

The Strategic Implications of Scale in Choice-Based

Conjoint Analysis

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Abstract

Choice-based conjoint (CBC) studies have begun to rely on simulators to forecast equilibrium prices for pricing, strategic product positioning, and patent/copyright valuations. While CBC research has long focused on the accuracy of estimated relative partworths of attribute levels, predicted equilibrium prices and strategic positioning are surprisingly and dramatically dependent on the magnitude of the partworths relative to the magnitude of the error term (scale). Although the impact of scale on the ability to estimate heterogeneous partworths is well-known, neither the literature nor current practice address the sensitivity of pricing and positioning to scale. This sensitivity is important because (estimated) scale depends on seemingly innocuous market-research decisions such as whether attributes are described by text or by pictures. We demonstrate the strategic implications of scale using a stylized model in which heterogeneity is modeled explicitly. If a firm shirks on the quality of a CBC study and acts on incorrectly observed scale, a follower, but not an innovator, can make costly strategic errors. We demonstrate empirically that image quality and incentive alignment affect scale sufficiently to change strategic decisions and affect patent/copyright valuations by hundreds of millions of dollars.

Keywords: *Conjoint analysis, market research, choice models, scale*

1. Scale Affects Patent/Copyright Valuations and Strategic Decisions

With an estimated 18,000 applications per year, conjoint analysis is one of the most-used quantitative market research methods (Orme 2014; Sawtooth Software 2015). Over 80% of these conjoint applications are choice-based. Firms routinely use choice-based conjoint analysis (CBC) to identify preferred product attributes in the hopes of maximizing profit—for example, General Motors alone spends 10s of millions of dollars each year (Urban and Hauser 2004). CBC is increasingly used in litigation and courts have awarded billion-dollar judgments for patent or copyright infringement based on CBC studies (Cameron, Cragg, and McFadden 2013; McFadden 2014; Mintz 2012).

Research in CBC has long focused on the ability to estimate accurate relative tradeoffs among product attribute levels. Improved question selection, improved estimation, and techniques such as incentive alignment all enhance accuracy of identified tradeoffs and lead to better managerial decisions. However, with the advancement of CBC simulators and faster computers, researchers, especially in litigation, have begun to use CBC studies to estimate price equilibria and the resulting equilibrium profits (e.g., Allenby et al. 2014). This use of CBC raises a new concern because, as shown in this paper, the calculated price equilibria depend critically on the relative error term, i.e., the magnitude of the part-worths relative to the magnitude of the error—called “scale” in the CBC literature. We define scale as inversely proportional to the standard deviation of the error as in Swait and Louviere (1993), recognizing that some authors define scale as proportional to the standard deviation of the error as in Sonnier, Ainslie, and Otter (2007, p. 315) or Train (2009, p. 40).

The implications of measured scale are significant. Both theoretically and empirically we show that scale affects strategic positioning decisions even when holding heterogeneity and unobserved attributes constant—a different phenomenon than that commonly analyzed in the strategic positioning literature. These implications of scale are important to practice because (1) pricing decisions, strategic positioning decisions, and patent/copyright valuations are based on the scale observed in a market-

research study, (2) observed scale depends upon market research decisions such as the realism of images or incentive alignment, and (3) observed scale can be adjusted, although relatively rarely done so in practice, to reflect how consumers will actually behave in the marketplace. The phenomenon we study has managerial significance. Managerial decisions based on market research, that incorrectly estimates scale, can be multimillion dollar mistakes. Patent/copyright valuations can differ by hundreds of millions of dollars depending upon scale.

We combine formal analysis with empirical research. Our formal analysis abstracts to a stylized model to illustrate how scale affects pricing decisions and strategic decisions regarding the configuration of the attributes of a product. In the stylized model, we account for heterogeneity in preferences explicitly and rule out unobserved variables. We do this so that, in the model, strategic positioning, pricing, and market research decisions are driven solely by a scale factor that is common to all consumers.

In our empirical application, we demonstrate that the basic insights apply to real applications. The empirical applications do not depend upon assumptions of two attributes, two levels, two firms, stylized heterogeneity in relative preferences, and homogeneous scale. Our stylized model posits that the quality of market research affects the scale upon which managerial or legal decisions are based. The empirical application demonstrates that the quality of market research does affect measured scale. For example, we show that differences in scale as induced by differences in market research quality can lead to different price decisions, different strategic decisions, and different patent/copyright valuations.

Besides illustrating the theoretical model *in vivo*, the empirical data suggest three important implementation issues (to the extent that the insights from our studies can be reproduced). First, while incentive alignment is known to affect data quality, the vast majority of published studies (and industry applications) use text-based attribute descriptions. Our data suggest that using realistic images rather than text has a larger impact on scale than incentive alignment. Second, most published studies and industry applications use scale as estimated from the CBC choice experiment and judge CBC studies us-

ing holdout sets within the same choice experiment. We demonstrate that strategic decisions might reverse if scale is adjusted for more realistic validations that mimic the marketplace. Third, while our theory focuses on scale, aspects of market research quality also affect relative partworths—a double whammy. The two effects reinforce one another in our application. Each separately and jointly have substantial implications for managerial decisions and patent/copyright valuations.

Our contribution is not that lower-quality market research can impact managerial decisions—this is intuitive. We demonstrate how market-research quality affects scale and we explore why measured scale affects pricing and positioning decisions. Neither the magnitude and direction of the errors, nor the large effect of seemingly minor market-research craft such as realistic images and incentive alignment are obvious without the insights from the stylized model. (At minimum, they are underappreciated in the academic literature and the vast majority of CBC applications.)

2. Typical Practice in CBC Studies and Recent Changes in Practice

Before introducing the mathematical definitions, we review briefly current practice and changes in current practice.

2.1. Typical Current Practice

In CBC, products (or services) are summarized by a set of levels of the attributes. For example, a smartwatch might have a watch face (attribute) that is either round or rectangular (levels), be silver or gold colored, and have a black or brown leather band. By varying the smartwatch attribute levels systematically within an experimental design, CBC estimates preferences for attribute levels, called “partworths,” which describe the differential value of the attribute levels. For example, one partworth might represent the differential value of a rectangular watch face relative to a round watch face.

Applied practice focuses on estimating accurately the relative partworths. For example, if rectangular and round watch faces are equally costly, but the partworth of a rectangular watch face is greater than the partworth of a round watch face for most consumers, then a typical recommendation

would be to launch a product with a rectangular watch face. The relative partworths can also be used to calculate willingness-to-pay (WTP) by comparing differences in partworths to the estimated price coefficient. For example, if a consumer's differential value between a rectangular and a round watch face is higher than the consumer's valuation of a \$100 reduction in the purchase price, firms typically infer that the consumer is willing-to-pay more than \$100 for a rectangular rather than a round watch face. (There are subtleties in this calculation due to the Bayesian nature of most estimates, but this is the basic concept.)

All of these calculations depend only on the (distribution of) relative partworths. If we double all partworths, including the price coefficient, for every respondent, the comparative value and WTP calculations remain unchanged. Furthermore, all interpretations are based on differences in partworths. However, reported estimated partworths also depend upon the magnitude of the error term (scale). The definition of scale that we use in this paper is inversely proportional to the standard deviation of the error term. See Swait and Louviere (1993); Sonnier, Ainslie, and Otter (2007), or Train (2009) among others for discussions of scale. (Some authors define scale as proportional to the error term, but the underlying concept does not change.) If scale varies among consumers, then the accuracy with which the relative partworths can be estimated depends upon accounting for heterogeneity in scale during estimation (e.g., Beck, Rose, and Hensher 1993; Fiebig, et al. 2010; Pancreas, Wang, and Dey 2016; Swait and Louviere 1993). Scale heterogeneity affects partworth estimation and comparisons and/or aggregation of respondents' WTP, but, once researchers account for heterogeneity, WTP does not depend upon a common (across respondent) increase or decrease in scale (e.g., Ofek and Srinivasan 2002, Eq. 15). The phenomenon we investigate is different from scale heterogeneity; we focus on the strategic implications of scale assuming accurate relative partworths and assuming the estimation accounts for scale heterogeneity. (To isolate scale effects in the stylized model, we assume relative partworths are accurate. This assumption is relaxed in the empirical analyses.)

The literature recognizes that sometimes estimated partworths need to be adjusted to better represent marketplace choices. One approach is to adjust scale and relative partworths to match market shares and use the adjusted scale and partworths in simulations (e.g., Gilbride, Lenk, and Brazell 2008). The adjustments are motivated by predictive ability rather than strategic implications. A second approach adjusts scale directly or in a procedure known as randomized first choice (RFC) in which an additive error is included in the simulations. RFC automatically determines the random perturbations to yield "approximately the same scale factor as the [logit] model" (Huber, Orme, Miller 1999). Scale adjustments are easy to implement, but usage is reported as rare—users almost always stick with the scale observed in the CBC estimation (Orme 2017). Many users report that market data, as a benchmark to adjust scale and relative partworths, are often not available, e.g., for innovations, or not relevant to the simulated markets. Our formal model and empirical illustration suggest that adjustments are critical and should be used much more often than they are currently. We also provide an alternative adjustment that does not require market data.

2.2. Current Practice is Changing: The Implications of Price Equilibria

WTP provides valuable diagnostic information for pricing and attribute-level decisions and has been used to motivate and interpret valuations in patent/copyright cases (e.g., Cameron, Cragg, and McFadden 2013; McFadden 2014; Mintz 2012), but because WTP does not account for competitive response, WTP does not indicate how actual marketplace prices will respond to new products or changes in a product's attributes (Orme 2014, p. 90-91, Orme and Chrzan 2017, p. 194). Due to the influence of game theory in marketing science, CBC simulators are beginning to consider competitive response. For example, if an innovator introduces a rectangular watch face and a follower responds with a round watch face (and all other attributes are held constant), then CBC simulations can be used to calculate the Nash price equilibrium. Based on equilibrium prices, simulators can calculate the follower's most-profitable response (rectangular vs. round) to the innovator's new product. As a result, pricing and stra-

tegic positioning decisions (which attribute levels to choose relative to competition) can be made based on equilibrium price changes not just WTP. Allenby et al. (2014) propose that these methods be used to value patents and copyrights. Courts recognized the issue as early as 2005 for class-action cases (e.g., Whyte 2005, albeit not CBC).

We show that scale (and scale adjustment) plays a central role when calculating price equilibria and predicting optimal competitive reactions and, hence, affects managerial pricing decisions, strategic positioning decisions, and estimates of patent/copyright valuations. We illustrate the magnitude of the managerial implications. (Sawtooth Software estimates that 80% of managerial CBC applications consider competition in market simulations, although the explicit calculation of equilibrium price is relatively new (Orme 2017).)

3. General Formulation and Basic Notation

We begin with notation for a fully heterogeneous model because the empirical studies in §9 use a fully heterogeneous model. (Different consumers can have different partworths and scale.) Appendix 1 summarizes notation for both the heterogeneous model and a more-stylized formal model. While empirical studies, including ours, can have many attributes and many levels for each attribute, we focus in the stylized model on a single attribute with two levels. This focus in the stylized model is sufficient to illustrate the impact of scale and is consistent with Irmen and Thisse (1998, p. 78), who analyze markets in which firms compete on multiple dimensions and conclude that “differentiation in a single dimension is sufficient to relax price competition and to permit firms to enjoy the advantages of a central location in all other characteristics.” Our stylized model also applies to simultaneous differentiation of a composite of multiple dimensions, say a silver smartwatch with a rectangular face and a black leather band vs. a gold smartwatch with a round watch face and a metal band. Both stylistically and empirically we hold all attributes other than our focal attributes constant across products.

3.1. Formal Definitions

To match typical applications of CBC, we focus on discrete (horizontal) levels of an attribute that we label r and s . A product can have either r or s , but not both. If mnemonics help, think of r as round, regular, routine, ruby, or rust-colored and s as square, small, special, sapphire, or scarlet. While the empirical model can handle many firms, it is sufficient for the stylized model to focus on two firms, each of which sells one product. We allow an “outside option” to capture other firms and products that are exogenous to the strategic decisions of the two-firm duopoly. The assumption of two products in the market is not critical to the insights. In §9.2, we show that the insights apply when there are more than two products, more than one attribute, and more than two levels.

Let u_{ij} be consumer i 's utility for Firm j 's product, let u_{io} be i 's utility for the outside option, and let p_j be product j 's price. Let β_{ri} and β_{si} be i 's partworths for attribute levels r and s , respectively, and let δ_{rj} and δ_{sj} be indicator functions for whether or not Firm j 's product has r or s , respectively. Let η_i indicate i 's preference for price, let ϵ_{ij} be an extreme value error term with variance $\pi^2/6\gamma_i^2$. If the error terms are independent and identically distributed, we have the standard logit model for the probability, P_{ij} , that consumer i purchases Firm j 's product (relative to Firm k 's product and the outside option):

$$u_{ij} = \beta_{ri}\delta_{rj} + \beta_{si}\delta_{sj} - \eta_i p_j + \epsilon_{ij}$$

$$P_{ij} = \frac{e^{\gamma_i(\beta_{ri}\delta_{rj} + \beta_{si}\delta_{sj} - \eta_i p_j)}}{e^{\gamma_i(\beta_{ri}\delta_{rj} + \beta_{si}\delta_{sj} - \eta_i p_j)} + e^{\gamma_i(\beta_{ri}\delta_{r,k \neq j} + \beta_{si}\delta_{s,k \neq j} - \eta_i p_{k \neq j})} + e^{\gamma_i u_{io}}}$$

To identify the model in estimation, we cannot simultaneously estimate β_{ri} , β_{si} , η_i , and γ_i . We must impose one constraint for identification. The constraint varies in the literature. McFadden (2014) constrains the price coefficient, η_i to unity. Sonnier, Ainslie, and Otter (2007) parameterize the model such that η_i is unity, but set $1/\mu_i = \gamma_i$ as the price coefficient. The alternative parameterization has no effect for maximum-likelihood estimation, but requires that Sonnier, Ainslie, and Otter (2007) adjust the

prior distributions for μ_i when computing Bayesian posterior distributions. For the stylized model, we prefer to parameterize scale as proportional to γ_i rather than inversely proportional to μ_i because we believe it is more intuitive if greater γ_i implies the "signal-to-noise" ratio is larger. §9.3 summarizes the empirical implications of the Sonnier, Ainslie, and Otter parameterization as well as a parameterization due to Allenby, et al. (2014). The basic insights remain unchanged.

In our stylized model, we focus on the effect of a common scale that may be affected by market-research quality. In the stylized model, we assume relative partworths are not affected by market-research quality. (The impacts on relative partworths are well-studied and not new to this paper.) The McFadden (2014), Sonnier, Ainslie, and Otter (2007), and Allenby, et al. (2014) parameterizations have equivalent theoretical implications for the focus of the stylized model.

When market-research quality affects both scale and relative partworths (§8.9), researchers may prefer a different definition of scale. For example, many empirical researchers define scale as the sum of the estimated importances. (The importance of an attribute is defined as difference between the largest and smallest partworth of an attribute.) Such alternative definitions are mathematically equivalent to the scale factor in the stylized model because the stylized model assumes accurate relative partworths. Empirically, we use random assignment among conditions to identify how differences in market-research quality affect scale. The same experimental design can be used for alternative definitions of scale.

For the remainder of the theory development, we parameterize the model such that $\eta_i = 1$. In this parameterization, the β 's are now relative partworths and γ_i is scale. With this parameterization, the CBC logit model for the stylized model becomes:

$$(1) \quad P_{ij} = \frac{e^{\gamma_i(\beta_{ri}\delta_{rj} + \beta_{si}\delta_{sj} - p_j)}}{e^{\gamma_i(\beta_{ri}\delta_{rj} + \beta_{si}\delta_{sj} - p_j)} + e^{\gamma_i(\beta_{ri}\delta_{r,k \neq j} + \beta_{si}\delta_{s,k \neq j} - p_{k \neq j})} + e^{\gamma_i u_{io}}}$$

If V is the market volume (including volume due to the outside option), c_j is the marginal cost

for product j , C_j is Firm j 's fixed cost, and $f(\beta_{ri}, \beta_{si}, \gamma_i)$ is the probability distribution over the relative partworths and scale (posterior if Bayesian), then the profit, π_j , for Firm j is given by:

$$(2) \quad \pi_j = V(p_j - c_j) \int P_{ij} f(\beta_{ri}, \beta_{si}, \gamma_i) d\beta_{ri} d\beta_{si} d\gamma_i - C_j$$

(Empirically, if all estimates are Bayesian, we use the posterior distribution in the standard way.)

For the purposes of this paper, we assume that c_j does not depend on the quantity sold nor the choice of r or s . These assumptions can be relaxed and do not reverse the basic intuition in this paper.

(The effect of the relative cost of r or s is well-studied, e.g., Moorthy 1988.)

3.2. Interpretation and Implications of the Error Term

The error term in CBC has many interpretations and implications. It has been interpreted as inherent stochasticity in consumer choice behavior and/or sources that are unobservable to the researcher, such as unobserved heterogeneity, unobserved attributes, functional misspecifications, or consumer stochasticity that is introduced by the CBC experiment (e.g., due to fatigue; e.g., Manski 1977; Thurstone 1927). We are most interested in what happens to the (observed) relative magnitude of the error term, and consequently scale, when the quality of the CBC study changes, say by the addition of more-realistic images or incentive alignment. To address this issue, we assume that the firm acts strategically on a CBC study anticipating the price equilibria implied by the CBC study. However, after the firm selects its positioning strategy (say a silver vs. gold smartwatch) and launches its product, the prices are set by market forces. (The innovator assumes that the follower will act on equivalent market research. The follower, acting second, need only observe the innovator's position.)

If the firm acts on market research it believes to be correct, and the innovator assumes the follower has equivalent market-research, the firm will anticipate a price equilibrium based on the scale it believes to be true and will choose its position optimally. But the actual realized equilibrium prices may differ if the firm's beliefs about scale are not sufficiently accurate. The mechanism by which market pric-

es adjust after positioning decisions is similar to that expressed by Hotelling (1927) and others. The mechanism is slightly different from the more-common simplifying assumption in modern game theory that “firms compete non-cooperatively in product specifications with instantaneous adjustment to the Nash equilibrium prices” (Economides 1996, p. 67). The difference is necessary because, unlike typical models, the firm may act based on market research it only believes to be accurate. From Hotelling (1927, p. 48-49):

But understandings between competitors are notoriously fragile. Let one of these business men, say B, find himself suddenly in need of cash. Immediately at hand he will have a resource: Let him lower his price a little, increasing his sales. His profits will be larger until A decides to stop sacrificing business and lowers his price to the point of maximum profit. B will now be likely to go further in an attempt to recoup, and so the system will descend to the equilibrium position. Here neither competitor will have any incentive to lower his price further, since the increased business obtainable would fail to compensate him.

Because actual sales and equilibrium prices depend on how consumers react to the products' chosen positions after the products are introduced to the market, we need the concept of a “true” scale that represents how the marketplace reacts. We purposefully do not define “true” scale as a philosophical construct—it is defined as the scale that best represents how consumers actually react in the marketplace. Practically, we expect the “true” scale to be finite because of inherent stochasticity (e.g., Bass 1974), but our stylized theory allows “true” scale to approach infinity. Our model admits many explanations of inherent uncertainty. The stylized model only needs to assume that, even with the best possible market research, the firm’s prediction of consumer behavior includes a (possibly zero) error term. The firm never needs to learn this true scale, although it is easy to imagine a structural model by which it is estimated. The market reaches the equilibrium price because firms adjust price to a Nash equilibrium after launch.

In our stylized model, we assume that the error term (in estimation) captures imperfections in the CBC study (if any) plus any residual uncertainty. We explicitly constructed the stylized model to rule

out unobserved attributes and unobserved heterogeneity. Our formal model differs from prior models in the positioning literature because we explicitly model heterogeneity in the partworths that are relevant to the positioning decisions. For “true” scale, the error term is limited to other imperfections in market research (if any). With explicit heterogeneity we seek to rule out explanations that the firms act strategically upon heterogeneity as in de Palma, et al. (1985, p. 780) who state: “the world is pervasively heterogeneous, and we have made it clear how, in a particular model, this restores smoothness [that leads to differentiation].” Anderson, de Palma, and Thisse (1999), de Palma, Ginsburgh, Papageorgiou, and Thisse (1985), de Palma, Ginsburgh, and Thisse (1987), and Rhee, et al. (1992) restore minimum differentiation to Hotelling’s line with uncertainty about heterogeneity in preferences (partworth heterogeneity in our model). In their model, each consumer’s utility depends on the transport cost from the firm to the consumer’s position on the line and on an error-like term reflecting the firm’s uncertainty about the heterogeneity in consumer’s tastes (and other attributes the consumer may value). Model parameters do not vary by consumers, hence aggregate demand is given by a logit-like function. When the unobserved heterogeneity is sufficiently large, the firm’s predictions are imprecise leading to reduced price competition and minimum differentiation. We note that their “error” term captures unobserved heterogeneity in consumer preferences rather than any stochasticity that remains after heterogeneity and missing attributes are fully modeled. The intuition we develop is related to their intuition, but more general and specifically adapted to the realities of CBC studies.

Explicit heterogeneity complicates the proofs immensely, but, we hope, the stylized model provides new practical insight for CBC studies used for pricing decisions, strategic positioning decisions, and patent/copyright valuations.

4. Stylized Formal Model with Two-Segment Heterogeneity

Although heterogeneity in partworths and scale is important for empirical estimation (Fiebig, et al. 2010; Salisbury and Feinberg 2010), and used in our empirical studies, our stylized model includes,

but simplifies heterogeneity. We focus on two mutually exclusive and collectively exhaustive consumer segments with different relative partworths. This level of heterogeneity is sufficient to enable two firms to target different segments and sufficient to illustrate the strategic effects of scale. The strategic effects survive the more-general empirical applications in §8 and §9 which use standard estimation procedures (hierarchical Bayes CBC, including three related specifications).

We label the segments R and S , with segment sizes R and S , respectively. Partworths vary between segments, but are homogeneous within segment ($\beta_{ri} = \beta_{rR}$ and $\beta_{si} = \beta_{sR}$ for all i in segment R ; $\beta_{ri} = \beta_{rS}$ and $\beta_{si} = \beta_{sS}$ for all i in segment S). While scale varies among consumers in the empirical applications, in the stylized model we seek to focus on a common scale adjustment that might vary among CBC studies of different quality. For this purpose, it is sufficient to assume scale is constant across consumers such that $\gamma_i = \gamma$ for all i .

We seek to investigate tradeoffs that firms make between (1) a differentiated strategy in which each firm targets a different segment and (2) an undifferentiated strategy in which both firms target the same segment. To do so, we need one segment to be more attractive than the other. It is sufficient to model relative influence of a segment by its size, R or S . We need partworths to vary between segments. It is sufficient that their relative values reverse ($r > s$ in one segment and $s > r$ in the other segment). Although the partworths differ between segments, it would be redundant to also vary the magnitude of partworth differences, thus we set $\beta_{rR} = \beta_{sS} = \beta^h$ and $\beta_{RS} = \beta_{Sr} = \beta^\ell$. We set $\beta^h \geq \beta^\ell$ and $R \geq S$ without loss of generality. (We can also set $\beta^\ell = 0$ without loss of generality, but interpretations are more intuitive if we retain β^ℓ in the notation.)

The costs, c_j and C_j , affect strategic decisions in the obvious ways and need not be addressed in this paper. For example, a firm might require a minimum price such that $p_j \geq c_j$ or choose not to enter if C_j is too large. Such effects are well-studied and affect firm decisions above and beyond the strategic effect of scale. For focus, we normalize V to a unit market volume, set $C_j = 0$, and roll marginal costs

into price by setting $c_j = 0$.

We label the potential strategic positions for Firms 1 and 2, respectively, as either rr , rs , sr , or ss . For example, rs means that Firm 1 positions at r and Firm 2 positions at s . Because prices, market shares, and profits depend on these strategic positioning decisions, we subscript prices, shares, and profits accordingly. For example, p_{1rr} is Firm 1's price in a market in which Firm 1's position is r and Firm 2's position is r . With this notation, Equations 1 and 2 simplify as illustrated in Equation 3 for rr . Equations for Firm 2, for Segment S, and for other positioning strategies are derived similarly.

$$(3) \quad P_{R1rr} = \frac{e^{\gamma^{true}(\beta^h - p_{1rr})}}{e^{\gamma^{true}(\beta^h - p_{1rr})} + e^{\gamma^{true}(\beta^h - p_{2rr})} + e^{\gamma^{true}u_o}}$$

$$\pi_{1rr} = Rp_{1rr}P_{R1rr} + Sp_{1rr}P_{S1rr}$$

In Equation 3, γ^{true} is the "true" scale—the scale that describes how consumers react in the marketplace. Scale homogeneity enables us to abstract from detailed estimation issues to focus on the effect of market-research quality on firms' decisions. (We relax this assumption empirically.)

5. The Effect of Scale on Equilibrium Prices and Strategic Positioning Decisions

5.1. Asymmetric Competition: Minimum versus Maximum Differentiation

The study of minimum versus maximum differentiation has a rich history in both economics and marketing. Hotelling (1929) proposed a model of minimum differentiation in which consumers are uniformly distributed along a line and two firms compete by first choosing a position (attribute level) and then a price. After demonstrating that the price equilibrium did not exist in Hotelling's model, d'Aspremont, Gabszewicz, and Thisse (1979) proposed quadratic transport costs and obtained an equilibrium of maximum differentiation—firms choose strategic positions at opposite ends of the line. We extend and modify their model to model heterogeneity explicitly in order to study the practical implications of market research quality.

Many other researchers explore Hotelling-like models to derive conditions when differentiation

is likely and when it is not (e.g., Eaton and Lipsey 1975; Eaton and Wooders 1985; Economides 1984; Graitson 1982; Johnson and Myatt 2006; Novshek 1980; Sajeesh and Raju 2010; Shaked and Sutton 1982; Shilony 1981). In these formal models, differentiation is driven by the heterogeneity of consumer preferences—something we hold constant.

In marketing, Thomadsen (2007) shows how asymmetries in attribute levels lead one firm to favor maximum differentiation in physical location while another favors minimum differentiation. Gal-or and Dukes (2003) show that a two-sided market (commercial media serving consumers and advertisers) reverses the differentiation found in d'Aspremont, Gabszewicz, and Thisse (1979). Guo (2006) extends an attribute-based analysis to forward looking consumers who observe one of two product attributes. In Guo's model, consumers anticipate probabilistically future valuations for the other product attribute. In these models, heterogeneity in preferences (partworths) drives strategic behavior with respect to prices and profits.

In the language of CBC, all of these papers focus on the distribution of relative partworths or on the partworths of unobserved or uncertain attributes. Although many models include error terms, none analyze the effect of imperfect market research or inherent stochasticity. We show that these phenomena alone can drive firms' decisions on differentiation. We begin by showing the firm's beliefs about scale drive strategic positioning, but this is a means to an end. It is not our final result. We use this intermediate step to develop the machinery to understand the impact of market-research quality of CBC studies on forecast equilibrium prices and strategic positioning decisions.

5.2. Basic Game to Demonstrate the Impact of True Scale (Inherent Stochasticity)

The price-positioning game is consistent with key references in the strategic positioning literature (§5.1), and realistic for most markets. Temporarily, we assume the firms believe they know γ^{true} , which may be either finite or approach infinity. Based on this knowledge, the firms first choose their product positions (r or s) sequentially, and then the market sets prices. (If the firms are correct in their

beliefs, they correctly anticipate equilibrium prices.) The positioning decisions, once made, are not easily reversible, perhaps due to production capabilities or ephemeral advertising investments. Without loss of generality, Firm 1 is the innovator and Firm 2 is the follower. The innovator enters assuming that the follower will choose its positions optimally. (We abstracted away from entry decisions by setting $c_j = C_j = 0$.) After the firms have entered, Nash equilibrium prices, if they exist, are realized. (This two-stage game will be embedded in another game in §7 in which firms know that market research may be imperfect and choose whether to invest in higher-quality market research prior to making strategic positioning decisions. We address the relationship to simultaneous entry in §6.4 and §7.6. We use * to indicate Nash equilibrium prices, shares, and profits.

The equilibria we obtain, and strategies that are best for the innovator and follower, have the flavor of models in the asymmetric competition literature (minimum vs. maximum differentiation), but with two important differences. (1) Our results are not driven by unobserved heterogeneity or strategically-relevant unobserved attributes. (2) Our results are focused on providing a structure to understand and evaluate the impact of improvements in CBC methods (market-research quality). The formal structure can be used as a practical tool to evaluate whether improvements, such as more-realistic images or incentive alignment, affect strategic decisions.

Although we believe that, ex post, the qualitative effects are intuitive, they have not been discussed in the CBC literature. It is well known that scale affects logit-model-based market shares because the logit curve is steeper in both attributes and price when scale increases, but we could find no discussion of the implications of scale for price equilibria or strategic positioning. Papers, which advocate the use of estimated equilibrium prices for managerial decisions or patent/copyright valuations, do not discuss sensitivity to scale.

5.3. Price Equilibria in Heterogeneous Logit Models (as in CBC)

We are not the first to investigate price equilibria in logit models. Choi, DeSarbo and Harker

(1990) demonstrate that price equilibria exist if partworths are homogeneous and consumers are not overly price-sensitive. Their condition (p. 179) suggests that price equilibria are more likely to exist if there is greater uncertainty in consumer preferences—a result consistent with our model which, in addition, accounts for heterogeneity. Choi and DeSarbo (1994) use similar concepts to solve a positioning problem with exhaustive enumeration. Luo, Kannan and Ratchford (2007) extend the analysis to include heterogeneous partworths and equilibria at the retail level. They use numeric methods to find Stackelberg equilibria if and when they exist.

We cannot simply assume that price equilibria exist and are unique. For example, Aksoy-Pierson, Allon, and Federgruen (APAF, 2013) warn that price equilibria in heterogeneous logit models may not exist. APAF generalize the analyses of Caplin and Nalebuff (1991) to establish sufficient conditions for price equilibria to exist, to be unique, and to be given by the first-order conditions. The APAF conditions apply to typical HB CBC studies (APAF, §6). Because these conditions apply empirically, we check existence and uniqueness in our empirical studies.

5.4. Equilibria in the Price Subgame

We begin the analysis of our stylized model with implicit first-order conditions for the realized prices recognizing that the market will set prices based on the true scale.

$$\frac{\partial P_{R1rr}}{\partial p_{1rr}} = \frac{-\gamma^{true} e^{\gamma^{true}(\beta^h - p_{1rr})} [e^{\gamma^{true}(\beta^h - p_{2rr})} + e^{\gamma^{true}u_o}]}{[e^{\gamma^{true}(\beta^h - p_{1rr})} + e^{\gamma^{true}(\beta^h - p_{2rr})} + e^{\gamma^{true}u_o}]^2} = -\gamma^{true} P_{R1rr} (1 - P_{R1rr})$$

$$\frac{\partial \pi_{1rr}}{\partial p_{1rr}} = RP_{R1rr} + SP_{S1rr} - \gamma^{true} p_{1rr} \{RP_{R1rr} (1 - P_{R1rr}) + SP_{S1rr} (1 - P_{S1rr})\}$$

Using these relationships, we obtain implicit equations for the equilibrium prices and the corresponding equilibrium profits. Similar equations apply for rs , sr , and ss and Firm 2:

$$(4a) \quad p_{1rr}^* = \frac{1}{\gamma^{true}} \frac{RP_{R1rr}^* + SP_{S1rr}^*}{RP_{R1rr}^* (1 - P_{R1rr}^*) + SP_{S1rr}^* (1 - P_{S1rr}^*)}$$

$$(4b) \quad \pi_{1rr}^* = \frac{1}{\gamma^{true}} \frac{(RP_{R1rr}^* + SP_{S1rr}^*)^2}{RP_{R1rr}^*(1 - P_{R1rr}^*) + SP_{S1rr}^*(1 - P_{S1rr}^*)}$$

Differentiating further, we obtain implicit second-order and cross-partial conditions (given in Appendix 2, existence and uniqueness section). Using these conditions, we establish that interior equilibria exist and are unique given (mild) sufficient conditions. We test these conditions for our empirical analyses and for all illustrative examples in Online Appendix 3.1. In all cases, the equilibria in the illustrative example exist and are unique. In our data, the empirical equilibria exist for most posterior draws and, when they exist, they are unique.

6. Sensitivity of Valuations and Strategic Decisions

In this section, we temporarily assume the firm believes it knows the true scale based on its market research. The true scale can be either finite (inherent uncertainty) or approach infinity (no inherent uncertainty). We explore how scale affects equilibrium prices, strategic positioning decisions, and patent/copyright valuations. In §7, we use these results to explore what happens when the firm bases its decisions on CBC market research.

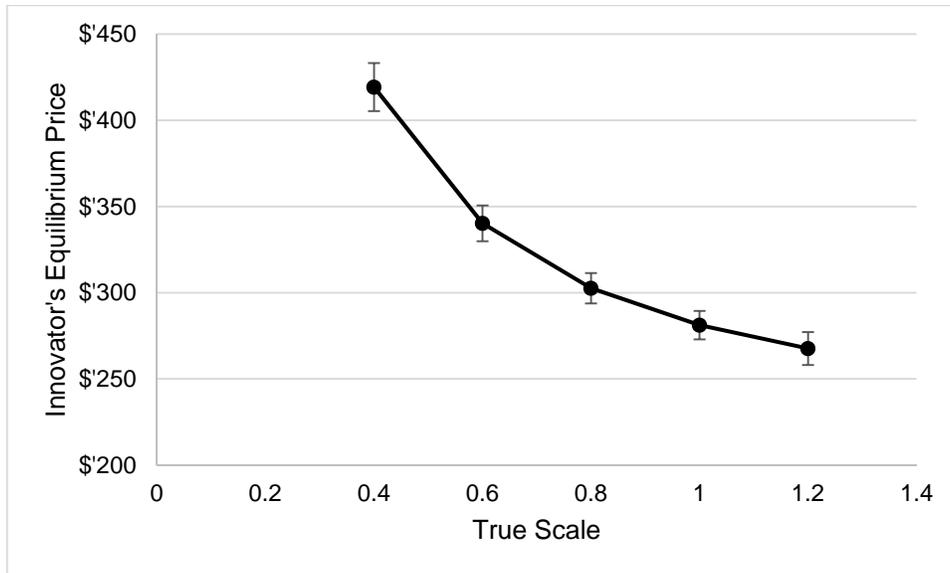
6.1. Scale Affects the Price Equilibrium that are Calculated—Illustration

Consider the probability that a consumer in Segment R chooses the innovator's product given positions rs . By assumption, the relative partworths for r vs. s do not change, nor does the relative preference for r (or s) vs. price. However, the impact of these preference differences depends upon γ^{true} . A larger γ^{true} makes P_{R1rs} more sensitive to both attribute differences and price differences; a smaller γ^{true} makes P_{R1rs} less sensitive. As firms react to one another, the net effect of a larger γ^{true} will drive the equilibrium price downward.

As an illustration, we plot the equilibrium price of Firm 1 as a function of γ^{true} using relative partworths we obtained in our empirical study about smartwatches (details in §9). Figure 1 is based on a CBC simulation with two firms whose products differ on the shape of the watch-face (round vs. rectan-

gular). We calculate the (counterfactual) price equilibria for each level of γ^{true} . (We address practical methods to compute equilibrium prices in typical CBC studies in §8.9.) In Figure 1, we chose the range of the scales to be typical of those reported in the literature and in our empirical studies. (Over the range in Figure 1, equilibrium prices are monotonically decreasing in scale, but there is no guarantee that they do not increase slightly as $\gamma^{true} \rightarrow \infty$. Indeed, they do so in the illustrative example in Online Appendix 3.1.)

Figure 1. Predicted Equilibrium Price Depends Upon the Scale of the CBC Study (Data from Empirical Study of Smartwatches; Error Bars are Posterior Standard Deviations)



This wide difference in (predicted) equilibrium prices has managerial and litigation implications. For example, suppose that a firm’s CBC study reports $\gamma = 0.4$ but the market acts according to $\gamma^{true} = 1.0$, then the firm would likely earn substantially less profit than it expects. We show in §7 that these differences in equilibrium prices might also lead a follower to choose a less profitable strategic position (attribute level). We also can estimate the magnitude of this impact for patent/copyright valuations.

Assume that the smartwatch price swing in Figure 1 applies to smartphones. (Smartphone prices are higher so that this will likely under-estimate the effect.) We can use publicly available data to get an

idea of the impact that scale would have had if equilibrium prices been used to motivate damages in the first Apple v. Samsung trial about smartphone patents (Mintz 2012; WTP, which is scale independent, was used in the 2012 trial). Using estimates of over 20 million infringing Samsung smartphones (The Verge 2012), the calculated price swing of more than \$130 implies a swing of more than \$2.6 billion in revenue. Patent/copyright valuations are based on profit differences, not revenue. Unfortunately, margins are subject to “protective orders.” If the predicted multi-year profit due to the infringement were only a small fraction of the revenue swing implied by Figure 1, damages could easily vary by tens of millions or even hundreds of millions of dollars depending upon the scale of the CBC analyses used to calculate those damages. This swing is in the order of magnitude of the jury awards in the Apple v. Samsung patent infringement cases (Mintz 2012).

6.2. True Scale Affects the Relative Profits of the Firms’ Positioning Strategies

To understand the effect of true scale on firms’ positioning strategies (choice of attribute levels in equilibrium), we examine how profit-maximizing attribute levels change as true scale increases from small to large. Because the functions are continuous, we need only show the extremes. Result 1 shows that for sufficiently low true scale, price moderation through differentiation does not offset the advantage of targeting the larger segment and choosing the attribute level with the highest WTP (ratio of partworth differences). The proof is driven by the fact that the logit curve becomes flatter as $\gamma^{true} \rightarrow 0$. When price is endogenous, common intuition is not correct. All shares, including the outside option, do not tend toward equality as $\gamma^{true} \rightarrow 0$. The endogenous increase in equilibrium prices offsets this effect. Instead, while the innovator and follower shares move closer to one another, the equilibrium prices increase and reduce shares relative to the outside option. The proof demonstrates that all of the countervailing forces balance to favor rs for the innovator and rr for the follower. We provide details in Appendix 2.

Result 1. For sufficiently low true scale ($\gamma^{true} \rightarrow 0$), the follower prefers not to differentiate

whenever the innovator positions for the larger segment ($\pi_{2rr}^* > \pi_{2rs}^*$). However, the innovator would prefer that the follower differentiate ($\pi_{1rs}^* > \pi_{1rr}^*$) and, if the follower were to differentiate, the innovator would earn more profits than the follower ($\pi_{1rs}^* > \pi_{2rs}^*$).

We now show that the firms prefer different strategic positions if true scale is sufficiently high. It is sufficient that (1) the relative partworth of r is larger than the relative partworth of the outside option and (2) the relative partworth of the outside option is at least as large as the relative partworth of s . With these conditions, market shares are sufficiently sensitive to price for large γ^{true} . Shares for differentiated markets become more extreme, the equilibrium prices are driven down, and shares increase relative to the outside option. The countervailing forces balance to favor rs for the follower.

Result 2. Suppose β^h is sufficiently larger than u_o and $u_o \geq \beta^\ell$. Then, there exists a sufficiently large γ^{true} such that the follower prefers to differentiate whenever the innovator positions for the larger segment ($\pi_{2rs}^ > \pi_{2rr}^*$). Differentiation earns more profits for the innovator than no differentiation ($\pi_{1rs}^* > \pi_{1rr}^*$), and those profits are larger than the profits earned by the follower ($\pi_{1rs}^* > \pi_{2rs}^*$).*

Together Results 1 and 2 establish that, if the innovator targets the larger segment, then the follower will choose to differentiate (s) when true scale is sufficiently high and will choose not to differentiate (r) when true scale is sufficiently low. All that remains is to show is that, in equilibrium, the innovator will target the larger segment. While this may seem intuitive from Results 1 and 2, we need Results 3 and 4 to establish the formal result.

Result 3. Among the undifferentiated strategies, both the innovator and the follower prefer to target the larger segment ($\pi_{1rr}^ = \pi_{2rr}^* > \pi_{1ss}^* = \pi_{2ss}^*$).*

Result 4. Suppose β^h is sufficiently larger than u_o and $u_o > \beta^\ell$. Then, there exists a sufficiently large γ^{true} such that the innovator prefers to differentiate by targeting the larger segment ra-

ther than the smaller segment ($\pi_{1rs}^* > \pi_{1sr}^*$).

6.3. Equilibrium in Product Positions

Results 1 to 4 establish necessary and sufficient conditions to prove the following propositions.

Proposition 1. For low true scale ($\gamma^{true} \rightarrow 0$), the innovator (Firm 1) targets the larger segment (r) and the follower chooses not to differentiate. It also targets the larger segment (r).

Proposition 2. If β^h is sufficiently larger than u_o and if $u_o \geq \beta^l$, then there exists a sufficiently large γ^{true} such that the innovator targets the larger segment (r) and the follower chooses to differentiate by targeting the smaller segment (s).

Because the profit functions are continuous (see also APAF), Propositions 1 and 2 and the Mean Value Theorem imply that there exists a γ^{cutoff} such the follower is indifferent between rr and rs .

Numerically, for a wide variety of parameter values, the profit functions are smooth, the cutoff value is unique, and $\pi_{2rs}^* - \pi_{2rr}^*$ is monotonically increasing in γ^{true} . We have not found a counterexample.

We now have the machinery to address the issue of why scale is an important consideration when CBC simulators are used for pricing decisions, strategic positioning decisions, and copyright/patent valuations.

6.4. Simultaneous Games

Realistically, one firm almost always acts first and, in doing so, earns the ability to choose the favored position, r . However, for completeness, it is worth considering a simultaneous game. If γ^{true} is small, then the first-stage positioning equilibrium will be rr , just as in the sequential game. However, if γ^{true} is large, there is an indeterminacy in the sense that both firms prefer r if the other firm were to choose s . Post hoc both rs and sr are Nash equilibria in the sense that once these positions are chosen, there are no unilateral incentives to change position. The key role played by our decision to describe entry by a sequential game is to break this indeterminacy.

7. Implications for Investing in the Quality of CBC Studies

7.1 Aspects of Market-Research Quality in CBC Studies

We reviewed the conjoint-analysis papers in *Marketing Science* from the last 15 years (2003-2017). Forty-six (46) papers addressed new estimation methods, new adaptive questioning methods, methods to motivate respondents, more efficient designs, non-compensatory methods, and other improvements. Mostly, papers focused on the improved estimation of relative partworths or implied managerial interpretations. Five of the papers address scale (or a related concept for non-CBC papers) explicitly, and of those five, three focus on more accurate estimation, one on brand credibility, and one on peer influence. None discuss the strategic (price or positioning) implications of scale. See Online Appendix 3.8.

While there is much focus on which CBC questions to ask and efficient designs, there is substantially less focus on data-quality issues such as the realism of the stimuli. Most papers do not report whether stimuli are text-only, pictorial, or animated, but of those that do, the vast majority are text-only. While interest in incentive alignment is growing, no papers discuss the impact of either realistic stimuli or incentive alignment on the scale observed for the estimation data. Furthermore, in practice, defaults in software lead most applications to text-only stimuli without incentive alignment.

Higher “quality” in CBC can be expensive. Some firms, such as Procter & Gamble, Chrysler, or General Motors are sophisticated and spend substantially on CBC. For example, some CBC studies invest 10s of thousands of dollars to create realistic animated descriptions of products and attributes complete with training videos. Incentive alignment can also be expensive: one CBC study gave 1 in 20 respondents \$300 toward a smartphone and another gave every respondent \$30 toward a streaming-music subscription (Koh 2014; McFadden 2014). Firms routinely use high-quality Internet panels, often paying as much as \$5-10 for each respondent and up to \$50-60 for hard-to-reach respondents. Our review of the literature suggests that firms believe that each of these investments increases the accuracy with which rela-

tive partworts are estimated. On the other hand, many firms reduce market research costs by using text-only attribute descriptions, less-sophisticated methods, convenience samples, and small sample sizes. Many “quality” decisions are driven by software defaults. We show that managerial implications of these “quality” decisions (or defaults) are not trivial. (Empirically, we illustrate the effects for two aspects of “quality,” realistic images and incentive alignment. These aspects of quality are illustrative; the theory applies to any aspect of a CBC study that improves data quality.)

7.2. Modeling Decisions with Respect to Market Research Quality

In §6, we temporarily assumed the firm believed the true scale to be accurate (defined as γ^{true}). True scale was the scale that described how consumers would react to r , s , and price in the market. We are interested in what happens if the firms (or testifying experts) shirk on their investments in the quality of CBC studies. We define two additional constructs. $\gamma^{market\ research}$ is the scale estimated by the CBC study. $\gamma^{market\ research}$ may or may not equal the true scale. $\gamma^{asymptotic}$ is the scale that the firm would obtain with the highest possible level of data quality and the best questions and estimation methods. Hopefully, $\gamma^{asymptotic} \cong \gamma^{true}$, but that is not guaranteed. We need not assume the firms ever learn γ^{true} . We need only assume that the firms experience equilibrium prices after they launch their products. (For reasons outside the model, such as improving future market research or launching products in different categories, the firms may be motivated to uncover why equilibrium prices differ from predictions.)

We embed the game from §6 into a larger game. We assume that if the firm invests more in the CBC study, its estimate of scale becomes better, that is, $|\gamma^{market\ research} - \gamma^{true}|$ becomes smaller. To focus on scale in the stylized model, we assume all (reasonable) CBC studies estimate the relative partworts correctly so that the firm knows that $r > s$ in R , $s > r$ in S , and $R > S$. (In §9, we investigate a double-whammy whereby market-research quality affects both estimated scale and estimated relative partworts. Because it is obvious and well-researched how errors in estimating relative partworts lead

to errors in pricing, positioning, and valuations, we focus the stylized model on scale.)

It is sufficient to illustrate the phenomenon in the stylized model if we consider lower-quality and higher-quality CBC studies such that $\gamma^{higher} = \gamma^{asymptotic} = \gamma^{true}$ for the higher-quality study and $\gamma^{lower} \neq \gamma^{true}$ for the lower-quality study. (In §9, we show that investments in more-realistic images and incentive alignment lead to scale estimates that more accurately reflect how consumers in a marketplace behave.) We seek to understand the implications of the firm's decisions on market-research quality. Thus,

1. The innovator decides whether to invest in the lower-quality or the higher-quality study.
(To focus on scale, we assumed that any quality CBC study reveals correctly that $r > s$ in R , $s > r$ in S , and $R > S$.)
2. The innovator completes its CBC study and observes $\gamma^{market\ research}$. The innovator assumes that the follower acts as if it has the same beliefs about $\gamma^{market\ research}$.
3. Based on its observed $\gamma^{market\ research}$, the innovator announces and commits to either r or s .
4. The follower decides whether to invest in the lower-quality or the higher-quality study.
(By assumption any quality study reveals that $r > s$ in R , $s > r$ in S , and $R > S$.)
5. The follower completes its CBC study and observes $\gamma^{market\ research}$. (The innovator has already acted; the follower observes the innovator's position, r or s .)
6. Based on its observed $\gamma^{market\ research}$, the follower announces and commits to either r or s . (Because the innovator has acted, the follower need not assume anything about the innovator's belief about $\gamma^{market\ research}$.)
7. Both firms launch their products. The market determines sales and price based on γ^{true} —the scale that best describes consumer response. The firms realize their profits.

It will be obvious in §7.3 that the follower could have made its market research decision before

learning of the innovator’s positioning—such a game ordering would give the same results. Commitment to r or s implicitly assumes that positioning decisions are “sticky,” expensive, or based on know-how, patents, or copyrights. Once made, the firm cannot change its positioning even when the market price, market shares, and profits are not as forecast. Propositions 1 and 2 give us sufficient insight to understand the innovator’s and the follower’s market-research-quality decisions.

7.3 Innovator’s Strategic Positioning Decision Does Not Depend Upon Observed Scale

The innovator chooses to target the larger segment (r) in both Propositions 1 and 2, thus the innovator makes the same decision whether $\gamma^{market\ research} = \gamma^{true}$ or $\gamma^{market\ research} \neq \gamma^{true}$. Because the innovator’s strategic positioning decision is independent of the observed scale, investing in a higher-quality CBC study has no effect on the innovator’s positioning strategy. (We state and prove the result formally in Appendix 2.) The insight is consistent with recommendations in product development (e.g., Urban and Hauser 1993, Ulrich and Eppinger 2004). These texts advise innovators to use market research to identify the best attributes, but also advise that the accuracy need only be sufficient for a GO/NO-GO decision.

7.4. Follower’s Strategic Positioning Decision Depends Upon Observed Scale

If a naïve follower underinvests in CBC studies, and if either $\gamma^{lower} < \gamma^{cutoff} < \gamma^{true}$ or $\gamma^{lower} > \gamma^{cutoff} > \gamma^{true}$, then the follower makes a strategic error by choosing the wrong strategic position (the wrong attribute level). (We state and prove the result formally in Appendix 2.) For example, if $\gamma^{cutoff} < \gamma^{true}$, then Proposition 1 implies that the most profitable attribute level for the follower is r . However, if the follower acts on $\gamma^{market\ research} = \gamma^{lower}$, and if $\gamma^{lower} < \gamma^{cutoff}$, then, by Proposition 2, the follower will choose the less profitable attribute level, s . In some cases, the naïve follower may underinvest in CBC studies, but get lucky, say if $\gamma^{true} < \gamma^{cutoff}$ and $\gamma^{lower} < \gamma^{cutoff}$. The first inequality implies s is the follower’s most profitable attribute level and the second inequality implies the follower chooses s . The important insight is that, if the naïve follower underinvests in the

quality of a CBC study, then it is relying on luck to make the right decision.

Empirically, shirking on the quality of CBC market research can either increase or decrease $\gamma^{market\ research}$ relative to γ^{true} . For example, all else equal, we might expect that a text-based CBC study would predict marketplace choices less precisely (lower scale) than a CBC study based on realistic stimuli—the firm might underestimate scale ($\gamma^{market\ research} < \gamma^{true}$) with a text-based study. However, consumers might also answer text-based questions more consistently than realistic-stimuli-based questions—the firm might overestimate scale ($\gamma^{market\ research} > \gamma^{true}$) if the firm relies only on CBC choice sets, not corrected for validation. If the firm shirks on both market-research quality and no validation correction, it cannot know *a priori* whether the increase in observed scale due to the easier task counteracts the decrease in observed scale because the task represents the market less well. The amount by which $\gamma^{market\ research}$ differs from γ^{true} is an empirical question. (We provide empirical examples in §8.)

From a practical standpoint, if the cost of higher quality is small compared to the expected opportunity loss from making the wrong positioning decision, then the follower should invest in higher quality CBC studies.

7.5. Sophisticated Bayesian Follower’s Decision on Investments in CBC Studies

As firms become more sophisticated, they might take the market-research decision a step further and use Bayesian decision theory to decide whether to invest in higher-quality or lower-quality market research. For example, if the follower can invest K dollars in higher-quality market research to learn γ^{true} , the firm can compare expected profits, from acting optimally on γ^{true} , to expected profits based on the prior distribution of γ^{true} . If higher-quality market research updates the prior, the calculations could take this into account. The sophisticated follower decides among two actions. In the first potential action, the follower invests in higher-quality market research, observes a posterior distribution for γ^{true} , and chooses r or s accordingly. In the second potential action, the follower does not invest in

higher-quality market research and chooses r or s based which strategic position maximizes expected profits (expectation over its prior on γ^{true}). If the difference in profit for the two actions exceeds K , the follower invests in higher-quality market research. Otherwise, the follower makes do with lower-quality market research. The calculations are straightforward and provide no incremental insight. See example in Online Appendix 3.2.

7.6. Simultaneous Game

If the firms enter simultaneously (and we have some mechanism to resolve the indeterminacy), then both firms can make strategic positioning mistakes if they shirk on market-research quality and misestimate γ^{true} .

7.7. Illustrative Example

In Online Appendix 3.1, we provide an illustrative numerical example with $\beta^h = 2$, $\beta^l = 1$, $u_o = 1$, and $R = 0.55$. (R programs are available from the authors.) The effect of γ^{true} on equilibrium prices is similar to that observed for the empirical data in Figure 1. For the vast majority of the range of scale, especially in the range we observe in empirical data, equilibrium prices (and profits) decrease with scale. Prices increase slightly as $\gamma^{true} \rightarrow \infty$. The latter is a result of multiple offsetting forces when the market approaches extreme behavior—very small increases in price relative to competition have large impacts on market shares. As predicted, differentiated positions are most profitable when γ^{true} is large and undifferentiated positions are most profitable when γ^{true} is small. In the illustrative example, $\gamma^{cutoff} \cong 1$. For the illustrative example, opportunity losses for choosing an incorrect strategic position can be quite large; patent/copyright valuations can be off by a factor of five.

7.8. Summary of the Relationship between Scale and Market Research Decisions

The stylized model illustrates that the follower (sequential game) or both firms (simultaneous game) can make strategic errors if they shirk on market research and misestimate the true scale. Similarly, testifying experts, who cannot assure the court that $\gamma^{market\ research} \cong \gamma^{true}$, could provide pa-

tent/copyright valuations that are inaccurate. Our examples suggest that such strategic errors can cost firms tens of millions or hundreds of millions of dollars and that patent/copyright valuations can be of by the same order of magnitude. The effect is real, CBC studies used in pricing and positioning decisions or in litigation often forego realistic images and incentive alignment and shirk in many other ways.

8. Empirical Test: Smartwatches

Stylized models, by their very nature, abstract from empirical applications to focus on a new insight. In our case, we focus on (1) the effect of a common measured scale on strategic positioning, pricing, patent/copyright valuations, and market research investments and (2) the effect of market-research quality on those decisions and valuations. It is reasonable to ask whether the phenomena we study are sufficiently strong that they are observable in empirical applications. In any empirical application, we expect that there will be more than one strategic attribute, more than two levels, more than two products, that heterogeneity is not limited to two segments, that scale is heterogeneous, and that partworths are also heterogeneous. Finally, we want to demonstrate that scale differences ($\gamma^{market\ research}$) depend upon aspects of market research quality and that the differences are sufficiently large to change strategic positioning decisions, pricing decisions, or patent/copyright valuations. By varying aspects of market-research quality according to an experimental design and keeping the same attributes and levels in all conditions, we seek to rule out unobserved attributes and unobserved heterogeneity as alternative explanations. Using counterfactual simulations, we show that errors in measured scale can drive strategic decisions even if there are no changes in the relative partworths and if firms do not react to unobserved attributes.

To achieve these goals, we undertake CBC studies in a realistic product category using multiple attributes, some with more than two levels. We vary two representative aspects of market research quality, incentive alignment and image realism, while holding other aspects constant (and attempt to

maintain the non-varying aspects at professional-level quality).¹ We further obtain a measure of external validity in the form of marketplace-like choices to demonstrate that measured scale depends upon whether scale is adjusted based on validation choices, or whether, as is common practice, scale is based purely on (internal) choices among profiles. We show that the phenomena and insights apply empirically. In addition, we gain important managerial insight on the impact of incentive alignment, stimuli realism, and validation corrections.

8.1. Smartwatch CBC Studies

The product category was smartwatches. We focused on four attributes: case color (silver or gold), watch face (round or rectangular), watch band (black leather, brown leather, or matching metal color), and price (\$299 to \$449). Following industry practice, we held all other attributes constant, including brand and operating system. Our focus on three smartwatch attributes and price is sufficient to test the generalizability of the stylized model; an industry study might vary more attributes. (In §9.2, we examine price equilibria in two studies with more attributes and a multitude of levels.) Empirically, we designed our stimuli so that any unobserved attributes are unlikely to vary among experimental conditions, i.e., higher-vs.-lower quality CBC studies. By assumption for the worlds we simulate, unobserved attributes, even if present, are not used strategically for positioning decisions.

We used sixteen choice sets for estimation (and two for internal validation) with three profiles per choice set. We included the outside option via a dual-response procedure. These settings are typical for current industry applications (Meissner, Oppewal, and Huber 2016; Wlömert and Eggers 2016). We followed standard survey design principles including extensive pretesting (28 respondents in the higher-quality study and 38 in the lower-quality study) to assure that (1) the questions, attributes, and tasks were easy to understand, (2) that the manipulation of research quality between respondents was not

¹ The two varied aspects were part of a larger experimental design. The other aspects were randomized. For brevity, we focus on the two most impactful aspects of quality. Details on the less impactful aspects are available from the authors.

subject to demand artifacts, and (3) that respondents did not report basing decisions on any attributes that were not varied.

8.2. Higher-Quality Aspects: Image Realism and Incentive Alignment

We varied image realism and incentive alignment in a 2 x 2 between-subjects design. These aspects of market-research quality are chosen as illustrative—we expect the stylized model to provide insight for other aspects of market-research quality such as the representativeness of the respondents, the completeness of the product attributes, the type of questions (simple versus dual-response), the number of choice tasks, the number of profiles per choice task, the quality of respondent training, and the quality of partworth estimation. We chose incentive alignment because of the growing academic interest in incentive alignment and because of its proven impact on predictive ability, e.g., Ding (2007), Ding, Grewal, and Liechty (2005), and Ding, et al. (2011). We chose image quality because the product-development literature suggests visual depictions and animations provide nearly the same results as physical prototypes and that rich visual representations are more realistic than text and more likely to evoke marketplace-like responses from respondents (e.g., Dahan and Hauser 2002; Dahan and Srinivasan 2000; Vriens, et al. 1998). Recent research in *Marketing Science* suggests that conjoint analysis with physical prototypes provides different partworth estimates than less-realistic stimuli (e.g., Dzyabura, Jagabathula, and Muller 2018). Despite these recommendations, our review of the *Marketing Science* literature (§7.1) suggests that most academic and industry studies either use text-only stimuli or do not report the type of images they use.

Image realism. After the screening questions, respondents entered the CBC section. After a training task (not used in estimation), each respondent chose among three smartwatches profiles and then indicated whether or not he or she would purchase the smartwatch. Respondents in the realistic-image experimental cells saw high-realism images (Figure 2). To make the images more realistic, the respondent could toggle among a detailed view, a top view, and an app view (not shown in Figure 3).

Respondents in the lower-quality image cells saw only text-based stimuli (with simple images), and could not toggle among views. See Figure 3.

Figure 2. Higher-Quality Study: Choice-Based Dual Response Task
(The images were animated allowing respondents to toggle views.)

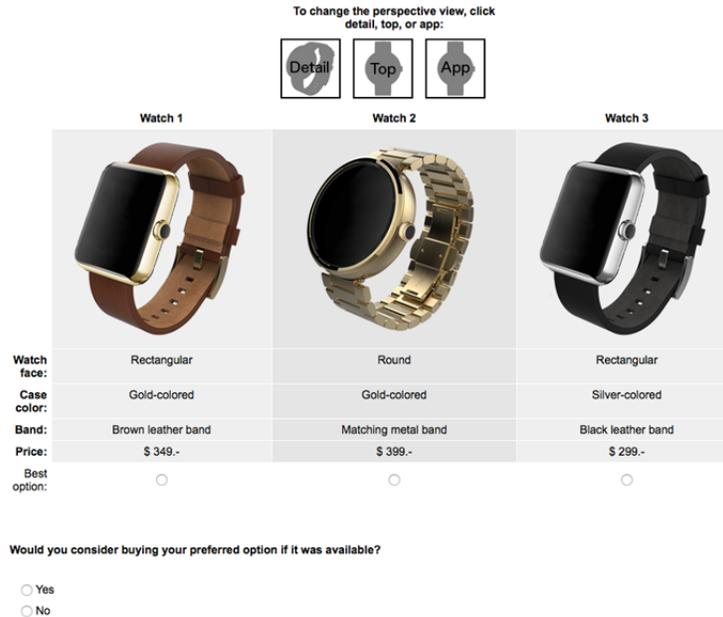
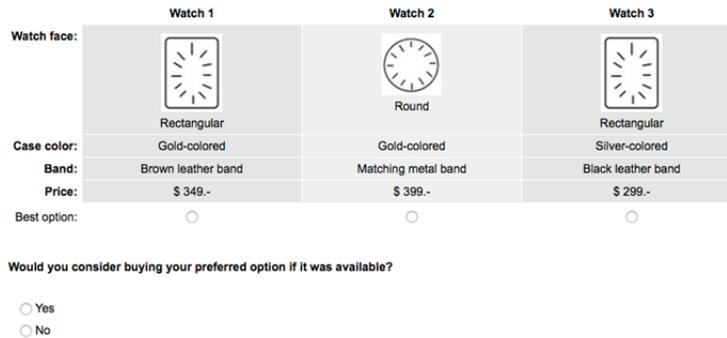


Figure 3. Lower-Quality Study: Choice-Based Dual Response Task (no ability to toggle)



Incentive alignment. In the incentive-aligned experimental cells, respondents saw an animated video to induce incentive alignment.² Specifically, respondents were told that some respondents (1 in 500) would receive a smartwatch and/or cash with a combined value of \$500—based on their answers to the survey. Image realism in the video was matched to image-realism in the experimental cell. See

² The incentive-alignment video is available at <https://www.youtube.com/watch?v=DBLPfRJo2Ho>.

Figure 4. Respondents who were not in incentive-aligned experimental cells received the same cash offer, but the cash was not tied to their answers.

Figure 4. Incentive Alignment Screenshot from the Higher-Quality Study

There is one more thing. We are giving away one of these products you are evaluating. Please watch the following video to learn more.



Conditions:

- 1 out of every 500 respondents will get the \$ 500.- budget to buy the smartwatch
- If you are the winner we will offer you a watch with the specific watch face, case color, band, and price you have chosen in a randomly determined choice set.
- This watch will be from your preferred brand: Apple
- If your exact preference will not be available on the market in about three weeks from now, we will match it as close as possible based on the available options and from what we know about your preferences.
- In order to take part in the lottery we need to know your email address that we will use only to contact you if you have won. We will not contact you otherwise and will not share it with anyone else.
- The reward can only be transferred to the USA, Canada, or Europe.
- Remember, you must be one of the lucky winners to be eligible for this gift.

You will be able to proceed after watching this video.

8.4. Validation Task

External validation is a challenge. The ideal to which we strive is a test of whether the CBC model predicts the choices consumers would make if the hypothetical profiles were to become real products in the marketplace. Although we can get hints from the marketplace, many of the hypothetical profiles will never be market tested. Instead, we seek to mimic marketplace choices by creating a “market” that approximates the marketplace as closely as feasible while controlling for unmodeled marketing actions. As an initial demonstration of the importance of validation scale adjustment, it is sufficient that the validation choice task be perceived as closer to marketplace choices than within-study holdout tasks. If observed scale varies substantially between scale adjusted to the validation task (tested here) and scale based on the estimated partworths (typical practice), then we demonstrate that validation adjustment is an important managerial consideration.

We created a marketplace with twelve smartwatches and an outside option. (Twelve smartwatches represents all possible design combinations.) Smartwatches varied on case color, watch face, watch band, and price. Starting three weeks after the two CBC studies, respondents in all experimental cells were given an incentive-aligned opportunity to choose either one of the twelve smartwatches or the outside option. Marketplace market shares were not available for the smartwatch profiles in our experiment, but, in practice, researchers might consider other validation adjustments such as those proposed by Gilbride, Lenk, and Brazell (2008) or Wlömert and Eggers (2016).

8.5. Sample

Our sample was drawn from a professional panel.³ We screened the sample so that respondents expressed interest in the category, were based in the US, aged 20-69, and agreed to informed consent as required by our internal review boards. We also screened out respondents who already owned a smartwatch. Such screening is reasonable for our research purpose. Respondents in both studies received standard panel incentives for participating in the study.

Overall, 1,693 respondents completed the first wave of studies and, of these, 1,147 completed the validation study (68%). We only considered respondents in the analyses who completed the validation study and removed respondents who always chose the outside option. There were no significant differences between the studies and the exclusion of respondents ($p = 0.86$). The final sample size was 1,044 with sample sizes varying from 248 to 275 among conditions. To illustrate the effect of market-research quality on $\gamma^{market\ research}$ and strategic decisions, we focus on comparisons among the highest-quality experimental cell (realistic images, incentive alignment) and the lowest-quality experimental cell (text-only and no incentive alignment). Later, to examine managerial implications, we compare the effect of realistic images to the effect of incentive alignment. (To be consistent with §7, we label the two

³ Peanut Labs is an international panel with 15 million pre-screened panelists from 36 countries. Their many corporate clients cumulatively gather data from approximately 450,000 completed surveys per month. Peanut Labs is a member of the ARF, CASRO, ESOMAR, and the MRA and has won many awards: web.peanutlabs.com.

studies in this subsection as higher- and lower-quality instead of highest- and lowest quality.)

8.6. Estimation of the Standard CBC Model

We adopt a standard HB CBC estimation model consistent with the stylized model. We begin by summarizing the model to indicate how the partworths and scale coefficients are estimated. We then describe how the (randomized) experimental manipulations enable us to identify a common scale-adjustment factor that depends upon market-research quality. The basic utility model generalizes that used for the stylized model (recall that u_{ij} is consumer i 's utility for product profile j and p_j is the price). For notational simplicity, we state the utility for binary attributes recognizing the standard generalization to multilevel attributes (as in our empirical CBC studies). If profile j has attribute k , then $a_{jk} = 1$, otherwise $a_{jk} = -1$. (It does not matter whether we use dummy coding or effects coding.) The utility model is:

$$u_{ij} = \sum_{k=1}^K \gamma_i (\beta_{ki} a_{jk} - p_j) + \epsilon_{ij}$$

where the probability of choosing each profile (or the outside option) is given by the standard logit model analogous to that used for the stylized model. This empirical model is similar to that used by Sonnier, Ainslie, and Otter (2007) with the exception that Sonnier, Ainslie, and Otter estimate $\mu_i = 1/\gamma_i$ rather than γ_i . Allenby et al. (2014) do not consider the effect of market-research quality on scale and, hence, do not specify a scale factor in the utility model. They need not set $\eta_i = 1$ for identification and estimate a price coefficient. Empirically, the implications, for the phenomena we study, from the three specifications are not significantly different (Online Appendices 3.5 and 3.7).

Like Allenby et al. (2014, p. 436), we use a hierarchical estimation that assumes the observed data are given by the choice model (as a function of the β_{ki} 's, γ_i 's, and a_{jk} 's). The β_{ki} 's and $\ln(\gamma_i)$'s are distributed multivariate normal. The second-stage prior is the standard Normal-Inverted-Wishart conditionally conjugate prior as in Rossi, et al. (2005). Allenby et al. (2014) use the standard relatively diffuse

prior for the β_{ki} 's, but modify the prior for $\ln(\gamma_i)$ to be more diffuse ($\ln(\eta_i)$ in their model). Details are provided in Online Appendix 3.4 and in Allenby et al. (2014), who provide graphical motivation.

To avoid misspecification errors, we tested for interaction effects. We did not detect significant improvements in model fit; our final model is based on main effects. All settings not specified by Allenby et al. followed standard procedures in Sawtooth Software (2015). For example, we used 10,000 burn-in iterations and a subsequent 10,000 iterations to draw partworths, from which we kept every 10th draw. All summaries, profits, and other reported quantities are based on the posterior distributions.

8.7. Identification of Scale

As reviewed in §3.1, utility is unique to a positive linear transformation. Without an experimental design (or panel data), scale cannot be identified independently of the β_{ki} and η_i . To examine whether market-research quality affects estimated scale, we normalize a scale-adjustment to 1.0 for the lower-quality experimental cell and for profiles in the estimation choice tasks. Let $Q_i^h = 1$ if respondent i was exposed to the higher quality condition (0, otherwise) and $V_i = 1$ for respondent i 's validation task (0 for the estimation tasks). Then, for all other experimental cells and for the respondents' choice in the validation data, we specify the utility model as follows. Following Allenby, et al. (2014) we use an exponential transformation to assure that all scale factors are positive.

$$u_{ij} = \sum_{k=1}^K \gamma^{condition} \gamma_i (\beta_{ki} a_{jk} - p_j) + \epsilon_{ij}$$

$$\text{where } \ln(\gamma^{condition}) = \lambda_{Q^h} Q_i^h + \lambda_V V_i + \lambda_{Q^h V} Q_i^h V_i.$$

The common shift in scale is thus identified by external variation in experimental cell and/or estimation vs. validation choice tasks. Thus, we identify a scale adjustment for each experimental cell (other than the lower-quality cell) and for those choices based on validation rather than standard CBC tasks. With two experimental cells and estimation and validation choice tasks, we identify three relative

scale adjustments (higher-quality estimation, lower-quality validation, and higher-quality validation—all relative to lower-quality estimation).

This specification enables us to use all of the data simultaneously and rigorously. We can also use an ad hoc method in which we estimate parameters for each experimental cell using the CBC choice tasks, then use a single-parameter logit model to estimate a scale adjustment between choice tasks and the validation task. When we did so, the more-rigorous methods are highly correlated with the ad hoc methods ($\rho=0.995$).

8.8. CBC Market-Research Quality and Validation Affect Scale as Observed by the Firm

Our first research questions are (1) whether differences in market-research quality affect the scale upon which the firm relies (lower- versus higher-quality) and (2) whether scale estimated from CBC choice tasks is different from scale adjusted with validation data. The posterior means and standard deviations of the scale-adjustment posterior distributions are given in Table 1.

Table 1. Posterior Means of Scale Adjustment
(standard deviations in parentheses; full posterior available from the authors)

	Lower-Quality Study	Higher-Quality Study
Scale is based on estimation data	1.00 ^a (n.a.)	0.86 (0.06)
Scale is adjusted to validation task	0.35 (0.04)	0.61 (0.06)

^a Normalized to 1.00 for identification.

First, we notice that if scale is based on the CBC estimation choice tasks only, the lower-quality study appears to be more precise (lower scale-adjustment factor for the higher-quality study). In the majority of posterior draws (99%), respondents were more consistent in answering text-based questions without incentive alignment than they were in answering questions based on more-realistic images with incentive alignment. If these were the only data available, the firm might conclude that investments in market-research quality were counterproductive.

Although this comparison may, at first, seem counterintuitive, we believe it is intuitive a posteriori. When scale is based on estimation data only, it is primarily a measure of internal reliability. Respondents might be extremely consistent and have well-defined preferences among text-based descriptions, but choices among profiles with text-only attributes (and no incentive alignment) may not reflect respondents' choices in the marketplace. (By extension, tests on hold-out choice tasks may not test the ability to predict marketplace choices and may also be misleading. Our empirical data confirm this.) Our data suggest the effect can occur and that firms should be cautious when making strategic decisions without proper validation data.

We next examine scale adjustment for the validation task. Overall, the scale for the validation task is lower. As expected, higher-quality market research greatly enhances validation-based scale. This is a consistent finding across all posterior draws. As a robustness check on the impact of higher quality on scale adjustment, we examine predictive ability. Both hit rates and uncertainty explained (U^2 , Hauser 1978) for the validation task are substantially improved for the higher quality study—hit rates increase from 24% to 39% (chance is 7.7%) and U^2 increases from 0.16 to 0.33. There was no draw in which the lower-quality study performed better.

The effects appear to be robust. For example, when we use a mixture of normal distributions to estimate upper-level heterogeneity or random splits of the sample or the choice tasks used for the estimation, we obtain the same basic results. When using a mixture approach, internal scale improves slightly with more components relative to a single normal distribution (internal fit measures, i.e., U^2 , increase by about 1%), but the scale-adjustment factors remain consistent. Scale adjustment factors are also not affected when using less data, neither via splits of the sample nor choice tasks, only their posterior standard deviations increase. (See §9.3 for robustness to alternative model specifications.)

In our data, validation adjustment decreases measured scale but validation-adjusted scale is larger for higher quality—the effects move in opposite directions. It is feasible that, by luck, the two

effects might offset, even though they do not do so in our data ($0.61 \neq 1.0$). A firm would be ill-advised to rely on luck for the two effects to offset. Furthermore, there is no guarantee that the two effects move in opposite directions.

8.9. The Empirical Data Produce Strategic Effects Analogous to the Stylized Model

We now examine whether the phenomena predicted by the stylized model can be reproduced using partworths from a typical HB CBC study. This section seeks to examine whether the stylized model applies when we relax that assumptions of two segments, two-levels, homogeneity of partworths within segment, and homogeneity of scale. In §8.10, we demonstrate that lower-quality market research affects relative partworths as well as scale—a double whammy. In this section, we isolate the scale effect to avoid confounding it with any effect of market-research quality on the relative partworths. To do so, we create counterfactuals for “true” scale by holding the distribution of relative partworths constant as we vary the scale adjustment. (Relative partworths and scale still vary among respondents.)

We use the CBC simulator to examine whether scale affects strategic positioning decisions for smartwatch color (silver vs. gold). We use the root-finding method described in Allenby et al. (2014) to find the price equilibria. In order to avoid extrapolation beyond the price range used in the CBC experiment, we cap prices at the upper limit of the data (\$449). For illustration, we chose scale-adjustment values consistent with Table 1 and near the strategic cutoff point at which the optimal strategy switches from differentiation to non-differentiated offerings. Empirically, $\gamma^{cutoff} \cong 0.6$.

Table 2, summarizes the positioning equilibria with unit demand and zero costs. The equilibria exist in the majority of draws and appear to be unique. Because more respondents preferred silver to gold (65.7%) than vice versa, the analogy to the stylized model is $r = \text{silver}$, even though “ r ” is mnemonically cumbersome for silver.

As the scale-adjustment decreased from $\gamma^{true} = 0.8$ to $\gamma^{true} = 0.4$ (holding the distribution of relative partworths constant), the positioning equilibrium shifted from differentiated positions (silver,

gold) to undifferentiated positions (silver, silver). Empirically, for validation-adjusted scale, the firm would differentiate if it based decisions on the higher-quality study and not differentiate if it based decisions on the lower-quality study. If we assume that the market is 10 million units, then positioning based on misestimating the true scale would result in a \$85 million opportunity loss for the follower. For comparison, the Apple Watch sold 11.9 million units in 2016 (Reisinger 2017).

We obtained similar results when we used CBC simulators for watch face (rectangular vs. round) and watch strap (black vs. brown or other combinations) or alternative model specifications. In all counterfactual tests using empirical HB CBC partworths, the market always shifted from differentiated to undifferentiated as “true” scale decreased through a critical value, γ^{cutoff} . We conclude that there are examples where the stylized theory applies to empirical data with heterogeneous relative partworths and scales.

Strategic decisions can also depend upon whether estimated scale is adjusted with validation choices. For the lower-quality study, estimation-based scale implies differentiation while validation-adjusted scale does not. If we assume that the higher-quality-validation-adjusted scale is closest to the true scale, then the lower-quality-estimation-based scale gets the right strategic decision by luck, but for the wrong reasons—the two effects offset. The key point is that strategic errors can result from relying on lower-quality studies and/or from not adjusting scale with validation tasks. The firm does not know *a priori* what, if any, strategic errors it will make if it shirks on market-research quality and validation adjustment.

Table 2. “True” Scale Affects Strategic Positioning with HB CBC Partworths
 (Relative partworths are heterogeneous, but the same in higher- and lower-scale markets.
 In this table, γ^{true} is the scale-adjustment factor which is proportional to scale.)

Higher-Scale ($\gamma^{true} = 0.8$)		Follower’s Position	
		Silver	Gold
Innovator’s Position	Silver	$\pi_{1rr}^* = 72.7$	$\pi_{1rs}^* = 110.8$
		$\pi_{2rr}^* = 72.7$	$\pi_{2rs}^* = 81.2$
	Gold	$\pi_{1sr}^* = 81.2$	$\pi_{1ss}^* = 62.8$
		$\pi_{2sr}^* = 110.8$	$\pi_{2ss}^* = 62.8$
Lower-Scale ($\gamma^{true} = 0.4$)		Follower’s Position	
		Silver	Gold
Innovator’s Position	Silver	$\pi_{1rr}^* = 112.6$	$\pi_{1rs}^* = 132.9$
		$\pi_{2rr}^* = 112.6$	$\pi_{2rs}^* = 106.6$
	Gold	$\pi_{1sr}^* = 106.6$	$\pi_{1ss}^* = 100.2$
		$\pi_{2sr}^* = 132.9$	$\pi_{2ss}^* = 100.2$

8.10. Double-Whammy: CBC Quality Also Affects the Relative Partworths

Our stylized model (and §8.9) focused on the impact of a common scale adjustment—the phenomena investigated in this paper. However, empirically, the quality of the CBC study might also affect the relative partworths. To examine this double whammy, we drew 1,000 times from the posterior distributions to compare the distributions of relative partworths between studies. The importances of the attributes are given by Table 3. (Recall that importance is the largest partworth minus the smallest partworth for each attribute.) The price coefficient for \$150 (the price range in the experiment) is normalized to 1.0 in Table 3 so that the importances are relative to price. The effect of market-research quality on relative partworths reinforces the effect on scale, except for color which is not significantly different between conditions. The estimated importances relative to price are larger for watch band and watch face when market-research quality is higher. This gives firms another incentive to differentiate.

One interpretation is that the higher-quality study encouraged respondents to evaluate attribute importances more carefully (see also Vriens, et al. 1998). However, we cannot rule out situations where greater respondent motivation and more realistic descriptions cause respondents to decrease valuations of attribute importances. As with market-research and validation adjustments, the firm should not rely on luck for the effects to offset.

**Table 3. Posterior Means of Relative Importances
(standard deviations in parentheses; full posterior available from the authors)**

	Lower-Quality Study	Higher-Quality Study
Color	1.39 (0.05)	1.33 (0.05)
Watch Band	1.88 (0.07)	2.46 (0.08)
Watch Face	1.38 (0.06)	1.60 (0.07)
Price	1.00 (n.a.)	1.00 (n.a.)

9. Further Considerations for Managerial Insight and Implementation

9.1. Realistic Images vs. Incentive Alignment

The purpose of §8 was to examine whether the phenomena highlighted in the stylized model apply in a realistic empirical application and to demonstrate that market-research quality could affect $\gamma^{market\ research}$. We now examine which of realistic images or incentive alignment has the largest impact on $\gamma^{market\ research}$. Table 4 provides the posterior means of the scale adjustments for realistic-images, incentive-alignment, and the interaction.

The results suggest that realistic images impacts scale at least as much as incentive alignment—more for validation-based adjustments. The relative improvement due to realistic images is about three times that of incentive alignment for validation-based adjustments: $0.53 - 0.35 = 0.18$ versus $0.41 - 0.35 = 0.06$. Interactions increase both effects. This is an important managerial recommenda-

tion because, while incentive alignment is gaining traction in academia and in practice, published and applied studies use predominately text-based images only.

Table 4. Posterior Means of Scale Adjustment for Realistic Images and Incentive Alignment (standard deviations in parentheses; full posterior available from the authors)

	Text-only images, no incentive alignment	Main effect of realistic images	Main effect of incentive alignment	Realistic-image x incentive alignment
Scale is based on estimation data	1.00 ^a (n.a.)	0.89 (0.04)	0.96 (0.04)	0.86 (0.06)
Scale is adjusted to validation task	0.35 (0.04)	0.53 (0.05)	0.41 (0.04)	0.61 (0.06)

^a Normalized to 1.00 for identification.

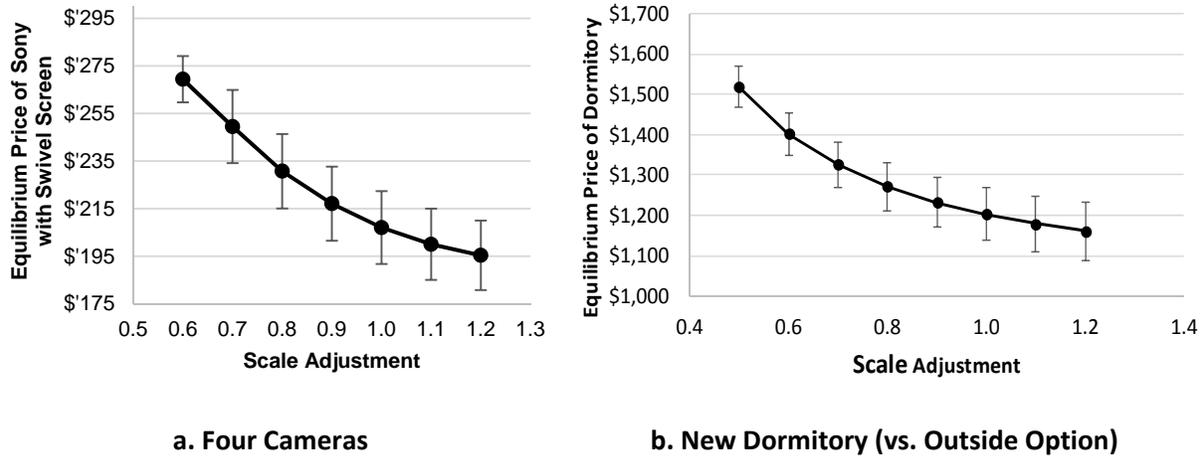
9.2. More Products and/or More Attributes

The smartwatch application focused on two products and four attributes. The estimation and equilibrium calculations are feasible for more products, more attributes, and more levels. For example, Allenby et al. (2014) estimate their HB CBC model and calculate price equilibria for a market with four brands and seven attributes representing a total of seventeen levels, plus an outside option. Although Allenby et al. (2014) do not consider scale effects, we use their data to estimate counterfactuals for scale adjustment. Figure 6a demonstrates that the price equilibria can be identified and that the effect of scale on price equilibria is conceptually similar to that observed for the smartwatch application (Figure 1). Plots for the other three brands are similar.

We also use data from a nationwide CBC study of student preferences for dormitories. (Summary in Online Appendix 3.3.) In this study, there were seven attributes representing a total of twenty-four levels. These data replicate a CBC study that a US university used to design new dormitories. In this case, there is but one “firm,” but it competes against average rents in the immediate vicinity of the university, i.e., the outside option. The new dormitories are sufficiently large (and built at the request of city government) that their presence affects those rents. Figure 6b plots the price equilibria and posteri-

or standard deviations of a specific dormitory characterized by all attributes (1 bedroom, 20 min walk commute, access to grocery stores, queen size bed, no amenities and no parking) as a function of counterfactual values of scale adjustments. The equilibria are feasible to compute in the majority of draws; the implications are conceptually similar to those from the smartwatch application.

Figure 5. Predicted Equilibrium Price as a Function of Scale for Camera and Dormitories



9.3. Alternative Specification of Scale

Sonnier, Ainslie, and Otter (2007) estimate the HB CBC model parameterizing scale as $\mu_i = 1/\gamma_i$. They demonstrate that estimates of mean WTP, as defined by the ratio of partworth differences to the price coefficient, depend whether researchers use the Allenby et al. (2014) specification or a specification in which $\eta_i \equiv 1$ and β_{ki}/η_i and μ_i are estimated directly. The differences in (ratio-based) WTP estimates are driven by the fact that the posterior of WTP is a ratio of two normally-distributed posteriors. This phenomenon is acknowledged in many patent/copyright valuations when experts report the median WTPs, rather than mean WTPs. The Allenby et al. (2014) equilibrium calculations seek to finesse the median-vs.-mean debate by defining valuations as changes in equilibrium prices or profits.

We compare empirical estimates obtained from the specification in the stylized model ($\eta_i = 1$ and γ_i log-normally distributed), the Allenby et al. (2014) specification (η_i log-normally distributed) or the Sonnier, Ainslie, and Otter (2007) specification ($\eta_i = 1$ and $\mu_i = 1/\gamma_i$ log-normally distributed). The

posterior means of the scale-adjustments vary slightly, but well within posterior confidence intervals. The implications of the stylized model are not dependent upon the empirical specification. As anticipated by Sonnier, Ainslie, and Otter (2007, p. 315-317), the ratio-based WTP posterior means vary more for the Allenby et al. (2014) specification. Medians reduce this variation to a certain extent. The ratio-based WTP posterior means and medians are almost identical between the stylized-model-based estimates and the Sonnier, Ainslie, and Otter (2007) model. See Online Appendix 3.7.

9.4. Computation Feasibility

The empirical equilibrium calculations require that we solve a fixed-point problem for every draw from the posterior distribution of partworths. This procedure is computationally intensive, but feasible. For example, on a standard Apple MacBook Pro computer with 2.7 GHz Intel Core i5 processor and 8 GB memory, using programs written in R, the equilibrium prices for Figure 6a with five alternatives and $n = 10,000$ draws of the hyper-parameters were computed in an average of 2.85 seconds per draw (standard deviation = 0.99 seconds, ≈ 48 minutes for 1,000 draws) and the equilibrium prices for Figure 6b with two alternatives and $n = 534$ consumers were computed in an average of 0.04 seconds per draw (standard deviation = 0.02 seconds, ≈ 44 seconds for 1,000 draws).

10. Managerial Implications and Future Research

Many previous papers establish (1) that quality improvements enhance the accuracy with which the relative partworths can be estimated and (2) that accounting for heterogeneity in scale enhances accurate estimation of the relative partworths. This paper demonstrates that market-research quality affects observed scale and that observed scale affects strategic positioning decisions, predicted equilibrium prices, and patent/copyright valuations. Managerial recommendations and/or patent/copyright valuations are extremely sensitive to the quality of the CBC study on which those decisions or valuations are based. Shirking on market-research quality can have implications in the range of hundreds of millions of dollars. It might surprise many that, even if the relative partworths are unaffected, decisions on

market-research quality can affect observed scale substantially and dramatically affect strategic positioning decisions, pricing decisions, and patent/copyright valuations.

Empirically, our initial tests have the following implications, each of which can have substantial profit implications. Specifically,

- incentive-alignment has a large impact on scale consistent with its known effect on partworth accuracy and predictive ability,
- image realism can have an even larger impact. The impact of realistic images, while obvious post hoc, is often not considered an issue in academic research, most CBC applications, or most litigation valuations.
- adjusting (observed) scale with validation may reverse strategic positioning decisions, prevent misleading pricing recommendations, or avoid misestimation of patent/copyright valuations. While academic authors suggest adjusting scale to market share, the practice is not widespread, nor is it common in typical applications or patent/copyright valuations. Most CBC studies (and papers) evaluate the quality of the data or estimation based only on held-out choices among profiles using the same format as for estimation.
- incentive alignment and image realism also affect relative partworth estimates. The former is now well-known, but the latter underappreciated. In our application, the effects on scale and relative partworth estimates reinforce one another, but there is no guarantee that reinforcement generalizes.
- while it is true that some effects can offset, e.g., the impact of market-research quality and the impact of validation scale adjustment, firms should not rely on luck for making the correct strategic decision.

Our stylized model abstracts aspects of empirical applications. Our empirical examples suggest the abstractions do not affect the basic insights. We sought proof-of-concept empirical demonstrations

for two aspects of market-research quality and one validation task. The theory extends to other aspects of market-research quality and other validation tasks. We hope that researchers explore whether our empirical observations generalize to other product categories—observations such as the interaction of market-research-quality and estimation-vs.-validation on scale adjustment or the reinforcing nature of scale and partworth estimation. We also hope that researchers further explore sensitivity, both theoretically and empirically, to the assumptions that were necessary for the stylized model.

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Appendix 1: Summary of Notation

i	indexes consumers.
j	indexes firms. Firm 1 is the innovator; Firm 2 is the follower.
c_j	Firm j 's marginal cost.
C_j	Firm j 's fixed costs.
r	a product attribute. We can think of r as red (or rose, regular, round, or routine).
s	a product attribute. We can think of s as silver (or sapphire, small, square, or special). A firm's product can have either r or s . It cannot have both or neither.
p_j	Firm j 's price.
p_{jrr}^*	Nash equilibrium price for Firm j given that Firm 1 chooses r and Firm 2 chooses r . Define p_{jrs}^* , p_{jsr}^* , and p_{jss}^* analogously.
P_{ijrr}	probability that consumer i purchases product from Firm j given that Firm 1 chooses r and Firm 2 chooses r . Define P_{ijrs} , P_{ijsr} , and P_{ijss} analogously.
P_{Rjrr}	probability that a consumer in segment R purchases product from Firm j given that Firm 1 chooses r and Firm 2 chooses r . Define P_{Rjrs} , P_{Rjrs} , P_{Rjss} , P_{Sjrs} , P_{Sjrs} , and P_{Sjss} , analogously.
R	size of Segment R (We use italics for the size of the segment; non-italics to name the segment.)
S	size of Segment S.
K	market research cost for high-quality study (sophisticated follower)
WTP	willingness to pay
u_{ij}	utility that consumer i perceives for Firm j 's product.
u_{io}	utility that consumer i perceives for the outside option
u_o	utility of outside option for Segments R and S.
u_{Rj}	utility of Firm j 's product among consumers in segment R.
u_{Sj}	utility of Firm j 's product among consumers in Segment S.
V	Number (measure) of consumers.
β_{ri}	relative partworth for r for consumer i .
β_{si}	relative partworth for s for consumer i .
β_{rR}	relative partworth of r for all $i \in R$. Define β_{sR} , β_{rS} , and β_{sS} analogously.
β^h	higher partworth, $\beta_{rR} = \beta_{sS} = \beta^h$.
β^ℓ	lower partworth, $\beta_{rS} = \beta_{sR} = \beta^\ell$. Theory holds if β^ℓ normalized to zero, but is less intuitive.
β_{ki}	relative partworth for attribute k and consumer i
δ_{rj}	indicator function for whether Firm j 's product has attribute r . Define δ_{sj} analogously.
ϵ_{ij}	error term for consumer i for Firm j 's product. Errors are i.i.d. extreme value random variables.
η_i	price coefficient, normalized to 1 in the stylized model and the some empirical applications
γ_i	scale. Larger values imply smaller relative magnitude of the error term. ($\mu_i = 1/\gamma_i$).
γ	when scale is homogeneous.
$\gamma^{asymptotic}$	scale obtained with theoretically best quality market research
$\gamma^{condition}$	relative scale as affected by market-research quality
γ^{cutoff}	cutoff value for scale. $\gamma > \gamma^{cutoff}$ implies differentiation. $\gamma < \gamma^{cutoff}$ no differentiation.
γ^{higher}	scale estimated with higher-quality CBC study. γ^{lower} for the lower-quality CBC study.
γ^{true}	the true scale (sometimes γ for notational simplicity in proofs if not confused with γ_i .)
$\lambda_{Q^h}, \lambda_V, \lambda_{Q^hV}$	used to identify scale effects for market-research-quality conditions (Q_i^h, V_i indicators)
π_j	profits for Firm j .
π_{jrr}^*	profits for Firm j at the Nash equilibrium prices. Define π_{jrs}^* , π_{jsr}^* , and π_{jss}^* analogously.
$\Delta_2(\gamma)$	indicator of whether, for a given γ , it is more profitable for Firm 2 to differentiate.
Δ_{R2rr}	defined in the proof to Result 2. Δ_{S2rr} and other terms for rs , sr , and ss defined analogously.

Appendix 2. Proofs to Results and Propositions (provided for review)

Throughout this appendix, for notational simplicity, we drop the superscript on γ^{true} and write it simply as γ . Results in this appendix are stated in notational shorthand, but are the same as those in the text.

Result 1. For $\gamma \rightarrow 0$, $\pi_{2rr}^* > \pi_{2rs}^*$, $\pi_{1rs}^* > \pi_{1rr}^*$, and $\pi_{1rs}^* > \pi_{2rs}^*$.

Proof. This proof addresses first-order conditions. We address second-order and cross-partial conditions when we examine existence and uniqueness later in this appendix. As $\gamma \rightarrow 0$, the logit curve becomes extremely flat, which motivates a Taylor's Series expansion of market share around $\beta^h = \beta^\ell$. When $\beta^h = \beta^\ell$ the logit equations for the market shares are identical for Firm 1 and 2, identical for all strategies, rr, rs, sr, ss , and symmetric with respect to Firm 1 and Firm 2. Thus, at $\beta^h = \beta^\ell$ we have

$$p_{1rr}^* = p_{2rr}^* = p_{1rs}^* = p_{2rs}^* = p \text{ at } \beta^h = \beta^\ell$$

$$P_{R1rr}^* = P_{R2rr}^* = P_{R1rs}^* = P_{R2rs}^* = P \text{ at } \beta^h = \beta^\ell$$

$$P_{S1rr}^* = P_{S2rr}^* = P_{S1rs}^* = P_{S2rs}^* = P \text{ at } \beta^h = \beta^\ell$$

Because the prices and shares are identical, we have:

$$\pi_{1rr}^* = \pi_{2rr}^* = \pi_{1rs}^* = \pi_{2rs}^* = \frac{1}{\gamma} \frac{P}{1-P} \text{ at } \beta^h = \beta^\ell$$

Where the last step comes from substituting the equalities for P in Equation 4b from the text, and simplifying using $R + S = 1$. We obtain the optimal price by solving the following fixed-point problem in p :

$$\gamma p = \frac{1}{1-P} = \frac{2e^{-\gamma p} + e^{\gamma u_o}}{e^{-\gamma p} + e^{\gamma u_o}} \text{ using } P = \frac{e^{-\gamma p}}{2e^{-\gamma p} + e^{\gamma u_o}}$$

Because the right-hand side is decreasing in p on the range $[1, 1.5]$ there will be exactly one solution in the range of $\gamma p \in [1, 1.5]$ for small γ . We compute the partial derivatives of the P 's at $\beta^h = \beta^\ell$:

$$\frac{\partial P_{R2rr}}{\partial \beta^h} = \gamma P(1-2P) \equiv \gamma \Delta_{R2rr}, \quad \frac{\partial P_{S2rr}}{\partial \beta^h} = 0 \equiv \gamma \Delta_{S2rr}$$

$$\frac{\partial P_{R2rs}}{\partial \beta^h} = -\gamma P^2 \equiv \gamma \Delta_{R2rs}, \quad \frac{\partial P_{S2rs}}{\partial \beta^h} = \gamma P(1-P) \equiv \gamma \Delta_{S2rs}, \quad \Delta = \beta^h - \beta^\ell$$

We now use a Taylor's series expansion with respect to β^h . Using standard mathematical arguments higher order terms that are $O(\gamma^2)$ or higher vanish as $\gamma \rightarrow 0$. (The ratio of terms $O(\gamma^2)$ or higher to terms $O(\gamma)$ goes to zero as $\gamma \rightarrow 0$.) Substituting the expressions for the partial derivatives into the first-order conditions (Equation 4b), multiplying by γ , and using the above notation, we obtain:

$$\gamma \pi_{2rr}^* = \frac{[R(P + \gamma \Delta_{R2rr} \Delta) + S(P + \gamma \Delta_{S2rr} \Delta)]^2 + o(\gamma^2)}{R(P + \gamma \Delta_{R2rr} \Delta)((1-P) - \gamma \Delta_{R2rr} \Delta) + S(P + \gamma \Delta_{S2rr} \Delta)((1-P) - \gamma \Delta_{S2rr} \Delta) + o(\gamma^2)}$$

$$\gamma\pi_{2rr}^* = \frac{P^2 + 2\gamma\Delta(R\Delta_{R2rr} + S\Delta_{S2rr}) + O(\gamma^2)}{P(1-P) + \gamma\Delta(1-2P)(R\Delta_{R2rr} + S\Delta_{S2rr}) + O(\gamma^2)}$$

Similarly,

$$\gamma\pi_{2rs}^* = \frac{P^2 + 2\gamma\Delta(R\Delta_{R2rs} + S\Delta_{S2rs}) + O(\gamma^2)}{P(1-P) + \gamma\Delta(1-2P)(R\Delta_{R2rs} + S\Delta_{S2rs}) + O(\gamma^2)}$$

Because all terms in the numerators and denominators of $\gamma\pi_{2rr}^*$ and $\gamma\pi_{2rs}^*$ are clearly positive, the condition for $\gamma\pi_{2rr}^* > \gamma\pi_{2rs}^*$ for $\gamma \rightarrow 0$ becomes:

$$\begin{aligned} P^2 + 2\gamma\Delta(R\Delta_{R2rr} + S\Delta_{S2rr})[P(1-P) + \gamma\Delta(1-2P)(R\Delta_{R2rs} + S\Delta_{S2rs})] \\ > P^2 + 2\gamma\Delta(R\Delta_{R2rs} + S\Delta_{S2rs})[P(1-P) + \gamma\Delta(1-2P)(R\Delta_{R2rs} + S\Delta_{S2rs})] \end{aligned}$$

After simplification and ignoring terms that are $O(\gamma^2)$, this expression reduces to:

$$\gamma\Delta P[(R\Delta_{R2rr} + S\Delta_{S2rr}) - (R\Delta_{R2rs} + S\Delta_{S2rs})][2 - 3P + 2P^2] > 0$$

We need only show that both terms in brackets are positive. We show the first term in brackets is positive because:

$$(R\Delta_{R2rr} + S\Delta_{S2rr}) - (R\Delta_{R2rs} + S\Delta_{S2rs}) = (RP(1-P) - RP^2) - (SP(1-P) - SP^2) > 0$$

The last step follows from $R > S$. We show the second term is positive because its minimum occurs at $P = \frac{3}{4}$ and its value at this minimum is $2 - 3P + 2P^2 = \frac{7}{8}$. Thus, $2 - 3P + 2P^2$ is positive for all $P \in [0,1]$.

To prove that $\pi_{1rs}^* > \pi_{1rr}^*$ for $\gamma \rightarrow 0$ we use another Taylor's series expansion and simplify by the same procedures that recognize that higher-order terms vanish. Most of the algebra is the same until we come down to the following term in brackets (now reversed because rs is more profitable for Firm 1 than rr as $\gamma \rightarrow 0$). Taking derivatives gives:

$$\frac{\partial P_{R1rr}}{\partial \beta^h} = \gamma P(1-2P) \equiv \gamma\Delta_{R1rr} \quad \frac{\partial P_{S1rr}}{\partial \beta^h} = 0 \equiv \gamma\Delta_{S1rr}$$

$$\frac{\partial P_{R1rs}}{\partial \beta^h} = \gamma P(1-P) \equiv \gamma\Delta_{R1rs} \quad \frac{\partial P_{S1rs}}{\partial \beta^h} = -\gamma P^2 \equiv \gamma\Delta_{S1rs}$$

The corresponding expression in brackets becomes (for $\gamma\pi_{1rs}^* - \gamma\pi_{1rr}^*$):

$$(R\Delta_{R1rs} + S\Delta_{S1rs}) - (R\Delta_{R1rr} + S\Delta_{S1rr}) = (RP(1-P) - SP^2) - (RP(1-P) - RP^2) > 0$$

where the last step is true because $R > S$.

By exploiting symmetry, we have $\pi_{1rr}^* = \pi_{2rr}^*$, yielding the result that $\pi_{1rs}^* > \pi_{1rr}^* = \pi_{2rr}^* > \pi_{2rs}^*$. \square

Lemma 1. $\gamma p_{1rs}^* < (1 - P_{R1rs}^*)^{-1}$ and $\gamma p_{2rs}^* < (1 - P_{S2rs}^*)^{-1}$. Related conditions hold for rr , ss , and sr .

Proof. We use the first-order conditions (for rs) as illustrated in Equation 4a. All terms are positive, so we cross multiply. After cross multiplying, the first expression is equivalent to $RP_{R1rs}^*(1 - P_{R1rs}^*) +$

$SP_{S1rs}^*(1 - P_{S1rs}^*) > RP_{R1rs}^*(1 - P_{R1rs}^*) + SP_{S1rs}^*(1 - P_{R1rs}^*)$, which is true if $P_{R1rs}^* > P_{S1rs}^*$. The latter holds whenever $\beta^h > \beta^\ell$ for all γ by substituting directly into the logit equation. We prove the second expression by using the first-order conditions for p_{2rs}^* . Related expressions hold for other positionings. For example, for the rr positions, $\gamma p_{1rr}^* < (1 - P_{R1rr}^*)^{-1}$ and $\gamma p_{2rr}^* < (1 - P_{R2rr}^*)^{-1}$.

Result 2. Suppose β^h is sufficiently larger than u_o and $u_o \geq \beta^\ell$. Then, there exists a sufficiently large γ such that $\pi_{2rs}^* > \pi_{2rr}^*$, $\pi_{1rs}^* > \pi_{1rr}^*$, and $\pi_{1rs}^* > \pi_{2rs}^*$.

Proof. In this proof we examine the first-order conditions. Second-order and cross-partial conditions are addressed when we consider existence and uniqueness later in this appendix. We first recognize that:

$$P_{S1rs} = \frac{e^{\gamma(\beta^\ell - p_1)}}{e^{\gamma(\beta^\ell - p_1)} + e^{\gamma(\beta^h - p_2)} + e^{\gamma u_o}}, P_{R1rs} = \frac{e^{\gamma(\beta^h - p_1)}}{e^{\gamma(\beta^h - p_1)} + e^{\gamma(\beta^\ell - p_2)} + e^{\gamma u_o}}$$

$$P_{R2rs} = \frac{e^{\gamma(\beta^\ell - p_2)}}{e^{\gamma(\beta^h - p_1)} + e^{\gamma(\beta^\ell - p_2)} + e^{\gamma u_o}}, P_{S2rs} = \frac{e^{\gamma(\beta^h - p_2)}}{e^{\gamma(\beta^\ell - p_1)} + e^{\gamma(\beta^h - p_2)} + e^{\gamma u_o}}$$

$$P_{R2rr} = \frac{e^{\gamma(\beta^h - p_2)}}{e^{\gamma(\beta^h - p_1)} + e^{\gamma(\beta^h - p_2)} + e^{\gamma u_o}}, P_{S2rr} = \frac{e^{\gamma(\beta^\ell - p_2)}}{e^{\gamma(\beta^\ell - p_1)} + e^{\gamma(\beta^\ell - p_2)} + e^{\gamma u_o}}$$

When γ is large relative to β and u_o , $P_{S1rr} = P_{S2rr} \approx 0$, $P_{R2rs} \approx 0$, and $P_{S1rs} \approx 0$. Substituting and using algebra to simplify the first-order conditions gives us:

$$\gamma p_{2rs}^* = \frac{RP_{R2rs}^* + SP_{S2rs}^*}{RP_{R2rs}^*(1 - P_{R2rs}^*) + SP_{S2rs}^*(1 - P_{S2rs}^*)} \cong \frac{SP_{S2rs}^*}{SP_{S2rs}^*(1 - P_{S2rs}^*)} = \frac{1}{1 - P_{S2rs}^*}$$

We substitute the logit model directly for P_{S2rs} and simplify algebraically to obtain:

$$\gamma p_{2rs}^* \cong \frac{e^{\gamma(\beta^h - p_{2rs}^*)} + e^{\gamma(\beta^\ell - p_{1rs}^*)} + e^{\gamma u_o}}{e^{\gamma(\beta^\ell - p_{1rs}^*)} + e^{\gamma u_o}} \cong e^{\gamma(\beta^h - u_o - p_{2rs}^*)} + 1$$

As γ gets large and positive, the effect of γ as an exponent is much larger than the effect of γ as a multiplier, thus the expression in parentheses in the exponent must converge toward zero for the equality to hold. As the expression approaches zero, the solution to this fixed point problem approaches $p_{2rs}^* = \beta^h - u_o - \epsilon$ where $\epsilon > 0$, ϵ is but a fraction of $\beta^h - u_o$, and $\epsilon \rightarrow 0$ as $\gamma \rightarrow \infty$. Thus, $\pi_{2rs}^* = p_{2rs}^*(RP_{R2rs}^* + SP_{S2rs}^*) \cong p_{2rs}^* SP_{S2rs}^* \cong SP_{S2rs}^*(\beta^h - u_o - \epsilon)$. (The first expression is by the definition of π_{2rs}^* .) Substituting p_{2rs}^* into the expression for P_{S2rs}^* , we get:

$$P_{S2rs}^* = \frac{e^{\gamma(u_o + \epsilon)}}{e^{\gamma(u_o + \epsilon)} + e^{\gamma u_o} + e^{\gamma(\beta^\ell - p_{1rs}^*)}} > \frac{e^{\gamma(u_o + \epsilon)}}{e^{\gamma(u_o + \epsilon)} + e^{\gamma u_o} + e^{\gamma(u_o - p_{1rs}^*)}} > \frac{1}{3}$$

Thus, using $P_{R2rs} \approx 0$, the solution to the fixed point problem, and the definition of π_{2rs}^* , we have shown that π_{2rs}^* is greater than $S(\beta^h - u_o)/3$ as $\gamma \rightarrow \infty$. (P_{S2rs}^* actually gets close to 1 and π_{2rs}^* gets close to $S(\beta^h - u_o)$ as $\gamma \rightarrow \infty$, but we only need the weaker lower bound.)

Thus, for sufficiently large γ (relative to β^h and u_o), the solution of π_{2rs}^* is greater than $S(\beta^h - u_o)/3$. Similar arguments establish that $\pi_{1rs}^* \cong R(\beta^h - u_o - \epsilon)P_{R1rs}^*$ and that π_{1rs}^* is greater than $R(\beta^h - u_o)/3$. (Recall that $\epsilon \rightarrow 0$ as $\gamma \rightarrow \infty$.)

We examine the price equilibrium when both Firm 1 and Firm 2 choose r . We first recognize that, by symmetry, $p_{1rr}^* = p_{2rr}^*$. Hence,

$$P_{R2rr} = \frac{e^{\gamma(\beta^h - p_{2rr}^*)}}{2e^{\gamma(\beta^h - p_{2rr}^*)} + e^{\gamma u_o}} \text{ and } P_{S2rr} = \frac{e^{\gamma(\beta^\ell - p_{2rr}^*)}}{2e^{\gamma(\beta^\ell - p_{2rr}^*)} + e^{\gamma u_o}}$$

We seek to show that there is a p_{2rr}^* , with the properties that $p_{2rr}^* \ll \beta^h - u_o$ and $p_{2rr}^* < u_o$, which satisfies the first-order conditions. In this case, as $\gamma \rightarrow \infty$, $P_{R2rr}^* \cong \frac{1}{2}$. The first-order conditions become:

$$\gamma p_{2rr}^* = \frac{RP_{R2rr}^* + SP_{S2rr}^*}{RP_{R2rr}^*(1 - P_{R2rr}^*) + SP_{S2rr}^*(1 - P_{S2rr}^*)} \cong \frac{\frac{1}{2}R + SP_{S2rr}^*}{\frac{1}{4}R + SP_{S2rr}^*(1 - P_{S2rr}^*)} \leq \frac{\frac{1}{2}R + S}{\frac{1}{4}R} = 2 + 4\frac{S}{R} < 6$$

The third to last step, setting $P_{S2rr}^* = 1$ for the inequality, is possible because the fraction increases in P_{S2rr}^* to obtain its maximum at $P_{S2rr}^* = 1$, as shown with simple calculus. Thus, if p_{2rr}^* satisfies the first-order conditions, then $p_{2rr}^* < 6/\gamma$. Putting the upper bound on π_{2rr}^* together with the lower bound on π_{2rs}^* , $\pi_{2rr}^* < \frac{6}{\gamma}(\frac{1}{2}R + S) < \frac{9}{2\gamma} < \frac{1}{3}S(\beta^h - u_o) < \pi_{2rs}^*$ for γ sufficiently large. (We use the condition that $\beta^h > u_o$ by a sufficient amount.) We establish $\pi_{1rr}^* < \pi_{1rs}^*$ by similar arguments recognizing that, by symmetry, $\pi_{1rr}^* = \pi_{2rr}^*$, and using the proven result that π_{1rs}^* is greater than $\frac{1}{3}R(\beta^h - u_o)$ for sufficiently large γ . \square

Result 3. $\pi_{1rr}^* = \pi_{2rr}^* > \pi_{1ss}^* = \pi_{2ss}^*$.

Proof. We examine the equations for the segment-based market shares to recognize that $P_{R1rr}(p_{1ss}^*, p_{2ss}^*) = P_{S1ss}(p_{1ss}^*, p_{2ss}^*)$ and $P_{S1rr}(p_{1ss}^*, p_{2ss}^*) = P_{R1ss}(p_{1ss}^*, p_{2ss}^*)$, and $P_{S1ss}(p_{1ss}^*, p_{2ss}^*) > P_{R1ss}(p_{1ss}^*, p_{2ss}^*)$. Thus, $\pi_{1rr}^* = \pi_{1rr}(p_{1rr}^*, p_{2rr}^*) \geq \pi_{1rr}(p_{1ss}^*, p_{2ss}^*) = p_{1ss}^*[RP_{R1rr}(p_{1ss}^*, p_{2ss}^*) + SP_{S1rr}(p_{1ss}^*, p_{2ss}^*)] = p_{1ss}^*[RP_{S1ss}^* + SP_{R1ss}^*] > p_{1ss}^*[SP_{S1ss}^* + RP_{R1ss}^*] = \pi_{1ss}^*$. The second inequality is by the principle of optimality. The last inequality uses $R > S$ and $P_{S1ss}^* > P_{R1ss}^*$. The equalities, $\pi_{1rr}^* = \pi_{2rr}^*$ and $\pi_{1ss}^* = \pi_{2ss}^*$, are by symmetry. \square

Result 4. Suppose β^h is sufficiently larger than u_o and $u_o \geq \beta^\ell$. Then, there exists a sufficiently large γ such that $\pi_{1rs}^* > \pi_{1sr}^*$.

Proof. By symmetry, we recognize that $\pi_{1sr}^* = \pi_{2rs}^*$. In the proof to Result 2 we established that $\pi_{2rs}^* \cong S(\beta^h - u_o)P_{S2rs}^*$ and $\pi_{1rs}^* \cong R(\beta^h - u_o)P_{R1rs}^*$ because $\epsilon \rightarrow 0$ as $\gamma \rightarrow \infty$. We also see that the fixed-point problems are identical for p_{1rs}^* and p_{2rs}^* , thus, as $\gamma \rightarrow \infty$, $p_{1rs}^* \cong p_{2rs}^*$, which implies that $P_{R1rs}^* \cong P_{S2rs}^*$. Putting these relationships together implies that $\pi_{1rs}^* \cong R(\beta^h - u_o)P_{R1rs}^* > S(\beta^h - u_o)P_{R1rs}^* \cong S(\beta^h - u_o)P_{S2rs}^* \cong \pi_{2rs}^* = \pi_{1sr}^*$ \square

Proposition 1. For $\gamma \rightarrow 0$, the innovator targets r and the follower targets r .

Proposition 2. If β^h is sufficiently larger than u_o and if $u_o \geq \beta^l$, then there exists a sufficiently large γ such that the innovator targets r and the follower targets s .

Proof. We prove the two propositions together. Result 1 establishes that $\pi_{2rr}^* > \pi_{2rs}^*$ as $\gamma \rightarrow 0$. Result 2 establishes that $\pi_{2sr}^* > \pi_{2rr}^*$ when γ is sufficiently large. Thus, if Firm 1 chooses r , Firm 2 chooses r as $\gamma \rightarrow 0$ and chooses s when γ gets sufficiently large.

To prove that Firm 1 always chooses r , we first consider the case where $\gamma \rightarrow 0$. If Firm 1 chooses r , then Firm 2 chooses r by Proposition 1. Suppose instead that Firm 1 chooses s , then Firm 2 will choose r . Firm 2 will choose r in this case because, by Result 1, $\pi_{1rs}^* > \pi_{1rr}^*$ and, by symmetry, $\pi_{2sr}^* = \pi_{1rs}^*$, hence $\pi_{2sr}^* > \pi_{1rr}^* = \pi_{2rr}^*$. If Firm 2 would choose r whenever Firm 1 chooses s , Firm 1 would earn π_{1sr}^* . But $\pi_{1sr}^* = \pi_{2rs}^*$ by symmetry and $\pi_{2rs}^* < \pi_{2rr}^* = \pi_{1rr}^*$ by Result 1. Thus, Firm 1 earns more profits (π_{1rr}^*) by choosing r than the profits it would obtain (π_{1sr}^*) by choosing s .

We now consider the case where γ is sufficiently large. Suppose Firm 1 chooses r , then Firm 2 will choose s by Result 2. Firm 1 receives π_{1rs}^* . Suppose instead that Firm 1 chooses s , then Firm 2 will choose r because $\pi_{2sr}^* = \pi_{1rs}^*$ by symmetry and $\pi_{1rs}^* > \pi_{1ss}^* = \pi_{2ss}^*$ under the conditions of Result 2. Thus, if Firm 1 chooses s it receives π_{1sr}^* . Because $\pi_{1rs}^* > \pi_{1sr}^*$ by Result 4, Firm 1 will choose r . \square

Existence and Uniqueness. The existence and uniqueness arguments require substantial algebra. To avoid an excessively long appendix, we provide the basic insight. Detailed calculations are available from the authors. The proofs to Results 1-4 rely on the first-order conditions, thus we must show that a solution to the first-order conditions, if it exists, satisfies the second-order conditions. The second-order conditions for the rs positions are. We seek to show they are negative at equilibrium.

$$\frac{\partial^2 \pi_{1rs}^*}{\partial p_{1rs}^2} = -\gamma R P_{R1rs}^* (1 - P_{R1rs}^*) [2 - \gamma p_{1rs}^* (1 - 2P_{R1rs}^*)] - \gamma S P_{S1rs}^* (1 - P_{S1rs}^*) [2 - \gamma p_{1rs}^* (1 - 2P_{S1rs}^*)]$$

We use Lemma 1 to substitute $(1 - P_{R1rs}^*)^{-1}$ for γp_{1rs}^* . The former is a larger value, so if the conditions hold for the larger value, they hold for γp_{1rs}^* . Algebra simplifies the right-hand side of the second-order condition to $-\gamma \{R P_{R1rs}^* (1 - P_{R1rs}^*) + S P_{S1rs}^* (1 - P_{S1rs}^*) (1 - 2P_{R1rs}^* + 2P_{S1rs}^*)\}$. With direct substitution in the logit model, recognizing $p_{1rs}^* \geq p_{2rs}^*$, we show $P_{S2rs}^* \geq P_{R1rs}^* \geq P_{R2rs}^* \geq P_{S1rs}^*$. (We show $p_{1rs}^* \geq p_{2rs}^*$ with implicit differentiation of the first-order conditions with respect to R .) These inequalities imply that $P_{S1rs}^* < \min\{P_{R1rs}^*, 1 - P_{R1rs}^*\}$. Hence, $R P_{R1rs}^* (1 - P_{R1rs}^*) \geq S P_{S1rs}^* (1 - P_{S1rs}^*)$ whenever $R > S$. Thus, the second-order condition is more negative than $-\gamma S P_{S1rs}^* (1 - P_{S1rs}^*) (2 - 2P_{R1}^* + 2P_{S1}^*) < 0$. We repeat the analysis for p_{2rs}^* using a sufficient technical condition that either $P_{S2rs}^* \leq \frac{1}{2}$ or that the ratio of S/R is above a minimum value. (The condition, not shown here, requires only $S > 0$ as $\gamma \rightarrow \infty$.) Although our proof formally imposes the technical sufficient condition, we have not found any violation of the second-order conditions at equilibrium, even with small S . Thus, with a (possible) mild restriction on S , the second-order conditions are satisfied whenever the first-order conditions hold.

We now establish that the second-order conditions are satisfied on a compact set. We begin by showing algebraically that $(1 - P_{R1rs}^*)^{-1}$ is decreasing in p_{1rs} and that it decreases from a finite positive value, which we call $F_{R1rs}(p_{1rs} = 0) > 1$. As $p_{1rs} \rightarrow \infty$, $(1 - P_{R1rs}^*)^{-1}$ decreases to 1. But γp_{1rs} increases from 0 to ∞ , thus there must be a solution to $\gamma p_{1rs} = (1 - P_{R1rs}^*)^{-1}$ for every p_{2rs} . Call this solution $p_{1rs}^m(p_{2rs})$. Because $(1 - P_{R1rs}^*)^{-1}$ is decreasing in γp_{1rs} , it must be true that $\gamma p_{1rs} \leq (1 - P_{R1rs}^*)^{-1}$ for all $p_{1rs} \in [0, p_{1rs}^m(p_{2rs})]$. Using similar arguments we show there exists a $p_{2rs}^m(p_{1rs})$ such that $\gamma p_{2rs} \leq (1 - P_{S2rs}^*)^{-1}$ for all $p_{2rs} \in [0, p_{2rs}^m(p_{1rs})]$. Together $p_{1rs} \in [0, p_{1rs}^m(p_{2rs})]$ and $p_{2rs} \in [0, p_{2rs}^m(p_{1rs})]$ define a compact set that is a subset of $p_{1rs} \in [0, p_{1rs}^m(0)]$ and $p_{2rs} \in [0, p_{2rs}^m(0)]$. ($p_{1rs}^m(p_{2rs})$ is continuous and decreasing in p_{2rs} and $p_{2rs}^m(p_{1rs})$ is continuous and decreasing in p_{1rs} ; $p_{1rs}^m(p_{2rs}), p_{2rs}^m(p_{1rs}) > 0$.) We have already established that $P_{S2rs}^* \geq P_{R1rs}^* \geq P_{R2rs}^* \geq P_{S1rs}^*$ when $p_{1rs}^* \geq p_{2rs}^*$. If we restrict the compact set to $p_{1rs} \geq p_{2rs}$ and the price difference is not too large, we have $P_{S2rs} \geq P_{R1rs} \geq P_{R2rs} \geq P_{S1rs}$ on the set. This simplifies the proof, but is not necessary. Thus, we can choose a compact set such that $\gamma p_{1rs} = (1 - P_{R1rs}^*)^{-1}$, $\gamma p_{2rs} \leq (1 - P_{S2rs}^*)^{-1}$, and $P_{S2rs} \geq P_{R1rs} \geq P_{R2rs} \geq P_{S1rs}$ on the set. This set contains the interior solution to the first-order conditions. Using arguments similar to those we used for the equilibrium prices, we establish that the second-order conditions hold on this compact set. If necessary, we impose a weak technical condition on S/R . This implies that both profit functions are concave on the compact set. Concavity on a compact set guarantees that the solution exists and, by the arguments in the previous paragraph, that the solution is an interior solution. Numerical calculations, for a wide variety of parameter values, suggest that the second-order conditions hold on the compact set, that the second-order conditions hold outside the set (the restrictions are sufficient but not necessary), that the second-order conditions hold for prices satisfying $p_{2rs} > p_{1rs}$, and that, at equilibrium, the second-order conditions hold for all S .

The proof for the rr positions follows arguments that are similar to those for the rs positions. We do not need the technical condition on S because $P_{S2rr}^* \leq \frac{1}{2}$ implies that $S > 0$ is sufficient. The compact set is simpler because $p_{1rr}^* = p_{2rr}^*$ by symmetry. The proofs for the sr and ss positions use related conditions and follow the logic of the proofs for the rs and rr positions. \square

Uniqueness requires that we examine the cross-partial derivatives, illustrated here for rs :

$$\frac{\partial^2 \pi_{1rs}}{\partial p_{1rs} \partial p_{2rs}} = \gamma R P_{R1rs} P_{R2rs} [1 - \gamma p_{1rs} (1 - 2P_{R1rs}^*)] + \gamma R P_{S1rs} P_{S2rs} [1 - \gamma p_{1rs} (1 - 2P_{S1rs}^*)]$$

Restricting ourselves to the a compact set as in the existence arguments, we can use $\gamma p_{1rs} \leq 1/(1 - P_{R1rs}^*)$, $\gamma p_{2rs} \leq 1/(1 - P_{S2rs}^*)$, and $P_{S2rs} \geq P_{R1rs} \geq P_{R2rs} \geq P_{S1rs}$. We substitute to show that, when the cross-partial derivative is positive (similar conditions and a similar proof applies when it is negative):

$$\begin{aligned} & \left| \frac{\partial^2 \pi_{1rs}}{\partial p_{1rs}^2} \right| - \left| \frac{\partial^2 \pi_{1rs}}{\partial p_{1rs} \partial p_{2rs}} \right| \\ & \geq \frac{\gamma}{1 - P_{R1rs}^*} \{ R P_{R1rs} (1 - P_{R1rs}^*)^2 + R (1 - P_{R1rs} - P_{R2rs}) P_{R1rs}^2 \\ & \quad + S P_{S1rs} (1 - P_{S1rs}^*) (1 - P_{R1rs}^*) + S P_{S1rs} (1 - P_{S1rs} - P_{S2rs}^*) (2P_{S1rs} - P_{R1rs}^*) \} \end{aligned}$$

We substitute further to show the third term on the right-hand side is larger than the, possibly negative, fourth term. Hence, the cross-partial condition is positive for π_{1rs} on the compact set. The cross-partial

condition for π_{2rs} is satisfied with a technical condition on S . Numerical calculations, for a wide variety of parameter values, suggest that the cross-partial conditions hold on the compact set, that the cross-partial conditions hold outside the set (the restrictions are sufficient but not necessary), that the cross-partial conditions hold for prices satisfying $p_{2rs} > p_{1rs}$, and that, at equilibrium, the cross-partial conditions hold for all S .

In summary, subject to (possible) technical conditions on the magnitude of S , we have proven that interior-solution price equilibria exist and are unique. At minimum, we have shown that this is true for many, if not most, markets. We have proven that the equilibria exist and are unique for markets satisfying the technical conditions on S/R . \square

Corollary 1. Firm 1 selects r for both γ^{lower} and γ^{higher} .

Proof. The result follows directly from Proposition 1 and Proposition 2. Firm chooses r if $\gamma^{lower} < \gamma^{cutoff}$ by Proposition 1 and chooses r if $\gamma^{lower} \geq \gamma^{cutoff}$ by Proposition 2. Thus, Firm 1 chooses r independently of γ^{lower} . We use the same arguments to show that Firm 1 chooses r independently of γ^{higher} . (The result also requires continuity of the profit functions, proven elsewhere.) \square

Corollary 2. If Firm 2 acts on γ^{lower} and $\gamma^{lower} \neq \gamma^{true}$, then Firm 2 might choose the strategy that does not maximize profits.

Proof. We provide two examples. If $\gamma^{lower} < \gamma^{cutoff}$ and $\gamma^{true} > \gamma^{cutoff}$, then, if Firm 2 acts on γ^{lower} it will choose s by Proposition 1, but the profit-maximizing decision is r by Proposition 2. If $\gamma^{true} < \gamma^{cutoff} < \gamma^{lower}$, then Firm 2 will choose r when its profit-maximizing decision is to choose s . The word “might” is important. Firm 2 might choose the correct strategy, even if $\gamma^{lower} \neq \gamma^{true}$ when both γ^{lower} and γ^{true} are on the same side of γ^{cutoff} . \square

Appendix 3. Online Appendices (not provided for review for brevity)

1. Numerical example for profits, market shares, equilibrium prices, first-order conditions and second-order conditions for stylized model.
2. Numerical example of market-research decisions by a sophisticated follower.
3. Attribute descriptions and example choice tasks for dormitory CBC study.
4. Brief summary of the McFadden-based (stylized-model based), Sonnier, Ainslie, and Otter (2007), and Allenby, et al. (2014) HB CBC specifications.
5. Comparison of estimates for scale adjustment factors from the McFadden-based (stylized-model based), Sonnier, Ainslie, and Otter (2007), and Allenby et al. (2014) specifications. Including alternative estimations accounting for gender, for split-sample, for split choice task, for a mixtures of normal distributions.
6. Posterior distributions for scale adjustment factors and attribute importances.
7. Posterior WTP estimates for McFadden-based (stylized-model based), Sonnier, Ainslie, and Otter (2007) and Allenby et al. (2014) specifications.
8. Additional citations: Five *Marketing Science* papers that discuss scale explicitly.