

A Shortcut to Efficiency?
Implications of the Small but Stratified World*

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Draft: 25-Aug-08

Word Count: 12,435

* Authorship is alphabetical. Thanks to Matt Bothner, Ron Burt, Damon Centola, Steve Kahl, Damon Phillips and to seminar participants at the University of Chicago and Washington University for useful feedback and advice. Any remaining foolishness is our fault.

A Shortcut to Efficiency: Implications of the Small but Stratified World

Abstract

Small World research (Watts and Strogatz 1998) has shown that “rewiring” just a few ties to be shortcuts across an otherwise clustered network structure results in a dramatic decline in the average distance between nodes but has only a modest effect on the degree of clustering. Accordingly, Small World structures seem to strike an attractive balance between two social goals: efficiency and community. But we show that the purported efficiency gains of the Small World may not be realized, even when the “contagions” being transmitted are quite “simple” (cf., Centola and Macy 2007). The problem is that Small Worlds rely on a small number of middlemen and are thus highly stratified. We show that structural inequality in Small World graphs can lead to dramatic outcome inequality, as reflected in the returns that actors receive from exchanging information with each other. And we show that this outcome inequality characteristic of the Small World also makes such structures inefficient when: (a) actors will not transmit valuable resources without getting equal value in return; and/or (b) actors are limited in their capacities for transmission. Under these very general conditions and especially in the connected structures studied in Small World research, the efficiency gains from the Small World are “hogged” by a small number of middlemen positioned on the shortcuts, who act as bottlenecks and thereby cause most others to do worse than in a more clustered structure. Our analysis thus demonstrates that the trade-off between efficiency and community is difficult to avoid, and the culprit is inequality.

Introduction

As research on social networks has progressed, attention has shifted from understanding the relative advantages of positions within a given network structure to clarifying the relative advantages of entire network structures. This shift can be attributed in part to the increased computational power that affords the analysis of large-scale structures and in part to the theoretical difficulties that result from focusing exclusively on within-structure variation without clarifying the implications of a structural configuration for a population as a whole. For instance, while it is generally advantageous for individual actors to become brokers, by minimizing the degree to which their contacts are connected (Burt 1992), the system as a whole would likely suffer if everyone follows this advice (Buskens and van den Rijt 2008; see also Gabbay and Zuckerman 1998: 196-198). Similarly, while weak ties can be helpful in overcoming the insularity of a closed community (Granovetter 1973), one would hardly recommend that all community members concentrate on developing weak ties. These observations point generally to the importance of understanding how different structural configurations shape outcomes for a system, and they specifically hint at the benefits of role differentiation, whereby some actors specialize as “locals” and others as “cosmopolitans” (e.g., Merton 1968; Gouldner 1957; Lazarsfeld, Berelson, and Gaudet, 1944).

With this agenda in mind, the recent resurgence of research on the “small world phenomenon” (Watts 1999a,b; Watts and Strogatz 1998) is noteworthy, as it suggests that network structures can be quite efficient when just a few cosmopolitans are added to a mix that otherwise consists of individuals with a highly local orientation. Let us define a structure as more efficient insofar as resources flow quickly from actors in one part of a structure to actors in more distant parts of a structure who desire or need them. Given this definition, it would seem that the most efficient social structure is maximally dense, with all actors linked to everyone else. But this is unachievable when the structure includes many actors, each of whom has limits on the amount of time and energy that they can devote to relationships. When such constraints operate, the most efficient social structure would seem to be a network where everyone can reach everyone else in the smallest number of intermediaries or “steps.” Such a structure is one where social relationships develop with little regard for interpersonal distinctions, as if network

connections develop “at random.” However, if such undifferentiated structures are indeed the most efficient structures we could reasonably achieve, this conclusion is distressing because the social structures we observe are far from random, with various homophilic tendencies (whereby actors with similar tastes or who share demographic categories are more likely to form and retain links with one another; see McPherson, Smith-Lovin, and Cook 2001; Reagans 2005) producing significant clustering of relations into “local neighborhoods.” Such tendencies towards highly clustered networks seem to imply that significant intervention is required to make the social structures that dominate our world more efficient.

The signal contribution of Watts and his colleagues (Watts 1999a, b; Watts and Strogatz 1998; see also Newman 2000; Newman and Watts 1999) has been to show that in fact, only minimal intervention is required to transform a structure from one that is highly inefficient, due to local clustering, to one that approximates the efficiency of a random network. To illustrate, consider the three 40-node graphs depicted in figure 1. As discussed below, and following past practice in this literature, these networks were constructed to eliminate differences in the “degree” or size of actors’ ego-networks (all actors have four “neighbors” or contacts) and the degree of connectivity in the graph (all graphs are built around a lattice such that everyone can “reach” everyone else through $n \geq 1$ intermediaries). The “Clustered World” network of figure 1a and the “Random World” network of figure 1c represent polar extremes in terms of degree of local clustering, as indicated by the reduction in the “Clustering Coefficient” (*CC*)-- i.e., the mean “neighborhood density” or proportion of a node’s neighbors that are linked-- from 50% to 2.5%. And this elimination of clustering brings about a reduction in the “Characteristic Path-Length” (*L*)-- the average or minimal path-length (or “geodesic”) between any given pair of nodes in the graph-- from 5.25 steps in Clustered to 2.57 steps in Random. Insofar as efficiency increases as *L* is reduced, and insofar as the high degree of clustering in real-world networks implies that they more closely resemble Clustered than they do Random, it would appear that very drastic interventions are required to make networks more efficient.

FIGURE 1 ABOUT HERE

In fact, Watts and his colleagues have shown that such drastic restructuring may not be necessary to achieve high levels of efficiency. The “Small World” network of figure 1b, which involves the minimal number (2) of “rewires” possible while keeping the number of links constant, produces a drop in L to from 5.25 to 4.31, which represents 35% of the reduction from Clustered to Random, at the expense of a reduction in CC to just 0.45 or 10% of the reduction from Clustered to Random. And as shown in figure 2, just a few more such rewires (in this case, on networks of 100 nodes) produces approximately the same L as the most random structures even while retaining a high degree of clustering.¹

FIGURE 2 ABOUT HERE

To be sure, and as Watts and others emphasize, the fact that just a few shortcuts are needed to produce efficient networks is hardly a blessing when interpersonal contact allows deleterious contagions to spread. And Lazer and Friedman (2007) show that short-term efficiency can be problematic in the long term because it limits the systemic diversity necessary for long term improvements in routines and practices (cf., Uzzi and Spiro 2005). But insofar as members of a system gain from the sharing of certain pieces of information or other desirable resources, the results presented in these graphs convey the apparent good news that networks that are highly clustered -- and which are widely thought provide the benefits often associated with a sense of local community (see Vaisey 2007 for review; cf., Coleman 1988; Portes and Sensenbrenner 1993)-- can be highly efficient. Moreover, a spate of recent studies has recently shown that Small World (i.e., high CC /low L) networks are quite common (see e.g., Baum, Shipilov, and Rowley 2003; Davis, Yoo, and Baker 2003; Fleming, King, and Juda 2007; Kogut and Walker 2001; Uzzi and Spiro 2005; Watts 1999a, b) and thus seemingly quite efficient while also preserving community.

But note that the prediction that Small World networks approximate the efficiency of Random networks relies on the assumption that the efficiency of a network is predicted by the average minimum path-length or L . Centola and Macy (2007) provide the first reason for doubt on that score. They note that using L as a predictor of efficiency

¹ Note that, for illustration purposes, the location of the shortcut in figure 1b was chosen to maximize the reduction in L produced. The results in figure 2 were derived using an algorithm that allows the location of shortcuts to be chosen randomly.

necessarily involves the assumption that a single link between two actors is sufficient for a resource to flow from one to the other-- in particular., that resources flow as readily through the “narrow bridges” created by shortcuts (where a bridge is considered narrow when the two nodes share no common contacts) as they do through wider bridges. This assumption seems problematic in the case of “complex contagions,” which Centola and Macy define as pieces of information that are not credible or persuasive unless they are confirmed by multiple sources. Centola and Macy show that if one assumes that confirmation is required for flow to occur, Small World networks are no longer efficient. In short, their analysis shows that the expectation that the Small World is efficient requires an assumption that often does not hold—i.e., that the mere existence of a path from i to j implies that information held by one will eventually reach the other.

There is also a second reason for doubting the efficiency of Small World networks, which is the subject of the current paper. In particular, observe that just as rewiring a Clustered World network to transform it into a Small World introduces narrow paths into a network that had consisted entirely of wide paths, it also introduces *variation* in the distribution of path-distances from a given node to all other nodes. Observe in figure 1 that while there is no variation in g (the geodesic or minimal path-length between two nodes) in the Clustered World ($SD_g=0$) and minimal variation in g in the Random World ($SD_g=0.05$), the Small World exhibits substantial variation ($SD_g=0.65$)-- from highly central actors like Jill and Don (who average 3.3 steps to others) to peripheral actors like Sue and Tina (who average 5.3 steps to others, just as they do in the Clustered World). The graphs in figure 3 illustrate how Small World graphs represent an extreme in such stratification or “structural inequality.” Thus, while introducing shortcuts into a clustered structure can produce a substantial reduction in path-distances while retaining a high degree of clustering, this same intervention produces substantial structural inequality.

FIGURE 3 ABOUT HERE

The recognition that Small Worlds are highly stratified raises a key question: How does such structural inequality affect the prediction that the Small World is an efficient structure for the transmission of “simple contagions” (i.e., those that do not require confirmation and thus can presumably travel over narrow bridges; Centola and Macy

2007)? To explore this question, we begin by specifying a simple computational model that validates the prediction that the efficiency of transmission is a function of L , thereby illustrating the appeal of the Small World in striking a balance between community and efficiency (where simple contagions are concerned). But this model also shows the Small World is marked by a spike in outcome inequality, which reflects the fact that the gains in efficiency rely disproportionately on a few middlemen who are positioned on or near the shortcuts and who are able to earn rents while sharing information with others. We then show that this outcome inequality implies a reduction in efficiency under two conditions that are quite widespread in the real world: (a) when actors will not be motivated to pass on valuable information without receiving information in return; and (b) when actors cannot transmit all that they know in a single interaction. We show that when either or both these conditions apply, the reliance of Small World structures on a few middlemen can actually reduce efficiency because such middlemen act as bottlenecks that choke off efficiency gains and render peripheral actors worse off. We conclude by discussing the key implication of our analysis-- that rather than presenting a way of escaping the trade-off between efficiency and community, the stratification of Small Worlds helps us better appreciate why such a trade-off cannot be escaped completely.

Baseline model: Efficiency and Outcome Inequality in the Small World

As discussed above, Small World networks have been proposed as attractive structural configurations due to their ability to achieve high efficiency in the transmission of desirable, simple contagions while retaining significant clustering. The basis for the expectation that Small Worlds will transmit simple contagions efficiently lies in the sharp reduction in the average geodesic or minimum path-length (L) that is produced when just a few relationships in a clustered network are rewired, as illustrated in figure 2. While this prediction seems intuitive, it is important to specify the assumptions necessary for justifying it. And once these assumptions are clarified, we can then investigate whether the prediction is robust to alternative assumptions. Thus, we begin by using these assumptions to simulate information diffusion across structures that vary from Clustered to Random. We use this simulation to check the internal validity of the basic prediction that: (a) a Small World is substantially more efficient, in facilitating the flow of simple

contagions, than is a Clustered World; and (b) that additional rewires produce more limited gains in efficiency. In short, we investigate whether these assumptions indeed carry the implication that the trend line reflected in the graph of L in figure 2 corresponds to the level of efficiency achieved.

This model and all that follow in the paper share the following core assumptions (cf., Reagans and Zuckerman 2008a):

1. *Links as fixed pipes*: The network links represent two-way channels through which resources can flow between linked actors.
2. *Resources as codified information*: The resources are “bits” of “information” that are:
 - a. “Nonrival,” in that they are always retained even after they have been transmitted, thus allowing each actor to transmit a bit to multiple contacts (e.g., Buskens and Yamaguchi 1999; Romer 1990) and
 - b. “Simple contagions” in that they are always transmitted without distortion or ambiguity and their value does not require social confirmation or proof (Centola and Macy 2007).
3. *Uniform valuation*: All actors place the same valuation on all bits.

To test the prediction that efficiency tracks L , we measure *inefficiency* as the average number of interactions in a system that is required for a bit to flow from one node to all other nodes in the graph-- i.e., for the piece information to diffuse completely. To model this, we: (a) randomly select one node to be the source node s in a given simulation by endowing that actor with the bit b to be transferred; (b) we randomly cycle through the nodes i without replacement; (c) for each i , we randomly select one of i 's neighbors j ; (d) if i possess b but j does not, we transfer b from i to j , and vice versa if j has b but i does not.

The main results from these simulations are presented in figure 4. As with the results in figure 3 and in later analyses, these results are averaged across sets of twenty 100-node network graphs, and each node has a degree (number of contacts or neighbors) of 4. And the sets of graphs vary according to r , the proportion of rewired links, as

delineated across the horizontal axis of figure 4. The Clustered World is the base case of $r=0.0\%$, while the Small World corresponds to the minimal proportion rewired, which in the case of a 100-node/degree=4 graph is $r=0.5\%$ (2 out of 400 links are rewired). Note that while all $r=0$ graphs are the same, the set of graphs within a higher r -level (including the Small World graphs) differ because the links chosen for rewiring are selected randomly. Thus, while the Small World depicted in figure 1b is deliberately chosen to maximize the reduction in L , the effect on L will be less when the rewiring involves pairs of nodes that were already proximate to one another.

We see from figure 4 that the association between r and efficiency is log-linear in a manner that is consistent with the basic prediction, as stated above. In particular, we see that the transformation of the Clustered World into a Small World produces a dramatic reduction in inefficiency (or improvement in efficiency), in line with the reduction in L shown in figure 2. Complete information diffusion requires an average of 1277 interactions when $r=0$ but requires only 1104 interactions when $r=0.5\%$, an improvement in efficiency of 13.6%. And just as suggested by the trend in L displayed in figure 2, there are diminishing returns to additional rewires. If 1% of the original ties is rewired, complete information diffusion takes 990 interactions, which represents a 10.4% gain over the first improvement. Further, 75.3% of the possible gain in efficiency associated with a shift from a clustered to a random network is achieved when only 5% of the ties in the clustered network has been rewired. And this dramatic efficiency gain comes at the expense of just a 26% decline in clustering (from $CC=.50$ when $r=0$ to $CC=.37$ when $r=5\%$). Thus, the results illustrate that if there are benefits that accrue to system members from high degrees of clustering—and which might be summarized under the term “community,” Small World networks seem to strike an attractive balance between community and efficiency.

FIGURE 4 ABOUT HERE

Yet figure 4 also tells a different story, which corresponds to the pattern in figure 3-- i.e., Small Worlds can be remarkably efficient (under assumptions 1-3), but they are also the most unequal of all worlds. In short, the measures of *outcome* inequality in figure 4 (standard deviation and coefficient of variation of total surplus) closely track the measures of *structural* inequality shown in figure 3. We see that the initial rewiring

simultaneously produces: (a) a dramatic reduction in path-length, while retaining a high degree of clustering, thus leading to the dramatic improvement in efficiency, as shown in figure 4; and (b) a dramatic spike in stratification, whereby some nodes become more central than others, which leads to a dramatic spike in variation in surplus.

To generate the latter result, we followed Reagans and Zuckerman (2008a) and built into the simulation the rule, that each time a bit is transmitted from i to j , some amount of a general medium of exchange called “dollars” flows in the opposite direction. We call this “bit-for-dollar” exchange. The dollar amount d lies between the “buyer’s” (j) willingness-to-pay and the “seller’s” (i) willingness-to-sell. For simplicity, we assume that the bits are costless to produce and distribute, so the seller’s willingness-to-sell is set equal to zero (i.e., the seller will sell for any price above zero but prefers as high a price as possible). And following assumption 3, we assume that the willingness-to-pay for all actors j for all bits is \$1. The price that j pays i for b is then modeled as decreasing (at a decreasing rate) in the number of alternative sources from which actor j can obtain b at the time of the focal transaction:

$$d_{jib} = \text{wtp}_{jb} * \frac{0.5^{S_{jb}}}{0.5} = \$1 * \frac{0.5^{S_{jb}}}{0.5}. \quad (1)$$

where S_{jb} is the number of j ’s alternative sources for bit b at the time of the interaction with i . So if the buyer j has no alternative sources, j will pay the monopoly price of \$1, which gives the seller i all available surplus. The revenue of \$1 minus the willingness-to-sell of \$0 equals a seller surplus of \$1. The willingness-to-buy of \$1 by the buyer minus the price paid of \$1 equals a buyer surplus of \$0. But if j has one alternative source, both buyer and seller will enjoy a surplus of \$0.50; and the buyer’s share will climb at the expense of the seller (at a decreasing rate) as the number of alternative sources available to the buyer increases.²

There will be no variation in surplus (i.e., no outcome inequality, if outcomes are measured in terms of surplus) if all nodes in a network are equally likely to: (a) generate buyer surplus, due to having multiple sources for a bit being acquired; and (b) generate

² Clearly, the functional form for the price discount is merely a heuristic device. We adopt it because has the desired properties for such a price function (see e.g., Marsden 1983: 704) and because it serves as a convenient way of representing how the terms of exchange should vary with variation in the number of alternative sellers available to a buyer. Note that this approach ignores the possibility that price may be increasing in the number of alternative targets or buyers available to a seller. Reagans and Zuckerman (2008a) show that incorporating this modification reinforces the power of a middleman.

seller surplus, by having numerous opportunities to offer a bit to others, especially when those contacts have no alternative sources for the bit. Such is clearly the case in the Clustered World, as illustrated in figure 1a, and demonstrated in the low levels of outcome inequality at $r=0$ in figure 4. Each actor can garner some seller surplus if she acquires a bit before her neighbors because she can then “sell” the bit to at least one of her neighbors at monopoly prices. However, the high degree of redundancy in the Clustered World allows the bit to diffuse quickly within the local neighborhood and so her monopoly erodes quickly. Moreover, since all nodes are equally central (and more generally, they are all role-equivalent; see Wasserman and Faust 1994) in such structures, there is no reason to expect any particular node to consistently earn greater seller surplus across simulations. That is, in a Clustered World, no one actor is better off than her peers. And the Random World also displays little outcome inequality of this sort. In this case, as illustrated by figure 1c, some nodes may be slightly more central than others and thus derive a slight advantage in garnering seller-surplus. But insofar as there tend to be numerous alternative routes by which a bit can flow, such advantages are slight.

By contrast, the shortcuts that produce Small World graphs (i.e., reduce L while retaining high CC) and make them efficient in these simulations by reducing the average “travel time” across the structure, simultaneously increase outcome inequality. This follows because, if a bit is to travel the quickest route, it will flow through the middlemen who lie on such shortcuts, and this means that such actors will often be in the position to sell a bit to neighbors who have no alternative sources for it. Accordingly, we see in table 1 that central actors such as Jill and Don earn more (seller and thus, total) surplus than do peripheral actors like Sue and Tina, with such actors as Ned and Ron, who are less distant from the shortcut, earning intermediate surplus.³

³ Note that middlemen such as Jill and Don actually earn lower *buyer* surplus than do actors such as Sue and Tina whose contacts are highly redundant with one another (i.e., directly and indirectly linked). As explained by Reagans and Zuckerman (2008a), this represents the weakness inherent in having nonredundant contacts—i.e., such contacts will be less likely to possess the same resources at a given moment, thus limiting ego’s ability to drive price down as an acquirer or buyer of resources. In the simulations presented here, a middleman’s weakness as buyer is dwarfed by her power as seller because middlemen not only can charge higher prices but can also make many more sales. However, Reagans and Zuckerman (2008a) show that if we modify assumption 3 and instead assume that actors’ preferences are “homophilic” (i.e., they favor bits that originate in their “local” neighborhood), the seller surplus of middlemen can diminish to the point that their total surplus is in fact lower than such actors as Sue and Tina.

TABLE 1 ABOUT HERE

Barter is Harder: SW Inefficiency with Information as the Medium of Exchange

We have shown that under assumptions 1-3, the introduction of shortcuts into a clustered structure simultaneously increases its: (a) efficiency due to the decline in L ; and (b) outcome inequality, due to the rise in variation in g . Yet as discussed above, the increase in outcome inequality raises the question as to whether the predicted efficiency gains will in fact be realized. In short, the dramatic increase in outcome inequality in the Small World demonstrates that the efficiency of the Small World depends on a small number of middlemen who emerge as a consequence of the shortcuts created by the initial rewires; and this raises the question as to why we should expect such middlemen to be *motivated* to increase efficiency.

In the “bit-for-dollar” model, motivation was not an issue because we assumed that there is a general medium of exchange that actors can use to buy information. And we further assumed that: (a) exchange is frictionless-- i.e., exchange of information for the medium does not lower the likelihood that bits will be transmitted faithfully and accurately; (b) all actors have enough of that medium to buy the bits they want (i.e., no budget constraints); and (c) all actors always want more of that medium (i.e., no satiation or wealth effects). Alternatively, we could have eliminated motivation as an issue by assuming that actors transmit their information without asking anything in return. Such a “free transmission” assumption-- i.e., that information is passed from one actor to another without regard for what the first actor receives in exchange for that information-- has been widely adopted by researchers in the recent small world tradition as well as the “strength of weak ties” tradition upon which more-recent small world research builds (Granovetter 1973, 1974; Buskens and Yamaguchi 1999; Centola and Macy 2007; but see Burt 1992; Reagans and Zuckerman 2008a). And this assumption enjoys face validity, as actors often pass information without an explicit *quid pro quo*.

Yet there is good reason to explore the implications of assuming that actors will not be so altruistic, and that there is no general medium of exchange that satisfies the three conditions listed above. The reason for not assuming “free transmission” is straightforward. Insofar as a transmitter understands that the information has significant value for the recipient and/or transmission carries some cost to the transmitter, it is

awkward to expect the transmitter to give away that information without receiving something in return, at least in the future. Accordingly, Smith (2005) shows that information about job opportunities (the focus of Granovetter 1973, 1974) often does not flow from one actor to another if the first actor is concerned about the reputation cost incurred when the second actor applies for the job and does not perform well. Such cases of “exchange failure” seem quite common and suggest both that: (a) actors often refuse to pass on knowledge without receiving something valuable in return; and (b) that sometimes the “price” required for transmission is too high for the “buyer” to pay-- perhaps because the buyer is resource-poor or perhaps because there is no mutually agreeable medium of exchange, either monetary or in-kind, according to which the value of the knowledge can be made commensurate (Espeland and Stevens 1998). In other cases, however, actors do agree upon a medium of exchange-- e.g., deference toward the seller (e.g., Blau 1964; Emerson 1962: 39) or money (e.g., referral fees [e.g., Fernandez, Castilla, and Moore 2000]), as captured in the previous simulations.

Perhaps most commonly, however, *actors exchange information for other information*. Accordingly, we now modify the earlier simulation to analyze the efficiency of information-diffusion when “bits” are exchanged for other “bits” rather than for “dollars.” In order to model such “bit for bit” barter, we need to make an important modification to the previous model. As is typical in past research (e.g., Buskens and Yamaguchi 1999; Centola and Macy 2007), bits could not be exchanged for other bits in our previous model because only one bit was traveling through a network in a given simulation. That is, the “bit for dollar” model presented above involved “single bit diffusion.” But it seems more realistic to assume instead that multiple pieces of information are traveling through the network at a given moment—what we call, “simultaneous bit diffusion.” In particular, rather than randomly selecting a node to be the source for the single bit that will be transmitted through the network, we now model information-flow as simultaneous bit diffusion. This involves endowing all actors in the network with a single unique bit, and then running the simulation either until all actors become fully informed or the system reaches an equilibrium in which bits are no longer being transmitted. In the next section, we will show how the assumption of simultaneous-bit diffusion affects the results from bit-for-dollar exchange (cf., Reagans

and Zuckerman 2008a). But now we focus on the main benefit of incorporating this assumption, which is that it allows us to explore the implications of information *exchange*. In particular, rather than making the flow of bits in one direction contingent on the flow of dollars in the other direction, we model “bit-for-bit” barter by assuming instead that in order to motivate i to transmit a bit b_1 that j does not yet have to j , j must possess another bit b_2 that i does not yet possess, and transmit b_2 to i in exchange for receiving b_1 . If either i or j possess more than one bit that the other does not possess, we randomly select one such bit for exchange, so that all exchanges are one-for-one.

We expect that this change in the medium of exchange will have a critical effect on the prediction that the efficiency by which simple contagions diffuse tracks the reduction in L . In short, when information is exchanged for other information, an increase in stratification threatens to lower efficiency because middlemen become bottlenecks who stop trading once they are fully informed. And this cessation of information-flow will then cascade from the center to the periphery of the network, leaving peripheral actors especially frustrated in their attempts to obtain desired resources. To see this, observe first that full diffusion is rare under “bit-for-bit” exchange in any network structure because the first actor to obtain all available bits loses the motivation to pass on what he knows. Thus, when modeling “bit-for-bit” exchange, we measure efficiency not in terms of the *speed* by which full-diffusion happens but in terms of the *extent* of diffusion. In particular, we define efficiency in such systems as the median proportion of bits in the system that actors accumulate by the end of a simulation. The question then is which types of structures facilitate more extensive diffusion, where actors must have a bit to obtain a bit.

As with “bit-for-dollar” (or free transmission), Random Worlds are much more efficient by this criterion than are Clustered Worlds, as demonstrated in figure 5 (based on the network graphs from figure 1). The median member of these systems receives a total (over 1,000 simulations) of 38.4 of the 40 bits in the system, with the most informed member obtaining 39.6 bits and the least-informed receiving 34.7. These less-informed actors stop accumulating information when all their neighbors no longer need the bits they possess. But while such bottlenecks occur in a Random World, they are relatively rare because all actors can reach one other in a small number of short indirect steps. As a

result, the system is not dependent upon participation by any one node (or small number of nodes) and so, information can diffuse broadly even as some nodes cease to be interested in exchanging. By contrast, bottlenecks are much more common in a Clustered World, leading to a lower level of efficiency (median number of bits obtained=28.5). Complete diffusion is contingent upon how effectively information flows between proximate neighborhoods. Although all of the actors are role equivalent, it will often be the case that one actor will accumulate what his or her contacts know and stop sharing information. Such a node becomes a bottleneck in the flow of information between proximate neighborhoods, resulting in incomplete information diffusion. Note, however, that the gap between the most-informed (29.4) and the least-informed (27.9) is quite small, which reflects the fact that while bottlenecks form in each simulation, they tend to be equally likely to appear at any location in Clustered World. As with the Random World, no node or set of nodes enjoys an advantage in a Clustered World.

FIGURE 5 ABOUT HERE

By contrast, the challenge to efficiency in the Small World lies in the fact that bottlenecks are unevenly distributed. As shown by the previous model, the introduction of shortcuts allows middlemen to accumulate bits quickly. And when, as assumed by that model, middlemen can be motivated to share those bits with their neighbors, the predicted efficiency gains are realized. But we have seen that when information is exchanged for information, exchange failures can occur. In the Random and Clustered Worlds, such failures are evenly distributed throughout the system. But in the Small World, they occur systematically at the middlemen nodes that are produced by shortcuts. Once their neighbors no longer have any new bits of information to offer, middlemen stop sharing what they know. And since middlemen are more central to the flow of information, once they stop exchanging, they serve as bottlenecks that inhibit information diffusion. Accordingly, there is no longer a basis for expecting Small Worlds to be efficient (i.e., to attain a high proportion of bits diffused) under these conditions. Rather, the results in figure 5 show that the Small World network is not only much more unequal than either the Random or Clustered Worlds (minimum of 21.6 vs. maximum of 39.7 bits, which is even higher than in the Random World), but it can be *less efficient* than them as well, if we measure overall efficiency by the system median (24.6). The results in table 2

illustrate the reasons for this drop in efficiency. The shortcuts in the Small World network allow middlemen like Don and Jill to accumulate information from both sides of the network. And nodes like Ned and Ron benefit from being connected to Don and Jill. However, peripheral nodes like Tina and Sue are made worse off because Don and Jill accumulate (virtually) all of the information and then stop exchanging, with the cessation in exchange cascading through the system to the point that they end up with fewer bits than they do in a Clustered World. In short, *the outcome inequality of the small world impedes its efficiency* once we assume: (a) that actors must be motivated to transfer valuable resources; and (b) there is no general medium of exchange that satisfies the highly restrictive conditions modeled in the bit-for-dollar model.

Results that reinforce this conclusion can also be observed in the 100-node networks, as shown in figure 6. As before, the vertical axis in figure 6 is the proportion of the total number of bits accumulated and the horizontal axis is the proportion of the original network that has been rewired, though the distribution is curtailed at $r=20\%$ rewired for presentation purposes.⁴ And we see that as in the case of bit-for-dollar exchange (figure 4) on the same distribution of graphs, there is a general tendency towards greater efficiency as the graphs are rewired to become more random. Yet the initial rewires produce substantial increases in outcome inequality with essentially no change in efficiency. After the first rewire, the median proportion of bits increases very slightly (40.8 versus 41.33), while the distribution around the median increases substantially. And additional rewires actually reduce efficiency slightly (median of 40.4 at $r=1.5\%$), and only start to increase thereafter. In short, the Small World graphs are marked by negligible changes in efficiency with dramatic increases in outcome inequality (as measured by variation across actors in the amount of information obtained).

FIGURE 6 ABOUT HERE

The results in figures 7 and 8 provide additional insight into this effect. To recall, each r -level beyond $r=0$ is actually a set of graphs (with the previous results averaged over that set), which differ according to where the rewires occur. This produces variation within any r -level, in the degree to which the additional rewire lowers L -- and

⁴ If one extends the analysis to include $r>20\%$, the variance quickly disappears, with all actors obtaining virtually all the available bits.

correspondingly increases SD_g or COV_g . For example, while figure 1b depicts a shortcut that produces the largest decrease in L and the largest increase in SD_g (or COV_g), rewiring more proximate sets of nodes will produce smaller increases in structural inequality. This provides an opportunity to explore how various Small World (i.e., $r=0.5\%$) graphs vary in their effect on efficiency and outcome inequality.

Effectively, we have argued that the reduction in L from the Clustered World to the Small World is problematic because it is accompanied by an increase structural inequality, which thereby: (a) raises outcome inequality, either as surplus or as information that disproportionately flows to middlemen along the shortcuts; and (b) such outcome inequality limits predicted efficiency gains because the middlemen act as bottlenecks.

The results in figure 7, which sort the twenty Small World graphs according to the reduction in L (and correspondingly, rise in structural inequality), substantiate this interpretation. In particular, we see that shortcuts that traverse a longer distance in the Clustered World (i.e., producing a lower L) increase the mean proportion of bits obtained, but reduce the median proportion of bits obtained. When L is low (the rewired link traveled a long distance), the mean is larger than the median, which implies that a small number of actors accumulate a relatively large proportion of the bits--i.e., there is more information inequality. When L is high (the rewired link traveled a short distance), the median is larger than the mean, which implies less information inequality. The reference line in figure 7 is the median proportion of bits obtained when the network is clustered. The results carry the ironic implication that, when efficiency is measured from the standpoint of the typical member of the system, Small World structures are *less* efficient when the initial shortcut is “optimally” placed (as in figure 1b), in that they link otherwise distant actors. This result directly contradicts the basic prediction from small world literature to this point, which expects increases in efficiency from shortcuts that reduce L . The problem is that such shortcuts also make some actors much more powerful than others, and such actors may be not be altruists.

FIGURE 7 ABOUT HERE

The results in figure 8 provide additional insight into this implication. To produce figure 8, nodes within Clustered ($r=0$) and Small ($r=0.5\%$) World 100-node graphs were ranked in terms of the proportion of bits obtained within a given system, with a rank of 1

indicating that the node had accumulated the most bits and 100 indicating the least informed node. The solid line in figure 8 shows the distribution of the proportion of bits obtained across ranks in the Clustered World, while the dashed line shows the distribution in the Small World (averaged across the twenty graphs). We see that the medians of the two distributions are nearly identical (because the Small World graphs include some graphs with ‘suboptimally placed shortcuts’). And we see that while the bottom quartile is worse off due to the introduction of a shortcut, the top twenty-five actors are better off. The gains and losses are not symmetric. The gains experienced by actors at the very top of the distribution are much larger than the losses experienced by actors at the bottom of the distribution. This is consistent with the result in figure 7, whereby the increase in mean efficiency associated with decreases in L is coupled with declines in median efficiency. In sum, we have seen that the stratification produced by introducing shortcuts into a clustered world not only engenders outcome inequality but that such outcome inequality can lower efficiency as well, especially if: (a) we consider the effect on the typical member of a system; and (b) especially if the shortcut is optimally-placed.

In sum, a contrast of results from the two models we have presented-- “bit-for-dollar, single-bit diffusion” and “bit-for-bit simultaneous diffusion” indicate the conditions under which a Small World will indeed represent an attractive balance of clustering and efficiency (in the diffusion of simple contagions). Results from the first model indicated that the expected gains in efficiency from a Small World *can* be realized despite the outcome inequality that inheres in such structures insofar as the actors involved are altruists or there is a general medium exchange that satisfies the restrictive conditions discussed above. But results from the second model suggest that when those assumptions are not satisfied--as seems generally to be the case, efficiency will be compromised because the middlemen who are needed to ensure efficiency in the Small World become satiated; and once satiated are no longer interested in sharing information.

SW Inefficiency Due to Limited Bandwidth

Bit-for-Dollar, Simultaneous Diffusion. Before discussing the broader implications of the foregoing analysis, we now show that the structural inequality inherent in the Small

World can reduce efficiency *even when motivation is removed as an issue*. To see this, contrast the results in figure 9 with those in figure 4. To recall, the results in figure 4 derive from a “bit-for-dollar, single-bit diffusion” model, whereby one node is selected as a source node and then the bit is exchanged for dollars according to the price function in equation 1. The results in figure 9 derive from the same bit-for-dollar model, but under “simultaneous diffusion,” as in the previous bit-for-bit model. A comparison of results in figures 4 and 9 shows that this change in the mode of diffusion has a substantial and meaningful impact on the level of efficiency of the Small World relative to the Clustered World. Note first that the effect on outcome inequality, as measured by surplus, is effectively the same as under single-bit diffusion. The stratification of the Small World means that the middlemen along the shortcuts extract significantly more surplus from transmitting more bits at higher prices than do more peripheral actors. It is important to recall as well that, in evaluating the efficiency of these systems, that motivation is not an issue, just as it was not an issue in the first bit-for-dollar model. While actors may have to pay for the bits they obtain, they always have enough dollars to motivate the transmitters and so bits will diffuse completely. Accordingly, the measure of *inefficiency* in both cases is the mean number of interactions required for full diffusion. And yet, while motivation is not an issue in either set of simulations, the effect on inefficiency is quite different. While the results in figure 4 are consistent with the basic prediction that inefficiency should track the reduction in L , this is not the case for the results in figure 9. To be sure, it remains the case that overall, the more a structure is rewired to be more like a Random than a Clustered World, inefficiency is reduced. But the initial rewires that create Small World networks (i.e., high CC/L structures) actually bring about a spike in inefficiency under simultaneous diffusion whereas they reduce inefficiency under single-bit diffusion.

FIGURE 9 ABOUT HERE

Why does the structural inequality of the Small World reduce inefficiency even when motivation is removed as an issue? The answer is that the middlemen will still act as bottlenecks as long as one assumes that actors have limited *bandwidth* or capacity for transmitting information within a given time-interval. The type of bottleneck created by limited bandwidth stems not from middlemen’s unwillingness to pass on information

once they have their alters' information, but from the fact that that their selection of some bits for transmission *crowd out* other bits that could otherwise have been transmitted. To see this, consider how bits might flow through the Clustered and Small Worlds of figure 1. In the Clustered World structure, the set of bits flowing through one part of the graph (e.g., the “bottom”) at a particular point in time will not overlap with those flowing through another part of the graph (e.g., the “top”). But in the Small World, there may be considerable redundancy in such flows. Such overlap is created when the middlemen at either end of a shortcut (e.g., Don and Jill) transmit the same bits in the same direction (e.g., rightward, towards Sue). As a result, a peripheral actor such as Tina or Sue now can receive the same bits from multiple directions. Crowding in the channels of communication slows that rate at which information reaches more peripheral actors. This explains why there is an increase in inefficiency under simultaneous diffusion. In particular, from any actor's perspective, his alters will help him get information faster when they effectively divide their labor, with each one specializing in acquiring and transmitting different sets of bits.⁵ But while this specialization pertains in the Clustered World, the introduction of shortcuts hampers such specialization in the Small World. Thus, as shown in figure 10, the transformation of a Clustered World into a Small World *both increases outcome inequality and reduces efficiency*. While the middlemen along the shortcuts obtain all the bits much faster in the Small World (and they can earn significant rents in the process), the typical member of the system (measured either by the mean or the median) accumulates bits less quickly.

FIGURE 10 ABOUT HERE

This second bottleneck effect can be eliminated by widening the necks—i.e., by relaxing the assumption that actors transmit and receive a maximum of one bit during an interaction.⁶ Thus, we show in figure 11 results from an identical simulation, but with actors now assumed to have unlimited bandwidth. That is, they transmit everything they know during a single interaction. Under these conditions, the results closely track what

⁵ At the same time, the fact that such peripheral actors now have more options for obtaining the same bit explains why the spike in outcome inequality (which is driven in part by buyer surplus) is not as great in figure 9 as in figure 4.

⁶ This problem could also be alleviated if middlemen coordinated their transmissions such that they made sure not to transmit redundant flows of bits down parallel paths of the structure. Besides raising the issue of motivation anew, this solution requires unrealistic capacities for coordination.

we found under one-bit-at-a-time diffusion. While outcome inequality increases significantly, there are now substantial gains in efficiency from the first rewire, which are widely shared. This follows from the fact that if bandwidth is unlimited, it is no longer a problem if middlemen transmit the same bits along parallel paths because many bits can flow along those paths at the same time. Indeed, all structures are much more efficient under these rules. The question, however, is how realistic it is to assume that actors have unlimited bandwidth. Insofar as actors are in fact limited in how much they can transmit in a given encounter, we have seen that the stratification of Small Worlds is problematic because the middlemen that are counted on to provide efficiency gains can add inefficiency *even when they are fully motivated to pass on what they know*. The problem is that if they are likely to pass on limited amounts of the same information when it would be better (for more peripheral actors) to specialize.

FIGURE 11 ABOUT HERE

Overloaded and Unmotivated. Our final simulations follow on the last simulation in that bandwidth is removed as a constraint, while we reintroduce the issue of motivation. These analyses are motivated in part by a potential objection one could raise to our conclusion that when actors are not altruists (and there is no medium of exchange that satisfies the restrictive assumptions discussed above), the stratification of Small World will eliminate and can even be said to reverse the expected efficiency gains. In particular, insofar as bandwidth is limited in the bit-for-bit barter models presented above, it could be that the decline in efficiency resulted not from the middlemen's lack of motivation to pass on what they knew, but from a version of the crowding-out effect demonstrated in the last section. Thus, we now lift the bandwidth restriction to better isolate how an actor's demand for compensation (in the form of new information) to motivate him to transmit information affects overall system efficiency in the Small World versus other worlds.

We model unlimited-bandwidth bit-for-bit barter in two ways, with results shown in figures 12 and 13. In each case, the results may be directly compared with those in figure 6, which are also derived from bit-for-bit barter, but with the actors limited to one-for-one transactions. We now loosen this restriction. First, to produce the results in figure 12, we require only that the number of bits that are transmitted in each direction be

equal. For instance, if i and j each have three bits that the other does not yet possess, they each transmit three bits to one another. But whereas the removal of the bandwidth restriction restores the promised efficiency of the Small World when motivation is not an issue (figure 11), it actually *exacerbates* the inefficiency of the Small World when actors must be motivated to transmit information. Thus, we see that whereas the median proportion of bits obtained is essentially flat after the first few rewires when bandwidth is restricted (figure 6), there is a clear decline when bandwidth is unrestricted (figure 12). This reflects the fact that while actors can now become informed faster, middlemen become fully informed even faster. The problem of motivating middlemen has not gone away and the bottleneck that they represent now operates to significantly limit the spread of information. That is, the cascades of exchange-cessation happen sooner.

FIGURE 12 ABOUT HERE

The results in figure 13 model the effect of motivation in a slightly different way, but with substantially the same effects. Whereas the previous bit-for-bit models assume that actors are willing to trade whenever they acquire a number of bits equal to the number they are being asked to transmit, our final simulation follows the bit-for-dollar simulations in supposing that i will not consider it an equal trade if j offers him bits that he can obtain from other contacts k in return for bits that j cannot obtain from anyone but i . That is, we assume that prices can be expressed in barter exchanges as well, and that the price i must pay j declines in the number of options i has for acquiring any bits that j offer. To introduce prices into the bit-for-bit framework, we: (a) consider all bits that each member of a dyad i and j could share during an interaction; (b) count the number of alternative sources or options i has for each bit that j possesses but i lacks, and vice versa; and (c) mark either i or j as the more powerful actor depending on which one has the most sources taken over all bits that could be exchanged. If neither party has additional sources (e.g., at the start of the interaction) or if the maximum number of sources held by i (taken over the bits that j possesses and i lacks) equals the maximum number of sources held by j (taken over the bits that i possesses and j lacks) then the simulation proceeds as in the prior simulation—i.e., i and j exchange as many bits as they can as long as it is an equal trade. But if one party is more powerful than the other, we assume that the more powerful party will demand extra bits, with the additional number equal to the number of

additional sources it has. For example, if i and j each have two bits that the other seeks, but if i has an alternative source for one of those bits but j has no alternatives for either, then i will receive j 's two bits and j will receive only 1 (randomly selected). We further assume that the exchange will not be completed if the “weaker” actor involved in the interaction does not have enough information to meet these terms of trade. Since middlemen are more likely to have multiple sources, we are essentially allowing middlemen to exercise their power by devaluing their contacts' bits relative to their own, an assumption which has face validity. Structurally advantaged actors often believe that their time and resources are worth more, even when talent and ability have very little to do with their structural position (Gould 2002).

FIGURE 13 ABOUT HERE

As shown in figure 13, these simulation rules produce similar effects. In particular, the initial rewires of the Clustered World reduce efficiency, when such efficiency is measured as the median proportion of bits that are obtained by the end of the simulation. Of course, while most actors do worse, some do much better. In particular, the rewires allow the middlemen along the shortcuts of the Small World ($r=0.5\%$) to obtain an average of 92.3 of the 100 bits. This compares with only 81.2 bits in the prior simulation, which did not incorporate relative power. Thus, we have shown that the stratification of the Small World can be expected to reduce efficiency when either, and especially if *both* of the following very general conditions apply: (a) actors are limited in their bandwidth, or capacity for transmitting information; and (b) actors need to be motivated to transmit the information in their possession.

Conclusion

Until recently, the global network properties that define the “Small World” --i.e., short path-lengths and high clustering--were thought to be in opposition (Watts and Strogatz, 1998: 440). The signal contribution of Watts and his colleagues (Watts and Strogatz 1998; see also Newman 2000; Newman and Watts 1999; Watts 1999a, b) was to demonstrate that as one moves from a clustered to a random network, both global network properties decline but the average path-length declines at a much faster rate. Consequently, it is possible to have a network with a relatively high degree of clustering

and relatively short path-lengths. This insight is sociologically interesting, at least in part, because it suggests that Small World networks can satisfy two social goals that are often used to evaluate social systems: “community” and “efficiency.” We defined a structure as more efficient insofar as resources flow quickly from actors in one part of a structure to more distant actors who desire or need them. Defining what is meant by community is notoriously difficult (see Vaisey 2007 for review), and is outside the scope of the present paper. For this discussion, however, it is sufficient to note that: (a) there are various collective resources that are frequently labeled with the term community (e.g., Coleman 1988; Portes and Sensenbrenner 1993); and (b) if the production of such resources cannot be reduced to network clusters, such clustering at least facilitates their production (Etzioni 2001). Thus, if adding shortcuts to a clustered network preserves community benefits while increasing efficiency, Small World networks appear to strike an attractive balance between community and efficiency.

In the foregoing analysis, we join an emerging line of research that tempers the enthusiasm for Small World structures with the recognition that shortcuts are often unreliable carriers of resources, at least when they are “manned” by human agents. Centola and Macy (2007) provided the first reason for doubting the efficiency-enhancing possibilities of shortcuts. They argue that the shortcuts characteristic of the Small World are too narrow to support the spread of an idea or innovation that requires confirmation from multiple sources. Like Centola and Macy, we: (a) point to an unrecognized structural implication of relying on shortcuts to achieve efficiency--the structural inequality they inherently produce; and (b) argue that sensitivity to either or both of two aspects of human nature-- self-interestedness and limited processing capacity-- implies that this structural inequality will make Small World structures inefficient, at least from the standpoint of the typical member. In short, the very same shortcuts that make efficiency and community more achievable in the Small World also introduce structural inequality and thus set the stage for outcome inequality. If the predicted efficiency gains are to be realized, valuable resources must travel across the bridges middlemen provide. But we have shown that the typical actor in a Small World structure actually obtains less information than in a highly clustered structure once we assume that middleman (and actors more generally): (i) require (the promise of) information to motivate them to

transmit information; and/or (ii) are limited in their capacity for transmitting information, even if fully motivated to do so. Thus, rather than striking an attractive balance between efficiency and community, our analysis suggests rather that it is very hard for social systems to achieve both high degrees of efficiency and community. The reason for such difficulties is that the attempt to achieve efficiency while retaining community involves a necessary increase in structural inequality. Such structural inequality in turn produces outcome inequality; and such outcome inequality will hamper efficiency if we assume actors are self-interested and/or limited in their capacities.

Of course, one could reasonably challenge the realism of the assumptions we have adopted and incorporated into the simulations we presented. We concede that these models are highly stylized and can provide insight only insofar as the assumptions capture an important aspect of real-world behavior. Insofar as real-world behavior departs from our assumptions, we will have done analytic violence to that reality. At the same time, if we are to realize the substantial promise of social network analysis (and sociological theory more generally) for understanding the properties of different social positions and systems, some set of simplifying assumptions must be adopted. Moreover, we believe that the assumptions we have adopted here are more realistic than the alternatives that had (implicitly) been adopted in the prior literature, and that they represent a good starting point for future research. To see this, it is useful to consider the implications of our analysis for how a “social engineer” might try to avoid the barriers to achieving high degrees of community and efficiency we have identified.

One set of measures would be behavioral. In particular, the social engineer might try to select agents or modify (i.e., teach) existing agents to overcome the limitations that we have assumed. That is, one might try to populate the world (and the middleman position in particular) with altruists who do not have limitations on their capacities for processing information. Clearly, much education is invested in the development of information-processing capacities, and various modern technologies and institutions are often touted as enlarging such capacities beyond those of any individual. Moreover, actors who occupy middleman positions can often be expected to develop additional capacities for absorbing and transferring knowledge (see Reagans and Zuckerman 2008b; cf., Burt 2008; Reagans and McEvily, 2003; Cohen and Levinthal 1990). And yet, even

with the enlargement of such capacities, it still seems reasonable to assume that individual capacities will always strain against the enormous (and increasing) volume of information in modern society.

Note further that even if we believe that our social engineer could avoid the problem that the actors have limited capacities, it is not clear how he could populate his world with altruists. Certainly, Hillel's famous dictum, "If I am only for myself, what am I?"⁷ reflects an ethic that is widely shared and sometimes followed. But of course, such appeals to altruism would not be made if altruism came easily to humans, especially in resource-constrained environments. Moreover, self-interestedness in moderation is no vice, as reflected in Hillel's preceding remark-- "If I am not for myself, who will be for me?"-- and as famously noted by Adam Smith. Thus, even were it possible for our social engineer to banish self-interestedness, it might not even be advisable for him to do so.

Perhaps, however, the social engineer could avoid the inefficiency produced by shortcuts, not by selecting or modifying the human beings who populate the middleman position, but by modifying the rules of exchange. In particular, the "bit-for-dollar" models we presented, in which bandwidth limitations did not restrict information-flow (either because a single bit was flowing at a time, or because each actor had unlimited bandwidth), did not suffer from lower efficiency in the Small World because middlemen were fully motivated by their earnings in "dollars." But while it is in fact quite common for information to be exchanged for a general medium of exchange (hence the term, "knowledge economy"), such currencies seem typically-- and perhaps, inherently-- to fail to satisfy the three conditions necessary discussed above: (a) no frictions or costs incurred in exchange; (b) no budget constraints; and (c) no satiation effects. The challenge of eliminating budget constraints seems the most obvious, especially if we focus on money as the relevant medium of exchange. And while the issue of satiation may not appear to pertain in the case of money, both budget constraints and satiation are quite salient if we focus on what is perhaps the most common currency for obtaining information: deference (see e.g., Blau 1964; Emerson 1962: 39). In particular, insofar as deference has zero-sum properties (i.e., deference to one party has value only insofar as it implies lack of deference to others), budget constraints inhere in such an exchange

⁷ Mishna tractate Avot, 1:14.

system. Moreover, insofar as the value of deference is increasing in the status of the deferrer, the high-status actor will necessarily be satiated--i.e., unmotivated-- by deference from low-status actors. In sum, while it may be reasonable to assume or even engineer a system based on the exchange of information for a general medium of exchange, it seems unreasonable to expect that such a medium will satisfy the conditions necessary to motivate the middlemen along shortcuts to help Small Worlds reach their potential efficiency.

A final approach is structural. In particular, if the problem is that middlemen reduce efficiency by serving as bottlenecks, our engineer could try and reduce the network's dependence on these middlemen. As the structure is rewired to include more shortcuts, efficiency gains are more certain because system level outcomes are less sensitive (i.e., more robust) to the behavior of a small number of central individuals. But insofar as there is a host of "community" benefits that emerge in highly clustered structures, such a strategy means foregoing such benefits because introducing more shortcuts produces a decline in clustering. Thus, while previous research had suggested that Small World networks strike an attractive balance between community and efficiency (for simple contagions), our analysis of the "Small World Phenomenon" suggests that the trade-off between the two objectives is very difficult to avoid because attempts to achieve both efficiency and community are likely to falter without addressing the issue of inequality.

Discussion

Both Efficiency and Inefficiency from Connecting the Disconnected?

We conclude by discussing two issues that relate to the value shortcuts provide to a system. The first involves a seemingly technical issue that, at least in a restricted sense, points to an important set of conditions under which adding a shortcut does in fact increase efficiency, even under the assumptions adopted above. In the models presented above, we followed conventional practice in our models by focusing on sets of graphs that were all fully connected (i.e., each node can reach every other node via $n \geq 1$ intermediaries). The advantage of such structures for documenting the effect of shortcuts is that aggregate functions of path-distance (L) are comparable across all such graphs. By

contrast, one cannot quantify the effect of, say, rewiring the “disconnected cavemen” graph in figure 14a to create the “connected caveman” graph in 14b because path-length is undefined for dyads that cannot reach one another. And yet, one lesson of our analysis and that of recent research (Centola and Macy, 2007; Lazer and Friedman, 2007) is that we can learn more about the performance of a system by shifting from structural measures that are *predicted* to govern such system outcomes as efficiency (e.g., L) to more direct measures of efficiency. In particular, it is possible to run our models on the graphs in figure 14 and measure the change in the extent of diffusion. The results from such analyses show that, unsurprisingly, the connected caveman graph is more efficient. In particular, actors in the disconnected caveman graph can accumulate at most 5 bits of information under any rule of exchange, while all actors in the connected cavemen graph average at least 11 bits, with the middlemen averaging 21 bits, under any of the unpriced bit-for-bit models. Ironically then, it would seem that the research strategy adopted by Small World researchers actually underestimates the value of shortcuts for enhancing efficiency. Moreover, there is good reason to focus on the type of transformation depicted in figure 14 since it is widely recognized that the most valuable bridges are those that connect otherwise disconnected groups (e.g., Granovetter 1973).

TABLE 14 ABOUT HERE

And yet, there at least three reasons why one should be cautious in inferring from these results that there is in fact no trade-off between efficiency and community. First, these gains in efficiency apply only to “simple contagions” that do not require validation from a second source (Centola and Macy 2007). Second, the introduction of such shortcuts still introduces significant outcome inequality into erstwhile equal systems. Consequently, even if actors like Bob are better off in absolute terms, they may now suffer from relative deprivation if they use one another as a frame of reference. And finally, there are good reasons to wonder whether the structural supports for community will remain in place despite this transformation. That is, while it may be reasonable to suppose that the rewiring of clusters that were already open to the outside (as in the Clustered Worlds examined above) does not dramatically reduce the community supported by such clusters, the opening up of a previously isolated cluster is likely to

have a much larger impact. In particular, the relative deprivation that results from the increase in inequality seems to threaten a community's sense of cohesion.

Valuing and Achieving Community, Efficiency, and Inequality

We close by considering the implications of modifying one assumption we have adopted throughout our analysis-- the "uniform valuation" assumption that all resources are valued equally by all actors in the system. This assumption supposes that all resources are valued equally by all actors in the system. Such an assumption is inherent in the supposition that a system can be judged by its efficiency for transmitting resources. After all, if actors in different clusters were not interested in resources located in distant clusters, the efficient flow of such resources would have no value for them. Yet in fact, this uniform valuation assumption frequently does not hold. As Reagans and Zuckerman (2008a,b) argue, tastes are often "homophilic" in the sense that actors value resources that originate in their local neighborhood more than they value more exotic resources. And especially insofar as the clustering that is characteristic of social structures is a product of homophily (see McPherson et al., 2001; Reagans 2005), it would seem that homophilic valuation is the more general condition. Indeed, one could argue that if the uniform valuation assumption in fact governed behavior in Small World structures, there would be no reason for the clusters to persist. That is, the stability of a Small World structure would seem to depend on there being sufficient demand for the resources that flow across the shortcuts as well as the community-based resources that are available locally.

Note finally the implications of shifting the assumption that there is demand for both community and efficiency within a system. If one assumes that there is demand only for local, community-based resources, the problems associated with inequality disappear. This stems from the fact that, as discussed by Reagans and Zuckerman (2008a,b), middlemen cannot earn seller surpluses if members of different clusters are not interested in the resources that travel across shortcuts. Thus, it is the desire for greater efficiency that introduces the potential for inequality, much as high degrees of inequality can impede such efficiency. And now assume instead that demand is purely uniform-- with no interest in local, community-based resources. Under such conditions, clustering

should disappear completely, with both the inequality and the inefficiency of Small World structures being ameliorated. In fact, it seems more reasonable to assume that we are generally somewhere in the middle, with both community and efficiency being prized. And if that is so, the attempt to achieve efficiency together with community must face the challenge of inequality.

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Table 1
 Illustrative Nodal Results for Bit-for-Dollar Exchange in the Clustered, Small, and Random 40-Node Graphs of Figure 1: Averages across 1,000 Simulations of Diffusion of Each Bit*

Clustered World					
	N of interactions until fully informed	Total Surplus	Buyer Surplus	Seller Surplus	N of Bits Sold
Don	209.01	\$.97	\$.42	\$.55	.97
Tina	210.68	\$.98	\$.42	\$.56	.98
Ned	208.71	\$.98	\$.42	\$.56	.99
Jill	207.93	\$.98	\$.42	\$.56	.98
Ron	207.39	\$.97	\$.42	\$.55	.97
Sue	206.16	\$.98	\$.41	\$.56	.98
Mean (SD)	208.15 (2.32)	\$.98 (.01)	\$.42 (.01)	\$.56 (.02)	.98 (.03)
Small World					
Don	142.30	\$1.27	\$.29	\$.97	1.38
Tina	203.85	\$.86	\$.46	\$.41	.80
Ned	148.53	\$1.06	\$.36	\$.70	1.13
Jill	140.11	\$1.28	\$.30	\$.98	1.38
Ron	154.17	\$.97	\$.40	\$.57	1.00
Sue	202.58	\$.88	\$.45	\$.43	.83
Mean (SD)	177.33 (20.41)	\$.98 (0.15)	\$.41 (.04)	\$.56 (.15)	.98 (.15)
Random World					
Don	107.31	\$1.05	\$.37	\$.68	1.07
Tina	110.25	\$1.04	\$.38	\$.66	1.04
Ned	108.04	\$1.00	\$.39	\$.61	1.01
Jill	109.03	\$.96	\$.40	\$.57	.97
Ron	109.81	\$1.01	\$.38	\$.63	1.02
Sue	110.83	\$1.01	\$.39	\$.62	1.02
Mean (SD)	110.30	\$0.98 (.06)	\$.40 (.03)	\$.58 (.09)	.98 (.09)

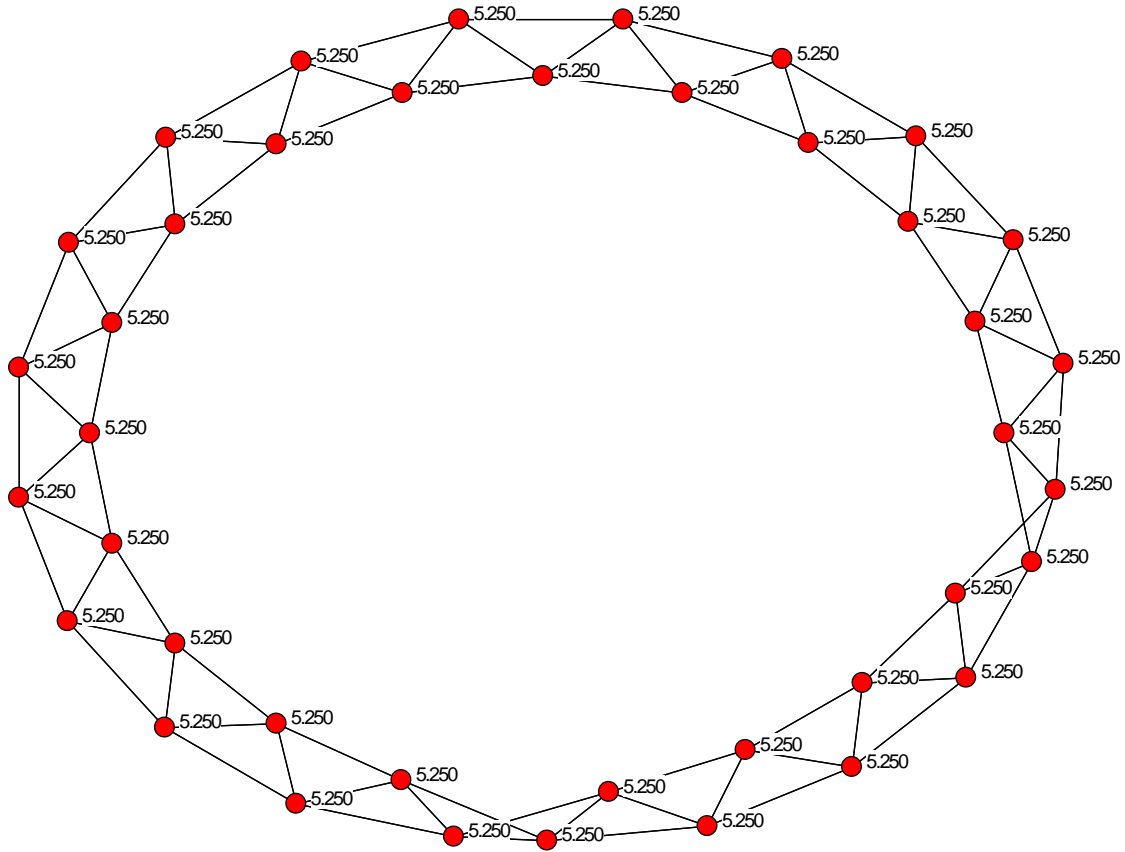
* Results are from 1,000 sets of simulations of “bit-for-dollar” information exchange in the “Small”, “Clustered,” and “Random” networks portrayed in figure 1. The names correspond to the nodes depicted in the Clustered World. Each set of simulations pertains to the diffusion of a “bit” that originates in one of the 40 nodes. Results for each node exclude the simulations for which it was the source bit. This makes the average number of bits distributed= $39/40=0.98$, and since the total surplus available in each transaction is \$1.00, this makes the average total surplus= $39/40=$0.98$ as well.

Table 2
 Illustrative Nodal Results for Bartered Exchange in the
 Clustered and Small World of Figure 1*

	Mean Number of Bits Obtained by end of Simulation		
	Clustered	Small	Random
Don	28.62	39.53	36.98
Tina	28.61	21.99	39.12
Ned	28.71	34.51	38.49
Jill	28.61	39.62	38.90
Ron	28.60	29.45	39.14
Sue	28.77	23.03	38.70
Mean (SD)	28.54 (0.29)	27.09 (5.71)	38.11 (1.23)

* Results are from 1,000 sets of simulations of “bit-for-bit” information exchange in the “Small”, “Clustered,” and “Random” networks portrayed in figure 1. The cell values represent the mean total amount of bits received by the end of the simulation. The names correspond to the nodes depicted in the Clustered World. Each set of simulations pertains to the diffusion of a “bit” that originates in one of the 40 nodes. Results for each node exclude the simulations for which it was the source bit.

Figure 1a
The Clustered World: Percentage of Links Rewired=0%



Graph Properties

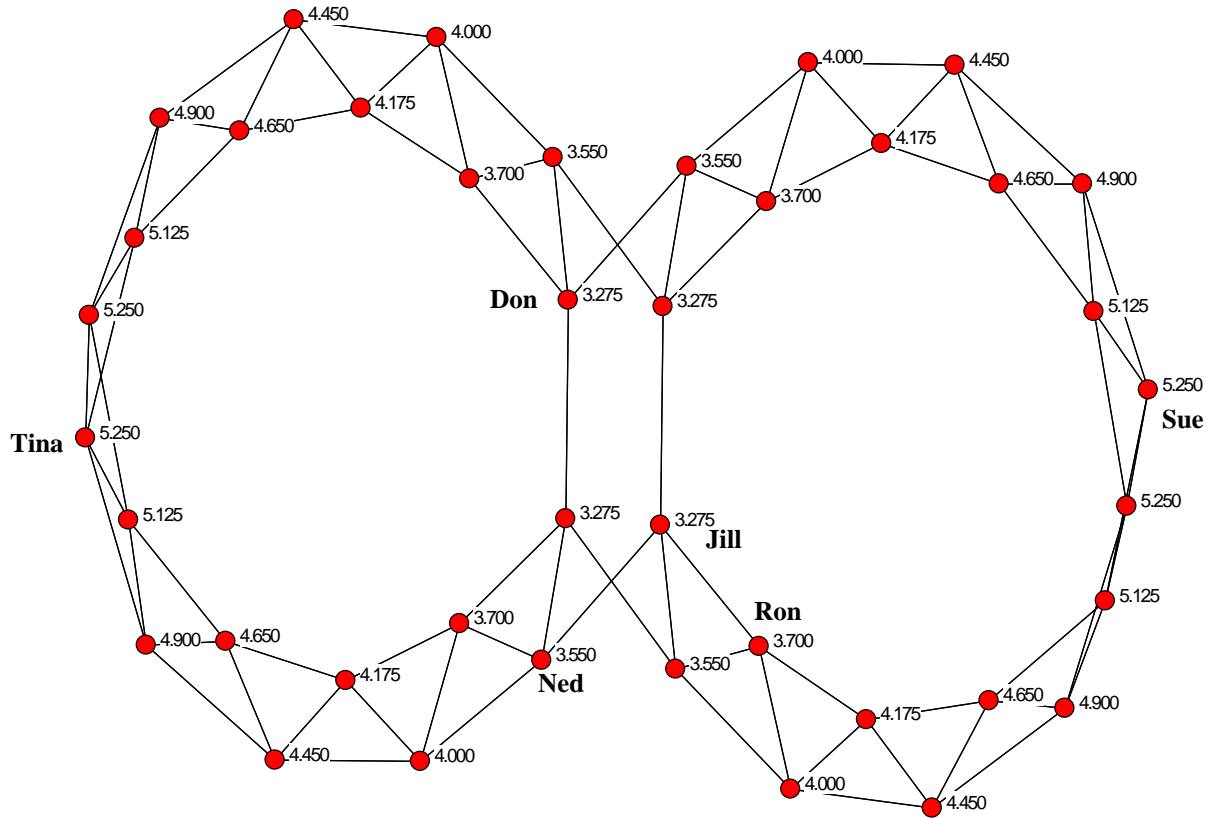
Two node-level properties are calculated, the first is represented by the labels in the graph:

- 1) g = the mean, taken across all other nodes in the network, of the minimum number of steps from that node to the other nodes.
- 2) nd = the density of the node's neighborhood—i.e., the ratio of actual links among a node's contacts relative to possible.

Four global properties characterize this Clustered World network:

L (Mean g)= 5.250
SD_ g (Standard deviation of g)=0
CC (mean nd) = .500

Figure 1b
The Small World: Percentage of Links Rewired=2.5%



Graph Properties

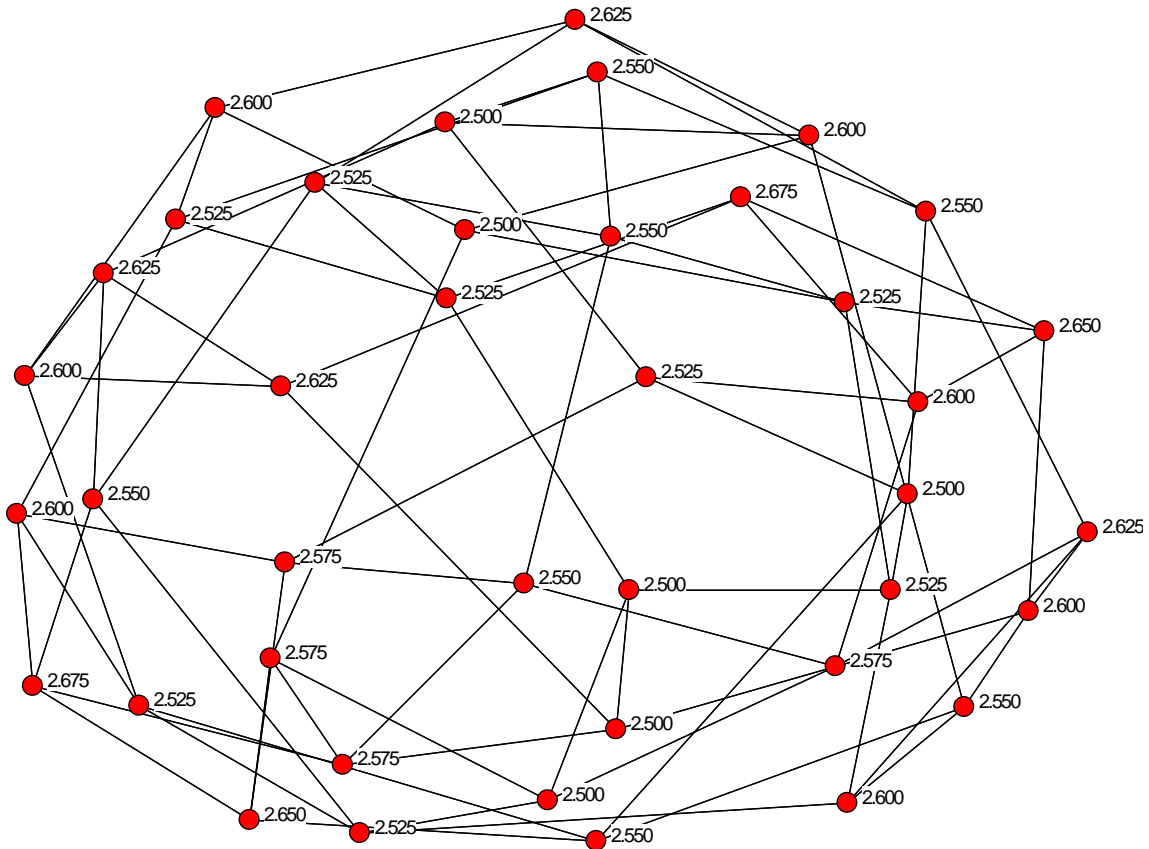
Two node-level properties are calculated, the first is represented by the labels in the Small World graph:

- 3) g = the mean, taken across all other nodes in the network, of the minimum number of steps from that node to the other nodes.
- 4) nd = the density of the node's neighborhood—i.e., the ratio of actual links among a node's contacts relative to possible.

Four global properties characterize this Small World network:

L (Mean g)= 4.307
 SD _{g} (Standard deviation of g)=0.653
 CC (mean nd) = .045

Figure 1c
The Random World: Percentage of Links Rewired=100%



Graph Properties

Two node-level properties are calculated, the first is represented by the labels in the graph:

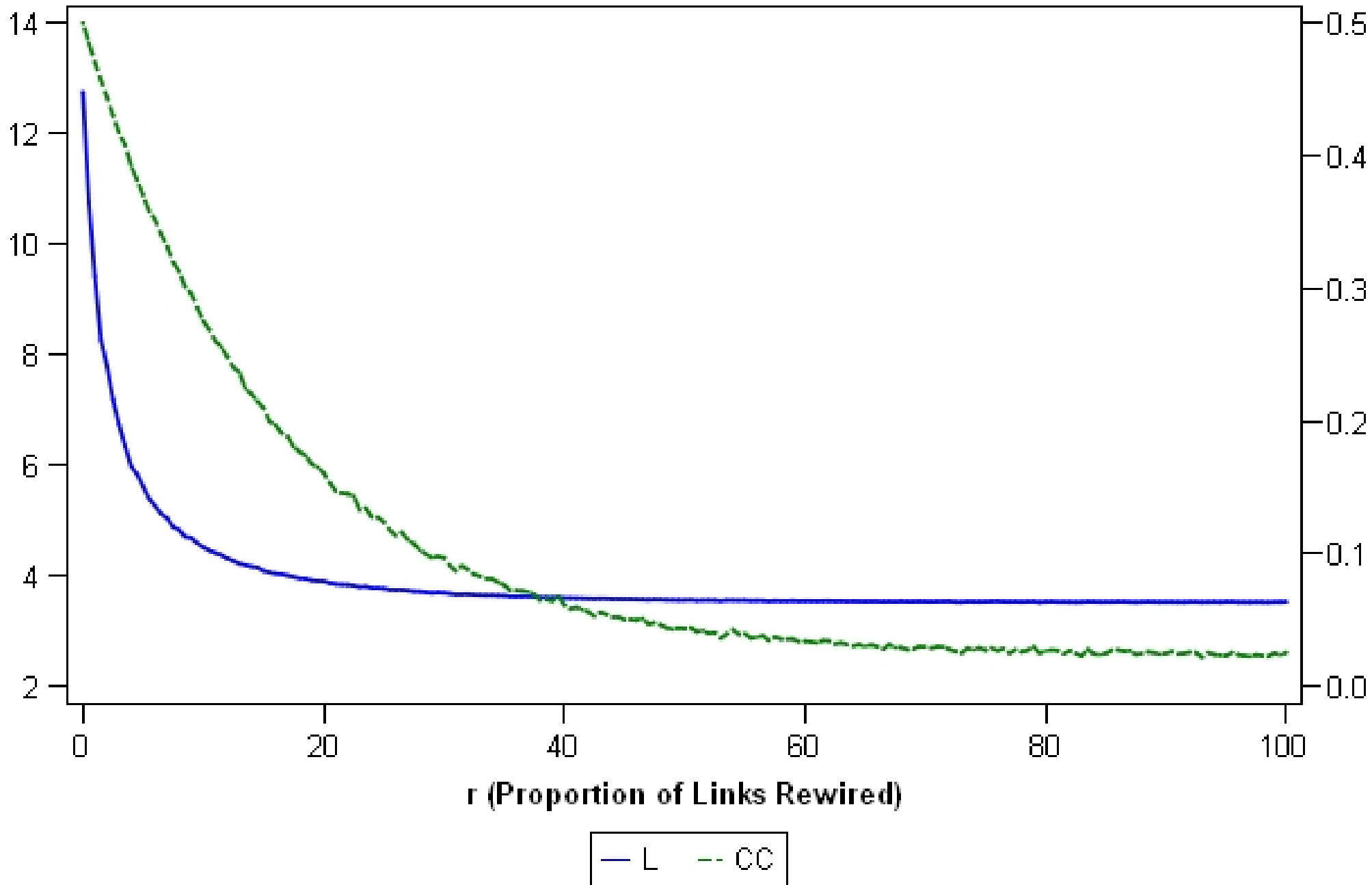
- 1) g = the mean, taken across all other nodes in the network, of the minimum number of steps from that node to the other nodes.
- 2) nd = the density of the node's neighborhood—i.e., the ratio of actual links among a node's contacts relative to possible.

Four global properties characterize this Random World network:

L (Mean g)= 2.57
 SD $_g$ (Standard deviation of g)= 0.05
 CC (mean nd) = 0.025

Figure 2: L and CC by r, 100-Node Degree=4 Graphs

L (Mean Geodesic or Characteristic Path Length) **CC (Mean Neighborhood Density or Clustering Coefficient)**

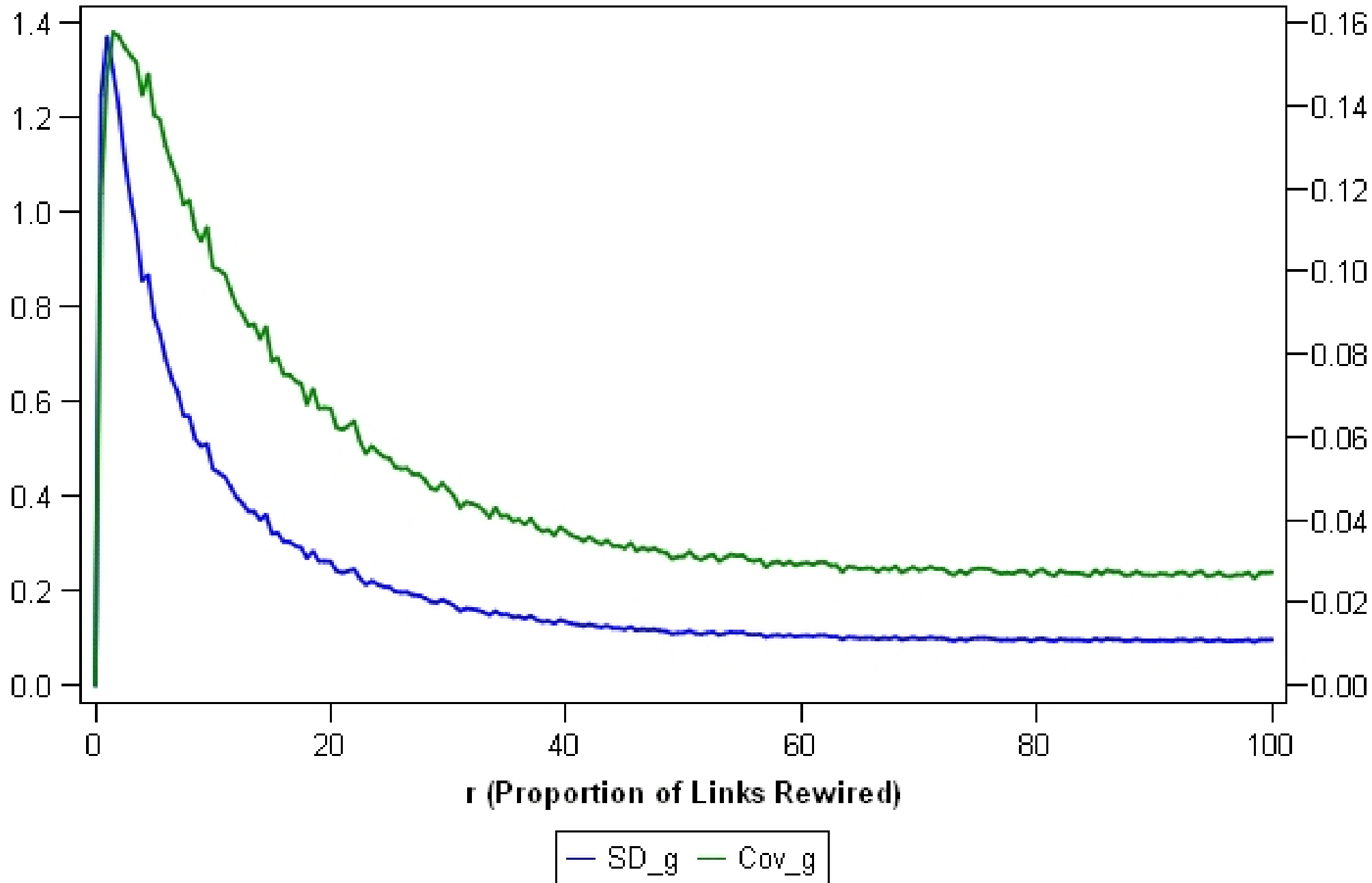


Each data point reflects an average over 100 graphs generated for the given proportion of rewired links

Figure 3: SD_g and COV_g by r, 100-Node Degree=4 Graphs

SD_g (Standard Deviation of the Geodesic)

COV_g (Coefficient of Variation of the geodesic)

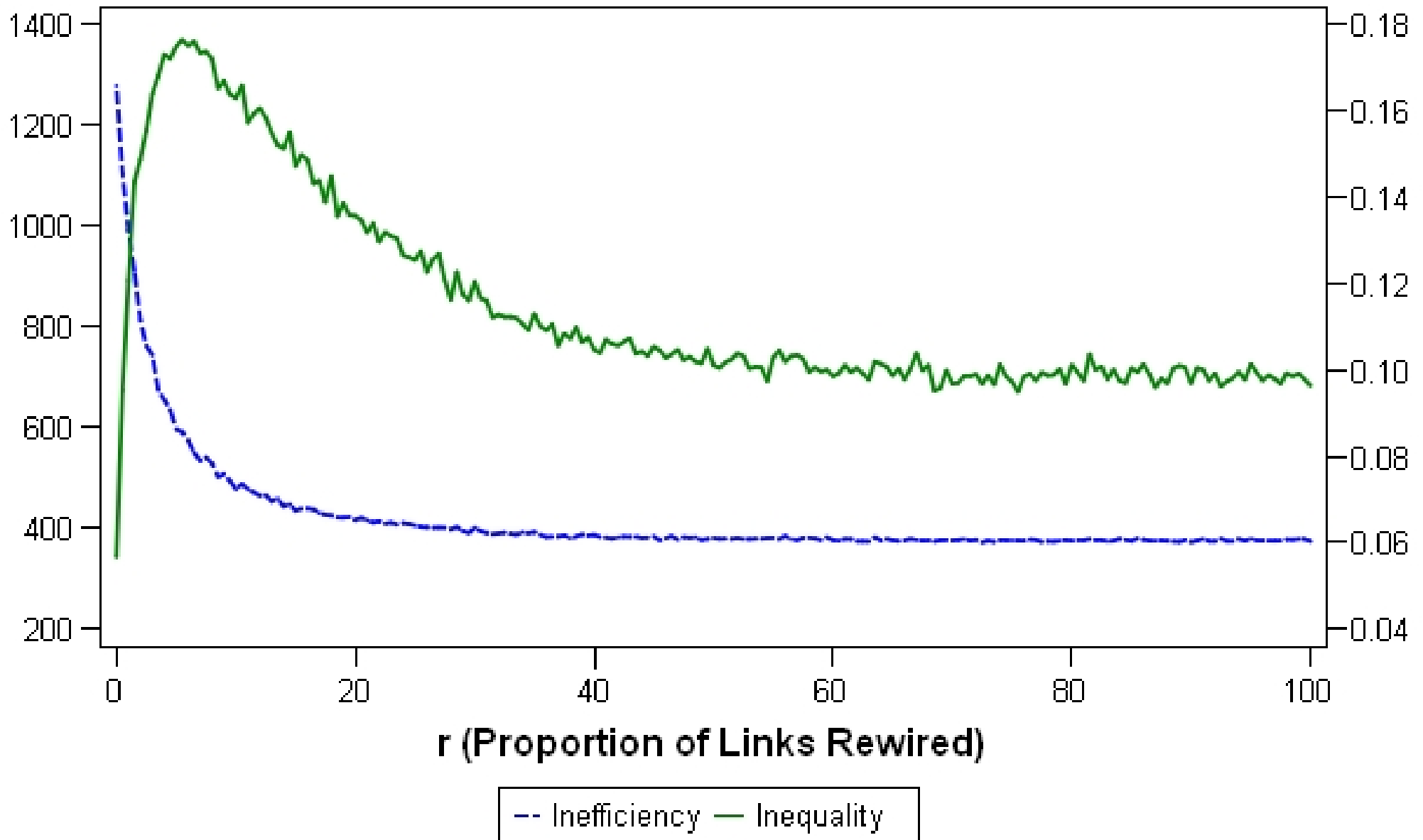


Each data point reflects an average over 100 graphs generated for the given proportion of rewired links

**Figure 4: Inefficiency and Inequality by r , 100-Node, Degree=4
Graphs: "Bit for Dollar" Exchange, Single-Bit Diffusion**

Inefficiency (Mean Interactions Until Full Diffusion)

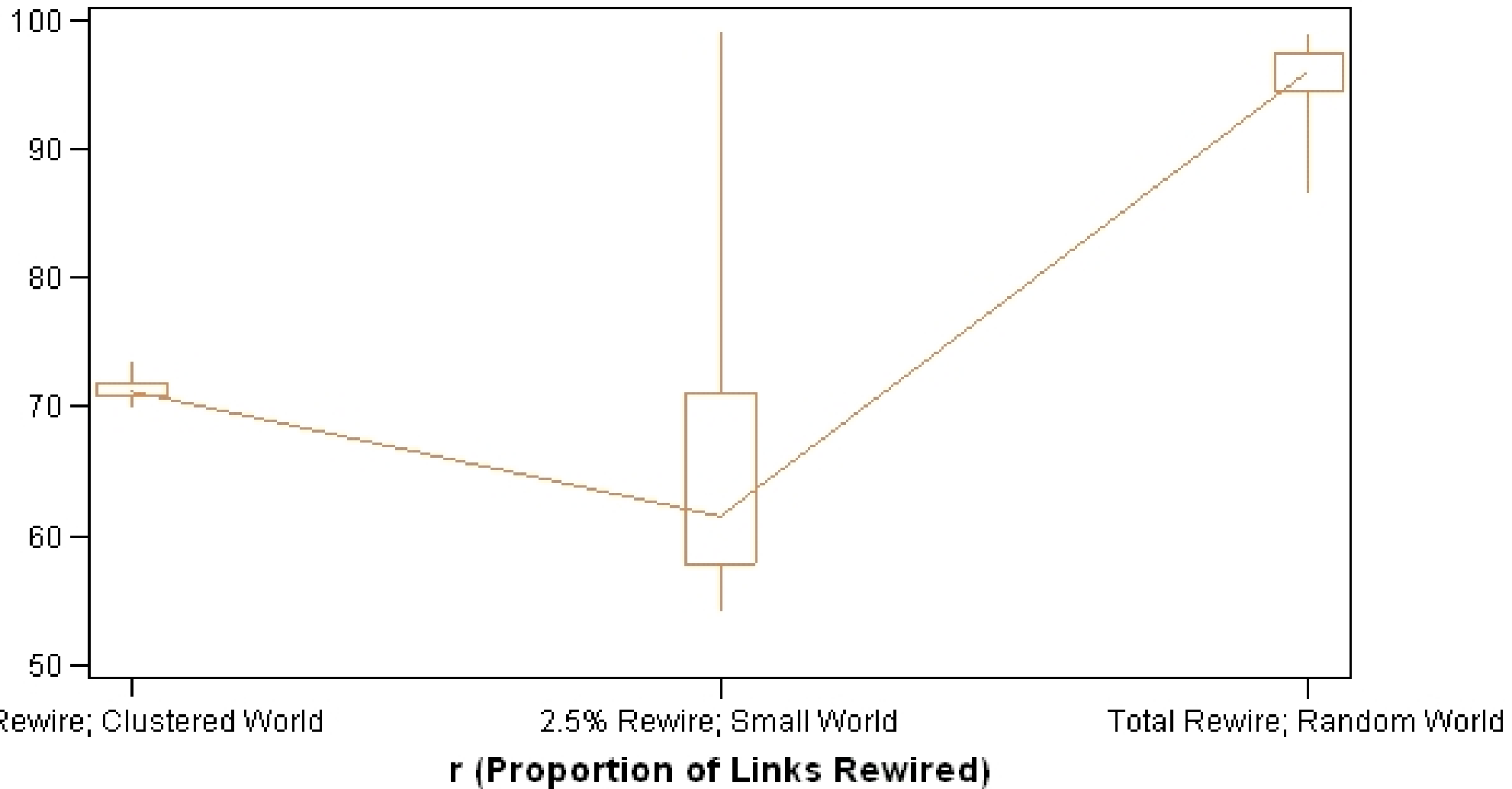
Inequality (Standard Deviation of Total Surplus)



Each data point reflects results averaged over 50 simulations conducted on 20 networks

Figure 5: Distribution of Bits Obtained by Proportion Rewired, "Bit for Bit" Barter in the 40-Node, Degree=4 Graphs Depicted in Figure 1

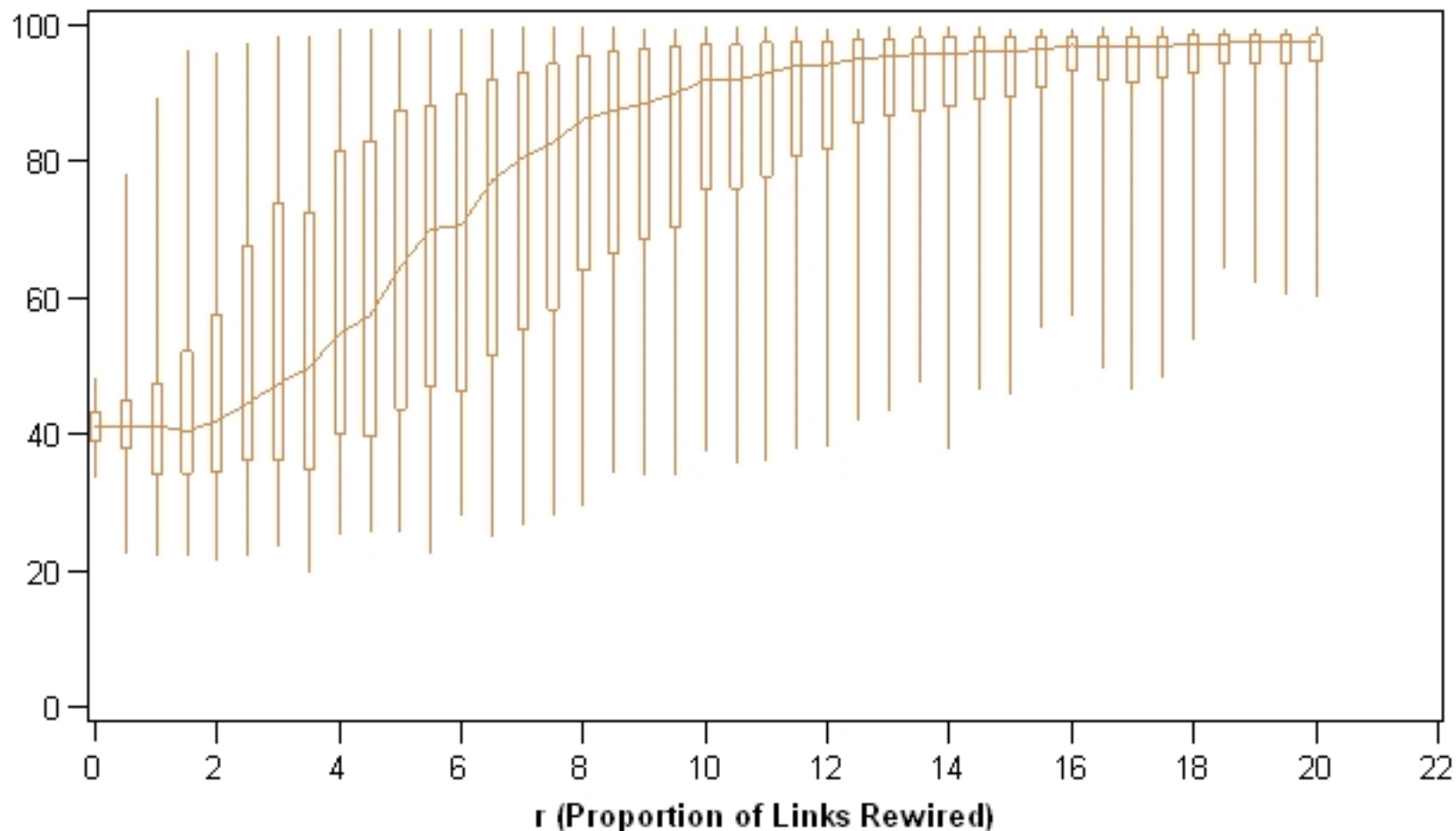
Proportion of Bits Obtained



For each box plot, the whiskers extend from the maximum to the minimum of the distribution, the boxes run from the first to the third quartile and the lines connect the medians. Results are averaged over 1000 simulations.

**Figure 6: Distribution of Bits Received by r ,
“Bit for Bit” Barter in 100-Node Degree=4 Graphs**

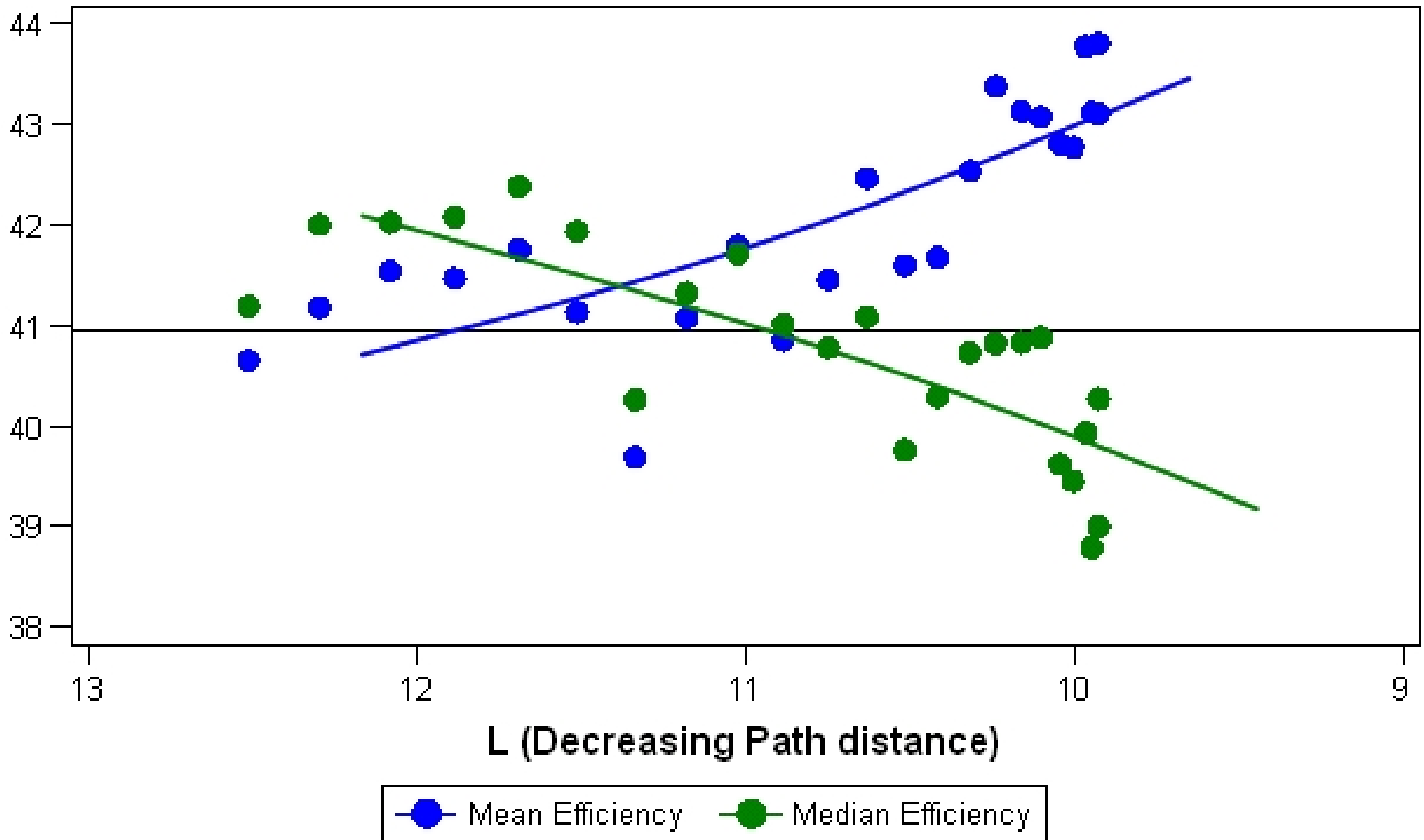
Proportion of Bits Obtained



For each box plot, the whiskers extend from the maximum to the minimum of the distribution, the boxes run from the first to the third quartile and the lines connect the medians. Each data point reflects results averaged over 50 simulations conducted on 20 networks

Figure 7: Mean and Median Efficiency by L in 100-Node, Degree=4 Small World ($r=0.5$) Graphs; "Bit-for-Bit" Barter

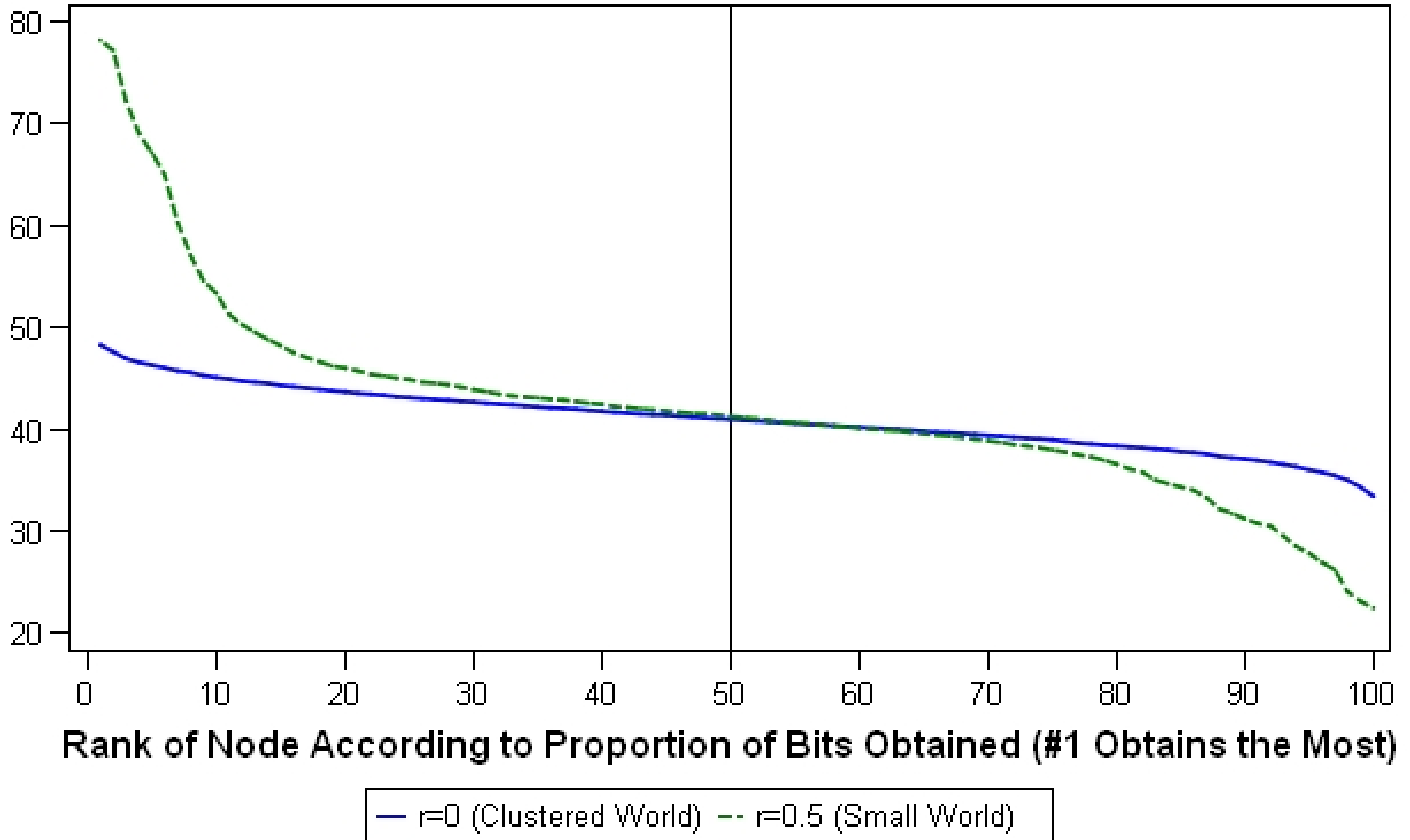
Efficiency: Mean/Median Proportion of Bits Obtained



The fitted lines are cubic splines that minimize a linear combination of the sum of squares of the residuals of fit. The reference line is the median efficiency for the $r=0$ (Clustered) graph. Each data point reflects results averaged over 50 simulations conducted on 20 networks.

Figure 8: Comparison of Information Distribution for $r=0\%$ (Clustered) and $r=0.5\%$ (Small World) in 100-Node Degree=4 Graphs; “Bit-for-Bit” Barter

Proportion of Bits Obtained by the Node

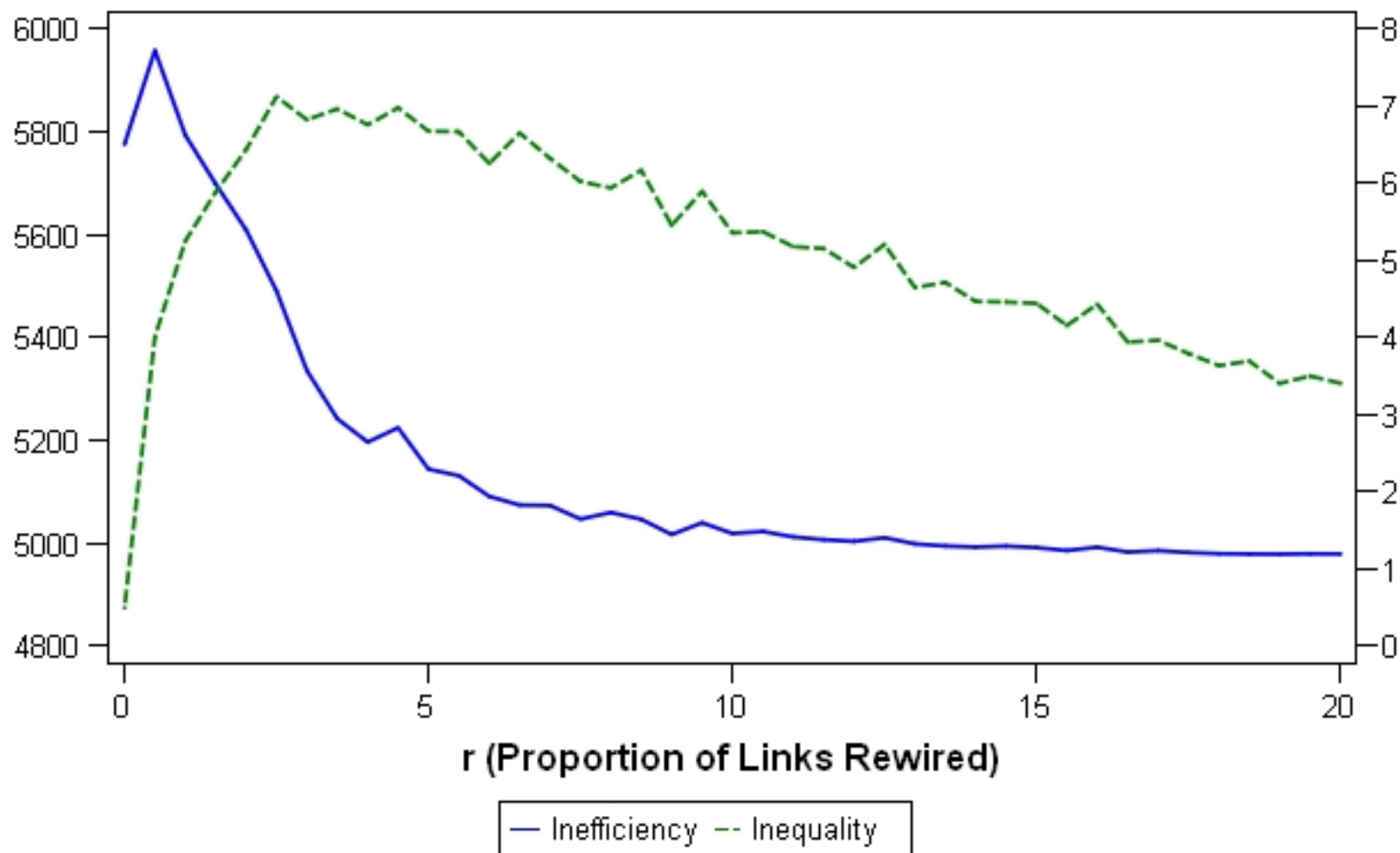


Each data point reflects results averaged over 50 simulations conducted on 20 networks.

Figure 9: Inefficiency and Inequality by r , 100-Node, Degree=4 Graphs: "Bit for Dollar, Limited Bandwidth" Exchange with Simultaneous Diffusion of 100 Bits

Inefficiency (Mean Interactions Until Full Diffusion)

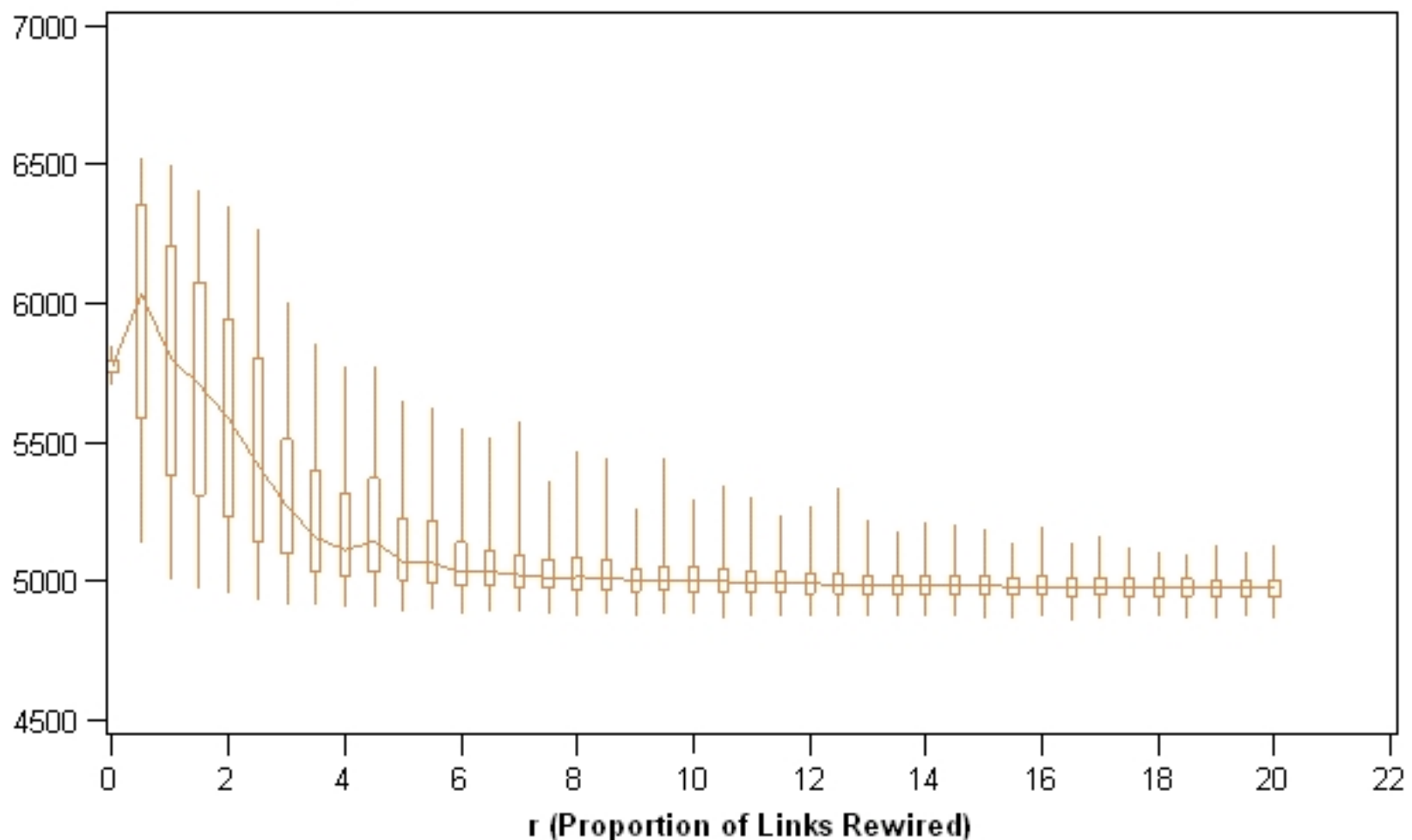
Inequality (Standard Deviation of Total Surplus)



Each data point reflects results averaged over 50 simulations conducted on 20 networks

Figure 10: Distribution of N of Interactions until Fully Informed by r , 100-Node, Degree=4 Graphs; "Bit for Dollar, Limited Bandwidth" Exchange, Simultaneous-Diffusion

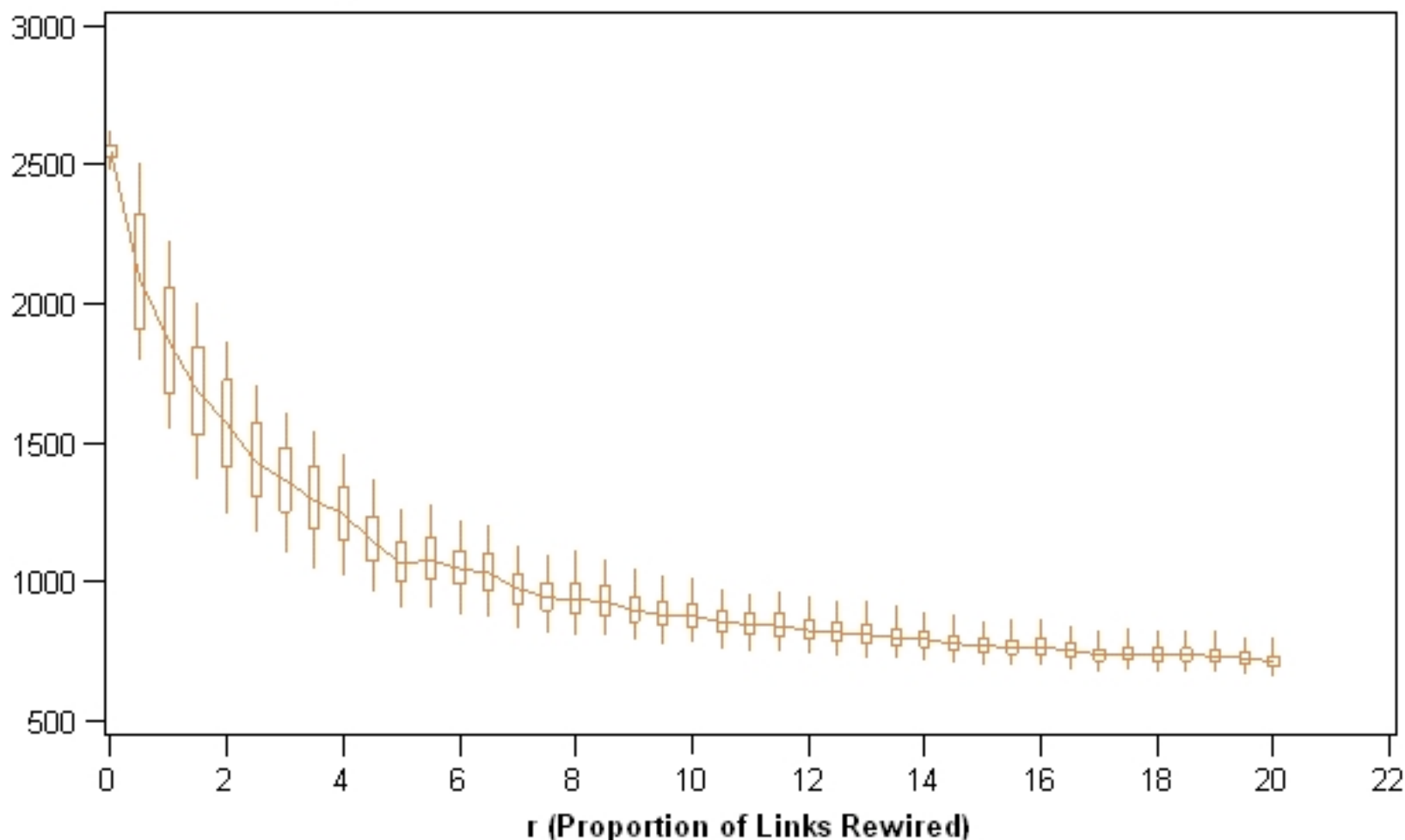
N interactions until fully informed



For each box plot, the whiskers extend from the maximum to the minimum of the distribution, the boxes run from the first to the third quartile and the lines connect the medians. Each data point reflects results averaged over 50 simulations conducted on 20 networks

Figure 11: Distribution of N of Interactions until Fully Informed by r , 100-Node, Degree=4 Graphs; “Bit for Dollar, Unlimited Bandwidth” Exchange, Simultaneous-Diffusion

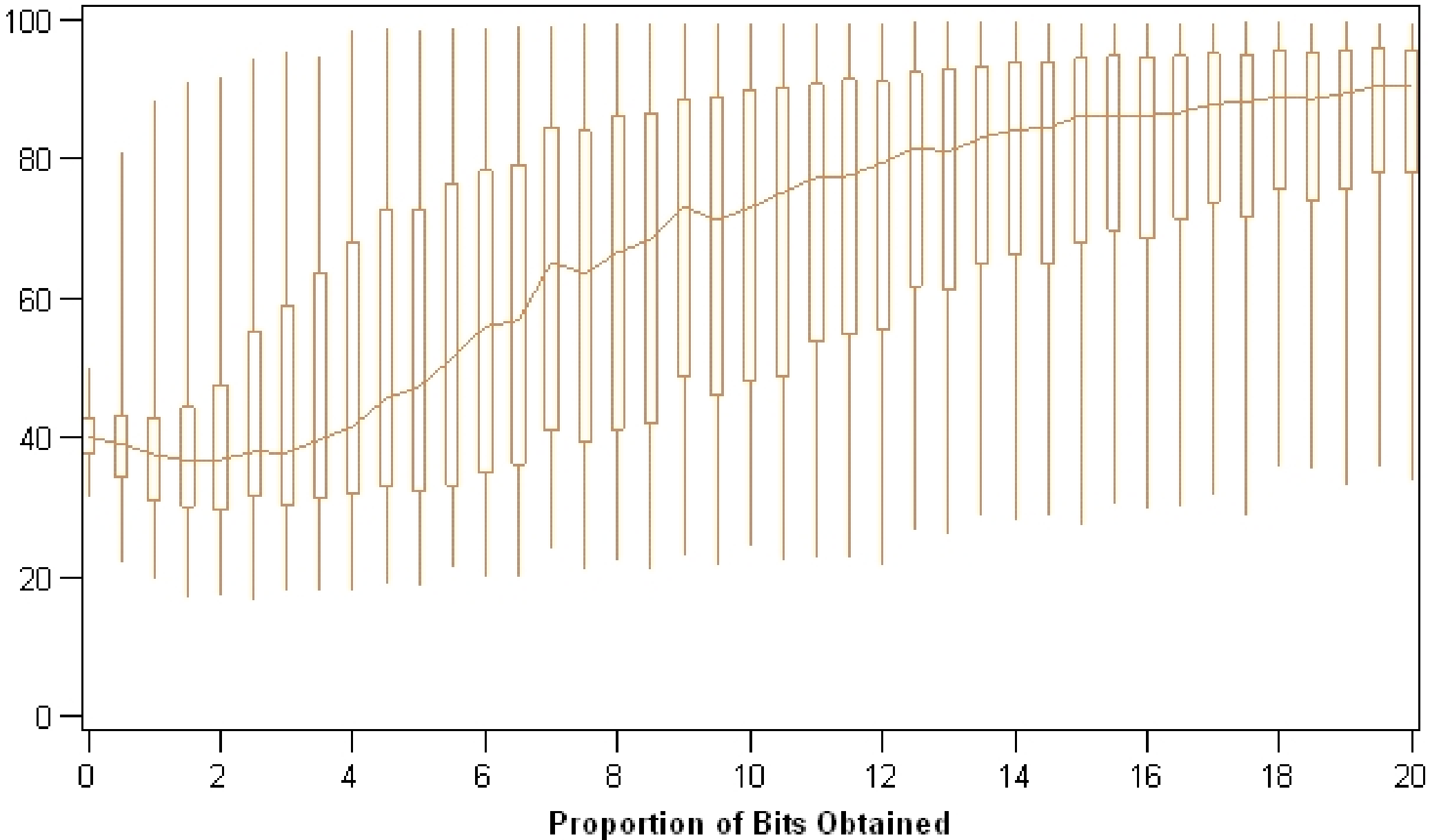
N interactions until fully informed



For each box plot, the whiskers extend from the maximum to the minimum of the distribution, the boxes run from the first to the third quartile and the lines connect the medians. Each data point reflects results averaged over 50 simulations conducted on 20 networks

Figure 12: Distribution of Bits Received by r , “Bit for Bit, Unpriced, Unlimited Bandwidth” Barter in 100-Node Degree=4 Graphs

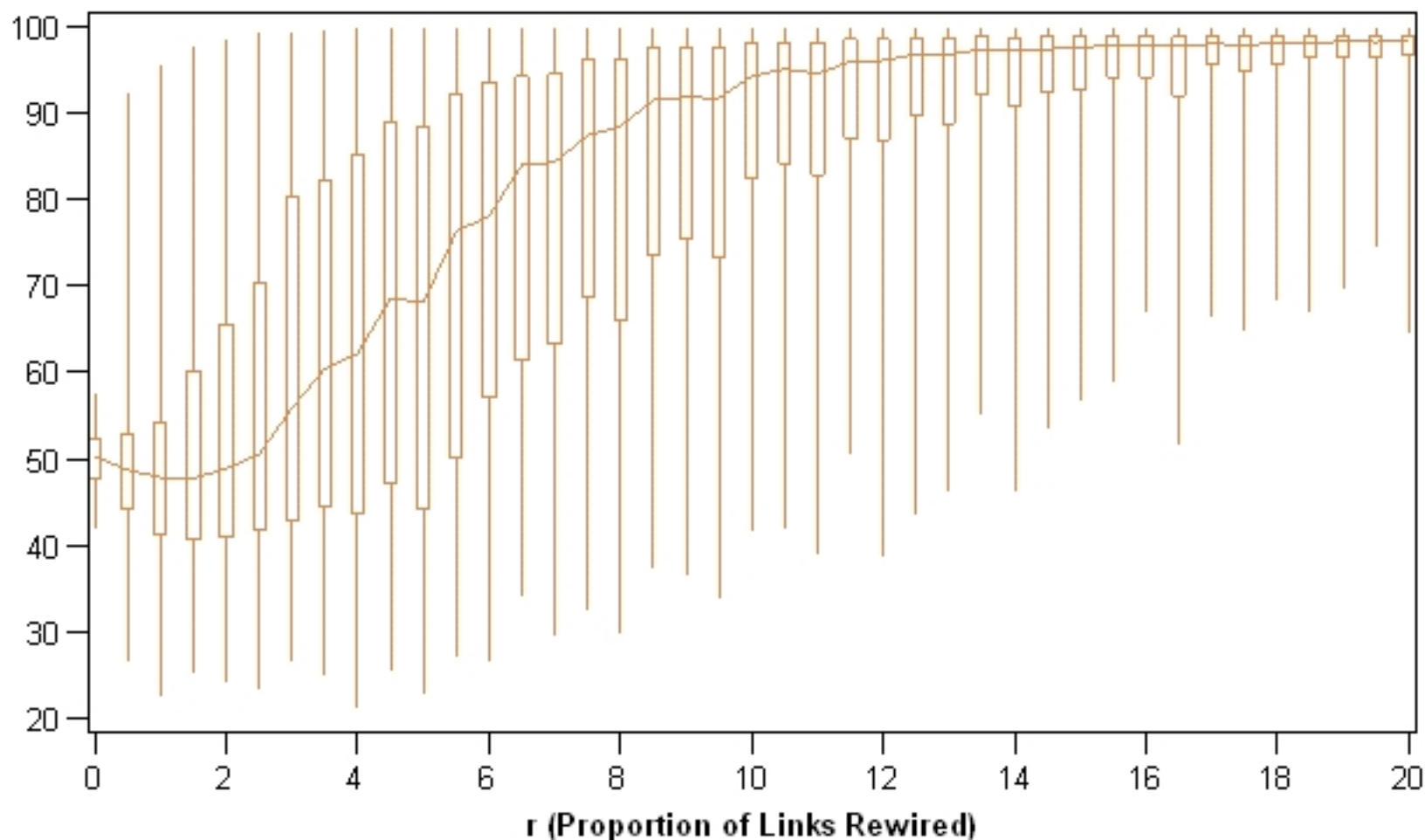
Proportion of Bits Obtained



For each box plot, the whiskers extend from the maximum to the minimum of the distribution, the boxes run from the first to the third quartile and the lines connect the medians.

Figure 13: Distribution of Bits Received by r , “Bit for Bit, Priced, Unlimited Bandwidth” Barter in 100-Node Degree=4 Graphs

Proportion of Bits Obtained



For each box plot, the whiskers extend from the maximum to the minimum of the distribution, the boxes run from the first to the third quartile and the lines connect the medians.

Figure 14a: “Disconnected Caveman” World

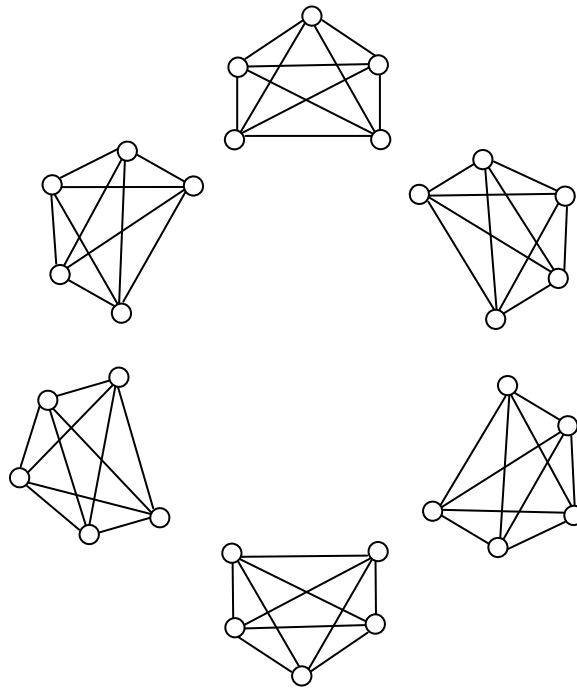


Figure 14b: “Connected Caveman” World

