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The Value Priority Hypotheses for Consumer Budget Plans

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Based on the behavioral sciences and mathematical programming, we hypothesize that consumers rank durables by a value (or net value) priority approximated by utility per dollar (or utility minus price) and plan to choose items in that order up to a budget cutoff. This paper derives these hypotheses and develops a convergent linear programming procedure to estimate utility. Using primary field data on reservation prices, purchase probabilities, lottery orders, and combination prizes, we estimate utilities and compare the hypotheses to 215 actual budget plans. LISREL V analysis provides further support for the hypotheses.

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Purchases of major consumer goods such as automobiles, home computers, and video cassette recorders account for substantial budget outlays by consumers. Such purchases have a major impact on national economic conditions and represent a challenging research issue. Scientific interest is strong because major consumer goods purchases depend upon inter-category comparisons (e.g., auto versus home computers) and upon the impact of limited consumer budgets. Managerial interest is strong because understanding the effects on families' purchases of relative price and competitive entry along with recession, inflation, and tax policy is critical for established products. In developing new durable goods, managerial and research attention is high because new product development costs are large (e.g., in automobiles such costs can exceed one billion dollars), and because key strategic decisions must be made prior to new product launch.

This article seeks to increase our understanding of consumer purchasing decisions. We focus on those consumer goods, usually durables, that consumers include in their budget plans and we examine marketing science hypotheses of how goods are prioritized within a budget. Our hypotheses are derived from mathematical programming and economic reasoning as modified by behavioral science considerations. The hypotheses represent what we believe are dominant effects and are proposed as a reasonable approximation to describe consumer planning behavior. They also provide a basis for future research.

In particular, this paper reviews the value priority hypothesis for consumer purchases, introduces a rival variation—the net value priority hypothesis, and discusses their interrelationships based on economics, marketing, and management science. We describe data collected to test the hypotheses and linear programming procedures to estimate the underlying model from the data. We then test the hypotheses by comparing their predictions to actual consumer budget plans, and we provide convergent tests with an alternative estimation procedure, LISREL V. We close with a discussion of some managerial implications.

VALUE PRIORITY HYPOTHESIS

We begin with the single period consumer model. Appendix A discusses how the model can be extended to multiple periods that include borrowing, savings, depreciation, operating costs, trade-ins, and interproduct complementarity. In a single period the consumer faces a fixed budget that s/he must allocate. For some goods, s/he plans explicitly, for others s/he does not. For the sake of simplicity, we will call planned-for items *durable goods*, although such items can include major expenditures such as those for vacations or tuition. Let g_j be the number of items of "durable" good j s/he purchases; g_j, is usually 0 (no purchase) or 1 (e.g., purchase one automobile), but it can be any integer (e.g., purchase two color televisions). Following standard economic theory (e.g., Rosen 1974) let y be a summary of the consumer's allocation to other goods (e.g., \$5,000 to household products), and let B be the consumer's budget. Let $U(.,\ .,\ \ldots)$ be the consumer's utility function and let p_j be the price s/he expects to pay for "durable" good j. Then the consumer's decision problem is represented mathematically as:

$$\begin{aligned} &\text{maximize} && U(g_1, g_2, g_3, \ldots, g_n, y) && \textbf{(MP1)} \\ &\text{subject to:} && p_1 g_1 + p_2 g_2 + \ldots + p_n g_n + y \leq B. && \end{aligned}$$

MP1 is the standard microeconomic consumer behavior model. Depending upon the functional form of the utility function, the solution to MP1 can involve complex nonlinear searches of all possible combinations of goods purchases. Exact solution of MP1 may be difficult for even the most advanced mathematical programming computer algorithms; thus, it is unlikely that consumers solve MP1 in its full complexity for everyday purchase decisions.

Various scientific disciplines including new economic theory (Heiner 1983), information processing theory (Sternthal and Craig 1982; Bettman 1979), social psychology (Johnson and Tversky 1983), mathematical psychology (Tversky and Kahneman 1974), and marketing science (Shugan 1980) suggest modifications to MP1. A variety of authors propose the simpler model that consumers establish and follow a buying order for durables. See, for example, Brown, Buck, and Pyatt (1965), Clarke and Soutar (1982), Dickson, Lusch, and Wilkie (1983), Kasulis, Lusch, and Stafford (1979), and Paroush (1965).

Such a prioritized buying order is consistent with a modified MP1. Suppose that the consumer can assign to each good a marginal utility, u_j, that represents the amount of utility s/he gets from possessing that durable good.[1] (We assume that u_j can be ratio scaled.) If the consumer considers more than one unit of the durable good, we assign values u_{j1}, u_2,..., etc. to the first, second, etc. units of good j with the usual assumption that $u_{j1} > u_{j2}$, etc. However, to simplify exposition we temporarily assume that g_j is at most one item. This is not a restriction in the theory.[2] MP1 now becomes MP2.

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[1] Technically this is an assumption of separability (Blackorby, Primont, and Russell 1975). Separability is a common assumption in economic modeling because it makes the general process (MP1) feasible to model. This becomes important when, as in this paper, we also seek an empirical realization of the model. In theory, one could include complementarity by making one utility, u_j, a function of which other goods are purchased, e.g., $u_j = u_j(g_k$ for all $k \neq j)$. Of course, for any practical problem the general function, $u_j(\cdot)$, must be simplified to, say, pairwise complementarity. For example, in Appendix A, Equation Al, u_{j1} could be a function of previous purchases. We leave such extensions for future research.

[2] If we allow g_j to be any integer, MP2 becomes max ; Σ_j, $\Sigma_k\, u_{jk}\delta_{jk} + u_y(y)$, s.t. Σ_j, $\Sigma_k\, p_j\delta_{jk} + y \leq B$ where $\delta_{jk} = 1$ iff $g_j \geq k$. Alternatively, we can redefine goods such that the $k+1^{st}$ item of good j has a different index than the k^{tb} item. See also Appendix A.

$$\text{maximize } u_1g_1 + u_2g_2 + \ldots + u_ng_n + u_y(y) \qquad \textbf{(MP2)}$$
$$\text{subject to: } p_1g_1 + p_2g_2 + \ldots + p_ng_n + y \leq B$$

where $u_y(y)$ is the marginal utility of allocating y dollars to nondurables.

MP2 is now a mathematical programming problem called the "knapsack" problem. If the g_j were not restricted to be discrete—that is, if you could buy a fractional automobile—its solution (called the "greedy" algorithm) is well known (e.g., Gass 1969, p. 204): allocate the budget to goods in order of u_j/p_j as long as u_j/p_j is greater than the budget cutoff, $\lambda = \partial u_y(y)/\partial y$, evaluated at the budget constraint.[3] Even when purchases are restricted to be discrete, "greedy" algorithms are excellent heuristics (Cornuejols, Fisher, and Nemhauser 1977; Fisher 1980).

The "greedy" algorithm is simple, yet it provides an excellent approximation to the optimization of MP2 across a variety of situations. We posit that this heuristic provides a reasonable approximation to describe consumer purchasing behavior. There is a simple behavioral interpretation of the mathematical result: that the criterion, u_j/p_j, of utility per dollar is a measure of "value." Thus, we call our proposition the value priority hypothesis:

> *Value Priority Hypothesis.* The consumer purchases durable goods in order of value as long as their value is above some cutoff, λ, which represents the value of spending an additional dollar on nondurable goods. Furthermore, value is measured by utility per dollar.

For example, suppose a consumer is considering a microwave oven, a video cassette recorder, an auto mobile, a personal computer, a snow blower, and home improvements. S/he would consider the pleasure and usefulness—i.e., utility—obtained from owning the best choice from each category, consider the price of the best choice, and rank them according to value as shown below.

Microwave oven
Video cassette recorder
Automobile
----------------Budget constraint (λ)
Personal computer
Snow blower
Home improvements

The consumer would purchase first the microwave oven (and some nondurables up to $u_{microwave}/p_{microwave}$), then the video cassette recorder, then the automobile. At this point s/he would find that the three durables (plus the corresponding nondurables) exhaust his/her budget. If s/he were to borrow or otherwise obtain additional funds, the next durable s/he would purchase would be a personal computer.

Of course, actual purchasing behavior is more complex, depending upon unexpected events as well as planning (e.g., Dickson and Wilkie 1978), but we feel that the value priority hypothesis is a good, first order explanation. It has roots in the econometric (Paroush 1965) and management science (Keon 1980) literatures and is consistent with small-sample, exploratory focus group semantics (Bertan and Hauser 1982) such as "you get what you pay for," "I want my money's worth," "good value for the money spent," "I want the most car for my money," "when you buy a car you shop value," and so on.

Appendix A shows that the value priority hypothesis extends beyond the simple single period model. For example, in a multiperiod problem with borrowing (saving) and depreciation, the "value" becomes the depreciated time stream of utility divided by the price in current dollars. Operating costs become an addition to price, discounted over time; replacements (trade-ins) are incorporated by computing net utility gain and net price; and complementarity is approximated by first order dependence. The budget constraints for each period are related by interest rates.

ANOTHER VIEWPOINT: NET VALUE PRIORITY

There are two components to the value priority hypothesis: the ordering by value and the means by which value is computed. In the previous section, we treated value as utility per dollar, but in brand choice price is often treated as an attribute. For example, models using conjoint analysis (Green and Srinivasan 1978), perceptual mapping (Hauser and Koppelman 1979), and logit analysis (McFadden 1974) have all included price as another (linear) explanatory variable. Srinivasan (1982) argues that this is a good representation if we recognize that the criterion, $u_j - \lambda p_j$, is the Lagrangian solution to MP2 when the problem is one of brand choice where one and only one good is chosen. He then argues that $u_j - \lambda p_j$ may be a more robust representation than u_j/p_j when price itself carries utility, such as in conspicuous consumption or when perceptions of quality are based on price.

Thus, a variation of the value priority hypothesis is that consumers order durable goods by net value where net value is the surplus of utility over price; that is, $u_j - \lambda p_j$. The net value priority hypothesis can be derived by examining the dual program to the mathematical program, MP2. (For those readers unfamiliar with dual linear programs see Gass 1969 or Appendix B.) Let λ continue to be the dual variable of the budget constraint and let y_j be the dual variables associated with the implicit

[3] Mathematically, λ is a complex function of all the variables of the problem. For our purposes, we need not evaluate it, we need only that it exists. For a given set of utilities it is quite easy to construct an algorithm that finds λ by iteratively allocating the budget between durables and nondurables according to maximum u_j/p_j or $\partial u_y(y)/\partial_y$. Alternatively, we can scale all utilities relative to λ.

constraints of $g_j \leq 1.0$.[4] Then the dual program is (Gass 1969, p. 90):

$$\begin{array}{lll} \text{minimize} & B\lambda + y_1 + y_2 + \ldots + y_n & \qquad \textbf{(MP3)} \\ \text{subject to:} & \lambda p_j + y_j \geq u_j/p_j & \text{for all } j \\ & \lambda = \partial u_y(y)/\partial y & \text{at optimum} \end{array}$$

By the duality theorem of linear programming (Gass 1969, p. 90), the solution of MP3 equals the solution of MP2 when purchases of fractional goods are allowed. Rearranging the constraints of MP3, we get $y_j \geq u_j - \lambda p_j$. If fractional goods are allowed, the complementary slackness theorem applies (Gass 1969, p. 99). By this theorem, $y_j = u_j - \lambda p_j$ is greater than zero in the solution to MP3 if and only if $g_j = 1$ in the solution to MP2. Also, y_j equals zero if and only if $g_j < 1$.

We obtain a behavioral interpretation of MP3 by recognizing that the simplex multiplier, y_j is the shadow price of the constraint $g_j \leq 1$. That is, y_j is the value at the margin to the consumer of buying more of durable j, or the value of relaxing the constraint that durables are discrete. Complementary slackness says that net value, $u_j - \lambda p_j$, is greater than zero if and only if a good is purchased. Net value is less than (or equal to) zero if less than one unit of a good is purchased.

Together, MP2 and MP3 suggest the behavioral interpretation that net value is the appropriate criterion if the consumer focuses on the marginal benefit of purchasing more of a given durable; value (utility per dollar) is the appropriate criterion if the consumer focuses on his or her overall budget allocation problem. Both are reasonable theoretical descriptive hypotheses. Empirical data will shed further light on both hypotheses.

In summary, value priority and net value priority are two reasonable hypotheses about consumer durable purchasing. Both are derived by assuming that the consumer uses an heuristic decision rule to maximize utility subject to a budget constraint and implicit constraints that fractional items are not available. Both imply that a consumer will order his or her durable purchasing (within budget) according to a simple criterion based on value. The two hypotheses differ in how this criterion is computed. Value priority focuses on the budget constraint and postulates a criterion of utility per dollar. Net value priority focuses on the marginal value of the next purchase and postulates a criterion of net value gained. We now examine data on both hypotheses.

DATA

The value priority and net value priority hypotheses are formulated for decision making units such as individuals or families. To test their implications one requires data on budget allocations of individual decision making units. In March 1983 we were given the opportunity to apply and test our hypotheses. An American automobile manufacturer planned to introduce a new automobile in the Spring of 1984 and, among other things, wanted to know with which durable products the automobile would compete. The new automobile was a luxury model for upscale consumers; hence, competition from vacations, second homes, pools, boats, and college tuition was a management concern.

The data collection was part of an ongoing project. The questioning procedure about to be described is based on focus groups in Boston, Massachusetts and Troy, Michigan in June 1982; automobile-show interviews in Boston, Massachusetts in November 1981, 1982, and 1983; a pretest (30 consumers) in Troy, Michigan in June 1982; a mini-test (40 consumers) in Phoenix, Arizona in January 1983; and a series of informal tests throughout the period. Throughout these tests the questioning procedure evolved to represent reasonable tradeoffs among measurement of the hypothesized concepts and feasibility of measurement.

The consumer tasks were administered with trained and experienced professional interviewers. The consumer tasks took approximately 50 minutes and were the opening part of a larger, two-hour interview in which respondents were paid $25 for their time (see Hauser, Roberts, and Urban 1983 for details on the full interview). The 174 respondents were chosen at random from the Cincinnati, Ohio area, but in proportion to their previous purchases of automobiles similar to the automobile of interest. For 12 percent of the interviews, both husbands and wives participated in making joint budget allocations.

Since our hypotheses and the data are at the level of the individual consumer, this data should be sufficient for an initial test of the value priority hypotheses. However, the specific durables and the magnitude of the budgets are not generalizable to the U.S. population because our sample was weighted towards potential luxury car buyers. Furthermore, our analyses are limited to any extent that luxury car buyers are different in their budgeting processes.

Budget Task

To obtain budget information, we gave consumers a deck of cards in which each card represented a potential purchase. For example, these cards included college tuition, vacations, home improvements, major clothing purchases, landscaping, cameras and accessories, furniture, home fuel savings devices, dishwashers, color televisions, stereo systems, jewelry, and so on. After an extensive pretest, we were able to identify 52 items that accounted for most purchases. (Consumers were given blank cards for additional purchases.)

Consumers first sorted these cards according to whether they (a) now owned the durable, (b) would consider purchasing it in the next three years, or (c) would not consider purchasing it in the next three years. (Pretests indicated that three years was a reasonable

[4] We can extend this to any integer value for g_j by the techniques of Footnote 2.

budgeting period.) Consumers next considered pile a—currently own—and removed those items they would either replace or supplement by buying an additional unit. Finally, they selected from pile b—would consider—and from the replacement/additional pile those items for which they would specifically budget and plan. These items are now their budgetable goods.

Consumers then allocated these items to the years 1983, 1984, and 1985 and ordered the items according to priority within each year. This rank order of items becomes our measure of their budget allocation. We estimate utilities with other data, described later, and attempt to forecast the measured rank-order buying priorities.

Explanatory Measures

Obtaining utility measures that can be used to infer value among product categories is a difficult task. Almost every utility measurement procedure of which we are familiar—including conjoint analysis, preference regression, logit analysis, expectancy values, and von Neumann-Morgenstern assessment—measures utility within a product category. In a series of pre-test measurements in 1981, 1982, and 1983, we tried over a dozen different methods including directly scaled (0–100 scale) points on "utility" and on "value," constant sum paired comparisons among items, and constant sum allocations among all items. We found four measures that appeared to be feasible and that provided meaningful tasks to the consumer. These four measures were included in our interviews.

None of the four measures was an explicit measure of utility. However, for each consumer measure, we use the value priority hypotheses to infer relationships among utilities. Details, as well as strengths and weaknesses of the measures, are given in the estimation section below. The measures were:

> *Reservation Price.* The consumer was asked to specify the minimum price at which he, she, or they would no longer purchase the durable.
>
> *Purchase Probability.* The consumer was asked to estimate the probability that he, she, or they would actually purchase the durable in the period of interest (0 to 10 "Juster" scale; see Juster 1966).
>
> *Lottery Order.* The consumer was asked to imagine that he, she, or they had won a lottery and would be allowed to select a prize. They were then to rank the durable allocated to each year in the order corresponding to the order in which he, she, or they would choose a prize in the lottery. Note that this ordering will usually be different than the budget allocation ordering because price is not to be considered in this task.
>
> *Combination Lottery Prizes.* The consumer was again told that he, she, or they had won a lottery, but this time, the task was to choose among two pairs of prizes. For example, the consumer(s) might be asked to choose among receiving either (a) the first and fourth ranked prize, or (b) the second and third ranked prize. Consumers were asked up to eight such pairs or combination for each budget year.

Example Respondent

Table 1 lists the actual data obtained from one respondent. This respondent, a 30-year-old married woman with three children and a $35,000 per year family income, has six durable goods in her 1985 budget. For example, she expects to purchase a $5,000 automobile with a probability of 0.70. This durable good is ranked first in the lottery prize question and has a reservation price of $10,000. If price were not an issue, she would rather have the automobile plus a freezer than paid tuition plus a vacation. There are three tables such as Table 1 for each respondent, one for each year.

ESTIMATION: CONVERGENT LINEAR PROGRAMMING

Each of the measures in Table 1 provides information about utility values, but none is a direct measure of utility. For example, the purchase probability might be a nonlinear function of utility and of λ, while the lottery order and combination lottery prizes provide only rank-order information about utility.

Because two data types—lottery orders and combination lottery prizes—are rank-order relationships and because the other data types are continuous and nonlinear, traditional methods based on continuous, linear relationships may not be appropriate or, at least, must be modified. We present in this section a modified linear programming (LP) procedure that can incorporate rank-order and continuous data types in a single convergent estimation procedure. A later section will present an alternative estimation procedure that uses covariance analysis (LISREL V). In that section the relative predictive capabilities of the two procedures are examined and the convergent indications about our hypotheses are discussed.

The Basic Idea

The idea behind convergent LP estimation is quite simple. Each datum implies a relationship either among various utility values or between a utility value and the datum. The relationship varies by data type. Our goal is to select utility values such that all relationships are satisfied. However, in the presence of measurement error and approximation error, it is unlikely that we will be able to satisfy all relationships simultaneously. Thus, for each datum—say, a lottery prize answer—we may be able to satisfy the relationship only approximately. The amount by which we cannot satisfy the relationship we call "error." Thus, we choose utility values to minimize a weighted sum of errors where the weights (chosen by the analyst) allow us to put a different emphasis on different data types. This minimization of errors can be accomplished with a linear program. The objective function is the weighted sum of errors, and the constraints are the relationships implied by each datum. In general terms this is (LP1):[5]

$$\text{minimize } W_1^* \text{ (errors based on reservation price answers)}$$
$$+W_2^* \text{ (errors based on purchase probability answers)}$$
$$+W_3^* \text{ (errors based on lottery order answers)}$$
$$+W_4^* \text{ (errors based on combination lottery prize answers)}$$

subject to relationships implied by the value priority (or net value priority) model. We now illustrate the specific mathematical relationships.

[5] It is useful to distinguish between the mathematical programs, MP1, MP2, and MP3, which are the consumers' budget problems, and the linear program, LP1, which is the analyst's estimation problem.

TABLE 1
DATA FROM EXAMPLE RESPONDENT

Durable	*Price ($)*	*Reservation price ($)*	*Purchase probability*	*Lottery order*
Automobile	5,000	10,000	.70	1
Furniture	2,000	4,000	.60	2
Tuition	2,000	5,000	.99	3
Movie camera	500	1,000	.60	4
Vacation	1,000	1,500	.70	5
Freezer	300	500	.50	6

Combination lottery prizes

1. Automobile, Freezer	>	Tuition, Vacation
2. Automobile, Vacation	>	Tuition, Camera
3. Tuition, Vacation	>	Furniture, Freezer
4. Tuition, Freezer	>	Camera, Vacation
5. Freezer, Vacation	>	Camera
6. Tuition	>	Camera, Freezer
7. Tuition, Freezer	>	Furniture

NOTE: > symbolizes "preferred to."

Reservation Price Relationships

The reservation price is the price at which the durable good leaves the budget. Thus, if r_j and u_j are the reservation price and utility of the j^{th} item, then the value priority hypothesis implies:

$$u_j/r_j = \lambda \quad \textbf{(1)}$$

because at the reservation price, the j^{th} item falls just below the budget cutoff, λ. To include Equation 1 as a relationship in LP1, we define "errors based on reservation price answers" as the absolute value of the difference between u_j/r_j and λ, that is, $|u_j/r_j - \lambda|$. In linear programming mathematics, this becomes:

$$\text{errors based on reservation price answers} = e_{r_j}^+ + e_{r_j}^- \quad \textbf{(2)}$$

To assure a consistent scale of errors across data types in LP1 we multiply through by r_j. The constraint relationships become:

$$u_j - e_{r_j}^+ + e_{r_j}^- = \lambda r_j \quad \textbf{(3)}$$
$$u_j, e_{r_j}^+, e_{r_j}^- \geq 0.$$

Equations 2 and 3 are the standard LP formulation for minimizing absolute error (see, e.g., Gass 1969, p. 320). If values for u_j and λ are estimated and u_j/r_j exceeds λ, only $e_{r_j}^+$ will take on a positive value because minimization of Equation 2 in LP1 forces $e_{r_j}^-$ to zero. If λ exceeds *to* u_j/r_j, only $e_{r_j}^-$ will be positive.

Since the LP seeks to minimize $e_{r_j}^+ + e_{r_j}^-$, and since it can simultaneously set u_j and λ, one trivial solution is to set all variables equal to zero. We avoid this problem by recognizing that utility, and hence λ, are ratio scales and are thus unique to a positive constant. Thus, we can set one utility value, or λ, arbitrarily. In our formulations, we set $\lambda = 1$, thus scaling everything relative to λ. When $\lambda = 1$, the net value priority hypothesis implies $u_j - \lambda r_j = u_j - r_j = 0$, which implies the same constraint as Equation 3 above. This is consistent with the complementary slackness theorem and the interpretation of MP2 and MP3. The duality theorem implies that at optimum, for a given *B,* the items in the budget as implied by the optimal solution are the same. However, the priority order predicted by value and net value may be quite different. This will be discussed in a later section.

Purchase Probability Relationships

The purchase probability is the consumer's estimate of the probability that the durable good will actually be purchased in the budget period. It is based on the utility and price of the durable good but also upon unobserved events that make the purchase more or less favorable. If these unobserved events represent observation error, then according to the value priority model, the probability of purchasing good *j* is given by:

$$L_j = Prob\{u_j/p_j + \text{error} \geq \lambda\} \quad \textbf{(4)}$$

That is, the likelihood of purchase (L_j) is the probability that the value (u_j/p_j) is greater than the budget constraint (λ) after adjusting for error. If we multiply through the p_j to assure consistent scaling in LP1, and assume that the resulting observation error is distributed with a double exponential probability distribution, then Equation 4 becomes the logit model shown in Equation 5, where β is a parameter to be estimated.

$$L_j = \frac{\exp\{\beta u_j - \lambda\beta p_j\}}{\exp\{\beta u_j - \lambda\beta p_j\} + 1} \quad \textbf{(5)}$$

For the derivation, see McFadden 1974. Equation 5 can be linearized by dividing through by (1 – L_j) and taking logarithms.

Finally, we again use the standard LP formulation for minimizing absolute error to obtain the objective function and constraint relationships for purchase probability. For the criterion function in LP1:

$$\text{errors based on purchase probability answers} = e_{l_j}^{+} + e_{l_j}^{-} \quad \textbf{(6)}$$

and the associated constraint is:

$$u_j - (\beta^{-1})\{\log[L_j/(1 - L_j)]\} - e_{l_j}^{+} + e_{l_j}^{-} = \lambda p_j$$
$$u_j, \beta^{-1}, e_{l_j}^{+}, e_{l_j}^{-} \geq 0. \quad \textbf{(7)}$$

In these equations, L_j and p_j are observed and u_j, β, $e_{l_j}^{+}$ and $e_{l_j}^{-}$ are variables. As before, we establish the scale by setting λ = 1, and as before, constraint 7 also estimates utilities for the net value priority hypothesis.

Lottery Orders

The lottery order is a ranking of the durable goods according to their usefulness or desirability to the consumer. As such, the lottery order implies a rank order on the magnitude of the utilities. For example, if u_1 is the utility of the first ranked durable, u_2 is the utility of the second ranked durable, and so on, then the lottery orders imply:

$$u_1 > u_2 \quad \textbf{(8)}$$
$$u_2 > u_3 \quad \text{etc.}$$

The reader will notice that this data and the constraints implied by Equation 8 are similar to the LP conjoint analysis algorithm LINMAP as proposed by Srinivasan and Shocker (1973). The only difference is that we are interested in the utilities of alternative durable goods whereas Srinivasan and Shocker were interested in the utilities of factorial combinations of product characteristics.

Following similar methods, we count errors only when the inequality relationships are violated; that is,

$$\text{lottery order error} = (1 - \delta_{jk})e_{ojk}^{+} + (\delta_{jk})e_{ojk}^{-} \quad \textbf{(9)}$$
$$u_j - u_k - e_{ojk}^{+} + e_{ojk}^{-} = 0 \quad \textbf{(10)}$$
$$u_j, u_k, e_{ojk}^{+}, e_{ojk}^{-} \geq 0$$

where

$$\delta_{jk} = \begin{cases} 1 \text{ if } j \text{ is preferred to } k \\ 0 \text{ if } k \text{ is preferred to } j \end{cases}$$

In Equations 9 and 10, the (0, 1) variable, δ_{jk}, is the "answer" to the lottery order question that tells us which product is preferred as a prize in the lottery. Because the relationship is specified directly in terms of utility, Equations 9 and 10 apply for both the value priority and net value priority hypotheses. Unlike Srinivasan and Shocker (1973), we need not worry about the scaling of the utilities because the scaling is already established by the constraints associated with the reservation price and/or purchase probability data.[6]

Combination Lottery Prizes

The combination lottery prize questions imply rank-order relationships among pairs of utilities. For example, if the combination of goods 1 and 4 is preferred to the combination of goods 2 and 3, then:

$$u_1 + u_4 \geq u_2 + u_3 \quad \textbf{(11)}$$

Objective functions for the paired comparison lottery error

$$(1-\delta_m)e_{cm}^{+} + (\delta_m)e_{cm}^{-} \quad \textbf{(12)}$$

and formal constraints similar to 10 can be established for each combination lottery question, *m*. For ease of exposition, we do not repeat them here.

Summary

The estimation LP is now to minimize the weighted sum of errors, given by Equations LP1, 2, 6, 9, and 12, subject to the constraints of 3, 7, 10, and 11. For example, for the six durable goods in Table 1, there are six reservation price relationships, six probability relationships, five lottery order relationships, and seven combination lottery prize relationships, totalling 24 constraints and 24 independent errors in the objective function. Because of the complementary slackness and

[6] This implies that either the weight associated with reservation price (W_1 in LP1) and/or with purchase probabilities (W_2 in LP1) must be non-zero to establish scaling in terms of λ.

duality theorems, LP1 applies for both the value priority and the net value priority hypotheses.

PREDICTING BUDGET PLANS

The data on reservation prices, purchase probabilities, lottery orders, and combination lottery prizes give us the ability to estimate the utilities of the goods in an individual's or a family's budget: If the value priority hypothesis and/or the net value priority hypothesis is a reasonable descriptive representation of consumer purchasing behavior, then the rank order of "value" ("net value")—that is, estimated utility divided by price (minus price)—should provide an estimate of the consumer's rank-order buying priorities. It will not be perfect due to measurement and approximation errors. We formulate a predictive test by comparing the estimated utilities (divided by or minus price) to the consumer's budget priorities.

TABLE 2
EXAMPLE PREDICATIVE TEST

Durable	Estimated utility	Price (000s)	Utility ÷ price (000s)	Value priority order	Actual budget priority order
Automobile	10.00	5.0	2.0	3	4
Furniture	4.00	2.0	2.0	3	3
Tuition	10.27	2.0	5.1	1	2
Movie Camera	1.22	0.5	2.5	2	1
Vacation	1.50	1.0	1.5	5	6
Freezer	0.30	3.0	1.0	6	5

NOTE: Correlation of estimate with budget priority: Spearman p = 0.87, Kendall = 0.69

Our predictive test is a comparison of budget plans as predicted by the value priority hypotheses to budget plans as stated directly by the consumers. Because of uncertain and unexpected events such as change in the economy, shortage or surplus of raw materials, unexpected raise or bonus, loss of employment, change of residence, and so on, actual purchases over the three years may differ from budget plans (Dickson and Wilkie 1978). By comparing predicted plans with actual plans we examine the value priority hypotheses as approximate explanations of how consumers believe they will act. We will illustrate our predictive tests in the next section.

A different predictive test would compare predicted plans with actual purchases. Such a test has the strength of external validity but the weakness of confounding the effect of plans and of unexpected events. Since it was not feasible to observe three years of actual purchases (and unexpected events) within our research project, we must leave such tests to future research.

Example Predictive Test

Consider the data in Table 1 and suppose we place equal weight on each data type—that is, $W_1 = W_2 = W_3 = W_4$. Applying convergent LP estimation provides the estimates of utility shown in the second column of Table 2. Dividing by price (third column) gives the estimates. in the fourth column of Table 2. Notice that the estimated utilities would predict that this consumer would rank tuition as her first budget priority (value = 5.1), a movie camera as her second budget priority (value = 2.5), and a freezer as her last budget priority (value = 1.0).

In comparing the budget priority predicted by the estimated utilities to the budget priority actually observed, it must be remembered that the observed budget priorities were not used in the estimation. Thus the comparison in Table 2 is a test of predictive ability, not of data-fitting ability. Comparing rank orders implied by the data in the fifth column to the sixth column of Table 2, we see that the predictions are reasonable but not perfect. Tuition and the movie camera are predicted and observed to be the top two items, but estimated value predicts tuition as the top priority while the consumer feels that the movie camera is her top priority. Overall, the Spearman rank-order correlation of the predicted rank from utility per dollar (column 5) and the actual rank (column 6) is 0.87, while the Kendall rank-order correlation is 0.69.

TABLE 3
VARYING WEIGHTS ON TYPES OF INPUT DATA
FOR EXAMPLE RESPONDENT

Weighting scheme	Spearman correlation
Equal weights to all four types	.87
Reservation price weighted heavily	.31
Purchase probability weighted heavily	.82
Lottery order weighted heavily	.82
Paired lottery prizes weighted heavily	.87

NOTE: "Weighted heavity means the relevant weight is 100 times more than others. Weights are not set equal to zero to maintain scaling as discussed in the text.

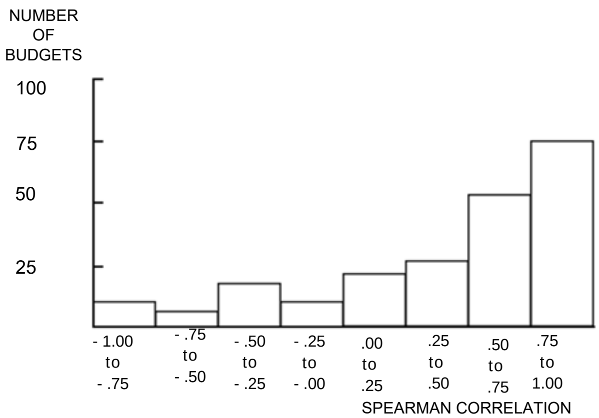
However, equal weighting of the data types is not the only choice. For example, Table 3 indicates the results we obtained by using each data source separately.[7] For this consumer, it appears that the purchase probabilities, lottery orders, and paired lottery prizes each alone provides reasonable estimates of budget priorities; however, in this case, reservation price data do not appear to be as good as the other measures. In fact, if we drop reservation prices and use equal weights on the other three data sources, we get a higher rank-order correlation—0.93—than if we use all four data sources.

Testing the net value priority hypothesis proceeds similarly. The only difference is that we subtract price (in $000s) from the estimated utility rather than divide by price. For example, for the automobile the net value criterion is 10.00 – 5.00 = 5.00, which turns out to be ranked second. For equal Ws for this respondent the net value priority hypothesis produces a Spearman rank-order correlation of 0.54. Thus, for this respondent (with equal Ws), the value priority hypothesis appears to predict better than the net value priority hypothesis. Unfortunately, because the tests are of different

[7] We report only the Spearman correlation for ease of exposition. Results are similar to Kendall's r. This applies for the remainder of the paper.

hypotheses rather than of nested hypotheses, we cannot be rigorous and state whether this difference is statistically significant.

FIGURE A
DISTRIBUTION OF SPEARMAN CORRELATION OF
ACTUAL AND PREDICTED BUDGET PRIORITY ORDERS
FOR VALUE PRIORITY HYPOTHESIS



Predictive Tests Across Individuals for the Value Priority Hypothesis

Our sample yields 522 potential budgets (174 families x 3 years). Sixty budgets (11.5 percent) had one or more values missing for either an explanatory or a predictive measure. These were spread across measures and demographics and did not appear to represent a systematic bias in measurement. Of the remaining 462 budgets, 247 had 0, 1, or 2 durables planned. Although the value priority (or net value priority) hypothesis applies to such small budgets, predictions would be perfect by definition for 0 or 1 items, and perfect by chance 50 percent of the time for 2 items in a budget. We felt that this would bias our results upward artificially, so we restricted ourselves to the more difficult task of predicting the 215 budgets which contained at least 3 durables.[8]

We applied the predictive tests as illustrated in Tables 1, 2, and 3 to each individual's (or family's) budgets in the resulting sample. To investigate the relative effectiveness of various measures, we estimated utilities for each individual (or family) for equal weights ($W_1 = W_2 = W_3 = W_4$.), for weighting heavily each data source (as in Table 3), and for weighting heavily combinations of data sources (e.g., reservation prices and purchase probabilities). Even with today's mainframe computers and efficient LP software, it was not feasible computationally to search all possible combinations of *W*s.

We summarize the data in two ways. To examine the value priority and net value priority hypotheses we report predictions based on the best set of *W*s (from our limited search as described above) for each individual or family. Then, to examine the relative merits of each data source, we keep the *W*s the same for all individuals and families. Other means of summarizing the data provide the same qualitative implications and are noted, as appropriate.

FIGURE B
DISTRIBUTION OF SPEARMAN CORRELATION OF
ACTUAL AND PREDICTED BUDGET PRIORITY ORDERS
FOR NET VALUE HYPOTHESES

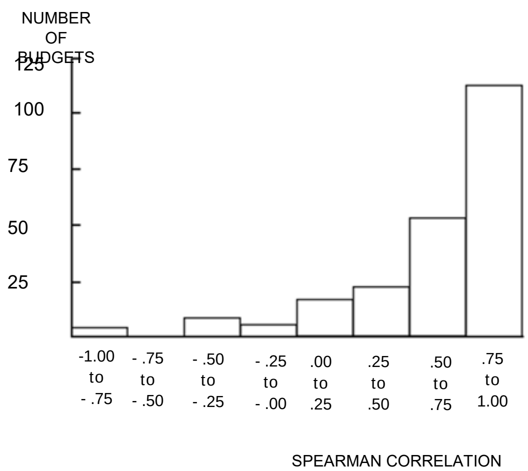


Figure A reports the Spearman correlations of the actual and predicted budget priorities for the value priority hypothesis. It is based on the best *W*s for each individual (or family), but we use the same weights for all his, her, or their budgets. Overall, the value priority hypothesis seems reasonable. Despite potential measurement error and conservative reporting due to eliminating the favorable budgets of O, l, or 2 items, roughly 83 percent of the budgets have positive correlations, 60 percent have correlations of 0.50 or better, the 35 percent have correlations of 0.75 or better. Significance levels are complex (because many ties are possible), vary by budget (the number of items in each budget varies), and do not apply between hypotheses. There is no single overall critical value that can be applied to Figure A.

Comparison of the Value Priority and Net Value Priority Hypotheses

Figure B reports the Spearman correlations of actual and predicted budget priorities for the net value hypothesis. The net value priority hypothesis also appears to be a reasonable description of consumer purchasing behavior. Roughly 91 percent of the budgets

[8] Of these annual budgets, 84 had three items, 54 had four items, 35 had five items and 22 had six items. The remainder had seven or more items up to a high of 12 items. We detected no systematic bias based on the number of items in a budget.

have positive correlations, 84 percent have correlations of 0.50 or better, and 51 percent have correlations of 0.75 or better. The net value hypothesis appears to do somewhat better than the value hypothesis.

TABLE 4
COMPARISON OF VALUE PRIORITY AND NET VALUE PRIORITY HYPOTHESES

Weighting scheme	Budgets: Value priority predicts best	Budgets: Net value priority predicts best
Number of budgets	56	122
Average number of products/budget	4.57	4.61
Average number of autos/budget	0.48	0.36
Average price of products in budget	$3278	$3635
Average reservation price of product in budget	$4232	$4349
Average age	43.7	44.1
Average income	$36,200	$36,300

Examining consumer by consumer the Spearman correlations of actual and predicted budget plans, 122 budgets (57 percent) were predicted better by net value priority, 56 budgets (26 percent) were predicted better by value priority, and 37 budgets (17 percent) were predicted equally well by both. As Table 4 suggests, we found no significant demographic differences that suggest when one hypothesis predicts better than the other.

In summary, both hypotheses do well, neither is rejected, and both are retained for future empirical testing. Although net value priority does better in our tests, the issue is very complex because of the theoretical interrelationships among the hypotheses (through the duality theorem). We will interpret our results in light of these interrelationships in a later section.

Variation Across Alternative Weightings of Data Types

Figure C reports distributions of Spearman correlation of actual and predicted budget priority orders for each of the data sources. For example, if we make W_1 much larger than W_2, W_3, and W_4, we emphasize reservation prices as the primary data source, as in panel 1.[9] The Figure C results are based on weights that do not vary across individual families. Overall, predictions based on reservation prices do better than random (67 percent are positive), but not nearly as well as they did in Figure B. This is not surprising because reservation price is a complex concept for many consumers, causing the quality of data to vary across consumers. Our example respondent appears to have understood the concept, but other respondents clearly did not. For example, some consumers gave a reservation price of $2001 for an item with an expected price of $2000. Other consumers found it hard to imagine the item staying the same as the price rises.

Figure C also reports the results for emphasizing data on purchase probabilities, lottery orders, and combination lottery prize answers. Of the data sources, purchase probabilities are clearly the best indicators of budget priorities (81 percent positive and 60 percent with correlations of 0.50 or better). Lottery orders and combination lottery prizes (67 percent and 69 percent positive, respectively) do about as well as reservation prices. Results for combinations of two, three, and four data sources tend to be in the range of those in Figure C. Those results also suggest that of the four data sources, purchase probabilities tend to predict budget priorities best.[10]

Although purchase probability measures appear to be the best indicators of budget priorities, Figure B suggests that consumers do vary in their abilities to answer any given question format. We recommend a convergent estimation approach that utilizes all four data sources. Convergent linear programming is one such approach; we will illustrate another in a later section.

Summary of Predictive Tests

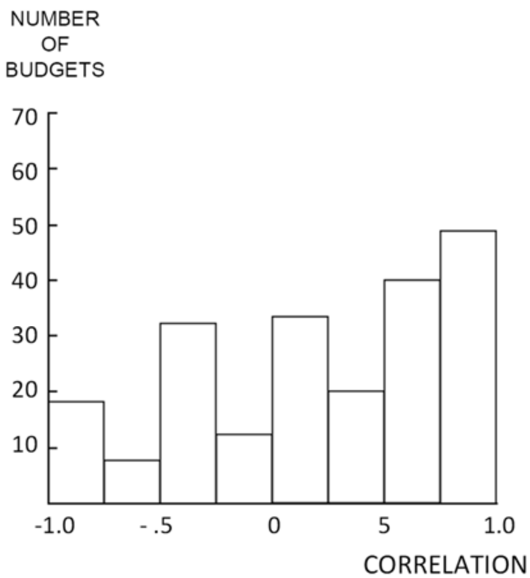
Based on convergent linear programming estimation with all four data sources, we are able to estimate utility values for durable products which, with price, forecast well consumers' budget orders. We feel that this is reasonable preliminary evidence that the hypotheses are good first order approximations to consumers' purchasing of durable goods. Elaborations of the hypotheses (Appendix A) may improve the approximation and predict better. The comparison of value priority and net value priority shows that both criteria predict well consumers' budget priorities. Net value priority (focusing on the marginal increase in net utility) does better than value priority (focusing on the budget constraint), but the results do vary by individuals and/or families. We found no systematic reason for the variations, but further research may suggest some hypotheses. Finally, consumers do vary in their ability to respond to complex utility questions, suggesting that utility is best measured with multiple questions and with at least one form of convergent estimation.

[9] We report the results for net value priority here. Results for value priority are qualitatively the same and in about the same relationship, as summarized by Figures A and B.

[10] Figure C gives one indication of how well purchase probabilities alone would predict budget plans.

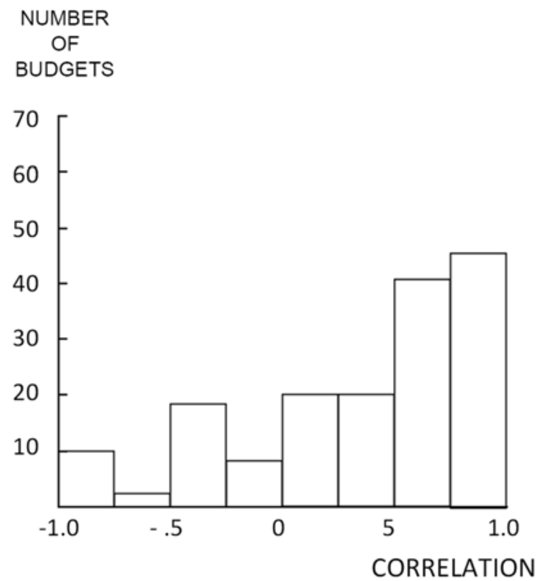
FIGURE C

DISTRIBUTIONS OF SPEARMAN CORRELATION OF ACTUAL AND PREDICTED BUDGET PRIORITY ORDERS FOR FOUR DIFFERENT DATA SOURCES



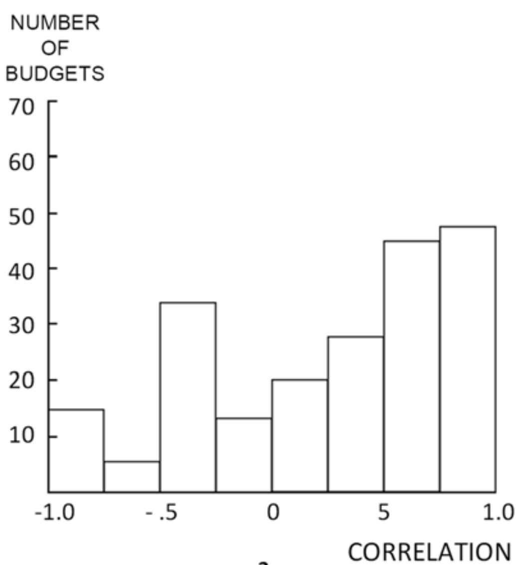
1

Prediction based on reservation price



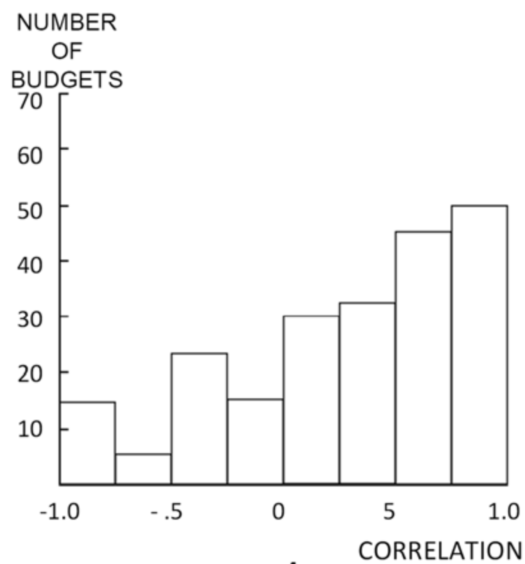
2

Prediction based on reservation price



3

Prediction based on reservation price



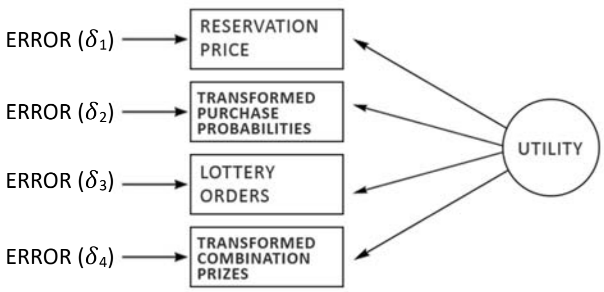
4

Prediction based on reservation price

ALTERNATIVE ESTIMATION PROCEDURE: LISREL V

Convergent linear programming is one way to incorporate multiple data sources. Its strengths are that it can readily accommodate both ordinal and cardinal measures and that the theoretical relationships suggested by the value priority hypotheses can be represented exactly within the structure. Furthermore, it is readily applied on a consumer-by-consumer basis to identify potential heterogeneities in response to question format and/or planning. Its disadvantages are that: (1) the cost of searching all combinations of weights (W_1, W_2, W_3, and W_4) to find a single best fit is prohibitive, and (2) statistical properties of linear programming estimation are not well known.

FIGURE D
MEASUREMENT MODEL FOR LISREL V ESTIMATION



NOTE: Data sources (boxes) are indicators of the unobservable utility values (circle).

There are other estimation procedures that use multiple data sources. Each has its relative strengths and weaknesses. We select one such estimation procedure to demonstrate that our basic result—the reasonableness of the value priority hypotheses—is robust with respect to the estimation procedure. The procedure we choose is covariance analysis as implemented by Jöreskog and Sörbom's (1981) LISREL V (see Heise 1975, Duncan 1975, and Bentler and Bonett 1980 for more details on covariance analysis). The advantages of LISREL V are that (1) a best set of weights (in the maximum likelihood sense) can be found for the proper emphasis among data sources, and (2) the statistical properties are well known when normality conditions hold. The disadvantages are that (1) because LISREL is very sensitive to departures from normality (see Jöreskog and Sörbom 1981 page I.39), it may not do well with our ordinal data or with our transformed probabilities, and (2) the estimation is infeasible for small sample sizes, as would be the case with individual-by-individual analyses (see Bentler and Bonett 1980, p. 591). Since the relative advantages and disadvantages of LISREL V compensate those of convergent linear programming estimation, the resulting estimation provides valuable insight into the value priority hypotheses.

Basic Estimation Model

The LISREL V analysis corresponding to convergent LP estimation is the measurement model shown in Figure D. The data sources (boxes) are indicators of the unobservable utility values (circle); thus each measurement—say, a reservation price—can be thought of as resulting from the unobserved utility value and a measurement error (δs in Figure D). The goal of LISREL V is to estimate the correlations (known as factor loadings) relating the observables to the unobserved utility and then to use the structure to estimate a quantity (known as a factor score) for the unobserved utility. We use the theoretical relationships as implied by the value priority hypotheses to specify the appropriate transformations of the raw measurements.

Measures

Based on the value priority hypotheses, the appropriate measures are:

1. Reservation prices as implied by Equation 3 with $\lambda = 1$.
2. Logit transformed probabilities as implied by Equation 7 with $\lambda = 1$. We allow the estimation to determine the scaling constant β^{-1}.

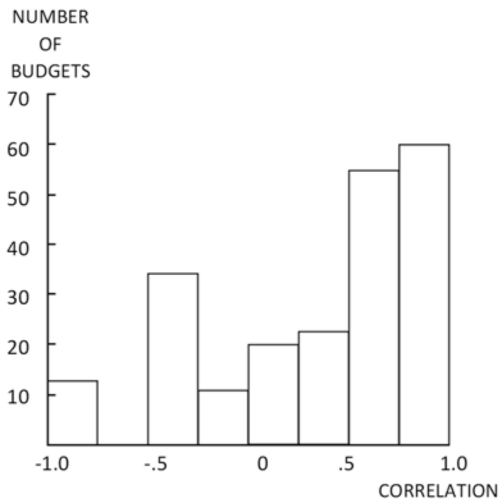
3.

4. Lottery orders. These orderings are rank-order measures and may violate strict normality assumptions, but they are monotonic in utility.
5. Combination lottery prizes. The rank-order relationships implied by Equations 11 and 12 are complex, dependent on each individual, and interrelated with lottery orders. They are not readily handled by the linear equations of LISREL V. We use as a surrogate the number of times a durable is chosen from the set of combinations. This measure is clearly monotonic in utility. Again, normality is a concern.

Estimation Results

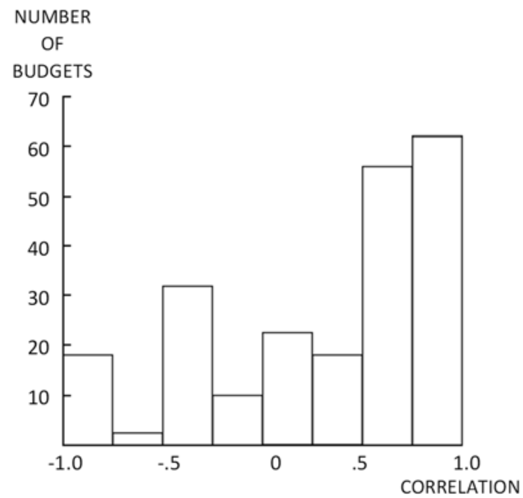
FIGURE E

DISTRIBUTION OF SPEARMAN CORRELATION OF ACTUAL AND PREDICTED BUDGET PRIORITY ORDERS FOR LISREL MODEL AND TWO DIFFERENT HYPOTHESES



1

Predictions based on value priority hypothesis



2

Predictions based on value priority hypothesis

The maximum likelihood estimation results are shown in Table 5. The estimation is based on 932 observations corresponding to the total number of budgeted items in the 215 budgets. Overall, the measurement model does remarkably well. The goodness of fit index—which "is independent of sample size and relatively robust against departures from normality" (Jörgskog and Sörbom 1981, page I.41)—suggests that 99.9 percent of relative covariance is accounted for by the model. Even adjusted for degrees of freedom, this measure is 98.9 percent. The coefficient of determination for the overall model is 92.4 percent, suggesting high overall reliability of the measurement model. The chi-squared value is low—0.87, indicating that no addition of free parameters would improve the model significantly.[11]

For the specific measures, reservation prices have the highest reliability followed by transformed probabilities. Both have excellent asymptotic t-statistics. The rank order measures fare less well, with low reliabilities but good *t*-statistics. Normalized residuals (not shown) are reasonable for the cardinal measures but do depart somewhat for the rank-order measures. Since the latter is to be expected, the t-statistics are acceptable, and we desire a comparison with the convergent LP estimation, we retain all measures in the model.

Predictive Tests

Based on the measurement model in Table 5, we use the LISREL V factor score regressions[12] to estimate utility for each durable in each budget. We then divide by price to forecast value priorities or subtract price to forecast net value priorities. As we did with the linear programming forecasts, we compare the LISREL V forecasts to actual consumer budget orders (see Figure E).

The LISREL V predictive results appear comparable to the convergent LP estimates that do not vary by family: 73 percent of the correlations are positive and 55 percent are 0.50 or better for the value priority hypothesis, while 71 percent are positive and 54 percent are equal or above 0.50 for the net value priority hypothesis.[13] These correlations are better than those obtained in Figure C for reservation prices, lottery orders, or combination prizes as single measures, and almost as good as those obtained for purchase probabilities. Of course, LISREL V does not do as well as the family-by-family estimates in Figures A and B. Based on the similarity of the LISREL V predictive results to the convergent LP predictive results, we have more confidence in our proposition that at least one of

[11] The model in Table 5 accounts for measurement correlation among the rank-order measures as suggested by the corresponding modification index (Jöreskog and Sörbom 1981, p. III.19). Without the extra free parameter, all coefficients and statistics except the chi-squared (40.1 with 2 d.f.) are virtually identical to those in Table 5.

[12] The factor score coefficients are 0.805, 0.180, 0.026, and 0.015, respectively, for the four measures.

[13] The value priority hypothesis does slightly better with LISREL V than does the net value priority hypothesis, but this result is probably not significant. The best comparison between the hypotheses remains the family-by-family analysis (review Table 4).

the value priority hypotheses is a reasonable first-order model of consumer budget planning.[14]

TABLE 6
AN EXAMPLE BUDGET WITH THREE ITEMS

Item	Utility	Price ($000s)	u_j/p_j	Value priority	$u_j - p_j$	Net value priority
Home improvement	.994	0.60	1.7	3	.394	1
Landscaping	.657	0.30	2.2	2	.357	2
Food processor	.328	0.08	4.1	1	.248	3

NOTE: u_j is the utility of item j, and p_j is the price.

DISCUSSION

The value priority hypothesis and the net value priority hypothesis are models of how consumers allocate their budgets to goods. Both hypotheses are derived from the standard economic model of maximizing utility subject to a budget constraint. However, both recognize the evidence from a variety of scientific disciplines suggesting that behavior as observed may differ from behavior as prescribed. Both hypotheses imply that the consumer (or family) uses a simple heuristic that leads to near-optimal behavior under a wide variety of conditions. This heuristic is to rank order durables according to value (or net value), and purchase items in that order up to and including a budget cutoff. The two hypotheses differ only in their derivation of the numeraire by which durables are ranked. The empirical evidence for the hypotheses presented earlier suggests that both are reasonable; we are comfortable with proposing both for further testing. Figures A and B also suggest that the net value priority hypothesis may be the better predictor. However, before embracing the net value hypothesis, there are a number of cautions worth considering. These include the complex interrelation ship between the two hypotheses, the distinction between descriptive and prescriptive theories, and the empirical observation of high rank-order correlation between u_j/p_j and $u_j - p_j$, all of which are discussed in the following sections.

Interrelationships Between the Hypotheses

If durables were not discrete, then the duality and complementary slackness theorems would imply that the optimal solutions to MP2 (value priority) and MP3 (net value priority) are the same. This means that for a given budget, B, and budget cutoff, λ^*, the two criteria—$u_j/p_j \geq \lambda^*$ and $u_j - \lambda\, p_j \geq 0$—would yield the same overall budget. (This can also be seen by dividing through by p_j in the net value criterion.) This does not mean that the order of planned purchasing—the rank order of u_j/p_j and $u_j - \lambda^* p_j$—will be the same.

Consider three items—a home improvement, land scaping, and a food processor—that are part of the budget of one of our respondents, a married 37-year old male with two children and a $65,000 family in come. Following the convergent linear programming estimation procedure described earlier, Table 6 displays the utilities scaled such that $\lambda = 1$. Net value priority predicts the order as shown: the home improvement, the landscaping, and then the food processor. Value priority predicts the reverse order, For these three items the respondent actually planned the home improvement, the landscaping, and then the food processor. For this consumer, net value priority appears to be a better descriptive model (review Table 4 for more general results).

Assumption of Stable λ

The example above does not indicate what would happen if the budget, B, the utilities, u_j, or the availability of products changed. The value priority criterion, u_j/p_j, would not change. On the other hand, the net value criterion would remain unchanged only if λ^* did not change. However, λ^*—which equals $\partial u_j(y^*)/\partial y$ at the optimum solution to MP3—may change if B or the u_j's change.[15] If the change were sufficiently dramatic, the net value ordering could change. Thus the net value priority hypothesis assumes that λ^*, or at least the consumer's perceived λ, changes slowly. This assumption is worth testing.

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[14] Curiously, LISREL V selects reservation price as the most reliable measure, but the predictive tests in Figure D (a measure of validity) suggest that purchase probabilities may predict better. Further research might estimate a more complex structural model including the dependent variable in the estimation. We did not do this because we felt it more appropriate to have an independent test of predictive ability that did not use the budget orders in the estimation, and because the dependent measure was at best an ordinal measure that clearly causes problems with LISREL V. Furthermore, our goal is to test the value priority hypotheses, not to compare the relative merits of linear programming and LISREL V. Finally, a full comparison of technique would best be done with multiple measures of the dependent variable in a variety of contexts.

[15] A change in λ^* would affect our scaling convention of $\lambda = 1$. The utilities are a function of λ^* and may themselves change if we change the budget problem yet restrict λ to be 1.0. For the comparative forecasts in this paper, the budget problem does not change; hence, the restriction, $\lambda = 1$, is not critical for our predictive tests. It could become critical in other situations.

TABLE 7
REVISED BUDGETS

Budget items	Utility	Price
Value priority		
Food processor	.328	$80
Convection oven	.328	$80
Landscaping	.657	$300
Total	1.313	$460
Net value priority		
Home improvement	.994	$600
Total	.994	$600

Descriptive vs. Prescriptive Hypotheses

We stated the value priority and the net value priority hypotheses as descriptive hypotheses. They may or may not lead the consumer to the "best" decisions. For example, consider the consumer discussed above and suppose there were another durable—say, a tabletop convection oven—with the same utility and price as the food processor. Then, for $600 of budget allocations, the two hypotheses recommend the allocations indicated in Table 7. Prescriptively (if our utilities are accurate), the consumer would have been better off (more utility and less money) using the value priority than net value priority. Such examples are easy to create. Indeed, if there were no integer constraints on durable purchases, the value priority algorithm is, prescriptively, the best algorithm. Even with integer constraints, it does not do badly and has reasonable worst-case properties[16] (Cornuejols et al. 1977; Fisher 1980). However, we can also create examples to favor net value priority over value priority, so again, we must interpret all prescriptive results with caution.

Correlation

The value priority and net value priority hypotheses have quite different behavioral interpretations. However, they may be difficult to distinguish from observations of behavior because the two criteria, u_j/p_j and $u_j - p_j$, have high rank-order correlations. To illustrate this, we drew 10,000 random values of u_j and p_j from a uniform distribution, each of five times. The resulting linear correlation[17] of log (u_j/p_j)—which is monotonic in u_j/p_j —and $u_j - p_j$ was quite high: 0.87. This suggests an even higher rank-order correlation. Because of this correlation, we must interpret with caution any empirical comparisons of observed budget plans. This does not mean the hypotheses are indistinguishable; for example, verbal protocols or process-tracing technology may be able to distinguish between the hypotheses. In summary, the evidence favoring at least one of the hypotheses as a description of durable purchasing is positive. However, comparisons between the hypotheses must be made with caution and subject to further testing.

SUMMARY AND IMPLICATIONS

Based on our data, estimation, and predictive tests, and subject to future research we posit that:

1. The ranking of goods according to a budget priority appears to be a reasonable descriptor/predictor of consumer budget plans.
2. Both value and net value provide reasonable approximations to the numeraire by which durables are ranked.
3. It is feasible to measure utility across categories if multiple convergent measures are used.
4. Convergent LP estimation is feasible, provides reasonable estimates of utility, and appears consistent in predictive ability with LISREL V.
5. Consumers vary in the heuristic numeraire—value or net value—they use for ranking.
6. Consumers vary in their ability to answer specific question types.

The last postulate is no surprise to the behavioral researcher who faces often the difficult task of estimating unobserved constructs. It does provide a caution to the market researcher or management analyst faced with limited measurement budgets who wants to forecast durable purchases. Convergent measurement is probably necessary.

Our value priority hypotheses are a first step in understanding consumer budget planning. They are reasonable descriptors/predictors of actual budget plans, but they do not model explicitly the mental processes leading to utility formation and information processing. A useful direction for research would be the investigation of information processing theories to explain how consumers form utility judgments and whether value, net value, or another hypothesis is the best description of how they integrate utility and price to form budget plans. Our postulates and analyses raise a number of other researchable issues as well. Among these are:

1. Two-stage predictive tests comparing predicted plans to actual plans and to actual purchase
2. Predictive tests collecting measures on all evoked goods and predicting which will be in the budget plans

[16] The theoretical worst case is a factor of two. For example, with a budget of $1000 and two products of utility 5.02 and 5.00 that cost $501 and $500, respectively, the optimum is two units of the second product rather than one unit of the first product. But if the budget can be relaxed or nondurables purchased, this worst-case result is mitigated.

[17] We seek to demonstrate the rank-order correlation of u_j/p_j and $u_j - p_j$. Thus, we seek monotonic transformations of either or both variables such that the linear correlation is maximized. The logarithmic transformation is effective and provides a reasonable lower bound on the maximum rank correlation. Technically, some transformation is necessary because the mean and variance of (u_j/p_j) are both infinite when u_j and p_j are independent and identically distributed uniform random variables. In addition, the logarithmic transformation gives the intuitive interpretation that log (u_j/p_j) = log u_j – log p_j, which we expect to be related to $u_j - p_j$.

3. Alternative methods such as verbal protocols and process tracing that can distinguish between the value and net value hypotheses without being subject to the bane of high rank-order correlation between hypotheses
4. Collection of data such as perceived interest rates, depreciation rates, and operating and maintenance costs that could test an elaborated model such as that in Appendix A.

These are a few of the many unanswered questions that can be addressed in future research.

Managerial Implications

We close on a practical note. The value priority hypotheses can be and are useful in forecasting sales in existing durable product classes. Once utilities are estimated for a sample of consumers, we can forecast the implications of new products, improved products, or changes in prices or economic conditions. New or improved products change utilities, and economic conditions change the budgets.[18]

TABLE 8
DURABLE GOODS COMPETING WITH AUTOMOBILES

Durable	Percent
1. School tuition 1983	96.4
2. Vacation 1983	92.8
3. Home improvement (minor)	84.0
4. Major clothing	78.8
5. Landscaping	77.8
6. School tuition 1984	76.7
7. Gifts/Donations	76.0
8. Cameras and Accessories	70.6
9. Furniture	68.0
10. Home fuel savings device	67.7
11. Home improvement (major)	67.3
12. Vacation 1984	64.2
13. Dishwasher	63.2
14. Color television	59.1
15. Stereo system	57.9
16. Jewelry	55.6
17. House	53.3
18. Oven	50.0
19. Movie/Video camera	50.0
20. Video tape recorder	46.9
21. Refrigerator/Freezer	46.2
22. School tuition 1985	45.0
23. Home computer	44.7
24. Vacation 1985	37.0

NOTE: The percents signify the percentage of budgets in which the indicated item was ranked above an automobile when both when both the item and an automobile were in the budget.

For each consumer we compute the value criterion (or net value criterion if λ does not change) and recompute the buying order. For example, a megabyte personal computer, a digital stereo/VCR, or a mini-van may have high enough value (or net value) to enter the budget of some consumers. The percent of consumers who now budget for the new product form is a forecast of its category sales.

The measurement system described in this paper has been used at General Motors and has provided valuable managerial insight into which durable goods compete most with luxury automobiles. For example, Table 8 lists those goods that were ranked above automobiles when both were in the budget. Recent automobile design and marketing campaigns have been based on budget priority analyses that consider product improvements or advertised image (less maintenance, improved comfort, and so on). Such changes are designed to increase the utility of an auto purchase and thereby move it up in the buying priority. The value priority model is also used to determine if the introduction of a new automobile will improve the position of an automobile purchase in the ordering. After having consumers drive the new automobile in a clinic environment, the new automobile's utility is measured relative to the respondent's current first choice automobile to determine if it is higher in the respondent's priority ordering than the previous automobile. Application work is continuing to ascertain the managerial usefulness of the value priority hypotheses in managing new and established durable consumer goods.

APPENDIX A

Multiple Period Hypotheses

The value priority hypothesis and the net value priority hypothesis are readily extendable. The equations for the value priority hypothesis were derived in Hauser and Urban (1982). We restate them here in condensed form and indicate how they apply to the net value priority hypothesis.

Let u_{ji} be the utility of the i^{th} item of the j^{th} good, p_{jt} be the expected price of that good at time t, δ_{jit} be a zero-one indicator of whether the i^{th} items of good j is purchased. Note $\delta_{jit} > 0$ only if $\delta_{j,i-l,t} = 1$. Let $u_y(y_t)$ be the utility of spending y_t on nondurables in time t. Let B_t be the consumer's budget constraint in time t, D_t be his/her debt in time t, and b_t be the amount borrowed (saved) in that period. Let d_j be the depreciation rate for good j and r be the interest rate. Let c_{jn} be the operating and maintenance cost of durable j, n periods after purchase. The consumer's problem (MP4) is as follows. Maximize:

$$\sum_{t=1}^{T} \sum_j \sum_i \left[\sum_{q=0}^{T-1} d_j^q u_{ji} \delta_{jit}\right] + \sum_{t=1}^{T} u_y(y_t)$$

subject to:

$$\begin{aligned} &\sum_j p_{jr}\left(\sum_i \delta_{jit}\right) + \sum_{n=1}^{T-1} c_{jn} \delta'_{jlt} + y_t - b_t \leq B_t \\ &D_t = D_{t-1}(1+r) + b_t, \quad D_T = 0 \text{ for all } t \\ &\delta_{jit} = 0, 1; \ \delta'_{jis} = 1 \ \text{ iff } \ \delta_{jit} = 1 \ and \ s > t. \end{aligned}$$

The value priority criterion for the LP relaxation of the integer constraints now becomes:

$$u_{ji}\left[\sum_{q=0}^{T-t} d_j^q / (1+r)^{T-t}\right] / \left[p_{jt} + \sum_{n=1}^{T-t} c_{jn}\right] \qquad \textbf{(A1)}$$

Finally, trade-ins are handled by computing net depreciated utility gain divided by net price, and pairwise complementarity is added with hierarchical dependence of u_{ji} on another good, k. For the net value hypothesis, the criterion is (numerator of A1) – μ (denominator of A1) where μ is the simplex multiplier of the debt constraint, $D_T = 0$.

APPENDIX B

Dual Linear Program: Non-Technical Summary

An important concept in linear programming is that for every linear program, there is a related dual linear program. The variables of the dual are known as simplex multipliers, or shadow prices. Each variable of the dual corresponds to a constraint in the original linear program and represents the "sensitivity" of relaxing the constraint—i.e., the amount by which the objective function would change if that constraint were relaxed. If the original linear program is a maximization problem, then the dual program has as its objective the minimization of a weighted sum of the dual variables. The weights are the constants in the constraints of the original linear program. The constraints of the dual are based on the constraints and objective function of the original linear program. For example:

	Original		Dual
max	$c_1x_1 + c_2x_2$	min	$b_1u_1 + b_2u_2$
subject to:	$a_{11}x_1 + a_{12}x_2 \leq b_1$	s.t.	$a_{11}u_1 + a_{21}u_2 \geq c_1$
	$a_{21}x_1 + a_{22}x_2 \leq b_2$		$a_{12}u_1 + a_{22}u_2 \geq c_2$
	$x_1, x_2 \geq 0$		$u_1, u_2 \geq 0$

Note that u_1, corresponds to the first constraint in the original program and represents the value of relaxing that constraint.

The duality theorem states the amazing result that the optimal values of the object functions of the two linear programs are identical. Complementary slackness states that if a dual variable has a non-zero value in the optimal solution to the dual, then the corresponding constraint in the original program must be binding, and vice versa. For a more complete and technical exposition see Gass (1969) or any linear programming text. Note that the dual of the dual is the original linear program.

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