resulting $\hat{E}_1$ is 0; we find $\hat{E}_1^{\mathrm{D}} = 0$ if $E \leq \frac{1}{\overline{s}_1^{\mathrm{D}}} - 1$.

To find the Es for which $\hat{E}_0^{\text{D}} = 0$ we use $\hat{E}_1 = 0$ in $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_0}$ and check when the resulting $\hat{E}_0$ is 0; we find $\hat{E}_0^{\text{D}} = 0$ if $E \leq \frac{-\underline{c}_0^{\text{D}}}{\underline{s}_0^{\text{D}}}$, where $\underline{s}_0^{\text{D}} := \frac{\beta^2 D_0}{M_{D_0}}$, $\underline{c}_0^{\text{D}} := \underline{s}_0^{\text{D}} - \frac{G_D}{M_{D_0}} - 1$. So $\hat{E}_0^{\text{D}} = \hat{E}_1^{\text{D}} = 0$ if $E \leq \frac{-\underline{c}_0^{\text{D}}}{\underline{s}_0^{\text{D}}}$.

- $\hat{E}_1^{\text{D}} = 0 < \hat{E}_0^{\text{D}}$. We already know that $\hat{E}_1^{\text{D}} = 0$ and $\hat{E}_0^{\text{D}} = \underline{s}_0^{\text{D}} E + \underline{c}_0^{\text{D}}$ for $\frac{-\underline{c}_0^{\text{D}}}{\underline{s}_0^{\text{D}}} < E \leq \frac{1}{\bar{s}_1^{\text{D}}} - 1$. We also know that for $\hat{E}_1 = 0 < \hat{E}_0$ the constraint $D_0 \left(1 + \hat{E}_0\right) + D_1 \left(1 + \hat{E}_1\right) \leq 1$ does not bind, so can be ignored.

- $\hat{E}_0^{\text{D}} > 0$ and $\hat{E}_1^{\text{D}} > 0$ and no constraints bind. We know in this case optimal beliefs are $\hat{E}_0^{\text{D}} = \bar{s}_0^{\text{D}} (1 + E) - 1 > \bar{s}_1^{\text{D}} (1 + E) - 1 = \hat{E}_1^{\text{D}}$. For these values, $D_0 \left(1 + \hat{E}_0\right) + D_1 \left(1 + \hat{E}_1\right) \leq 1$ does not bind for $E \leq \frac{1}{D_0 \bar{s}_0^{\text{D}} + D_1 \bar{s}_1^{\text{D}}} - 1$, so we combine this with the condition for $\hat{E}_0^{\text{D}} > 0$, i.e., $E > \frac{1}{\bar{s}_1^{\text{D}}} - 1$.

- $\hat{E}_0^{\text{D}} > 0$ and $\hat{E}_1^{\text{D}} > 0$, and $D_0 \left(1 + \hat{E}_0\right) + D_1 \left(1 + \hat{E}_1\right) \leq 1$ binds, which happens when $E > \frac{1}{D_0 \bar{s}_0^{\text{D}} + D_1 \bar{s}_1^{\text{D}}} - 1$. We use $E = \frac{1}{D_0 \bar{s}_0^{\text{D}} + D_1 \bar{s}_1^{\text{D}}} - 1$ in the unconstrained optimal beliefs, to get $\hat{E}_0^{\text{D}} = \frac{\bar{s}_0^{\text{D}}}{D_0 \bar{s}_0^{\text{D}} + D_1 \bar{s}_1^{\text{D}}} - 1 > \frac{\bar{s}_1^{\text{D}}}{D_0 \bar{s}_0^{\text{D}} + D_1 \bar{s}_1^{\text{D}}} - 1 = \hat{E}_1^{\text{D}}$.

To summarize, we have

$$\begin{cases} \hat{\mathbb{E}}_0^{\text{D}}[\eta] = 0 & = & \hat{\mathbb{E}}_1^{\text{D}}[\eta] = 0 & \text{if} & & \mathbb{E}[\eta] & \leq \mu_L^{\text{D}} \\ \hat{\mathbb{E}}_0^{\text{D}}[\eta] = \underline{s}_0^{\text{D}} \mathbb{E}[\eta] + \underline{c}_0^{\text{D}} & > & \hat{\mathbb{E}}_1^{\text{D}}[\eta] = 0 & \text{if} & \mu_L^{\text{D}} < & \mathbb{E}[\eta] & \leq \mu_I^{\text{D}} \\ \hat{\mathbb{E}}_0^{\text{D}}[\eta] = \bar{s}_0^{\text{D}} (1 + \mathbb{E}[\eta]) - 1 & > & \hat{\mathbb{E}}_1^{\text{D}}[\eta] = \bar{s}_1^{\text{D}} (1 + \mathbb{E}[\eta]) - 1 & \text{if} & \mu_I^{\text{D}} < & \mathbb{E}[\eta] & \leq \mu_U^{\text{D}} \\ \hat{\mathbb{E}}_0^{\text{D}}[\eta] = \frac{\bar{s}_0^{\text{D}}}{D_0 \bar{s}_0^{\text{D}} + D_1 \bar{s}_1^{\text{D}}} - 1 & > & \hat{\mathbb{E}}_1^{\text{D}}[\eta] = \frac{\bar{s}_1^{\text{D}}}{D_0 \bar{s}_0^{\text{D}} + D_1 \bar{s}_1^{\text{D}}} - 1 & \text{if} & \mu_U^{\text{D}} < & \mathbb{E}[\eta] & , \end{cases} \tag{B.3}$$

where $\mu_L^{\text{D}} := \frac{-\underline{c}_0^{\text{D}}}{\underline{s}_0^{\text{D}}}$, $\mu_I^{\text{D}} := \frac{1}{\bar{s}_1^{\text{D}}} - 1$, and $\mu_U^{\text{D}} := \frac{1}{D_0 \bar{s}_0^{\text{D}} + D_1 \bar{s}_1^{\text{D}}} - 1$.

Comparing the well-being with a non-binding and with a binding deadline: We compare the well-being from optimal beliefs that implement a non-binding deadline, found in Case A, with the well-being from optimal beliefs that implement a binding deadline, found in Case B. Regarding the critical values for E, algebra shows that $\mu_L^{\text{ND}} - \mu_L^{\text{D}} \propto \mu_I^{\text{D}} - \mu_I^{\text{ND}} \propto \mu_U^{\text{ND}} - \mu_U^{\text{D}} \propto \beta - \phi$.

Focusing first on the case $\beta > \phi$, we compare $\mathcal{W}$ for various values of E:

1. If $E \leq \mu_L^{\text{D}}$, then $\hat{E}_0^{\text{ND}} = \hat{E}_1^{\text{ND}} = \hat{E}_0^{\text{D}} = \hat{E}_1^{\text{D}} = 0$, so $\mathcal{W}^{\text{ND}} = \mathcal{W}^{\text{D}}$.

2. If $\mu_L^{\mathrm{D}} \leq E \leq \mu_L^{\mathrm{ND}}$, $\hat{E}_0^{\mathrm{D}} = \underline{s}_0^{\mathrm{D}} E + \underline{c}_0^{\mathrm{D}}$ is the only belief that changes from above. Since the binding-deadline case is less constrained than above, we conclude that $\mathcal{W}^{\mathrm{ND}} < \mathcal{W}^{\mathrm{D}}$.

3. If $\mu_L^{\mathrm{ND}} \leq E \leq \mu_I^{\mathrm{ND}}$, $\hat{E}_1^{\mathrm{ND}} = \underline{s}_1^{\mathrm{ND}} E + \underline{c}_1^{\mathrm{ND}}$ is the only belief that changes from above. Algebra shows that $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}}$ is a quadratic in E whose leading term and the value where the extremum is attained have opposite signs, so we have two cases:

   - $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}}$ is a concave quadratic in E, hence the values of E such that it is positive are a convex set, so to show that $\mathcal{W}^{\mathrm{ND}} < \mathcal{W}^{\mathrm{D}}$ for $\mu_L^{\mathrm{ND}} < E \leq \mu_I^{\mathrm{ND}}$, we need to show it for the endpoints. We showed this above for μ_L^{ND}, and we now show it for μ_I^{ND}: Plugging for $E = \mu_I^{\mathrm{ND}}$, we calculate $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}} \propto \beta - \phi > 0$.
   - $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}}$ is a convex quadratic in E and the value of E at which the minimum is attained is negative, hence not in $\left[\mu_L^{\mathrm{ND}}, \mu_I^{\mathrm{ND}}\right]$ since $\mu_L^{\mathrm{ND}} = \frac{-\underline{c}_1^{\mathrm{ND}}}{\underline{s}_1^{\mathrm{ND}}} = \frac{\beta^2+\beta\phi+\phi^2}{\beta^2(\beta+\phi)} > 0$. So $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}}$ is increasing for $E \in \left[\mu_L^{\mathrm{ND}}, \mu_I^{\mathrm{ND}}\right]$, so having shown that $\mathcal{W}^{\mathrm{ND}} < \mathcal{W}^{\mathrm{D}}$ for μ_L^{ND} implies it is also true for the whole range.

4. If $\mu_I^{\mathrm{ND}} \leq E \leq \mu_I^{\mathrm{D}}$, $\hat{E}_0^{\mathrm{ND}} = \overline{s}_0^{\mathrm{ND}}(1+E) - 1$ and $\hat{E}_1^{\mathrm{ND}} = \overline{s}_1^{\mathrm{ND}}(1+E) - 1$ are the only beliefs that change from above. Algebra shows that $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}}$ is a quadratic in E, whose leading term and the value where the extremum is attained have opposite signs, so we have the same two cases as above, and to show that $\mathcal{W}^{\mathrm{ND}} < \mathcal{W}^{\mathrm{D}}$ for $\mu_I^{\mathrm{ND}} < E \leq \mu_I^{\mathrm{D}}$, we need to show it at the endpoints. We already showed this for μ_I^{ND}, and we show it below for μ_I^{D}.

5. If $\mu_I^{\mathrm{D}} \leq E \leq \mu_U^{\mathrm{D}}$, $\hat{E}_0^{\mathrm{D}} = \overline{s}_0^{\mathrm{D}}(1+E) - 1$ and $\hat{E}_1^{\mathrm{D}} = \overline{s}_1^{\mathrm{D}}(1+E) - 1$ are the only beliefs that change from above. Algebra shows $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}} \propto \beta - \phi > 0$.

6. Note that $\mathcal{W}^{\mathrm{D}}$ and $\mathcal{W}^{\mathrm{ND}}$ both consist of a part that depends on subjective and a part that depends on objective beliefs; the latter is the same for both, so we ignore it. We have shown that for $E > \mu_U^{\mathrm{D}}$, optimal beliefs $\hat{E}_0^{\mathrm{D}}$, $\hat{E}_1^{\mathrm{D}}$ remain at their level at $E = \mu_U^{\mathrm{D}}$, but optimal beliefs $\hat{E}_0^{\mathrm{ND}}$, $\hat{E}_1^{\mathrm{ND}}$ have not hit the bound $\omega_1 \leq 1$ yet. So the part of $\mathcal{W}^{\mathrm{D}}$ that depends on subjective beliefs remains constant, while the corresponding part of $\mathcal{W}^{\mathrm{ND}}$ decreases as E increases beyond μ_U^{D}. Since we have already shown that $\mathcal{W}^{\mathrm{D}} > \mathcal{W}^{\mathrm{ND}}$ for $E \leq \mu_U^{\mathrm{D}}$, this implies that $\mathcal{W}^{\mathrm{D}} > \mathcal{W}^{\mathrm{ND}}$ also holds for $E > \mu_U^{\mathrm{D}}$.

We conclude that for $\beta > \phi$, $\mathcal{W}^{\text{ND}} \leq \mathcal{W}^{\text{D}}$ ($\mathcal{W}^{\text{ND}} < \mathcal{W}^{\text{D}}$) for all E (for $E > \mu_L^{\text{D}}$). We work identically to show that for $\beta < \phi$ the opposite is true, in which case a deadline is never optimal and optimal beliefs are as in Equation B.2 (note that these beliefs need to satisfy $D_0 \left(1 + \hat{E}_0^{\text{ND}}\right) + D_1 \left(1 + \hat{E}_1^{\text{ND}}\right) \leq B_0 \left(1 + \hat{E}_0^{\text{ND}}\right) + B_1 \left(1 + \hat{E}_1^{\text{ND}}\right)$, because otherwise a deadline would be chosen at $t = 0$, which is suboptimal; indeed they do, since $\hat{E}_0^{\text{ND}} \leq \hat{E}_1^{\text{ND}}$ and $B_1 - D_1 = D_0 - B_0$). Finally, for $\beta = \phi$, $\mathcal{W}^{\text{ND}} = \mathcal{W}^{\text{D}}$ and either set of beliefs is optimal.

B.2 Detailed proofs

In this section, we present detailed proofs of selected propositions in the paper.

B.2.1 Proof of Proposition 4

B.2.1.1 Step 1 – Optimal work given arbitrary deadline and arbitrary beliefs

Combining a deadline of the form $w_1 \geq \psi$ with the result from Proposition 1, the optimal work is $w_1^* \left(\left\{\hat{\mathbb{E}}_t\right\}, \psi\right) = \min\left\{1, \max\left\{\psi, B_1 \left(1 + \hat{E}_1\right)\right\}\right\}$.

B.2.1.2 Step 2 – Optimal deadline given optimal work and arbitrary beliefs

Since a deadline $\psi \notin \left[w_1^*, 1\right]$ will be ignored at $t = 1$, at $t = 0$ the agent chooses deadline $\psi \in \left[w_1^*, 1\right]$ to maximize V_0. So at $t = 1$ the agent will optimally choose $w_1^* \left(\left\{\hat{\mathbb{E}}_t\right\}, \psi\right) = \psi$. Manipulating V_0, we have

$$V_0 \propto -\frac{1}{2}\left\{\psi^2 + D_0\left[\left(1+\hat{E}_0\right)^2 + \hat{\Sigma}_0 - 2\left(1+\hat{E}_0\right)\psi\right] + D_1\left[\left(1+\hat{E}_1\right)^2 + \hat{\Sigma}_0 - 2\left(1+\hat{E}_1\right)\psi\right]\right\}$$
$$\frac{\mathrm{d}V_0}{\mathrm{d}\psi} \propto -\left\{\psi - D_0\left(1+\hat{E}_0\right) - D_1\left(1+\hat{E}_1\right)\right\},$$

where $D_0 := \frac{\beta^2+\phi^2}{\beta^2+\phi^2+\beta+\phi+\beta\phi}$, $D_1 := \frac{\beta\phi}{\beta^2+\phi^2+\beta+\phi+\beta\phi}$, so imposing $\psi \in \left[w_1^*, 1\right]$:

$$\begin{cases} \psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = B_1\left(1+\hat{E}_1\right) & \text{if } D_0\left(1+\hat{E}_0\right) + D_1\left(1+\hat{E}_1\right) \leq B_1\left(1+\hat{E}_1\right) \leq 1 \\ \psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = D_0\left(1+\hat{E}_0\right) + D_1\left(1+\hat{E}_1\right) & \text{if } B_1\left(1+\hat{E}_1\right) \leq D_0\left(1+\hat{E}_0\right) + D_1\left(1+\hat{E}_1\right) \leq 1 \\ \psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = 1 & \text{otherwise.} \end{cases}$$

B.2.1.3 Step 3 – Optimal beliefs given optimal work and optimal deadline

The optimality of complete overconfidence trivially follows from the assumption of quadratic utility, so we turn our attention to optimal expectations. They satisfy $B_1\left(1+\hat{E}_1\right) \leq 1$, $D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right) \leq 1$. If not, from our expressions for $w_1^*\left(\left\{\hat{\mathbb{E}}_t\right\}, \psi\right)$, $\psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right)$, we see that optimal work at $t=1$ would be 1, and optimal beliefs could become more optimistic, yielding anticipatory benefits without altering behavior, so without cost. So we need only consider two cases.

First, let $\psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right)=B_1\left(1+\hat{E}_1\right)$; working as in Section A.2, we find the same optimal beliefs, which indeed satisfy $D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right) \leq B_1\left(1+\hat{E}_1\right) \leq 1$.

Second, let $\psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right)=D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right)$; in what follows, we find the optimal beliefs and check when the condition $B_1\left(1+\hat{E}_1\right) \leq D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right) \leq 1$ is satisfied.

Substituting for $w_1^*\left(\left\{\hat{\mathbb{E}}_t\right\}, \psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right)\right)$ in $\mathcal{W}$ and differentiating w.r.t. $\hat{E}_0$, $\hat{E}_1$, we have

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_0} = -M_{D_0}\left(1+\hat{E}_0\right)-G_D\left(1+\hat{E}_1\right)+\beta^2 D_0(1+E) \tag{B.4}$$

$$\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_1} = -G_D\left(1+\hat{E}_0\right)-M_{D_1}\left(1+\hat{E}_1\right)+\beta^2 D_1(1+E), \tag{B.5}$$

where

$$\begin{aligned} G_D &:= FD_0D_1-\phi^2 D_1-\beta\phi D_0 \\ M_{D_0} &:= FD_0^2+\phi^2(1-2D_0) \\ M_{D_1} &:= FD_1^2+\beta\phi(1-2D_1), \end{aligned}$$

and simple algebra shows that $G_D < 0$, $M_{D_0} > 0$, and $M_{D_1} > 0$.

Ignoring the constraints and setting the derivatives to 0 we find

$$\begin{aligned} \hat{E}_0^{\dagger} &= \bar{s}_0^{\mathrm{D}}(1+E)-1 \\ \hat{E}_1^{\dagger} &= \bar{s}_1^{\mathrm{D}}(1+E)-1, \end{aligned}$$

where $\bar{s}_0^{\mathrm{D}} := \frac{\phi^2 D_1^2+\beta\phi D_0(1-D_1)}{M_{D_0}M_{D_1}-G_D^2}\beta^2$ and $\bar{s}_1^{\mathrm{D}} := \frac{\phi^2 D_1(1-D_0)+\beta\phi D_0^2}{M_{D_0}M_{D_1}-G_D^2}\beta^2$, and where we can show that $M_{D_0}M_{D_1}-G_D^2 > 0$ and $\hat{E}_0^{\dagger} \geq \hat{E}_1^{\dagger}$. In addition, we note that $D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right) \leq 1$ binds only if $\hat{E}_0$, $\hat{E}_1$ are both positive. If not, we would have $\hat{E}_0 = \frac{1-D_0-D_1}{D_0}$ and $\hat{E}_1 = 0$ since $\hat{E}_0^{\dagger} \geq \hat{E}_1^{\dagger}$, but then $\mathcal{W}$ could be increased by raising $\hat{E}_1$ by some $\varepsilon > 0$ and lowering $\hat{E}_0$ by $\frac{D_1}{D_0}\varepsilon$, since at $\hat{E}_0 = \frac{1-D_0-D_1}{D_0}$ and $\hat{E}_1 = 0$ we have $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_1}-\frac{D_1}{D_0}\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_0} = \frac{D_1}{D_0^2}\phi^2(1-D_0-D_1) > 0$.

Imposing $\hat{E}_0 \geq 0$, $\hat{E}_1 \geq 0$, and $D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right) \leq 1$, but still ignoring $B_1\left(1+\hat{E}_1\right) \leq D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right)$, the possible optimal beliefs are:

- $\hat{E}_0^{\dagger\dagger} = \hat{E}_1^{\dagger\dagger} = 0$. In the interior we have $\hat{E}_0 \geq \hat{E}_1$, so to find the Es for which $\hat{E}_1^{\dagger\dagger} = 0$ we use the interior $\hat{E}_0$ in $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_1}$ and check when the resulting $\hat{E}_1$ is 0; we find $\hat{E}_1^{\dagger\dagger} = 0$ if $E \leq \frac{1}{\bar{s}_1^{\mathrm{D}}} - 1$. To find the Es for which $\hat{E}_0^{\dagger\dagger} = 0$ we use $\hat{E}_1 = 0$ in $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_0}$ and check when the resulting $\hat{E}_0$ is 0; we find $\hat{E}_0^{\dagger\dagger} = 0$ if $E \leq \frac{-\underline{c}_0^{\mathrm{D}}}{\underline{s}_0^{\mathrm{D}}}$, where $\underline{s}_0^{\mathrm{D}} := \frac{\beta^2 D_0}{M_{D_0}}$, $\underline{c}_0^{\mathrm{D}} := \underline{s}_0^{\mathrm{D}} - \frac{G_D}{M_{D_0}} - 1$. So $\hat{E}_0^{\dagger\dagger} = \hat{E}_1^{\dagger\dagger} = 0$ if $E \leq \frac{-\underline{c}_0^{\mathrm{D}}}{\underline{s}_0^{\mathrm{D}}}$.

- $\hat{E}_1^{\dagger\dagger} = 0 < \hat{E}_0^{\dagger\dagger}$. We already know that $\hat{E}_1^{\dagger\dagger} = 0$ and $\hat{E}_0^{\dagger\dagger} = \underline{s}_0^{\mathrm{D}} E + \underline{c}_0^{\mathrm{D}}$ for $\frac{-\underline{c}_0^{\mathrm{D}}}{\underline{s}_0^{\mathrm{D}}} < E \leq \frac{1}{\bar{s}_1^{\mathrm{D}}} - 1$. We also know that for $\hat{E}_1 = 0 < \hat{E}_0$ the constraint $D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right) \leq 1$ does not bind, so can be ignored.

- $\hat{E}_0^{\dagger\dagger} > 0$ and $\hat{E}_1^{\dagger\dagger} > 0$ and no constraints bind. We know in this case optimal beliefs are $\hat{E}_0^{\dagger\dagger} = \bar{s}_0^{\mathrm{D}}(1+E) - 1 > \bar{s}_1^{\mathrm{D}}(1+E) - 1 = \hat{E}_1^{\dagger\dagger}$. For these values, the constraint $D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right) \leq 1$ does not bind for $E \leq \frac{1}{D_0\bar{s}_0^{\mathrm{D}}+D_1\bar{s}_1^{\mathrm{D}}} - 1$, so we combine this with $\hat{E}_0^{\dagger\dagger} > 0$, i.e., $E > \frac{1}{\bar{s}_1^{\mathrm{D}}} - 1$.

- $\hat{E}_0^{\dagger\dagger} > 0$ and $\hat{E}_1^{\dagger\dagger} > 0$ and the constraint $D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right) \leq 1$ binds, which happens when $E > \frac{1}{D_0\bar{s}_0^{\mathrm{D}}+D_1\bar{s}_1^{\mathrm{D}}} - 1$. We use $E = \frac{1}{D_0\bar{s}_0^{\mathrm{D}}+D_1\bar{s}_1^{\mathrm{D}}} - 1$ in the unconstrained optimal beliefs, to get $\hat{E}_0^{\dagger\dagger} = \frac{\bar{s}_0^{\mathrm{D}}}{D_0\bar{s}_0^{\mathrm{D}}+D_1\bar{s}_1^{\mathrm{D}}} - 1 > \frac{\bar{s}_1^{\mathrm{D}}}{D_0\bar{s}_0^{\mathrm{D}}+D_1\bar{s}_1^{\mathrm{D}}} - 1 = \hat{E}_1^{\dagger\dagger}$.

To summarize, ignoring $B_1\left(1+\hat{E}_1\right) \leq D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right)$, we have

$$\begin{cases} \hat{\mathbb{E}}_0^{\dagger\dagger}[\eta] = 0 & = & \hat{\mathbb{E}}_1^{\dagger\dagger}[\eta] = 0 & \text{if} & & \mathbb{E}[\eta] & \leq \mu_L^{\mathrm{D}} \\ \hat{\mathbb{E}}_0^{\dagger\dagger}[\eta] = \underline{s}_0^{\mathrm{D}}\mathbb{E}[\eta] + \underline{c}_0^{\mathrm{D}} & > & \hat{\mathbb{E}}_1^{\dagger\dagger}[\eta] = 0 & \text{if} & \mu_L^{\mathrm{D}} < & \mathbb{E}[\eta] & \leq \mu_I^{\mathrm{D}} \\ \hat{\mathbb{E}}_0^{\dagger\dagger}[\eta] = \bar{s}_0^{\mathrm{D}}(1+\mathbb{E}[\eta]) - 1 & > & \hat{\mathbb{E}}_1^{\dagger\dagger}[\eta] = \bar{s}_1^{\mathrm{D}}(1+\mathbb{E}[\eta]) - 1 & \text{if} & \mu_I^{\mathrm{D}} < & \mathbb{E}[\eta] & \leq \mu_U^{\mathrm{D}} \\ \hat{\mathbb{E}}_0^{\dagger\dagger}[\eta] = \frac{\bar{s}_0^{\mathrm{D}}}{D_0\bar{s}_0^{\mathrm{D}}+D_1\bar{s}_1^{\mathrm{D}}} - 1 & > & \hat{\mathbb{E}}_1^{\dagger\dagger}[\eta] = \frac{\bar{s}_1^{\mathrm{D}}}{D_0\bar{s}_0^{\mathrm{D}}+D_1\bar{s}_1^{\mathrm{D}}} - 1 & \text{if} & \mu_U^{\mathrm{D}} < & \mathbb{E}[\eta] & \end{cases} \tag{B.6}$$

where $\mu_L^{\mathrm{D}} := \frac{-\underline{c}_0^{\mathrm{D}}}{\underline{s}_0^{\mathrm{D}}}$, $\mu_I^{\mathrm{D}} := \frac{1}{\bar{s}_1^{\mathrm{D}}} - 1$, and $\mu_U^{\mathrm{D}} := \frac{1}{D_0\bar{s}_0^{\mathrm{D}}+D_1\bar{s}_1^{\mathrm{D}}} - 1$.

Now we impose $B_1\left(1+\hat{E}_1\right) \leq D_0\left(1+\hat{E}_0\right)+D_1\left(1+\hat{E}_1\right)$. Using simple algebra, we see that $D_0 \leq B_1 - D_1$, so we observe the following:

- The constraint binds for $E < \mu_L^{\mathrm{D}}$, since for $\hat{E}_0^{\dagger\dagger} = \hat{E}_1^{\dagger\dagger} = 0$ it is violated.

- If the constraint stops binding for some $\mu_{L'}^{\mathrm{D}} \in \left(\mu_L^{\mathrm{D}}, \mu_I^{\mathrm{D}}\right]$, then it binds $\forall\ E \le \mu_{L'}^{\mathrm{D}}$ and does not bind for any $E > \mu_{L'}^{\mathrm{D}}$, because i) in $\left(\mu_L^{\mathrm{D}}, \mu_I^{\mathrm{D}}\right]$, $\hat{E}_0^{\dagger\dagger}$ is increasing in E and $\hat{E}_1^{\dagger\dagger}$ is constant, so the constraint is relaxed as E increases; and ii) substituting from Equation B.6, the constraint becomes $(B_1 - D_1)\,\overline{s}_1^{\mathrm{D}} \le D_0 \overline{s}_0^{\mathrm{D}}$ for $E > \mu_I^{\mathrm{D}}$, i.e., it does not depend on E, so if it does not bind at μ_I^{D}, it does not bind above it.

- If the constraint does not stop binding in $\left(\mu_L^{\mathrm{D}}, \mu_I^{\mathrm{D}}\right]$, then it binds for all values of E, because by our argument above, if it binds at μ_I^{D}, it binds above it.

So there are two possibilities: the constraint binds for all E, or it only binds up to $\mu_{L'}^{\mathrm{D}} \in \left[\mu_L^{\mathrm{D}}, \mu_I^{\mathrm{D}}\right]$. So we check if it is satisfied for the values of $\hat{E}_0^{\dagger\dagger}$, $\hat{E}_1^{\dagger\dagger}$ for $E > \mu_U^{\mathrm{D}}$ (see Equation B.6); the constraint becomes $(B_1 - D_1)\,\overline{s}_1^{\mathrm{D}} \le D_0 \overline{s}_0^{\mathrm{D}}$, which is equivalent to $0 \le \beta^3 - \phi^3 + \beta\phi^2$, which is equivalent to $\beta \ge \overline{\beta}_L(\phi)$ for $\overline{\beta}_L(\phi) := \frac{2}{3}\sqrt[3]{\sqrt{\frac{31}{108}} + \frac{1}{2}}\phi \approx 0.68233\phi$. Next, we examine these two cases.

<u>$\boldsymbol{\beta \ge \overline{\beta}_L(\phi)}$</u>

In this case, the constraint $(B_1 - D_1)\left(1 + \hat{E}_1\right) \le D_0\left(1 + \hat{E}_0\right)$ binds only up to a $\mu_{L'}^{\mathrm{D}} \in \left[\mu_L^{\mathrm{D}}, \mu_I^{\mathrm{D}}\right]$. Since $\hat{E}_1^{\dagger\dagger} = 0$ for $E \le \mu_I^{\mathrm{D}}$, using it in the constraint, we have $\hat{E}_0^{\mathrm{D}} = \frac{B_1 - D_1 - D_0}{D_0}$ and $\hat{E}_1^{\mathrm{D}} = 0$ for $E \le \mu_{L'}^{\mathrm{D}}$. For $E > \mu_{L'}^{\mathrm{D}}$ optimal beliefs are as in Equation B.6. Note that $\mu_{L'}^{\mathrm{D}}$ is the E such that the constraint binds with optimal beliefs for the range $\mu_L^{\mathrm{D}} < E \le \mu_I^{\mathrm{D}}$ (see Equation B.6); we find $\mu_{L'}^{\mathrm{D}} := \frac{1}{\underline{s}_0^{\mathrm{D}}}\left(\frac{B_1 - D_1 - D_0}{D_0} - \underline{c}_0^{\mathrm{D}}\right)$. Before we compare the well-beings with and without a binding deadline, we determine the ordering of the cutoffs; we already know $\mu_L^{\mathrm{ND}} < \mu_U^{\mathrm{ND}}$ and $\mu_{L'}^{\mathrm{D}} < \mu_I^{\mathrm{D}} < \mu_U^{\mathrm{D}}$.

Determine ordering of E cutoffs.

- We have $\mu_U^{\mathrm{ND}} - \mu_U^{\mathrm{D}} = -\frac{\phi}{\beta^2}\frac{\phi^5 + 3\beta\phi^4 + 5\beta^2\phi^3 + 2\beta^3\phi^2 + \beta^4\phi - \beta^5}{(\beta+\phi)^2\left(\beta^4 + 2\beta^2\phi^2 + \beta\phi^3 + \phi^4\right)}$. Let the numerator be C_1; then $\frac{5}{\beta}C_1 - \frac{\mathrm{d}C_1}{\mathrm{d}\beta} > 0$. So $C_1 < 0 \Rightarrow \frac{\mathrm{d}C_1}{\mathrm{d}\beta} < 0$, and so $C_1 > 0, \forall\beta < \overline{\beta}_U(\phi)$, and negative otherwise. $\overline{\beta}_U(\phi)$ is a multiple of ϕ, and solving numerically we find $\overline{\beta}_U(\phi) \approx 2.6491\phi$.

- We have $\mu_L^{\mathrm{ND}} - \mu_{L'}^{\mathrm{D}} = -\frac{\phi}{\beta^2}\frac{\phi^6 + (1+\beta)\phi^5 + (2+3\beta)\beta\phi^4 + 2(2+\beta)\beta^2\phi^3 + \beta^3\phi^2 + \beta^4\phi - \beta^5 - \beta^6}{\left(\beta^2+\phi^2\right)^2(\beta+\phi)(1+\beta+\phi)}$. Let the numerator be C_2; then $\frac{5}{\beta}C_2 - \frac{\mathrm{d}C_2}{\mathrm{d}\beta} > 0$. So $C_2 < 0 \Rightarrow \frac{\mathrm{d}C_2}{\mathrm{d}\beta} < 0$, and so $C_2 > 0, \forall\beta < \overline{\beta}(\phi)$, and negative otherwise. We find numerically that $C_2\left(\beta = \overline{\beta}_L(\phi)\right) > 0$, $C_2\left(\beta = \overline{\beta}_U(\phi)\right) <$

0, so $\overline{\beta}_L(\phi) < \overline{\beta}(\phi) < \overline{\beta}_U(\phi)$. In addition, implicitly differentiating C_2 and using $C_2\left(\beta = \overline{\beta}(\phi)\right) = 0$, we can show $\overline{\beta}(\phi)$ is increasing in ϕ.

- Algebra shows that $\mu_L^{\text{ND}} < \mu_I^{\text{D}}$.

- We have $\mu_U^{\text{ND}} - \mu_I^{\text{D}} = \frac{\phi}{\beta}\frac{\beta(1+\phi)(\beta^3+\phi^3+2\beta\phi^2)+\phi^2(3\beta^3+\beta^2\phi+\beta+\phi+\phi^2)}{(\beta+\phi)^2(\beta^4+\phi^3+\phi^4+2\beta^2\phi^2+\beta\phi^2+\beta\phi^3)} > 0$.

Thus, we have shown that:

$$\begin{cases} \mu_L^{\text{ND}} < \mu_{L'}^{\text{D}} < \mu_I^{\text{D}} < \mu_U^{\text{ND}} < \mu_U^{\text{D}} & \text{if } \overline{\beta}_L(\phi) < \beta \le \overline{\beta}(\phi) \\ \mu_{L'}^{\text{D}} < \mu_L^{\text{ND}} < \mu_I^{\text{D}} < \mu_U^{\text{ND}} < \mu_U^{\text{D}} & \text{if } \overline{\beta}(\phi) < \beta \le \overline{\beta}_U(\phi) \\ \mu_{L'}^{\text{D}} < \mu_L^{\text{ND}} < \mu_I^{\text{D}} < \mu_U^{\text{D}} < \mu_U^{\text{ND}} & \text{if } \overline{\beta}_U(\phi) < \beta \end{cases}$$

Compare the well-beings. The difference in $\mathcal{W}$ with and without a deadline is

$$\begin{aligned} \mathcal{W}^{\text{D}}-\mathcal{W}^{\text{ND}} = \;& \tfrac{1}{2}F\left(B_0\left(1+\hat{E}_0^{\text{ND}}\right)+B_1\left(1+\hat{E}_1^{\text{ND}}\right)\right)^2-\tfrac{1}{2}F\left(D_0\left(1+\hat{E}_0^{\text{D}}\right)+D_1\left(1+\hat{E}_1^{\text{D}}\right)\right)^2 \\ & +\tfrac{1}{2}\left(\phi^2\left(1+\hat{E}_0^{\text{ND}}\right)^2+\beta\phi\left(1+\hat{E}_1^{\text{ND}}\right)^2\right)-\tfrac{1}{2}\left(\phi^2\left(1+\hat{E}_0^{\text{D}}\right)^2+\beta\phi\left(1+\hat{E}_1^{\text{D}}\right)^2\right) \\ & +\left(\phi^2\left(1+\hat{E}_0^{\text{D}}\right)+\beta\phi\left(1+\hat{E}_1^{\text{D}}\right)+\beta^2(1+E)\right)\left(D_0\left(1+\hat{E}_0^{\text{D}}\right)+D_1\left(1+\hat{E}_1^{\text{D}}\right)\right) \\ & -\left(\phi^2\left(1+\hat{E}_0^{\text{ND}}\right)+\beta\phi\left(1+\hat{E}_1^{\text{ND}}\right)+\beta^2(1+E)\right)\left(B_0\left(1+\hat{E}_0^{\text{ND}}\right)+B_1\left(1+\hat{E}_1^{\text{ND}}\right)\right). \end{aligned} \tag{B.7}$$

Before considering the three sub-cases, $\beta \in \left(\overline{\beta}_L(\phi), \overline{\beta}(\phi)\right]$, $\beta \in \left(\overline{\beta}(\phi), \overline{\beta}_U(\phi)\right]$, and $\beta > \overline{\beta}_U(\phi)$ separately, we make a few general observations about $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$:

- Optimal beliefs are piece-wise linear in E and $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ is a quadratic in beliefs, so $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ is a differentiable piecewise quadratic in E.

- For $E \le \min\left\{\mu_{L'}^{\text{D}}, \mu_L^{\text{ND}}\right\}$, where $\hat{E}_t^{\text{ND}} = \hat{E}_t^{\text{D}} = 0$, algebra shows $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}} = -\frac{\beta\phi^3(2\beta^2+\beta\phi+2\phi^2)}{2\left[(\beta^2+\phi^2)(1+\beta+\phi)\right]^2} < 0$.

- For $E > \max\left\{\mu_U^{\text{ND}}, \mu_U^{\text{D}}\right\}$, where $w_1 \le 1$ binds in both cases, algebra shows $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}} = -\frac{\phi}{2}\frac{\phi^5+3\beta\phi^4+5\beta^2\phi^3+2\beta^3\phi^2+\beta^4\phi-\beta^5}{(\beta+\phi)^2(\beta^4+2\beta^2\phi^2+\beta\phi^3+\phi^4)} = \mu_U^{\text{ND}} - \mu_U^{\text{D}}$, so i) if $\beta > \overline{\beta}_U(\phi)$, then $\mu_U^{\text{D}} < \mu_U^{\text{ND}}$ and $\mathcal{W}^{\text{ND}} < \mathcal{W}^{\text{D}}$ for $E > \mu_U^{\text{ND}}$, or ii) if $\beta \le \overline{\beta}_U(\phi)$, then $\mu_U^{\text{ND}} \le \mu_U^{\text{D}}$ and $\mathcal{W}^{\text{D}} \le \mathcal{W}^{\text{ND}}$ for $E > \mu_U^{\text{D}}$.

We now consider the three sub-cases in detail:

1. For $\overline{\beta}_L(\phi) < \beta \leq \overline{\beta}(\phi)$, we have $\mu_L^{\text{ND}} < \mu_{L'}^{\text{D}} < \mu_I^{\text{D}} < \mu_U^{\text{ND}} < \mu_U^{\text{D}}$. Using beliefs for $E = \mu_U^{\text{ND}}$ from Equations A.1, B.6 and $E = \mu_U^{\text{ND}}$ in $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$:

$$\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}} \propto -\left(\phi^5 + 3\beta\phi^4 + 5\beta^2\phi^3 + 2\beta^3\phi^2 + \beta^4\phi - \beta^5\right),$$

which is negative for $\beta < \overline{\beta}_U(\phi)$; this shows $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$ for $E = \mu_U^{\text{ND}}$. We already know that $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$ for $E > \mu_U^{\text{D}}$. Also $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ is decreasing in $\left[\mu_U^{\text{ND}}, \mu_U^{\text{D}}\right]$, since the component in $\mathcal{W}$ that does not depend on beliefs is common to $\mathcal{W}^{\text{D}}$ and $\mathcal{W}^{\text{ND}}$ so it drops out, and the component that depends on beliefs is constant for $\mathcal{W}^{\text{ND}}$ because beliefs have hit the bound $w_1 \leq 1$, but decreasing for $\mathcal{W}^{\text{D}}$. So we conclude that $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$ in the whole range $\left[\mu_U^{\text{ND}}, \mu_U^{\text{D}}\right]$. Finally, $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}} < 0$ up to μ_L^{ND}, and concave in $\left[\mu_L^{\text{ND}}, \mu_{L'}^{\text{D}}\right]$ since the optimal beliefs that implement the non-binding deadline become less constrained. Then, we can conclude that $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$ for all E, since the only two possibilities are i) $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ is concave in $\left[\mu_{L'}^{\text{D}}, \mu_I^{\text{D}}\right]$, but then even if $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ is convex in $\left[\mu_I^{\text{D}}, \mu_U^{\text{ND}}\right]$, it can only have one root in this range, which would imply that $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}} > 0$ at μ_U^{ND}, and we have shown this is not true; and ii) $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ is convex in $\left[\mu_{L'}^{\text{D}}, \mu_I^{\text{D}}\right]$, so it is also convex in $\left[\mu_I^{\text{D}}, \mu_U^{\text{ND}}\right]$ since the optimal beliefs that implement the binding deadline become less constrained, which implies that it can only have one root (in one or the other range), which would imply that $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}} > 0$ at μ_U^{ND}, and we have shown this is not true. Thus, for $\overline{\beta}_L(\phi) < \beta \leq \overline{\beta}(\phi)$, a binding deadline is never chosen.

2. For $\overline{\beta}(\phi) < \beta \leq \overline{\beta}_U(\phi)$, we have $\mu_{L'}^{\text{D}} < \mu_L^{\text{ND}} < \mu_I^{\text{D}} < \mu_U^{\text{ND}} < \mu_U^{\text{D}}$. We know $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}} < 0$ up to $\mu_{L'}^{\text{D}}$ and convex in $\left[\mu_{L'}^{\text{D}}, \mu_L^{\text{ND}}\right]$, since the optimal beliefs that implement the binding deadline become less constrained. Also, as in the previous sub-case we can show that $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$ in the whole range $\left[\mu_U^{\text{ND}}, \mu_U^{\text{D}}\right]$. So the possibilities are:

   - If $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ has a root in $\left[\mu_{L'}^{\text{D}}, \mu_L^{\text{ND}}\right]$, it also has one in $\left[\mu_L^{\text{ND}}, \mu_U^{\text{ND}}\right]$, since $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$ at μ_U^{ND}.
   - $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ has one root in $\left[\mu_L^{\text{ND}}, \mu_I^{\text{D}}\right]$ and one in $\left[\mu_I^{\text{D}}, \mu_U^{\text{ND}}\right]$ or two roots in $\left[\mu_L^{\text{ND}}, \mu_I^{\text{D}}\right]$, since $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$ at μ_U^{ND}.
   - $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ has no roots, and so $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$ for all E.

3. For $\overline{\beta}_U(\phi) < \beta$, we have $\mu_{L'}^{\text{D}} < \mu_L^{\text{ND}} < \mu_I^{\text{D}} < \mu_U^{\text{D}} < \mu_U^{\text{ND}}$. Using beliefs for $E = \mu_U^{\text{D}}$ from

Equations A.1, B.6 and $E = \mu_U^{\mathrm{D}}$ in $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}}$:

$$\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}} \propto -\left(\phi^5 + 3\beta\phi^4 + 5\beta^2\phi^3 + 2\beta^3\phi^2 + \beta^4\phi - \beta^5\right),$$

which is positive for $\beta > \overline{\beta}_U(\phi)$. An analogous argument to the one used in sub-case 1 shows that $\mathcal{W}^{\mathrm{D}} > \mathcal{W}^{\mathrm{ND}}$ in the whole range $\left[\mu_U^{\mathrm{D}}, \mu_U^{\mathrm{ND}}\right]$. Also $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}} < 0$ up to $\mu_{L'}^{\mathrm{D}}$, and convex in $\left[\mu_{L'}^{\mathrm{D}}, \mu_L^{\mathrm{ND}}\right]$ since the optimal beliefs that implement the binding-deadline become less constrained. Thus, $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}}$ has an odd number of roots. But since it is piece-wise quadratic, it can only have up to one root in $\left[\mu_{L'}^{\mathrm{D}}, \mu_L^{\mathrm{ND}}\right]$ and up to two roots in each of $\left[\mu_L^{\mathrm{ND}}, \mu_I^{\mathrm{D}}\right]$ and $\left[\mu_I^{\mathrm{D}}, \mu_U^{\mathrm{D}}\right]$, for a total of up to five roots; but if it has two roots in either of the latter two ranges, it cannot have two roots in the other. So we conclude that $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}}$ has either one or three roots. Plotting all the possible cases in which there are three roots, we see that the common characteristic in all these cases is that $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}}$ has a minimum in $\left[\mu_I^{\mathrm{D}}, \mu_U^{\mathrm{D}}\right]$ and this minimum is negative. Plugging the optimal beliefs for this range in $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}}$, we find that $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}}$ is a quadratic with extreme value $\mathcal{W}^{\mathrm{D}} - \mathcal{W}^{\mathrm{ND}} = \frac{1}{2}\frac{\phi^2(\beta^3+\beta^4+\phi^3+\beta^2\phi^2+\beta\phi+3\beta\phi^2+3\beta^2\phi+2\beta^3\phi)}{\beta^3+\beta^4+\phi^3+\phi^4+2\beta^2\phi^2+\beta\phi+3\beta\phi^2+3\beta^2\phi+2\beta\phi^3+2\beta^3\phi} > 0$. So we conclude $\mathcal{W}^{\mathrm{D}} < \mathcal{W}^{\mathrm{ND}}$ for $E < \underline{\mu} \in \left[\mu_{L'}^{\mathrm{D}}, \mu_U^{\mathrm{D}}\right]$ and $\mathcal{W}^{\mathrm{D}} \geq \mathcal{W}^{\mathrm{ND}}$ otherwise.

In conclusion, for $\beta \geq \overline{\beta}(\phi)$ we have $\mathcal{W}^{\mathrm{D}} \geq \mathcal{W}^{\mathrm{ND}}$ for $E \in M(\beta, \phi)$ where $M(\beta, \phi)$ is a (possibly empty) convex set, and $\mathcal{W}^{\mathrm{D}} < \mathcal{W}^{\mathrm{ND}}$ for all other values. Note that both in this case ($\beta \geq \overline{\beta}_L(\phi)$) and in the case below ($\beta < \overline{\beta}_L(\phi)$), whenever $\mathcal{W}^{\mathrm{D}} < \mathcal{W}^{\mathrm{ND}}$ the optimal expectations are $\hat{\mathbb{E}}_t^{\mathrm{ND}}$ from Equation A.1, as long as they satisfy $D_0\left(1 + \hat{E}_0^{\mathrm{ND}}\right) + D_1\left(1 + \hat{E}_1^{\mathrm{ND}}\right) \leq B_1\left(1 + \hat{E}_1^{\mathrm{ND}}\right)$, because otherwise a deadline would be chosen at $t = 0$, which is suboptimal. Since $\hat{E}_0^{\mathrm{ND}} = 0$ and $B_1 > D_0 + D_1$, this condition is trivially satisfied.

$\beta < \overline{\beta}_L(\phi)$

Here, the constraint $(B_1 - D_1)\left(1 + \hat{E}_1\right) \leq D_0\left(1 + \hat{E}_0\right)$ binds for all E. So optimal beliefs implementing the binding deadline must always be proportional, so they are both constants or both proportional to E. Given that we have the constraints $\hat{E}_0 \geq 0$, $\hat{E}_1 \geq 0$ and $w_1 \leq 1$, we conclude that there are values of E, $\mu_{I'}^{\mathrm{D}}$ and $\mu_{U'}^{\mathrm{D}}$ to be defined below, that partition the E space in regions: for $E \leq \mu_{I'}^{\mathrm{D}}$ optimal beliefs do not depend on E because $\hat{E}_0 \geq 0$ and $\hat{E}_1 \geq 0$ bind, for $\mu_{I'}^{\mathrm{D}} < E \leq \mu_{U'}^{\mathrm{D}}$ optimal beliefs are proportional to E, and for $\mu_{U'}^{\mathrm{D}} < E$ optimal beliefs do not depend on E because $w_1 \leq 1$ binds. We now determine $\mu_{I'}^{\mathrm{D}}$ and $\mu_{U'}^{\mathrm{D}}$, and the optimal beliefs in the various ranges of the

E space.

Equations B.4 and B.5 give the F.O.C. of $\mathcal{W}$ w.r.t. $\hat{E}_0$ and $\hat{E}_1$ when ignoring the constraints $\hat{E}_0 \geq 0$, $\hat{E}_1 \geq 0$ and $w_1 \leq 1$. Forming the Lagrangian, $\mathcal{L}$, to account for $(B_1 - D_1)\left(1 + \hat{E}_1\right) \leq D_0\left(1 + \hat{E}_0\right)$, but still ignoring the other constraints, the F.O.C. are:

$$
\begin{aligned}
\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\hat{E}_0} &= -M_{D_0}\left(1 + \hat{E}_0\right) - G_D\left(1 + \hat{E}_1\right) + \beta^2 D_0 (1 + E) + \lambda D_0 \\
\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}\hat{E}_1} &= -G_D\left(1 + \hat{E}_0\right) - M_{D_1}\left(1 + \hat{E}_1\right) + \beta^2 D_1 (1 + E) - \lambda (B_1 - D_1).
\end{aligned}
$$

Combining these F.O.C. with the constraint, we get

$$
\hat{E}_0^{\mathrm{D}} = \frac{\frac{B_1 - D_1}{D_0} D_0 + D_1}{2G_D + \frac{B_1 - D_1}{D_0} M_{D_0} + \frac{D_0}{B_1 - D_1} M_{D_1}} \beta^2 (1 + E) - 1 \tag{B.8}
$$

$$
\hat{E}_1^{\mathrm{D}} = \frac{D_0 + \frac{D_0}{B_1 - D_1} D_1}{2G_D + \frac{B_1 - D_1}{D_0} M_{D_0} + \frac{D_0}{B_1 - D_1} M_{D_1}} \beta^2 (1 + E) - 1. \tag{B.9}
$$

These are the optimal expectations in $\mu_{I'}^{\mathrm{D}} < E \leq \mu_{U'}^{\mathrm{D}}$, where no other constraints bind.

Clearly $\hat{E}_0^{\mathrm{D}} > \hat{E}_1^{\mathrm{D}}$, so $\hat{E}_1 \geq 0$ binds first. Setting $\hat{E}_1^{\mathrm{D}} = 0$ in Equation B.9, we have $\mu_{I'}^{\mathrm{D}} := \frac{1}{\beta^2} \frac{2G_D + \frac{B_1 - D_1}{D_0} M_{D_0} + \frac{D_0}{B_1 - D_1} M_{D_1}}{D_0 + \frac{D_0}{B_1 - D_1} D_1} - 1$. So for $E \leq \mu_{I'}^{\mathrm{D}}$, we have $\hat{E}_1^{\mathrm{D}} = 0$, which substituted in $(B_1 - D_1)\left(1 + \hat{E}_1\right) \leq D_0\left(1 + \hat{E}_0\right)$ yields $\hat{E}_0^{\mathrm{D}} = \frac{B_1 - D_1 - D_0}{D_0}$.

Using interior optimal beliefs from Equations B.8 and B.9 in $w_1^*\left(\left\{\hat{\mathbb{E}}_t\right\}, \psi^*\left(\left\{\hat{\mathbb{E}}_t\right\}\right)\right) = D_0\left(1 + \hat{E}_0\right) + D_1\left(1 + \hat{E}_1\right)$, i.e., the optimal work given the optimal binding deadline, and setting it to 1, we find $\mu_{U'}^{\mathrm{D}} := \frac{1}{\beta^2} \frac{2G_D + \frac{B_1 - D_1}{D_0} M_{D_0} + \frac{D_0}{B_1 - D_1} M_{D_1}}{2D_0 D_1 + \frac{B_1 - D_1}{D_0} D_0^2 + \frac{D_0}{B_1 - D_1} D_1^2} - 1$. Using $E = \mu_{U'}^{\mathrm{D}}$ in these beliefs, we get $\hat{E}_0^{\mathrm{D}} = \frac{1}{B_1} \frac{B_1 - D_1}{D_0} - 1$, $\hat{E}_1^{\mathrm{D}} = \frac{1}{B_1} - 1$ for $E > \mu_{U'}^{\mathrm{D}}$.

Determine ordering of E cutoffs We know $\mu_L^{\mathrm{ND}} < \mu_U^{\mathrm{ND}}$, $\mu_{I'}^{\mathrm{D}} < \mu_{U'}^{\mathrm{D}}$. Also:

- $\mu_L^{\mathrm{ND}} - \mu_{I'}^{\mathrm{D}} = -\frac{\phi^2}{\beta^2} \frac{\beta^5 + \beta^4\phi + \beta^4 + 2\beta^3\phi^2 + 2\beta^3\phi + 2\beta^2\phi^3 + 3\beta^2\phi^2 + \beta\phi^4 + 2\beta\phi^3 + \phi^5 + \phi^4}{\left(\beta^2 + \phi^2\right)^2 (\beta + \phi)(\beta + \phi + 1)} < 0$.

- Algebra shows that $\mu_U^{\mathrm{ND}} > \mu_{I'}^{\mathrm{D}}$.

- $\mu_U^{\mathrm{ND}} - \mu_{U'}^{\mathrm{D}} = -\frac{\phi^2}{\beta^2} \frac{\left(\beta^2 + \beta\phi + \phi^2\right)^2}{\left(\beta^3 + \beta^2\phi + \beta\phi^2 + \phi^3\right)^2} < 0$.

Thus, $\mu_L^{\text{ND}} < \mu_{I'}^{\text{D}} < \mu_U^{\text{ND}} < \mu_{U'}^{\text{D}}$.

Compare the well-beings $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ is as given in Equation B.7, so as already argued, is piecewise quadratic and continuously differentiable. We show that $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$ everywhere (so a deadline is never optimal and optimal expectations are $\hat{\mathbb{E}}_t^{\text{ND}}$):

1. If $E \leq \mu_L^{\text{ND}}$, optimal expectations are $\hat{E}_0^{\text{ND}} = \hat{E}_1^{\text{ND}} = 0$ and $\hat{E}_0^{\text{D}} = \frac{B_1 - D_1 - D_0}{D_0} > \hat{E}_1^{\text{D}} = 0$. Plugging these in, we find $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}} = -\frac{1}{2}\frac{\beta\phi^3\left(2\beta^2+\beta\phi+2\phi^2\right)}{\left(\beta^2+\phi^2\right)^2(\beta+\phi+1)^2} < 0$.

2. If $\mu_L^{\text{ND}} < E \leq \mu_{I'}^{\text{D}}$, the optimal beliefs without a deadline become less constrained, so $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ is concave (so $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$) in this range.

3. If $\mu_{U'}^{\text{D}} < E$ optimal expectations are $\hat{E}_0^{\text{ND}} = \frac{1}{B_1} - 1 < \frac{1}{B_1}\frac{B_1 - D_1}{D_0} - 1 = \hat{E}_0^{\text{D}}$ and $\hat{E}_1^{\text{ND}} = 0 < \frac{1}{B_1} - 1 = \hat{E}_1^{\text{D}}$, and optimal work are $w_1^{\text{ND}} = w_1^{\text{D}} = 1$ since $w_1 \leq 1$ binds. So actions are the same but the binding-deadline case has more pessimistic beliefs, hence $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$.

4. Given that $\mathcal{W}^{\text{D}} < \mathcal{W}^{\text{ND}}$ for $E \notin \left[\mu_{I'}^{\text{D}}, \mu_{U'}^{\text{D}}\right]$ and $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ is concave for $\mu_L^{\text{ND}} < E \leq \mu_{I'}^{\text{D}}$, $\mathcal{W}^{\text{D}} > \mathcal{W}^{\text{ND}}$ anywhere in $\left[\mu_{I'}^{\text{D}}, \mu_{U'}^{\text{D}}\right]$ requires $\mathcal{W}^{\text{D}} - \mathcal{W}^{\text{ND}}$ convex in $\left[\mu_{I'}^{\text{D}}, \mu_U^{\text{ND}}\right]$, but this necessitates that it is decreasing and concave in the left neighborhood of $\mu_{U'}^{\text{D}}$, which contradicts differentiability at $\mu_{U'}^{\text{D}}$.

B.2.2 Proof of Proposition 5

Finding the optimal beliefs We work as in Sections B.2.1.1 and B.2.1.2, to find $w_1^*\left(\left\{\hat{\mathbb{E}}_t\right\}, \psi\right) = \min\left\{1, \max\left\{\psi, B_1\left(1+\hat{E}_1\right)\right\}\right\}$ and

$$\begin{cases} \psi^{*,\text{ED}}\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = B_1\left(1+\hat{E}_1\right) & \text{if } D_0(1+E)+D_1\left(1+\hat{E}_1\right) \leq B_1\left(1+\hat{E}_1\right) \leq 1 \\ \psi^{*,\text{ED}}\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = D_0(1+E)+D_1\left(1+\hat{\mathbb{E}}_1[\eta]\right) & \text{if } B_1\left(1+\hat{E}_1\right) \leq D_0(1+E)+D_1\left(1+\hat{E}_1\right) \leq 1 \\ \psi^{*,\text{ED}}\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = 1 & \text{otherwise,} \end{cases}$$

where $\psi^{*,\text{ED}}\left(\left\{\hat{\mathbb{E}}_t\right\}\right)$ is the optimal externally-imposed deadline, given expectations $\hat{\mathbb{E}}_t$.

We are interested in the optimal beliefs that implement a binding deadline, i.e., the case $B_1\left(1+\hat{E}_1\right) \le D_0(1+E)+D_1\left(1+\hat{E}_1\right) \le 1$. Working as in Section A.4, we find

$$\begin{aligned}\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_0} &= -\phi^2\left\{\left(1+\hat{E}_0\right)-\left[D_0\left(1+E\right)+D_1\left(1+\hat{E}_1\right)\right]\right\}\\ \frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_1} &= -\beta\phi\left(1-D_1\right)\left(1+\hat{E}_1\right)+\beta^2 D_1\left(1+E\right)+\phi^2 D_1\left(1+\hat{E}_0\right).\end{aligned}$$

Imposing $\hat{E}_0 \ge 0$, we have $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_0} < 0$, so $\hat{E}_0^{\text{ED}} = 0$, and so "interior" optimal $\hat{E}_1$ is $\hat{E}_1^{\dagger} = \frac{\beta^2(1+E)+\phi^2}{M_{D_1}} D_1 - 1$. Now impose all constraints:

- We check if $B_1\left(1+\hat{E}_1\right) \le D_0\left(1+E\right)+D_1\left(1+\hat{E}_1\right)$ binds for some E with interior beliefs. Substituting $\hat{E}_0^{\text{ED}}$ and $\hat{E}_1^{\dagger}$ in the constraint, we can write it as $\frac{(B_1-D_1)\frac{\phi^2 D_1}{M_{D_1}}}{D_0-(B_1-D_1)\frac{\beta^2 D_1}{M_{D_1}}} \le 1+E$. Algebra shows the denominator less the numerator of the LHS is positive, so since $E \ge 0$, the constraint does not bind.

- We check if $\hat{E}_1 \ge 0$ binds for any E. Setting $\hat{E}_1^{\dagger} = \frac{\beta^2(1+E)+\phi^2}{M_{D_1}} D_1 - 1 = 0$, we see the constraint binds for E below $\mu_I^{\text{ED}} := \frac{1}{\beta^2}\left(\frac{M_{D_1}}{D_1}-\phi^2\right)-1 = \frac{\beta+\phi}{\beta^2} > 0$.

- We check if $B_1\left(1+\hat{E}_1\right) \le D_0\left(1+E\right)+D_1\left(1+\hat{E}_1\right)$ binds for some $E < \mu_I^{\text{ED}}$. We substitute $\hat{E}_0^{\text{ED}} = \hat{E}_1^{\text{ED}} = 0$ into the constraint, to find that it is satisfied with equality at $\mu_L^{\text{ED}} := \frac{B_1-D_1-D_0}{D_0} = \frac{\beta\phi}{\left(\beta^2+\phi^2\right)(1+\beta+\phi)}$. Algebra verifies $\mu_L^{\text{ED}} \le \mu_I^{\text{ED}}$. But it turns out the constraint cannot be satisfied as E drops below μ_L^{ED} because: i) since $\hat{E}_0^{\text{ED}} = \hat{E}_1^{\text{ED}} = 0$, we cannot reduce $\hat{E}_0^{\text{ED}}$ or $\hat{E}_1^{\text{ED}}$; ii) since $B_1 \ge D_1$, raising $\hat{E}_1^{\text{ED}}$ does not help; and iii) raising $\hat{E}_0^{\text{ED}}$ does not help. Thus, no beliefs implement the externally-imposed binding deadline for $E \le \mu_L^{\text{ED}}$.

- We check if $D_0\left(1+E\right)+D_1\left(1+\hat{E}_1\right) \le 1$ binds. Using interior optimal beliefs in it, we find that it binds for $E > \mu_U^{\text{ED}} := \frac{M_{D_1}-\phi^2 D_1^2}{D_0 M_{D_1}+\beta^2 D_1^2} - 1$. Using $E = \mu_U^{\text{ED}}$ in the interior beliefs, we get $\hat{E}_0^{\text{ED}} = 0 < \frac{\beta^2+\phi^2 D_0}{D_0 M_{D_1}+\beta^2 D_1^2} D_1 - 1 = \hat{E}_1^{\text{ED}}$ for $E > \mu_U^{\text{ED}}$.

So optimal beliefs implementing the externally-imposed deadline satisfy $\hat{E}_0^{\text{ED}} = 0$,

$$\begin{cases} \hat{\mathbb{E}}_1^{\text{ED}}\left[\eta\right] = 0 & \text{if} \quad \mu_L^{\text{ED}} \le \mathbb{E}\left[\eta\right] \le \mu_I^{\text{ED}} \\ \hat{\mathbb{E}}_1^{\text{ED}}\left[\eta\right] = \frac{\beta^2(1+\mathbb{E}[\eta])+\phi^2}{M_{D_1}} D_1 - 1 & \text{if} \quad \mu_I^{\text{ED}} < \mathbb{E}\left[\eta\right] \le \mu_U^{\text{ED}} \\ \hat{\mathbb{E}}_1^{\text{ED}}\left[\eta\right] = \frac{\beta^2+\phi^2 D_0}{D_0 M_{D_1}+\beta^2 D_1^2} D_1 - 1 & \text{if} \quad \mu_U^{\text{ED}} < \mathbb{E}\left[\eta\right] \end{cases}$$

where $\mu_L^{\text{ED}} := \frac{B_1 - D_1 - D_0}{D_0}$, $\mu_I^{\text{ED}} := \frac{1}{\beta^2}\left(\frac{M_{D_1}}{D_1} - \phi^2\right) - 1$, $\mu_U^{\text{ED}} := \frac{M_{D_1} - \phi^2 D_1^2}{D_0 M_{D_1} + \beta^2 D_1^2} - 1$. For $E < \mu_L^{\text{ED}}$, a binding deadline cannot be implemented and optimal beliefs satisfy $\hat{E}_0^{\text{ED}} = \hat{E}_1^{\text{ED}} = 0$.

We have shown $\hat{E}_0^{\text{ED}} = 0 \leq \hat{E}_1^{\text{ED}}$, i.e., beliefs become more pessimistic over time. So to prove optimism, we just need to show $\hat{E}_1^{\text{ED}} \leq E$. For $E \leq \mu_I^{\text{ED}}$, we have $\hat{E}_1^{\text{ED}} = 0 < E$. For $\mu_U^{\text{ED}} < E$, we have $\hat{E}_1^{\text{ED}} = \frac{\beta^2 + \phi^2 D_0}{D_0 M_{D_1} + \beta^2 D_1^2} D_1 - 1 < \mu_U^{\text{ED}}$. For $\mu_I^{\text{ED}} < E \leq \mu_U^{\text{ED}}$, as a function of E, $\hat{E}_1^{\text{ED}}$ is a straight line segment whose endpoints lie below the line E, so $\hat{E}_1^{\text{ED}} < E$.

Having determined optimal expectations $\hat{\mathbb{E}}_t^{\text{ED}}$, we define $\psi^{\text{ED}} := \psi^{*,\text{ED}}\left(\left\{\hat{\mathbb{E}}_t^{\text{ED}}\right\}\right)$.

Outsider's deadline is stricter than the agent's deadline To show this, we need to show $\psi^{\text{ED}} \geq \psi^{\text{D}}$, i.e., $D_0\left(E - \hat{E}_0^{\text{D}}\right) \geq D_1\left(\hat{E}_1^{\text{D}} - \hat{E}_1^{\text{ED}}\right)$. Straightforward algebra shows this is true for interior beliefs (so also for beliefs above the interior). Now we show it is true for all remaining beliefs for which a binding self-imposed deadline is optimal. From Section B.2.1, we know that $\hat{E}_1^{\text{D}} \geq 0$ binds first as E becomes smaller, then $D_0\left(1 + \hat{E}_0^{\text{D}}\right) \geq (B_1 - D_1)\left(1 + \hat{E}_1^{\text{D}}\right)$ binds, and finally $\hat{E}_0^{\text{D}} \geq 0$ binds. We also know that once either of the latter two constraints binds, the self-imposed deadline is not optimally chosen because it yields weakly lower well-being, so we just need to check what happens when $\hat{E}_1^{\text{D}} \geq 0$ binds. But we already know from Section B.2.1 that $\hat{E}_0^{\text{D}} \leq E$, hence $D_0\left(E - \hat{E}_0^{\text{D}}\right) \geq D_1\left(\hat{E}_1^{\text{D}} - \hat{E}_1^{\text{ED}}\right)$.

Outsider's deadline is smaller than w_1^{RE} We have shown that beliefs are optimistic; combined with $D_0 + D_1 < B_1$, this trivially proves $\psi^{\text{ED}} < w_1^{\text{RE}}$.

B.3 Additional proofs

B.3.1 Additional proofs for Section 1

Here, we provide a formal proof of the claim made in Section 1.3, that a cost of belief distortion modeled as a quadratic cost that is increasing in the (absolute) difference between objective and subjective expectations about the random variable in our model, η, leads to less optimistic, but still optimistic, beliefs. The assumption of quadratic payments is made for tractability.

In terms of notation, optimal quantities for the case with a belief distortion cost contain a "dc" in their superscripts.

Claim B.1. *(Costs of belief distortion)*
A cost $-\frac{1}{2}\left(\mathbb{E}_t[\eta]-\hat{\mathbb{E}}_t[\eta]\right)^2$ *at time t that is increasing in the absolute difference between objective and time t subjective expectations about the random variable* η, $\left|\mathbb{E}_t[\eta]-\hat{\mathbb{E}}_t[\eta]\right|$, *results in time t beliefs that are less optimistic, i.e.,* $\hat{\mathbb{E}}_0^{\mathrm{ND,dc}}[\eta] > \hat{\mathbb{E}}_0^{\mathrm{ND}}[\eta]$ *and* $\hat{\mathbb{E}}_1^{\mathrm{ND,dc}}[\eta] > \hat{\mathbb{E}}_1^{\mathrm{ND}}[\eta]$, *but still optimistic, i.e.,* $\hat{\mathbb{E}}_0^{\mathrm{ND,dc}}[\eta] < \mathbb{E}[\eta]$ *and* $\hat{\mathbb{E}}_1^{\mathrm{ND,dc}}[\eta] < \mathbb{E}[\eta]$.

Proof of Claim B.1 The only change from the setup in Section 1 is that now Equation 3 becomes

$$U_t := \hat{\mathbb{E}}_t\left[\sum_{\tau\geq t}\phi^{\tau-t}\left(u(w_\tau)-\frac{1}{2}\left(\mathbb{E}_\tau[\eta]-\hat{\mathbb{E}}_\tau[\eta]\right)^2\right)\right]. \tag{B.10}$$

We first work as in the proof of Proposition 1 in Section A.1 to find the optimal amount of work w_1 in period 1. The agent chooses w_1 at $t=1$ to maximize V_1. Using Equations 1 and B.10, V_1 becomes

$$\hat{\mathbb{E}}_1\left[u(w_1)+(\beta+\phi)\,u(1+\eta-w_1)-\frac{1}{2}\left(E-\hat{E}_1\right)^2\right],$$

which is concave in w_1. Using $u(w)=-\frac{1}{2}w^2$, the F.O.C. yields $w_1^\dagger\left(\left\{\hat{\mathbb{E}}_t\right\}\right)=B_1\left(1+\hat{E}_1\right)$, where $B_1=\frac{\beta+\phi}{1+\beta+\phi}$ as before, and imposing $w_1\leq 1$ yields $w_1^{*,\mathrm{dc}}=\min\left\{1, B_1\left(1+\hat{E}_1\right)\right\}$.

Now we work as in the proof of Proposition 2 in Section A.2. Repeating the argument there, we can show that optimal expectations satisfy $B_1\left(1+\hat{E}_1\right)\leq 1$, so we can substitute $w_1 = B_1\left(1+\hat{E}_1\right)$ and $w_2=1+\eta-w_1$ into $\mathcal{W}$. Using Equations 2 and B.10 to substitute for V and U, respectively, in $\mathcal{W}$, and doing some algebra, we can write

$$\begin{aligned}\mathcal{W} \;=\; & \mathbb{E}\left[(\beta+\phi)\,u(w_1)+\phi^2\hat{\mathbb{E}}_0\left[u(1+\eta-w_1)\right]+\beta\phi\hat{\mathbb{E}}_1\left[u(1+\eta-w_1)\right]+\beta^2u(1+\eta-w_1)\right]\\ & -\frac{1}{2}\left[\left(E-\hat{E}_0\right)^2+(\beta+\phi)\left(E-\hat{E}_1\right)^2\right].\end{aligned}$$

The first line of the RHS of this equation is simply the well-being in the absence of a cost of belief distortion. So $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_t}$ equals its counterpart in the case without a cost of belief distortion, plus a constant times $E-\hat{E}_t$, which is positive given that beliefs are optimistic. Thus, optimal beliefs with a cost of belief distortion are less optimistic than without a cost of belief distortion. It is obvious that optimal beliefs will still be optimistic, since $E-\hat{E}_t$ is positive only if beliefs are optimistic, while it would be negative if beliefs were pessimistic.

B.3.2 Additional proofs for Section 6

In this section, we provide formal proofs of two claims made in Section 6: first, that an incentive for the speed of task completion modeled as a payment at $t = 2$ that is quadratic and decreasing in total work, $w_1 + w_2$, leads to more optimistic beliefs; and second, that an incentive for the accuracy of task duration prediction modeled as a payment at $t = 2$ that is quadratic and decreasing in the (absolute) difference between objective and subjective expectations about task duration, $1 + \eta$, leads to less optimistic beliefs. The assumption of quadratic payments is made for tractability.

In terms of notation, optimal quantities for the case with an incentive for the speed of task completion contain an "s" in their superscripts, while optimal quantities for the case with an incentive for the accuracy of task duration prediction contain an "a" in their superscripts.

Claim B.2. *(Incentive for speed of task completion)*
A payment $-\frac{1}{2}(w_1 + w_2)^2$ *at* $t = 2$ *that is decreasing in total work,* $w_1 + w_2$*, makes beliefs (weakly) more optimistic, i.e.,* $\hat{\mathbb{E}}_0^{\text{ND,s}}[\eta] \leq \hat{\mathbb{E}}_0^{\text{ND}}[\eta]$ *and* $\hat{\mathbb{E}}_1^{\text{ND,s}}[\eta] \leq \hat{\mathbb{E}}_1^{\text{ND}}[\eta]$.

Proof of Claim B.2 The only change from the setup in Section 1 is that now Equation 3 becomes

$$U_t := \hat{\mathbb{E}}_t \left[\phi^{2-t} \left(-\frac{1}{2}(w_1 + w_2)^2 \right) + \sum_{\tau \geq t} \phi^{\tau - t} u(w_\tau) \right]. \tag{B.11}$$

We first work as in the proof of Proposition 1 in Section A.1, to find the optimal amount of work w_1 in period 1. The agent chooses w_1 at $t = 1$ to maximize V_1. Using Equations 1 and B.11, V_1 becomes

$$\hat{\mathbb{E}}_1 \left[u(w_1) + (\beta + \phi) u(1 + \eta - w_1) + (\beta + \phi) \left(-\frac{1}{2}(1 + \eta)^2 \right) \right],$$

which is concave in w_1. Using $u(w) = -\frac{1}{2}w^2$, the F.O.C. yields $w_1^\dagger\left(\left\{\hat{\mathbb{E}}_t\right\}\right) = B_1\left(1 + \hat{E}_1\right)$, where $B_1 = \frac{\beta+\phi}{1+\beta+\phi}$ as before, and imposing $w_1 \leq 1$ yields $w_1^{*,\text{s}} = \min\left\{1, B_1\left(1 + \hat{E}_1\right)\right\}$.

Now we work as in the proof of Proposition 2 in Section A.2. Repeating the argument there, we can show that optimal beliefs satisfy $B_1\left(1 + \hat{E}_1\right) \leq 1$, so we can substitute $w_1 = B_1\left(1 + \hat{E}_1\right)$ and $w_2 = 1 + \eta - w_1$ into $\mathcal{W}$. Using Equations 2 and B.11 to substitute for V and U, respectively, in $\mathcal{W}$, and doing some algebra, we can write

$$\begin{aligned}\mathcal{W} &= \mathbb{E}\Big[(\beta+\phi)\,u(w_1)+\phi^2\hat{\mathbb{E}}_0\,[u(1+\eta-w_1)]+\beta\phi\hat{\mathbb{E}}_1\,[u(1+\eta-w_1)]+\beta^2u(1+\eta-w_1)\Big] \\ &\quad +\beta^2\mathbb{E}\left[-\frac{1}{2}\,(1+\eta)^2\right]+\phi^2\hat{\mathbb{E}}_0\left[-\frac{1}{2}\,(1+\eta)^2\right]+\beta\phi\hat{\mathbb{E}}_1\left[-\frac{1}{2}\,(1+\eta)^2\right].\end{aligned}$$

The first line of the RHS of this equation is simply the well-being in the absence of an incentive for speed. So differentiating, we see that for $t = 0$ and for $t = 1$, $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_t}$ equals its counterpart in the case without an incentive for speed, minus a constant times $1 + \hat{E}_t$. Thus, optimal beliefs with an incentive for speed are (weakly, since the constraint $\hat{E}_t \geq 0$ may bind) more optimistic than without an incentive for speed.

Claim B.3. *(Incentive for accuracy of task duration prediction)*
A payment $-\frac{1}{2}\left(\mathbb{E}\,[\eta]-\hat{\mathbb{E}}_u\,[\eta]\right)^2$ *at* $t = 2$ *that is decreasing in the absolute difference between objective and time u subjective expectations about task duration,* $\left|\mathbb{E}\,[\eta]-\hat{\mathbb{E}}_u\,[\eta]\right|$*, where u is* 0 *or* 1*, makes time u beliefs less optimistic, i.e.,* $\hat{\mathbb{E}}_0^{\mathrm{ND,a}}\,[\eta] > \hat{\mathbb{E}}_0^{\mathrm{ND}}\,[\eta]$ *or* $\hat{\mathbb{E}}_1^{\mathrm{ND,a}}\,[\eta] > \hat{\mathbb{E}}_1^{\mathrm{ND}}\,[\eta]$.

Proof of Claim B.3 The only change from the setup in Section 1 is that now Equation 3 becomes

$$U_t := \hat{\mathbb{E}}_t\left[\phi^{2-t}\left(-\frac{1}{2}\left(E-\hat{E}_u\right)^2\right)+\sum_{\tau\geq t}\phi^{\tau-t}u\,(w_\tau)\right]. \tag{B.12}$$

Working exactly as in the proof of Claim B.2 above, we find that the optimal work at time 1 is $w_1^{*,\mathrm{a}} = \min\left\{1,\, B_1\left(1+\hat{E}_1\right)\right\}$.

Again working exactly as in the proof of Claim B.2 above, we can write the well-being as

$$\begin{aligned}\mathcal{W} &= \mathbb{E}\Big[(\beta+\phi)\,u(w_1)+\phi^2\hat{\mathbb{E}}_0\,[u(1+\eta-w_1)]+\beta\phi\hat{\mathbb{E}}_1\,[u(1+\eta-w_1)]+\beta^2u(1+\eta-w_1)\Big] \\ &\quad -\frac{1}{2}\,(\beta^2+\beta\phi+\phi^2)\left(E-\hat{E}_u\right)^2.\end{aligned}$$

The first line of the RHS of this equation is simply the well-being in the absence of an incentive for accuracy. So $\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}\hat{E}_t}$ equals its counterpart in the case without an incentive for accuracy, plus $(\beta^2+\beta\phi+\phi^2)\left(E-\hat{E}_u\right)$, which is positive given that beliefs are optimistic. Thus, optimal beliefs with an incentive for accuracy are less optimistic than without an incentive for accuracy.