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The Optimal Exploration and Production of Nonrenewable Resources

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Most studies of nonrenewable resource production and pricing assume there is a fixed reserve base to be exploited over time, but in fact, with economic incentives reserves can be increased. Here we treat the reserve base as the basis for production and exploratory activity as the means of increasing or maintaining reserves. "Potential reserves" are unlimited, but as depletion ensues, given amounts of exploratory activity result in ever smaller discoveries. Given these constraints, resource producers must simultaneously determine their optimal rates of exploratory activity and production. We solve this problem for competitive and monopolistic markets and show that if the initial reserve endowment is small, the price profile will be U-shaped; at first production will increase as reserves are developed, and later production will decline as both exploratory activity and the discovery rate fall.

I. Introduction

The exploitation of an exhaustible resource from a fixed reserve base has by now received considerable attention. Hotelling (1931) first demonstrated that with constant marginal extraction costs, price minus marginal cost should rise at the rate of discount in a competitive market and rent (marginal revenue minus marginal cost) should rise at the rate of discount in a monopolistic market.[1] The monopoly price will initially be higher (and later will be lower) than the competitive price, but the extent to which the two prices will differ depends on the level of production

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[1] For other derivations and interpretations of Hotelling's results, see Herfindahl (1967) and Gordon (1967); for further discussion, see Solow (1974).

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cost and the particular way in which demand elasticities change as the resource is depleted.[2] If the extraction costs rise as the resource is depleted, both the monopolist and competitor will be more "conservationist," that is, they will set prices that are initially higher but that grow less rapidly relative to the case of constant extraction cost.[3]

More recent work has extended the basic Hotelling model in a number of directions.[4] There has been particular concern about the effects of uncertainty (over the resource reserve base, the appearance of substitutes for the resource, and changes in demand) on the rate of extraction. As one would expect, a resource should be extracted more slowly (by a monopolist or a competitor) when the reserve base is not known with certainty. The characteristics of extraction paths under reserve uncertainty have been examined by Gilbert (1976*a*), Heal (1978), and Loury (1976). Dasgupta and Stiglitz (1976), Heal (1978), and Hoel (1976) studied optimal extraction paths when a substitute for the resource may be introduced at some uncertain future time, under alternative market structures for both the resource and the substitute. Gilbert (1976*a*, 1976*b*) examined the use and value of exploratory activity to obtain a better estimate of the size of the fixed reserve base.

These studies all examined how producers should exploit a fixed reserve base over time, without considering where the reserve base came from in the first place. Producers are not "endowed" with reserves but instead must develop them through the process of exploration. Thus the early history of resource use—which for oil and gas occurred some 20–40 years ago, but for uranium and bauxite is still going on today—involves a period of reserve discovery and development and relatively little production. During this period production will gradually increase, rather than steadily decrease as in the Hotelling model and its variants. For resources like bauxite, where depletion is not likely to be an important factor for many years, or uranium, where reserve levels in the near future are of greater international policy concern than levels in the distant future, this early period is in fact more interesting than the later period of declining production.

Even in the later stages of resource use, there is really no "fixed" reserve base (in an economically meaningful sense) to be exhausted over time. Given the economic incentives, reserves can be maintained or increased through further exploration—even though the physical returns

[2] This is examined by Stiglitz (1976) and Sweeney (1975). Stiglitz shows that if extraction costs are zero and the demand elasticity is constant, the monopoly and competitive-price trajectories will be the same.

[3] The case of rising extraction costs has been examined by Heal (1976), Levhari and Leviatan (1976), and Solow and Wan (1976). Price trajectories for several empirical examples have been calculated by Pindyck (1978).

[4] For a general development and presentation of most of the recent results in the economics of exhaustible resources, see Dasgupta and Heal (1978); for a survey, see Peterson and Fisher (1976).

to exploration decrease as "depletion" ensues. It therefore makes more sense to think of resources like oil and uranium as being "nonrenewable," rather than "exhaustible."

In this paper we view exploratory activity as the means of accumulating or maintaining a level of reserves, and we treat depletion by assuming that reserve additions ("discoveries") resulting from exploratory activity fall as cumulative discoveries increase. The desired level of reserves depends in part on the behavior of production costs. If production costs were independent of reserves (and if there were no uncertainty about the discoveries resulting from exploratory activity), producers would postpone much of their exploratory activity (thereby discounting its cost) and maintain no reserves. In fact, production costs rise as reserves decline, although the exact relationship between the two may be complex.[5] Thus producers must simultaneously determine optimal levels of exploratory activity and production—resulting in an optimal reserve level—that balance revenues with exploration costs, production costs, and the "user cost" of depletion.

The design of an optimal exploration strategy to accumulate reserves has already been examined by Uhler (1975, 1978), who calculated an optimal rate of exploratory effort assuming a fixed price for the resource. The price (and rate of production) of the resource, however, will change over time, and the optimal production rate and exploration rate are interrelated. Here we examine exploration and production simultaneously and study the joint dynamics of the two. This will enable us to describe the entire price and reserve profile for a resource—from the early period of reserve development, increasing production and decreasing price, to the later period of rising price and the eventual winding down of both exploration and production.

II. Exploration and Production under Competition and Monopoly

We consider first competitive producers of a nonrenewable resource.[6] Producers take the price p as given and choose a rate of production q from

[5] For resources like oil and gas, at the level of individual pools and fields lower reserves mean higher extraction costs as the rate of physical output per unit of capital equipment declines and eventually as secondary and tertiary recovery techniques are needed. Even at the aggregate level, however, reserve depletion will be accompanied by higher average extraction costs since lower-cost deposits are usually produced first, and of those individual deposits with similar cost characteristics, reserves per deposit will on average be lower when aggregate reserves are lower. For many mineral resources extraction costs will similarly increase as higher-cost deposits are tapped and as deeper mines must be utilized for individual deposits.

[6] We are ignoring the problem of common access. In effect we are assuming here that there are a large number of identical firms that all ignore each other, or, equivalently but more realistically, that a state-owned company has sole exploration and production rights and sets a competitive price.

a proved reserve base R. The average cost of production $C_1(R)$ increases as the proved reserve base is depleted. Additions to the proved reserve base occur in response to the level of exploratory effort w.[7] The rate of flow of additions to proved reserves depends on both w and cumulative reserve additions x, that is, $\dot{x} = f(w, x)$ with $f_w > 0$ and $f_x < 0$. Thus, as exploration and discovery proceed over time, it becomes more and more difficult to make new discoveries. The cost of exploratory effort $C_2(w)$ increases with w. We assume that $C_2''(w) \geq 0$ and that the marginal discovery cost, $C_2'(w)/f_w$, increases as w increases.[8] We further assume that $C_1(R) \rightarrow \infty$ as $R \rightarrow 0$. The producer's problem, then, is as follows:

$$\max_{q,w} W = \int_0^\infty [qp - C_1(R)q - C_2(w)]e^{-\delta t}\, dt \tag{1}$$

subject to

$$\dot{R} = \dot{x} - q \tag{2}$$

$$\dot{x} = f(w, x) \tag{3}$$

and

$$R \geq 0, q \geq 0, w \geq 0, x \geq 0. \tag{4}$$

The solution of this optimization problem is straightforward. The Hamiltonian is

$$H = qpe^{-\delta t} - C_1(R)qe^{-\delta t} - C_2(w)e^{-\delta t} + \lambda_1[f(w, x) - q] + \lambda_2 f(w,x). \tag{5}$$

Note that H is a linear function of q but in general a nonlinear function of w. Differentiating H with respect to R and x gives the dynamic equations for λ_1 and λ_2:

$$\dot{\lambda}_1 = C_1'(R)qe^{-\delta t} \tag{6}$$

and

$$\dot{\lambda}_2 = -(\lambda_1 + \lambda_2)f_x. \tag{7}$$

From (5) we see that each producer should produce either nothing or at some maximum capacity level, depending on whether $pe^{-\delta t} - C_1(R)e^{-\delta t} - \lambda_1$ is negative or positive. Since this expression depends on the price p, market clearing will ensure that

$$pe^{-\delta t} - C_1(R)e^{-\delta t} - \lambda_1 = 0. \tag{8}$$

[7] w might represent the number of exploratory wells drilled, or it might be an index of drilling footage adjusted for depth.

[8] Note that $C_2'(w)$ and f_w are, respectively, the additional cost and the additional discoveries associated with one more unit of exploratory effort.

Note that λ_1 is the change in the present value of future profits resulting from an additional unit of reserves. λ_1 is always positive, but $\dot{\lambda}_1$ is negative, since $C_1'(R)$ is negative by assumption. We can see then that at some point production ceases (generally before proved reserves become zero), even though further exploration could yield more reserves.

Differentiating (8) with respect to time, substituting (2) for $\dot{R}$, and equating with (6) gives us the equation describing the dynamics of the price path:

$$\dot{p} = \delta p - \delta C_1(R) + C_1'(R)f(w, x). \tag{9}$$

Observe that price rises more slowly than in the case of production without exploration.[9] Note also that if $C_1'(R)$ is zero—that is, if production costs do not depend on reserves—the rate of change of the price path is unaffected by exploration and is identical with that in the standard constant-cost Hotelling problem. The level of the price path, however, will be affected by exploration; since "planned" reserves (i.e., the total amount of the resource available for production, including what will ultimately be discovered) are greater than initial reserves, our producer can set the initial price at a lower level. Price trajectories with and without exploration are shown for constant extraction costs in figure 1.

We can now determine the optimal rate of exploration by setting $\partial H/\partial w = 0$, and substituting in equation (8) for λ_1. This yields the following equation for λ_2:

$$\lambda_2 = \frac{C_2'(w)}{f_w}e^{-\delta t} - pe^{-\delta t} + C_1(R)e^{-\delta t}. \tag{10}$$

Using equations (8) and (10), we can rewrite equation (7) as:

$$\dot{\lambda}_2 = -\frac{f_x}{f_w}C_2'(w)e^{-\delta t}. \tag{11}$$

Differentiating equation (10) with respect to time and substituting (2), (3), and (9) for $\dot{R}$, $\dot{x}$, and $\dot{p}$ yields:

$$\begin{aligned}\dot{\lambda}_2 = {} & -C_2'(w)\frac{f_{wx}\cdot f}{(f_w)^2}e^{-\delta t} + \frac{f_w C_2''(w) - C_2'(w)f_{ww}}{(f_w)^2}\dot{w}e^{-\delta t} \\ & -\delta\frac{C_2'(w)}{f_w}e^{-\delta t} - C_1'(R)qe^{-\delta t}.\end{aligned} \tag{12}$$

[9] I showed in an earlier paper (1978) that if extraction costs rise as reserves fall, but there is no exploration, price follows the equation $\dot{p} = \delta p - \delta C_1(R)$. Note, however, that the introduction of exploration does not make our producer more conservationist. Given any initial reserve level R_0, total production will be larger if there was no exploration, so that price can begin at a lower level and rise more slowly over a longer period of time.

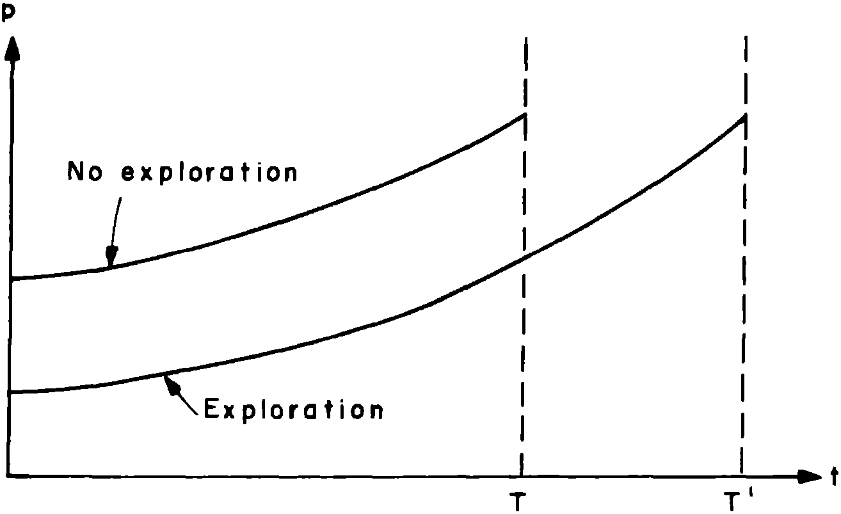

FIG. 1.—Price paths for constant extraction costs

Equating this with (11) and rearranging gives us an equation that describes the dynamics of exploratory effort:

$$\dot{w} = \frac{C_2'(w)[(f_{wx}/f_w) \cdot f - f_x + \delta] + C_1'(R)qf_w}{C_2''(w) - C_2'(w)\dfrac{f_{ww}}{f_w}}. \qquad (13)$$

The characteristics of the boundary conditions for equations (9) and (13) depend on whether or not $C_2'(0)/f_w(0)$ is zero. Suppose first that $C_2'(0)/f_w(0) = 0$.[10] At a terminal time T (when production ceases), further exploratory effort is of no value, so w must be zero. A second boundary condition is obtained from the transversality condition; since there is no terminal cost associated with cumulative discoveries x, $\lambda_2(T) = 0$. Then from equation (10) and the fact that $C_2'(0)/f_w(0) = 0$, we have that $p_T = C_1(R_T)$, that is, price rises and reserves fall (raising extraction costs) until the profit on the last bit of the resource is just zero.[11]

[10] This will be the case for most empirically supportable functional forms for $C_2(w)$ and $f(w, x)$, including our empirical example in the Appendix.

[11] Note that w and q must become zero at the same time, i.e., there cannot be an interval $T_1 \leq t \leq T$ for which $w = 0$ but $q > 0$. When $w = 0$, $\dot{\lambda}_2 = 0$, but since $\lambda_2(T) = 0$, $\lambda_2(T_1) = 0$. Then $p_{T_1} - C_1(R_{T_1}) = C_2'(0)/f_w(0) = 0$, so that no additional profit can be made from further extraction.

Finally, we see from equation (8) that λ_1, discounted rent, falls to zero at time T.[12]

Now suppose that $C_2'(0)/f_w(0) = \phi > 0$. In this case exploratory effort will become zero before production does. Let $T_1 < T$ be the time at which exploratory effort becomes zero. From the transversality condition, $\lambda_2(T) = 0$ as before. Also, as long as $w = 0$, $\dot{\lambda}_2 = 0$, so that $\lambda_2(T_1) = 0$. Then, from (10) and (8), for $t \geq T_1$, $p - C_1(R) = \lambda_1 e^{\delta t} = \phi$, $\dot{\lambda}_1 = -\delta\lambda_1$, and, using (6), $C_1'(R)q = -\delta\phi$. This describes the behavior of w, q, and p and says that w becomes zero at a time T_1 just as (1) $p - C_1(R) \rightarrow \phi$ and (2) $-C_1'(R)q/\delta \rightarrow \phi$. Then, for $t \geq T_1$ both $p - C_1(R)$ and $C_1'(R)q$ remain constant, so that p, C_1, and C_1' rise as q falls.[13] Finally, note that conditions (1) and (2) can be interpreted by recognizing that new reserves can have value by being extracted and sold, or by being stored, thereby reducing extraction costs. Thus the last additional unit of reserves should be discovered when its marginal discovery cost (ϕ) equals (1) the net revenue that would be obtained by extracting and selling the unit and (2) the storage value of the unit, that is, the PDV of all resulting future extraction cost savings.

Given particular functional forms for f, C_1, and C_2, and a demand function relating p and q, equations (9) and (13) can be solved together with the boundary conditions described above to yield optimal paths for price (and hence production) and exploratory effort. The particular pattern of exploratory effort, price, and production that will result depends critically on the initial value of reserves. The intertemporal trade off in exploration involves balancing the gain from postponing exploration (so that its cost can be discounted) with the loss from higher current production costs resulting from a lower reserve base. If initial reserves are large so that $C_1(R)$ is small, most exploration can be postponed to the future, whereas if initial reserves are small, exploration must occur early on so as to increase the inventory of proved reserves. In this latter case production will increase initially (as price falls), and later reserves and production will fall as exploratory effort diminishes. We will examine the behavior of price and exploratory effort in more detail in Section III of this paper.

Let us now turn to the case of a monopolistic producer. The monopolist also chooses q and w to maximize the sum of discounted profits in equation

[12] This is analogous to the recent result of Heal (1976)—that if a higher-cost backstop technology exists for a resource, the rent component of price for the lower-cost resource supply will decline toward zero as the low-cost stock is exhausted. Note that it differs quite sharply from the constant discounted rent in Hotelling's original constant extraction cost model.

[13] If demand for the resource becomes zero at some maximum price $\bar{p}$ and $C_1'(R) \rightarrow \infty$ as $R \rightarrow 0$, then p asymptotically approaches $\bar{p}$ as q and R asymptotically approach zero.

(1), but faces a demand function $p(q)$, with $p'(q) < 0$. Equations (6) and (7) still apply, but maximizing H with respect to q yields

$$\lambda_1 = MR_t e^{-\delta t} - C_1(R)e^{-\delta t}, \tag{14}$$

with $MR = p + q(dp/dq)$. Differentiating (14) with respect to time and equating with (6) gives us the equation describing the dynamics of marginal revenue:

$$\dot{MR} = \delta MR - \delta C_1(R) + C_1'(R)f(w, x). \tag{15}$$

Again note that if extraction costs do not depend on the reserve level, marginal revenue follows the same differential equation as in the standard Hotelling problem, that is, marginal revenue net of extraction cost rises at the rate of discount. Given any initial reserve level, however, exploration permits the initial price (and marginal revenue) to be lower since the total quantity that can be extracted will be greater.

Maximizing H with respect to w and substituting (14) for λ_1 gives us an expression for λ_2:

$$\lambda_2 = \frac{C_2'(w)}{f_w} e^{-\delta t} - MR_t e^{-\delta t} + C_1(R)e^{-\delta t}. \tag{16}$$

Differentiating this with respect to time, and equating with (7), yields the differential equation for w:

$$\dot{w} = \frac{C_2'(w)[(f_{wx}/f_w) \cdot f - f_x + \delta] + C_1'(R)qf_w}{C_2''(w) - C_2'(w)\dfrac{f_{ww}}{f_w}}. \tag{17}$$

This is identical to equation (13), but this does not mean that the pattern of exploratory effort is the same in the monopoly and competitive cases. As long as q is initially lower for the monopolist, $\dot{w}$ will be larger, since $C_1'(R)$ is negative. Thus, whether initial proved reserves are small or large, we would expect the monopolist to initially undertake less exploratory activity than the competitor, but later undertake more.[14]

III. The Behavior of Optimal Exploration and Production

In the solution of the typical exhaustible resource problem for a competitive market, price rises slowly over time as reserves are depleted, so that

[14] Unless extraction costs are zero and the elasticity of demand is constant, in which case both price and exploratory activity will be the same for the monopolist and the competitor. Stiglitz demonstrated (1976), for the case of production without exploration, that these special conditions result in monopolistic and competitive price trajectories that are identical. When extraction costs are zero the differential equations for price (in the competitive case) and for marginal revenue (in the monopoly case) do not depend on reserves or exploratory activity, so that price (and quantity) trajectories are again identical. Since eqq. (13) and (17) are identical, the trajectories for exploratory effort will also be the same for the monopolist and the competitor.

demand is choked off just as the last unit is extracted (if extraction costs are constant) or just as the profit on the last unit extracted becomes zero (if extraction costs rise as reserves decline). In our model of a nonrenewable resource, the price profile will depend on the initial level of reserves and on the magnitude and behavior of extraction costs. If initial reserves are large enough so that extraction costs are low, price will slowly rise over time as in the Hotelling solution. On the other hand, if reserves are initially very small (as in the early history of resource use) and extraction costs indeed depend on reserves, price will begin high, fall as reserves increase (as a result of exploratory activity), and then rise slowly as reserves decline. Let us examine these alternative solutions in more detail.

Case 1: Price Steadily Increasing

We saw earlier (see fig. 1) that if extraction costs are constant, the form of the solution is the same as in the standard Hotelling problem. However, even if extraction costs depend on reserves, price can increase steadily if initial reserves are large enough.

With initial reserves large, $C_1(R)$ and $C_1'(R)$ will be small, so that $\dot{p}$ will be positive—in fact the rate of growth of p will be just slightly below the discount rate. In addition, $\dot{w}$ will be positive initially. To see this, observe that the denominator of the right-hand side of (13) is always positive, while the first term in the numerator is positive and the second term is very small.[15] Thus w will begin growing from some very low level (when reserves are large, new discoveries are not needed initially, so that the cost of exploration can be postponed and thus discounted). Since initially there are almost no discoveries, reserves will fall. Reserves will fall more and more slowly, however, as exploration increases. At some point after reserves have become small enough, $\dot{w}$ will become negative, as $C_1'(R)$ becomes large, and exploration will decline toward zero as most of the reserves are used up. Price will increase until demand is choked off just as profit on the last unit of the resource is zero and just as exploratory activity becomes zero. At this point the resource has not been "exhausted," but it no longer pays to explore for new reserves. This pattern of exploratory activity and reserves is shown by the solid lines in figure 2.

Suppose extraction costs are small relative to price and to the cost of exploration. Then there is no value in holding a large stock of reserves, and most exploratory activity will be postponed until near the end of the planning horizon. This is illustrated by the dotted lines in figure 2.

[15] By assumption, $d/dw[C_2'(w)/f_w] > 0$. Then, since $f_w > 0$, $(C_2'' - C_2' f_{ww}/f_w) > 0$. Since w is small initially, $(f_{wx}/f_x)f - f_x < \delta$, and since R is large, $C_1'(R)$ is small.

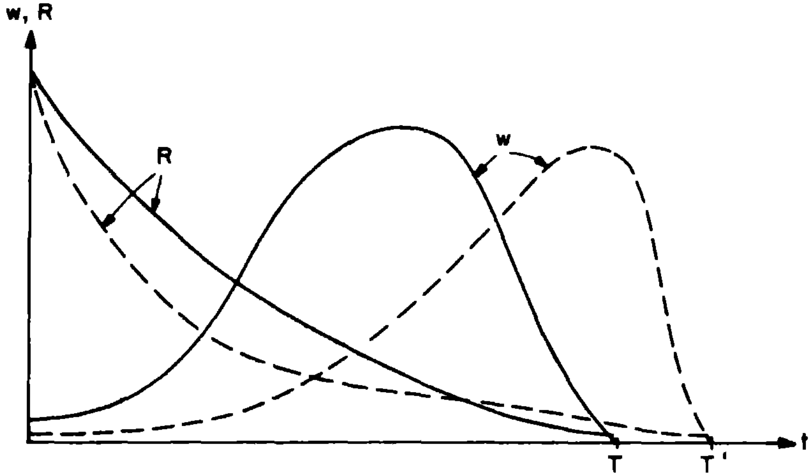

FIG. 2.—Exploratory activity and proved reserves—initial reserves large

Case 2: U-Shaped Price Path

This is a more interesting case and would usually apply if we wished to describe an entire history of resource use. If reserves are initially very small, price will begin declining from a high level, since $C_1(R)$ and $C_1'(R)$ are large in magnitude. Exploration will also begin declining from some high level, again because $C_1'(R)$ is a large negative number. Reserves will at first increase in response to exploration, but in the later stages of resource use they will decrease as exploration diminishes and the average product of exploration decreases. As reserves decrease price will increase, until demand, exploratory activity, and the profit on the last extracted unit of the resource all become zero simultaneously. This is illustrated in figure 3.

If extraction costs are small, exploration can decline more rapidly since there is no need to build up as much reserve. Later, as production increases, $\dot{w}$ can become positive; exploration then increases so that the stock of reserves does not fall to zero too quickly. Finally, as the returns from exploration diminish, $C_1'(R)$ will dominate the numerator of (13), $\dot{w}$ will become negative, and exploration will fall to zero. Price will again follow a U-shaped path (as long as extraction costs still depend on the level of reserves). This is illustrated by the dotted lines in figure 3.

These two cases can be summarized by the phase diagram in figure 4. From equations (2) and (3) we see that the $\dot{R} = 0$ isocline is nearly vertical for large values of R, but as R becomes small, q becomes small, so that this isocline bends in toward the origin. From equation (13) it is clear that the $\dot{w} = 0$ isocline will be downward sloping, since increased R and increased w both make $\dot{w}$ larger. Note that this isocline will shift

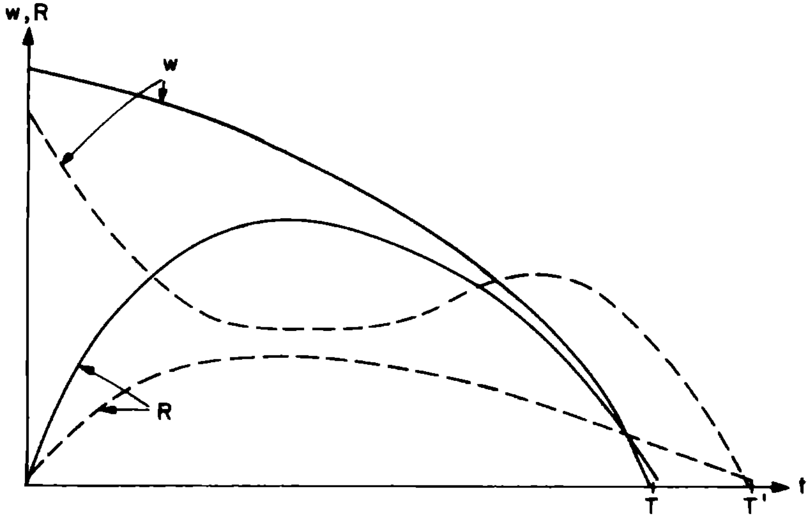

FIG. 3.—Exploratory activity and proved reserves—initial reserves small

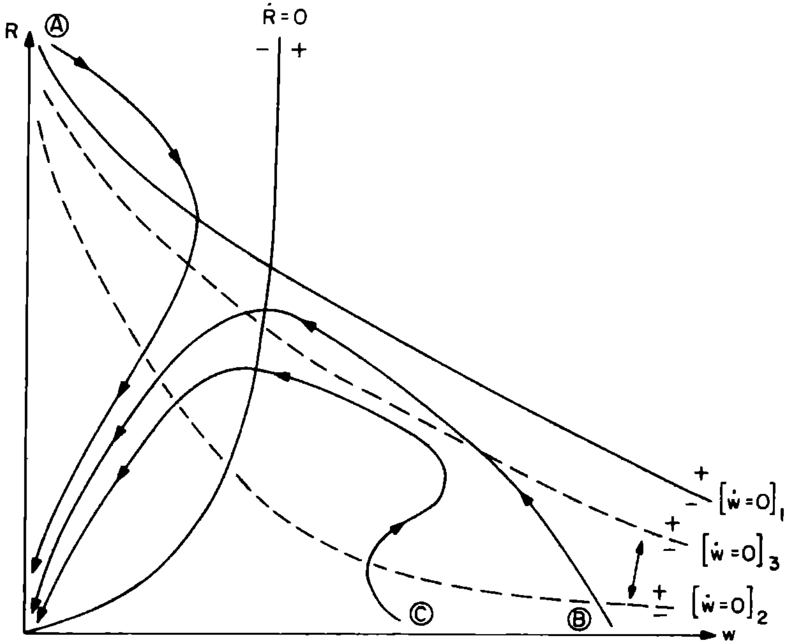

FIG. 4.—Phase diagram and optimal trajectories

to the left if q decreases or if cumulative discoveries x increases, and it is closer to the origin if extraction costs are relatively low. In the figure, $(\dot{w} = 0)_1$ corresponds to large extractions costs, while $(\dot{w} = 0)_2$ and $(\dot{w} = 0)_3$ correspond to relatively low extraction costs, with q small and/or x large for $(\dot{w} = 0)_2$, and the opposite for $(\dot{w} = 0)_3$.

If reserves are initially large, the optimal trajectory is given by curve A, where reserves always decrease, with exploration increasing and then decreasing. If reserves are initially small, the optimal trajectory depends on extraction costs. If extraction costs are large, exploration will be at a higher level and will continually decrease, as in B. If extraction costs are small, exploration can decrease, increase, and decrease again, as in C. Here the trajectory crosses the $\dot{w} = 0$ isocline so that $\dot{w}$ becomes positive, the isocline shifts to the right as q increases so that $\dot{w}$ becomes negative again, and reserves keep falling as the isocline moves back to the left as q decreases and x increases.

The characteristics of exploration and production are further illustrated by a numerical example presented in the Appendix. In that example a model is estimated and solved for crude oil exploration and production in the Permian region of Texas.

IV. The Case of No Depletion

For some nonrenewable resources (e.g., bauxite), depletion can effectively be ignored (at least for intermediate-term analysis). If the returns from exploration do not decline as cumulative discoveries increase, that is, if $f_x = 0$, production can go on indefinitely. In this case there will be an initial transient period during which reserves approach some long-run steady-state level $\bar{R}$ and after which steady-state exploration $\bar{w}$ results in discoveries just equal to steady-state production $\bar{q}$. This can be seen from the phase diagram in figure 5. Since $f_x = 0$, increases in cumulative discoveries will not result in a shift of the $\dot{w} = 0$ isocline. Trajectories A and B (large initial reserves and small initial reserves, respectively) lead to a long-run equilibrium of constant reserves and production. Any other trajectory leads to reserves and a level of exploration that grow large without limit, or else to a decline in reserves and cessation of production.

We can examine the characteristics of this steady state by setting f_x and $\dot{w}$ equal to 0 in equation (13). From this we obtain

$$\frac{C_2'(w)}{f_w} = -\frac{C_1'(\bar{R})\bar{q}}{\delta}. \tag{18}$$

The right-hand side of (18) is the present discounted value of the annual flow of extraction-cost savings resulting from one extra unit of reserves. If this quantity is less than the marginal discovery cost incurred in

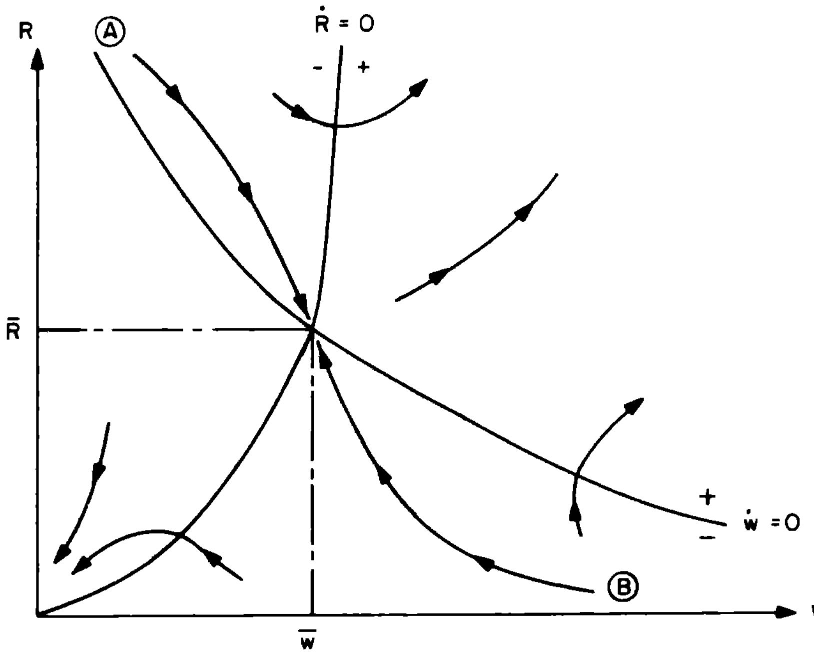

FIG. 5.—Phase diagram for case of no depletion

maintaining that extra unit of reserves (the left-hand side of [18]), profits would be greater with a level of exploration below the steady-state level, and indeed, we will have $\dot{w} > 0$, $w < \bar{w}$, and $R > \bar{R}$. Similarly, if this quantity is greater than the marginal discovery cost, we will have $\dot{w} < 0$, $w > \bar{w}$, and $R < \bar{R}$. In the first case the initial reserve level is larger than necessary, and in the second case it is too small.

We can also see that the optimum steady state $\bar{w}$, $\bar{R}$, and $\bar{q}$ are independent of initial reserves. Since $\dot{R} = 0$ in the steady state, $\bar{q} = f(\bar{w})$. Then under competition, p is taken as given and $\bar{w}$ is chosen to maximize profit:

$$\max_{\bar{w}} \Pi = pf(\bar{w}) - C_1(\bar{R})f(\bar{w}) - C_2(\bar{w}). \tag{19}$$

Setting $\partial\Pi/\partial\bar{w} = 0$ gives us a relationship between $\bar{w}$, $\bar{R}$, and $\bar{p}$:

$$\bar{w} = g(\bar{R}, \bar{p}). \tag{20}$$

Since $\dot{w} = 0$, we have from equation (13)

$$\delta C_2'(\bar{w}) + C_1'(\bar{R})f(\bar{w})f'(\bar{w}) = 0. \tag{21}$$

Finally, we have

$$f(\bar{w}) = \bar{q} \tag{22}$$

and

$$\bar{p} = p(\bar{q}). \tag{23}$$

Thus equations (20), (21), (22), and (23) provide a unique solution for $\bar{w}$, $\bar{R}$, $\bar{q}$, and $\bar{p}$ that is independent of the initial conditions. This can be thought of as a "Golden Rule" of reserve accumulation; whatever "endowed" initial reserves are, they will be increased (or, if they are very large, allowed to decline) until a profit-maximizing steady-state level is reached.

V. Measuring Resource Scarcity

In the United States, policymakers often use estimated "potential reserves" of oil, natural gas, and various minerals as a measure of resource scarcity. This, of course, implies viewing these resources as exhaustible, which as we have argued makes little economic sense. But even if such resources were exhaustible, the volume of potential reserves does not provide a useful measure of scarcity, since it does not reflect the difficulty of actually obtaining these reserves. As Fisher (1977) points out, an appropriate scarcity measure "should summarize the sacrifices required to obtain a unit of the resource." If by a resource we mean the raw material in the ground, "rent" (i.e., the difference between price and marginal extraction cost in a competitive market, and the difference between marginal revenue and marginal extraction cost in a monopolistic market) represents the opportunity cost of resource extraction and better reflects resource scarcity.

In this paper we have argued that most mineral resources can be best thought of as nonrenewable but inexhaustible, so that "potential reserves" has little meaning as a scarcity measure. On the other hand, "rent" provides a scarcity measure that is particularly appropriate and even applies to resources for which there is little or no depletion. To see this, rearrange equation (8) for price in the competitive case:

$$p = C_1(R) + \lambda_1 e^{\delta t}. \tag{24}$$

The second term on the right-hand side of this equation is undiscounted rent, and by setting $\partial H/\partial w$ equal to 0, we see that it has two components:

$$\lambda_1 e^{\delta t} = \frac{C_2'(w)}{f_w} - \lambda_2 e^{\delta t}. \tag{25}$$

The second term on the right-hand side of (25) is the shadow price of an additional unit of cumulative discoveries, and it measures the impact of this additional unit on future marginal discovery costs. We would usually expect λ_2 to be negative, since discoveries today result in an increase in the amount of exploratory effort that will be needed to obtain future discoveries.[16]

One might ask why both the marginal discovery cost and the opportunity cost of additional cumulative discoveries should be included in a measure of scarcity, rather than simply lumping discovery cost together with extraction cost and using only the last term in (25) to measure scarcity. Note from equation (11) that (assuming λ_2 is negative) λ_2 is positive, so that the discounted value of this opportunity cost becomes smaller in magnitude over time—as the *actual* value of marginal discovery cost grows. The reason is that once marginal discovery cost has become very large—and the resource is very scarce—resource use decreases as potential future profits become small, so that the opportunity cost of additional discoveries is small. For example, it might be that 30 years from now the marginal discovery cost of oil will exceed $100 per barrel, at which time oil will be extremely scarce, even though the opportunity cost of additional discoveries will be small. Thus the full rent of equation (25) should be used to measure scarcity.

It must be emphasized that rent is the appropriate scarcity measure only if we are referring to the resource in situ.[17] If we are referring to the resource as a factor of production (or consumption good), price is a more appropriate scarcity measure since extraction costs are indeed part of the sacrifices required to obtain the resource. In fact, as shown by Fisher (1977), Heal (1976), and in our model above, if extraction costs rise rapidly enough as depletion occurs, rent can fall over time. This simply implies, however, that the opportunity cost of resource extraction is falling because resource use is decreasing as extraction costs (and therefore price) rise, so that in an in situ context, the resource is indeed becoming less scarce.

VI. Concluding Remarks

We have argued that many "exhaustible" resources could be better thought of as inexhaustible but nonrenewable and that the optimal rates of exploration and production for these resources are interrelated and must be jointly determined. Exploratory activity is chosen to build the reserve

[16] As Fisher (1977) and Uhler (1975, 1976) point out, additional cumulative discoveries might initially result in a decrease in the amount of exploratory effort needed to obtain future discoveries by providing geological information. In this case, λ_2 would be positive initially and would later become negative as the effects of depletion offset the informational gains from cumulative discoveries. Uranium is a resource for which λ_2 might conceivably be positive today, but for most other resources of policy interest (and particularly oil and gas), λ_2 is negative.

[17] Rent is still an imperfect measure of scarcity, however, in that it ignores external costs such as the environmental damage resulting from resource exploration and production.

base up to a level that reduces extraction costs and then is adjusted over time so as to trade off cost savings from postponed exploration with savings from lower extraction costs and revenue gains from greater total production. The pattern of optimal exploratory activity thus depends highly on initial reserve levels and on rates of depletion.

Viewing a resource in this way has enabled us to describe the entire history of its use, from the early period of reserve accumulation to the eventual winding down of exploration and production. We saw that if the initial reserve endowment is small, the price profile will be U-shaped, rather than steadily increasing as in the Hotelling model and its variants. This helps explain the fact that the real prices of many nonrenewable resources have fallen over the years. For example, the secular decline of oil prices prior to the formation of OPEC, and the decline in the real price of bauxite prior to the cartelization of the world bauxite market can be attributed to the significant increases in the proved reserves of those resources that allowed production to steadily increase.

In the later stages of resource use (or throughout, if the initial reserve endowment is large) price will increase over time as in the Hotelling model. However, the introduction of exploratory activity has the effect of reducing the rate of increase of price (so that observed rates of growth of resource rents below market interest rates need not be indicative of monopoly power). Finally, we saw that in the development of a new resource for which depletion is not significant (but for which exploration and reserve accumulation are necessary), an optimal steady-state reserve level should be reached that is independent of any initial reserve endowment.

Obviously our approach ignored a number of important problems, including the effect of common access, market structures other than monopoly and perfect competition, the effects of government controls, and the effect of uncertainty. This last factor is perhaps the most important deficiency of this paper. Any representation of the response of discoveries to exploratory activity will be an uncertain one, both in terms of specification and estimated parameters, and the presence of uncertainty could significantly alter the "optimal" rates of exploration and production.

Appendix

A Numerical Example

In order to examine numerically the characteristics of the competitive and monopoly solutions, we have specified and estimated functional forms for $f(w, x)$, $C_1(R)$ and $C_2(w)$, using data for oil in the Permian region of Texas over the period 1965–74.

We assume that average production cost increases hyperbolically as the proved reserve base goes to zero, that is, $C_1(R) = m/R$. In 1966 extraction costs were \$1.25 per barrel and Permian reserves were 7,170 million barrels, so we set $m = 8{,}960$.

We represent the level of exploratory activity by the number of exploratory and development wells drilled each year. Over the years, the cost per well exhibited mild economies of scale. Measuring C_2 in millions of 1966 dollars we obtain (t-statistics in parentheses):

$$\frac{C_2(w)}{w} = \underset{(5.09)}{0.0670} + \underset{(2.43)}{103.2/w} \tag{A1}$$

$$R^2 = .458 \qquad \text{SE} = .0039 \qquad F(1, 7) = 5.90$$

Our discoveries function is of the form $f(w, x) = Aw^{\alpha}e^{-\beta x}$, $\alpha, \beta > 0$. We use a constructed series for reserve additions as the left-hand side variable, and obtain the following fitted equation:[18]

$$\log \text{DISC} = \underset{(0.77)}{2.389} + \underset{(1.53)}{0.599} \log w - \underset{(-5.86)}{.0002258x} \tag{A2}$$

$$R^2 = .837 \qquad \text{SE} = 0.172 \qquad F(2, 7) = 17.93$$

Here both DISC and x are measured in millions of barrels.

We complete the specification with a linear market demand function with price elasticity of -0.1 at a price of \$3.00 and production of 600 million barrels:[19]

$$q = 660 - 20p. \tag{A3}$$

To obtain numerical solutions for this example, we write difference equation approximations to our differential equations for w and p (or MR in the monopoly case), choose a discount rate of .05, and substitute in our estimated functions. Adding the accounting identities for x and R, we repeatedly simulate the resulting model, varying the initial conditions for p_0 and w_0 until the terminal condition that w, q, and average profit all become zero simultaneously is satisfied. (Note from eqq. [A1] and [A2] that $C_2'(0)/f_w(0) = 0$.)

Solutions for the competitive and monopoly cases are given in tables A1 and A2.[20] The competitive price is initially lower but later higher than the monopoly price. Since monopoly production is initially lower, fewer discoveries are needed to maintain the reserve base, so exploratory effort is smaller. Competitive exploration and production cease after about 55 years, but monopoly exploration and production continue for an additional 37 years since average production is smaller. At the points of termination, cumulative discoveries are about the same in both cases.

[18] Actual crude oil reserve additions consist of three components: new discoveries, extensions, and revisions. Although new discoveries and extensions have a strong dependence on well drilling and cumulative reserve additions, revisions behave like a random process with a mean value several times (6.0 in the Permian region) the mean value of discoveries plus extensions. We therefore obtain our constructed series by multiplying our data on discoveries plus extensions by the ratio of the mean value of reserve additions to the mean value of discoveries plus extensions.

[19] This reflects elasticity estimates for the 1960s, a period during which real oil prices were roughly constant at about \$3.00. Elasticity estimates for today's higher prices are in the range of -0.2 to -0.5, consistent with the equation. Eq. (A3) is also consistent with a backstop price of \$33, at which demand becomes zero as oil is replaced with alternative energy sources.

[20] It should be stressed that these are approximate solutions; the terminal conditions are not met exactly, since iterating over the initial conditions can become computationally costly. However, except for the last 2 or 3 years, these solutions are quite close to the true optimal.

TABLE A1

SOLUTIONS TO COMPETITIVE CASE

Year	Production (10^6 Barrels/ Year)	Price ($/Barrel)	Rent* ($/Barrel)	Wells Drilled	Reserves (10^6 Barrels)	Cumulative Discoveries (10^6 Barrels)	Profits (10^6 $/Year)
1965	552.0	5.400	4.150	9,353	7,170	0.0	1,556
1966	557.0	5.146	4.177	4,779	9,243	2,630	1,901
1967	554.9	5.254	4.284	4,120	9,648	3,590	2,019
1968	551.9	5.402	4.488	3,794	9,801	4,295	2,118
1969	548.5	5.573	4.661	3,612	9,822	4,865	2,209
1970	544.7	5.760	4.842	3,511	9,763	5,350	2,298
1975	522.1	6.891	5.883	3,554	8,892	7,138	2,729
1980	493.6	8.317	7.147	4,014	7,659	8,433	3,154
1985	458.8	10.05	7.990	4,681	4,361	9,500	3,550
1990	417.0	12.14	10.391	5,411	5,117	10,428	3,867
1995	368.1	14.59	12.350	5,978	3,994	11,247	4,039
2000	312.4	17.37	14.413	6,012	3,031	11,960	3,996
2005	251.0	20.44	16.452	5,062	2,243	12,552	3,684
2010	185.4	23.72	18.201	2,968	1,623	12,993	3,071
2015	113.6	27.31	19.584	669.8	1,159	13,244	2,075
2020	22.04	31.89	22.120	3.074	917.1	13,308	383.7
2021	.18	32.99	23.233	3.415	917.9	13,309	−99.03

* Marginal discovery cost and opportunity cost of additional cumulative discoveries.

TABLE A2

SOLUTIONS TO MONOPOLY CASE

Year	Production (10^6 Barrels/Year)	Price (\$/Barrel)	Rent* (\$/Barrel)	Wells Drilled	Reserves (10^6 Barrels)	Cumulative Discoveries (10^6 Barrels)	Profits (10^6 \$/Year)
1965	303.0	17.85	1.450	3,618	7,170	0.0	4,681
1966	304.8	17.75	1.427	2,293	8,353	1,488	4,828
1967	305.2	17.73	1.448	1,871	8,851	2,291	4,876
1968	305.1	17.74	1.499	1,641	9,137	2,883	4,901
1969	304.9	17.75	1.538	1,495	9,310	3,360	4,916
1970	304.5	17.77	1.588	1,396	9,409	3,764	4,925
1975	301.8	17.90	1.843	1,197	9,360	5,230	4,932
1980	297.9	18.10	2.194	1,200	8,901	6,269	4,909
1985	293.1	18.34	2.597	1,295	8,276	7,120	4,868
1990	287.1	18.64	3.097	1,460	7,576	7,868	4,811
2000	271.5	19.42	4.375	1,983	6,116	9,198	4,639
2010	249.5	20.52	6.142	2,746	4,720	10,403	4,359
2020	219.8	22.00	8.432	3,592	3,489	11,511	3,928
2030	181.5	23.92	11.226	3,994	2,479	12,495	3,313
2040	134.8	26.25	14.226	3,050	1,699	13,281	2,519
2050	77.60	29.11	17.241	801.6	1,123	13,751	1,481
2055	40.05	30.99	19.396	145.7	934.9	13,842	743.2
2058	12.33	32.38	21.706	64.55	891.2	13,864	167.6

* Marginal discovery cost and opportunity cost of additional cumulative discoveries.

Both cases could be characterized by curve C in the phase diagram of figure 4. The initial reserve base is too small, and therefore well drilling begins at a high level (quickly increasing reserves as the price drops and production briefly rises), falls (to a level sufficient to maintain these reserves for some years), slowly rises (as depletion ensues), and then, over the last 15 or 20 years, falls to zero (as production falls to zero, and proved reserve falls to the level at which extraction cost approaches the cut-off price of \$33).

If oil in Texas were nondepletable, exploratory activity, production, price, and reserves would approach steady-state levels determined from equations (20), (21), (22), and (23).[21] Solving these equations, we find that for the competitive case, $\bar{w} = 913$ wells per year, $q = 651.4$ million barrels per year, $\bar{p} = 43¢$ per barrel, and $\bar{R} = 54.1$ billion barrels, and for the monopoly case, $\bar{w} = 288$, $q = 326$, $\bar{p} = \$16.70$, and $\bar{R} = 43.0$. In the competitive case, well drilling would begin at a high level and then decline toward the steady-state value of 913 wells per year, as reserves rise to 54 billion barrels. The resulting discoveries would just be sufficient to replace the steady-state production. The steady-state price (43¢) is equal to the sum of the marginal extraction cost (17¢) and the marginal cost per barrel (for an additional barrel of steady-state production) of well drilling (26¢).

We compared the optimal values of well drilling and price to their historical values and to their optimal myopic values, that is, the values that would occur if future depletion were ignored but the reserve-production ratio were maintained at its initial level (12.0).[22] Optimal well drilling is initially much larger than actual well drilling (so that optimal reserves are larger than actual reserves) but is close to the actual in later years, while the optimal price is always at least \$2.00 above the actual price.

It might be that oil producers were myopic. The myopic price is just below the actual price, and the myopic pattern of well drilling more closely follows the actual data. Producers might have ignored the future gains from reduced production costs that would have resulted from higher initial well drilling and might have ignored the opportunity-cost component of rent in determining output. Alternatively, producers might have properly looked to the future but taken risk into account in making their calculations. One "certainty-equivalent" rule of thumb for treating risk is to increase the discount rate, and from equations (9) and (13) we can see that this would reduce the initial optimal price and also reduce initial optimal well drilling.

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[21] In the monopoly case, eq. (20) is obtained by substituting (23) into (19) before maximizing with respect to w.

[22] We take the competitive myopic price to be the sum of marginal production cost and average well-drilling cost. We use average (with respect to output) drilling cost rather than marginal cost because average costs decline with output in our model. Thus this competitive price corresponds to zero profits. Average drilling cost will depend on the amount of exploration needed to maintain the reserve-production ratio, and this will rise over time as depletion ensues. It is easy to show that if the reserve-production ratio is to be constant, the discoveries needed in each period are: $\Delta x_t = R_{t-1}(q_t/q_{t-1}) + q_{t-1} - R_{t-1}$, so that necessary well drilling is given by $w_t = A^{-1/\alpha}e^{(\beta/\alpha)x_t}[R_{t-1}(q_t/q_{t-1}) + q_{t-1} - R_{t-1}]^{1/\alpha}$.

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