



CPI COMPETITION POLICY
INTERNATIONAL

Volume 7 | Number 1 | Spring 2011

Jeffrey Rohlfs' 1974 Model of
Facebook: An Introduction
with
*A Theory of Interdependent
Demand for a Communications
Service*
by Jeffrey Rohlfs

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Jeffrey Rohlfs' 1974 Model of Facebook: An Introduction

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I. Introduction

Jeffrey Rohlfs' pioneering 1974 study of demand in the presence of network externalities,¹ which make each actor's demand for some good or service depend in part on whether others purchase it, laid the foundation for a huge academic literature that has had a major impact on antitrust policy. The government's case in *U.S. v. Microsoft*, for instance, relied heavily on network externality arguments.

In most of the post-Rohlfs network-effects literature, buyers are modeled as making long-term product or technology choices because those choices either involve the purchase of significant durable goods or create switching costs. Examples include the choices between VHS and Betamax VCRs, or between Apple and Wintel computers, or the choice to purchase an early fax machine.

In contrast, Rohlfs presents a model that seems better suited to analysis of new Internet-based businesses that rely on network effects, like Facebook and YouTube. These businesses provide services rather than durable goods, and their customers are not required to make long-term commitments. Switching costs are at most moderate, and customers can often participate in multiple competing networks at the same time. (In the terminology of the recent, related literature on two-sided markets, they can “multi-home.”²) Thus I think the Rohlfs paper deserves to be read carefully on its own, apart from the literature it helped to launch.

*Howard W. Johnson Professor of Economics and Management, Massachusetts Institute of Technology. I am indebted to David Evans and Joe Farrell for helpful comments, but errors and opinions are mine alone. It is a particular pleasure to help increase awareness of the Rohlfs paper both because its insights have shaped some of my own recent work (as discussed below) and because I remember Jeff fondly from our time together in graduate school at MIT.

Rohlf's fundamental assumption was that the amount any individual firm or household would be willing to pay for a telecommunications service would depend on the set of other individuals with whom they could use that service to communicate. This effect, what we now call a *direct* network effect or externality, seems to have been recognized from the first days of the telephone industry. (The recent two-sided markets literature, in contrast, focuses mainly on *indirect* network externalities, in which participation by members of one customer group makes a platform more attractive to members of another customer group.) Rohlf's was not the first formal analysis to incorporate this assumption: his paper cites earlier articles by Artle & Averous and by Squire that do so.³ Those papers were concerned with optimal telecommunications pricing, however, while Rohlf's generally took price as given and provided a deeper, more general analysis of the implications of network externalities for market demand—without, it should be noted, ever using the term “network externalities.”

ROHLF'S GENERALLY TOOK PRICE AS GIVEN AND PROVIDED A DEEPER, MORE GENERAL ANALYSIS OF THE IMPLICATIONS OF NETWORK EXTERNALITIES FOR MARKET DEMAND—WITHOUT, IT SHOULD BE NOTED, EVER USING THE TERM “NETWORK EXTERNALITIES.”

The next section provides a guide to Rohlf's analysis. It is, I hope, a bit easier to digest than the paper it attempts to exposit, but it is intended mainly for economists and lawyers who are tolerant of formal reasoning. None of its equations are essential for Section III, which describes the impact Rohlf's paper has had and considers some of its implications for both economic analysis of and competition policy toward Facebook-like businesses.

II. A New Telecommunications Service

Writing as a Member of the Technical Staff at Bell Laboratories, Rohlf was mainly concerned with the problem of launching a new telecommunications service (his main example, in 1974, was a video communications service) and thus with disequilibrium situations. His most general model (in Section 2) begins with the assumption that, all else equal, if any individual i subscribes to the telecommunications service under consideration, her utility will not be decreased and may be increased if any other individual j also subscribes. Since in the 1970s customers of the Bell System could only rent telephones and other terminal equipment, it was natural for Rohlf to assume that subscriptions to a new service would involve only a per-period price, with no fixed cost of subscribing or unsubscribing.

Under these assumptions, Rohlf defines an *equilibrium user set*: taking prices and all other individuals' status as given, each individual in such a set wishes to continue to subscribe to the service, and each individual not in the set does not wish to subscribe. This is a natural Nash equilibrium concept, but note that it

rules out both explicitly coordinated behavior and non-myopic behavior that depends on expectations of others' future actions. Since decisions to subscribe or unsubscribe could be reversed at little, if any, cost in the regulated telecommunications sector circa 1974, it is not unreasonable to assume that expectations about the future would not affect an individual's current actions. As we discuss below, however, coordination devices could nonetheless be both privately and socially valuable.

Rohlfs notes that it is a fundamental feature of this model that equilibrium user sets are not generally unique. Consider, for example, a population of four individuals in which individuals 1 and 2 are willing to pay a lot to be able to communicate with each other but neither has any interest in individual 3 or individual 4. Suppose the situation is symmetric, so that individuals 3 and 4 are eager to communicate with each other but care nothing for 1 or 2. Then, if the service is of no value unless it enables you to communicate with somebody you care about, and if its price is low enough, there will be four equilibrium user sets: the null set, individuals 1 and 2, individuals 3 and 4, and all four individuals.

Whenever the service is of no value to any individual unless at least one other individual subscribes, the null set, in which no one subscribes, is always an equilibrium user set. Similarly, in more general settings there may also be a dramatic difference between the largest and smallest equilibrium user sets, and, as Rohlfs

notes, "In a practical situation, this difference may mean the difference between marketing success and failure."⁴

REAL MARKETS WITH LARGE NUMBERS OF PARTICIPANTS RARELY LEAP INSTANTLY TO EQUILIBRIUM, PARTICULARLY WHEN THEY INVOLVE NOVEL GOODS OR SERVICES.

Real markets with large numbers of participants rarely leap instantly to equilibrium, particularly when they involve novel goods or services. To reflect this, Rohlfs considers a

broad class of adjustment processes in which, out of equilibrium, each individual who wishes to subscribe or unsubscribe does so with some finite lag. In general, the equilibrium set to which such processes converge depends on the initial set of subscribers and, possibly, on the details of the adjustment process. In particular, Rohlfs notes, "It may be critical whether or not the disequilibrium non-users subscribe before the disequilibrium users drop out."⁵

In order to obtain sharper results, Rohlfs turns in his Section 3 to the special case in which the utility of the service to any one individual depends only on the number of other individuals who also subscribe, which he refers to as the *uniform calling* model, and the relationship is linear. This portion of the paper has perhaps been the most influential. Let f be the fraction of the total relevant population that subscribes. Then for a given positive price, p , a typical individual i will want to subscribe to the service if and only if

$$fw_i - p \geq 0, \text{ or } w_i \geq p/f. \tag{1}$$

where w_i is a non-negative constant. For a large population, one can treat the w_i as distributed according to a smooth probability density function, $h(w)$.

Given all the assumptions in the preceding paragraph, the shape of $h(w)$ and the corresponding distribution function, $H(w)$, will determine the characteristics of market demand. Taking price as given, if the fraction of the population currently subscribing is f , the fraction of the population who will want to subscribe once they know this, f^* , is just the fraction of the population for which equation (1) is satisfied:

$$f^*(f|p) = 1 - H(p/f). \quad (2)$$

Because distribution functions are non-decreasing, the function $f^*(f)$ is non-increasing in p and non-decreasing in f . If $f^*(f_i|p) = f_i$, then, given p , there is an equilibrium user set consisting of the fraction f_i of the population with the largest values of w . As long as price is positive, the null set ($f = 0$) is always an equilibrium user set, since $H(\infty) = 1$ for any distribution function. There may be no equilibrium user sets with $f > 0$, or there may be one or more such sets. If $H(p) = 1$, so that everyone in the population is willing to pay price p , then $f^*(1|p) = 1$, and the whole population is an equilibrium user set.

To analyze disequilibrium situations, Rohlfs considers a general class of adjustment mechanisms according to which f increases over time if $f < f^*$, and f decreases over time if $f > f^*$. It is then easy to show that if there are equilibria with $f > 0$, stable and unstable equilibria must alternate.⁶ That is, if the null set is a stable (unstable) equilibrium, the next smallest, if it exists, must be unstable (stable), and so on. Rohlfs defines the *critical mass* problem for the new system as the problem of somehow reaching a level of f such that $f^* > f$ and the business is viable at the next highest stable equilibrium, to which the system will then tend over time if price remains fixed. He notes, however, that attaining the socially optimal equilibrium user set “may require ruinous (albeit temporary) promotional costs.”⁷ Moreover, he adds that although he naturally assumes in his analysis “that the product is viable, it is worth noting that in real life the seller would have no such guarantee.”⁸

If there are multiple stable equilibria, it may be advantageous for both the seller and its customers (for whom participation by others adds value) if the market attains an equilibrium with high participation rather than a lower equilibrium or even the null equilibrium in which there is zero participation. For this reason, devices to coordinate behavior may have both private and social value. Thus, for instance, Glen Weyl analyzes “insulating tariffs,” in which prices are carefully set as functions of participation levels so that the market is guided to the desired equilibrium.⁹ In practice, of course, particularly with highly innovative new products, the distribution of reservation prices is unknown and thus so are the available equi-

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libria. And, partly as a consequence, investors' enthusiasm for spending on costly coordination attempts is typically well curbed. On the other hand, it is not unusual for new products with network effects to raise price over time as participation grows and the product thus becomes more attractive,¹⁰ and Rohlfs considers strategies of this sort. One can think of these as developing and announcing pieces of insulating tariffs on the fly.

Much of the analysis in Rohlfs' Section 3 is devoted to an example in which w is distributed uniformly between zero and one, so that $H(w) = w$ over that interval. Then (2) becomes simply:

$$f^*(p|w) = \begin{cases} 1 - (p|f), & \text{for } 0 < p < f \leq 1. \\ 0, & \text{for } p > f. \end{cases} \quad (3)$$

Solving equation (3) for $f = f^*$, there are at most three distinct equilibria, which are shown in Figure 1:

$$f_0 = 0, f_1 = \frac{1 - \sqrt{1 - 4p}}{2}, \text{ and } f_2 = \frac{1 + \sqrt{1 + 4p}}{2}. \quad (4)$$

Note that if $p > 1/4$, the only equilibrium involves no subscribers, even though if everybody subscribed, 3/4 of them would gladly pay more than 1/4. The problem is that those willing to pay less than 1/4 even with the entire population subscribing would leave the service, reducing its attractiveness to those who remained, inducing further defections, and so on until the business spiraled down to zero.

The arrows in Figure 1 illustrate the dynamics of this example for some $p < 1/4$. The smallest equilibrium, f_0 , is stable; the next smallest, f_1 , is unstable, and the largest, f_2 , is stable. At the given price, the critical mass problem is to get the fraction of the population subscribing to above f_1 . If that can't be done, the subscriber base will inevitably shrink to zero, but if it is done, network effects will fuel organic growth to the largest equilibrium, f_2 .

In Section 4, Rohlfs generalizes the linear utility model in equation (1) to a situation in which the population can be divided into k groups. Initially he assumes that individuals in group i care only about the number of other individuals in group i who subscribe; then he allows for

inter-group externalities. In the latter case the analysis involves considerations of group-specific critical mass levels, and the ultimate equilibrium reached from any specific starting point depends on the details of both that point and the adjustment process.

ROHLFS' SECTION 5 CONSIDERS
TWO APPROACHES TO
SOLVING THE START-UP PROBLEM
FOR A NEW SERVICE.

Rohlfs' Section 5 considers two approaches to solving the start-up problem for a new service: 1) free service to a carefully selected group of people for a limited period of time, and 2) a low introductory price that is raised over time. If being

a subscriber has value only to the extent that it enables communication (rather than, say, because it is a status symbol), the hard part is getting two or more individuals with high values of w to subscribe at a positive price. In theory, at least, this might be done by offering the service for free for a limited time to a targeted group, then raising price just above zero so that at least a few high- w individuals find it optimal to remain subscribers.

Once this has been done successfully, however, at least in the uniform calling case, price can then be increased gradually over time according to some function $p^g(f)$. As long as $p^g(f) < f(1 - f)$, f^* will exceed f from (3), and the subscriber base will grow. (Once again, it doesn't matter whether individuals anticipate later price increases or not, since myopic behavior is individually rational here.) When the optimal level of price is reached, price is constant thereafter, and the system adjusts to the higher, stable equilibrium corresponding to that price. These pricing policies are, as I noted above, broadly in the spirit of Weyl's insulating tariffs, but Rohlfs seems to view them as generally being developed as information about demand arrives, rather than as the results of *ex ante* optimization with demand known.

Rohlfs establishes a very neat result in this context. Suppose that at some point, after the price has been raised to $p_0 > 0$ and all those who find it optimal to unsubscribe have done so, a fraction $f_0 > 0$ of the population finds it optimal to remain on the system. Then as long as $p^g(f)$ is less than $f(1 - f)$, as above, and the elasticity of $p^g(f)$ with respect to its argument does not exceed one, the system will grow and no subscriber will ever leave.¹¹

All this rests on the extremely unrealistic assumption that the distribution of the w_i is somehow known, of course. And, as Rohlfs shows, if the uniform calling assumption does not hold, devising a startup strategy necessarily becomes more complex even if all taste distributions are known, and the details of non-uniformity matter.

III. Impact and Implications

Jeffrey Rohlfs' 1974 paper has been widely cited—669 times according to Google Scholar—but its importance beyond telecommunications took some time to be recognized.¹² The first widely cited paper that I can find that cites the Rohlfs paper is a 1980 survey of the economic theory of clubs.¹³ The literature on network externalities, and with it citations of the Rohlfs paper, exploded in the mid-1980s with the publication of influential papers by Joseph Farrell and Garth Saloner and by Michael Katz and Carl Shapiro that acknowledged Rohlfs' contribution.¹⁴

As noted above, however, the literature that grew out of these papers deals with very different market environments than those considered by Rohlfs. In a widely cited 1994 survey, Michael Katz and Carl Shapiro note that in this literature's

analysis of technology adoption and product selection decisions, expectations and coordination play important roles.¹⁵ As I noted above, expectations and coordination do not appear at all in the Rohlfs analysis, with the exception that he considers increasing price paths as a device to attain a desirable equilibrium.

Katz and Shapiro's initial example of a technology adoption decision is the decision to buy a fax machine.¹⁶ In that decision expectations about the behavior of other potential fax machine owners are clearly important. Had fax machines existed in the early 1970s when Rohlfs was writing, however, they would have been available only for rent from the local telephone company, and expectations and coordination would have been of much less importance in decision-making about subscribing to fax services. Similarly, expectations and coordination were clearly important in making many of the product selection decisions between incompatible rival "hardware/software" systems that they describe as being "in the newspaper almost every day."¹⁷ These include the choice

between Beta and VHS video recording systems and among rival home video game systems. Again, if short-term rental of the "hardware" parts of these systems were available, expectations and coordination would have been much less important.

The focus of this post-Rohlfs network-effects literature on technology and product selection decisions with long-lasting consequences sent two important messages to competition policy-makers.

The first was implicit: technology adoption and product selection were generally modeled as discrete, once-and-for-all decisions that typically produced winner-take-all results. Not only is that how the choice between Beta and VHS seemed to most observers at the time, but to model multi-stage processes, in which expectations of future technologies and products (like DVDs and Blu-Ray and ...) could influence today's choices, would have involved considerable incremental complexity. The second was explicit: most theoretical analysis showed that market outcomes in markets with once-and-for-all competition and network effects could be seriously socially inefficient: buyers could find themselves selecting the wrong product or technology, and society could be locked-in to those bad choices for the foreseeable future.

Katz and Shapiro do counsel caution in their 1994 survey and note that, "In short, we are far from having a general theory of when government intervention is preferable to an unregulated market outcome."¹⁸ The literature they surveyed nonetheless suggested that, at the very least, competition authorities should pay particular attention to competition in industries with network effects to ensure that firms with short-run market power don't use anticompetitive behavior and network effects to build and lock in dominant positions for the long haul. As Carl

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Shapiro, then Deputy Assistant Attorney General for Economics in the Antitrust Division of the U.S. Department of Justice, put it in a January, 1996 speech:¹⁹

“Even more so than in other areas, antitrust policy in network industries must pay careful attention to firms’ business strategies, the motives behind these strategies, and their likely effects... Furthermore, antitrust enforcers must be alert in these industries because the very nature of the “positive feedback” cycle means that monopolization may be accomplished swiftly. And, once achieved, the network effects that helped create dominance may make it more difficult for new entrants to dislodge the market leader than in other industries lacking network characteristics.”

The Justice Department’s Sherman Act case against Microsoft, which rested heavily on arguments involving network effects, was filed in May, 1998.

All this suggests that the Rohlfs paper reprinted here may have become one of those classics that is often cited but rarely read. I think that is unfortunate, but not because the network-effects literature than began in the mid-1980s is wrong in any technical sense or inapplicable to some markets. Nor do I think the policy concerns it raised are not relevant in those markets or that the enforcement stance expressed by Professor Shapiro in the thoughtful speech just cited is inappropriate in those markets.²⁰

The point is that the network-effects literature that began in the mid-1980s does not seem to have much to say about markets in which product or technology choices do not have long-lasting consequences—markets like those analyzed by Rohlfs. Without durability, expectations are not critical to decision-making, and lock-in is much less likely, particularly if multi-homing is possible.

And, as David Evans and I have argued,²¹ “almost every day” one now reads about markets of the sort analyzed by Rohlfs. Consider social networks, for instance. There are clearly direct network effects in these businesses: the value of being a participant in any particular social network depends on who else is participating. (There are also indirect network effects in these businesses, since participants attract advertisers.) But it is easy to switch between networks or to participate in multiple networks at the same time. Thus despite

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its early network-effect-enhanced advantages, MySpace has been almost totally eclipsed by Facebook, which used a university-based launch strategy that could have been cribbed from Rohlfs' discussion of launching a new service in a population consisting of multiple groups with strong intra-group affinities. Network effects, while present, were clearly not the only important factors in competition between MySpace and Facebook.

It is particularly interesting to note that video communications services, of the sort that Rohlfs could only hypothesize in the early 1970s, are now generally available on the Internet. And, as Rohlfs assumed, network effects plainly matter in choosing which service to use. But deciding to use one or another at any point in time has essentially no long-term implications. I currently use software for two such services on my computer and could no doubt access others if I saw benefits from doing so. I have no reason to care which service will prove more popular in the future, as I can use any one when it is worth using and ignore it otherwise.

ROHLFS, HOWEVER, CONSIDERED
A DIFFERENT SUBSET OF MARKETS
WITH NETWORK EFFECTS,
ONE THAT I BELIEVE IS BECOMING
MORE IMPORTANT IN PART
BECAUSE OF THE INTERNET.

As a final example, consider smart phone operating systems. As I write this, there is a vigorous struggle going on between Apple and Google, with Microsoft and RIM also engaged. But this is not the PC market in the 1990s: consumers buy new smart phones fairly often, and

the costs of switching between phones based on different operating systems do not seem to be significant. Thus, even though consumers don't generally multi-home in this market as they easily can for social networks and video calling services, product selection does not involve a very durable commitment, and competition does not look like a one-shot, winner-take-all affair.

Thus, while Jeffrey Rohlfs' paper has been influential in calling the attention of the economics profession to markets with network externalities, both economists and policy-makers have tended, until recently, to focus on a particular subset of those markets—those in which it is at least arguably the case that anticompetitive behavior during critical periods of winner-take-all competition may lead to undesirable and long-lasting outcomes. In such markets, especially close attention by antitrust authorities during those critical periods is appropriate. Rohlfs, however, considered a different subset of markets with network effects, one that I believe is becoming more important in part because of the internet. In these markets switching costs are not important, and the key decisions do not involve purchases of big-ticket, long-lived durables. Accordingly, participation decisions can easily be reversed. Moreover, multi-homing is often possible in these markets, so even at the individual level competition is not a winner-take-all matter even for short periods of time.

This is not to say that the markets considered by Rohlfs may not raise novel and interesting competition policy issues, since the presence of externalities and the potential for multiple equilibria imply the possibility of departures from the textbook norm. But it is far from clear that such markets deserve especially close antitrust scrutiny.²² In any case, I believe Rohlfs' analysis deserves to be read carefully and to be both extended and applied by economists and, further, that its implications should be carefully considered by antitrust enforcement agencies as they increasingly deal with markets for which that analysis is relevant. ▼

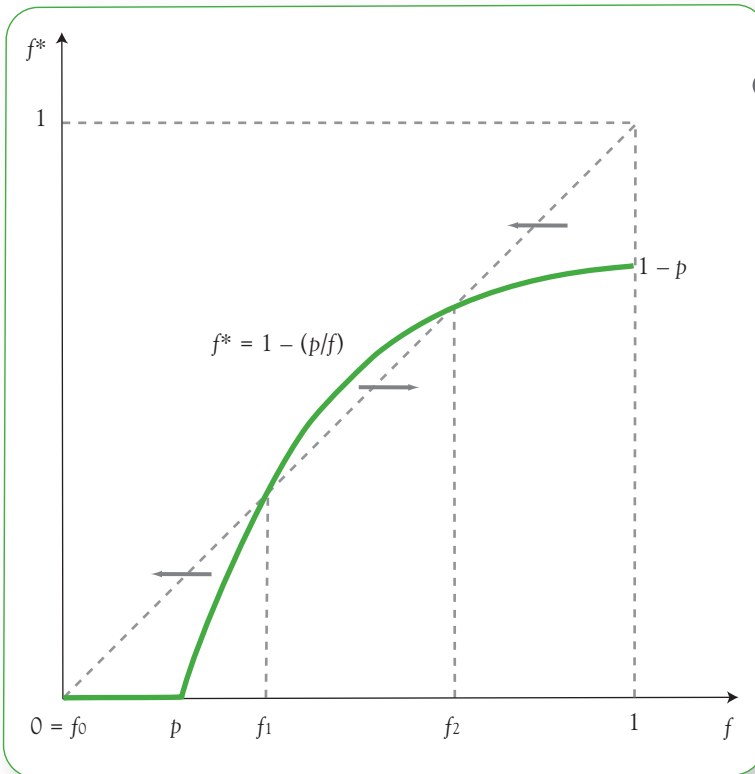


Figure 1

The Example from
Rohlfs' Section 3

- 1 Jeffrey Rohlfs, *A Theory of Interdependent Demand for a Telecommunications Service*, 5 BELL J. ECON. MGMT' T SCI. 5, 16-37 (1974), reprinted in this issue of COMPETITION POL'Y INT'L, (hereinafter "Rohlfs").
- 2 See, e.g., Jean-Charles Roche & Jean Tirole, *Two-Sided Markets: A Progress Report*, 37 RAND J. ECON. 645-67 (2006).

- 3 Roland Artle & Christian Averous, *The Telephone System as a Public Good: Static and Dynamic Aspects*, 4 BELL J. ECON. MGMT SCI. 89-100 (1973) and Lyn Squire, *Some Aspects of Optimal Pricing for Telecommunications*, 4 BELL J. ECON. MGMT SCI, 515-525 (1973).
- 4 Rohlfs, *supra* note 1, at 24.
- 5 *Id.*
- 6 Suppose some equilibrium point, f_0 , is stable. Then df^*/df must be less than one at that point, so that $f^* > f$ for f just below f_0 , and $f^* < f$ for f just above f_0 . But then in order for f and f^* to be equal for some $f_1 > f_0$, f^* must rise faster than f near f_1 , since f^* is approaching f from below. Thus df^*/df must exceed one at f_1 , from which it follows that that equilibrium is unstable. Repeating the argument starting at an unstable equilibrium establishes the result.
- 7 Rohlfs, *supra* note 1, at 32.
- 8 *Id.* at 33.
- 9 E. Glen Weyl, *A Price Theory of Two-Sided Platforms*, 100 AMER. ECON. REV., 1642-72 (2010).
- 10 A common alternative in the internet world is to keep price to consumers at zero but to degrade quality by adding advertising.
- 11 If at some value of f , household i finds it optimal to subscribe, it must be that $w_i \geq p^*(f)/f$. This household will remain a subscriber as f increases if $p^*(f)/f$ does not increase, which is equivalent to the requirement that the elasticity of p^* with respect to f not exceed unity. Rohlfs requires $p^*(f)$ to be concave because he assumes that it passes through the origin.
- 12 All citation counts are as of March 10, 2011.
- 13 Todd Sandler & John T. Tschirhart, *The Economic Theory of Clubs: An Evaluative Survey*, 18 J. ECON. LIT. 1481-1521 (1980). This article has 330 Google Scholar citations.
- 14 Joseph Farrell & Garth Saloner, *Standardization, Compatibility, and Innovation*, 16 RAND J. ECON. 70-83 (1985) and Michael L. Katz & Carl Shapiro, *Technology Adoption in the Presence of Network Externalities*, 94 J. Pol. Econ., 822-841. These papers have 1662 and 1713 Google Scholar citations, respectively. Interestingly enough, the most frequently cited of the first-round network externality papers by these authors (Michael L. Katz & Carl Shapiro, *Network Externalities, Competition, and Compatibility*, 73 AMER. ECON. REV., 424-440 (1985)), which has 4113 Google Scholar citations, does not cite the Rohlfs paper. It cites instead a later paper that in turn cites Rohlfs' paper, along with the related earlier papers by Artle & Averous and by Squire, cited here in note 3, *supra*.
- 15 Michael L. Katz and Carl Shapiro, *Systems Competition and Network Effects*, Journal of Economic Perspectives 8, 93-115 (1994). This article has 1621 Google Scholar citations. Katz and Shapiro also note that the literature they survey considers compatibility decisions, but these are beyond the scope of the present discussion.
- 16 *Id.* at 93.
- 17 *Id.* at 105.
- 18 *Id.* at 113.

- 19 Speech delivered by Carl Shapiro before the American Law Institute and American Bar Association, January 25, 1996, p. 6; available at <http://www.justice.gov/atr/public/speeches/0593.pdf>.
- 20 But see Richard Schmalensee, *Antitrust Issues in Schumpeterian Industries*, 90 AMER. ECON. REV. PAPERS AND PROCEEDINGS, 471-5 (2000) and David S. Evans & Richard Schmalensee, *Some Economic Aspects of Antitrust Analysis in Dynamically Competitive Industries*, INNOVATION POLICY AND THE ECONOMY, VOL. 6 (A. Jaffe, J. Lerner, & S. Stern, eds.) 2, 1-49 (2002).
- 21 David S. Evans & Richard Schmalensee, *Failure to Launch: Critical Mass in Platform Businesses*, 9 REV. NETWORK ECON., Article 1 (2010). That paper provides several brief case studies and generalizes the Rohlfs uniform calling analysis to platform businesses serving two customer groups, between which there are indirect network externalities. The nature of the critical mass constraint—"chicken and egg" or "chicken or egg"—is shown to depend on details of the adjustment process, and if there are multiple equilibria, stable equilibria and saddle-points alternate.
- 22 If network effects do lead to dominance, for instance, but that dominance can be quickly eroded by a better product either because participation decisions can be costlessly reversed or through multi-homing, the general presumption that network effects create troublesome entry barriers would not hold. Joe Farrell has pointed out (personal communication) that in some respects this sort of dominance resembles market-wide exclusive dealing contracts of very short duration.

A Theory of Interdependent Demand for a Communications Service

By Jeffrey Rohlfs*

The utility that a subscriber derives from a communications service increases as others join the system. This is a classic case of external economics in consumption and has fundamental importance for the economic analysis of the communications industry. This paper analyzes the economic theory of this kind of interdependent demand. We begin by defining “equilibrium user set” as a set of users consistent with all individuals’ (users and nonusers) maximizing their utilities. There are typically multiple equilibria at any given price and which equilibrium is attained depends partly on the static model, partly on the initial disequilibrium conditions, and partly on the disequilibrium adjustment process. Some general properties of equilibrium user sets are derived. Then we turn our attention to some specific models based on simple characterizations of communities of interest. The implications for pricing are discussed, with special reference to the problem of starting up a new communications service (e.g. a video communications service).

*This article was published in the Spring 1974 issue of the *Bell Journal of Economics and Management Science*. At the time, Jeffrey Rohlfs was a Member of the Technical Staff at Bell Labs, where his research interests included applied microeconomics, micro theory, and econometrics. He subsequently became Head of Economic Modeling Research at Bell Laboratories. Dr. Rohlfs received an A.B. in economics from Amherst College in 1965 and his Ph.D. in economics from M.I.T in 1969. In his introduction, Dr. Rohlfs wrote, “In the course of this study, I talked to many people, and their knowledge and ideas contributed to much of the analysis in this paper. M. Wish has been a collaborator in some previous related work, and he has greatly influenced my thinking. My analysis of the general problem was greatly stimulated by a discussion I had with E. Gilbert, who developed some preliminary results about the maximum equilibrium set. I have also had many profitable discussions with W. Ballamy, J. Berrier, A. Ciesielka, D. Deutsch, A. Gersho, E. Goldstein, A.H. McKeage, D. Mitra, and R. Sanders. F. Sinden, W. Taylor, N. Valcoff, M. Wilk, and E. Zajac provided helpful comments on previous written and oral presentations of the material.

I. Introduction

The utility that a subscriber derives from a communications service increases as others join the system. This is a classic case of external economics in consumption and has fundamental importance for the economic analysis of the communications industry. It suggests that although marginal cost pricing may be superior to allocated-cost formulae, it is still not completely appropriate.

This can be illustrated with respect to an historical policy of the industry: promoting universal service. This policy might be justified on the basis of marginal cost pricing, so long as new subscribers pay the incremental cost of expanding the system to accommodate them—even if they do not pay their “allocated” share of average costs. A still lower price, perhaps much lower, might be justified if the externalities are taken into account. The total benefits that *all* subscribers derive from the expansion of the service may be sufficient to justify the incremental costs—even if the subscribers are unwilling to pay the entire incremental costs.

Recently Artle and Averous¹ made what appears to be the first published analysis of these externalities in communications.² They formulate a simple model in which the incremental utility of the service to an individual depends only on the number of telephone subscribers—not on who they are. This is the uniform calling model discussed in Section 3 of this paper. They also assume that the cost of providing telephone service depends only on the number of subscribers. This enables them to derive and interpret the necessary conditions for a social welfare optimum. Their expression has some important similarities (but also some differences) for a social optimum with respect to a pure public good.

The authors then use these notions to develop a dynamic demand model. They show that interdependent demand can sustain continual growth in a stationary population with stationary income. The mechanism is as follows. New subscribers join. This increases the incremental utility of the service and induces marginal nonusers to join. That in turn induces further growth, etc., etc. The authors offer this as a possible explanation for the continual growth of telephone service observed in all empirical studies of the industry.

Squire studies the problem using a somewhat different model.³ He considers usage of the system as well as number of telephones and assumes that the cost of providing the service is a function of these two variables. Squire specifies individual demand curves (based on a fixed number of subscribers) for incoming and outgoing calls. This enables him to develop an expression for optimal usage of the system, based on a modified consumer-surplus concept. He then derives the optimal price per call (charged only to the person making the call) consistent with this optimal usage. He finally develops an expression for the optimal size of the system and the price per telephone consistent with this optimum.

This paper makes a much more detailed analysis of the demand side of the market than attempted by Artle and Averous or by Squire. We begin by defining

an “equilibrium user set” as a set of users consistent with all individuals’ (users and nonusers) maximizing their utilities. A basic result is that there are typically multiple equilibria at any given price. For example, a very small equilibrium user set may be consistent with utility maximization, since the smallness of the user set in itself makes the service relatively unattractive to potential users. However, a much larger user set may also be possible for the same population at the same price. In this case the largeness of the user set would make the service attractive and allow a high level of demand to be sustained. In any planning (public or private) for the communications service, special attention must be paid to which equilibrium user set is likely to be attained.

The next section of this paper develops a general theory of demand. It derives the following results:

- 1) The static model determines the attained equilibrium user set (at a given price) within a certain set of bounds.
- 2) A possibly narrower set of bounds (for a given price) is derived, given the initial user set.
- 3) Within the bounds defined in (2), the equilibrium attained (at a given price) depends entirely on the disequilibrium adjustment process.

The following two sections of the paper develop specialized models based on various simple characterizations of communities of interest. The simplest of all is the uniform calling pattern, which assumes that no one has any special community of interest (other than the entire population). This model is the only one in which the equilibrium theory can be developed in terms of the *number* of users, without paying attention to who they are. We can therefore define a demand curve, which turns out to have an inverted *U* shape. See Figure 1.

Zero demand is a stable equilibrium for all positive prices. The upward-sloping part of the inverted *U* consists of unstable equilibria and constitutes the “critical mass” of the service (at any given price). If the critical mass is exceeded demand expands to the downward sloping part of the inverted *U*. Points on the latter are stable equilibria and represent the maximum level of demand sustainable at a given price.

Unfortunately (for ease of analysis), the uniform calling pattern may not be very realistic. People typically belong to groups, each of which has a strong community of interest within itself. And they typically have a few principal contacts who alone account for a substantial part of their communication. These complications are briefly discussed in the section entitled “Nonuniform Calling Patterns.”

The final section of the paper discusses some implications of the preceding demand analysis for supply and pricing of the service. An important distinction is made between viability of service (existence of a nonnull equilibrium user set

that can be served with nonnegative profits) and the start-up problem (how to attain such a user set, starting from a small or null initial user set).

Viable nonnull equilibrium user sets (if they exist) are always superior to the null set from a static point of view. We can compare such sets to determine the static social optimum or the overall market equilibrium corresponding to a static supply model. However, this kind of analysis is incomplete and may be misleading without consideration of the start-up problem. Achieving the static optimal user set may require ruinous (albeit temporary) promotional costs.

Appropriate solutions to the start-up problem depend in large part on the demand model. In the uniform calling model, the start-up problem is simply a question of getting beyond the critical mass. Community of interest groups may make the practical start-up problem much easier, but they also introduce some special problems. If an individual's demand is contingent on a few principal contacts' being users, there may exist many small self-sufficient user sets. These allow the possibility of a long-term introductory program, in which the seller gradually expands the size and number of such sets.

This paper presents only a limited discussion of costs and supply. The reason is that costs of a communications service are very complex and merit a separate study in their own right. This is a very fruitful topic for future research.

II. General Theory of Demand

Let the population consist of n individuals. As in Artle and Averous' work, we define a set of binary variables:

$$q = \begin{cases} 0 & \text{if individual } i \text{ does not subscribe to the communications service} \\ 1 & \text{if the individual } i \text{ does subscribe to the communications service} \end{cases} \quad (1)$$

for $i = 1, \dots, n$.

We assume that there are also m other goods in the economy. To model interdependent demand, we specify a pair of utility functions for each individual:

$$U_i^0 = U_i^0(r_{i1}, \dots, r_{im}) \quad (2)$$

$$U_i^1 = U_i^1(q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_n, r_{i1}, \dots, r_{im}) \quad (3)$$

where

U_i^0 = Utility of individual i if he does not subscribe to the communications service,

U_i^1 = Utility of individual i if he does subscribe to the communications service, and

r_{ij} = Consumption of (noncommunications) good j by individual i .

Equations (2) and (3) implicitly assume independent utilities with respect to all goods in the economy *other* than the communications service in question. In addition we make the usual monotonicity assumptions:

$$\frac{\partial U_i^k}{\partial r_{ij}} \geq 0 \text{ for all } j \text{ and } > 0 \text{ for some } j; \text{ and} \tag{4}$$

$$U_i^0 \leq U_i^1 \tag{5}$$

for all $i, k, q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_n, r_{i1}, \dots, r_{im}$.

We also make a specialized assumption applicable to a communications service:

$$\frac{\partial U_i^1}{\partial q_m} \tag{6}$$

for all $i \neq w, q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_n, r_{i1}, \dots, r_{im}$. That is, a subscriber's utility never decreases as additional individuals subscribe (and none drop out).

This seems like a reasonable working assumption. We can, of course, imagine some exceptions: e.g., the value of the service to others would probably be lessened if a large number of life insurance salesmen subscribed to the service to solicit other subscribers. However, we assume that such occurrences are the exception rather than the rule—that, in general, the availability of a communications link is not detrimental to either party.

We assume utility maximization, which we analyze in two steps. (1) We evaluate the maxima of U_i^0 and U_i^1 (with respect to r_{i1}, \dots, r_{im}) subject to individual i 's budget constraint. Let us denote these maxima as \hat{U}_i^0 and \hat{U}_i^1 . (2) We then compare \hat{U}_i^0 and \hat{U}_i^1 to see if the individual demands the communication service. This defines a demand variable for each individual:

$$q_i^D = \begin{cases} 0 & \text{if } \hat{U}_i^0 > \hat{U}_i^1 \\ 1 & \text{if } U_i^0 \leq \hat{U}_i^1 \end{cases} \tag{7}$$

for $i = 1, \dots, n$.

The basic methodology of this paper is to ignore interrelationships between the communications market and other markets and concentrate on relationships *within* the communications market. Thus, we make the *ceteris paribus* assumption that prices of all goods other than the communications service are fixed and that each individual has a fixed budget constraint. This allows us to express the demand variables as functions of price and the set of subscribers:

$$q_i^D = q_i^D(p, q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_n) \tag{8}$$

for $i = 1, \dots, n$, where p = the price of the communications service.

It follows from previous assumptions that all the q_i^D are monotonically decreasing (equality allowed) with respect to p . That is, an increase in p can never change q_i^D from 0 to 1; a decrease in p can never change q_i^D from 1 to 0. However a change in p may have no effect on q_i^D . It also follows from previous assumptions that all the q_i^D are monotonically increasing (equality allowed) with respect to all q_w ($w \neq i$).

A. EQUILIBRIUM USER SETS

Naturally, there is a correspondence between demanding the service and being a subscriber. We define an *equilibrium user set* as a set of users such that

$$q_1 = q_1^D(p, q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_n) \quad (9)$$

for all i .⁴ Thus, in equilibrium all users demand the service; all nonusers do not demand it.

Equation (9) defines equilibrium with respect only to the demand side of the market. It describes user sets that are consistent with utility maximization at a given price. These constitute necessary but not sufficient conditions for an overall market equilibrium. The latter additionally requires that the user set and price be consistent with some specified model of supply behavior.

For fixed $p = \bar{p}$, equations (9) are a system of n equations in n binary variables. Such a system does *not* generally have a unique solution. In fact, unique solutions did not arise in any of the simple models investigated in this paper (except in the trivial case where price is so high that there can never be any demand at all).

Consequently, the equation

$$q = q^D, \quad (10)$$

where

$$q = \sum_{i=1}^n q_i \text{ and}$$

$$q = \sum_{i=1}^n q_i^D$$

may be indeterminate (for fixed \bar{p}). That is, it may either hold or fail to hold depending on which set of users constitutes the sum q .

For this reason, the general theory of interdependent demand cannot be developed in terms of the sum q . It is necessary to work with the individual q_i . The basic analytical concept is not the demand curve—i.e. the equilibrium pairs (q, p) —but rather equilibrium user sets.

B. DISEQUILIBRIUM ANALYSIS

Given that several equilibrium user sets exist for a given price (*ceteris paribus*), it is important to know which ones (if any) are most likely to occur. This requires analyzing what happens if the market is initially in disequilibrium. Our procedure is as follows. We specify a very general disequilibrium adjustment process. We then investigate the extent to which the user sets resulting from this process depend on the static model, the extent to which they depend on the initial disequilibrium conditions, and the extent to which they are indeterminate, depending on a more detailed specification of the adjustment process.

In this section, we restrict our attention to the demand side of the market and assume a given price for the communications service. We further assume that adjustments of consumption in other markets can be made rapidly and costlessly. This seems like a reasonable simplifying assumption, allowing us to analyze disequilibria in the communications market without considering possible disequilibria in the rest of the economy.

Now suppose there is an arbitrary user set. It may be based on utility maximization for current or previous states of the world, past selling efforts of the supplier of the service, or anything else. We assume that adjustments to this user set occur according to the following adjustment process. (1) An individual in equilibrium ($q_i^D = q_i$) never changes his status from user to nonuser or vice versa. This is reasonable, since such a change would always reduce his utility (except in the knife-edge case where $\hat{U}_i^0 = \hat{U}_i^1$, in which case the change in status has no effect on utility). (2) The length of time an individual can remain continually in disequilibrium ($q_i^D \neq q_i$) is bounded. He eventually must change his status. This is also reasonable, since the change always increases his utility (except in the knife-edge case where $\hat{U}_i^0 = \hat{U}_i^1$, in which case the change in status has no effect on utility).

The adjustment process is essentially a model of utility maximization with inertia. It is very general in that it makes no assumption about the speed of adjustment. This speed may vary from individual to individual. It may depend on the user set or actions of the seller. Or it may change over time.

A limitation of this process is that it does not allow individuals to collude and subscribe together. This is relatively unimportant if an individual's demand is contingent on a large user set, since such collusion would be difficult with very large groups. However (as will be seen later) we do have to consider relaxing the assumption in models where an individual's demand is contingent on a few of his principal contacts' being users.

It is important to note that the adjustment process does *not* necessarily converge to an equilibrium user set. Consider the following example. A demands the service if and only if B is a user; B demands the service if and only if C is a user; C demands the service if and only if A is a user. Suppose the initial user set is A.

One possible version of the adjustment process is as follows. C joins because A is a user. Then A disconnects because B is not a user. Then B joins because C is a user. Then C disconnects because A is not a user. Then A joins because B is a user. Then B disconnects because C is not a user. We are now back to the original user set, and the process can be repeated indefinitely.

Nevertheless, the user sets resulting from the adjustment process can be bounded, as shown in the following theorems:

Theorem 1: If the initial user set is the entire population, the adjustment process can only *remove* individuals from the user set; no individual can ever be added who has previously dropped out.

Proof: If the entire population is an equilibrium user set no one is added or removed, and the theorem is satisfied. If the entire population is *not* an equilibrium set, let r_1, r_2, \dots represent the sequence of individuals who change status. (If individuals change status simultaneously, we list them in arbitrary order.) Now r_1 must be a removal (not an addition), since the entire population consists of users, and there is no one left to be added. Given that r_1, \dots, r_k are removals, r_{k+1} must also be a removal for the following reason. The only possible additions would be the individuals r_1, \dots, r_k . But all of these dropped out (and therefore did not demand the service) when the user set contained the current user set. Thus, they cannot demand the service according to the monotonicity assumption.

It follows that all of the r_i must be removals. Q.E.D.

Theorem 2: If the initial user set is the population, the adjustment process converges to an equilibrium user set in finite time.

Proof: The process must converge in finite time for the following reason. All changes of status are removals. Since only n (the size of the population) individuals can be removed, there are at most n changes in status. These must all occur in finite time.

After all the changes in status occur, no user can fail to demand the service, for otherwise the process would continue. Moreover, no nonuser can demand the service because of the monotonicity condition. Thus, the final user set is an equilibrium user set. Q.E.D.

Theorem 3: If the initial user set is the entire population, the adjustment process converges to the union of all equilibrium user sets regardless of the order of removals.

Proof: Let X be an arbitrary equilibrium user set; let R be the equilibrium result after individuals r_1, \dots, r_k have been removed according to the adjustment process. X cannot contain r_1 , since r_1 was removed when the user set was the entire population (and hence contained X). Since X does not contain r_1 , it cannot contain

r_2 either. (The entire population minus r_1 contains $X - r_1$.) Nor can X contain r_3, \dots, r_k . Thus, $X \subseteq R$.

But X is an arbitrary equilibrium user set. Thus, R contains all equilibrium user sets. Since R is itself an equilibrium user set, it is the union of all equilibrium user sets. Q.E.D.

This set will hereafter be referred to as the “maximum equilibrium user set.”

By entirely symmetrical reasoning we can establish the following. If the initial user set is null,

1. the adjustment process can only add individuals, and no one is ever removed who previously joined;
2. the adjustment process converges to an equilibrium user set in finite time;
3. the adjustment process converges to the intersection of all equilibrium user sets regardless of the order in which individuals are added.

This set will hereafter be referred to as the “minimum equilibrium user set.”

It is important to note that the minimum and maximum equilibrium user sets need not be the same. For example, consider the following simple model. Each individual demands the service if three of his five principal contacts are users. The minimum equilibrium user set is null. There are no users; so no one has three of his five principal contacts as users; so no one demands the service. The maximum equilibrium set is the entire population. Everyone has all five of his principal contacts as users; so everyone demands the service. In addition, there may be any number of equilibrium user sets between these two extremes, depending on the distribution of principal contacts.

In this example, the static model tells us practically nothing about what equilibrium will actually be attained. We only know that it will be zero or 100 percent or somewhere in between. This is an extreme case but in general the static model determines the actual equilibrium only within certain bounds—the minimum and maximum equilibrium sets. Moreover, in all of the models investigated in this paper there exists the possibility that these bounds may be far apart. In a practical situation, this difference may mean the difference between marketing success and failure.

The above theorems show that if the initial user set is sufficiently large, convergence to the maximum equilibrium user set is assured (according to the assumed adjustment process). If the initial user set is sufficiently small, convergence to the minimum equilibrium set is assured. For intermediate initial disequilibrium user sets, the actual equilibrium attained may also depend on a more detailed specification of the adjustment process than given above.⁵ It may be crit-

ical whether or not the disequilibrium nonusers subscribe before the disequilibrium users drop out.

C. PARTICULAR INITIAL USER SET.

The minimum and maximum equilibrium user sets provide bounds on user sets that are possible for *any* version of the adjustment process described above.⁶ This subsection provides bounds on user sets that can be attained from a *particular* initial user set for *any* version of the adjustment process.

Let S be an arbitrary initial user set. We now define the following two adjustment sequences.

Optimistic sequence

1. First all nonusers who demand the service subscribe in arbitrary order but no users drop out. This converges to the same user set \bar{S} , irrespective of the order in which individuals subscribe. (Proof is analogous to that of Theorem 3.)
2. Then all users who do not demand the service drop out in arbitrary order. This converges to the same user set S , irrespective of the order in which individuals drop out. Moreover, \bar{S} is an equilibrium user set. (Proof analogous to Theorems 2 and 3.)

Pessimistic sequence

1. First all users who do not demand the service drop out in arbitrary order, but no nonusers subscribe. This converges to the same user set \underline{S} , irrespective of the order in which individuals drop out. (Proof analogous to Theorem 3).
2. Then all nonusers who demand the service subscribe (in arbitrary order). This converges to the same user set \underline{S} , irrespective of the order in which individuals subscribe. Moreover, \underline{S} is an equilibrium user set. (Proof analogous to Theorems 2 and 3.)

Now let R_1, R_2, \dots be a sequence of user sets resulting from applying an arbitrary version of the adjustment process to S . As previously discussed, this sequence need not converge to an equilibrium user set. However, we can place the following bounds on the sequence:

1. $\underline{S} \subseteq R_i \subseteq \bar{S}$ for all i . This follows directly from the monotonicity assumption.
2. After some finite period of time, $\underline{S} \subseteq R_i \subseteq \bar{S}$ for all i .

Proof: Let x_1, \dots, x_k be a sequence of individuals who drop out in part (2) of the optimistic sequence. Since x_i does not demand the service given user set \bar{S} , he cannot demand it given any user set R_i . Thus, if x_i is a user, he is continually in

disequilibrium. He must drop out in finite time and can never thereafter rejoin. Once x_i drops out we can apply the same reasoning sequentially to x_2, \dots, x_m . Thus, after some finite period of time, all $R_i \subseteq \bar{S}$. The proof that $S \subseteq \bar{R}_i$ is exactly symmetrical. Q.E.D.

Thus, the optimistic and pessimistic sequences define bounds on user sets attainable from a particular initial user set. These bounds may (or may not) be considerably narrower than the bounds provided by the minimum and maximum equilibrium sets. In any event, within these bounds the equilibrium use set attained depends entirely on a detailed specification of the adjustment process.

D. ADDITIVE UTILITIES.

In order to proceed further we must make more assumptions. To simplify the problem we propose a model of additive utilities. That is, we define a vector \underline{f} and a matrix \underline{V} such that

$$U_i^0 = f_i(r_{i1}, \dots, r_{im}) \tag{11}$$

$$U_i^1 = f_i(r_{i1}, \dots, r_{im}) + \sum_{j \neq i} v_{ij} q_j, \tag{12}$$

where v_{ij} ($i \neq j$) is the incremental utility to individual i of a communications link with individual j , ($v_{ij} \geq 0$).

The additive model assumes that these incremental utilities do not depend on consumption of other goods or on other communications links available to the individual. These do seem to be reasonable simplifying assumptions, but there are some problems with them. The growth of telephone service has had fundamental effects on social and business customs, and these would not be captured in an additive model. It has also resulted in substantial changes in communities of interest, which are assumed to be fixed in equation (12). However, the additive model would be commensurately better for analyzing smaller differences in market penetration or for analyzing a service that does not provide so revolutionary an improvement in communications as did the invention of the telephone.

Equation (12) also assumes that the service has no value except to communicate with others who have the service. The service is worthless if no one else subscribes. This assures that the null set is an equilibrium user set at any positive price.

This assumption sounds reasonable enough, but there are some possible exceptions. An individual may have noncommunications applications for the hardware. If the service is new, he may find it prestigious or derive self-satisfaction from being an innovator. However, these kinds of considerations go beyond the scope of this paper.

The additivity assumption is quite useful and allows us to derive a convenient expression for q_1^D as shown below.

The maxima \hat{U}_i^0 are defined by the *ceteris paribus* conditions and do not depend on anything in the communications industry. Maximizing equation (12) with respect to r_{i1}, \dots, r_{im} , subject to individual i 's budget constraint, we obtain

$$\hat{U}_i^1 = \hat{U}_i^0 - h_i(p) + \sum_{j \neq i} v_{ij} q_j \tag{13}$$

for some function h_1 where $h_1(0) = 0, h_1(p) > 0$ for all i .

It follows that

$$q_i^D = \{0 \text{ if } \sum_{j \neq i} v_{ij} q_j < h_i(p) \\ 1 \text{ if } \sum_{j \neq i} v_{ij} q_j \geq h_i(p)\} \tag{14}$$

where $v_{ij} \geq 0$ for all i, j .

We also assume constant marginal utility of money for a given individual. This means that $h_i(p)$ is a linear function:

$$h_i(p) = b_i p. \tag{15}$$

We can therefore write equation (14) as follows:

$$q_i^D = \{0 \text{ if } \sum_{j \neq i} w_{ij} q_j < p \\ 1 \text{ if } \sum_{j \neq i} w_{ij} q_j \geq p\} \tag{16}$$

where $w_{ij} = \frac{v_{ij}}{b_i}$ for all $i \neq j$.

E. FURTHER SIMPLIFICATION.

Both the monotonicity and the additivity assumptions greatly simplify the problem (at some cost in realism). However, we still must deal with the matrix \underline{V} which is the size of the population squared. Thus for a city with a population of one million, \underline{V} would have one trillion entries. Clearly, further simplification is required. The following two sections consider some possibilities for breaking the problem down to manageable size.

III. Uniform Calling Pattern

The preceding section began by considering the problem in its full generality and considered some reasonable kinds of simplifying assumptions. We now take the opposite approach, beginning with a very simple model and then relaxing assumptions to make the model more complicated and realistic.

In this section we assume that all the (off-diagonal) elements in any single row of \underline{V} are equal. This implies that no one has any special community of interest

other than the entire population. The *number* of subscribers affects an individual's demand, but he does not care who these subscribers are.

This may in fact be a reasonable approximation (for some purposes). We might reason that the individual communicates with a large number of people during the course of a year, many of whom he does not know in advance. The number of users may be as good a proxy as any for the incremental utility he derives from the service.

However, it is also true that most people belong to groups, each of which has a community of interest within itself. They also typically have a few principal contacts with whom they communicate more than with others. Thus, their demand for a communications service would depend on how many members of their community of interest group and which of their principal contacts subscribe to that service.

In any event, the uniform calling model seems like a good place to begin developing the theory. (This model is also adopted by Artle and Averous and Squire.)⁷ It allows some strong results to be derived and provides some useful insights about interdependent demand. Results from the uniform calling model also provide convenient reference points for analyzing more complex models, which are briefly discussed in the next section of this paper.

The uniform calling model allows us to write equation (15) as follows for a large population:

$$q_i^D = \begin{cases} 0 & \text{if } fw_i < p \\ 1 & \text{if } fw_i \geq p \end{cases}, \quad (17)$$

where $f =$ the user fraction (q/n), and

$$w_i = \sum_{j \neq i} w_j.$$

This in turn allows individuals to be ordered in terms of their demand for the service. That is, if $w_i \geq w_j$ (i 's demand exceeds j 's), individual i is in every equilibrium set that contains j .

A. DEMAND CURVE.

Since individuals can be ordered as above, every equilibrium user set consists of *all* individuals (i) for whom $w_i \geq$ some K . Similarly, for any q , there is at most one equilibrium user set with q members; i.e., the q people with the highest value of w_i . (If more than one person has the minimum w_i in the user set, all persons with that w_i must be in the user set for it to be an equilibrium.)

Thus, every equilibrium user set can be uniquely characterized by q , the number of members in it. We can develop the equilibrium theory for this model in terms

of the sum q , without specifying the individual elements q_i . In particular, we can define a demand curve; i.e., the locus of all the pairs (q,p) for which there exists an equilibrium user set (which would have to be the q people with highest w_i).

This gives us a convenient way of looking at the relationship between price and the equilibrium user sets. However, it is important to note that equation (10) is still indeterminate, and we must be careful in applying the demand curve in disequilibrium situations.

B. AN EXAMPLE.

Before discussing the general properties of such a demand curve, let us consider a specific example. Suppose the population is large, and w_i is distributed uniformly between 0 and 100 over the population. For the marginal individual

$$w_i = 100(1 - f). \quad (18)$$

For an equilibrium at $0 < f < 1$, fw_i for the marginal individual must equal p . [See equation (17).] Thus, the demand curve is the locus of points where

$$100f(1 - f) = p \quad (19)$$

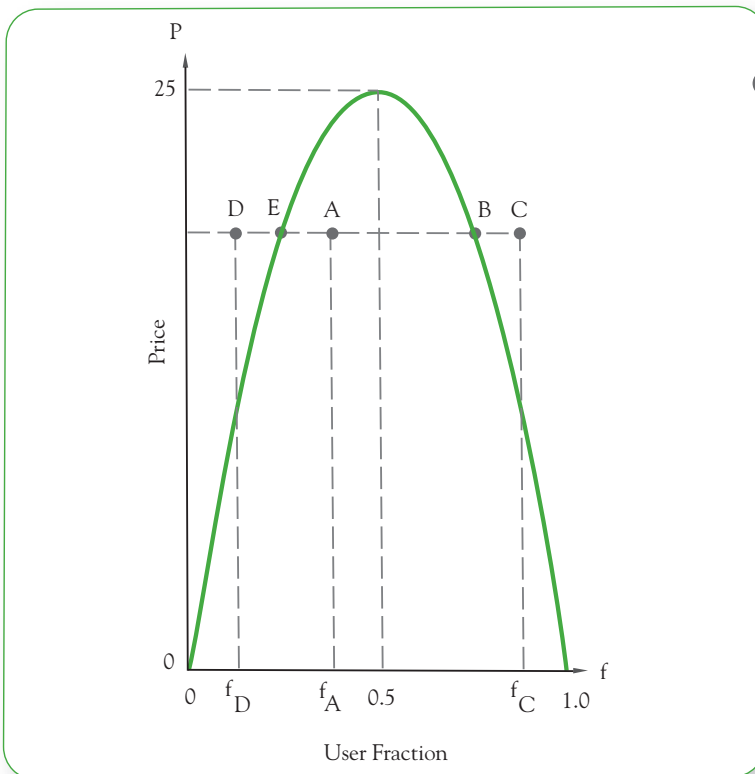


Figure 1

Demand Curve
for Uniform
Calling Pattern

As previously mentioned⁸ the null set ($f = 0$) is an equilibrium user set for all $p > 0$. For an equilibrium at ($f = 1$), p must be less than or equal to fw_i for all individuals. But the minimum of w_i and hence fw_i is 0. Thus, the only equilibrium is $p = 0$.

Figure 1 shows the demand curve. It consists of the entire positive p -axis plus an inverted parabola going through $(0,0)$ and $(1,0)$ and having a maximum at $(0.50,25)$.

The maximum equilibrium set is the right-hand side of the parabola for $0 < p \leq 25$; it is null for $p > 25$.

For small p , there is an enormous difference between the minimum and maximum equilibrium user sets. Thus, the actual equilibrium attained (for small p) depends critically on the initial disequilibrium condition and the disequilibrium adjustment process.

C. DISEQUILIBRIUM ANALYSIS.

The following analysis of disequilibrium is based on the adjustment process proposed above.⁹ As before, we assume a fixed price and restrict our attention to the demand side of the market. We first consider the special case in which the initial disequilibrium users are those with the highest w_i . We then consider the general case by examining arbitrary perturbations from an initial equilibrium.

D. INITIAL USERS HAVE HIGHEST w_i .

This subsection assumes that the initial user set consists of all individuals for whom $w_i \geq$ some K . This is necessarily true if the users form an equilibrium user set for any \bar{p} . Thus, the results apply to any disequilibrium brought about by a price change from an initial equilibrium.

Suppose we are originally in disequilibrium at A (in Figure 1), underneath the parabola. Given the user fraction f_A , the equilibrium price is higher than the actual price. All users are satisfied, but some nonusers would prefer to become users. If p remains constant, the user fractions will ultimately increase to B .

Suppose we are originally in disequilibrium at C . Given f_C , actual price exceeds equilibrium price and f declines to B .

Suppose we are originally in disequilibrium at D . Given f_D , actual price exceeds the equilibrium price. So, f declines. As f declines, the discrepancy between actual and equilibrium price increases until the market achieves equilibrium at $f = 0$.

In all these cases, the order in which individuals join or drop out is immaterial. The optimistic and pessimistic sequences are equivalent, and all versions of the adjustment process converge to the same equilibrium user set.

In general, the positive p -axis and the downward sloping part of the parabola in figure 1 consist of stable equilibria. The upward sloping part of the parabola consist of unstable equilibria.

The upward sloping part of the parabola can be regarded as a “critical mass” for the service. That is, for any positive price below the maximum of the parabola, the market must be forced to some initial disequilibrium beyond the critical mass before the service can grow by itself. The higher the price, the higher is the critical mass.

E. ARBITRARY PERTURBATIONS FROM EQUILIBRIUM.

The preceding analysis does *not* necessarily apply for arbitrary initial conditions. An initial user set of γ people may converge to very different equilibria depending on who those γ people are and on a more detailed specification of the adjustment process (than given above¹⁰).

For example, suppose the initial user set of γ people contains none of the γ people with highest w_i . Suppose it contains the people with the next γ highest values of w_i . Even though the initial market penetration is γ , the market can achieve a critical mass of 2γ if the γ nonusers with highest w_i all subscribe before any of the initial users drop out (optimistic sequence). However, if all the initial users who do not demand the service drop out before any nonusers subscribe (pessimistic sequence), the market may fail to achieve a critical mass much lower than γ .

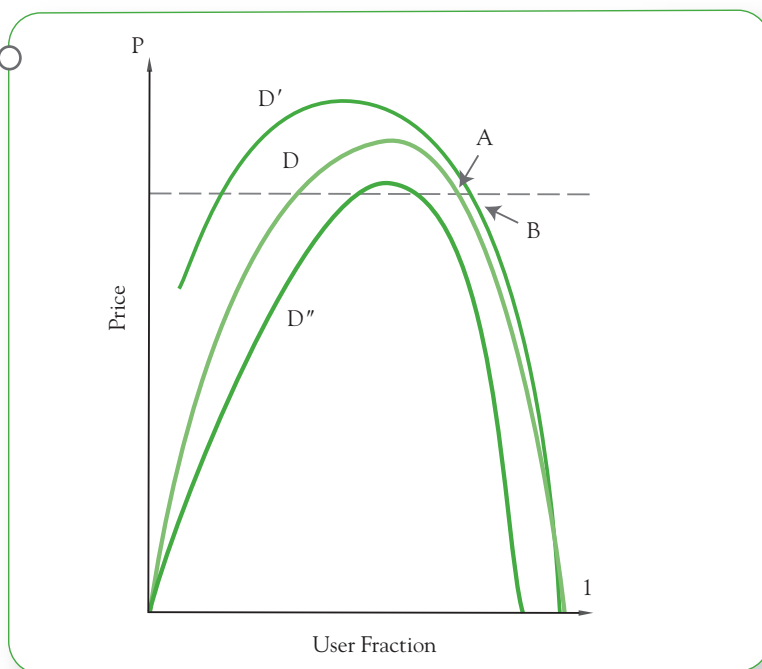
The results of the preceding subsection do apply for small but arbitrary perturbations (in the user set) from an original stable equilibrium. For example, suppose the market is originally in the stable equilibrium at A in Figure 2, and a set (R) of nonusers subscribes. We can analyze this by constructing a demand curve conditional on all members of R being users (D' in Figure 2). The perturbation of R becoming users may cause additional users to subscribe. However, no matter what the adjustment process is (subject to the rules laid down above¹¹), demand can never expand beyond B . And at B (or any point between B and A), only people in the *original* equilibrium set demand the service. Thus, the market eventually goes back to A (so long as the price remains fixed).

In the general case the perturbation may involve users dropping out as well as nonusers joining. We analyze this by constructing D' as before and D'' , demand conditional on those who drop out being nonusers. The market might converge to $f = 0$, if the perturbation is large and brings demand below critical mass. However, if the perturbation is sufficiently small, D'' will be sufficiently close to D that demand cannot go below critical mass. Thus, the market must return to the initial equilibrium.

This reasoning also applies to the stable equilibria at $f = 0$. If the perturbation is sufficiently small, D' will be sufficiently close to D that critical mass cannot be achieved, and the market must return to the initial equilibrium.

Figure 2

Arbitrary
Perturbations
from Equilibrium



F. GENERAL PROPERTIES OF THE MODEL.

Some properties of the above example apply generally to all uniform calling models. The entire positive p -axis always consists of stable equilibria. The demand curve always has an upward-sloping part, which constitutes the critical mass for the service for initial users sets with maximal w . It always has a downward sloping part (perhaps vertical), which consists of stable equilibria. However, both parts need not be unique, and the demand curve may be jagged. This allows the possibility of many stable equilibria for a given price.

IV. Nonuniform Calling Patterns

This section considers some models that are more complex than the uniform calling model. Some specific results are presented, but they are naturally not so strong as those of the previous section. Our primary objective is to point out the analytical complexities in such models and suggest some ways of dealing with them.

A. COMMUNITY OF INTEREST GROUPS.

Suppose the population consists of k groups ($k \leq n$). We assume that an individual has the same community of interest with everyone in the same group. However, this community of interest may be different for different groups. Mathematically, we assume that if individuals j and m are in the same group, $v_{ij} = v_{im}$ for all i .

B. DISJOINT GROUPS.

The simplest case is disjoint groups. That is, $v_{ij} = 0$ unless i and j are in the same group. In such a model, we can consider each group as a separate population, and all the analysis of the previous section carries over. A critical mass can be defined for each group in terms of market penetration within that group. For given \bar{p} the maximum possible number of stable equilibria is 2^k (unless the demand curve for some groups is jagged). The equilibria are characterized by which of the k groups achieve their critical mass.

C. JOINT GROUPS.

We now consider the case of joint groups, where v_{ij} does not necessarily equal zero for i and j in different groups. In this model, the incremental utility of the service to an individual is a function of g_i (and p). This is illustrated in Figure 3 for the case of the two groups.

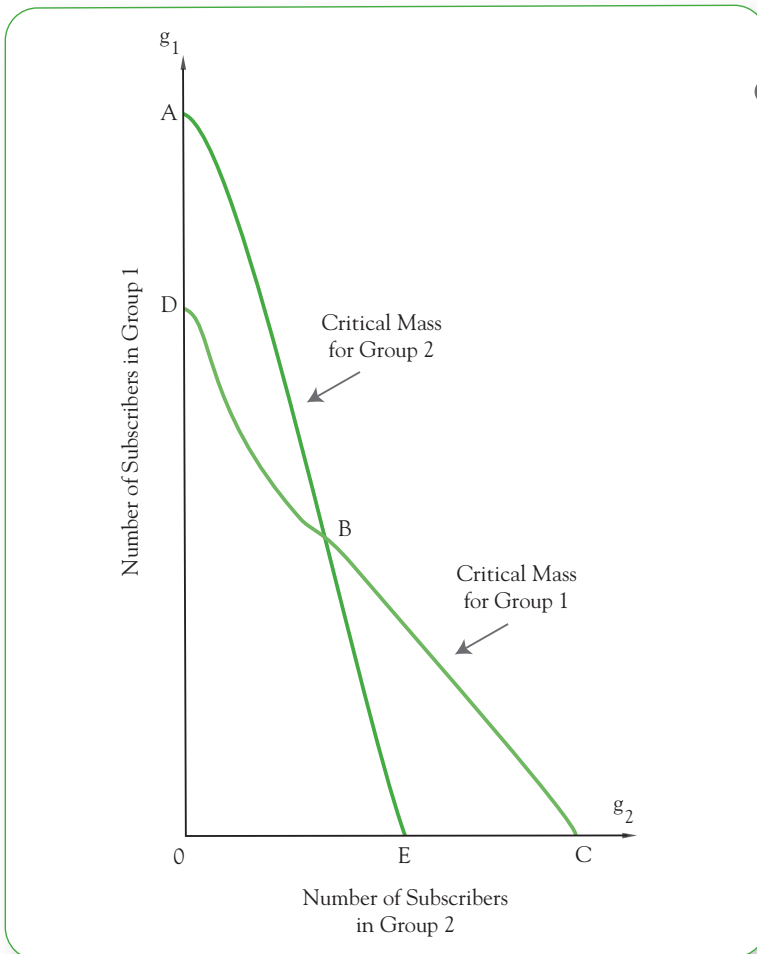


Figure 3

Critical Masses for Model with Communities of Interest

If the initial user set in Figure 3 consists of individuals with the highest w_i in each group (a weaker condition than requiring initial users to have the highest w_i in the population), we have the following result. If all groups achieve their critical mass (initial market penetration outside *ABCO* in Figure 3), the service will expand to the maximum equilibrium set. If no group achieves its critical mass (initial market penetration within *DBEO* in Figure 3), the service will collapse to the minimum equilibrium set. If some groups achieve their critical mass but others do not (initial market penetration within *ABD* or *BCE* in Figure 3), the service may expand to the maximum equilibrium set, collapse to the minimum equilibrium set, or achieve equilibrium somewhere in between. Which of these occurs depends on the parameters of the static model, the initial market penetration in each group, and the disequilibrium adjustment process. An upper bound on the number of stable equilibria is 2^k (unless the demand surface is jagged). Each equilibrium is characterized by which of the k groups achieve their critical mass and which do not.

D. FURTHER REFINEMENTS.

Introducing further refinements into the model is straightforward. As before, we somehow divide the population in k groups such that within each group individuals can be ordered in terms of their demand for the service. For greater realism, we could have a large value for k , but that has the drawback of requiring us to deal with a k -dimensional quantity vector and to contend with the possibility of many different equilibria (for each price).

E. FEW PRINCIPAL CONTACTS.

An individual's demand may depend primarily on which of his few principal contacts are users. A basic analytical tool for studying such demand is the "self-sufficient" user set; i.e., a set of individuals, each of whom demands the service conditional on all others in the set being users. An equilibrium user set must, of course, be self sufficient, but the converse is not necessarily true. Someone outside the self-sufficient set may demand the service if everyone in the set has it.

All self-sufficient sets necessarily belong to the maximum equilibrium user set. Moreover, if the entire self-sufficient set is contained in the initial disequilibrium user set, then the entire self-sufficient set is necessarily part of the final equilibrium user set.

In any practical problem, we could never hope to have a complete empirical list of principal contacts. The way to proceed in such cases is to specify a probability distribution which indicates (approximately) how likely various configurations of principal contacts are. This leads to some interesting combinatorial analysis but goes beyond the scope of this paper.

V. Some Implications for Supply and Pricing

Costs of providing a communications service depend on the consistency of the user set as well as its size. These costs are in themselves fully as complex as interdependent demand. Such complexities go beyond the scope of this paper, which is a study of interdependent *demand*. Nevertheless, it is useful to look at some implications of the preceding sections for supply and pricing of the service. We assume that the service is provided by a (regulated) monopoly and specify a very general cost function:

$$C = C(q_1, \dots, q_n), \quad (20)$$

where C is strictly monotonic (equality *not* allowed) in all its arguments.

We investigate various kinds of pricing strategies, some involving short-run losses. However, we assume that in (long-run) equilibrium, the monopoly must earn nonnegative profits. Some of the pricing strategies considered involve discriminatory pricing, but we do not assume that perfect discrimination is necessarily possible. Indeed, perfect discrimination would surely not be possible in any realistic situation. Consequently, the nonnegative profit restriction may be inconsistent with Pareto optimality. It is nevertheless consistent with existing real-world institutions.

We can now make a crucial distinction for planning supply of a communications service; i.e., the difference between “viability of the service” and “the start-up problem.” Viability is determined solely by the static model. It means that there exists a nonnull equilibrium user set that can be served with nonnegative profits. (We also refer to such a user set as “viable.”) The start-up problem is a dynamic consideration. It refers to the costs and practical difficulties of attaining a viable user set, starting from a small or null initial user set.

From a static point of view, any viable user set is superior to the null user set (in the sense that the supplier of the service and all users are at least as well off as before and possibly better off. Nonusers’ utilities are unchanged).¹² If there are several viable user sets, they can be compared to determine the social optimum (subject to the nonnegative profit restrictions). We can also determine the overall market equilibria consistent with various static supply models.

However, this kind of static analysis is incomplete and may in fact be misleading. We must also consider the dynamic aspects; i.e. the start-up problem. If the initial user set is small or null, the static social optimum may require ruinous (albeit temporary) promotional costs. Thus, all things considered, a smaller user set, or perhaps even the null set, may be superior.

The remainder of this section discusses various possible solutions to the start-up problem. We consider the case of a new service (e.g., a video communications service) and assume that the initial user set is null. This is necessarily an equilib-

rium user set at all positive prices. Thus, the service, even though viable, cannot get started by itself. It requires some positive action by the seller, probably involving temporary losses. The next three subsections consider some possible start-up strategies in the context of the various demand models we have studied in the previous sections.

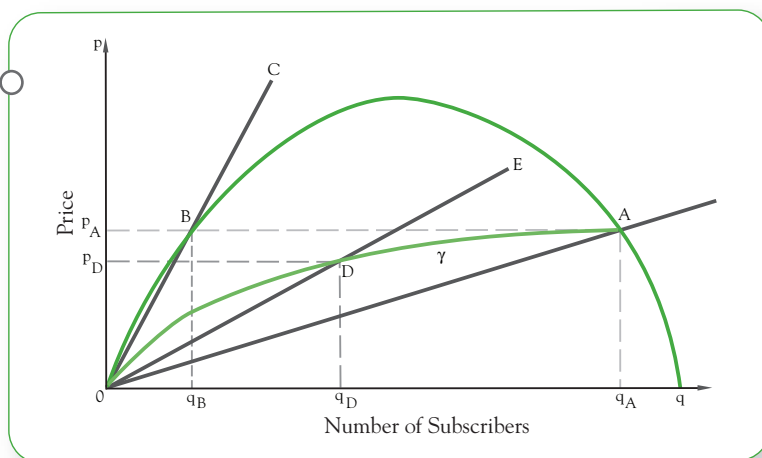
Although we assume in this analysis that the product is viable, it is worth noting that in real life the seller would have no such guarantee. There is always a risk involved in introducing a new product or service. The seller generally faces certain losses when the product or service is first introduced and the *prospect* of future profits. In a regulatory environment, the supplier must consider how the regulators will respond if the service succeeds and how they will respond if it fails.

A. UNIFORM CALLING PATTERN.

Let us assume that the demand curve is a (nonjagged) inverted U, and suppose the desired nonnull equilibrium set is A in Figure 4. The long-run optimal price is P_A , but some method must be contrived to get beyond the critical mass.

Figure 4

Solving the Start-Up Problem



B. DIRECT APPROACH.

The most direct approach is to give the service free to a selected group of people for a limited time. For this method to succeed, the initial user set must, of course, be sufficiently large to achieve critical mass. Half measures are worse than useless. If critical mass is not achieved, the whole effort will be a complete failure, and demand will eventually contract to zero.

The success of this approach may also depend on how the initial user set is selected. The discussion above¹³ shows that the optimal initial user does not necessarily consist of those with maximal w_i . (In the example, an alternative choice

of y people allowed a critical mass of $2y$ to be achieved.) However, if any other set is chosen, there should be some assurance that the initial users will not discontinue the service before those with higher w_i subscribe. If they do discontinue, the start-up effort will have failed to achieve anything.

C. LOW INTRODUCTORY PRICE.

Another way to start up the service is to have a low introductory price. The price could then be raised as the number of subscribers increased. There are any number of ways this could be accomplished. We can represent the introductory program as an expansion path in (q,p) space; e.g., γ in Figure 4. The expansion path shows how p increases as q increases.

The expansion path must pass through the origin to get the service started in this simple model. If, in addition, the expansion path is always concave downward, the introductory program has a very desirable property. Regardless of the order in which individuals enter the market, no individual ever subscribes and then later discontinues service after price and quantity rise along the expansion path. The proof is as follows. Suppose an individual enters at D . That means he is willing to pay p_D to have the service available for q_D of his communication. It follows from the uniform calling assumption that he would be an equilibrium user at *any* point along the straight line OE through D . But the concavity assumption assures that the expansion path γ is always below OE for $p > p_D$, $q > q_D$. Thus, if the individual joins at D , he cannot drop out at any point on γ to the right of D . Q.E.D.

The above condition can be very important if the cost of connecting an individual to the network is large.

Perhaps the most interesting program with a low introductory price is usage-proportional pricing. The remainder of this subsection investigates that plan under various assumptions.

We first consider the possibility that all subscribers have equal usage, proportional to the number of subscribers. It follows that price is also proportional to number of subscribers [instead of proportional to a quadratic function of number of subscribers as in the equilibrium model, equation (19)], and the expansion path of the service is a straight line.

One possibility would be to let $p = p_A \frac{q}{q_A}$. In that case demand would expand along OA . Price would increase automatically as q increased. When q reached the optimum, q_A , price would just equal its optimum, p_A .

An alternative is to let $p = p_A \frac{q}{q_B}$. Demand would expand along OB until the critical mass was reached. Then, for further expansion price should be fixed at $p_A(-\epsilon)$, and demand would expand along BA .

Unfortunately, the equal-usage assumption may not be very realistic. If not, usage-proportional pricing would exclude individuals with a lot of low value usage and would admit (for a low price) individuals with a small amount of high value usage.

This might not be a bad idea, even in the long run. In general, usage-proportional pricing would be appropriate if the network were being used at capacity and costs were closely related to total usage, not necessarily to the number of subscribers. A fixed price per subscriber would be appropriate if the network were not being used at capacity and much equipment had to be committed for each subscriber.

The latter is perhaps more typical of a new communications service. Thus, usage-proportional pricing would probably result in some inefficiencies and misallocation of resources. Whether these inefficiencies are substantial or trivial is an empirical question. In any event they should be compared to the inefficiencies of other start-up programs; e.g., the direct approach discussed in the previous subsection.

For a mature service the price should probably have a higher fixed component and a lower usage component. However, the externalities in consumption still have to be taken into account.¹⁴

A program with a low introductory price relies more on market processes than does the direct approach discussed in the previous subsection and does not depend so critically on the managers' judgment. In particular, the managers are not required to determine which individuals have the highest w_i . The individuals select themselves by choosing to subscribe to the service at the offered price.

D. COMMUNITY OF INTEREST GROUPS.

Community of interest groups may greatly reduce the practical difficulty of starting up the service. Maximum equilibrium demand may be achieved even if the initial user set is small—so long as that set exceeds the critical masses for some community of interest groups.

At the same time, community of interest groups place a greater burden on whatever procedure is used to select the initial users. If the initial user set is selected by managers, they must know what the community of interest groups are and decide how many individuals to select from each. In some circumstances, it would be optimal to select everyone from the same group; in other cases a more even spread would be optimal. In any event, the managers must make this choice, and the success of the program may be greatly influenced by how well they choose.

Community of interest groups also place a greater burden on the market process for programs involving a low introductory price. The expansion path

involves price and g_1, \dots, g_k (instead of q as in Figure 4). Efficiency requires that k different linear combinations of the g_i be concave downward. (The equivalent condition to γ 's being concave downward in Figure 4.) But we have only one control variable, p . Clearly, it may be impossible to satisfy all k concavity conditions.

Thus, any program based on a (single) low introductory price may be inefficient in the sense that some individuals may join at the low introductory price but later drop out as the price rises. If this problem is serious, the seller may find it advantageous (and perhaps necessary) to use discriminatory pricing to assure that only "permanent" users join.

E. FEW SUBSTANTIAL CONTACTS.

If an individual's demand depends on his principal contacts' being users, the start-up problem may be fundamentally different from that discussed above. It may be unnecessary for there to be hundreds or thousands of users before an individual demands the service. A user population of two or three may be self-sufficient—if they are the right two or three people.

These small self-sufficient sets do not necessarily promote further growth but they do allow a different kind of approach to starting up the service. The seller can begin by establishing small self-sufficient user sets. He can then gradually expand the size and number of these sets until the desired equilibrium user set is attained or until the service starts to grow by itself. The practicality of this method depends primarily on the size of the (minimum) self-sufficient user sets.

The smallest possible self-sufficient user sets consists [sic] of two mutual contacts for whom both v_{ij} and $v_{ji} \geq p$. If there are many such pairs, i, j in the population, the service will start up, expanding beyond the minimum equilibrium set, with little or no help. Selling the service to i and j requires only getting the two together. And they may organize themselves and agree to subscribe to the service (contrary to the disequilibrium adjustment process assumed above¹⁵). After i and j both subscribe to the service, they may attract other individuals, and the service can grow further.

In fact, such growth from self-sufficient sets of two probably accounts in large part for the success in starting up telephone service. Indeed, telephone made a substantial penetration of the market while it was entirely a private-line service.

However, even if the service is viable, its incremental utility may be insufficient for very many people to demand the service to communicate with a single principal contact. If this is the case, the start-up problem is more difficult, and we must deal with larger self-sufficient user sets.

A self-sufficient set of three mutual contacts might also be able to organize itself and have all three members agree to subscribe together. However, this

becomes progressively more difficult and unlikely as the size of the self-sufficient sets increases. It would be especially difficult if all the members of the set were not mutual contacts and no one knew all the other members of the set.

The seller might be able to gather data and determine self-sufficient use sets. He could then try to sell the service to everyone in such a set simultaneously. Naturally this is more difficult, the larger is the self-sufficient set. In fact, the difficulties of organizing even six to eight people and getting them all to agree to a joint purchasing decision may be far from trivial.

Nevertheless, the seller might do well to gather data on communications patterns and try to determine self-sufficient user sets. But it may be necessary to combine this with some other kind of start-up program. This could take the form of the direct approach or the low introductory price previously discussed. However, it is also possible to have a continual program in which each new subscriber is offered a low rate until the seller can connect a self-sufficient set of users that contains him. This kind of program would be effective if new users had high values of w_i but little community of interest with the current user set. ▼

REFERENCES

1. ARTLE, R. AND AVEROUS, C. "The Telephone Service as a Public Good: Static and Dynamic Aspects." *The Bell Journal of Economics and Management Science*, Vol. 4, No. 1 (Spring 1973), pp. 89-100.
2. MARRIS, R. *The Economic Theory of "Managerial" Capitalism*. New York: Free Press, 1964.
3. SQUIRE, L. "Some Aspects of Optimal Pricing for Telecommunications." *The Bell Journal of Economics and Management Science*, Vol. 4, No. 2 (Autumn 1973), pp. 515-525.

1 See [1].

2 An earlier attempt to model interdependent demand is Marris [2]. Marris develops a general theory of demand for new products but does not consider communications in particular. The interdependent aspects of demand have a much different interpretation in his analysis than in this paper or in Artle and Averous [1] or Squire [3] (discussed later in the text). Also, irreversibility plays a much larger role in Marris' analysis. Nevertheless, Marris develops some of the same concepts used in this paper; e.g. critical mass.

3 In [3]

4 We can also define equilibrium user sets with respect to any given set of discriminatory prices; i.e., a set of users such that

$$q_i = q_i^D(p_i, q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_n) \tag{9a}$$

for all i , where p_i is the price charged to individual i .

5 P. 22

6 P. 22

7 In [1] and [3], respectively.

8 P. 26

9 P. 22

10 P. 22

11 P. 22

12 Of course, this issue becomes much more complicated if we consider interrelationships with other markets and possible income redistribution.

13 P. 29

14 See Squire [3].

15 P. 22.