The Generalized Little’s Law and an Asset Picking System to Model and Maintain a Customized Investment Portfolio: a Working Prototype

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This research project bridges three major business disciplines:

- **Operational Research**
- **Finance**
- **Financial Engineering**
Research Project Structure

A portfolio of financial assets is thought as a queue: when an asset is bought it joins the queue, when it is sold it leaves the queue.

An adviser working with a client uses decision rules to trade-off risk vs. expected return over time.

The LL (Little’s Law) handles the queues.

The GLL (Generalized Little’s Law) tracks the assets and their attributes.

The APS (Asset-Picking-System) selects assets and # of shares per asset.

The GLL-APS model generates a customized financial portfolio over time.
Little’s Law (LL)

\[ L = \lambda W \]

- \( L \) Average number of investments \((I_i)\) over a period \( t \in [0, T] \)
- \( \lambda \) Average arrival rate of \((I_i)\) to the portfolio during that period (investments/week)
- \( W \) Average length of time (weeks) that an \((I_i)\) stays in the portfolio during \([0, T]\)

LL in action:

- \((I_i)\) join the Queue
- \((I_i)\) leave The Queue
Theorem LL.2 on [0, T] for standard LL (Little 2011)

Fig. 1 - \( n(t) \) vs. \( (t) \) showing a sample path \( (\omega) \) of the number of investments \( n(t) \) in a portfolio, \( \omega \), over an observation period, \( t \in [0, T] \), with finite \( (T) \), \( \Rightarrow L = \lambda W \).
Corollary C.1 to LL.2: Extension

Let:

a) $k = 1, 2, \ldots, K$
index sets of mutually exclusive $(I_i)$

b) \( \{L_k, \lambda_k, W_k\} \) parameters of LL for the (I) class $k$

then

\[
L_k = \lambda_k W_k
\]

defining
\[
\lambda = \Sigma_k \lambda_k
\]
\[
L = \Sigma_k L_k
\]
\[
W = \Sigma_k (\lambda_k / \lambda) W_k
\]

we have:
\[
L = \lambda W
\]

Practical meaning of C.1:
in a portfolio
with many classes of $(I_i)$
we can put an individual class $k$
(i.e. Index Bonds) in isolation
and use LL.2

Other activities may affect
the values of $(L_k), (W_k)$
and $(\lambda_k)$ BUT not LL relationship

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Corollary C.2 to LL.2: Nesting

Example:

\[
\text{CLASS } k = 5 \\
\text{Municipal Bonds (MB) with two main features:} \\
- \text{no federal income taxes} \\
- \text{may be state taxes}
\]

Track (MB) by State since States are mutually exclusive

A Subscript (s) designates state within the class of MB

Art of Model Building suggests Wisdom of Keeping things simple

Arguments establish C.2 and go through to nest a new attribute within MB class
Corollary C.3 to LL.2: Flexibility

Theorem LL.2 encourages flexibility

A useful thinking of C3:
A check-marked class and a different period [0,T] define a new subclass

Adviser suggestions

\[ I_i = \text{Bond, Stock, or other} \]
\[ I^* = \text{particular check-marked class of} \ (I_i) \text{ when enters the portfolio} \]

Advisor:
- Monitor \( I^* \) within \([0,T]\)
- Apply LL.2 over \([0,T]\) to check-marked class
- Make changes at time \((t)\)
- Evaluate LL.2 results up to \((t)\)
- Recommend next

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Expands scope and concept of LL

\[ H = \lambda G \]

EXPAND SCOPE, given:
\( f_i(t) \) over a period of time \([0,T]\)
unique to each investment \((I_i)\),
\( f_i(t) = 0 \)
extcept when \((I_i)\) is in the portfolio.

EXPAND CONCEPT TO A VECTOR OF \((I_{iM})\), denoted by:
\( Fi(t) = \{ f_{i1}(t), f_{i2}(t),..,f_{im}(t),.., f_{iM}(t)\} \), where \((i)\) indicates the investment and \((M)\) the attribute.