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Optimal Beliefs, Asset Prices, and the Preference for Skewed Returns

By Markus K. Brunnermeier, Christian Gollier, and Jonathan A. Parker*

This paper studies portfolio holdings and asset prices in an economy in which people's natural tendency to be optimistic about the payout from their investments is tempered by the ex post costs of basing their portfolio decisions on incorrect beliefs. We show that this model can generate the following three stylized facts.

First, households' portfolios are not optimally diversified according to various theoretical-based measures (Marshall E. Blume, Jean Crockett, and Irwin Friend 1974; William N. Goetzmann and Alok Kumar 2001; Laurent E. Calvet, John Y Campbell, and Paolo Sodini 2006; Stephanie Curcuru et al. forthcoming). The costs of this lack of diversification appear to be modest. Most households hold a well-diversified portfolio of mutual funds and also a significant number of shares of one or two additional stocks.[1]

Second, and part of the evidence for the first fact, household portfolios are tilted toward stocks with identifiable attributes, and, in particular, toward holdings of individual stocks with positive skewness. Further, undiversified households hold individual stocks that have relatively high idiosyncratically skewed returns, and their portfolios have relatively high idiosyncratically skewed returns (Todd Mitton and Keith Vorkink forthcoming).

Finally, positively skewed assets tend to have lower returns. This is true for stocks in the US stock market in general (Yijie Zhang 2005) as well as for specific well-studied examples, such as the value-growth premium and the long-run underperformance of IPOs.[2]

This paper argues that these three patterns are observed because human beings want to believe what makes them happy and want to make good decisions that lead to good outcomes in the future. We consider an exchange economy with two periods and complete markets in which households with log utility invest in the first period and consume in the second period. We show that these patterns arise in this economy when investors hold beliefs that optimally trade off the ex ante benefits of anticipatory utility against the ex post costs of basing investment decisions on biased beliefs.

Our model of beliefs follows the optimal expectations framework of Brunnermeier and Parker (2005, henceforth BP). We assume that people behave optimally given their beliefs, choosing portfolios that maximize their expected present discounted value of utility flows.[3] Because investors care about expected future utility flows, they are happier if they overestimate the probabilities of states of the world in which their investments pay off well. But such optimism would lead to suboptimal decision making and lower levels of utility on average ex post. Optimal beliefs trade off these competing

* Brunnermeier: Bendheim Center for Finance, Princeton University, Princeton, NJ 08544 (e-mail: markus@princeton.edu); Gollier: (LERNA), University of Toulouse, Place Anatole France, 31042 Toulouse cedex, France (e-mail: gollier@cict.fr); Parker: Princeton University, Princeton, NJ 08544 (e-mail: jparker@princeton.edu).

[1] Further, and complementary evidence for our purposes, household income risk is not insured fully by households across groups of households, where moral hazard would seem an implausible reason for this failure (Orazio Attanasio and Steven J. Davis 1996).

[2] There is also complementary evidence from gambling behavior. Lotteries are highly skewed assets, and the demand for lottery tickets rises with probability controlling for expected return. And, in parimutuel betting on races, in which the bettors determine returns in equilibrium, longshots have lower expected returns than favorites.

[3] Annette Vissing-Jorgensen (2004) shows that differences in investor's self-reported beliefs about future market returns (or internet stock returns) are highly significantly correlated with the share of their portfolio in equities (or in Internet stocks).

forces. People's beliefs maximize the objective expectation of their well-being, the average of their expected present discounted value of utility flows. This economic model of beliefs balances the anticipatory benefits of optimism against the costs of basing actions on distorted beliefs. Because the costs of small deviations from optimal behavior are of second order, and the anticipatory benefits of biases in probabilities typically are of first order, optimal subjective and objective probabilities differ. Christian Gollier (2005) and BP study portfolio choice and asset prices in incomplete markets (and a two-state complete market example). This paper derives a general characterization in a complete markets economy.

In terms of portfolios, we show, in Section I, that an investor with optimal expectations does not fully diversify his portfolio but instead biases upward (a lot) his subjective beliefs about the likelihood of one state, and biases downward (a little) his subjective beliefs about the likelihood of all the remaining states. He does this because there is a natural complementarity between believing a state more likely and purchasing more of the asset that pays off in that state. Once a state is perceived as more likely, one wants more consumption in that state, and once one has more consumption in that state, one wants even more to believe that that state is more likely. We further show that an investor chooses to be optimistic about the states associated with the most skewed Arrow-Debreu securities: either the least expensive state (when states are equally likely) or the least likely state (when state prices are actuarially fair), or the least expensive and least likely state when these coincide (in general). This happens because low-price and low-probability states are the cheapest states in which to buy consumption, and so distort consumption in the rest of the states the least (for a given bias). Thus, portfolios are not perfectly diversified, households overinvest in the most skewed assets, and household portfolios have positively skewed returns.

In general equilibrium, we show, in Section II, that investors tend to be optimistic about different states. Thus, investors' portfolios have idiosyncratically skewed returns and consumption insurance appears to be incomplete. In terms of asset prices, this preference for skewed returns has price effects. *Ceteris paribus*, states with relatively small probabilities tend to have relatively low expected returns.[4]

All proofs are contained in an Appendix available at www.e-aer.org/data/may07/P07015_app.pdf.[5]

I. Portfolio Choice with Optimal Expectations

The economy has two periods. There are S possible states of the world in period 2, with state s having objective probability $\pi_s > 0$. An investor with subjective beliefs $\hat{\pi}_s$ allocates his wealth among a complete set of Arrow-Debreu securities in the first period and consumes the payoff from this portfolio in the second period. A person's investment choices, $\mathbf{c} = \{c_1, c_2, \ldots, c_S\}$, maximize his expected utility given his subjective beliefs, $\hat{\boldsymbol{\pi}} = \{\hat{\pi}_1, \hat{\pi}_2, \ldots, \hat{\pi}_S\}$:

$$V_1 = \max_{\mathbf{c}} \sum_{s=1}^{S} \hat{\pi}_s \ln(c_s) \quad (1)$$

subject to $\sum_{s=1}^{S} p_s c_s = 1$ and $c_s \geq 0$,

where $p_s > 0$ is the price of the Arrow-Debreu security yielding one unit in state s, and initial wealth is normalized to unity.[6] Optimal portfolio choices exist and are unique:

$$c_s^*(\hat{\pi}) = \frac{\hat{\pi}_s}{p_s}. \quad (2)$$

A. *Optimal Beliefs*

But what are the investor's subjective beliefs? One assumption is that people hold rational expectations, an extreme assumption typically made for its tractability and for the discipline it provides. Further, the argument goes, since objective beliefs lead to the best decisions and thus the highest average present discounted value of utility, people have the incentive to

[4] Nicholas C. Barberis and Ming Huang (2007) show that the exogenous belief distortions of Prospect Theory make similar predictions.

[5] A more extended analysis and proofs can be found in Brunnermeier, Gollier, and Parker (2007).

[6] While here we assume $p_s > 0$. Section II endogenizes prices and derives this as a result.

acquire information and learn rationally so that their beliefs should have a general tendency to converge to objective probabilities.

But, in fact, objective beliefs do not lead to the highest expected present discounted value of utility flows. An investor can increase V_1 by holding distorted beliefs, trading on these, and then anticipating high average future utility. But biased beliefs come at a cost. A person who makes objectively poor investment decisions has lower utility ex post, $V_2 = \ln c$, on average. Our theory balances these effects. It trades off the anticipatory benefits of optimism against the utility losses caused by decisions based on optimistic beliefs. Further, this approach provides discipline: biases in beliefs are determined endogenously by the economic environment.

Formally, each investor's beliefs maximize his well-being, defined as the average expected utility across periods 1 and 2, when actions are optimal, given subjective beliefs. That is, $\hat{\boldsymbol{\pi}}$ maximizes $\frac{1}{2}E[V_1 + V_2]$ subject to the constraints that the $\hat{\pi}_s$ are probabilities, and that portfolio choices are optimal given $\hat{\boldsymbol{\pi}}$. This well-being function is similar to that proposed in Andrew J. Caplin and John Leahy (2000), and analogous arguments support our use of this function. Optimal beliefs maximize the Lagrangian:

$$(3) \quad \mathcal{L} = \sum_{s=1}^{S} \hat{\pi}_s \ln c_s^*(\hat{\boldsymbol{\pi}}) + \sum_{s=1}^{S} \pi_s \ln c_s^*(\hat{\boldsymbol{\pi}}) - \mu \left[\sum_{s=1}^{S} \hat{\pi}_s - 1 \right]$$

(and subject to $\hat{\pi}_s \geq 0$). Beliefs impact well-being directly through anticipation of future flow utility and indirectly through their effect on portfolio choice.[7]

Because $c_s^*(\hat{\boldsymbol{\pi}})$ is continuous in subjective probabilities, $\mathcal{L}$ is also, and, since probability spaces are compact, optimal beliefs exist. Further, if $\hat{\pi}_s = 0$, $c_s^*(\hat{\boldsymbol{\pi}}) = 0$, the investor would get infinite negative utility if state s is realized.

PROPOSITION 1 (Existence of interior optimal beliefs): *Optimal subjective probabilities,* $\hat{\boldsymbol{\pi}}^*$, *exist and are positive:* $0 < \hat{\pi}_s^* < 1$ *for all s.*

Turning to the characterization of behavior, the first-order conditions for beliefs are

$$(4) \quad \frac{\pi_s}{\hat{\pi}_s} - \ln \frac{\pi_s}{\hat{\pi}_s} = \mu - 1 + \ln \frac{p_s}{\pi_s} \text{ for all } s.$$

And the second-order conditions (reorganized) are

$$(5) \quad \hat{\pi}_s \left[1 - \frac{\pi_{s'}}{\hat{\pi}_{s'}} \right] \leq \hat{\pi}_{s'} \left[\frac{\pi_s}{\hat{\pi}_s} - 1 \right] \text{ for all } s \neq s'.$$

By Proposition 1, the first-order conditions have real solutions with $0 < \hat{\pi}_s < 1$ for all s. The left-hand side of each first-order condition is a convex function of $\pi_s / \hat{\pi}_s$ with its minimum at $\hat{\pi}_s = \pi_s$, and the right-hand side is independent of beliefs. Thus, each first-order condition has one or two solutions.[8] If for state s, the right-hand side equals one, then objective beliefs are the only possible solution. Otherwise, the right-hand side is greater than one, and there are two solutions to the first-order condition, one with a positive bias and one with a negative bias. From the second-order condition, if beliefs about the probability of s' are biased upward, so that $\pi_{s'} / \hat{\pi}_{s'} < 1$, then $\pi_s / \hat{\pi}_s > 1$ for all $s \neq s'$, so that beliefs about the probabilities of all other states are biased downward. Further analysis of the program shows that objective beliefs are optimal beliefs only if $S = 2$, and $\pi_1 = \pi_2$, and $p_1 = p_2$.

PROPOSITION 2: *If* $S = 2$, $\pi_1 = \pi_2$, *and* $p_1 = p_2$, *objective beliefs are optimal. Otherwise:*

(i) one and only one state has upward-biased subjective probability, and all other states have downward-biased subjective probability: $\exists\, s'$ *such that* $\hat{\pi}_{s'}^* > \pi_{s'}$ *and* $\hat{\pi}_s^* < \pi_s$ *for all* $s \neq s'$;

[7] This approach is a "frictionless extreme" in the sense that only ex post costs limit biases in beliefs. Additional factors might also constrain biases.

[8] The equations specified by (4) are Lambert W functions in $\pi_s / \hat{\pi}_s$, and, in general, no closed-form solution exists.

(ii) among states with downward-biased subjective probabilities, states with larger price-probability ratios (economy-wide stochastic discount factors) are biased down by larger factors: for s'', $s' \in \{s : \hat{\pi}_s^* < \pi_s\}$, $\pi_{s'}/\hat{\pi}_{s'}^* > \pi_{s''}/\hat{\pi}_{s''}^*$ *if and only if* $p_{s'}/\pi_{s'} > p_{s''}/\pi_{s''}$ *and* $\pi_{s'}/\hat{\pi}_{s'}^* = \pi_{s''}/$ *if and only if* $p_{s'}/\pi_{s'} = p_{s''}/\pi_{s''}$.

The result that the investor biases upward the probability of only one state comes from a natural complementarity between the subjective belief about a state and the level of consumption in that state. Once a state is perceived as more likely, one wants more consumption in that state, and once one has more consumption in that state, there are greater benefits to believing that that state is more likely. The second part of the proposition is driven by the same force. An investor purchases less consumption in a more expensive state, and so has a greater incentive to believe that the more expensive state is unlikely to occur.

For the remainder of this section, we rule out the knife-edge case that delivers rationality.

ASSUMPTION 1: *Either* $S > 2$ *or* $\pi_s \neq 1/2$ *or* $p_1 \neq p_2$.

We now characterize which state an investor is optimistic about. The benefits of optimism about a state are related to the consumption purchased in that state and the costs are related to the objective misallocation of consumption across states. The costs are second-order, so for an infinitesimal change in beliefs a person should bias upward the probability of the state in which he has the most consumption. Starting from objective beliefs, this is the cheapest state in terms of price-probability ratio.

Analogously, optimal expectations, which are not infinitesimal deviations from rational expectations, tend to bias upward the probability of the cheapest state because extra consumption in that state requires the least decrease in consumption in the remaining states, where "cheap" refers to a combination of low price and low ratio of price to probability. If all states have the same ratio of price to probability, the investor biases upward the probability of the lowest price (and probability) state. If states have equal objective probabilities but vary in price, then the investor overestimates the probability of the least expensive state because this requires the smallest reduction in consumption in the other states.[9]

PROPOSITION 3:

(i) If all states have the same price-probability ratio, $p_s/\pi_s = m$ *for all s, the investor overestimates the probability of (one of) the state(s) with the lowest probability.*

(ii) If all states are equally likely, $\pi_s = \pi$ *for all s, then the investor overestimates the probability of (one of) the state(s) with the lowest price-probability ratio.*

(iii) If one state has both the lowest probability and the lowest price-probability ratio, then the investor overestimates the probability of this state.

(iv) For any state, there exist $\overline{m}$ *and* $\underline{m}$, *such that for a sufficiently low price* $p_s \leq \underline{m}\pi_s$, *optimal beliefs overestimate the probability of this state,* $\hat{\pi}_s^* > \pi_s$, *and for a sufficiently high price* $p_s \geq \overline{m}\pi_s$, *optimal beliefs underestimate the probability of this state,* $\hat{\pi}_s^* < \pi_s$.

B. *Optimal Portfolio Choice*

While beliefs are interesting, our ultimate interest is in explaining prices and quantities, that is, returns and portfolios.

First, consider the case of actuarially fair prices, $p_s/\pi_s = m$ for all s. Under rational expectations, the optimal portfolio is risk free. For optimal beliefs, equations (2) and (4) imply first-order conditions $1/(mc_s) + \ln(mc_s) = \mu - 1 + \ln m$ for all s, where the Lagrange multiplier μ is such that $\overline{c}\pi_{s'} + \underline{c}\sum_{s \neq s'} \pi_s = 1/m$ and $(\underline{c}, \overline{c})$ are the two solutions to this equation: $\overline{c}$ is the consumption level in the state with positively biased subjective probability, and $\underline{c}$ is the consumption level in the remaining states. The following corollaries follow directly.

[9] Other effects are present—the elasticity of consumption to beliefs, and the curvature of the utility function matter—but this is the strongest effect here.

COROLLARY 1 (Preference for skewness): *If $p_s/\pi_s = m$ for all s, then the investor prefers the most skewed assets. The investor buys $\bar{c}$ of one of the Arrow-Debreu securities that pays off with the smallest probability and $\underline{c} < \bar{c}$ of each of the remaining securities.*

COROLLARY 2 (Two fund separation): *If $p_s/\pi_s = m$ for all s, then the investor holds a portfolio consisting of the risk-free asset (an equal amount of all Arrow-Debreu securities) and an additional positive amount of one and only one of the most skewed securities.*

These corollaries match two of the empirical findings described at the start of the paper. First, investors are well diversified except for investing in one asset. Second, both the return on the additional asset they hold and the return on their portfolios are positively skewed.

When prices are not actuarially fair, investors still do not optimally diversify and invest more than the investor with rational expectations in securities with skewed returns. The latter occurs both because investors tend to be optimistic about states with low probabilities and prices (Proposition 3), and because pessimism is more severe for states with high prices (Proposition 2(ii)). In general, diversification, preferred by an agent with rational expectations, would destroy skewness, preferred by an agent with optimal expectations.

As we now show, equilibrium prices tend to make different investors optimistic about different states, and so portfolios in equilibrium tend to be heterogeneous and have idiosyncratically skewed returns.

II. Asset Pricing in an Exchange Economy with Optimal Expectations

We consider an exchange economy with a unit mass of investors, $S > 2$, and aggregate per capita endowment in each state of C_s. Due to space constraints, we consider an example that illustrates the general characteristics of optimal expectations equilibria. In this economy, portfolios are heterogeneous across investors, portfolio returns are idiosyncratically skewed, and securities with positively skewed returns have lower expected returns.

DEFINITION: *An optimal expectations equilibrium is a portfolio $\mathbf{c}^i$ and beliefs $\hat{\boldsymbol{\pi}}^i$ for each agent i and prices $\mathbf{p}$ such that: (a) each agent's portfolio is optimal given his beliefs and prices; (b) each agent's beliefs maximize his well-being; (c) the market for each asset clears.*

Before analyzing a more complex environment, consider an economy with equally probable states, $\pi_s = \pi$, and no aggregate risk, $C_s = C = 1$. Suppose that prices are actuarially fair, $p_s = p$. Each investor biases upward the subjective probability of one state, purchases $\bar{c}$ of the Arrow-Debreu security associated with this state, and purchases $\underline{c} < \bar{c}$ of the Arrow-Debreu security associated with the remaining downward-biased states (where $\bar{c}$ and $\underline{c}$ are as defined in the previous section). This is an equilibrium if an equal share of agents are optimistic about each state, so that demand for consumption is equal across states, and each asset's price is $p = 1/S$. This equilibrium is locally stable, in the sense that a small change in prices would lead all investors to bias up the subjective probabilities of the cheapest states (Proposition 3(ii)), which would lead to excess demand for consumption in these states and a (relative) increase in price for the cheapest states.

Now consider similar economies in which the variation in the aggregate endowment across states is "not too large." An equilibrium with actuarially fair prices exists as long as there exist different shares of agents who are optimistic about each state so that the demand for each asset matches the supply. Thus, in economies with equally probable states and low aggregate risk, prices are fair and agents hold heterogeneous beliefs, overinvest in different skewed assets, and thus hold portfolios with idiosyncratically skewed returns. In the corresponding rational expectations equilibrium, investors' portfolios would be homogeneous and perfectly diversified, $c_s = C_s$. Further, also unlike in the rational expectations equilibrium, aggregate risk, in limited amounts, is not priced.[10] People have an interest in risk, and a small amount of aggregate risk satisfies this desire without changing prices.

[10] The equality of probabilities across states is key for this results. With unequal probabilities, aggregate risk is typically priced.

Finally, as aggregate endowment risk increases, beliefs become less heterogeneous.

PROPOSITION 4 (Heterogenous portfolios and idiosyncratic skewness): *Suppose that* $\pi_s = \pi$ *for all s. For any vector* $(C_1,\ldots, C_S)$ *of aggregate endowment such that* $\underline{c} \leq C_S \leq \bar{c}$ *and* $\Sigma_{s=1}^{S} C_s = \bar{c} + (S - 1)\underline{c}$, *there exists an optimal expectations equilibrium with the following characteristics: (i) prices are actuarially fair:* $p_s = p$ *for all s; (ii) for all s, a fraction* $\lambda_s = (C_s - \underline{c})/(\bar{c} - \underline{c})$ *of investors buys* $\bar{c}$ *of the Arrow-Debreu security associated to state s and* $\underline{c}$ *of the security for every other state, where* $\underline{c}$ *and* $\bar{c}$ *are defined in Section I.*

Having established this result, we now construct our example that matches all three stylized facts discussed in the introduction. Consider an economy with some unlikely states and some likely states. At actuarially fair prices, each investor would bias upward his probability of one of the unlikely states. Analogous to Proposition 4, this is an equilibrium if there exists shares of investors who are optimistic about each unlikely state such that the market clears. For example, if $\pi_s = \pi^A$ and $C_s = C^A := (1/\underline{s})\bar{c} + [1 - (1/\underline{s})]\underline{c}$ for $s \leq \underline{s}$ and $\pi_s = \pi^B > \pi^A$ and $C_s = C^B := \underline{c}$ for $s > \underline{s}$, then an equilibrium with fair prices exists in which $1/\underline{s}$ investors bias up their probabilities for states $s \leq \underline{s}$. But if the endowments across states are not so different, then prices, p_s, in the unlikely states must be higher so that demand for output is also relatively lower in these states, and hence the expected returns of the most skewed Arrow-Debreu securities are lower.

PROPOSITION 5 (Underperformance of skewed assets): *For a small reduction in* $C^A - C^B$ *such that* p_s *does not change for* $s > \underline{s}$, p_s *increases for* $s \leq \underline{s}$ *so that: (i) the securities with the more skewed returns have lower expected returns than in a rational expectations equilibrium; (ii) the securities with the more skewed returns have relatively lower expected returns,* $\pi^A/p_s < \pi^B/p_{s'}$ *for all* $s \leq \underline{s}$ *and* $s' > \underline{s}$.

This equilibrium fits all three stylized facts: (a) portfolios are heterogeneous and not perfectly diversified; (b) each investor overinvests in one security that is more skewed than the average security and his portfolio return is more skewed than the market return; (c) more positively skewed securities have lower returns. These results relate to the use of co-skewness as a pricing factor. As C^A varies (and in richer environments), the relative importance of idiosyncratic skewness and aggregate skewness for asset prices varies. Finally, consumption insurance appears incomplete, but not because of missing markets or moral hazard, but because households optimally choose to hold risk.

How important are the assumptions of our example? First, if the low-probability states have much larger endowments than the other states ($C_s > C^A$), then we still match (a) and (b), but the expected returns on the most skewed assets are higher for the usual reason that investors discount payouts in states with high aggregate endowment. Even in this case, however, Proposition 5(*i*) implies that the returns on the most skewed assets are lower than in a rational expectations equilibrium. Second, the assumption that some probabilities are equal is not essential. If probabilities differ among high-probability states, nothing changes. If probabilities differ among low-probability states, prices would have to be higher for lower probability states for investors to remain indifferent among (perhaps a subset) of states and for portfolios to be heterogeneous.

The desire for skewness can also impact the market return. If bad aggregate states have low probabilities, as for disasters or peso problems, then it is possible for the desire for skewness to increase the equity premium as investors seek to avoid negative skewness.

In conclusion, the natural human tendency toward optimism tempered by the real costs of poor decisions implies that people hold heterogeneous, underdiversified portfolios to attain skewed returns, and that this behavior reduces the returns of positively skewed assets.

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