# THE EFFECT OF ITEM SIMILARITY ON CHOICE PROBABILITIES

## A COLLECTION OF WORKING PAPERS FROM A CONFERENCE AT QUAIL ROOST

June 17-20, 1981

Joel Huber
Coordinator

# ABSTRACT

## AGENDAS AND CHOICE PROBABILITIES

by

John R. Hauser

Massachusetts Institute of Technology

and

Amos Tversky

Stanford University

Agendas, both implicit and externally imposed, have many strategic implications in marketing science. For example, advertising copy often attempts to influence the order in which alternative products are compared. One goal of the original 7-up "uncola" campaign was to shift 7-up from being considered with other medicinal drinks to being considered with colas. The placement of products in stores and the choice of distribution channel are other applications. Consider the importance of "end-aisle" displays which place a product apart from competitors or consider the success of "L'eggs" and subsequent entrants which encourage consumers to consider hosiery as another grocery store purchase. One does not have to look beyond the power of the House Ways and Means Committee in the U.S. Congress to recognize the influence of agendas on the outcome of group choice.

Externally imposed constraints on agendas are important but we cannot neglect implicit or self-imposed agendas. For example, an industrial seller of computer peripherals may face a buying process of multiple buying centers who sequentially consider various aspects of the seller's equipment and successively eliminate some manufacturers from consideration based

on whether or not they fulfill the needs (specifications) of the buying center. We will show that the order in which aspects are considered can influence the outcome of the purchase decision.

Another implicit agenda problem is the identification of market structure. Selecting the "right" market to enter greatly enhances the likelihood and magnitude of a new product success. Selecting the wrong market can doom a new product to failure. One component of market selection is the measurement techniques and analytical models used to determine the structure of the market hierarchy. For example, consider the simplified hypothetical market hierarchy for the television market shown in figure 1. If, as shown, the consumer first chooses among 'consoles' and 'portables' then among 'color' and 'black and white', a manufacturer of color consoles may favor color portables so he can compete in both "markets" and not cannibalize his existing product line. Once such market structure is understood, the innovating firm can better select a market on the basis of sales potential, penetration, scale, input, reward, risk, and match to the organization's capabilities.

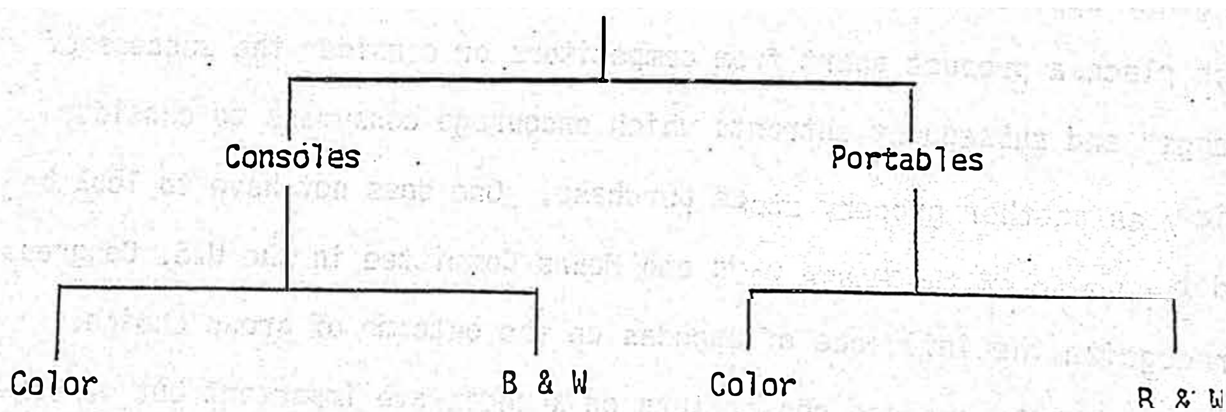



Figure 1: *Simplified Hypothetical Implicit Agenda for the Television Market*

This paper investigates some of the theoretical properties of agendas and their effect on choice probabilities. Our purpose is not to construct normative marketing science models, rather to investigate relationships, structure, and identification of agendas with respect to three general cognitive processing choice rules. As such we hope to provide the mathematical theory for future strategic models.

*Processing Rules*

To model the effect of agendas we model the human (organizational) decision maker as successively selecting subsets of alternatives from some offered set until only one alternative is left. (He can, of course, simply select a set which contains but one product.) Within this structure there are three components to the decision process:

1. Admissible sets, i.e., the choice sets which are allowable within the model's structure.
2. Agendas, i.e., the order of consideration in selecting among choice sets.
3. Processing rules, i.e., rules by which aspects of choice alternatives determine the probability that an admissible choice set is selected at each level within the agenda.

These three components completely specify choice probabilities within an offered set. In this paper we consider four processing rules, three of which are generalizations of the first.

*Constant Ratio Model (CRM),* Also known as independence of irrelevant alternatives (IIA) and as Luce's axiom is probably the most familiar processing rule. CRM States that an alternative is selected with probability proportional to the measure of the alternative, independently of the choice set. As such, CRM is agenda-free. All subsets are admissible but we need only be concerned with the full offered set and all singletons (sets containing only one alternative). CRM is a useful model but leads to many logical problems. See Tversky (1972).

*Elimination by Aspects (EBA)* concentrates on aspects of choice alternatives rather than choice alternatives. For example, an aspect of a television might be 'color'. EBA postulates that an aspect is selected with probability proportional to the aspect's measure and all alternatives not containing the aspect are eliminated. The process continues until but one alternative is left. Any set of alternatives which share an aspect are admissible. EBA eliminates many of the logical problems of CRM, but can be quite complex requiring $2^n$ parameters if all subsets are admissible. (n is the number of alternatives.)

*Hierarchical Elimination Model (HEM)* specifies choice among partitions of choice sets. At each level in an agenda, the decision maker is faced with a choice among mutually exclusive and collectively exhaustive sets of choice alternatives (partitions). If HEM holds, the proability that a choice set is selected is proportional to the sum of the measures of all aspects which are associated with at least one alternative in the choice set excluding aspects shared across partitions. The decision proceeds through a succession of partitions until one alternative is left.

*Hierarchical Balance Model (HBM)* also specifies choice among partitions. However, the probability that a choice set is selected is proportional to the sum of the aspect measures of each alternative in the partition excluding aspects shared across all alternatives in the partitions. The difference between HEM and HBM is subtle but important. In HEM we assume the decision maker only looks at the unique aspects of a partition. Hence, if an aspect, say, 'color', is contained in an alternative in each set of the partition, HEM does not include it even if some alternatives in a set do not contain the aspect. In HBM we assume the decision maker looks at alternatives within a partition. He excludes only those aspects contained in all alternatives in the offered set. Hence, if an aspect is in each set of a partition it will be counted unless it appears in all alternatives in the offered set. The appendix gives examples of CRM, EBA, HEM, and HBM. (The terminology of HBM comes from an analogy to a series of balance pans.)

Each of the last three models, EBA, HEM, and HBM, are logical generalizations of CRM and each model overcomes the logical problems associated with CRM. Each rule uses different cognitive processing, but, a priori, we cannot specify which model best describes human (or organizational) choice behavior. Before further examining the relationships among the models we specify certain structures of admissible sets that are given special consideration.

*Agendas*

We can represent an agenda as a directed graph. The source is the offered set. Each singleton (single alternative) is a terminus. Flows proceed from each node to proper subsets which are admissible. For example, if {x,y,v,w} is the offered set and {x,y},{v,w},{x,v},{y,w},{x},{y},{v}, and {w} are admissible then one possible graph is shown in figure 2. We show later that figure 2 is a graph of a full, balanced design.

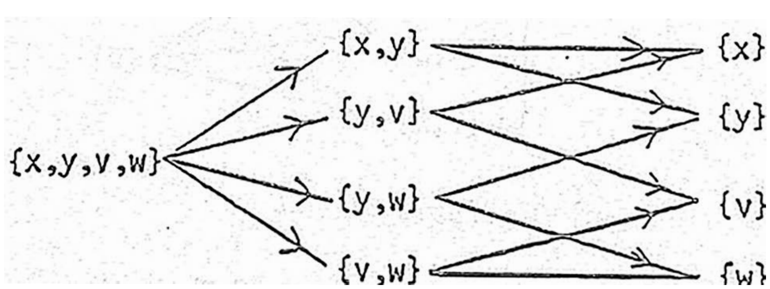


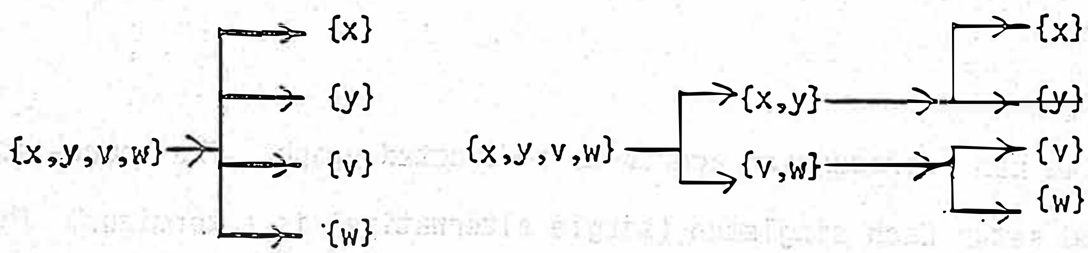

Figure 2: *A Graph of EBA on a Full Balanced Design*

*Graph of CRM.* A parsimonious representation of CRM is a "bush" with no intermediate nodes. See figure 3a.

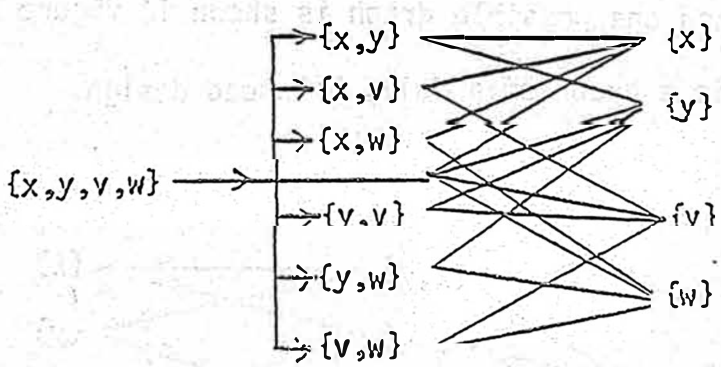
*Graph of HEM, HBM.* If we know the agenda, then both HEM and HBM appear as tree structures. See figure 3b. We show that various aspect structures (preference trees, full, balanced design) restrict the allowable graphs to certain trees.

*Graph of Similarity Models.* Recently a number of choice models have been proposed which extend CRM by incorporating similarity measures. In graphical terms the admissible sets for these models are all doublets and singletons. All links are allowable, see figure 3c.

*Graph of EBA.* In the most general case for EBA, all proper subsets are admissible. Even for four alternatives EBA is so complex we have not drawn it here.




a) *Constant Ratio Model* b) *One Partition for HEM or HBM*




c) *Similarity Model*

Figure 3: *Graphic Interpretations of Three Agendas*

*Admissible Sets*

As discussed above EBA is a very complex model incorporating all potential subsets and links in a graph. Fortunately there are a number of useful restrictions on admissible sets that make EBA a more practical model for strategic decisions. Similarity models are one restriction requiring only $n(n+1)/2$ parameters compared to $2^n$ in a general EBA model. Hierarchies of partitions such as figure 3b are another class of useful restrictions. We are interested in two other important restrictions, preference trees and full, balanced designs. These are certainly not the only restrictions, however, they are a useful beginning to illustrate many phenomena.

*Preference Trees* are a subclass of partitions defined on aspects. In particular, if $x'$ is the set of aspects associated with alternative x then two alternatives, x and y, are part of a preference tree if either $x' \subset y'$, $y' \subset x'$, or $x' \cap y' = \Phi$. Conceptually a preference tree is an assymetric tree in which an aspect cannot appear on two or more distinct branches. Figure 4 is a preference tree. Preference trees turn out to be useful strategic concepts because (1) they have a natural agenda defined by the aspect structure and (2) they are a much more parsimonious representation than a general EBA model. For greater detail see Tversky and Sattath (1979).

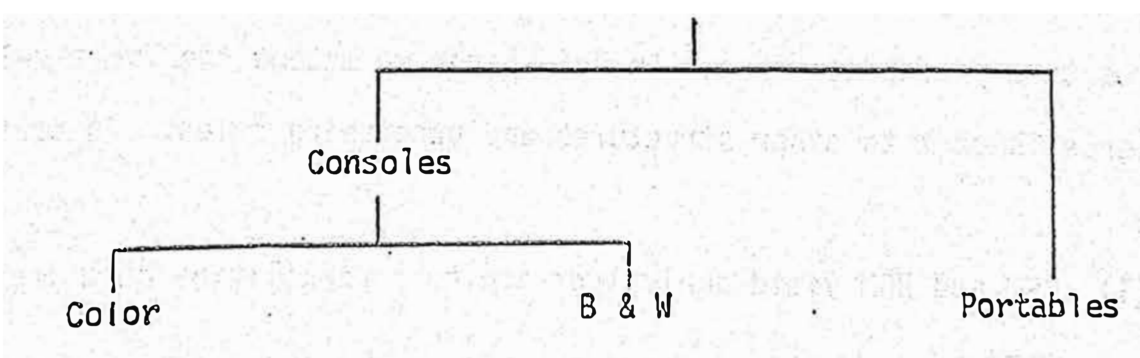



Figure 4: *Hypothetical Preference Tree (Assymetric Tree)*

*Full, balanced designs.* Not all marketing problems occur as preference trees. For example, many market structure problems, such as figure 1, are concerned with trees where aspects appear on more than one branch. Furthermore, many data collection procedures use factorial designs. In this paper we investigate full factorial designs. For example, consider two attributes, a and b, each with two levels, $a_1$, $a_2$, and $b_1$, $b_2$. Then a full factorial design consists of four products, x,y,v,w, defined such that $x' = \{a_1,b_1\}$, $y' = \{a_1,b_2\}$, $v' = \{a_2,b_1\}$ and $w' = \{a_2,b_2\}$. If we apply EBA to this design then the admissible sets are shown in figure 2. For example, if $a_1$ is the first aspect to be chosen then the decision maker selects the admissible set, {x,y}, from the offered set, {x,y,v,w}. We do not investigate fractional factorials, such as orthogonal arrays, but these are natural extensions of the theory developed here.

*Relationships Among Processing Rules*

EBA, HEM, and HBM are different cognitive processing rules, but they do not always imply different choice probabilities. For example, Tversky and Sattath (1979, Equivalence Theorem) show that EBA and HEM yield identical choice probabilities on a preference tree. Such a result is not obvious a priori, but once proven it has tremendous implications. For example, if we wish to distinguish experimentally among EBA and HEM then we cannot restrict ourselves to a preference tree. In this paper we extend the Tversky-Sattath Equivalence Theorem to other structures and processing rules. In particular, we show

(1) HEM and HBM yield equivalent choice probabilities on a preference tree.

(2) EBA and HEM yield equivalent choice probabilities on a full balanced design.

(3) HEM and HBM (and hence EBA and HBM) yield different choice probabilities on a full balanced design.

(4) All three rules, HEM, HBM, and EBA, yield potentially different choice probabilities on incomplete factorials. (eg., {x,y,v} in the above notation.)

At present we have investigated these theorems on two-by-two designs. By the time of the conference we should know the extent of their generalizability.

*Constrained Agendas*

HEM and HBM have associated agendas implied by definition. In general, EBA does not. Thus we consider a constrained version of elimination by aspects (CEBA) in which an agenda is imposed. In CEBA, the probability of choosing a partition is equal to the sum of the choice probabilities of the alternatives in the partition where the choice probabilities satisfy EBA on the offered set. With this definition we can compare the effects of externally imposed agendas on HEM, HBM, and CEBA.

*Invarience.* Tversky and Sattath (1979) show that if the aspect structure is a preference tree then CEBA choice probabilities are unaffected if and only if, the constrained agenda is compatible with the agenda implied by the preference tree aspect structure. (Two agendas are compatible if there is a third agenda that is a refinement of both). As shown in figure 2, full, balanced designs also imply specific agendas by their aspect structure. For example, if $x' = \{a_1, b_1\}$ and $w' = \{a_2, b_2\}$ , EBA would never yield the choice set {x,w} from a full, balanced offered set, {x,y,v,w}. It turns out that choice probabilities of a full, balanced design are unaffected if, and only if, the constrained agenda is compatible with the agenda implied by the aspect structure. We are still investigating the generalization of this theorem to any aspect structure.

*Dissimilar groupings.* If choice probabilities can be affected by agendas then it is possible that a marketing manager can increase the likelihood that his product will be chosen by imposing an agenda. For example, he can influence an agenda by advertising his product as particularly competitive to another product. Witness the advertising battle between Xerox copiers ("It's just as good as a xerox.") and Savin copiers ("Xerox will probably buy one before you do.").

Tversky and Sattath (1979) show that if x',y', and z' form a preference tree with {x,y} on one branch and {z} on the other branch, then, for CEBA, grouping the dissimilar alternative, z, with one of the pair, say x, always benefits x, harms z, and is immaterial to y. We can expect parallel effects on full, balanced designs.

In fact, suppose that x,y,v, and w form a full, balanced design as defined above. Suppose further that $a_1 > a_2$ and $b_1 > b_2$. (Actually, these inequalities are defined on the measures of the aspects, not the aspects.) Then a dissimilar grouping is an imposed agenda that forces a choice among {x,w} and {y,v} followed by a choice within the chosen set. We show that, for CEBA, such a dissimilar grouping hurts x and helps w. (Note that x is always the greater probability alternative.) This would imply that Savin benefits from being associated with Xerox while Xerox is harmed by being associated with Savin.

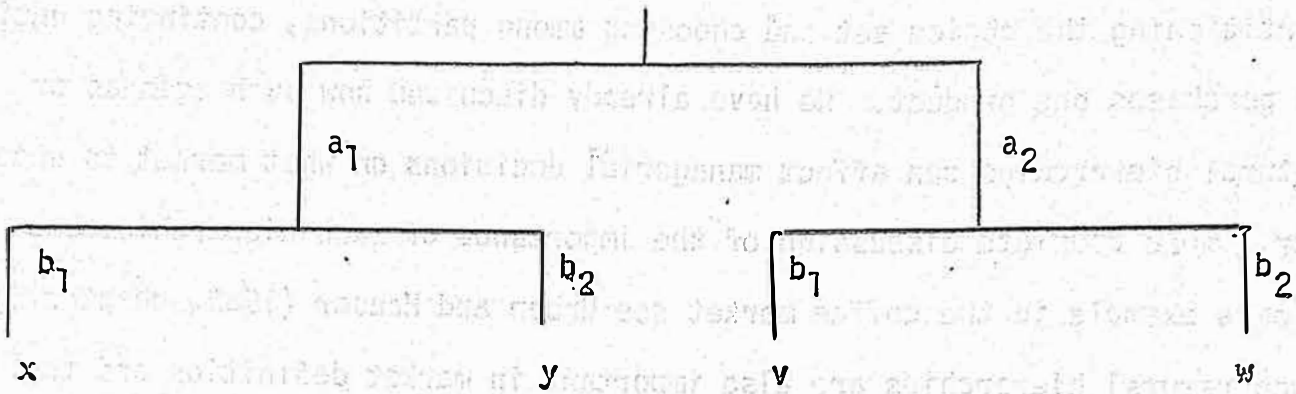
From our results on invariance we know that CEBA alters choice probabilities if the agenda is not compatible with the aspect structure. On a full, balanced design a dissimilar grouping is not compatible with the aspect structure, thus CEBA is not necessarily equivalent to HEM for dissimilar groupings. This raises the question of how dissimilar groupings affect HEM and HBM. We show that, for HEM, a dissimilar grouping hurts x and helps w, but, for HBM the direction of change, hurt or help, depends on the relative magnitude of $a_1$, $a_2$, $b_1$, and $b_2$.

*Identification of Natural Hierarchies*

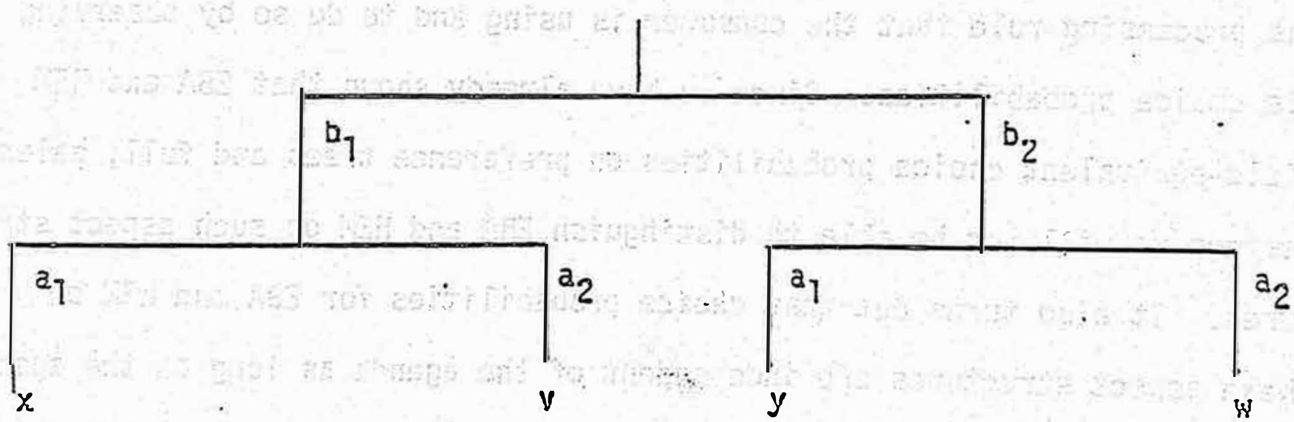
In constrained agendas the marketing manager acts to influence the agenda that consumers use to choose among products in an offered set. But suppose the consumer himself simplifies a complex choice problem by partitioning the choice set and choosing among partitions, continuing until he purchases one product. We have already discussed how such agendas or natural hierarchies can affect managerial decisions on what market to enter. For a more complete discussion of the importance of such hierarchies and a case example in the coffee market see Urban and Hauser (1980, chapter 5). Such natural hierarchies are also important in market definition and the identification of strategic business units for strategic planning.

Mathematically, the problem is to identify the implicit agenda and the processing rule that the consumer is using and to do so by observing his choice probabilities. Since we have already shown that EBA and HEM yield-equivalent choice probabilities on preference trees and full, balanced designs we will not be able to distinguish EBA and HEM on such aspect structures. It also turns out that choice probabilities for EBA and HEM on these aspect structures are independent of the agenda as long as the agenda is compatible with the aspect structure. Thus, for preference trees or full, balanced designs we can not, in theory, distinguish EBA from HEM or identify their implicit agendas. On the other hand, HBM differs from both EBA and HEM and its choice probabilities depend on the agenda.

*Agenda identification.* The mathematical problem is illustrated in figure 5. (We begin with a full, balanced design because of its importance in marketing research.) For a full, balanced 2x2 design, two agendas are possible. The consumer can first decide among $a = \{a_1, a_2\}$ then $b = \{b_1, b_2\}$ as shown in figure 5a or he can first decide among b then a as shown in figure 5b. Call figure 5a, a|b; call figure 5b, b|a. Our problem is to distinguish a|b from b|a by observing choice behavior.




a) Schematic of the agenda, a|b




b) Schematic of the agenda b|a

Figure 5: *Two Possible Agendas for a 2 x 2 Design*

*Discriminal statistics.* Let $P(x|\{x,y\})$ be the probability that alternative x is chosen from the set $\{x,y\}$. Let $R(x,y) = P(x|\{x,y\})/P(y|\{x,y\})$ then define $Z_1$ such that:

$$Z_1 = \ln [R(x,y)\ R(y,w)\ R(w,v)\ R(v,x)]$$

We show that for HBM if $Z_1 > 0$ then $a|b$, if $Z_1 < 0$ then $b|a$, and if $Z_1 = 0$ the agenda degenerates to CRM which we write, $a \sim b$. We can also define:

$$Z_2 = \ln [R(x,y)\ R(w,v)] \quad \text{and} \quad Z_3 = \ln [R(y,w)\ R(v,x)]$$

and show that for HBM:

i) $Z_2 = 0$ and $Z_3 \geq 0$ if $a|b$

ii) $Z_2 \leq 0$ and $Z_3 \geq 0$ if $a \sim b$

iii) $Z_2 \leq 0$ and $Z_3 = 0$ if $b|a$

Thus, if there were no measurement error we could distinguish $a|b$, $b|a$, and $a \sim b$ by observing $Z_1$ or $Z_2$ and $Z_3$. (Note that the tests are not truly independent since $Z_1 = Z_2 + Z_3$.)

We can generalize these tests to more complex designs and determine whether a precedes b in the agenda $(a\|b)$ even if other aspect pairs intervene. The tests are straightforward generalizations of the 2 x 2 tests where the discriminal statistics become averages of quartic comparisons in which each quartic comparison holds all aspect pairs but a and b fixed. Thus we can distinguish agendas for full, balanced designs $(2^m)$ with arbitrary m.

*Error theory.* We rarely measure consumer behavior error-free. Furthermore, even if HBM holds, we expect it will hold within some confidence interval rather than exactly. Thus we assume log-normal error on the ratios of choice probabilities, $R(x,y)$'s. We use this error theory to derive a battery of three t-tests to distinguish $a|b$, $b|a$, and $a \sim b$.

The T-battery distinguishes among agendas, but the probability ratios may still not satisfy the axioms of HBM even for the identified agenda. To address this issue we apply the Hauser-Shugan (1980) transitivity F-tests to either $Z_1$, $Z_2$, or $Z_3$ where the average is taken over repetitions. Since $Z_1 = 0$ for $a \sim b$, $Z_2 = 0$ for $a|b$, and $\overline{Z}_3 = 0$ for $b|a$ we use the transitivity F-tests to determine if the appropriate $\overline{Z}_i$ is closer to zero (at some confidence level) than would be true if HBM did not hold.

*Identification of Processing Rules*

The preceding statistical tests distinguish HBM from EBA, HEM, and CRM. (The case $a \sim b$ is actually the hypothesis that either EBA or HEM or CRM holds.) We know that EBA can not be distinguished from HEM on preference trees or full, balanced designs, but that does not mean the two processing rules are always identical.

*CEBA vs. HEM.* It turns out that we can identify different processing rules by using constrained agendas. Consider a full, balanced design as defined above and let $P([x,w] \mid \{x,y,v,w\})$ be the probability that the set $\{x,w\}$ is chosen from $\{x,y,v,w\}$ under a constrained agenda forcing a choice among $\{x,w\}$ versus $\{y,v\}$. If we define $Z_4 = P([x,w] \mid \{x,y,v,w\}) - P([y,v] \mid \{x,y,v,w\})$ then it is not hard to show that $Z_4 > 0$ for CEBA and $Z_4 = 0$ for HEM. (Remember $a_1 > a_2$ and $b_1 > b_2$.) Thus $Z_4$ becomes a discriminal statistic for CEBA versus HEM. An error theory is being investigated.

*EBA vs. CRM.* Let $R(x,w \mid \{x,y,v,w\}) = P(x \mid \{x,y,v,w\}) / P(w \mid \{x,y,v,w\})$ and define $Z_5 = R(x,w \mid \{x,y,v,w\}) - R(x,w \mid \{x,w\})$. Then we can show that for a full, balanced design as defined above, $Z_5 > 0$ for EBA and $Z_5 = 0$ for CRM. An error theory is being investigated for the discriminal statistic $Z_5$.

*Summary*

This paper investigates agendas and their relationship to four cognitive processing rules. We begin with a general structure in which a choice process is defined as a directed graph from sets of alternatives to admissible subsets and show that a natural definition of an agenda is the order of choice among partitions. Our first results concern the equivalence and non-equivalence of the various processing rules (CRM, EBA, HEM, and HBM) on preference trees and full, balanced designs.

We next investigate externally imposed agendas and show that only a certain class of agendas, those compatible with the aspect structure, do not affect choice probabilities. We make this result normative by showing that dissimilar groupings, for both CEBA and HEM on a full, balanced design, help the weaker alternative.

Agendas can also be implicit. Thus we derive a series of statistic tests to distinguish among agendas for HBM and to determine whether HBM is a reasonable interpretation of observed choice probabilities. (Natural, compatible agendas on full, balanced designs do not affect choice probabilities for HEM and EBA).

Finally, we investigate discriminal statistics to distinguish CEBA from HEM and to distinguish EBA from CRM.

*References*

Hauser, J.R. and S.M. Shugan, "Intensity Measures of Consumer Preference", *Operations Research,* vol. 28, No. 2, March-April 1980, pp. 278-320.

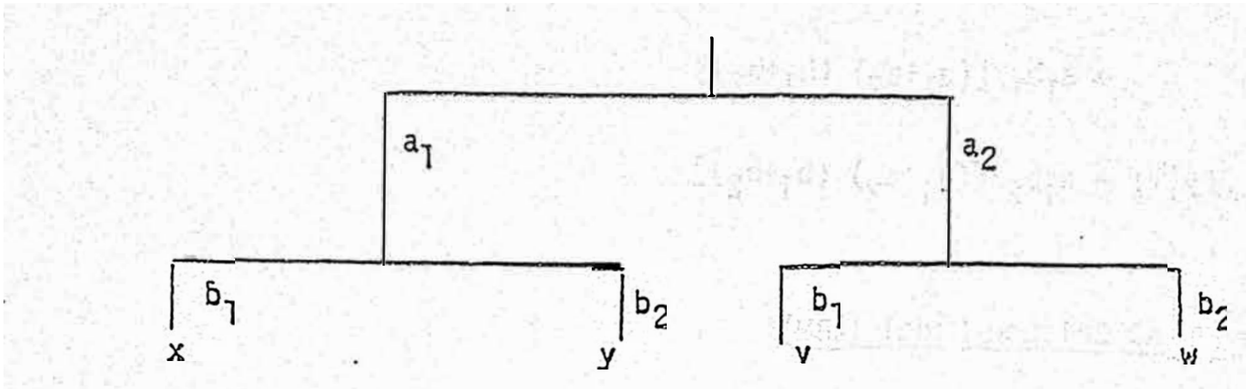
Tversky, A., "Elimination-by-Aspects: A Theory of Choice", *Psychological Review,* vol. 79, No. 4, 1972, pp. 281-299.

Tversky, A. and S. Sattath, "Preference Trees", *Psychological Review,* vol. 86, No. 6, pp. 542-573.

Urban, G.L. and J.R. Hauser, *Design and Marketing of New Products,* (Prentice Hall: Englewood Cliffs, N.J.) 1980.

*Appendix: Example Calculations for CRM, EBA, HEM, and HBM*

Consider the following tree, let T = {x,y,v,w,}:




Constant Ratio. Model (CRM)

$$P(x|T) = (a_1+b_1) /[(a_1+b_1) + (a_1+b_2) + (a_2+b_1) + (a_2+b_2)]$$

$$P(y|T) = (a_1+b_2) /[(a_1+b_1) + (a_1+b_2) + (a_2+b_1) + (a_2+b_x)]$$

Elimination by Aspects (EBA)

$$P(x|T) = a_1 \; P(x|\{x,y\}) + b_1 \; P(x|\{x,v\})$$

$$= a_1 \; [b_1/(b_1+b_2)] + b_1 \; [a_1/(a_1+a_2)]$$

$$= a_1b_1/[(a_1+a_2) \; (b_1+b_2)]$$

where

$$a_1+a_2+b_1+b_2 = 1$$

$$P(y|T) = a_1b_2/[(a_1+a_2)(b_1+b_2)]$$

Hierarchical Elimination Model (HEM)

$$P(x|T) = [a_1/(a_1+a_2)]\ [b_1/(b_1+b_2)]$$

$$= a_1b_1/[(a_1+a_2)\ (b_1+b_2)]$$

$$P(y|T) = a_1b_2/[(a_1+a_2)\ (b_1+b_2)]$$

Hierarchical Balance Model (HBM)

$$P(x|T) = \{(a_1+b_1+b_2)/[(a_1+b_1+b_2) + (a_2+b_1+b_2)]\}\ [b_1/(h_1+b_2)]$$

$$P(y|T) = [(a_1+b_1+b_2)/[(a_1+b_1+b_2) + (a_2+b_1+b_2)]\}\ [b_2/(b_1+b_2)]$$