

Internet Appendix for “A Unified Theory of Tobin’s q , Corporate Investment, Financing, and Risk Management”^{*}

Patrick Bolton[†]

Hui Chen[‡]

Neng Wang[§]

This appendix is organized as follows. Section A examines our model’s implication for the risk and return on equity, in particular, on how beta changes with the firm’s cash holdings. Section B studies the comparative statics of the stationary cash-capital ratio distributions. Section C calculates the endogenous average financing costs. Section D presents a model that endogenizes the credit line limit, and Section E illustrates the stationary distribution for the general case where the firm has access to a credit line. Finally, Section F derives the boundary conditions and outlines our numerical procedure.

A. Risk and return

In this section, we investigate how the firm’s investment, financing, and cash management policies affect the risk and return of the firm. Livdan, Saprizza, and Zhang (2009) also study the effect of financing constraints on stock returns. Their model does not allow for cash accumulation, which is

^{*}Citation format: Bolton, Patrick, Hui Chen, and Neng Wang, 2010, Internet Appendix to “A Unified Theory of Tobin’s q , Corporate Investment, Financing, and Risk Management,” Journal of Finance [vol x], [xxx-xxx], [http://www.afajof.org/IA/\[year\].asp](http://www.afajof.org/IA/[year].asp). Please note: Wiley-Blackwell is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

[†]Columbia University, NBER and CEPR. Email: pb2208@columbia.edu. Tel. 212-854-9245.

[‡]MIT Sloan School of Management and NBER. Email: huichen@mit.edu. Tel. 617-324-3896.

[§]Columbia Business School and NBER. Email: neng.wang@columbia.edu. Tel. 212-854-3869.

the focus of our analysis.

In order to highlight the impact of financing constraints on the firm's risk and returns, we adopt the benchmark asset pricing model (CAPM), which measures the riskiness of an asset with its market beta. We use r_m and σ_m to denote the expected return and volatility of the market portfolio.

Without financial frictions (the MM world), the firm implements the first-best investment policy. Its expected return is constant and is given by the classical CAPM formula:

$$\mu^{FB} = r + \beta^{FB} (r_m - r), \quad (\text{IA.1})$$

where

$$\beta^{FB} = \frac{\rho\sigma}{\sigma_m} \frac{1}{q^{FB}}, \quad (\text{IA.2})$$

and ρ is the correlation between the firm's productivity shock dA and returns of the market portfolio.

We can derive an analogous *conditional* CAPM expression for the instantaneous expected return $\mu^r(w)$ of a *financially constrained firm* by applying Ito's lemma (see Duffie (2001)):

$$\mu^r(w) = r + \beta(w) (r_m - r), \quad (\text{IA.3})$$

where

$$\beta(w) = \frac{\rho\sigma}{\sigma_m} \frac{p'(w)}{p(w)} \quad (\text{IA.4})$$

is the conditional beta of the financially constrained firm.

Our analysis highlights how idiosyncratic risk affects the beta of a financially constrained firm. Idiosyncratic risk, as systematic risk, causes earnings fluctuations and induces underinvestment for firm facing external financing costs. Thus, through its effect on $p(w)$ and $p'(w)$, idiosyncratic risk affects beta.

Equation (IA.4) implies that the beta for a financially constrained firm is monotonically decreasing with its cash-capital ratio w . The cash-capital ratio w has two effects on the conditional beta: first, an increase in w relaxes the firm's financing constraint and reduces underinvestment. As a result, the risk of holding the firm is lower. Second, the firm's asset risk is also reduced as a

result of the firm holding a greater share of its assets in cash (whose beta is zero). Both channels imply that the conditional beta $\beta(w)$ and the required rate of return $\mu^r(w)$ decreases with w .

Interestingly, when w is sufficiently high, the beta for a firm facing external financing costs can be even lower than the beta for the neoclassical firm (facing no financing costs). We may illustrate this point by rewriting the conditional beta as follows:

$$\beta(w) = \frac{\rho\sigma}{\sigma_m} \frac{p'(w)}{(p(w) - w) + w} = \frac{\rho\sigma}{\sigma_m} \frac{p'(w)}{q_a(w) + w}, \quad (\text{IA.5})$$

where $q_a(w) = p(w) - w$ is the firm's *average q* (the ratio of the firm's *enterprise value* and its capital stock). Although $q_a(w) < q^{FB}$ and $p'(w) > 1$, the second term, w , in the denominator of $\beta(w)$ can be so large that $\beta(w) < \beta^{FB}$. Intuitively, as a financially constrained firm hoard cash to reduce external financing costs, the firm beta becomes a weighted average of the asset beta and the beta of cash (zero). With a large enough buffer stock of cash holdings relative to its assets, this firm can be even safer than neoclassical firms facing no financing costs and holding no cash.

Panel A of Figure IA.1 plots the firm's value-capital ratio $p(w)$ for three different levels of idiosyncratic volatility (5%, 15%, 30%). The other parameter values for this calculation are $r_m - r_f = 6\%$, $\sigma_m = 20\%$, and the systematic volatility is fixed at $\rho\sigma = 7.2\%$ (assuming $\rho = 0.8$ and $\sigma = 9\%$). As expected, it shows that firm value is higher and the payout boundary \bar{w} is lower for lower levels of idiosyncratic volatility.

Insert Figure IA.1 About Here

Panel B plots the *marginal value of cash* $p'(w)$ for the same three levels of idiosyncratic volatility. It shows, as expected, that $p'(w)$ is decreasing in w for each level of idiosyncratic volatility. The figure also reveals that for high values of w , the marginal value of cash $p'(w)$ is higher for higher levels of idiosyncratic volatility. But, more surprisingly, for low values of w the marginal value of cash is actually decreasing in idiosyncratic volatility. The reason is that when the firm is close to financial distress, a dollar is more valuable for a firm with lower idiosyncratic volatility, which is more likely to avoid raising external funds.

Panel C plots the *investment-capital ratio* for the three different levels of idiosyncratic volatility. We see again that for sufficiently high w , investment is decreasing in idiosyncratic volatility, whereas

for low w , it is increasing. That is, when w is low, firms with low idiosyncratic volatility engage in more asset sales. Again, this latter result is driven by the fact that a marginal dollar has a higher value for a firm with lower idiosyncratic volatility. Therefore, such a firm will sell more assets to replenish its cash holdings.

Panel D plots *conditional betas* normalized by the first-best beta: $\beta(w)/\beta_{FB}$. For the same firm, $\beta(w)$ is decreasing in the cash-capital ratio w . At low levels of w , the firm's normalized beta $\beta(w)/\beta_{FB}$ can approach a value as high as 1.8 for idiosyncratic volatility of 5%. On the other hand, $\beta(w)$ is actually lower than β^{FB} for high w . For example, the conditional beta can be as low as 60% of the first-best beta in the case of 30% idiosyncratic volatility. As we have explained above, this is due to the fact that a financially constrained firm endogenously hoards significant amounts of cash, a perfectly safe asset, so that the mix of a constrained firm's assets may actually be safer than the asset mix of an unconstrained firm, which does not hoard any cash.

The rankings of beta across the firms with different idiosyncratic volatility depends on w . For large cash-capital ratio w , the beta is increasing in the idiosyncratic volatility. However, when the level of w is low, firms with low idiosyncratic volatility actually have higher beta. The rankings of beta are driven by the ratio $p'(w)/p(w)$, which can be inferred from the top two panels.

We close this section by briefly considering the implications of costly external financing for the *internal rate of return* (IRR) of an investor who seeks to purchase shares in an all-equity firm for a fixed buy-and-hold horizon T . We use the same parameter values as the beta calculation above, except that we fix the total earnings volatility to $\sigma = 9\%$ (the benchmark value). Starting with a given initial value w_0 and fixing a holding period T , we simulate sample paths of productivity shocks. On each path, we use the optimal decision rules to determine the dynamics of cash holdings, investment, financing, and payout to shareholders and to compute the value of the firm at time T . We then compute the IRR for the simulated cash flows from the investment. We report the IRR solutions in Figure IA.2 for investment horizon (holding period) T ranging from 0 to 30 years and for firms with initial w ranging from the 5% lowest to the 75% highest cash-holding firms in the population (note that a firm among the 5% lowest cash inventory holders would have a cash-capital ratio smaller than $w = 0.09$). For an investor with a very short investment horizon (say, less than a year), buying shares in an all-equity firm among the 5% lowest holders of cash may require a

return as high as 9.1%, compared to 6.9% for an otherwise identical firm among the 25% highest holders of cash.

Insert Figure IA.2 About Here

B. Comparative statics

In this section, we conduct comparative static analysis of firm cash holdings and investment for the following six parameters: $\mu, \theta, r, \sigma, \phi, \lambda$. We divide these parameters into two categories. The first three (μ, θ, r) are parameters on the physical side and have direct effects on investment (see i^{FB} in Equation (7)); the rest (σ, ϕ, λ) only affect investment and firm value through financing constraints. We examine the effects of these parameters through their impact on the distributions of cash holdings and investment in Figure IA.3 and IA.4.

In Figure IA.3, the left panels (A, C, and E) plot the cumulative stationary distributions (CDF) of the cash holdings w , and the right panels (B, D, and F) plot the cumulative distributions of firm investments i . As Panel A highlights, when mean productivity increases (from $\mu = 16\%$ to $\mu = 18\%$) firms tend to hold more cash. That is, the cumulative distributions of firms for higher values of μ first-order stochastically dominate the distributions for lower values of μ . This is intuitive, since the return on investment increases with μ so that the shadow value of cash increases. Still, one might expect firms to spend their cash more quickly for higher μ as the value of investment opportunities rises, so that the net effect on firm cash holdings is ambiguous a priori. In our baseline model, the net effect on w of a higher μ is positive, because investment adjustment costs induce firms to only gradually increase their investment outlays in response to an increase in μ .

Insert Figure IA.3 About Here

The effect of an increase in μ on investment is highlighted in Panel B. Firms respond to an increase in μ by increasing investment. For $\mu = 16\%$ firms are disinvesting as $i(w)$ is negative for all firms. For $\mu = 17\%$ nearly all firms are making positive investments, with most firms bunched at an investment level of roughly $i(w) = 3.5\%$. Finally, for $\mu = 18\%$ most firms are investing close to $i(w) = 11\%$.

The effects of an increase in investment adjustment cost θ and interest rate r on cash holdings and investment are also quite intuitive. As Panel D shows, an increase in θ has a negative effect on investment. If firms invest less, one should expect their cash holdings to increase almost mechanically. However, this turns out not to be the case. Firms have a lower shadow value of cash if they anticipate lower future investment outlays. Therefore they end up holding less cash, as is illustrated in Panel C. Similar comparative statics hold for increases in the risk-free rate r : with higher interest rates firms invest less and therefore hold less cash. This is indeed the case, as is illustrated in Panel E and F.

The effects of an increase in the idiosyncratic volatility of productivity shocks are shown in Panels A and B of Figure IA.4, where the stationary distribution is plotted for values of $\sigma = 7\%$, $\sigma = 9\%$ and $\sigma = 11\%$. We change σ by changing the idiosyncratic volatility while holding the systematic volatility fixed, so that the risk-adjusted mean productivity shock μ is unaffected. Again, it is intuitive that firms respond to greater underlying volatility of productivity shocks by holding more cash. Higher cash reserves, in turn, tend to raise the average cost of investment, so that one might expect a higher σ to induce firms to scale back investment. Similarly, an increase in external costs of financing ϕ ought to induce firms to increase their precautionary cash holdings and to scale back their capital expenditures. This is exactly what our model predicts, as shown in Panel C and D. The effect of an increase in the carry cost λ ought to be to induce firms to spend their cash more readily, by disbursing it more frequently to shareholders or investing more aggressively. Interestingly, although cash holdings decrease with λ , as seen in Panel E, the net effect on investment is negative, as Panel F shows. A higher λ makes it more expensive for firms to maintain its buffer-stock cash holdings and indirectly raises the cost of investment.

Insert Figure IA.4 About Here

Finally, one clear difference between Figure IA.3 and IA.4 is that, unlike the physical parameters, the parameters σ, ϕ, λ have rather limited effects on investment. This result implies that firms can effectively adjust their cash/payout/financing policies in response to changes in financing or cash management costs, limiting the impact on the real side (investment).

C. Financing costs

While the financing constraint depends crucially on the magnitude of the fixed cost parameter ϕ , it is not easy to gauge what is a reasonable range for ϕ . Empirical studies estimating external financing costs have measured the average cost of external financing, defined as the ratio of total financing costs and the size of the equity issue:

$$AC = \frac{\phi}{m} + \gamma.$$

Importantly, the size of the issue m is endogenous and should be increasing in ϕ , as the firm seeks to lower the average cost of external funds by increasing m when ϕ is higher. Moreover, one expects m to be concave in ϕ as the marginal value of cash $p'(w)$ is decreasing in w . Both features are confirmed numerically in Panel A of Figure IA.5. Panel B shows that the average financing cost still rises with ϕ , but at a lower rate than if m was fixed. For our benchmark parameters ($\phi = 1\%$, $\mu = 18\%$), the average financing cost is about 20 cents per dollar.

Insert Figure IA.5 About Here

The fixed cost parameter ϕ of equity issuance ought to be larger for smaller firms, and one would expect smaller firms to have higher average costs of external financing, other things equal. However, smaller firms are also likely to grow faster and be less exposed to systematic risk. This means a higher μ for smaller firms, which raises the optimal issue size (relative to capital) m as highlighted in Panel A. Therefore, the relation between average issuance costs and firm size is ambiguous. Panel B of Figure IA.5 demonstrates this observation.

This discussion highlights the importance of heterogeneity and endogeneity issues when measuring issuance costs. It helps explain why there may not be a clear relation between firm size and average costs of equity issues in the data. It sheds light, in particular, on the empirical debate over the nature of scale economies in equity issuance, and whether equity issuance costs are primarily fixed or variable (see Lee et al. (1996), Calomiris and Himmelberg (1997), and Calomiris, Himmelberg, and Wachtel (1995)).

D. Endogenous credit line limit

We can endogenize the amount of credit line that a firm obtains by introducing a commitment fee ν on any unused credit line commitment, in addition to the spread α it pays on the used credit line. To maintain the homogeneity, we assume that the credit line commitment is proportional to the firm's capital stock. For illustration, we consider the case where there is no fixed cost for financing ($\phi = 0$). There is no difficulty to add the fixed cost.

We first solve for the value function for an exogenously chosen credit line limit c . Then, in the cash region ($w > 0$), the value function $p(w; c)$ satisfies:

$$rp = (i(w) - \delta) (p - wp') + ((r - \lambda)w - \nu c + \mu - i(w) - g(i(w))) p' + \frac{\sigma^2}{2} p'', \quad (\text{IA.6})$$

where νc is the commitment fee the firm pays for having the credit line commitment c .

In the credit line region ($w < 0$), $p(w)$ satisfies the following ODE:

$$rp = (i(w) - \delta) (p - wp') + ((r + \alpha)w - \nu(c + w) + \mu - i(w) - g(i(w))) p' + \frac{\sigma^2}{2} p'', \quad (\text{IA.7})$$

where $\nu(c + w)$ is the commitment fee for unused credit line $c + w$. The boundary conditions are similar to those in Section V, except that in the absence of fixed financing cost, the condition $p'(-c) = 1 + \gamma$ replaces the old boundary condition for $p(-c)$, and there is no longer a target cash-capital ratio m .

Next, the optimal amount of credit line commitment c^* that the firm chooses will depend on its current cash holding. For example, if the current cash holding is high, the firm is expected to stay in the cash region for a long time before accessing credit line. Other things equal, the firm will then choose a lower credit line limit to reduce the commitment fees. This choice in turn affect the firm's investment, payout, and financing policies. In particular, it will lead to a higher payout boundary \bar{w} . There is, however, a natural upper bound for the payout boundary \bar{w}_{max} and correspondingly a lower bound for the credit line limit c_{min}^* . For initial cash-capital ratio $w_0 > \bar{w}_{max}$, the firm will immediately pay out lump sum $w_0 - \bar{w}_{max}$, and the optimal credit line limit will be the same as when $w_0 = \bar{w}_{max}$.

Specifically, for an initial cash-capital ratio w_0 , the optimal credit line limit c^* is the one that maximizes the firm value:

$$c^* = \operatorname{argmax}_c p(w_0; c).$$

As a numerical example, we set the commitment fee $\nu = 0.2\%$. The remaining parameters are from Table I. Figure IA.6 illustrates the solution for initial cash-capital ratio $w_0 = 0$. The optimal credit limit is $c^* = 0.1425$, while the payout boundary is $\bar{w} = 0.0821$. We can then define a cash-credit line (limit) ratio of $\bar{w}/c^* = 0.58$. As conjectured, the credit line limit falls as w_0 rises, but the change is very small (with $c_{min}^* = 0.1400$).

Insert Figure IA.6 About Here

E. Stationary distribution in the case of credit line

In this section, we analyze the stationary distribution of firms with access to credit line. To understand the different behavior in the cash and the credit regions, we report the first four moments of the distribution plus the medians of the variables of interests (w , $i(w)$, $p'(w)$, $q_a(w)$, and $q_m(w)$) for both the credit region and cash region in Table IA.I. The most significant observation is that the availability of credit makes the firm's stationary distribution for these variables much less skewed and fat-tailed in the cash region. Because liquidity is more abundant with a credit line, the firm's marginal value of cash is effectively unity throughout the cash region. However, the skewness and fat-tails of the distribution now appear in the credit region (note, for example, the high kurtosis (126) for marginal q in the credit region). Although the firm has a credit line of up to 20% of its capital stock, it only uses about 4% of its line on average. The reason is that the firm does not spend much time around the credit line limit. The risk of facing a large fixed cost of equity induces the firm to immediately move away from its credit limit.¹

Insert Table IA.I About Here

The cash-capital ratio w , $i(w)$, $q_a(w)$, and $q_m(w)$ are all skewed to the left in the cash region, as in our baseline model without a credit line. The intuition is similar to the one provided in

¹DeAngelo, DeAngelo and Whited (2011) make a similar observation.

the baseline model. Moreover, because of the firm's optimal buffer-stock cash holding, there is effectively no variation in the cash region for the firm's investment and value. Note also that the mean and median of marginal q and average q are all equal to 1.189, up to the third decimal point. Even for the investment-capital ratio $i(w)$, the difference between its median and mean values only appear at the third decimal point.

Unlike in the cash region, not only is the marginal value of credit $p'(w)$ skewed to the left, but so is marginal q in the credit region. The left skewness of marginal q and $p'(w)$ are both driven by the fact that every so often the firm hits the credit limit and incurs large financing costs. In other words, there is much more variation in the credit region than in the cash region for marginal q and the marginal value of liquidity $p'(w)$. As marginal q and the marginal value of liquidity move in the same direction in the credit region, there is, however, much less variation in $i(w)$, which is monotonically related to the ratio $q_m(w)/p'(w)$.

F. Derivations of boundary conditions and numerical procedure

We begin by showing that $P_W(K, W) \geq 1$. The intuition is as follows. The firm always can distribute cash to investors. Given $P(K, W)$, paying investors $\zeta > 0$ in cash changes firm value from $P(K, W)$ to $P(K, W - \zeta)$. Therefore, if the firm chooses not to distribute cash to investors, firm value $P(K, W)$ must satisfy

$$P(K, W) \geq P(K, W - \zeta) + \zeta,$$

where the inequality describes the implication of the optimality condition. With differentiability, we have $P_W(K, W) \geq 1$ in the accumulation region. In other words, the marginal benefit of retaining cash within the firm must be at least unity due to costly external financing. Let $\overline{W}(K)$ denote the threshold level for cash holding, where $\overline{W}(K)$ solves

$$P_W(K, \overline{W}(K)) = 1. \tag{IA.8}$$

The above argument implies the following payout policy:

$$dU_t = \max\{W_t - \overline{W}(K_t), 0\},$$

where $\overline{W}(K)$ is the endogenously determined payout boundary. Note that paying cash to investors reduces cash holding W and involves a linear cost. The following standard condition, known as *super contact* condition, characterizes the endogenous upper cash payout boundary (see e.g., Dumas (1991)):

$$P_{WW}(K, \overline{W}(K)) = 0. \quad (\text{IA.9})$$

When the firm's cash balance is sufficiently low ($W \leq \underline{W}$), under-investment becomes too costly. The firm may thus rationally increase its internal funds to the amount \underline{W} by raising total amount of external funds $(1 + \gamma)(\underline{W} - W)$. Optimality implies that

$$P(K, W) = P(K, \underline{W}) - (1 + \gamma)(\underline{W} - W), \quad W \leq \underline{W}. \quad (\text{IA.10})$$

Taking the limit by letting $W \rightarrow \underline{W}$ in (IA.10), we have

$$P_W(K, \underline{W}(K)) = 1 + \gamma. \quad (\text{IA.11})$$

Numerical procedure. We use the following procedure to solve the free boundary problem specified by ODE (13) and the boundary conditions associated with the different cases. First, we postulate the value of the free (upper) boundary \overline{w} , and solve the corresponding initial value problem using the Runge-Kutta method. For each value of \overline{w} we can compute the value of $p(w)$ over the interval $[0, \overline{w}]$. We can then search for the \overline{w} that will satisfy the boundary condition for p at $w = 0$. In the cases with additional free boundaries, including Case II and the model of hedging with margin requirements, we search for \overline{w} jointly with the other free boundaries by imposing additional conditions at the free boundaries.

References

- Calomiris, C. W., and C. P. Himmelberg, 1997, Investment banking costs as a measure of the cost of access to external finance, Working Paper.
- Calomiris, Charles W., Charles P. Himmelberg, and Paul Wachtel, 1995, Commercial paper, corporate finance, and the business cycle: a microeconomic perspective, *Carnegie-Rochester Conference Series on Public Policy* 42, 203–250.
- DeAngelo, Harry, Linda DeAngelo, and Toni Whited, 2011, Capital structure dynamics and transitory debt, *Journal of Financial Economics* 99, 235–261.
- Duffie, Darrell, 2001, *Dynamic Asset Pricing Theory* (Princeton University Press: Princeton) 3rd edn.
- Dumas, Bernard, 1991, Super contact and related optimality conditions, *Journal of Economic Dynamics and Control* 4, 675–685.
- Lee, Inmoo, Scott Lochhead, Jay R. Ritter, and Quanshui Zhao, 1996, The costs of raising capital, *Journal of Financial Research* 19, 59–74.
- Livdan, Dmitry, Horacio Sapriza, and Lu Zhang, 2009, Financially constrained stock returns, *Journal of Finance* 64, 1827–1862.

Table IA.I: CONDITIONAL MOMENTS FROM THE STATIONARY DISTRIBUTION OF THE CREDIT LINE MODEL

This table reports the population moments for cash-capital ratio (w), investment-capital ratio ($i(w)$), marginal value of cash ($p'(w)$), average q ($q_a(w)$), and marginal q ($q_m(w)$) from the stationary distribution in the case with credit line.

	cash capital ratio w	investment capital ratio $i(w)$	marginal value of cash $p'(w)$	average q $q_a(w)$	marginal q $q_m(w)$
A. credit region					
mean	-0.040	0.104	1.030	1.188	1.190
median	-0.030	0.108	1.023	1.188	1.189
std	0.034	0.015	0.023	0.001	0.002
skewness	-1.214	-3.071	4.186	-3.387	9.011
kurtosis	4.228	19.200	34.462	20.552	125.634
B. cash region					
mean	0.055	0.124	1.002	1.189	1.189
median	0.060	0.125	1.001	1.189	1.189
std	0.024	0.002	0.003	0.000	0.000
skewness	-0.569	-1.602	1.636	-2.146	-0.860
kurtosis	2.219	4.710	4.841	6.950	2.388

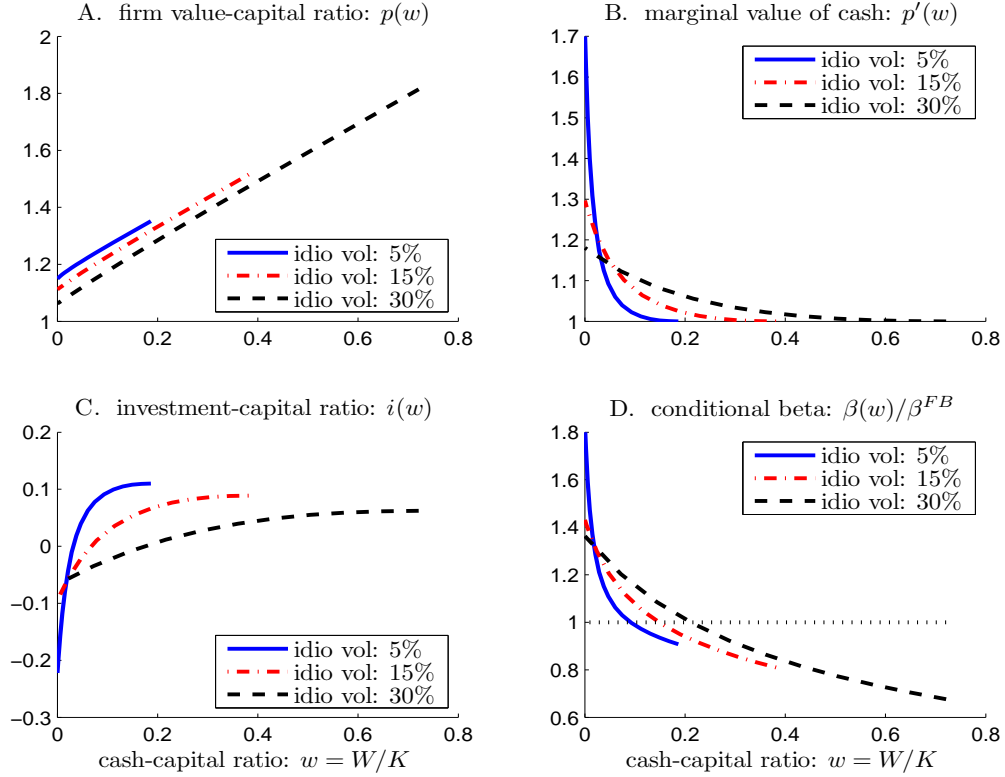


Figure IA.1: **Idiosyncratic volatility, firm value, investment, and beta.** In the refinancing case ($\phi = 1\%$), fixing all other parameters while using three different levels of idiosyncratic volatility (5%, 15%, 30%), this figure plots the firm value-capital ratio, marginal value of cash, investment-capital ratio, and the ratio of the conditional beta of a constrained firm to that of an unconstrained firm (first best). The right end of each line corresponds to the respective payout boundary.

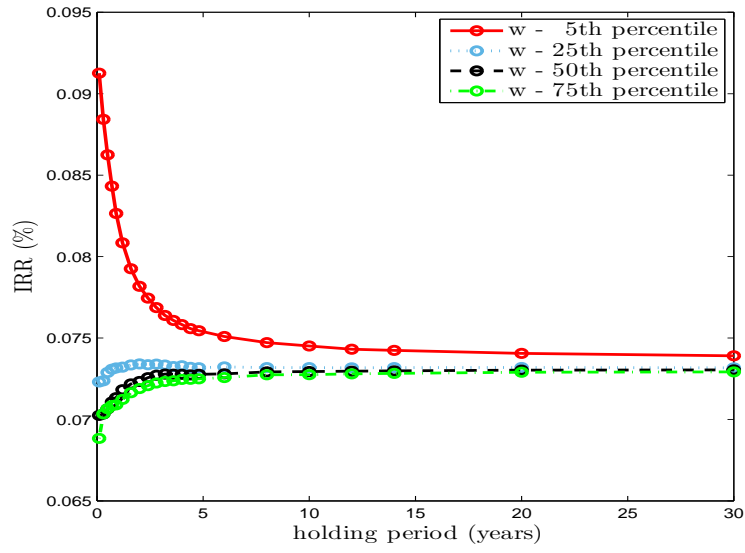


Figure IA.2: **Conditional IRR.** This figure plots the conditional internal rate of return from investing in the firm in Case II over different horizons at different levels of cash-capital ratios w . These values of w correspond to 5, 25, 50, 75th percentile of the stationary distribution.

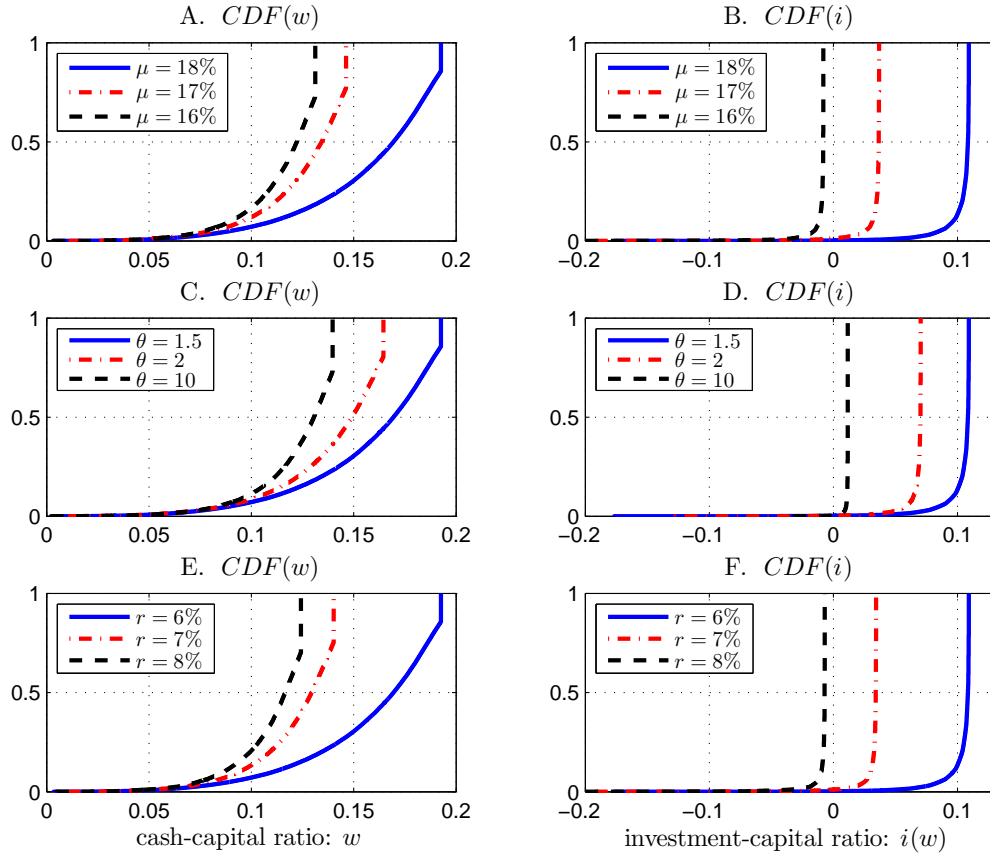


Figure IA.3: **Comparative statics I: μ , θ , and r .** This figure plots the cumulative distribution function for the stationary distribution of cash-capital ratio (w) and investment-capital ratio ($i(w)$) for different values of the mean of productivity shocks μ , investment adjustment cost θ , and interest rate r .

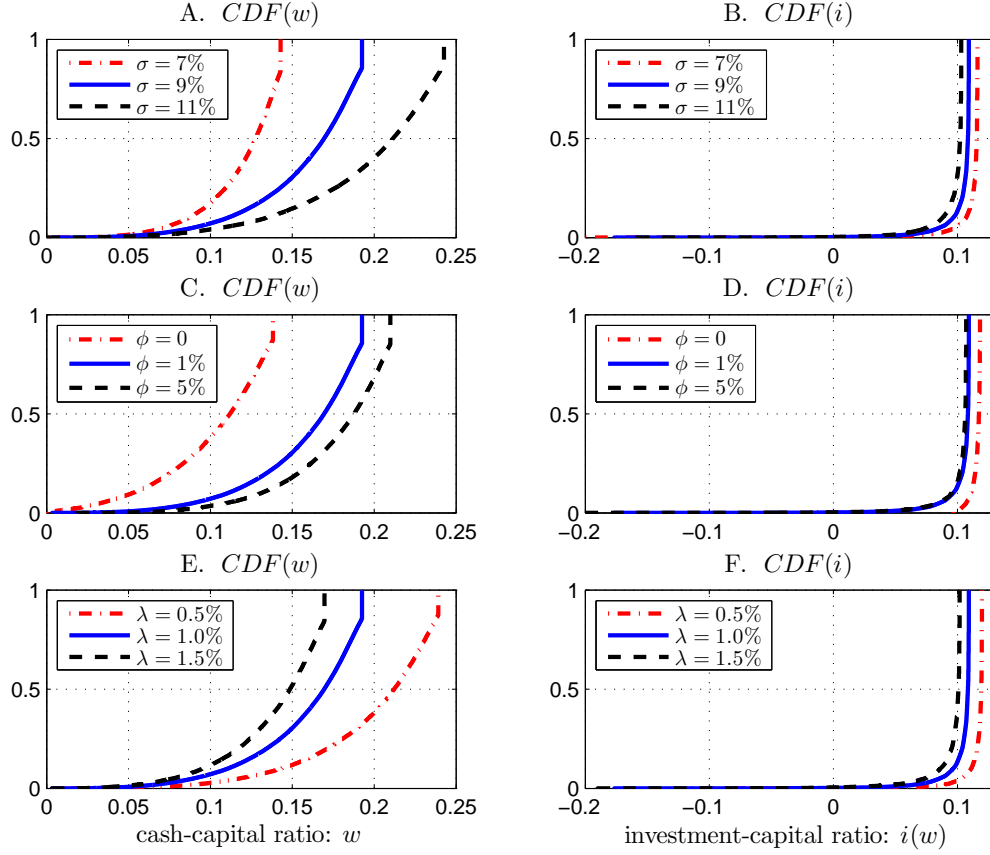


Figure IA.4: **Comparative statics II: σ , ϕ , and λ .** This figure plots the cumulative distribution function for the stationary distribution of cash-capital ratio (w) and investment-capital ratio ($i(w)$) for different values of the volatility of productivity shocks σ , fixed costs of external financing ϕ , and carry cost of cash λ .

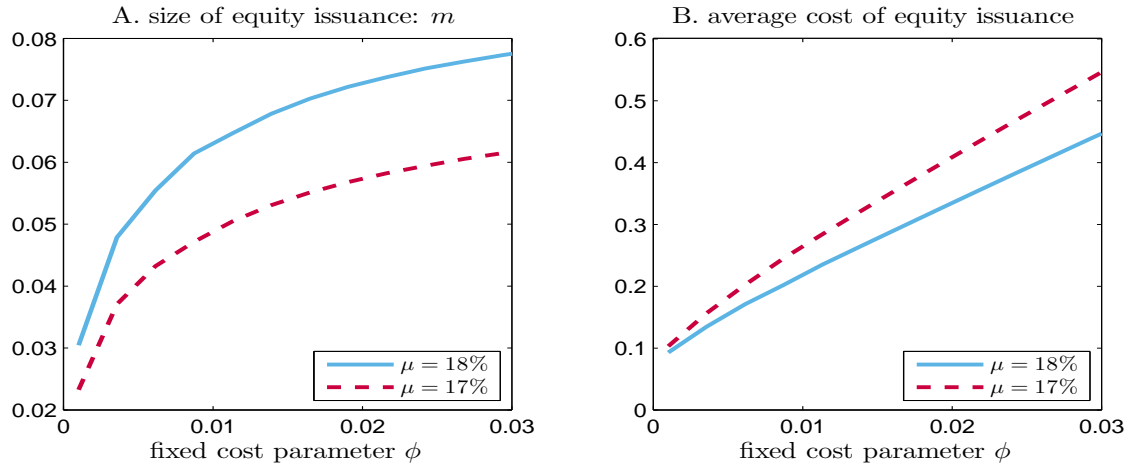


Figure IA.5: **Relative size and average cost of equity issuance.** This figure plots the size of equity issuance relative to capital (m) and the average cost of equity issuance (AC) for different levels of fixed cost of issuance and expected productivity. The remaining parameters are fixed at the benchmark values as reported in Table I.

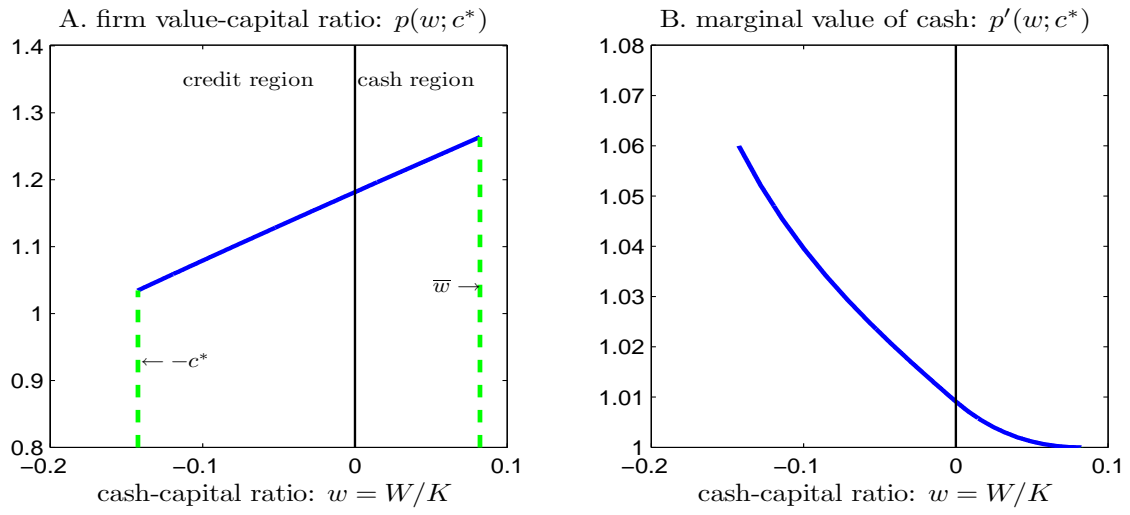


Figure IA.6: **Endogenous Credit Line.** This figure plots the solution for the case with endogenous credit line limit.