The Impact of Utility Balance and Endogeneity in Conjoint Analysis
Technical Appendix

Endogeneity Bias for the 2x2 Stylized Model

Proposition 1. For a simple problem involving two binary features, adaptation based on metric utility balance (1) biases partworth estimates upward and (2) biases smaller partworths proportionally more than larger partworths.

Proof. Following the text, we scale the low level of each feature to zero, let w_i be the partworth of the high level of feature *i*, denote the utility of a product with *feature 1* and *feature 2* by *u(feature 1, feature 2)*, and denote the estimates of the partworths with $\hat{w}_1$ and $\hat{w}_2$. The assumption of no interactions implies that the true utilities are:

$u(0, 0) = 0 \qquad u(1, 0) = p_1 \qquad u(0, 1) = p_2 \qquad u(1, 1) = p_1 + p_2$

Following the text we assume response error is an additive, zero-mean random variable, e, with probability distribution $f(e)$. Label the error associated the first question as e_{ub} and label errors associated with subsequent questions as either e_1 or e_2. Without loss of generality, consider the case where $2w_2 > w_1 > w_2 > 0$. With this assumptions the off-diagonal question, which compares {0,1} to. {1,0}, is the most utility-balanced first question. In this simple problem, adaptive metric utility balance implies the following sequence.

First question:	$\hat{w}_1 - \hat{w}_2 = w_1 - w_2 + e_{ub}$			
Second question:	$\hat{w}_1 = w_1 + e_1$	if	$e_{ub} < w_2 - w_1$	(Case 1)
	$\hat{w}_2 = w_2 + e_2$	if	$e_{ub} \geq w_2 - w_1$	(Case 2)

Suppose that $e_{ub} \geq w_2 - w_1$, then:

$$E[\hat{w}_2] = w_2 + E[e_2] = w_2$$

$$E[\hat{w}_1] = E[\hat{w}_2] + w_1 - w_2 + E[e_{ub} | e_{ub} \geq w_2 - w_1] = w_1 + E[e_{ub} | e_{ub} \geq w_2 - w_1] > w_1$$

Suppose that $e_{ub} < w_2 - w_1$, then:

$$E[\hat{w}_1] = w_1 + E[e_1] = w_1$$

$$E[\hat{w}_2] = E[\hat{w}_1] + w_2 - w_1 - E[e_{ub} | e_{ub} < w_2 - w_1] = w_2 - E[e_{ub} | e_{ub} < w_2 - w_1] > w_2$$

We now calculate the expected bias in w_1 and w_2.

$$E[bias\,in\,w_1] = \Pr\{e_{ub} \geq w_2 - w_1\} E[e_{ub} \mid e_{ub} \geq w_2 - w_1]$$

$$E[bias\,in\,w_1] = \left[\int_{w_2-w_1}^{\infty} f(e_{ub})de_{ub}\right]\left[\int_{w_2-w_1}^{\infty} e_{ub} f(e_{ub})\,de_h \Big/ \int_{w_2-w_1}^{\infty} f(e_{ub})de_{ub}\right] = \int_{w_2-w_1}^{\infty} e_{ub} f(e_{ub})de_{ub}$$

$$E[bias\,in\,w_2] = -\left[\int_{-\infty}^{w_2-w_1} f(e_{ub})de_{ub}\right]\left[\int_{-\infty}^{w_2-w_1} e_{ub} f(e_{ub})\,de_h \Big/ \int_{-\infty}^{w_2-w_1} f(e_{ub})de_{ub}\right] = -\int_{-\infty}^{w_2-w_1} e_{ub} f(e_{ub})de_{ub}$$

We obtain the following where the last step relies on $f(e_{ub})$ being zero-mean which makes the first integral negative and the second integral zero. The term in brackets is positive because $w_1 > w_2$ by assumption.

$$\frac{E[bias\,in\,w_2]}{w_2} - \frac{E[bias\,in\,w_1]}{w_1} = -\frac{1}{w_2}\int_{-\infty}^{w_2-w_1} e_{ub} f(e_{ub})de_{ub} - \frac{1}{w_1}\int_{w_2-w_1}^{\infty} e_{ub} f(e_{ub})de_{ub}$$

$$= -\frac{1}{w_2}\int_{-\infty}^{w_2-w_1} e_{ub} f(e_{ub})de_{ub} - \frac{1}{w_1}\int_{w_2-w_1}^{\infty} e_{ub} f(e_{ub})de_{ub} + \frac{1}{w_1}\int_{-\infty}^{w_2-w_1} e_{ub} f(e_{ub})de_{ub} - \frac{1}{w_1}\int_{-\infty}^{w_2-w_1} e_{ub} f(e_{ub})de_{ub}$$

$$= -\left(\frac{1}{w_2} - \frac{1}{w_1}\right)\int_{-\infty}^{w_2-w_1} e_{ub} f(e_{ub})de_{ub} - \frac{1}{w_1}\int_{-\infty}^{\infty} e_{ub} f(e_{ub})de_{ub} > 0$$

Thus, for the simple model, both partworths are biased upward <u>and</u> the larger partworth is biased relatively more than the smaller partworth.

Winner's Curse Technical Arguments

In the text we argue that the following equation updates partworths for ACA:

(1) $$\vec{w}_{q+1} - \vec{w}_q = (I + X_q' X_q)^{-1} \vec{x}_{q+1}' \frac{a_{q+1} - \hat{a}_{q+1}}{1 + \vec{x}_{q+1}(I + X_q' X_q)^{-1} \vec{x}_{q+1}'}$$

We pre-multiply this equation with a row vector of 1's, $\vec{e}$, and suppose for a moment the term, $\vec{e}(I + X_q' X')^{-1} \vec{x}_{q+1}'$, is of the same sign as $\vec{e}.\vec{x}_{q+1}'$ In the positive case, for example, Equation 1 implies that whenever the observed answer is larger than the expected answer, the sum of the partworths will increase. (This is the case because $\vec{e}(\vec{w}_{q+1} - \vec{w}_q)$ is the difference in the sum of the partworths). Attempting to balance utility based on estimated partworths increases the likelihood that an answer is "cursed." That is, the utility-balance criterion attempts to get $\hat{a}_{q+1}$ as close to zero as feasible and, in doing so, exploits random errors in the answers to the first q

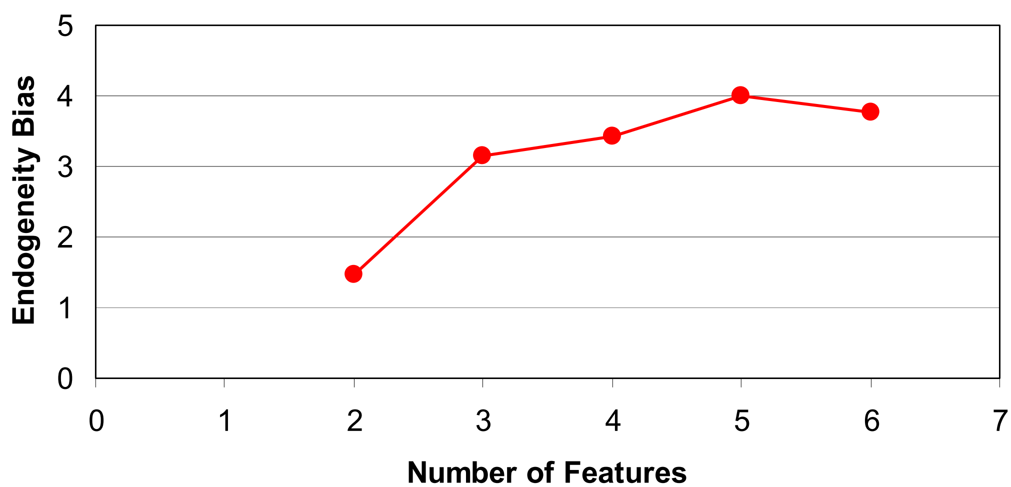
questions. When we observe a_{q+1} we are surprised (cursed) that it is larger than expected and this affects our estimate of $\vec{w}_{q+1}$.

Equation 1 provides the intuition, but it is not a proof (it assumes that $\vec{e}(I + X'_q X')^{-1}\vec{x}'_{q+1}$, is of the same sign as $\vec{e}.\vec{x}'_{q+1}$) and, technically, it applies only to OLS estimation. We might simplify Equation 1 for special X'_q matrices to approximate a general ACA matrix, but the equation remains approximate, not exact.[1] We have not been able to obtain a general analytic proof, rather we simulate reasonable-sized problems to gain insight on the practical implications of Equation 1.

Winner's Curse Simulations and Graphs

The text provides simulations which attempt to test the winner's curse when four features are allowed to vary. In this appendix we provide (1) an example where three features are allowed to vary and (2) a graph of expected bias as a function of the number of features that are allowed to vary. Please note that when there are an odd number of binary features varying, "same" questions are not feasible. This phenomenon is seen clearly in Figure A1. The graphs and the table use the same parameters as Table 3, with 200 simulated respondents.

Figure A1
Endogeneity Bias as a Function of the Number of Features that Vary



[1] One analytic solution is possible if $X_q{}'X_q$ is proportional to an identity matrix. ACA uses constraints to promote orthogonality in $X_q{}'X_q$, but, as we argue later, metric utility balance, which tends to make this matrix singular, works against an analytic solution.

Table A1
Simulations to Demonstrate the Winner's Curse -- Three-Variable Example

	Adaptive Utility-Balanced Questions	Random Questions
Percent "up" questions	59%	84%
Percent of "down" questions	41%	16%
Percent of "same" questions	–	–
Percent of "up" questions that are "cursed"	55%	44%
Percent of "down" questions that are "cursed"	24%	24%
Percent of "same" questions that are "cursed"	–	–
Evolution for "up" questions X percent "up"	0.17	-0.13
Evolution for "down" questions X percent "down"	0.20	0.43
Evolution for "same" questions X percent "same"	–	–
Overall bias	3.15*	-0.14

*Significant at 0.01 level

Heterogeneity and Selection Bias in Metric Utility Balanced Questions

Following the stylized model in the text, we first ask the question of {1, 0} vs. {0, 1} to obtain an unbiased observation of $w_1 - w_2 = a_{ub}$. We then use minimum estimated utility balance to design the second question.

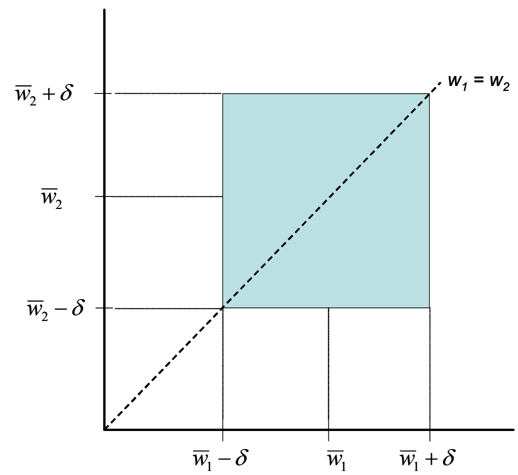
First question:	$\hat{w}_1 - \hat{w}_2 = w_1 - w_2$			
Second question:	$\hat{w}_1 = w_1$	if	$a_{ub} < 0 \Rightarrow w_1 \leq w_2$	(Case 1a)
	$\hat{w}_2 = w_2$	if	$a_{ub} > 0 \Rightarrow w_2 \leq w_1$	(Case 2a)

Suppose that $\vec{w}_1$ and $\vec{w}_2$ are both independently uniformly distributed with means $\overline{w}_1$ and $\overline{w}_2$. Define δ as the spread of the uniform distributions, i.e., w_1 varies from $\overline{w}_1 - \delta$ to $\overline{w}_1 + \delta$. Thus, δ indicates the magnitude of the heterogeneity. For ease of exposition we set $\overline{w}_1 = \overline{w}_2 = \overline{w}$.

Hence, by symmetry, $\Pr\{w_1 \leq w_2\} = \Pr\{w_2 \leq w_1\} = \frac{1}{2}$. Integrating over the region for which

$w_1 \leq w_2$, we obtain $E[w_1 | w_1 \leq w_2] = \dfrac{\frac{1}{6}(\overline{w}+\delta)^3 + \frac{1}{3}(\overline{w}-\delta)^3 - \frac{1}{2}(\overline{w}+\delta)(\overline{w}-\delta)^2}{2.\delta^2} = \overline{w} - \dfrac{\delta}{3}$. Similarly, $E[w_2 \mid w_2 \leq w_1] = \overline{w} - \delta/3$. Thus, the second question will ask about w_1 for half of the population and these answers will be downwardly biased by $\delta/3$. This selection bias will be mitigated by the first question, which is asked of the entire population and is not subject to selection bias. Nonetheless, the net result will be a downward bias in the mean of w_1 and this bias will be larger for populations that are more heterogeneous. These calculations are illustrated by the following figure. We encourage readers to explore other distributions for $f(w_1, w_2)$.

Figure A2
Illustration of the Stylized Model with Symmetric Uniform Distributions



Utility Balance and Choice-Based Questions

Following Arora and Huber (Equation 5) and Kanninen (Equation 9) we illustrate the impact of utility balance on efficiency with the binary logit model. In particular, for binary choice sets it is easy to show that:

$$\Sigma^{-1} = R\sum_{i=1}^{q}(\vec{x}_{i1} - \vec{x}_{i2})' P_{i1}(1-P_{i1})(\vec{x}_{i1} - \vec{x}_{i2}) = R\sum_{i=1}^{q}\vec{d}_i{}' P_{i1}(1-P_{i1})\vec{d}_i$$

where $\vec{d}_i$ is the row vector of differences in the features for the i^{th} choice set. In general, optimizing Equation 2 requires numerical means (e.g., Kanninen 2002), however, we can illustrate the basic intuition by examining the trace of Σ^{-1}. The trace is given by:

$$trace(\Sigma^{-1}) = \sum_{i=1}^{q}\sum_{k=1}^{K} d_{ik}^2 P_{i1}(1-P_{i1})$$

If we focus on one feature, say the K^{th} feature, and allow it to vary continuously, then the first-order conditions for the focal feature are:

$$d_{iK} = \left(p_K \sum_{k=1}^{K} d_{ik}^2 \right) \left(\frac{P_{i1} - P_{i2}}{2} \right) \text{ for all } i$$

where d_{ik} is the level of the k^{th} feature difference in the i^{th} binary question.

We first rule out the trivial solution of $d_{ik} = 0$ for all k, which would imply a choice between two identical alternatives. The only other solutions imply some utility imbalance because non-zero d_{iK} requires that $P_{i1} \neq P_{i2}$. Following Arora and Huber (2001) and Toubia, Hauser and Simester (2004), we use the magnitude of the partworths as a measure of response accuracy. Because $trace(\Sigma^{-1}) = \sum_{i=1}^{q} \sum_{k=1}^{K} d_{ik}^2 P_{i1}(1 - P_{i1})$ is separable in i, we focus on a single i and drop the i subscript. We let $\vec{p}$ be the partworths for choice-based questions and rewriting P_i in terms of the partworths, $\vec{p}$. We allowing m to scale the magnitude of the partworths and we obtain for a given i, $T \equiv trace_i(\Sigma^{-1}) = \sum_{k=1}^{K} d_k^2 \Big/ (e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}} + 2)$. We assume, without loss of generality, that $p_K > 0$ and $y_{K-1} \equiv \sum_{k=1}^{K-1} p_k d_k \leq 0$. Define $s_{K-1} \equiv \sum_{k=1}^{K-1} d_k^2$ and let d_K^* be the optimal d_K.

Lemma 1. For $p_K > 0$ and $y_{K-1} \leq 0$, $d_K^* \geq 0$.

Proof. Assume $d_K^* < 0$. If $a^* = p_K d_K^* + y_{K-1} < 0$, then $trace^* = (d_K^{*2} + s_{K-1})/((e^{ma^*} + e^{-ma^*} + 2)$. Consider $d_K^{**} = (-a^* - y_{K-1})/p_K > 0$ such that $a^{**} = -a^* > 0$. This assures that the denominator of the trace stays the same. Now $\left|d_K^{**}\right| = \left|(-a^* - y_{K-1})/p_K\right| > \left|(a^* - y_{K-1})/p_K\right| = \left|d_K^*\right|$ if $y_{K-1} < 0$ because both a^* and y_{K-1} are of the same sign. Thus, the numerator of the trace is larger and the denominator is unchanged and we have the result by contraction. In the special case of $y_{K-1} = 0$, $trace(d_K) = trace(-d_K)$, hence we can also restrict ourselves to $d_K \geq 0$.

Lemma 2. For $p_K > 0$, $y_{K-1} < 0$, then the trace has no minimum in d_K on (0, ∞).

Proof. $T'(d_K) = [2d_K(e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}} + 2) - \sum_k d_k^2 m p_K (e^{m\vec{d}\vec{p}} - e^{-m\vec{d}\vec{p}})]/(e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}} + 2)^2 \equiv h(d_K)/b(d_K)$. Then, $T'(d_K^*) = 0 \Rightarrow h(d_K^*) = 0$ and $T''(d_K^*) = h'(d_K^*)/b(d_K^*)$. $T''(d_K^*)$ will have the same sign as $h'(d_K^*) = 2(e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}} + 2) + 2d_K m p_K (e^{m\vec{d}\vec{p}} - e^{-m\vec{d}\vec{p}}) - 2d_K m p_K (e^{m\vec{d}\vec{p}} - e^{-m\vec{d}\vec{p}})$ $- \sum_k d_k^2 m^2 p_K^2 (e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}})$. Cancel the second and third terms. Using the FOC gives

$h'(d_K^*) = \sum_k d_k^2 m p_K (e^{m\vec{d}\vec{p}} - e^{-m\vec{d}\vec{p}})/d_K - \sum_k d_k^2 m^2 p_K^2 (e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}})$. Thus, $h'(d_K^*)$ and $H \equiv (e^{m\vec{d}\vec{p}} - e^{-m\vec{d}\vec{p}}) - m d_K p_K (e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}})$ have the same sign. Because $d_K^* \geq 0$ by assumption, the FOC imply that $e^{m\vec{d}\vec{p}} - e^{-m\vec{d}\vec{p}} \geq 0$, hence $m\vec{d}\vec{p} > 0$. If $m d_K p_K \geq 1$, then H < 0 and so is $T''(d_K^*)$. If $m d_K p_K < 1$, then $m\vec{d}\vec{p} < 1$ because $y_{K-1} < 0$. Thus, $e^0 < e^{m\vec{d}\vec{p}} < e^1$, hence $H < e - 1 - 2 m d_K p_K$. Thus, $H < 0$ if $m d_K p_K > (e-1)/2$. Call $h_0 = (e-1)/2$. If $m d_K p_K < h_0$, then $0 < m\vec{d}\vec{p} < h_0$ and $H < e^{h_0} - 1 - 2 m d_K p_K$, which is negative if $m d_K p_K > h_1 = (e^{h_0} - 1)/2$. For all *h* in (0,1], we have $(e^h - 1)/2 < h$, so by recursion we show that $H < 0$ if $m d_K p_K > h_\ell$ and h_ℓ converging to zero.

Proposition 2. When optimizing A-efficiency for binary choice, greater response accuracy implies greater utility balance.

Proof. Assume $p_K > 0$ *and* $y_{K-1} \leq 0$ without loss of generality and rewrite $T = u(d_K, m)/v(d_K, m)$ where $u(d_K, m) = \sum_k d_k^2$, $v(d_K, m) = (e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}} + 2)$. For $m_o > 0$, $d_K^*(m_o)$ satisfies $f \equiv u'/u = v'/v \equiv g$. The numerator, *u*, does not depend upon *m*. Taking derivatives yields $v'/v = m p_K (e^{m\vec{d}\vec{p}} - e^{-m\vec{d}\vec{p}})/(e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}} + 2)$ and $\partial(v'/v)/\partial m =$

$$\frac{[p_K(e^{m\vec{d}\vec{p}} - e^{-m\vec{d}\vec{p}}) + m p_K \vec{d}\vec{p}(e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}})][e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}} + 2] - m p_K \vec{d}\vec{p}(e^{m\vec{d}\vec{p}} - e^{-m\vec{d}\vec{p}})^2}{(e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}} + 2)^2}.$$

We rearrange the numerator to obtain the following expression: $p_K(e^{m\vec{d}\vec{p}} - e^{-m\vec{d}\vec{p}})(e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}} + 2) + 2 m p_K \vec{d}\vec{p}(e^{m\vec{d}\vec{p}} + e^{-m\vec{d}\vec{p}}) + 4 m p_K \vec{d}\vec{p}$, which is positive because $m\vec{d}\vec{p} > 0$ and $e^{m\vec{d}\vec{p}} - e^{-m\vec{d}\vec{p}} \geq 0$ as proven in Lemma 2. Hence, v'/v is increasing in *m*. Thus, $g(d_K^*(m_o), m_1) > g(d_K^*(m_o), m_o)$ for $m_1 > m_o$, which implies $f(d_K^*(m_o), m_1) = f(d_K^*(m_o), m_o)$ $= g(d_K^*(m_o), m_o) < g(d_K^*(m_o), m_1)$. Hence, $T'(d_K^*(m_o), m_1) < 0$.

We have shown that at $m = m_1$, the derivative to *T* with respect to d_K is negative at $d_K^*(m_o)$. By Lemma 2, it is non-positive for all $d_K > d_K^*(m_o)$. Thus, $d_K^*(m_1) < d_K^*(m_o)$. Because $y_{K-1} + d_K^*(m_o) p_K > 0$, $y_{K-1} + d_K^*(m_1) p_K > 0$, and $y_{K-1} < 0$, we have $0 < y_{K-1} + d_K^*(m_1) p_K < y_{K-1} + d_K^*(m_o) p_K$, which proves that utility balance increases.

Parameters for Simulations in Text

Table 1 . Ten binary partworths with true differences 10, 20, …, 100, e.g., ±5, ±10, …, ±50. Response error is normally distributed with standard deviation 20. Answers to self-explicated questions are necessary for ACA question selection, but are not used in the estimation. SE questions are unbiased with normally distributed noise, standard deviation 20. Estimation via OLS.

1,000 simulated respondents answering twenty questions each. Profiles differ on three attributes in the paired-comparison questions.

Table 3 and Figure 1. Ten binary partworths with true differences uniformly distributed on [0,100]. Response error is normally distributed with standard deviation 20. SE questions are unbiased with normally distributed noise, standard deviation 20. Estimation via ACA's OLS methods. 1,000 respondents answering ten paired-comparison questions each (in addition to the ten SE questions, bringing the total number of questions to 20).

Table 4. Ten binary partworths with true average differences 10, 20, …, 100. Individual respondent partworths are equal to the population mean plus a deviation that is drawn from a zero-mean normal distribution with standard deviation 20. Response error and SE questions as in the simulations for Table 1. 1,000 simulated respondents answering twenty questions each. Estimation is via OLS. In the first row of Table 4, ACA is used to select questions. In the second, third, and fourth rows the same first-row questions are used. In the second row, we redraw the true partworths from a normal distribution, but keep the response errors the same as they were in the first row. In the third row, we keep the true partworths the same as the first row, but redraw the response errors. In the fourth row, both the true partworths and the response errors are redrawn. Details are provided in Table A2. For comparison, Table A2a normalizes the mean partworths then computes the percent differences vs. the true population means.

Table A2: Detailed Results Underlying the Summaries in Table 4 from the Text

	True Population Means	Means Based on ACA Questions	Same Questions, Response Error Redrawn	Same Questions, Heterogeneity Redrawn	Heterogeneity and Response Error Redrawn
Handle	9.3	3.7	2.7	10.7	9.6
Price	20.0	8.6	8.3	20.1	19.8
Logo	29.6	15.1	14.5	31.5	30.8
Closure	39.1	20.7	20.8	40.4	40.4
Mesh pockets	49.4	28.7	28.0	50.6	49.9
PDA	60.0	35.4	35.2	61.1	60.9
Cell phone	68.9	41.9	41.6	69.1	68.9
Color	79.9	49.6	49.0	81.2	80.7
Size	89.5	55.4	55.6	90.0	90.2
Boot	99.9	63.0	62.4	101.7	101.1
Mean of estimates	54.6	32.2	31.8	55.6	55.2
Selection bias	–	-41%	-42%	2%	1%
Endogeneity bias	–	6.3	-0.5	6.9	0.1
t-test of endo. bias	–	7.1	-0.7	6.2	0.1

Table A2a: Normalized Percent Differences (vs. True Population) for Table A2

	True Population Means	Normalized Difference ACA Questions	Normalized Difference Same Questions, Response Error Redrawn	Normalized Difference Same Questions, Heterogeneity Redrawn	Normalized Difference Heterogeneity and Response Error Redrawn
Handle	9.3	-50.2%	-32.6%	12.8%	2.0%
Price	20.0	-28.8%	-27.2%	-1.5%	-2.2%
Logo	29.6	-16.0%	-13.6%	4.4%	2.8%
Closure	39.1	-8.8%	-10.3%	1.3%	2.1%
Mesh pockets	49.4	-2.8%	-1.6%	0.4%	-0.2%
PDA	60.0	0.6%	-0.1%	-0.1%	0.3%
Cell phone	68.9	3.6%	3.0%	-1.7%	-1.2%
Color	79.9	5.2%	5.2%	-0.3%	-0.2%
Size	89.5	6.6%	4.9%	-1.4%	-0.4%
Boot	99.9	7.1%	6.8%	-0.2%	0.0%
Correlation		0.93	0.89	-0.63	-0.36
t-test of correlation		7.32	5.67	-2.28	-1.08

Table A3
Empirical Test of Aggregate Selection Biases in ACA Questions

	Orthogonal Questions		ACA Questions		
Actual Features	**Average of Individual Estimates**	**Aggregate Estimates**	**Average of Individual Estimates**	**Aggregate Estimates**	**Normalized Difference in Aggregate Estimates**
Handle	28.0	29.3	55.5	16.4	19.0%
Price ($100 - $70)	54.0	53.3	59.6	17.8	-29.0%
Logo	24.3	23.3	18.4	0.9	-91.8%
Closure	13.3	15.1	22.8	6.1	-14.1%
Mesh pockets	7.3	8.7	10.4	1.0	-75.6%
PDA	9.4	10.9	2.6	8.3	62.0%
Cell phone	11.2	11.5	8.6	0.8	85.2%
Color	27.7	29.1	49.8	18.5	35.2%
Size	7.3	7.6	38.2	23.5	557.7%
Boot	22.2	22.4	25.6	6.0	-43.0%
Correlation. with Orthogonal.			0.75	0.39	0.30
t-statistic			3.18	1.18	-0.87

Comparison Of Adaptive Polyhedral Questions and Orthogonal Questions

The following graph uses the framework in Toubia, et. al. (2004) – four features at four levels each with partworths drawn from a normal distribution with mean 0.5*{-1, -1/3, 1/3, 1} and standard deviation equal to √(0.5*0.5)=0.5. Two hundred (200) simulated respondents chose from sixteen questions with four profiles per choice task based on logistic choice probabilities calculated from these partworths. Four features at four levels give twelve independent partworths. We normalize and order the true partworths, sort them into twelve categories, and compute the differences in errors for each category (errors for polyhedral HB minus errors for fixed orthogonal HB). The differences in errors are not significantly different at the 0.05 level.

Figure A2
HB Errors Do Not Vary Based on Question-Selection Method