Technical Appendix

for

"Precautionary Saving and Consumption Fluctuations"

A. GMM estimation of the parameters of the utility function

This appendix describes the estimation of the parameters of the utility function by GMM. The estimator uses grouped CEX data, as in Attanasio and Weber (1995), and the nonlinear theoretical moments implied by equation (2.1), as pioneered using aggregate data by Hansen and Singleton (1982).

A.1. The moments

Given J instruments, the Euler equation implies J moment conditions for each household:

$$E\left[\mathbf{h}\left(\boldsymbol{\delta}, \sigma, \mathbf{X}_{i,t+1}, \frac{C_{i,t+1}}{C_{i,t}}, \mathbf{Z}_{i,t}\right)\right] = 0 \tag{A.1}$$

where $E[\;]$ is the mathematical (unconditional) expectations operator and where

$$\mathbf{h}\left(\boldsymbol{\delta}, \sigma, \mathbf{X}_{i,t+1}, \frac{C_{i,t+1}}{C_{i,t}}, \mathbf{Z}_{i,t}\right) \equiv \left[\exp\left(\mathbf{X}_{i,t+1}\boldsymbol{\delta}\right)\left(\frac{C_{i,t+1}}{C_{i,t}}\right)^{-\frac{1}{\sigma}} R^{f}_{i,t+1} - 1\right]\mathbf{Z}_{i,t}. \tag{A.2}$$

Given ideal data, we would use the estimator

$$(\hat{\boldsymbol{\delta}}, \hat{\sigma}) \equiv \arg\min_{\{\boldsymbol{\delta},\sigma\}} \mathbf{g}(\boldsymbol{\delta}, \sigma; \mathbf{Y})'\boldsymbol{\Omega}\mathbf{g}(\boldsymbol{\delta}, \sigma; \mathbf{Y}) \tag{A.3}$$

where Y denotes the stacked matrix of all data, Ω is a JI by JI weighting matrix and $g(\delta, \sigma; Y)$ is the JI by 1 column vector of the empirical counterparts to each theoretical moment in equation (A.1). $g(\delta, \sigma; Y)$ consists of I stacked $J \times 1$ column vectors of

$$\mathbf{g}_i\left(\boldsymbol{\delta}, \sigma, \mathbf{Y}\right) = \frac{1}{T_i}\sum_{t=1}^{T_i}\mathbf{h}\left(\boldsymbol{\delta}, \sigma, \mathbf{X}_{i,t+1}, \frac{C_{i,t+1}}{C_{i,t}}, \mathbf{Z}_{i,t}\right) \tag{A.4}$$

where T_i is the number of observations available for household i.

We do not directly use equations (A.3) and (A.4) for three reasons.

First, the consistency of this estimator requires that the empirical moments converge to the theoretical moments, which, as is clear from (A.4), happens only as the time dimension on each household becomes large. In the CEX, T_i is fixed at a maximum value of three. Since T is so small, the properties of the estimator proposed in (A.4) are not reliably approximated by the asymptotic distribution. The CEX data does however contain short sequences of overlapping consumption growth of different households covering a long time period. We therefore use a weaker restriction and estimate our model from the restriction that average of the expectation errors across all households average to zero over time,[21] formally:

$$E\left[\frac{1}{I(t)}\sum_{i\in I_t}\mathbf{h}\left(\boldsymbol{\delta},\sigma,\mathbf{X}_{i,t+1},\frac{C_{i,t+1}}{C_{i,t}},\mathbf{Z}_{i,t}\right)\right]=0. \tag{A.5}$$

Condition (A.5) is implied by (A.1) but does not imply (A.1).

Second, household-level characteristics and particularly consumption growth are measured with error in the CEX. When consumption growth is measured with substantial error, and when the model is overidentified so that the moments are not all set to zero, the intertemporal elasticity of substitution, σ, is biased upwards by measurement error in consumption growth. This follows because measurement error increases the variance of the constructed innovation to marginal utility. Since, in finite samples, there is some covariance between the instruments and this residual, the estimation procedure raises σ to lower the variance of the innovation and thus the covariance with the moments. We address this issue and the third issue by partially aggregating the data.

Before proceeding however, it is worth noting that the estimate is consistent. Under the assumptions on measurement error used in the main body of the paper, substituting

[21] A better assumption would be that the expectation errors of *similar* households average to zero, but non-linear estimation does not converge with more moments.

the observed consumption series into our theoretical moment condition yields

$$\begin{aligned}
& E\left[\left(e^{\mathbf{X}_{i,t+1}\boldsymbol{\delta}}\left(\frac{C^*_{n,t+1}}{C^*_{n,t}}\right)^{-\frac{1}{\sigma}}\left(\frac{\mu_{i,t+1}}{\mu_{i,t}}\right)^{\frac{-1}{\sigma}} R^f_{i,t+1}-1\right)\mathbf{Z}_{i,t}\right] \\
= \;& E\left[E\left[\left(\frac{\mu_{i,t+1}}{\mu_{i,t}}\right)^{\frac{-1}{\sigma}}|\mathbf{Z}_{i,t}\right] E\left[e^{\mathbf{X}_{i,t+1}\boldsymbol{\delta}}\left(\frac{C^*_{i,t+1}}{C^*_{i,t}}\right)^{-\frac{1}{\sigma}} R^f_{i,t+1}|\mathbf{Z}_{i,t}\right]\mathbf{Z}_{i,t}\right]-E\left[\mathbf{Z}_{i,t}\right] \\
= \;& E\left[e^{\varphi} E\left[e^{\mathbf{X}_{i,t+1}\boldsymbol{\delta}}\left(\frac{C^*_{i,t+1}}{C^*_{i,t}}\right)^{-\frac{1}{\sigma}} R^f_{i,t+1}|\mathbf{Z}_{i,t}\right]\mathbf{Z}_{i,t}\right]-E\left[\mathbf{Z}_{i,t}\right] \\
= \;& E\left[\left(e^{\varphi+\mathbf{X}_{i,t+1}\boldsymbol{\delta}}\left(\frac{C^*_{i,t+1}}{C^*_{i,t}}\right)^{-\frac{1}{\sigma}} R^f_{i,t+1}-1\right)\mathbf{Z}_{i,t}\right].
\end{aligned}$$

If the true parameters are $\delta^0=(\delta^0_1, \delta^0_2, \ldots, \delta^0_K)'$ and σ_0, then the probability limits of the parameters that set the moment to zero when the consumption data are mismeasured are $(\delta^0_1-\varphi, \delta^0_2, \ldots, \delta^0_K)'$ and σ_0. GMM does not estimate consistently δ_1 – the mean level of impatience across households – but it does estimate consistently the remaining structural parameters of interest.

Third, the theoretical moments are too many to feasibly use in estimation. There are J (the number of predetermined variables) times I (the number of households) moment conditions and the predetermined variables may include individual information, aggregate variables, and characteristics of the distribution of individual variables. In an economy in which markets are incomplete, the evolution of the aggregate economy generally depends upon the distribution of wealth and income in the economy. In the extreme case, it may be that the information set that each household uses to form expectations contains its own household-level characteristics and the household-level characteristics of all other households in the economy.

We address these issues by partially aggregating the data. We assume that the economy consists of N groups of households that are able to pool their resources within each group to insure their risks completely from all shocks, except those that affect the group average. This assumption is made because it renders the estimator feasible: groups are observed over long time periods and averaging reduces the impact of measurement error.[22] We construct the expectation error by averaging $X_{i,t+1}$ and $\ln \frac{C_{i,t+1}}{C_{i,t}}$ within each group (denoted by n), thus mitigating the impact of measurement error. As to the subset of instruments, we assume that $Z_{n,t}$ contains only information on group n's average characteristics and aggregate information.

Our final estimator, based on the theoretical moment given in equation (A.5) but with groups, n, instead of households, is (A.3) where $\hat{\Omega}^{-1}$ is $J \times J$ and $g(\delta, \sigma; Y)$ is the $J \times 1$ column vector

$$\mathbf{g}(\boldsymbol{\delta}, \sigma, \mathbf{Y}) = \frac{1}{T} \sum_{t=1}^{T} \left[\frac{1}{N(t)} \sum_{n \in N_t} \mathbf{h}\left(\boldsymbol{\delta}, \sigma, \mathbf{X}_{n,t+1}, C_{n,t+1}^{GR}, \mathbf{Z}_{n,t}\right) \right] \tag{A.6}$$

where $Z_{n,t} \equiv \frac{1}{I(n,t)} \sum_{i \in I_{n,t}} Z_{i,t}$, $X_{n,t+1} \equiv \frac{1}{I(n,t)} \sum_{i \in I_{n,t}} X_{i,t+1}$, and $C_{n,t+1}^{GR} \equiv \prod_{i \in I_{n,t}} \left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{\frac{1}{I(n,t)}}$ where $I_{n,t}$ defines the set of $I(n,t)$ households in group n observed in t and $t+1$. We refer to $\frac{1}{T} \sum_{t=1}^{T} N(t)$ as the number of groups used in the construction of an estimator.

A.2. The weighting matrix and inference

Ω is the optimal weighting matrix so that the asymptotic distribution of estimator defined by equations (A.3) and (A.6) is normal with variance-covariance matrix given by

$$\left(\mathbf{G}' \boldsymbol{\Omega} \mathbf{G}\right)^{-1}$$

where

$$\mathbf{G} = \left(\frac{d\mathbf{g}(\boldsymbol{\delta}, \sigma, \mathbf{Y})}{d\boldsymbol{\delta}}, \frac{d\mathbf{g}(\boldsymbol{\delta}, \sigma, \mathbf{Y})}{d\sigma} \right)$$

and

$$\boldsymbol{\Omega} = E\left[\mathbf{g}(\boldsymbol{\beta}, \sigma; \mathbf{Y}) \mathbf{g}(\boldsymbol{\beta}, \sigma; \mathbf{Y})'\right]. \tag{A.7}$$

We construct an empirical counterpart to Ω, denoted $\hat{\Omega}$ that allows for an arbitrary degree of correlation across households within any time period. By allowing for this arbitrary correlation, we make inference consistently when there is an unknown degree

[22] Even in the presence of true panel data with a large time dimension this assumption may be required. Attanasio and Low (2000) show that group averaging improves the empirical performance of GMM estimation of the linear consumption Euler equation even when the true economy has no insurance of individual income shocks.

of market incompleteness. Since it is possible that the decision period of the household is finer than the period observed in the data, we also allow for arbitrary covariance among all innovations that share any of the months from which consumption growth is calculated. This flexible form also accounts for the adjustment that is necessary to compensate for the temporal correlation for any single group induced by mismeasurement of consumption. Finally, we allow for arbitrary heteroskedasticity across all observations.

Consider a household for whom consumption growth is observed from the three months ending in $t-3$ to the three months ending in t. This datum is constructed from monthly observations from $t-5$ through to t. It follows that all household observations for which the last month of an observed growth in consumption falls anywhere between $t-5$ to $t+5$ could potentially be correlated with our original household due to a common effect from an aggregate shock or consumption insurance.

For notational simplicity, first define a scalar expectation error for each group as $\varepsilon_{n,t+1} \equiv e^{\mathbf{X}_{n,t+1}\tilde{\boldsymbol{\delta}}} \left(\frac{C_{n,t+t}}{C_{n,t}}\right)^{-\frac{1}{\tilde{\sigma}}} R^f_{i,t+1} - 1$ where $\tilde{\delta}$ and $\tilde{\sigma}$ are estimates from a first-stage estimation that sets $\hat{\Omega} = I$. Second, let N be the number of cohorts; T_j the number of periods for which cohort j is observed;T_{ij} the number of periods over which both cohorts i and j are observed; and $Z_{i,t}$ the instrument vector for individual i at time t.

Our estimate of $\hat{\Omega}$ is:

$$\hat{\Omega} = \mathbf{P}_0 + \alpha_1 \sum_{v=1}^{5} \mathbf{P}_v + \alpha_2 \sum_{v=0}^{5} \left(\overline{\mathbf{P}}_v + \overline{\mathbf{P}}'_v\right)$$

where:

$$\mathbf{P}_v = \frac{1}{N} \sum_{j=1}^{N} \frac{1}{T_j} \sum_{t=v+1}^{T_j} \left(\mathbf{Z}_{j,t}\mathbf{Z}'_{j,t-v} u_{j,t} u_{j,t-v} + \mathbf{Z}'_{j,t-v}\ \mathbf{Z}_{j,t} u_{j,t} u_{j,t-v}\right)$$

$$\overline{\mathbf{P}}_v = \frac{1}{N} \sum_{j>i}^{N} \frac{1}{T_j} \sum_{t=v+1}^{T_j} \left(\mathbf{Z}_{j,t}\mathbf{Z}'_{i,t-v} u_{j,t} u_{i,t-v} + \mathbf{Z}_{j,t-v}\mathbf{Z}'_{i,t} u_{j,t-v} u_{i,t}\right)$$

and α_1 and α_2 are weights to ensure the positive definiteness of the estimated variance-covariance matrix. Thus P_v captures the set of possible correlations within a given cohort (other than variance); and P_v the set of possible correlations between cohorts. Finally we note that if markets are complete and groups fully insure consumption risk across groups, then contemporaneous cross-correlations should be unity apart from measurement error. This weighting matrix insures that, if this is the case, the optimal estimate and its variance covariance matrix are asymptotically the same an estimate derived from the model in which all households are assumed part of the same insurance group and $N = 1$.

A.3. Instruments used in estimation

In selecting instruments, $Z_{n,t}$, we choose variables that the household might use to forecast future wealth or preferences and/or that are relevant for predicting income risk. All instruments are constructed so that they are in the household information set at the start of period t for group n, allowing for time aggregation. We assume that households know the group-average change in family size and number of children that will occur between t and $t+1$, so these variables are included in $Z_{n,t}$. The instrument set includes the indicator variables for month and interview that are included in preferences, as well as the cohort indicators when they are included in preferences. We include a set of average information about the household: two indicator variables for whether a male head is present and whether a female head is present; four indicators for the possible states of married/single and with and without children; three indicator variables for whether there is a working male head, female head, or other; age; age squared; an indicator for older than 45; two indicator variables for whether there is a male or female head who is retired; two indicator variables for whether there is a male or female head who is a government employee; six indicator variables for whether there is a male of female head with less education than a high school degree, a high school degree, and some college but not a degree. We include six aggregate variables. The instruments contain the real interest rate for $t-1$ and its lag, and 4 variables that are out-of-sample predictions constructed from rolling regressions using monthly data from period $t-1$: the log of the unemployment rate for $t+1$ and its first difference; the real and nominal interest rates in the last month of $t+1$ (deflated using the NIPA aggregate deflator).

A.4. Parameter Estimates

We group households into 5-year birth cohorts, as done in Attanasio and Weber (1995), but estimate a nonlinear Euler equation. At this level of grouping, there are 33 birth cohorts and $1,901$ observations. Parameter estimates are summarized in Table A1, and the first column of results is those we use in the paper. The intertemporal elasticity of substitution is estimated to be 0.65 and statistically significantly. The coefficients on change in family size, change in number of children and change in female hours worked, the set of cohort dummies, and the set of seasonal dummies, are all economically and statistically significant. The specification tests do not reject the model (the Hansen J-statistic has p-value 0.56). The estimated σ is quite similar if we instead define cohorts by one year birth groups or interact the five-year groups with four education categories.[23] The demographics remain significant, but are less stable. At still finer definitions of cohort, estimated σ rise or the estimator does not converge. The model has similar behavior is cohort effects are not included. If we estimate the model in linearized form (ignoring precautionary saving), estimates of σ typically decline to half their previous values. In the linear model, our estimated coefficients on the demographic variables and labor supply variable are a little smaller than those in Attanasio and Weber (1995), although both sets of estimates vary substantially by specification.

Despite the fact that overidentification tests do not reject the model, misspecification could potentially remain a concern as the estimates from this model are used to construct our measure of precautionary savings at an individual household level, rather than at the synthetic cohort level at which estimates of preferences were obtained. That the Euler equation holds for a synthetic cohort is necessary but not sufficient for it to be satisfied by an individual household.

[23] Columns 2 and 3 report results from GMM using an identity weighting matrix because the dimension of Ω and the that fact that is has small eigenvalues makes it difficult to invert and the parameter estimates are quite sensitive to small varations in specification or sample. Thus we use the more robust estimator.

Table A1: Parameter Estimates from Consumption Euler Equations for Cohort Groupings

	Group: 5-year birth cohorts 33 Cohorts	Group: 1-year birth cohorts 165 Cohorts	Group: 5-year birth cohorts and 4 education groups 132 Cohorts
	1	2	3
Number of Observations:	1901	9670	7599
Coefficients:			
Real Interest Rate	0.652	0.639	0.659
	(0.142)	(0.094)	(0.168)
Change in Family Size	1.355	-0.004	1.055
	(0.250)	(0.512)	(0.587)
Change in Number of Children	-0.405	0.024	-0.275
	(0.085)	(0.181)	(0.180)
Change in Female Hours Worked	-0.043	-0.028	-0.064
	(0.016)	(0.008)	(0.015)
Specification Test:	24.35	37.16	48.49
	0.56	0.07	0.001

Note: Column 1 presents estiamtes from efficient (two-step) GMM; columns 2 and 3 from one-step GMM with an identity weighting matrix. All models also include indicator variables for cohort, month and interview.