

Optimal Capital Structure with Imperfect Competition*

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Abstract

We develop a model of optimal capital structure in imperfectly competitive product markets. The key feature of our model is that it endogenizes both financing and investment decisions. We show that in equilibrium the industry leader uses debt conservatively, while the competitor uses debt more aggressively and therefore defaults first. The model generates leverage choices and survival rates that are consistent with empirical evidence. We use the model to study within-industry dispersion of leverage as a function of industry characteristics, and find strong support for the key predictions of the model in the data.

Keywords: Capital structure, Product market competition, Option games, Real options

JEL Classification Codes: G13, G34, L13

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1. Introduction

Product market competition plays an important role in determining firms' investment and financing policies. This insight dates back to [Brander and Lewis \(1986\)](#), who recognize that the limited liability effect of debt can change firms' optimal production policies in the presence of imperfect competition. Most of the developed literature since then treats either financing choices or product market competition as exogenous.

In this paper, we develop a framework that endogenizes both investment and financing decisions, and explicitly model the interaction between two competing firms in the product market. Firms' leverage, entry, and default decisions are determined in a unified framework and are driven by the tax benefits of debt, expected bankruptcy costs, and strategic considerations resulting from the actions of competitors. The focal point of this paper is to determine how a competitive environment shapes firms' optimal financing decisions.

We start our analysis by building an industry with one active incumbent and focus on the optimal investment and financing policies of a new entrant, taking the incumbent's leverage as exogenously given. This scenario often arises in practice. For example, at the time of its founding in 1886, the Coca-Cola Company may not have anticipated the emergence of PepsiCo twelve years later in 1898, and therefore may have not factored this into its financing strategy. We show that the optimal leverage of the new entrant is nonlinear in the incumbent's leverage. If the incumbent is highly levered, then the new entrant has an incentive to strategically undercut the incumbent in leverage. By doing so, the entrant is able to force the incumbent to default earlier. We show that in certain circumstances the entrant issues substantially less debt than would be optimal by merely trading off tax benefits against bankruptcy costs. As a result, the optimal leverage can be up to 25% lower. This result contributes to our understanding of why some firms choose to have lower levels of debt relative to what is prescribed by stand-alone trade-off models of capital structure.¹

We proceed by endogenizing the investment and financing strategies of both firms by constructing the full equilibrium of the game. This corresponds to a scenario in which there are

¹See [Goldstein, Ju and Leland \(2001\)](#), [Strebulaev \(2007\)](#), [Hennessy and Whited \(2005\)](#), [Strebulaev and Yang \(2013\)](#), among others, for discussion on why the traditional trade-off theory of capital structure generates higher leverage ratios than the ones observed in the data.

two firms ready to enter a new market. When making their decisions, each of them takes the actions of its competitor into account. In equilibrium, one firm designates itself as the leader and enters first, while the other firm becomes the follower. The leader, facing a threat of pre-emption, is forced to invest earlier and to issue a modest amount of debt. The follower uses debt more aggressively and, as a result, defaults first. In equilibrium, the follower has a higher leverage ratio compared to the incumbent. This result holds true in the data: we show that new entrants on average have 13.12% higher leverage ratios relative to incumbents. In addition, we show that entrants' survival rates are on average lower relative to survival rates of incumbents. In particular, 5-year survival rates of entering firms are 2.6% lower and 10-year survival rates are 6.2% lower.

We next study the relation between equilibrium cross-sectional dispersion of leverage within industries. First, we show that industries with higher cash flow volatility are characterized by higher cross-sectional dispersion of leverage. High volatility of cash flow makes it optimal for both entering and incumbent firm to keep leverage lower. This increases the value of the option to wait and widens the relative distance between the entry thresholds of the leader and the follower, leading to a greater leverage dispersion. Second, industries with higher tax rates have lower cross-sectional dispersion of leverage. We show that this is due to the concavity of both optimal coupon payments and optimal leverage with respect to tax rates.

An important and novel feature of our model is that it distinguishes two effects of bankruptcies on firms. The first effect is due to direct costs of bankruptcy, such as legal and administrative fees. These costs affect only the defaulting firm. The second effect comes from the impact of bankruptcy on competing firms. In our model, bankruptcy reduces production capacity of the defaulting firms, which affects optimal investment decision of the competing firms through the demand function. This effect is commonly observed in practice. For example, as documented in [Asquith, Gertner and Scharfstein \(1994\)](#), the majority of firms sell some fraction of their assets in bankruptcy. In addition, customer demand may shift from the bankrupt firm to competing firms. [Lang and Stulz \(1992\)](#) document that in concentrated industries bankruptcy announcements lead to positive abnormal stock returns of the bankrupt firm's competitors. In the literature these costs are usually described as indirect costs of bankruptcy. To distinguish between indirect costs that affect rival firms from the costs that have no such effect (e.g.,

managerial distraction), we use the term “indirect spillover costs.”

We show that direct and indirect spillover costs have different effect on leverage choices of rival firms, and therefore differently affect leverage dispersion in industries. Higher direct costs decrease within-industry leverage dispersion. We show that this is due to higher sensitivity of the follower’s leverage with respect to direct costs of bankruptcy. With higher direct costs, both firms optimally choose to have lower leverage but the difference in their leverage choices is lower, which lowers the dispersion. On the other hand, indirect spillover costs have a positive effect on leverage dispersion. We show that spillover costs do not affect the difference in optimal entry decisions of rival firms. Therefore, in industries with low indirect costs and in industries with high indirect costs the difference in the optimal leverage between two competing firms is the same. At the same time, higher costs still make it optimal for firms to choose lower leverage. The combination of these two forces increases leverage dispersion in industries with higher indirect spillover costs of bankruptcy.

Finally, we show that the above results hold in the data. Industries with higher cash flow volatility have higher leverage dispersion, while industries with higher tax rates have lower leverage dispersion. To test the relation between bankruptcy costs and leverage dispersion, we use expected costs computed by [Glover \(2014\)](#) from a structural model imposed on the data. These costs capture both direct and indirect costs of bankruptcy, and therefore do not allow us to test whether they differently affect leverage dispersion. Most of the variation in the estimated expected costs of bankruptcy, however, is due to indirect costs rather than direct costs. We therefore expect to find a positive correlation between costs and dispersion, which we confirm in our statistical tests.

A prerequisite for our work is the work of [Lambrecht \(2001\)](#), which studies the impact of capital structure on the investment and exit decisions of firms in a duopoly. He determines the order in which levered firms leave the industry and examines which regime, the incumbent defaults first or the follower defaults first, is likely to dominate in an alternating monopoly-duopoly market structure. Most importantly – and different from our paper – [Lambrecht \(2001\)](#) takes leverage as exogenously given and does not model any determinants of debt levels. In his model, debt is costly because of the liquidation costs incurred in the event of default and also

because it reduces the firm's ability to outlive competitors.²

Although a significant step forward, our model is limited because we do not allow firms to readjust their capital structure once they have entered the market. Fully dynamic investment and financing games pose extreme technical challenges as the action space becomes infinite and depends on the future path of the opponents' actions.³ Our analysis is nevertheless important for two reasons. First, before attempting to build fully dynamic investment and financing competition games, it is important to understand the features of equilibrium in the simpler framework adopted in this paper. Second, to the extent that firms' capital structure changes infrequently, our analysis helps us to better understand strategic considerations over shorter horizons.

Our paper is related to the literature that studies interaction of firms' financing and investment decisions in a competitive environment. It bridges two strands of literature by using the dynamic contingent claims framework of [Leland \(1994\)](#) to model investment and optimal capital structure while explicitly incorporating an imperfectly competitive setting. Introducing product market competition adds an additional layer to the traditional trade-off between tax benefits and bankruptcy costs. We show that this new strategic effect plays an important role in capital structure decisions and can lead to significant deviations from financing policies that would be otherwise optimal in a single firm environment. Moreover, we show that ex ante identical firms optimally choose different debt ratios resulting in significant within-industry dispersion of leverage. A firm may optimally resort to a low debt ratio and forego some tax benefits, if its debt level provides a strategic advantage over its competitor.

Related literature studies capital structure decisions in a perfectly competitive industry. [Fries, Miller and Perraudin \(1997\)](#) and [Zhdanov \(2007\)](#) focus on aggregate uncertainty and equilibrium financing strategies under perfect competition. [Miao \(2005\)](#) incorporates a capital structure trade-off in the [Dixit \(1989\)](#) model and examines the evolution of a competitive industry when firms experience idiosyncratic technology shocks. Other related literature studies optimal financing decisions by taking into account a firm's relation with its suppliers. [Hennessy](#)

²Other important work includes [Maksimovic \(1988\)](#), who investigates the effect of debt financing on the ability of firms to collude in the context of a model of repeated oligopoly, and [Showalter \(1995\)](#), who extends the analysis of [Brander and Lewis \(1986\)](#) to the case of cost uncertainty and Bertrand competition.

³[Back and Paulsen \(2009\)](#) discuss related mathematical aspects of equilibria in dynamic investment games.

and Livdan (2009) study the financing decision of a firm that relies on an implicit contract with a supplier, and shows that leverage increases with supplier bargaining power. Chu (2012) adapts the Leland (1994) model to study the relationship between optimal leverage and supplier market structure. He finds that more supplier competition decreases a firm’s leverage.

The need to better understand firms’ financing and investment decisions in a competitive environment is dictated by the fact that empirically, the link between capital structure and product market competition has been shown to be important. Kovenock and Phillips (1997) report a significant effect of leverage on investment and plant closing when the industry is highly concentrated. They also report that increased debt makes rivals more aggressive, which follows from our model. Chevalier (1995) shows that competitors react to leveraged buyouts in the supermarket industry. In an empirical study of four industries, Phillips (1995) reports that product prices and quantities are related to industry debt ratios. MacKay and Phillips (2005) find that the distribution of firms’ leverage ratios depends on the industry structure, and that new entrants follow more aggressive debt financing policies compared to incumbents. They also find that most variation of financial structure is within industries, with industry fixed effects accounting for only 13% of variation of financial structure. This finding is important because it indicates that within-industry factors are a more important determinant of firms’ capital structure. Our paper provides a theoretical foundation by analyzing within-industry leverage dispersion and studying the economic mechanisms that make firms optimally adopt different financing policies. Finally, Campello (2006) documents that high debt levels lead to product market underperformance.

The remainder of the paper is organized as follows. The model is set up in the next section. Major propositions and equilibrium are derived in Section 3. Section 4 analyzes optimal financing and investment decisions of competing firms. In Section 5 we test predictions of our model in the data. Section 6 concludes. Technical details and proofs are given in the Appendix.

2. Model Setup

Consider an industry with two potential entrants. Each of the two firms has the technology to produce one unit of output per time period. To start production, a fixed irreversible investment

cost I must be incurred. We make the following assumptions that specify the conditions under which the firms operate.

Assumption 1. The instantaneous after-tax payoff to the equityholders of an active firm is given by

$$\pi_i(x) = (1 - \tau)[q_i(\theta_i)p(Q, x) - s_i], \quad (1)$$

where $q_i(\theta_i)$ is the per-period quantity produced by firm i , θ_i is a binary variable that equals 0 if the firm has defaulted and 1 otherwise, τ is the corporate tax rate, and $p(Q, x)$ is the price in the output market. The firm has a perpetual bond outstanding on which it pays an instantaneous coupon of $s_i \geq 0$. As long as the tax rate τ is strictly positive, firms find it attractive to issue debt because of the tax-deductibility of interest payments.

The price $p(Q, x)$ depends on the non-shock component of the industry's inverse demand function $D(Q)$ and on the current state of the demand shock x . The function $D(Q)$ is assumed to be monotonically declining and continuously differentiable. The stochastic demand shock influences the price multiplicatively, i.e.

$$p(Q, x) = xD(Q),$$

and follows a geometric Brownian motion with drift μ and volatility σ :

$$\frac{dx_t}{x_t} = \mu dt + \sigma dW_t,$$

where μ and $\sigma > 0$ are constant parameters and $(W_s)_{s \geq 0}$ is a standard Brownian motion.

Assumption 2. A firm in default faces partial liquidation and loses a fraction α of its production capacity. Thus, if default occurs, the firm produces only $1 - \alpha$ units of output per period thereafter, i.e. $q_i(1) = 1$ and $q_i(0) = 1 - \alpha$.

This assumption is motivated by the fact that the majority of firms in financial distress sell a fraction of their assets as documented in [Asquith, Gertner and Scharfstein \(1994\)](#). In addition, they document that many firms sell a very substantial fraction: 18 out of 102 firms in their sample selling more than 20%. In addition, firms selling assets in distress usually sell them

at significantly lower prices relative to market prices.⁴ This modeling assumption is similar to [Strebulaev \(2007\)](#), who also models asset sales in distress. An alternative, and possibly more realistic assumption would be to let the production assets revert to the pre-bankruptcy level after a lag. In this case the output would temporarily decrease to $1 - \alpha$, but eventually revert back to 1. This assumption would substantially complicate the model, while leading to similar qualitative results. We therefore assume that the post-bankruptcy production output is fixed at $1 - \alpha$.

Assumption 2 is different from the assumption used by [Lambrecht \(2001\)](#), who assumes that a bankrupt firm leaves the industry immediately. This assumption is inconsistent with the empirical evidence, especially in the U.S., where most large corporations in default are successfully reorganized under Chapter 11 of the bankruptcy code. For example, [Wruck \(1990\)](#) documents that over 97% of firms in financial distress successfully reorganize and emerge.⁵ In addition, firms can also avoid costly liquidation through private renegotiations.

Assumption 2 implies that there are six possible realizations of the profit parameter $q_i(\theta_i)D(q)$. We describe these six alternative states below.

When a firm is the only one operating in the industry (its rival has not yet exercised its entry option), its profit multiplier is given by $\pi_h = D(1)$, which indicates that there is one active firm in the industry operating at full capacity. If this firm defaults while its competitor is still inactive, then its profit falls to $\pi_b = (1 - \alpha)D(1 - \alpha)$. As profits of a firm in financial distress go down, the following should hold:

$$\pi_h > \pi_b. \tag{2}$$

The remaining four realizations of the profit parameter are as follows. First, $\pi_{hh} = D(2)$ corresponds to the case when both firms are active and neither has gone bankrupt. Second, $\pi_{bb} = (1 - \alpha)D(2 - 2\alpha)$ is the duopoly profit parameter when both players have defaulted and shrunk their production capacities. Third, $\pi_{hb} = D(2 - \alpha)$ is the profit parameter of a solvent (“healthy”) firm competing against the rival in default. Fourth, $\pi_{bh} = (1 - \alpha)D(2 - \alpha)$ is the

⁴See [Pulvino \(1998\)](#) for empirical evidence on asset fire sales in the airline industry, and [Shleifer and Vishny \(1992\)](#) for an equilibrium model of asset liquidation that leads to fire sales.

⁵See [Bernstein, Colonnelli and Iverson \(2019\)](#) for the recent statistics on Chapter 7 and Chapter 11 filings.

profit parameter of a bankrupt firm competing against the healthy rival.

Because $D(q)$ is monotonically declining, the following inequalities must hold: $\pi_{bh} < \pi_{bb}$ and $\pi_{hh} < \pi_{hb}$.⁶ It is also straightforward to see that $\pi_{bh} < \pi_{hb}$. In addition, since profits of a firm that defaults and sells a fraction of its assets decline, $\pi_{bh} < \pi_{hh}$ and $\pi_{bb} < \pi_{hb}$ hold. If it were not the case, then the firm would have reduced its capacity earlier, and the necessity to sell assets would represent a benefit rather than a cost for the firm’s stakeholders.

The following proposition specifies a sufficient condition for the desired relations between various profit parameters to hold.

Lemma 1 *If the non-shock component of the inverse demand function, $D(q)$, is a monotonically declining continuously differentiable function, such that*

$$D(q) + \frac{dD(q)}{dq} < 0 \tag{3}$$

holds for $q \in [1 - \alpha, 2]$, then the following inequalities obtain:

$$\pi_{bh} < \pi_{hh}, \pi_{bb} < \pi_{hb}, \pi_b < \pi_h. \tag{4}$$

Inequality (3) in Lemma 1 requires that $D(q)$ not be declining “too fast,” so that, keeping the competitive structure of the industry fixed, neither firm has an incentive to reduce its production capacity.

Because of the tax shields on interest payments, both firms have an incentive to borrow, so we expect $s_i > 0$, $i = 1, 2$. High debt levels may not be desirable because of the higher probability of default, the associated bankruptcy costs, and the necessity to sell a fraction of assets if bankruptcy occurs. Finally, debt also has a strategic effect, because in equilibrium the financing and investment decisions of both players are interrelated. For example, as we show below, relative debt levels determine the order in which the firms default. Also, the expected bankruptcy costs for a firm depend on *both* the production and financing decisions of its rival.

⁶Some anecdotal evidence suggests that the values of competing firms increase when their major rivals file for bankruptcy. For example, when K-Mart filed for Chapter 11 bankruptcy protection on January 22, 2002, the stock prices of its two major competitors, Wal-Mart and Target, both went up by about 3.2%. On that day the DJIA fell by 0.6% and the NASDAQ fell by almost 2.5%. For more systematic evidence see [Lang and Stulz \(1992\)](#).

New entry lowers the output price and, therefore, increases the probability of foreclosure. On the other hand, the default of one firm produces a favorable effect on the profit of the other.

At the time of entry the shareholders of an entering firm make their capital structure decision by choosing their coupon payment s_i . We assume that coupon payment remains constant throughout the life of the firm, until shareholders declare bankruptcy and transfer ownership to debtholders. Any dynamic adjustments in leverage are assumed to be prohibitively costly.⁷

We assume that all agents are risk-neutral and discount their future payoffs at a rate r .

Before proceeding to the formal analysis, it is useful to outline the general intuition behind the industry equilibrium. When the state of the stochastic demand shock x is low, so is the price that the firms expect to get for their products should either of them enter the industry, so they both prefer to stay aside.

However, as demand grows, the potential price in the output market reaches a threshold at which it is optimal for one firm to enter. The firm enters and becomes the “leader,” while the other automatically assumes the role of the “follower.” We do not designate one particular firm as a leader. As we show below, in equilibrium the payoffs to the leader and to the follower are identical, so the firms are ex ante indifferent between their roles.

3. Equilibrium

To derive the equilibrium strategies of both players, we solve the game backwards. First, assuming that both firms have already entered and are active, we focus on their optimal default strategies (subsection 3.1). Then we consider a situation when one firm (the leader) has entered the industry while the other (the follower) is still waiting, and focus on the optimal investment and financing policies of the follower (subsection 3.2). Finally, we construct the equilibrium of the whole game and solve for the optimal strategy of the leader (subsection 3.3).

⁷Introducing dynamic adjustments of capital structure in a context of product market competition is an interesting but technically very challenging extension of the model and is left for future research. The model developed in this paper can nevertheless be realistic given that transaction costs associated with issuing or retiring debt are significant, and firms in practice adjust leverage infrequently. See, for example, Korteweg and Strebulaev (2015).

3.1. Equilibrium Default Strategies

We start by considering an industry with two active levered firms and focus on their optimal default strategies. Throughout the paper, we consider an equity-based definition of default whereby shareholders inject funds into the firm as long as the value of equity is positive.⁸ Shareholders default on their debt obligation the first time shock x decreases below a certain threshold. At the time of default the value of equity is equal to zero.

As the following proposition indicates, there are two potentially optimal default thresholds for the shareholders of firm i : $x_b^d(\pi_{hh}, s_i)$, a threshold corresponding to the case when the other firm (firm j) is still in the solvent state and has not gone bankrupt, and $x_b^d(\pi_{hb}, s_i)$, optimal when the rival is already in the bankrupt state. In our notation, the subscript “b” stands for “bankrupt” and the superscript “d” stands for “duopoly.”

Proposition 1 *If firm i defaults before its rival firm j , which remains solvent and operates at full capacity, then it will do so at the first passage time of x to the optimal default trigger $x_b^d(\pi_{hh}, s_i)$:*

$$x_b^d(\pi_{hh}, s_i) = \frac{\beta_2}{\beta_2 - 1} \frac{s_i(r - \mu)}{\pi_{hh}r}. \quad (5)$$

On the contrary, if firm i defaults after its rival firm j , which is in the post-default state and operates at reduced capacity, then the optimal default trigger $x_b^d(\pi_{hb}, s_i)$ is given by

$$x_b^d(\pi_{hb}, s_i) = \frac{\beta_2}{\beta_2 - 1} \frac{s_i(r - \mu)}{\pi_{hb}r}, \quad (6)$$

where β_2 is the negative root of the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$.

The default thresholds in equations (5) and (6) have the following properties. The higher the coupon payment, the lower the growth rate of the demand shock x , and the lower its volatility, the higher the optimal default thresholds. Low growth rate and low volatility of the demand shock erode the value of the option to wait. The default thresholds also increase with the discount rate r . When r is high, the equityholders are more concerned about immediate losses than about potential future profits, and exercise their default option sooner. When its

⁸As in Mello and Parsons (1992) and Leland (1994), among others.

rival is bankrupt and operates at reduced capacity, a firm enjoys higher profits and therefore is less willing to default, therefore, $x_b^d(\pi_{hb}, s_i) < x_b^d(\pi_{hh}, s_i)$.

Proposition 1 identifies two potentially optimal default thresholds for a firm in a duopoly – one for the case when the rival is still solvent, the other one for the case when the rival has already defaulted. Proposition 1, however, does not provide any guidance regarding the order in which the firms default. If firm i is forced to default when firm j still operates at full capacity, then the best it can do is to default at $x_b^d(\pi_{hh}, s_i)$. The shareholders of firm i , however, may expect their rival to default soon. The rival’s default generates a positive jump in the instantaneous profit of firm i . In this case, it may be optimal for the shareholders of firm i not to default at $x_b^d(\pi_{hh}, s_i)$, but to wait until firm j goes bankrupt, and default later at $x_b^d(\pi_{hb}, s_i)$.

In general, two equilibria are feasible. In one of them firm i defaults at $x_b^d(\pi_{hh}, s_i)$, leading to an increase in firm j ’s profit. Firm j defaults later at $x_b^d(\pi_{hb}, s_j)$. In the other, firm j defaults at $x_b^d(\pi_{hh}, s_j)$, while firm i defaults at $x_b^d(\pi_{hb}, s_i)$. Proposition 1 does not identify which of the two potential equilibria prevails. It only states that if the shareholders of firm i believe that their rival is going to default at $x_b^d(\pi_{hh}, s_j)$, then the best they can do is to default at $x_b^d(\pi_{hb}, s_i)$, and vice versa.

The key observation is that the equilibrium strategies of both firms depend on their capital structures. For example, if firm i selects a very low coupon payment s_i , substantially lower than the coupon payment of its competitor s_j , then firm i ’s default threshold $x_b^d(\pi_{hh}, s_i)$ is also very low. In this case firm j will prefer to default at the first passage time of x to $x_b^d(\pi_{hh}, s_j)$, because the present value of the positive shift in profit resulting from the default of firm i is insignificant. However, if the two firms have similar coupon payments, then each of them may strictly prefer the scenario in which its competitor defaults at $x_b^d(\pi_{hh}, s_i)$, allowing the firm to enjoy the benefit of reduced competition.

We label the firm with the higher coupon rate as the “weaker” firm and the firm with the lower coupon rate as the “stronger” firm. Without loss of generality, we assume that $s_j > s_i$, making firm i stronger than firm j . The following proposition uses the logic of Lambrecht (2001) and reduces the set of feasible equilibrium strategies by focusing on subgame perfect equilibria on *connected* sets. In our model, a connected set equilibrium implies that in any subgame each

firm defaults at the first passage time of the stochastic shock to a certain threshold from above and there are no equilibria where a firm defaults when a certain threshold is reached by the shock x from below.

Proposition 2 states that the weaker firm always defaults first. Murto (2004) examines an extended class of equilibria for exit games in a duopoly and shows that when the underlying uncertainty is sufficiently high and there is a substantial asymmetry between the two firms there might exist additional equilibria on disconnected sets in which the weaker firm defaults first. We focus on connected strategies because in our model equilibria on disconnected sets exhibit unrealistic features that contradict economic intuition.⁹ In addition, when modeling equilibrium strategies in Section 4.2, we require that there are no disconnected strategy equilibria for the resulting parameter values.

Proposition 2 *Any subgame perfect Nash Equilibrium on a connected set involves the weaker firm j defaulting first at the first passage time of the stochastic shock x to the threshold $x_b^d(\pi_{hh}, s_j)$, and the stronger firm i defaulting second, at the first passage time of x to $x_b^d(\pi_{hb}, s_i)$.*

The economic intuition behind Proposition 2 is as follows. Firm j is the weaker firm, therefore its default thresholds are higher than those of firm i , i.e., $x_b^d(\pi_{hh}, s_j) > x_b^d(\pi_{hh}, s_i)$ and $x_b^d(\pi_{hb}, s_j) > x_b^d(\pi_{hb}, s_i)$. Let us assume for the moment that $x_b^d(\pi_{hb}, s_j)$ is hit while firm j is still solvent. Then there is no reason for the shareholders of firm j to continue retaining control of the firm, regardless of whether its rival has already defaulted or not. The shareholders of the weaker firm will default no later than at the first stopping time upon reaching $x_b^d(\pi_{hb}, s_j)$. The default of the weaker firm leads to a higher payoff to the shareholders of the stronger one. Therefore, the shareholders of the stronger firm will never want to default while x stays in a certain region above $x_b^d(\pi_{hb}, s_j) : x \in [x_b^d(\pi_{hb}, s_j), y)$, where y is some value of the stochastic shock greater than $x_b^d(\pi_{hb}, s_j)$. The optimal default time for the weaker firm is thus no later than upon hitting y for the first time. Applying similar arguments iteratively leads to the result established in Proposition 2. We give formal proof of this proposition in the Appendix.

⁹For example, a disconnected set equilibrium implies a positive relation between default probability and profitability in certain subgames, and implies a negative relation between probability of default and financial leverage in all subgames. Both of these predictions contradict economic intuition and available empirical evidence, for example in Campbell, Hilscher and Szilagyi (2008).

The final element in the model is the direct bankruptcy cost, η , which reduces the post-default value of the firm. Unlike parameter α , which represents the fraction of assets that must be sold in the event of default, and therefore affects total industry supply and its competitive structure, η is a direct cost faced by the claimholders of the defaulting firm. This direct cost does not affect the industry supply and the profits of competitors. The typical direct costs incurred by firms include legal, accounting, consulting, and other administrative expenses.

In the event of default, we assume that debtholders continue to employ the remaining assets but do not relever the firm. Therefore, the value of the firm in liquidation must equal to the value of the unlevered firm with reduced capacity. If $A(x)$ is the value of the unlevered firm, then the payoff to debtholders is then given by $(1 - \eta)A(x)$. For simplicity, we assume that the new owners do not use debt to finance the firm. Allowing them to do so would result in a higher value of the firm in default due to the tax benefits of debt. Thus, the refinancing option leads to higher values of debt, held by the original bondholders who take control of the firm in the event of bankruptcy. Technically this is equivalent to reducing the direct cost parameter η , and no additional insights can be obtained from giving the new owners the refinancing option.

The optimal default strategies of both firms depend on their capital structures. When the stronger firm forecloses, the debtholders of the weaker firm, who are now its new owners, experience a positive shock to their profits. Therefore, the post-default value of the weaker firm depends on the capital structure of the stronger firm. On the other hand, after the stronger firm defaults, the competitive environment of the industry remains unchanged forever. Therefore, the post-default value of the stronger firm is not a function of its rival's past leverage. The post-default values of both firms are established in the following proposition.

Proposition 3 *The post-default value of the stronger firm is given by*

$$A_i(x) = (1 - \eta)(1 - \tau) \frac{x\pi_{bb}}{r - \mu}, \quad (7)$$

while the post-default value of the weaker firm is

$$A_j(x) = (1 - \eta)(1 - \tau) \left\{ \frac{x\pi_{bh}}{r - \mu} - \left(\frac{x}{x_b^d(\pi_{hb}, s_i)} \right)^{\beta_2} \frac{x_b^d(\pi_{hb}, s_i)(\pi_{bh} - \pi_{bb})}{r - \mu} \right\}. \quad (8)$$

In (7) the term on the right-hand side is the present value of the after-tax profits of a firm operating at reduced capacity while its competitor is also in the post-default state, net of the proportional liquidation cost η . In (8) the first term in the curly brackets is the post-default value of the weaker firm if the stronger firm never defaults. The second term represents the increase in value due to a positive shock to profit at the time when the stronger firm defaults.

3.2. Optimal Strategy of the Follower

Once the equilibrium default strategies of both firms are established, the next step is to examine the optimal strategy of the follower (i.e., the firm that enters second) by considering an industry with one active firm (the leader). The follower can exercise its entry option at any time by paying the irreversible investment cost I . Immediately upon its entry the instantaneous profit parameters of both firms become equal to $D(2) = \pi_{hh}$.

We denote the optimal entry threshold of the follower by $x_{2e}(\theta_i)$.¹⁰ The entry threshold is determined below by solving the follower's optimization problem. The total value of the follower upon entry is given by the sum of its debt and equity values net of the investment cost I :

$$v_2(x_{2e}, s_2) = e_2(x_{2e}, s_2) + d_2(x_{2e}, s_2) - I. \quad (9)$$

The shareholders of the follower choose the entry threshold x_{2e} and the contractual coupon payment s_2 to maximize the total discounted value of their firm ex ante. To properly identify the sources of the follower's value, we need to have a closer look at the structure of the game and the options available to both players. If the initial state of the demand shock x_0 is very low and is not sufficient to attract entry into the industry then both firms will wait. The first time a certain threshold x_{1e} is reached by x from below, one of the firms designates itself as the leader and enters the industry. The other accepts the role of the follower and continues to wait. As uncertainty resolves, two different scenarios are possible:

1. Demand falls so much that it becomes optimal for the shareholders of the leading firm to default. The leading firm defaults, faces partial liquidation, and its instantaneous profit

¹⁰Note that the optimal entry threshold of the follower $x_{2e}(\theta_i)$ depends on θ_i , the current status of the leader (solvent or post-default), because the default of the leader affects the profits of both firms in the industry.

parameter falls from π_h to π_b . This leads to an increase in the potential after-tax payoff to the shareholders of the follower, as it jumps from $(1 - \tau)(x\pi_{hh} - s_2)$ to $(1 - \tau)(x\pi_{hb} - s_2)$. The follower enters later, at a stopping time upon reaching a threshold $x_{2e}(0)$.

2. Demand rises high enough to become optimal for the follower to join the leader while the latter still operates at full capacity. The instantaneous after-tax payoffs to the shareholders of both firms become $(1 - \tau)(x\pi_{hh} - s_i)$. The follower exercises its investment option at the first passage time of the shock x to an investment trigger $x_{2e}(1)$.

Since the order in which the two firms default is determined by their capital structures, the values of the follower's securities depend on whether the follower becomes the "weaker" or the "stronger" firm, i.e. whether its coupon payment s_2 is above or below that of the leader, s_1 . In general, three different cases are possible: $s_1 < s_2$, $s_1 > s_2$, and $s_1 = s_2$. Proposition 4 determines the values of the follower's debt and equity as functions of its entry point x_{2e} and its coupon payment s_2 in two alternative cases: when $s_1 < s_2$ (the follower is the weaker firm) and $s_1 > s_2$ (the follower is the stronger firm).

Proposition 4 *Assume that by the time of the follower's entry the leader has not yet defaulted ($\theta_1 = 1$). Then:*

1. *If $s_1 < s_2$ (the follower is the weaker firm), then the values of the follower's equity and debt are given, respectively, by*

$$e_2(x_{2e}, \theta_1 = 1) = (1 - \tau) \left[\frac{x_{2e}\pi_{hh}}{r - \mu} - \frac{s_2}{r} - \left[\frac{x_{2e}}{x_b^d(\pi_{hh}, s_2)} \right]^{\beta_2} \left(\frac{x_b^d(\pi_{hh}, s_2)\pi_{hh}}{r - \mu} - \frac{s_2}{r} \right) \right] \quad (10)$$

and

$$d_2(x_{2e}, \theta_1 = 1) = \frac{s_2}{r} + \left[\frac{x_{2e}}{x_b^d(\pi_{hh}, s_2)} \right]^{\beta_2} \left(A_j(x_b^d(\pi_{hh}, s_2)) - \frac{s_2}{r} \right), \quad (11)$$

where $A_j(x_b^d(\pi_{hh}, s_2))$ is the post-default value given by (8), and $x_b^d(\pi_{hh}, s_2)$ is the optimal default threshold of the follower given by (5);

2. *If $s_1 > s_2$ (the follower is the stronger firm), then*

$$e_2(x_{2e}, \theta_1 = 1) = (1 - \tau) \left\{ \frac{x_{2e}\pi_{hh}}{r - \mu} - \frac{s_2}{r} - \left[\frac{x_{2e}}{x_b^d(\pi_{hh}, s_1)} \right]^{\beta_2} \left(\frac{x_b^d(\pi_{hh}, s_1)(\pi_{hh} - \pi_{hb})}{r - \mu} + \right. \right. \\ \left. \left. \left[\frac{x_b^d(\pi_{hh}, s_1)}{x_b^d(\pi_{hb}, s_2)} \right]^{\beta_2} \left(\frac{x_b^d(\pi_{hb}, s_2)\pi_{hb}}{r - \mu} - \frac{s_2}{r} \right) \right\} \quad (12)$$

and

$$d_2(x_{2e}, \theta_1 = 1) = \frac{s_2}{r} + \left[\frac{x_{2e}}{x_b^d(\pi_{hb}, s_2)} \right]^{\beta_2} \left(A_i(x_b^d(\pi_{hb}, s_2)) - \frac{s_2}{r} \right), \quad (13)$$

where $A_j(x_b^d(\pi_{hh}, s_2))$ is the post-default value given by (7), and $x_b^d(\pi_{hb}, s_2)$ is the optimal default threshold of the follower given by (6).

In equation (10) the first two terms in the square brackets represent the value of the perpetual entitlement to the current flow of income, given by $(1 - \tau)(x_{2e}\pi_{hh} - s_2)$. The last term is the value of the shareholders' option to default, which is the product of the surplus created by this option and a stochastic discount factor $\left[\frac{x_{2e}}{x_b^d(\pi_{hh}, s_2)} \right]^{\beta_2}$. In equation (11) the first term is the value of a default-free bond, while the second term is the default premium. A similar interpretation applies to equations (12) and (13).

Note that the debtholders of the stronger firm are always better off, because the stronger firm defaults last so its debtholders receive their contractual payments for a longer period of time. It can be verified that for a given pair of s_1 and s_2 the value of debt of the stronger firm (13) exceeds that of the weaker firm (11). The implications for equity values are less straightforward. As discussed above, in some cases (in particular when the coupon payment of the competitor is low) the benefit of "outliving" the competitor may not offset the losses to be incurred while the competitor still operates at full capacity. Therefore the equityholders may be better off by assuming the role of the weaker firm.

Note that when $s_1 = s_2 = s$, the equityholders of both firms would prefer the other firm to default at $x_b^d(\pi_{hh}, s)$, and to default themselves at $x_b^d(\pi_{hb}, s)$.¹¹ Since both firms are identical, there is no way to distinguish which one defaults at $x_b^d(\pi_{hh}, s)$ and lets the other one default at $x_b^d(\pi_{hb}, s)$. It is therefore reasonable to assume that at the first passage time to $x_b^d(\pi_{hh}, s)$ each firm faces a 0.5 probability of bankruptcy. In that case the values of the follower's securities

¹¹It is straightforward to show that when $s_1 = s_2$ the equity value given by (12) is strictly greater than the one given by (10).

when $s_1 = s_2$ will be equal to the arithmetic averages of their values in the case when the follower defaults first at $x_b^d(\pi_{hh}, s_2)$, given by (10) and (11), and the case when the follower outlives the leader and defaults second at $x_b^d(\pi_{hb}, s_2)$, given by (12) and (13).

If the leader defaults while the follower is still inactive, then the instantaneous profit of the follower's shareholders upon entry is given by $(1 - \tau)(x\pi_{hb} - s_2)$. In this case the optimization problem of the follower is analogous to the problem of a firm operating in the monopoly environment. The capital structure that the leader had prior to its foreclosure becomes irrelevant for the follower's optimization program. The values of the follower's securities in this case are established by the following proposition.

Proposition 4a *Assume that when the follower enters, the leader is already in the post-default state ($\theta_1 = 0$). Then the values of the follower's equity and debt are given, respectively, by*

$$e_2(x_{2e}, \theta_1 = 0) = (1 - \tau) \left[\frac{x_{2e}\pi_{hb}}{r - \mu} - \frac{s_2}{r} - \left[\frac{x_{2e}}{x_b^d(\pi_{hb}, s_2)} \right]^{\beta_2} \left(\frac{x_b^d(\pi_{hb}, s_2)\pi_{hb}}{r - \mu} - \frac{s_2}{r} \right) \right] \quad (14)$$

and

$$d_2(x_{2e}, \theta_1 = 0) = \frac{s_2}{r} + \left[\frac{x_{2e}}{x_b^d(\pi_{hb}, s_2)} \right]^{\beta_2} \left(A_i(x_b^d(\pi_{hb}, s_2)) - \frac{s_2}{r} \right), \quad (15)$$

where $A_i(x_b^d(\pi_{hb}, s_2))$ is the post-default value given by (7), and $x_b^d(\pi_{hb}, s_2)$ is the optimal default threshold of the follower given by (6).

As before, there are two terms in (14) and (15). The first terms give the present values of the perpetual flows of income. The second terms represent the value of the equityholders' option to default and the corresponding negative change in the value of debt.

Now let us assume that the follower has not yet exercised its entry option. Once the leader has entered, there are two alternative investment strategies available to the follower. It can either wait until the leader defaults and enter at a later date, or it can join the leader while it is still solvent and operates at full capacity. The values of the follower's securities in these two cases are provided by Propositions 4 and 4a, respectively. Therefore, the value of the follower is given by the appropriately discounted weighted average of its values in the two scenarios.

Let x_{1e} be the entry trigger of the leader. Assume that $x_{1e} \in (x_b^m(\pi_h, s_1), x_{2e}(1))$, i.e., the leader's entry does not immediately lead to its default or to the entry of the follower. Denote

by $\mathcal{L}(x; z, y)$ the present value of \$1 to be received the first time x reaches the lower threshold z , conditional on x reaching z before reaching the upper threshold y . In addition, denote by $\mathcal{H}(x; z, y)$ the present value of \$1 to be received the first time that the industry shock x reaches the higher threshold y , conditional on x reaching y before the lower threshold z . The following proposition derives the value of the follower at the initial state.

Proposition 5 *If the leader enters at the first passage time of x to x_{1e} (such that $x_{1e} > x_0$) and issues debt with a coupon payment s_1 upon entry, then the value of the follower is given by*

$$V_2(x_0) = \left[\frac{x_0}{x_{1e}} \right]^{\beta_1} \{ \mathcal{H}(x_{1e}; x_b^m(\pi_h, s_1), x_{2e}(1)) [e_2(x_{2e}(1), \theta_1 = 1) + d_2(x_{2e}(1), \theta_1 = 1) - I] + \left[\frac{x_b^m(\pi_h, s_1)}{x_{2e}(0)} \right]^{\beta_1} \mathcal{L}(x_{1e}; x_b^m(\pi_h, s_1), x_{2e}(1)) (e_2(x_{2e}(0), \theta_1 = 0) + d_2(x_{2e}(0), \theta_1 = 0) - I) \}, \quad (16)$$

where β_1 and β_2 are respectively the positive and the negative roots of the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$, $x_b^m(\pi_h, s_1)$ is the optimal default threshold of the leader given that the follower has not yet invested, $d_2(x_{2e}(\theta_1), \theta_1)$ and $e_2(x_{2e}(\theta_1), \theta_1)$ are given by propositions 4 ($\theta_1 = 1$) and 4a ($\theta_1 = 0$), x_0 is the current state of the stochastic shock, and $x_{2e}(\theta_1)$ is the optimal entry threshold of the follower if the leader is in the solvent ($\theta_1 = 1$) or post-default ($\theta_1 = 0$) state.

The quantities $\mathcal{L}(X; z, y)$ and $\mathcal{H}(X; z, y)$ are defined as

$$\mathcal{L}(x; z, y) = (y^{\beta_1} x^{\beta_2} - y^{\beta_2} x^{\beta_1})(y^{\beta_1} z^{\beta_2} - y^{\beta_2} z^{\beta_1})^{-1}, \quad (17)$$

$$\mathcal{H}(x; z, y) = (x^{\beta_1} z^{\beta_2} - x^{\beta_2} z^{\beta_1})(y^{\beta_1} z^{\beta_2} - y^{\beta_2} z^{\beta_1})^{-1}. \quad (18)$$

Given the entry point of the leader x_{1e} and its coupon payment s_1 , the follower's objective is to maximize its value, V_2 , provided by Proposition 5. It does so by optimally choosing its entry thresholds $x_{2e}(\theta_1 = 0, 1)$ as well as its coupon payments $s_2(\theta_1 = 0, 1)$. Therefore, the follower's optimization problem is

$$x_{2e}^*(\theta_1 = 0, 1), s_2^*(\theta_1 = 0, 1) = \arg \max_{x_{2e}, s_2} V_2[x_{2e}(0), x_{2e}(1), s_2(0), s_2(1)]. \quad (19)$$

The optimization problem (19), however, can be decomposed into two separate problems. Once

the leader defaults, there is no more uncertainty about the instantaneous profit parameter of the follower upon its entry, and therefore finding the optimal values of $x_{2e}(0)$ and $s_2(0)$ constitutes an independent problem:

$$x_{2e}^*(0), s_2^*(0) = \arg \max_{x_{2e}(0), s_2(0)} [x_{2e}^{-\beta_1}(0)(e_2(x_{2e}(0), \theta_1 = 0) + d_2(x_{2e}(0), \theta_1 = 0) - I)], \quad (20)$$

where $x_{2e}^{-\beta_1}(0)$ is the appropriate discount factor, and $e_2(x_{2e}(0), \theta_1 = 0)$ and $d_2(x_{2e}(0), \theta_1 = 0)$ are the values of the follower's equity and debt, given by (14) and (15). The problem (19) can then be rearranged as

$$x_{2e}^*(1), s_2^*(1) = \arg \max_{x_{2e}(1), s_2(1)} (V_2(x_{2e}^*(0), x_{2e}(1), s_2^*(0), s_2(1))). \quad (21)$$

While this optimization problem needs to be solved numerically, it is a straightforward process given that all values in (12)-(18) are expressed in closed form.

3.3. Optimal Strategy of the Leader

Next, we study the firm that enters the market first and therefore becomes the leader. When making its decision, the leader takes into account the actions of the other firm that will follow, examined above. The equityholders of the leading firm are rational and anticipate the entry of the competitor when x reaches the corresponding investment threshold x_{2e} . At that moment the shareholders of the leader (if they have not defaulted before) experience a negative shock to their instantaneous profit, as it falls from $(1 - \tau)(x\pi_h - s_1)$ to $(1 - \tau)(x\pi_{hh} - s_1)$.

Therefore, the value of the leader comes from two different sources: the monopoly rents to be received before the entry of the follower and the stream of duopoly profits to be received thereafter. The following proposition establishes the value of the leader as a function of its entry trigger x_{1e} and its contractual coupon payment s_1 .

Proposition 6 *The value of the leader is given by*

$$V_1(x_0) = \left[\frac{x_0}{x_{1e}} \right]^{\beta_1} (e_1(x_{1e}) + d_1(x_{1e}) - I), \quad (22)$$

where

$$\begin{aligned}
e_1(x_{1e}) = & (1 - \tau) \left[\frac{x_{1e}\pi_h}{r - \mu} - \frac{s_1}{r} \right] + \\
& \mathcal{H}(x_{1e}; x_b^m(\pi_h, s_1), e_2(x_{2e}(1))) \left\{ e_1^*(x_{2e}(1)) - (1 - \tau) \left[\frac{x_{2e}(1)\pi_h}{r - \mu} - \frac{s_1}{r} \right] \right\} - \\
& \mathcal{L}(x_{1e}; x_b^m(\pi_h, s_1), e_2(x_{2e}(1))) (1 - \tau) \left\{ \frac{x_b^m(\pi_h, s_1)\pi_h}{r - \mu} - \frac{s_1}{r} \right\} \quad (23)
\end{aligned}$$

and

$$\begin{aligned}
d_1(x) = & \frac{s_1}{r} + \mathcal{H}(x_{1e}; x_b^m(\pi_h, s_1), e_2(x_{2e}(1))) \left[d_1^*(x_{2e}(1)) - \frac{s_1}{r} \right] + \\
& \mathcal{L}(x_{1e}; x_b^m(\pi_h, s_1), e_2(x_{2e}(1))) \left[A_1(x_b^m(\pi_h, s_1)) - \frac{s_1}{r} \right], \quad (24)
\end{aligned}$$

where $e_1^*(x_{2e}(1))$ and $d_1^*(x_{2e}(1))$ are the values of the leader's equity and debt at the time of the follower's entry, $\mathcal{L}(x; z, y)$ and $\mathcal{H}(x; z, y)$ are defined in equations (17) and (18), and $A_1(x_b^m(\pi_h, s_1))$ is the post-default value of the leader given by

$$\begin{aligned}
A_1(x_b^m(\pi_h, s_1)) = & (1 - \eta)(1 - \tau) \left\{ \frac{x_b^m(\pi_h, s_1)\pi_b}{(r - \mu)} + \right. \\
& \left. \left(\frac{x_b^m(\pi_h, s_1)}{x_{2e}(0)} \right)^{\beta_1} \left[\frac{x_{2e}(0)(\pi_{bh} - \pi_b)}{r - \mu} + \left(\frac{x_{2e}(0)}{x_b^d(\pi_{hb}, s_2)} \right)^{\beta_2} \frac{x_b^d(\pi_{hb}, s_2)(\pi_{bb} - \pi_{bh})}{r - \mu} \right] \right\}.
\end{aligned}$$

Equation (22) requires that the present value of the leader be equal to its future value upon entry multiplied by the corresponding discount factor $\left[\frac{x_0}{x_{1e}} \right]^{\beta_1}$. Equation (23) shows that the equity value at the time of entry has three components. The first term is the present value of the perpetual flows of monopoly income to equityholders. The second term accounts for the change in the value of equity caused by the entry of the follower, conditional on the leader remaining solvent. The third term accounts for the effect caused by the leader's default, if it happens before the follower's entry. Equation (24) can be interpreted similarly. The first term is the value of a default-free perpetual bond, while the second and the third terms account for the effect of the leader's entry and the debtholders' loss in the event of default.

Proposition 6 specifies the value of the leader as a function of its contractual coupon pay-

ment, its entry trigger x_{1e} , and the corresponding investment threshold of the follower. For any given entry trigger x_{1e} and coupon payment s_1 , there is a uniquely determined value of the follower $V_2^*(x_{1e}, s_1)$, derived in Proposition 5. To obtain this value, the equityholders of the follower optimally choose their investment trigger $x_{2e}^*(x_{1e}, s_1)$ and contractual coupon payment $s_2^*(x_{1e}, s_1)$. At the time of entry the leader issues its coupon to maximize the value of its securities. The maximum value of the leader upon entry is then given by

$$V_1^*(x_{1e}) = \max_{s_1} V_1 \{x_{1e}, s_1, x_{2e}^*(x_{1e}, s_1), s_2^*(x_{1e}, s_1)\}. \quad (25)$$

However, the leader cannot unconditionally maximize its value by varying its entry threshold x_{1e} , as it faces the threat of preemption from the follower. If for a given x_{1e} , the corresponding value of the follower exceeds the value of the leader, $V_2^*(x_{1e}) > V_1^*(x_{1e})$, then the leader has no incentive to enter. In this case the follower will make no attempt to preempt the leader. Let us denote by x_{1e}^* the solution to $V_1^*(x_{1e}) = V_2^*(x_{1e})$. If the current state of the demand shock x is below x_{1e}^* , then investment is not optimal and both firms stay aside. However, once x_{1e}^* is reached, one firm enters and designates itself as the leader, while the other becomes the follower. An important feature of the equilibrium is that the values of the leader and the follower are the same, and therefore the two firms are indifferent between these two roles. If it were not the case, then one of the firms (the one with the lower value) would have an incentive to deviate from its investment strategy. For example, if the leader remains idle when $x > x_{1e}^*$, then the follower has an incentive to invest and become the leader itself.

The above analysis does not specify which firm is going to lead and which will become the follower. Furthermore, if both firms invest at once, each will obtain the value below $V_1^*(x_{1e})$ (or $V_2^*(x_{1e})$). Given that one firm invests at $x = x_{1e}$, the optimal strategy of the other firm is to wait and invest later at $x = x_{2e}$. Therefore, simultaneous investment is a “mistake” from the perspective of both firms. This mistake can potentially occur if each firm decides to become the leader, and both invest at the same time. However, by constructing a discrete time version of the investment game and considering the limiting continuous time case it can be shown that in the continuous time setting the probability of simultaneous investment converges to zero. A technical analysis is given in [Fudenberg and Tirole \(1985\)](#).

4. Optimal Investment and Financing

4.1. Benchmark: The Case of a Single Firm

In this section we examine the optimal investment and capital structure of a monopolist, i.e. a firm that does not face the threat of subsequent entry into the industry. This establishes a benchmark for the case with more than one competing firms. The shareholders of a monopolist receive an instantaneous income flow of $(1 - \tau)(x\pi_h - s_m)$, where s_m is the coupon payment. The values of a monopolist's equity and debt upon entry are given by

$$e_m(x_{me}, s_m) = (1 - \tau) \left\{ \frac{x_{me}\pi_h}{r - \mu} - \frac{s_m}{r} - \left[\frac{x_{me}}{x_b^m(\pi_h, s_m)} \right]^{\beta_2} \left(\frac{x_b^m(\pi_h, s_m)\pi_h}{r - \mu} - \frac{s_m}{r} \right) \right\}, \quad (26)$$

and

$$d_m(x_{me}, s_m) = \frac{s_m}{r} + \left[\frac{x_{me}}{x_b^m(\pi_h, s_m)} \right]^{\beta_2} \left(A_m(x_b^m(\pi_h, s_m)) - \frac{s_m}{r} \right), \quad (27)$$

where x_{me} is the entry threshold of the monopolist, $x_b^m(\pi_h, s_m) = \frac{\beta_2}{\beta_2 - 1} \frac{s_m(r - \mu)}{\pi_h r}$ is its optimal default threshold, and $A_m(x_b^m(\pi_h, s_m)) = (1 - \tau)(1 - \eta) \frac{x_b^m(\pi_h, s_m)\pi_b}{r - \mu}$ is its post-default value. As before, the values of securities are given by the discounted perpetuities of the appropriate cash flows, truncated in the event of default. The first term in curly brackets in equation (26) represents the value of the perpetual profit flow to equityholders if they never default, the term in round brackets gives the payoff of the option to default, and $\left[\frac{x_{me}}{x_b^m(\pi_h, s_m)} \right]^{\beta_2}$ is the present value of \$1 to be received the first time x reaches the default threshold $x_b^m(\pi_h, s_m)$.

The shareholders maximize the ex ante present value of their securities. Therefore, their optimization problem is

$$\begin{aligned} V_m^*(x_{me}^*, s_m^*) &= \max_{x_{me}, s_m} V_m(x_{me}, s_m) \\ &= \max_{x_{me}, s_m} \left\{ \left[\frac{x_0}{x_{me}} \right]^{\beta_1} (d_m(x_{me}, s_m) + e_m(x_{me}, s_m) - I) \right\}. \end{aligned} \quad (28)$$

In (28) the term in round brackets is the total value of the firm's securities upon entry net of the investment cost I , while $\left[\frac{x_0}{x_{me}} \right]^{\beta_1}$ is the present value of \$1 to be received at the first passage time to x_{me} . Figure 1 presents the optimal entry trigger and the optimal leverage of

a monopolist as functions of the volatility of cash flows σ , the liquidation cost η , and the tax rate τ .

The base parameter values for Figure 1 are the following: $I = 4$; $\pi_h = 1$; $\pi_b = 0.8$; $\eta = 0.2$; $\tau = 0.15$; $r = 0.05$; $\mu = 0.01$; $\sigma = 0.2$. The risk-free rate reflects the historical average of treasury bonds rates. The growth rate of cash flows is selected to generate a dividend yield consistent with observed yields. Similarly, the value of the volatility parameter is chosen to match the (leverage-adjusted) asset return volatility of an average S&P 500 firm, as in [Strebulaev \(2007\)](#). The tax advantage of debt captures corporate and personal taxes and is set equal to $\tau = 0.15$.

Consistent with economic intuition, the optimal leverage of a monopolist is an increasing function of the tax rate and a decreasing function of the bankruptcy costs. High bankruptcy costs make debt financing less favorable, while high tax rates lead to greater tax shields on the interest payments and increase the benefits of debt, resulting in higher optimal leverage ratios. Figure 1 also reveals a negative relation between the volatility of the monopolist's cash flows and its optimal leverage ratio. An increase in volatility raises the probability of default and therefore leads to higher expected bankruptcy costs, making debt a less attractive source of financing.

As in standard real options models, higher volatility increases the value of the option to wait and therefore raises the optimal investment trigger. Consistent with this intuition, Figure 1 shows that the optimal entry threshold increases approximately by a factor of 4 when the volatility rises from 10% to 50%. The optimal entry threshold also increases with the tax rate. Ex ante, the objective of the equityholders is to maximize the total value of the firm's securities. An increase in the tax rate leads to a lower overall value of the firm. Therefore, it becomes optimal to wait longer before proceeding with investment.

With this benchmark single-firm case in mind, we now proceed to the main case where we analyze two competing firms.

4.2. The Main Case: Two Firms

4.2.1. Follower’s Optimal Investment and Financing

The follower enjoys the benefit of being able to observe the capital structure of the leader. The leader’s capital structure affects its equilibrium default strategy, which in turn influences the expected profit of the follower. This effect leads to different optimal investment and financing strategies of the follower depending on the leader’s leverage. Figures 2 and 3 illustrate the optimal investment and financing decisions of the follower. Figure 3a displays the relation between the optimal coupon rate of the follower and that of the leader. Since financial leverage has a more direct empirical counterpart than a coupon payment, Figure 3b displays the same relation for the leverage ratio of the follower. Figure 2 provides comparative statics results for the optimal entry threshold of the follower and the total value of its securities. In Figures 2 and 3 we use the following set of parameter values: $I = 4$; $\alpha = 0.15$; $D(q) = 3 - q$; $\eta = 0.2$; $\tau = 0.15$; $r = 0.05$; $\mu = 0.01$; $\sigma = 0.2$. We use the same parameters as in the benchmark case. In addition, Asquith, Gertner and Scharfstein (1994) document that on average, firms in financial distress sell 12% of their asset base, therefore we assume $\alpha = 0.15$.

Figure 3a reveals a non-monotonic relation between coupon payments of the leader and the follower. Generally, there are four different strategies of the follower corresponding to four different regions of the leader’s coupon s_1 . Those regions are denoted by A, B, C, and D in Figure 3. Below we explore the economic factors determining the strategies of the follower in each of these regions. We emphasize that the coupon payment of the incumbent s_1 affects *both* the optimal capital structure and entry threshold of the follower.

Note that there is always a strategic benefit of setting a coupon payment below that of the leader. By undercutting the leader in leverage, the follower becomes the “stronger” firm and outlives the leader, which results in greater values of the follower’s securities. However, there is a cost to such a strategy: if the leader’s coupon is very low, undercutting the leader in leverage will result in a substantial loss of tax benefits. The financing strategy of the follower depends on the trade-off between these two effects. When the leader’s coupon payment is quite low (Region A in Figure 3a), the tax benefits dominate, and the optimal strategy of the follower in Region A is to set its coupon relatively high and enjoy higher tax benefits, but accept the role

of the firm that defaults first. The optimal leverage of the follower is flat in this region, while its coupon rate is very slightly decreasing. Higher coupon payment of the leader results in its earlier default, which in turn leads to a higher value of the follower and lowers its optimal entry threshold, thereby reducing its coupon payment issued at the time of entry.

As the leverage of the leader increases, the loss of tax benefits resulting from undercutting the leader in leverage becomes smaller. At some point the strategic advantage to undercut the leader in leverage and reap the benefits of being the “stronger” firm outweighs the corresponding reduction in tax benefits. We then arrive in Region B in Figure 3. Note that as the leader’s coupon payment increases, the follower’s coupon increases one-to-one, so the slope in Region B in Figure 3a is one. However, this does not apply to market leverage ratios of the firms. Even though the coupon payments of the two firms are almost identical, by setting its coupon payment marginally below s_1 the follower forces the leader to accept the role of the “weaker” firm. The weaker firm defaults first, at a higher default trigger, so its equity value is lower than that of the stronger firm. Therefore, the market leverage of the leader in Region B is higher than that of the follower.¹² The graph in Figure 3b shows that at the border between Regions A and B the optimal leverage of the follower falls by about 25% from 58.5% to 44%. This result deserves specific attention because of the known feature of many single-firm capital structure models to predict higher leverage ratios than those empirically observed. The present analysis reveals that for some values of the leader’s leverage, the follower has a strategic incentive to set its leverage ratio considerably below the one resulting from the traditional trade-off between tax benefits and bankruptcy costs.

In Region C the leader’s coupon payment no longer presents a binding constraint for the follower’s optimization problem, as in Region B. In Region C the optimal entry threshold, optimal coupon payment, and optimal leverage ratio of the follower all decrease with the coupon payment of the leader. The higher the leader’s coupon payment, the sooner it will default and create more favorable conditions for the follower. This increases the total value of the follower’s securities upon entry and leads to an earlier exercise of the investment option. Earlier entry

¹²We look at the leverage-coupon relation rather than at the leverage-leverage relation, because the leverage of the leader is capped at 100%. For example, in Region D the leader defaults immediately upon the follower’s entry, so its market leverage is always 100%, which makes it impossible to derive any comparative static results in that region.

implies lower instantaneous profits and therefore leads to a lower optimal coupon payment. Also, because higher s_1 speeds up the default of the leader and the corresponding positive effect on the profit of the follower, it also increases the value of the follower's equity. Therefore, the optimal market leverage of the follower decreases with s_1 in Region C.

When the leader is highly levered, and the follower is confident that the leader is the weaker firm, the follower can adopt one of the two following strategies. It can either join the leader at an entry threshold high enough to make it attractive for both firms to stay in the industry, $x_{2e} > x_b^d(\pi_{hh}, s_1)$, or it can speed up the exercise of its investment option and enter at a lower threshold $x_{2e} \leq x_b^d(\pi_{hh}, s_1)$, so that the leader will be forced to leave the industry immediately upon the follower's entry. The former case corresponds to Region C, while the latter corresponds to Region D.

In Region D the coupon of the leader is so high that the follower has an incentive to force the leader into bankruptcy immediately. This incentive arises because of the higher profit parameter of the follower: $\pi_{hb} > \pi_{hh}$. To implement this strategy the follower has to enter earlier, at a lower realization of the stochastic shock x , otherwise the leader will have no incentive to default. However, as the leader's coupon payment s_1 increases, its optimal default threshold $x_b^d(\pi_{hh}, s_1)$ goes up as well. At some point, the incentive to force the leader into default dominates the cost of earlier entry, and we arrive in Region D.

In Region D the leader's default threshold represents a binding constraint for the follower's investment problem. This forces the follower to accelerate the exercise of its entry option. For example, in Figure 3b the optimal investment threshold of the follower falls from 0.302 to 0.255 when s_1 rises from 0.5 to 0.55. This result is consistent with Lambrecht (2001), who finds that in some cases entry of the follower can lead to immediate crowding out of the incumbent.¹³ Numerical results imply, however, that for this scenario to be plausible, the leader must be very highly levered. If the market is fully informed about the potential entry of the follower, then the equity value of the incumbent when x reaches the follower's entry threshold is exactly zero, as the market participants anticipate the incumbent's foreclosure, and therefore its market leverage is close to 100%. In the case of imperfect information, when the market believes that

¹³In our model the notion of "crowding out" should be interpreted as forcing to default and consequently sell a fraction of production assets.

the follower's entry is possible but is uncertain about its timing or probability, the value of the leader's equity is strictly positive. However, even in the extreme case, when investors expect the incumbent to operate in the monopoly environment forever, the leader's market leverage is about 66% in Region D for the base set of parameter values. In addition, we show in the next section that in equilibrium the leader has no incentive to set its coupon payment any higher than the equilibrium coupon payment of the follower. Therefore, Region D represents an off-equilibrium path. The leader can still find itself in the region if when making its capital structure decision some time ago, it did not anticipate subsequent entry into the industry.

As discussed above, in Region D the leader's default threshold $x_b^d(\pi_{hh}, s_1)$ represents a binding constraint for the follower's investment problem. Since $x_b^d(\pi_{hh}, s_1)$ is positively related to s_1 , so is the entry threshold of the follower x_{2e} . Higher entry threshold results in a higher optimal coupon s_2 , as displayed in Figure 3a. In Region D the leader defaults immediately upon the follower's entry, so its coupon payment s_1 has no effect on the optimal leverage of the follower, which is flat in Region D.

In summary, different strategies become optimal for the follower depending on the contractual coupon payment of the leader s_1 . The optimal leverage of the follower is flat in Regions A and D, increasing in Region B, and decreasing in Region C. The optimal entry threshold is slightly decreasing in Region A, decreasing in Regions B and C, and increasing in region D. In Region A, the follower is definitely the weaker firm, so the leader's coupon produces very little effect on the follower's strategy. In Region B the follower has a strategic incentive to undercut the leader in leverage. In Regions C and D the follower is certainly the stronger firm. In addition, in Region D the leader's coupon payment is so high that the follower has an incentive to enter with the intention to force the leader into bankruptcy immediately.

The above analysis has empirical implications. First, the optimal leverage of the new entrant does depend on the capital structure of the incumbents. This can explain diversity in capital structures among similar firms across industries. The optimal leverage of a firm is determined not only by its own characteristics, but also by the characteristics of other firms in the industry. Second, for some values of the incumbent's leverage the follower has a strategic incentive to undercut the incumbent in leverage. In this case the resulting optimal leverage may be substantially lower than the one obtained in the traditional trade-off framework.

4.2.2. Leader’s Optimal Investment and Financing

The leader faces the threat of preemption by the follower, and therefore has to speed up the exercise of its investment option. Figure 4 illustrates this intuition by displaying the values of the leader and the follower as functions of the leader’s entry threshold for the base parameter set. The solid line in Figure 4 provides the optimal value of the leader corresponding to its optimal leverage ratio as a function of its investment threshold. It is assumed in Figure 4 that the follower does not attempt to preempt the leader. The dotted line represents the maximum value of the follower. For the base set of input parameters the solution to the unconstrained optimization problem of the leader, assuming the follower makes no preemption attempt, is $x_{1e} = 0.19$. However, the equilibrium entry threshold of the leader is much lower: $x_{1e}^* = 0.124$. For any potential entry threshold of the leader exceeding x_{1e}^* , the follower has an incentive to enter first and preempt the leader’s entry. By doing so, “the follower” will designate itself as “the leader,” and the roles of the two competing firms will switch. In this case the maximum value that “the leader” can achieve is that of the follower. On the other hand, for any entry trigger below the equilibrium one, the follower has no incentive to preempt since in this region the value of the follower is strictly higher than that of the leader, as displayed in Figure 4, and therefore the leader is better off waiting. Thus, competition tends to speed up the exercise of firms’ investment options. This result is consistent with the extant literature exploring the interaction between firms’ investment decisions and competition in product markets.¹⁴

We now proceed to examine the *optimal financing* strategy of the leader. The shareholders of the leader have rational expectations and are aware of the consequences of their financing decisions on the subsequent actions of the follower (as illustrated in Figures 2 and 3). For example, if they set their coupon payment high then the follower will undercut the leader in leverage (Region B in Figure 3a.) In addition to the usual trade-off between tax benefits and bankruptcy costs, there is a strategic advantage of becoming the “stronger” firm by issuing a lower coupon payment. The stronger firm enjoys the prize of reduced competition in the future.

Two reasons make it optimal for the leader to select a relatively low coupon upon entry and commit to becoming the stronger firm at the cost of possibly losing some tax benefits that would result from a higher coupon payment. First, the leader is forced to enter at a lower investment

¹⁴See, for example, Grenadier (2002) and Baldursson (1998).

threshold than the follower, therefore making a lower coupon payment optimal. Second, the follower has the advantage of the second move. It has the option to observe the coupon of the leader and strategically undercut the leader in leverage, if necessary. The leader, however, has no such option, because the financing decisions of the two firms are not simultaneous. This makes Region B of coupon payments in Figure 3a less attractive for the leader. Regions C and D are associated with high expected bankruptcy costs.

Therefore, as the numerical results imply, the equilibrium coupon of the leader is at the right border of Region A. At this point the leader enjoys the maximum possible tax benefits while making it optimal for the follower to select a higher coupon and therefore to default first.¹⁵ Since the leader’s coupon payment is always lower than that of the follower, the same is true for the leverage ratios of the two firms (measured at the time of the follower’s entry): the leverage of a new entrant exceeds that of the incumbent. Finally, the high equilibrium coupon payment of the follower makes it the weaker firm and leads to its earlier default.

Figure 5 presents the optimal leverage ratios of the leader and the follower as functions of the cash flow volatility, tax rates, and bankruptcy costs for the base set of input parameters. Optimal leverage ratios of the leader and the follower are declining with volatility, increasing with tax rates, and declining with bankruptcy costs. These results are analogous to the monopoly case and should be interpreted similarly. Note that as the tax rate goes to zero, so do the optimal leverage ratios of both firms. If there are no tax benefits, all-equity financing becomes optimal.

In our model we assume zero costs of production for both firms. This assumption results in relatively high equilibrium market leverage ratios of both the leader and the follower. If there is a positive per-period cost of production, then the optimal leverage of the leader decreases. Since the leader enters earlier and since production costs do not depend on the entry trigger, earlier entry implies higher “operating leverage,” making financial leverage more costly.

4.3. Leverage dispersion

Finally, we study within-industry dispersion of leverage. We define it as the difference between the leverage of the follower and the leverage of the leader, normalized by the leverage of the

¹⁵The threat that the follower will undercut the leader in leverage presents a binding constraint.

leader:

$$\text{Dispersion} = \frac{L_{\text{follower}} - L_{\text{leader}}}{L_{\text{leader}}}.$$

Leverage ratios of both firms are measured at the time of the follower's entry.

Figure 6 shows the effects of cash flow volatility, tax rates, and bankruptcy costs on leverage dispersion. The first result is that dispersion increases in the volatility of cash flows. While high volatility has a negative effect on the leverage of both firms, it increases the value of the option to wait and widens the relative distance between the entry thresholds of the leader and the follower due to convexity of the threshold function with respect to volatility as illustrated in Figure 1. This results in a higher leverage dispersion.

Second, leverage dispersion decreases with tax rates because optimal coupon payment and optimal leverage are concave functions of the tax rates, as shown in Figure 5. Higher tax rates increase the value of tax shields making higher leverage optimal for both firms. But the relative distance between leverage choices of the two firms increases slower than the increase in leverage chosen by both firms, which leads to the lower dispersion of leverage.

Direct and indirect spillover costs of bankruptcy have different impact on leverage dispersion. Higher direct costs of bankruptcy decrease leverage dispersion. As Figure 5 shows, the follower's leverage is very sensitive to direct costs. For example, if direct costs of bankruptcy are 5%, follower's leverage is 0.70. When costs increase to 15%, leverage drops to 0.61. For comparison, the leader's leverage at 5% costs is 0.44, and at 15% it is 0.40. Therefore, as direct costs increase, the relative difference in leverages of two firms decrease, thereby decreasing leverage dispersion.

Indirect spillover bankruptcy costs increase leverage dispersion. Both the leader and the follower have similar sensitivity of their optimal leverage with respect to indirect costs. For example, as indirect costs increase from 15% to 25%, leader's leverage drops from 0.38 to 0.36 and follower's leverage falls from 0.58 to 0.56. In fact, over the entire range of feasible indirect costs, the relative distance between the two leverage ratios is nearly flat. Since leverage of both firms decline as costs increase but the relative distance remains the same, it increases leverage dispersion.

5. Empirical Analysis

In this section we bring theoretical predictions of the model to data. Our sample consists of all Compustat firms from 1961 until 2018. In Section 5.1 we study leverage choices and survival rates of incumbent and entering firms, and in Section 5.2 we study the determinants of within-industry leverage dispersion.

Table 1 defines variables used in our analysis. Summary statistics are shown in Table 2. We define leverage as the ratio of long-term debt plus debt in current liabilities over book value of assets. The average leverage in our sample is 0.30, which is similar to averages reported in the literature, for example Lemmon, Roberts and Zender (2008).

To measure cash flow volatility we use coefficient of variation of quarterly operating income before depreciation. Coefficient of variation (CV) is defined as the standard deviation over the absolute value of the mean. This normalization controls for the size differences and makes standard deviation comparable across firms. We adjust this measure by subtracting average CV in 2-digit SIC industries to control for seasonal variation in demand and earnings across industries. This definition and the process of constructing the variable follows Minton and Schrand (1999).

We obtain simulated marginal tax rates (MTR) from Graham (1996). There are two measures of MTRs: the first one is based on income before interest expense has been deducted, and the second one is based on income after the interest expense. The average tax rate for the first measure is 29.7% with moderate dispersion, while the second measure has a lower mean of 19.0% and significantly higher dispersion.

To test predictions of our model that are related to bankruptcy costs, we use estimates of expected costs obtained by Glover (2014). As control variables in our analysis, we use firm size, defined as the book value of assets, profitability, as measured by return on assets, and equity market-to-book ratio. All three variables are known to be correlated with firms financing decisions: smaller firms, more profitable firms, and high market-to-book firms have lower leverage.

In unreported results, we confirm prior findings in the literature that firms with higher cash

flow volatility, higher bankruptcy costs, and lower tax rates have lower leverage.¹⁶

5.1. Leverage and Survival Rates of New and Existing Firms

We start by classifying all firms into incumbents and entrants. Each year, firms that appear in Compustat for the first time and have positive book value of assets are classified as new entrants. The remaining firms, i.e. those who had reported positive book value of assets at least one year earlier, are classified as incumbents. This treatment of new versus existing firms is similar to how literature defines firm age as the number of years since a firm first appears in Compustat.¹⁷ Another common definition of firm age is the number of years since IPO date. Because Compustat tracks most companies as they first start filing with the SEC, which usually occurs around the IPO date, the two measures of firm age are highly correlated. We verify that our qualitative results are unchanged if we defining new firms relative to the year in which they go public.

As discussed in Section 4.2.2, optimal entering and financing decisions of the leader and the follower imply that leverage of incumbents should on average be lower than leverage of new entrants. Table 3 shows that average leverage of incumbents is 29.3% while the average leverage of entering firms is 33.1%. This result is similar to MacKay and Phillips (2005) who do a more detailed micro analysis of the manufacturing sector. Panel B of Table 3 shows that the difference between leverage of incumbents and entrants is more economically significant – 5.7% – in the most recent data from 2000 until 2018.¹⁸

In the equilibrium of our model since the entering firm uses more aggressive debt financing policy, it defaults first. Therefore, in the data we expect to see that survival rates of new entrants are lower than survival rates of incumbents. We find that 5-year survival rates for of new entrants are 2.6% lower, while 10-year survival rates are 6.2% lower. In more recent subsample, from 2000 until 2018, 5-year and 10-year differences in survival rates are 5.0% and

¹⁶See Section 4.1 and Figure 1 for the economic discussion. For empirical results see, for example, Lemmon, Roberts and Zender (2008).

¹⁷See for example Leary and Roberts (2010), among many others.

¹⁸We note that in Compustat there is a large influx of firms in year 1974. This increase is driven by either new firms that went public around 1974, or by increased coverage of existing firms by Compustat. After 1974, the number of new firms every year is stationary. If we compute leverage starting from 1974 instead of 1961, we get 30.3% for incumbents and 34.9% for entrants. The difference is both economically and statistically significant.

5.3%, respectively.

Overall, leverage ratios and survival rates of incumbents and entrants observed in the data are consistent with our model.

5.2. Determinants of Leverage Dispersion Within Industries

Our model produces a rich set of predications regarding cross-sectional leverage dispersion within industries as discussed above in Section 4.3. In particular, our theory implies that dispersion increases in cash flow volatility, decreases in tax rates, decreases in direct bankruptcy costs, and increases in indirect spillover bankruptcy costs.

Table 4 reports results of estimating linear regressions of leverage dispersion on industry characteristics. We estimate leverage dispersion at 4-digit SIC codes level. In addition, in order to test the relation between industry leverage dispersion and bankruptcy costs, we aggregate our results at the level of 17 industries defined in Fama and French (1988). The reason for this choice is due to availability of estimates of bankruptcy costs. We are using costs estimated by Glover (2014), who reports them for 17 Fama-French industries. Summary statistics for leverage dispersion are given in Table 2. Independent variables in regressions in Table 4 are first computed at the firm level and then averaged at the industry level.

The first two regressions in Panels A and B test for the equilibrium relationship between leverage dispersion and cash flow volatility. The main result is that leverage and cash flow volatility are strongly positively related, which confirms our theoretical result. In regression (2) we control for market-to-book, return on assets, and firm size. Since cash flow volatility affects profitability, valuation, and ultimately firm size, statistical relation between cash flow volatility and leverage dispersion weakens. In regression (2) in Panel A the result is significant at the 5% level, while in Panel B it remains significant at 1% level.¹⁹

Third and fourth regressions test for the equilibrium relationship between leverage and marginal tax rates from Graham (1996), computed based on income before the deduction of interest expense. Results show that there is a strong negative relation between leverage

¹⁹Industry leverage dispersion is negatively related to industry-average market-to-book, profitability, and firm size. Economically, this means that surviving firms in more mature industries – the ones that are characterized by lower MB, higher ROA, and larger firm size – have more similar leverage ratios.

dispersion and tax rates, which again confirms the prediction of our model. In regressions (5) and (6) we are using tax rates computed based on income after interest expense. Results are similar to regressions (3) and (4), but statistically they are marginally weaker.

Next, we test the relation between bankruptcy costs and leverage dispersion. As we discussed in Section 4.3, our model predicts that leverage dispersion is negatively affected by direct bankruptcy costs, but is positively affected by indirect bankruptcy costs that lead to reduced firm output. Empirically, testing these predictions is challenging for several reasons. First, expected bankruptcy costs are difficult to measure for firms that have not defaulted. Available estimates in the literature are either based on a small sample of defaulted firms, e.g., [Davydenko, Strebulaev and Zhao \(2012\)](#), or on modelling assumptions imposed on observed data, e.g., [Glover \(2014\)](#). Second, available estimates of bankruptcy costs typically combine both direct and indirect bankruptcy costs. Direct bankruptcy costs were estimated in early literature, e.g., [Warner \(1977\)](#), and typically show that these costs are low, with little variability across industries.

To test our theory, we take bankruptcy cost estimates from [Glover \(2014\)](#). These estimates combine both direct and indirect costs of bankruptcy. Since most of the variation in these costs across industries is likely driven by indirect costs, we expect that empirically these costs would be positively associated with leverage dispersion. Regressions (7) and (8) in Panel B confirm this. The first regression shows that bankruptcy costs are positively related to dispersion with a t -statistic of 2.36. After we control for growth, profitability, and firm size, this relation becomes weaker.

Taken together, results presented in Table 4 show determinants of observed leverage dispersion, while our theory provides economic mechanisms that explain observed patterns in the data.

6. Conclusion

In this paper we develop a model of optimal capital structure with imperfect competition. The main contribution of our work is to endogenize firms' investment, financing, and strategic decisions in the product market. Our model incorporates the traditional determinants of debt,

such as tax benefits and bankruptcy costs, and combines them with a strategic effect of debt due to a positive externality from the default of one firm on the profit of its competitor. This additional strategic effect makes the optimal financing and investment strategies of one firm dependent on those of its rival.

We first examine the optimal strategy of a new entrant seeking entry into an industry with an active firm. We show that competition in product markets can lead to a significant deviation from financing policies that are optimal in a single firm environment. A firm may optimally resort to a low debt ratio and forego a fraction of tax benefits, if its debt level gives it a strategic advantage over its competitors. This analysis demonstrates how a firm's optimal leverage is determined not only by its own characteristics, but also by the characteristics of other firms in the industry, and the overall industry structure.

We then study the optimal investment strategies of the two firms. The relation between the optimal investment threshold of the follower and the leverage of the incumbent is shown to be non-monotonic. For relatively low values of the leader's coupon payment, this relationship is negative, because a higher coupon payment implies earlier default. This, in turn, increases the expected value of the follower upon entry. However, if the leader is very highly levered it may default immediately upon the follower's entry. In this case the relation between the coupon payment of the leader and the follower's entry trigger becomes positive.

In equilibrium, the leader, facing the threat of preemption, is forced to invest earlier. Its optimal strategy is to issue less debt and consequently enjoy the benefits of becoming the stronger firm. The follower issues more debt and defaults first. This implies that incumbent firms may stay longer, and firms that enter later are more likely to default first. The equilibrium leverage ratio of the new entrant is higher than that of the incumbent. We confirm these results in the data, and show that new entrants are more highly levered and their survival rates are significantly lower.

Our results also imply that operationally identical firms optimally choose different debt ratios, which results in within-industry dispersion. We show that this dispersion can be economically significant, and depends on cash flow volatility, tax rates, and bankruptcy costs. We discuss economic intuition behind these results and show that they hold in the data.

Overall, our paper demonstrates the importance of competition in product markets in shap-

ing firms' optimal capital structures. It provides economic rationale for significant differences in optimal leverage ratios among operationally similar firms within an industry.

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Figure 1. Optimal entry and leverage of a monopolist.

This figure shows the optimal leverage ratio and entry threshold of a firm operating in the monopoly environment as functions of the volatility of cash flows, σ , the tax rate, τ , and the bankruptcy cost, η . The input parameters are set as follows: $I = 4$; $\pi_h = 1$; $\pi_b = 0.8$; $\eta = 0.2$; $\tau = 0.15$; $r = 0.05$; $\mu = 0.01$; $\sigma = 0.2$.

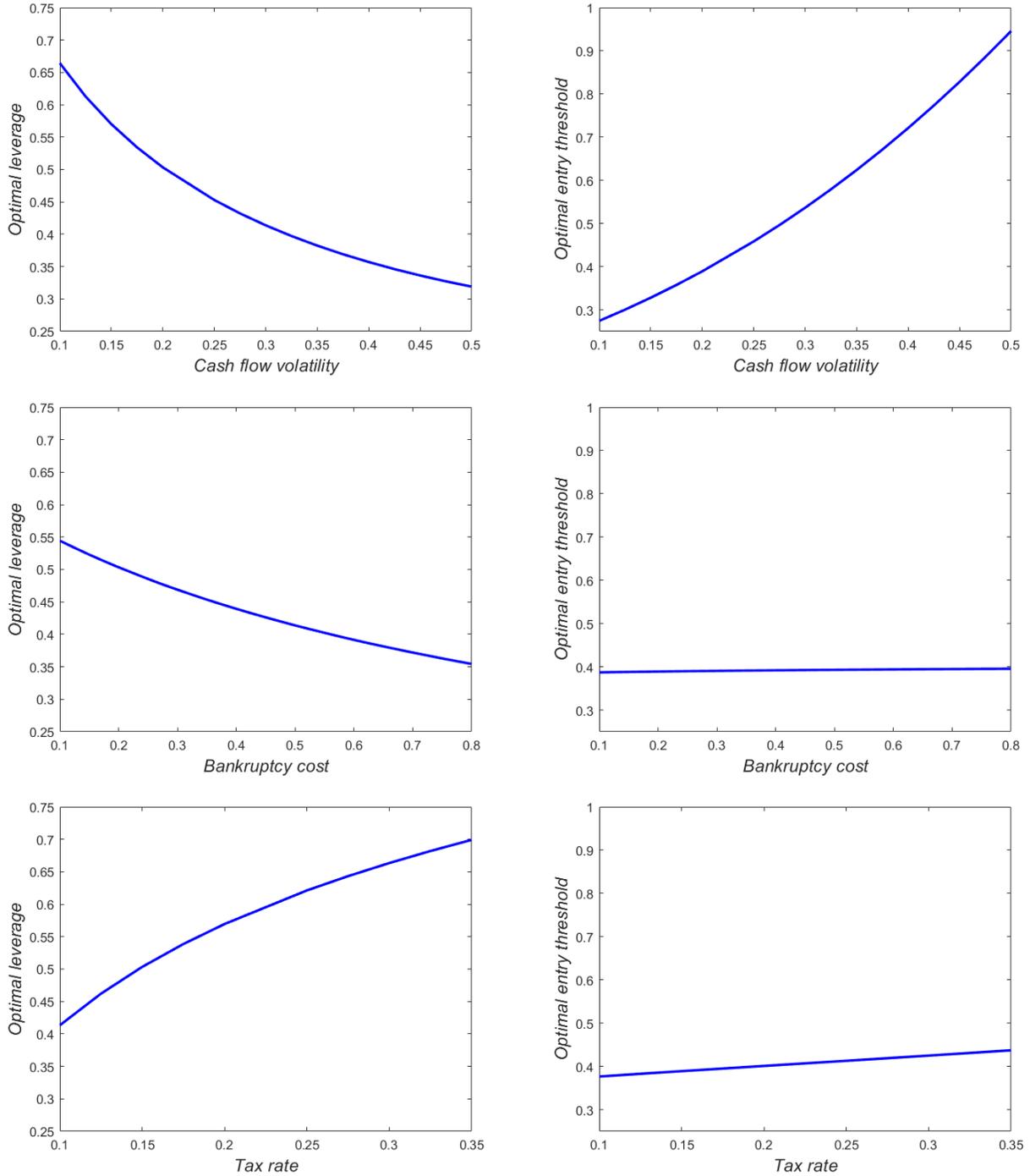


Figure 2. Optimal entry threshold and the total value of the follower.

This figure presents the optimal entry threshold and the total value of the follower's securities as functions of the leader's coupon payment for the following set of input parameters: $I = 4$; $\alpha = 0.15$; $D(q) = 3 - q$; $\eta = 0.2$; $\tau = 0.15$; $r = 0.05$; $\mu = 0.01$; $\sigma = 0.2$.

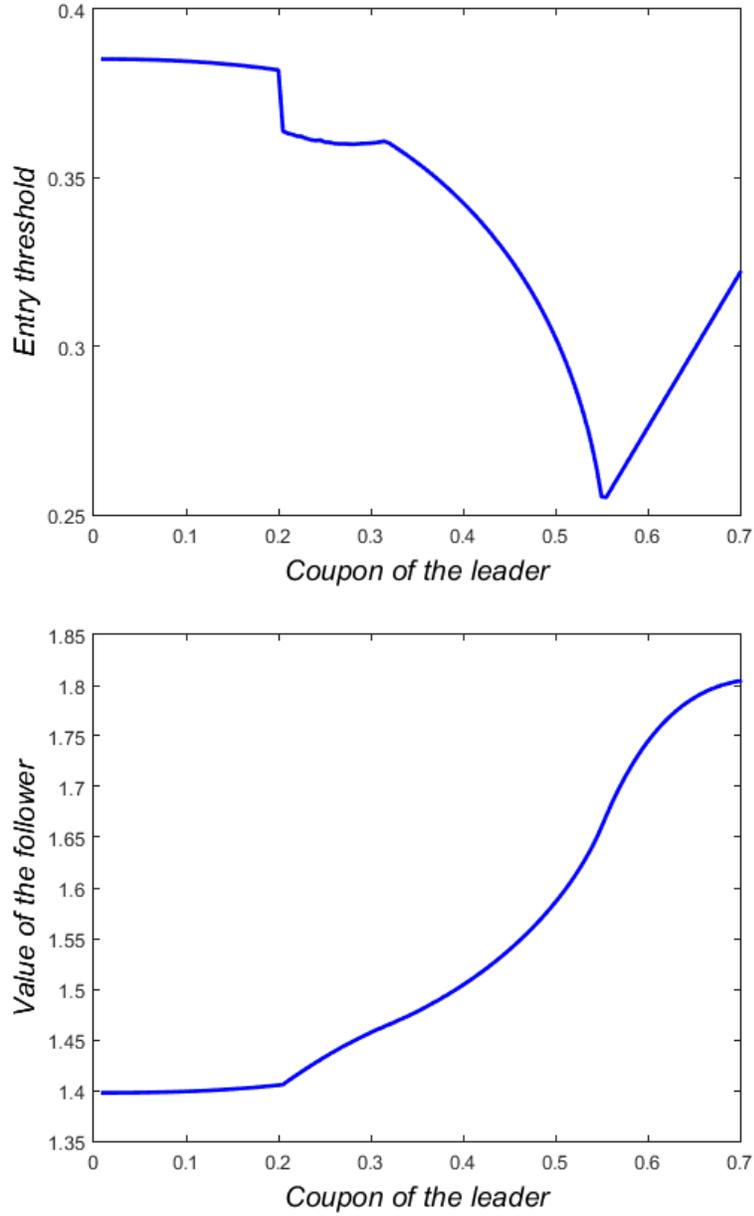


Figure 3a. Optimal coupon rate of the follower.

This figure presents the optimal coupon payment of the follower as a function of the coupon payment of the leader for the following set of input parameters: $I = 4$; $\alpha = 0.15$; $D(q) = 3 - q$; $\eta = 0.2$; $\tau = 0.15$; $r = 0.05$; $\mu = 0.01$; $\sigma = 0.2$.

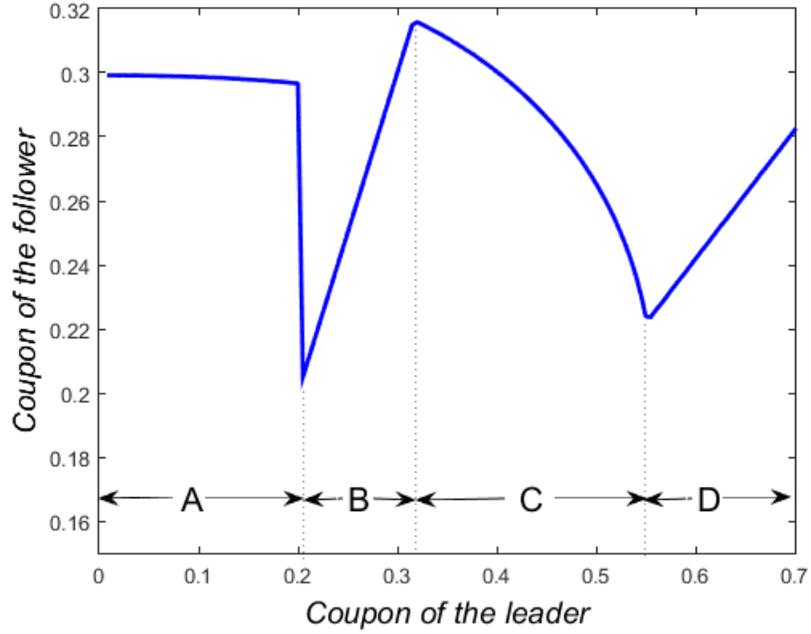


Figure 3b. Optimal leverage of the follower.

This figure presents the optimal leverage ratio of the follower as a function of the coupon payment of the leader for the base set of input parameters.

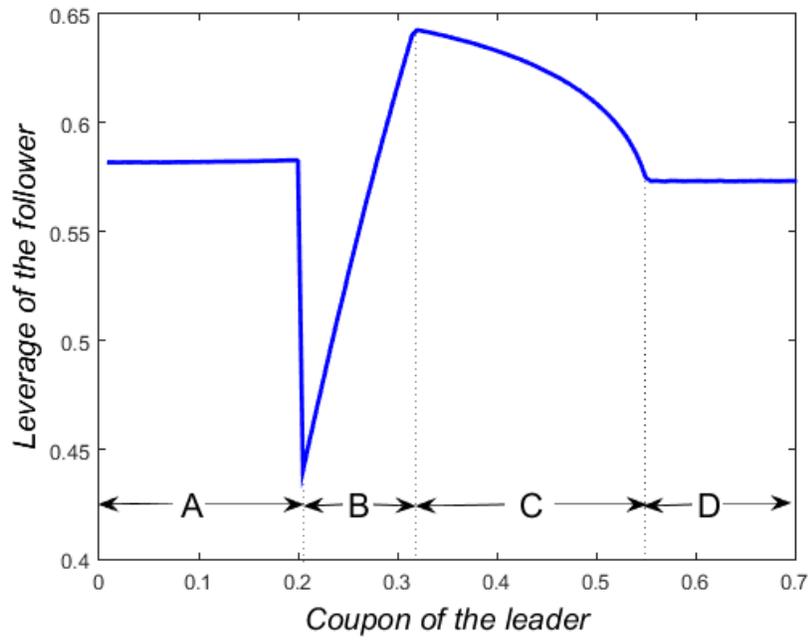


Figure 4. Values of the leader and the follower in a duopoly.

This figure shows the values of the leader and the follower in a duopoly as functions of the leader's entry threshold. The solid line represents the value of the leader; the dashed line represents the value of the follower. It is assumed that the follower does not attempt to preempt the leader's entry. The input parameters are set as in the base case environment. In addition, the initial state of the stochastic shock x_0 is set to 0.1.

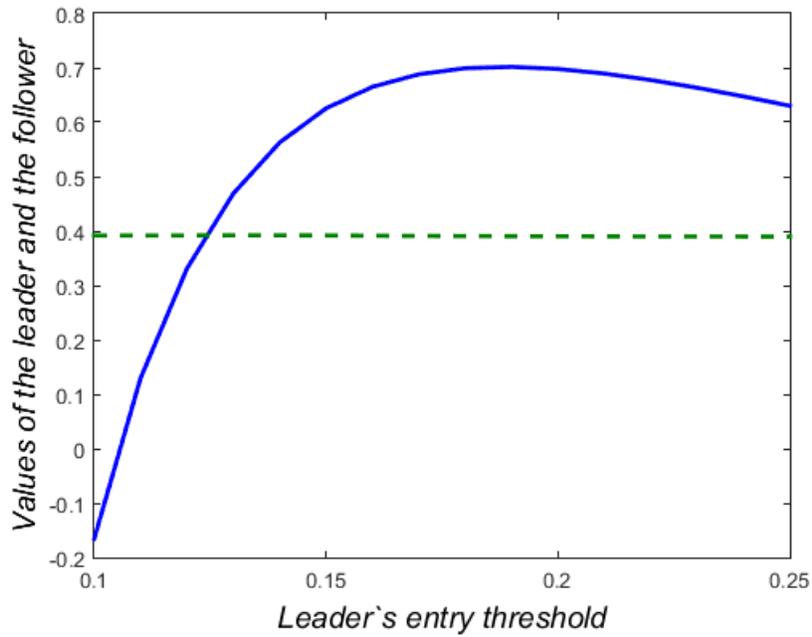


Figure 5. Optimal leverage ratios and entry thresholds of the leader and the follower with two competing firms.

This figure presents the optimal leverage ratios and entry thresholds of the leader and the follower as functions of cash flow volatility, tax rate, and direct and indirect spillover costs for the base set of input parameters. Solid lines provide the corresponding values of the leader, while the dashed lines represent the follower.

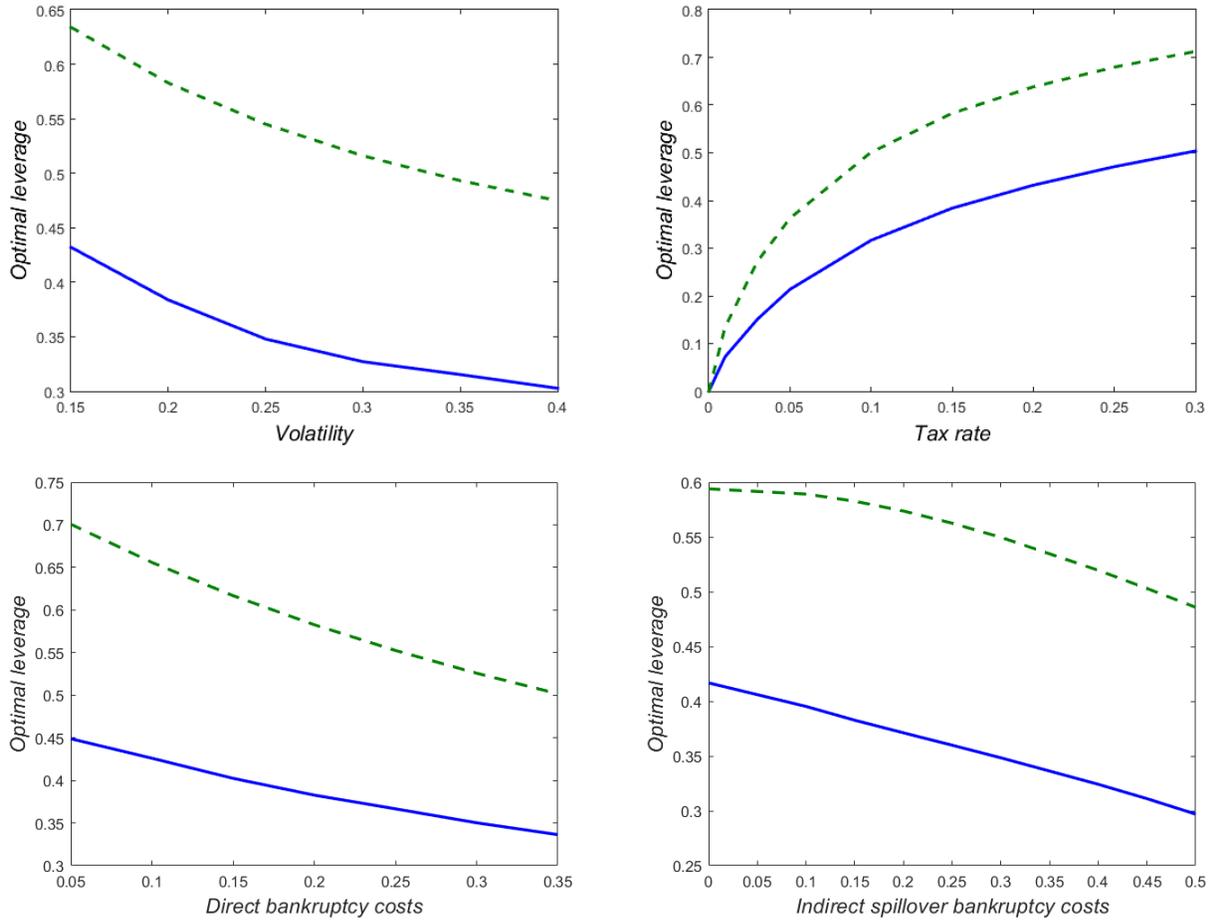


Figure 6. Industry Leverage Dispersion

This figure presents the dispersion of leverage as a function of cash flow volatility, tax rate, direct costs of bankruptcy, and indirect spillover costs of bankruptcy. Leverage dispersion is defined as the difference between leverage of the follower and leverage of the leader, normalized by the leverage of the leader.

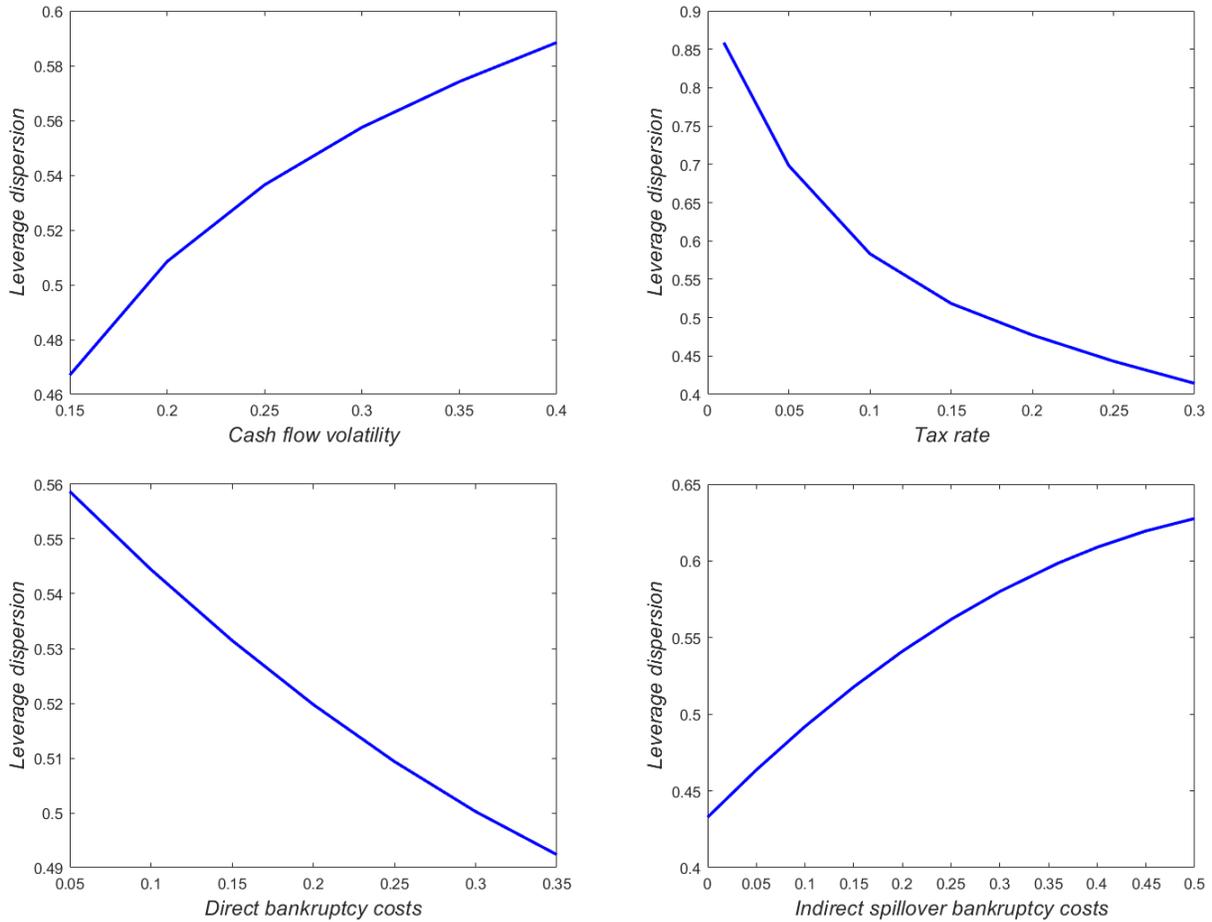


Table 1
Variable definitions

This table defines variables used in the empirical analysis of this paper. The source of data is Compustat, both at quarterly and annual frequencies. The sample covers years 1961 through 2018.

Variable	Definition
Leverage	The ratio of debt in current liabilities (DLC) plus long-term debt (DLTT) divided by the book value of assets (AT).
Cash flow volatility	This variable is constructed from quarterly operating income before depreciation (OIBDPQ). We first compute coefficient of variation, which is the standard deviation divided by the absolute value of the mean, over the proceeding 24 quarters. We require that the firm reports OIBDPQ in at least 15 quarters out of 24. We adjust this measure for the industry-average coefficient of variation, by subtracting its mean. This adjustment is done at the 2-digit SIC industries. This procedure closely follows Minton and Schrand (1999) .
MTR1	Simulated marginal tax rate (MTR) taken from Graham (1996) . This tax rate is based on income before interest expense has been deducted.
MTR2	Simulated marginal tax rate (MTR) taken from Graham (1996) . This tax rate is based on income after interest expense has been deducted.
Bankruptcy Costs	Bankruptcy costs are taken from Glover (2014) , and are estimated at the level of 17 industries defined in Fama and French (1988) .
MB	Market-to-book ratio. It is computed as the ratio of market value of a firm's equity to its book value of common equity (CEQ). Market value is computed by multiplying share price at the end of the fiscal year (PRCC_F) by the number of common shares outstanding (CSHO).
ROA	Return on assets. It is computed as the ratio of earnings before interest and taxes (EBIT) to the book value of assets (AT).
Firm size	Natural logarithm of book value of assets (AT).

Table 2
Summary Statistics

This table shows summary statistics for the variables used in the analysis. Variables are defined in Table 1. The last two rows show leverage dispersion measured within 4-digit SIC codes and within 17 industry groups defined by Fama and French (1988). The source of data is Compustat, both at quarterly and annual frequencies. The sample covers years 1961 through 2018. Column “SD” shows standard deviation, and columns “25P” and “75P” show 25th and 75th percentiles, respectively.

Variable	Mean	SD	25P	Median	75P
Leverage	0.301	0.397	0.050	0.219	0.408
Cash Flow Volatility	1.180	1.476	0.335	0.591	1.233
MTR1	29.7%	13.8%	22.3%	34.4%	36.0%
MTR2	19.0%	17.7%	0.8%	20.2%	35.0%
Bankruptcy Costs	0.425	0.050	0.374	0.422	0.463
Firm Size	4.65	2.78	2.77	4.58	6.54
ROA	-0.094	0.674	-0.024	0.055	0.113
Market-to-Book	2.33	3.92	0.82	1.49	2.78
Leverage dispersion, 4-digit SIC	0.261	0.211	0.140	0.200	0.298
Leverage dispersion, 17 FF	0.244	0.099	0.168	0.232	0.304

Table 3
Leverage and Survival Rates of New Entrants and Existing Firms

This table shows average leverage ratios and survival rates for new entrants and existing firms. In each year, new entrants are defined as firms for which Compustat reports non-missing value of book assets for the first time. The remaining firms for which book value of assets has been reported at least once before are classified as existing firms. Panel A reports results for the full sample. Panel B reports results for years 2000-2018. T-statistic is computed based on a two-sample t-test that allows for unequal variances (Welch's *t*-test).

Panel A: 1961-2018				
	Leverage	5-year survival rate	10-year survival rate	Average number of firms per year
Existing firms	29.3%	71.2%	55.9%	6,505
New entering firms	33.1%	68.6%	49.7%	590
Difference	3.8%	-2.6%	-6.2%	
T-statistic	14.60	10.03	16.08	
Panel B: 2000-2018				
	Leverage	5-year survival rate	10-year survival rate	Average number of firms per year
Existing firms	30.7%	59.8%	41.0%	9,312
New entering firms	36.4%	54.8%	35.7%	723
Difference	5.7%	-5.0%	-5.3%	
T-statistic	8.60	14.90	6.57	

Table 4
Industry Leverage Dispersion

This table shows results of estimating linear regressions of industry leverage dispersion on cash flow volatility, tax rates, bankruptcy costs, market-to-book, ROA, and firm size. All variables are defined in Table 1. Panel A shows results at the 4-digit SIC level. Panel B shows results at the level of 17 Fama and French (1988) industries defined based on SIC codes. Leverage dispersion is measured annually at the corresponding industry level. Independent variables are averages across firms in a given industry. T-statistics, shown in parenthesis, are clustered at the industry level. Bankruptcy cost estimates are taken from Glover (2014).

Panel A: Dispersion at the 4-digit SIC code level						
	(1)	(2)	(3)	(4)	(5)	(6)
Cash Flow Volatility	0.041 (8.83)	0.007 (2.18)				
MTR1			-0.695 (-14.51)	-0.237 (-5.60)		
MTR2					-0.535 (-13.43)	-0.154 (-4.72)
MB		-0.003 (-1.79)		-0.006 (-3.60)		-0.005 (-3.12)
ROA		-0.532 (-23.44)		-0.501 (-18.04)		-0.511 (-18.94)
Firm Size		-0.012 (-5.87)		-0.012 (-4.91)		-0.012 (-5.07)
Constant	0.242 (37.18)	0.325 (30.09)	0.505 (29.26)	0.415 (18.70)	0.400 (36.20)	0.373 (22.73)
R ²	0.024	0.379	0.077	0.371	0.068	0.375
Number of observations	17,091	17,086	13,641	13,626	13,922	13,907

Table 4
Industry Leverage Dispersion – Continued

Panel B: Dispersion at the level of 17 Fama and French (1988) industries

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cash Flow Volatility	0.114 (5.44)	0.037 (3.57)						
MTR1			-0.868 (-8.07)	-0.364 (-4.61)				
MTR2					-0.894 (-7.57)	-0.340 (-3.54)		
Bankruptcy Costs							0.297 (2.36)	0.094 (1.26)
MB		0.009 (1.39)		0.012 (2.01)		0.014 (2.45)		-0.001 (-0.25)
ROA		-0.614 (-6.29)		-0.549 (-5.45)		-0.517 (-5.46)		-0.619 (-8.37)
Firm Size		-0.003 (-0.43)		-0.014 (-1.39)		-0.011 (-1.29)		0.010 (2.49)
Constant	0.146 (5.88)	0.215 (6.85)	0.543 (15.42)	0.417 (6.70)	0.457 (16.92)	0.354 (6.84)	0.122 (2.24)	0.169 (3.79)
R2	0.199	0.693	0.271	0.679	0.382	0.680	0.024	0.723
Nobs	883	883	595	595	595	595	891	856

Appendix

Proof of Lemma 1

The set of inequalities (4) is equivalent to the following set:

$$\begin{aligned}(1 - \alpha)D(2 - 2\alpha) &< D(2 - a), \\ (1 - \alpha)D(2 - \alpha) &< D(2), \\ (1 - \alpha)D(1 - \alpha) &< D(1),\end{aligned}$$

or

$$(1 - \alpha)D(\gamma - \alpha) < D(\gamma) \tag{A.1}$$

for $\gamma \in \{1, 2 - a, 2\}$.

Assume that condition (3) holds for $q \in [1 - \alpha, 2]$. Then for any $\gamma \in \{1, 2 - a, 2\}$

$$D(\gamma) = D(\gamma - \alpha) + \int_{\gamma - \alpha}^{\gamma} \frac{dD(y)}{dy} dy \geq D(\gamma - \alpha) - \int_{\gamma - \alpha}^{\gamma} D(y) dy > D(\gamma - \alpha) - D(\gamma - \alpha)a,$$

which is equivalent to (A.1).

■

Proof of Proposition 1

Using standard arguments, it is straightforward to show that the Bellman equation corresponding to the optimization problem of the equityholders is:

$$re_i(x) = (1 - \tau)(x\pi_{hn} - s_i) + \frac{1}{dt}\mathbf{E}^x(de_i(x)), \tag{A.2}$$

where $e_i(x)$ is the equity value of firm i , \mathbf{E}^x is the expectation operator, $n = h$ if firm j (the rival) is in the healthy (solvent) state, and $n = b$ otherwise. Equation (A.2) states that the instantaneous rate of return on equity equals the instantaneous cash flows to equityholders plus the expected instantaneous change in the value of equity.

Equation (A.2) is equivalent to the following ODE:

$$\frac{1}{2}x^2\sigma^2e_{ixx}(x) + \mu xe_{ix}(x) - re_i(x) + (1 - \tau)(x\pi_{hn} - s_i) = 0.$$

The solution of this equation is given by

$$e_i(x) = Ax^{\beta_1} + Bx^{\beta_2} + (1 - \tau) \left[\frac{x\pi_{hn}}{r - \mu} - \frac{s_i}{r} \right], \tag{A.3}$$

where β_1 and β_2 are respectively the positive and the negative roots of the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$, and A and B are some constants to be determined.

As the level of the stochastic shock goes up to infinity, the probability of bankruptcy becomes negligibly small. Therefore, the value of the firm's equity must converge to the value of the

perpetual entitlement to the cash flows received by equityholders (ignoring default), i.e.

$$\lim_{x \rightarrow \infty} \frac{e_i(x)}{(1 - \tau) \left[\frac{x\pi_{hn}}{r - \mu} - \frac{s_i}{r} \right]} = 1.$$

It follows that A must equal zero. The optimal default threshold is obtained by using the following value-matching and smooth-pasting conditions:

$$e_i(x_i) = 0 = Bx_i^{\beta_2} + (1 - \tau) \left[\frac{x\pi_{hn}}{r - \mu} - \frac{s_i}{r} \right] \quad (\text{A.4})$$

and

$$e_{ix}(x_i) = 0 = \beta_2 Bx_i^{\beta_2 - 1} + \frac{(1 - \tau)\pi_{hn}}{r - \mu} = 0. \quad (\text{A.5})$$

Solving equations (A.4) and (A.5) together yields the desired result.

■

Proof of Proposition 2

First, it is useful to establish the relationship among different default thresholds. The following inequalities hold: $x_b^d(\pi_{hh}, s_j) > x_b^d(\pi_{hh}, s_i)$ and $x_b^d(\pi_{hb}, s_j) > x_b^d(\pi_{hb}, s_i)$. If $x_b^d(\pi_{hb}, s_j) > x_b^d(\pi_{hh}, s_i)$, then the only sustainable equilibrium is the one involving firm j defaulting at $x_b^d(\pi_{hh}, s_j)$. Therefore, the only non-trivial case is $x_b^d(\pi_{hh}, s_j) > x_b^d(\pi_{hh}, s_i) > x_b^d(\pi_{hb}, s_j) > x_b^d(\pi_{hb}, s_i)$.

Second, it is important to introduce the notion of a “reservation threshold” of the weaker firm, $x_r(s_j)$. The shareholders of the weaker firm are ex-post indifferent between defaulting at $x_b^d(\pi_{hh}, s_j)$ and $x_b^d(\pi_{hb}, s_j)$, provided that the other (stronger) firm defaults at x_r . Apparently, $x_b^d(\pi_{hb}, s_j) < x_r(s_j) < x_b^d(\pi_{hh}, s_j)$. The reservation threshold is a point such that the expected losses incurred in result of operating while competing with a “healthy” firm up until this point are exactly offset by the profits to be received later, once the rival has defaulted. Using standard arguments, it can be shown that $x_r(s_j)$ is given by the solution to the following equation

$$A(z) + B(z) = 0, \quad (\text{A.6})$$

where

$$A(z) = \frac{x_b^d(\pi_{hh}, s_j)\pi_{hh}}{r - \mu} - \frac{s_j}{r} - \left(\frac{x_b^d(\pi_{hh}, s_j)}{z} \right)^{\beta_2} \left(\frac{z\pi_{hh}}{r - \mu} - \frac{s_j}{r} \right)$$

is the expected loss to be incurred while competing with a healthy rival, and

$$B(z) = \left(\frac{x_b^d(\pi_{hb}, s_j)}{z} \right)^{\beta_2} \left[\frac{z\pi_{hb}}{r - \mu} - \frac{s_j}{r} - \left(\frac{z}{x_b^d(\pi_{hb}, s_j)} \right)^{\beta_2} \left(\frac{x_b^d(\pi_{hb}, s_j)\pi_{hb}}{r - \mu} - \frac{s_j}{r} \right) \right]$$

is the gain to be received afterwards. The existence and uniqueness of the solution to (A.6) follows from the monotonicity of $A(z) + B(z)$ and the corresponding boundary conditions. Indeed, one can easily notice that $A(x_b^d(\pi_{hb}, s_j)) < 0$, $B(x_b^d(\pi_{hb}, s_j)) = 0$, $B(x_b^d(\pi_{hh}, s_j)) > 0$,

$A(x_b^d(\pi_{hh}, s_j)) = 0$. Also,

$$\frac{dA(z)}{dz} = - \left(\frac{x_b^d(\pi_{hh}, s_j)}{z} \right)^{\beta_2} \left[\frac{\pi_{hh}(1 - \beta_2)}{r - \mu} + \frac{s_j \beta_2}{rz} \right] > 0$$

for $z < x_b^d(\pi_{hh}, s_j) = \frac{\beta_2}{\beta_2 - 1} \frac{s_j(r - \mu)}{\pi_{hh}r}$, and similarly $\frac{dB(z)}{dz} < 0$.

Therefore, if $C(z) = A(z) + B(z)$, then $C(x_b^d(\pi_{hb}, s_j)) < 0$, $C(x_b^d(\pi_{hh}, s_j)) > 0$, $\frac{dC(z)}{dz} < 0$ for $x_b^d(\pi_{hb}, s_j) < z < x_b^d(\pi_{hh}, s_j)$. The continuity of $C(z)$ then guarantees the existence and uniqueness of the reservation threshold $x_r(s_j)$.

If the reservation threshold $x_r(s_j)$ is greater than the default threshold of firm i , $x_b^d(\pi_{hh}, s_i)$, then the best that the shareholders of firm j can do is to default at $x_b^d(\pi_{hh}, s_j)$, since the other firm will definitely default only after the reservation threshold $x_r(s_j)$ has been hit. Therefore, the only non-trivial case left is when the relation among the various thresholds considered above is as follows:

$$x_b^d(\pi_{hh}, s_j) > x_b^d(\pi_{hh}, s_i) > x_r(s_j) > x_b^d(\pi_{hb}, s_j) > x_b^d(\pi_{hb}, s_i). \quad (\text{A.7})$$

Only if (A.7) holds may the equityholders of the weaker firm decide to default at $x_b^d(\pi_{hb}, s_j)$. However, we show below proof that it can never occur in any subgame perfect Nash Equilibrium. To prove that, it is useful to examine what happens if $x_b^d(\pi_{hb}, s_j)$ is hit, but neither of the firms has defaulted. At $x_b^d(\pi_{hb}, s_j)$ the shareholders of firm j default immediately as they have no more incentives to wait. (The only reason why they have not defaulted before is because they anticipated that firm i would default before $x_b^d(\pi_{hb}, s_j)$ was hit.)

Consequently, firm j is partially liquidated, resulting in a positive shock to the profit of firm i , whose shareholders are then guaranteed the expected payoff of

$$u = (1 - \tau) \left[\frac{x_b^d(\pi_{hb}, s_j)\pi_{hb}}{r - \mu} - \frac{s_i}{r} - \left(\frac{x_b^d(\pi_{hb}, s_j)}{x_b^d(\pi_{hb}, s_i)} \right)^{\beta_2} \left(\frac{x_b^d(\pi_{hb}, s_i)\pi_{hb}}{r - \mu} - \frac{s_i}{r} \right) \right].$$

Therefore, there exists a reservation threshold y_1 , such that the stronger firm is indifferent between defaulting at y_1 or waiting until $x_b^d(\pi_{hb}, s_j)$ is hit, and the weaker firm defaults (if it has not defaulted before). y_1 can be found as the root of the following (non-linear) equation:

$$(1 - \tau) \left[\frac{y_1\pi_{hh}}{(r - \mu)} - \frac{s_i}{r} - \left(\frac{y_1}{x_b^d(\pi_{hb}, s_j)} \right)^{\beta_2} \left(\frac{x_b^d(\pi_{hb}, s_j)\pi_{hh}}{(r - \mu)} - \frac{s_i}{r} \right) \right] + \left(\frac{y_1}{x_b^d(\pi_{hb}, s_j)} \right)^{\beta_2} u = 0.$$

The existence and uniqueness of y_1 follows from the same reasoning as was used above to prove the existence and uniqueness of the solution to (A.6).

Therefore, the stronger firm will never exit while x stays between $x_b^d(\pi_{hb}, s_j)$ and y_1 . The shareholders of the weaker firm are rational, and fully aware of the nature of the game. Therefore, they have no incentive to wait after y_1 is hit and they will default at y_1 , at the latest. (Note that this argument does not necessarily hold if one allows for equilibria on disconnected sets.) The same argument can be applied again to show that there exists another threshold $y_2 > y_1 > x_b^d(\pi_{hb}, s_j)$, such that the stronger firm will never default while $x \in (x_b^d(\pi_{hb}, s_j), y_2)$, and therefore the weaker firm must default at the first passage time to y_2 or earlier. Using the same argument iteratively, we construct an increasing sequence of thresholds y_k , constraining

the equilibrium default strategies of the levered firm. If

$$y_k > x_r(s_j)$$

holds for some k , then the only equilibrium strategy left for the weaker firm is to default at $x_b^d(\pi_{hh}, s_j)$.

To show that such k indeed exists, let us assume the contrary. Since the sequence y_k is increasing and (by assumption) bounded above by $x_r(s_j)$, it must converge to some limit $\bar{y} \leq x_r(s_j)$. Therefore, for any $\varepsilon > 0$ there exists some n , such that

$$y_{n+1} - y_n < \varepsilon. \quad (\text{A.8})$$

However, y_{n+1} by construction is given by the root of the following equation

$$(1 - \tau) \left[\frac{y_{n+1}\pi_{hh}}{r - \mu} - \frac{s_i}{r} - \left(\frac{y_{n+1}}{y_n} \right)^{\beta_2} \left(\frac{y_n\pi_{hh}}{r - \mu} - \frac{s_i}{r} \right) \right] + \\ (1 - \tau) \left(\frac{y_{n+1}}{y_n} \right)^{\beta_2} \left[\frac{y_n\pi_{hb}}{r - \mu} - \frac{s_i}{r} - \left(\frac{y_n}{x_b^d(\pi_{hh}, s_j)} \right)^{\beta_2} \left(\frac{x_b^d(\pi_{hh}, s_j)\pi_{hb}}{r - \mu} - \frac{s_i}{r} \right) \right] + \left(\frac{y_{n+1}}{x_b^d(\pi_{hh}, s_j)} \right)^{\beta_2} u = 0.$$

Apparently, when $y_n \rightarrow y_{n+1}$, $G(y_n, y_{n+1}) = \left[\frac{y_{n+1}\pi_{hh}}{r - \mu} - \frac{s_i}{r} - \left(\frac{y_{n+1}}{y_n} \right)^{\beta_2} \left(\frac{y_n\pi_{hh}}{r - \mu} - \frac{s_i}{r} \right) \right] \rightarrow 0$.

Therefore, for any $\delta > 0$ there exists $\varepsilon > 0$, such that $y_{n+1} - y_n < \varepsilon$ implies that $G(y_n, y_{n+1}) < \delta$, or

$$u < \delta(1 - \tau) \left(\frac{y_{n+1}}{x_b^d(\pi_{hh}, s_j)} \right)^{-\beta_2},$$

which cannot be true, since δ can be made arbitrarily small and y_{n+1} is by assumption bounded by $x_r(s_j)$.

■

Proof of Proposition 3

As established by Proposition 2, firm i , the stronger one, defaults last. After its default, there will be no more changes in the competitive environment, so its post-default value equals the present value of the perpetual flow of $(1 - \tau)x\pi_{bb}$, net of the proportional liquidation cost η :

$$A_i(x) = (1 - \eta)(1 - \tau) \frac{x\pi_{bb}}{r - \mu}.$$

The post-default value of firm j is given by

$$A_j(x) = (1 - \eta)(1 - \tau) \left(Bx^{\beta_2} + \frac{x\pi_{bh}}{r - \mu} \right),$$

subject to

$$A_j(x_b^d(\pi_{hb}, s_i)) = (1 - \eta)(1 - \tau) \frac{x_b^d(\pi_{hb}, s_i)\pi_{bb}}{r - \mu},$$

which implies that

$$B = \frac{1}{(x_b^d(\pi_{hb}, s_i))^{\beta_2}} \left(\frac{x_b^d(\pi_{hb}, s_i)\pi_{bb}}{r - \mu} - \frac{x_b^d(\pi_{hb}, s_i)\pi_{bh}}{r - \mu} \right)$$

and

$$A_j(x) = (1 - \eta)(1 - \tau) \left\{ \frac{x\pi_{bh}}{r - \mu} - \left(\frac{x}{x_b^d(\pi_{hb}, s_i)} \right)^{\beta_2} \left(\frac{x_b^d(\pi_{hb}, s_i)\pi_{bh}}{r - \mu} - \frac{x_b^d(\pi_{hb}, s_i)\pi_{bb}}{r - \mu} \right) \right\}.$$

■

Proof of Proposition 4

Let us focus on the case when $s_1 < s_2$ (the follower is the weaker firm). The values of the follower's securities when $s_1 > s_2$ can be found in a similar way. Using standard arguments, one can show that the follower's equity value is given by

$$e_2(x) = (1 - \tau) \left(\frac{x\pi_{hh}}{r - \mu} - \frac{s_2}{r} \right) + Bx^{\beta_2},$$

subject to the value-matching condition:

$$e_2(x_b^d(\pi_{hh}, s_2)) = 0,$$

where $x_b^d(\pi_{hh}, s_2)$ is the default threshold of the weaker firm, given by (5). Solving these two equations together yields the desired result.

The value of debt can be found similarly by combining

$$d_2(x) = \frac{s_2}{r} + Cx^{\beta_2}$$

with the value-matching condition

$$d_2(x_b^d(\pi_{hh}, s_2)) = A_j(x_b^d(\pi_{hh}, s_2)),$$

where $A_j(x_b^d(\pi_{hh}, s_2))$ is the post-default value of the weaker firm, given by (8).

■

Proof of Proposition 5

The value of the follower comes from two different sources: the value of the option to wait until the leader defaults and to enter later, and the value of the option to join the leader while it is in the solvent (healthy) state. The follower's strategy is given by a pair of its entry trigger $x_{2e}(\theta_1)$ and its contractual coupon payment s_2 . Suppose that the entry of the leader occurs at x_{1e} (and is observed by the follower).

While $x \in (x_b^m(\pi_h, s_1), x_{2e}(1))$, where $x_b^m(\pi_h, s_1)$ is the optimal default threshold of the leader corresponding to the case when the follower is still inactive, the total value of the follower's

securities, $V_2(x)$ satisfies the following ODE:

$$\frac{1}{2}x^2\sigma^2V_{2xx}(x) + \mu xV_{2x}(x) - rV_2(x) = 0 \quad (\text{A.9})$$

with the boundary conditions $V_2(x_{2e}(1)) = e_2(x_{2e}(1), 1) + d_2(x_{2e}(1), 1) - I$ and $V_2(x_b^m(\pi_h, s_1)) = \left[\frac{x_b^d(\pi_h, s_1)}{x_{2e}(0)}\right]^{\beta_1} (e_2(x_{2e}(0), 0) + d_2(x_{2e}(0), 0) - I)$.

The solution of (A.9) is

$$V_2(x) = Ax^{\beta_1} + Bx^{\beta_2}, \quad (\text{A.10})$$

where the constants A and B together with the optimal entry threshold are to be determined through the set of boundary conditions in the following way:

$$A = \frac{V_2(x_{2e}(1)) (x_b^m(\pi_h, s_1))^{\beta_2} - V_2(x_b^m(\pi_h, s_1))x_{2e}^{\beta_2}(1)}{x_{2e}^{\beta_1} (x_b^m(\pi_h, s_1))^{\beta_2} - (x_b^m(\pi_h, s_1))^{\beta_1} x_{2e}^{\beta_2}(1)}$$

and

$$B = \frac{V_2(x_b^m(\pi_h, s_1))x_{2e}^{\beta_1}(1) - V_2(x_{2e}(1)) (x_b^m(\pi_h, s_1))^{\beta_1}}{x_{2e}^{\beta_1} (x_b^m(\pi_h, s_1))^{\beta_2} - (x_b^m(\pi_h, s_1))^{\beta_1} x_{2e}^{\beta_2}(1)}.$$

Plugging the values of A and B into (A.10) yields:

$$\begin{aligned} V_2[x_{1e}(1)] &= Ax_{1e}^{\beta_1}(1) + Bx_{1e}^{\beta_2}(1) = \\ &\quad \{\mathcal{H}(x_{1e}; x_b^m(\pi_h, s_1), e_2(x_{2e}(1))) [e_2(x_{2e}(1), \theta_1 = 1) + d_2(x_{2e}(1), \theta_1 = 1) - I] + \\ &\quad \left[\frac{x_b^m(\pi_h, s_1)}{x_{2e}(0)}\right]^{\beta_1} \mathcal{L}(x_{1e}; x_b^m(\pi_h, s_1), e_2(x_{2e}(1))) (e_2(x_{2e}(0), \theta_1 = 0) + d_2(x_{2e}(0), \theta_1 = 0) - I)\}, \end{aligned} \quad (\text{A.11})$$

where the quantities $\mathcal{L}(x; z, y)$ and $\mathcal{H}(x; z, y)$ are given by (17) and (18).

Equation (A.11) gives the value of the follower at the time of the entry of the leader, i.e. at the first time x_{1e} is hit. The value of the follower corresponding to the current state of the stochastic shock x_0 is obtained by multiplying $V_2[x_{1e}(1)]$ by the appropriate discount factor:

$$V_2(x_0) = \left[\frac{x_0}{x_{1e}}\right]^{\beta_1} V_2(x_{1e}).$$

■

Proof of Proposition 6

The value of the leader is an appropriately weighted sum of its discounted payoffs to be received in the two alternative scenarios: 1) the follower enters while the leader has not yet defaulted, 2) the follower enters after the leader's foreclosure. The optimal default threshold $x_b^m(\pi_h, s_1)$ is chosen so as to maximize the leader's equity value.

Note that once the follower has entered (at x_{2e}), the values of the leader's securities can be obtained from Proposition 4 by replacing $e_2(\cdot)$ and $d_2(\cdot)$ with $e_1(\cdot)$ and $d_1(\cdot)$, s_1 with s_2 , and θ_1 with θ_2 . Let us call these values $e_1^*(x_{2e})$ and $d_1^*(x_{2e})$. Both $e_1^*(x_{2e})$ and $d_1^*(x_{2e})$ depend not

only on the coupon payment of the leader s_1 , but also on the anticipated coupon payment of the follower, s_2 , because the order in which the two firms default is determined by the relative magnitudes of their coupon payments.

After its own entry, but before the entry of the follower, the leader's equity value satisfies the following ODE:

$$\frac{1}{2}x^2\sigma^2e_{1xx}(x) + \mu xe_{1x}(x) - re_1(x) + (1 - \tau)(x\pi_h - s_1) = 0, \quad (\text{A.12})$$

with the boundary conditions $e_1(x_b^m(\pi_h, s_1)) = 0$, $e_1(x_{2e}) = e_1^*(x_{2e})$, and $\frac{de_1(x)}{dx}|_{x_b^m(\pi_h, s_1)} = 0$. Similarly, the value of debt is a solution to

$$\frac{1}{2}x^2\sigma^2d_{1xx}(x) + \mu xd_{1x}(x) - rd_1(x) + s_1 = 0, \quad (\text{A.13})$$

with the boundary conditions $d_1(x_b^m(\pi_h, s_1)) = A_1(x_b^m(\pi_h, s_1))$ and $d_1(x_{2e}) = d_1^*(x_{2e})$, where $A_1(x_b^m(\pi_h, s_1))$ is the leader's post-default value realized in the case when the leader defaults before the follower's entry occurs. $A_1(x_b^m(\pi_h, s_1))$ is not equal to the monopoly post-default value because there exists a possibility that the follower enters even after the leader has declared bankruptcy.

Solutions to (A.12) and (A.13) are

$$e_1(x) = \alpha_1 x^{\beta_1} + \alpha_2 x^{\beta_2} + (1 - \tau)\left(\frac{x\pi_h}{r - \mu} - \frac{s_1}{r}\right)$$

and

$$d_1(x) = \gamma_1 x^{\beta_1} + \gamma_2 x^{\beta_2} + \frac{s_1}{r},$$

which, together with the boundary conditions yield:

$$\begin{aligned} \alpha_1 &= \frac{\left\{e_1^*(x_{2e}) - (1 - \tau)\left(\frac{x_{2e}\pi_h}{r - \mu} - \frac{s_1}{r}\right)\right\} [x_b^m(\pi_h, s_1)]^{\beta_2} + (1 - \tau)\left(\frac{x_b^m(\pi_h, s_1)\pi_h}{r - \mu} - \frac{s_1}{r}\right)x_{2e}^{\beta_2}}{[x_b^m(\pi_h, s_1)]^{\beta_2}x_{2e}^{\beta_1} - [x_b^m(\pi_h, s_1)]^{\beta_1}x_{2e}^{\beta_2}}, \\ \alpha_2 &= \frac{-(1 - \tau)\left(\frac{x_b^m(\pi_h, s_1)\pi_h}{r - \mu} - \frac{s_1}{r}\right)x_{2e}^{\beta_1} - \left\{e_1^*(x_{2e}) - (1 - \tau)\left(\frac{x_{2e}\pi_h}{r - \mu} - \frac{s_1}{r}\right)\right\} [x_b^m(\pi_h, s_1)]^{\beta_2}}{[x_b^m(\pi_h, s_1)]^{\beta_2}x_{2e}^{\beta_1} - [x_b^m(\pi_h, s_1)]^{\beta_1}x_{2e}^{\beta_2}}, \\ \gamma_1 &= \frac{\left(d_1^*(x_{2e}) - \frac{s_1}{r}\right) [x_b^m(\pi_h, s_1)]^{\beta_2} - A_1(x_b^m(\pi_h, s_1) - \frac{s_1}{r})x_{2e}^{\beta_2}}{[x_b^m(\pi_h, s_1)]^{\beta_2}x_{2e}^{\beta_1} - [x_b^m(\pi_h, s_1)]^{\beta_1}x_{2e}^{\beta_2}}, \\ \gamma_2 &= \frac{A_1(x_b^m(\pi_h, s_1) - \frac{s_1}{r})x_{2e}^{\beta_1} - \left(d_1^*(x_{2e}) - \frac{s_1}{r}\right) [x_b^m(\pi_h, s_1)]^{\beta_1}}{[x_b^m(\pi_h, s_1)]^{\beta_2}x_{2e}^{\beta_1} - [x_b^m(\pi_h, s_1)]^{\beta_1}x_{2e}^{\beta_2}}. \end{aligned}$$

The post-default value $A_1(x_b^m(\pi_h, s_1))$ is given by

$$\begin{aligned}
A_1(x_b^m(\pi_h, s_1)) &= (1 - \eta) \left\{ \mathbf{E}^{x_b^m(\pi_h, s_1)} \int_0^{T(x_{2e}(0))} e^{-rt} (1 - \tau) x \pi_b dt + \right. \\
&\quad \mathbf{E}^{x_b^m(\pi_h, s_1)} \int_{T(x_{2e}(0))}^{T(x_b^d(\pi_{hb}, s_2))} e^{-rt} (1 - \tau) x \pi_{bh} dt + \\
&\quad \left. \mathbf{E}^{x_b^m(\pi_h, s_1)} \int_{T(x_b^d(\pi_{hb}, s_2))}^{\infty} e^{-rt} (1 - \tau) x \pi_{bb} dt \right\} = \\
&\quad (1 - \eta)(1 - \tau) \left\{ \frac{x_b^m(\pi_h, s_1) \pi_b}{(r - \mu)} + \right. \\
&\quad \left. \left(\frac{x_b^m(\pi_h, s_1)}{x_{2e}(0)} \right)^{\beta_1} \left[\frac{x_{2e}(0) (\pi_{bh} - \pi_b)}{r - \mu} + \left(\frac{x_{2e}(0)}{x_b^d(\pi_{hb}, s_2)} \right)^{\beta_2} \frac{x_b^d(\pi_{hb}, s_2) (\pi_{bb} - \pi_{bh})}{r - \mu} \right] \right\},
\end{aligned}$$

where \mathbf{E} is the expectation operator and $T(z)$ is the first passage time of the stochastic shock x to a threshold z .

The expected value of the leader corresponding to the initial state of the stochastic shock x_0 is given by its value upon entry multiplied by the appropriate discount factor $\left[\frac{x_0}{x_{1e}} \right]^{\beta_1}$:

$$V_1(x_0) = \left[\frac{x_0}{x_{1e}} \right]^{\beta_1} (e_1(x_{1e}) + d_1(x_{1e}) - I).$$

■