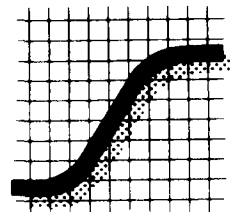# Note

COMPETITIVE PRICE AND POSITIONING STRATEGIES

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Brand positioning and brand pricing are important strategic decisions for marketing managers. Such decisions are interrelated and depend upon competitive brand positions and prices. However, any unilateral decisions may encourage repositioning and price adjustment by competitors thus leading to either new market equilibria or a price/positioning "war." This paper examines such price and positioning decisions in a competitive environment by extending the 'Defender' consumer model to more complex equilibria. We assume that the brands are already in the market and that positions are "sticky" in the sense that brands cannot leapfrog one another.

Among the insights for such incumbent brands are:

• if prices are symmetric and held constant, firms seek to reposition toward the center of the market,

• but price advantages by central firms can mediate that central tendency such that an equilibrium exists with positive profits;

• if brand positions are held constant, the Nash equilibria for prices exist and are unique, and

• category profits, the sum of brand profits, are maximized for maximum brand differentiation;

• for two and three brand markets the (subgame perfect Nash) equilibrium, where positioning decisions are made with foresight on the price equilibria, causes firms to unilaterally seek maximum brand differentiation; but

• for four or more brands greater foresight and/or cooperation appears necessary to reach the point of maximum brand differentiation.

These results apply where the Defender consumer model is an exact description of consumer response and where the competitive reaction assumptions hold. In any empirical situation such forces might be softened resulting in differentiated, but not maximally differentiated, brands.

(Pricing; Positioning)

1. Motivation

In 1983 Steven Shugan and I published a paper on defensive marketing strategy (Hauser and Shugan 1983) in which we examined how an existing brand should re-

spond to a competitive new product entry. Since then the consumer model has been tested empirically (Hauser and Gaskin 1984) and the full model has been applied in over 20 managerial situations in the United States, Europe, Brazil, and Japan (Hauser 1986a; Klein 1984, 1985; Foltz 1985). It appears to be a reasonable description of consumer behavior.

Recently a number of authors (Carpenter 1986; Eliashberg and Chatterjee 1985; Kumar and Sudharshan 1988; and Schmalensee and Thisse 1985) have discussed the original 'Defender' equilibrium assumptions.[1] An important question is which results hold if all brands, not just the attacker and defender, have full response capability. In this paper I seek to address this issue.

In parallel, recent advances by economists in the study of competition have called into question classical results such as Hotelling's principle of minimum differentiation (Hotelling 1929) and have suggested new ways of examining market structures when all firms have full response capability and foresight. See Kreps and Spence (1984), d'Aspremont, Gabszewicz, and Thisse (1979), Economides (1984, 1986a, b), Shaked and Sutton (1982), Lane (1980), Prescott and Visscher (1977), Eaton and Lipsey (1975), Eaton and Wooders (1985), Novshek (1980), and Axelrod (1984). I seek to add to these insights by examining some of the equilibrium implications of the Defender consumer model. Some results extend to this marketing model, others do not.

To highlight positioning issues and to keep the analysis feasible, I analyze the situation where brands are already in the market. Such mature markets represent a large portion of the situations faced by marketing managers. For example, repositioning an existing brand, changing advertising copy, adjusting price, or reformulating product features are all considerations in mature markets. In fact, many of the defensive strategy applications to date have been for situations in which a manager sought to understand his market and adjust his brand's positioning to obtain maximum profitability. In those applications, competitors' reactions were modeled judgmentally and the consumer model was used to forecast consumer reaction to the new strategies.

Topics such as the decisions by firms on whether and where to enter a market, the actions by firms to deter entry of competitive products, and the decisions to exit a market are left to future research. However, a precursor to understanding entry and exit is understanding how the market will behave after firms have entered the market.

This paper shows that the (Nash) equilibrium in prices exists for the Defender model and it is unique. (A formal proof is summarized and referenced.) If positioning decisions are made without considering the price equilibria, firms will cluster closer to one another than is optimal (similar to Hotelling 1929) and experience fierce price competition (similar to d'Aspremont et al. 1979 and Shaked and Sutton 1982). On the other hand, if price competition is considered, firms make more profit if they seek maximum differentiation to achieve "local monopolies." For two or three brand markets maximum differentiation is a stable subgame perfect Nash equilibrium. For four or more brands firms have unilateral incentives to deviate from maximum differentiation and hence some mechanism such as long-term motivation, prisoner's dilemma-like cooperation, or coalitions are needed to maintain stability. The paper closes with comments and hypotheses about empirical implications.

2. Analytical Model of Consumer Response

This section provides a brief review of the Defender model. For greater detail and empirical evidence see Hauser and Shugan (1983), Hauser and Gaskin (1984), Hauser (1986a, b) and Shugan (1986).

[1] In Hauser and Shugan (1983) we assumed implicitly that the entrant enters with perfect foresight as to the defenders' response. The paper then analyzed the subsequent defensive response. Thus, for these two brands the equilibrium has a von Stackelberg flavor. The paper discussed, but did not model explicitly, responses by other brands.

Basic Model

The consumer model is illustrated by Figure 1, a hypothetical representation of an industrial robotics market. Customers in this hypothetical market evaluate brands on the perceptual dimensions of 'Ease of Use' and 'Power.' These subjective evaluations are in turn based on physical product characteristics such as servo motors, computer chips and the like and on psycho-social cues such as advertising and salesforce messages. The firm decides on a perceptual position ('ease of use' and 'power') and chooses the optimal characteristics and advertising (salesforce) to achieve that position. See Hauser (1986b), Hauser and Simmie (1980), or Schmalensee and Thisse (1985) for details and examples. Thus, from the firm's perspective, there is a resulting cost associated with each perceptual position. Figure 1 shows two dimensions and three products. The model has been applied to more dimensions (e.g. Hauser and Gaskin 1984), but analytical results have been limited by tractability considerations to two dimensions. There is no theoretical limitation on the number of products.

The model assumes consumers choose the brand that maximizes value where value[2] is a linear combination of "per dollar" dimensions, e.g., w_1 * ('power' per dollar) + w_2 * ('ease of use' per dollar), where w_1 and w_2 are weights.

Consumers are assumed heterogeneous in tastes, that is, w_1 and w_2 vary by consumers. We have found it convenient to represent consumer tastes by the angle, α, that the indifference curve makes with the vertical axis. Mathematically, $\alpha = \arctan(w_2/w_1)$. If consumer tastes are distributed with density $f(\alpha)$, it is easy to show that the market share, m_j, of the jth brand, say ROCON, is just

$$m_j = \int_{\alpha_{IJ-}}^{\alpha_{IJ+}} f(\alpha)d\alpha \tag{1}$$

[2] This assumption can in turn be derived from the more fundamental assumptions that (1) consumers maximize utility subject to a budget constraint, (2) utility is separable by product category, and (3) utility is linear in perceived characteristics (see Hauser and Urban 1986 or Hauser 1986a). Alternatively the same model can be derived from Lancaster-like arguments (Lancaster 1971) if one considers explicitly the relationship among physical and perceived characteristics (Hauser and Simmie 1981).
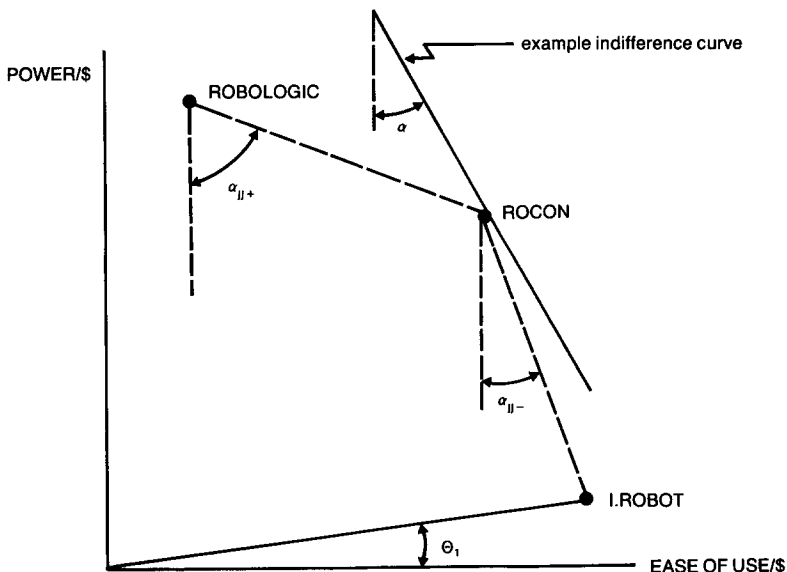

FIGURE 1.   Illustration of Defender Model in a Hypothetical Robotics Market Containing Three Firms and Two Dimensions.

where α_{jj+} is the angle (with the vertical axis) of the line connecting brand j to the next efficient brand (e.g., ROBOLOGIC) above brand j and α_{jj-} is the angle for the next efficient brand (e.g., I.ROBOT) below brand j. See examples in Figure 1.

This is the basic analytical model. Note that it does not make any assumptions about symmetry or returns to scale. However, in their analysis Hauser and Shugan (1983) assumed constant returns to scale, no fixed costs, and modeled, in addition, awareness and availability as functions of advertising and distribution spending. This paper continues to assume constant returns to scale and leaves to future elaboration the impacts of nonconstant returns, of fixed costs, and of awareness and availability.

Further assumptions for this paper. Intuitively, positioning decisions depend on the cost structure, the preference structure, and competition. For example, if 'power' were most costly than 'ease of use' we would expect firms to seek positioning nearer 'ease of use'. Similarly if consumer tastes, $f(\alpha)$, favor 'ease of use' we would expect firms to position toward 'ease of use'. See Hauser (1986b) for examples of these effects. However, if all competitors have freedom of movement, the tendency may not be so obvious. For example, with the Hotelling model and with a reasonable cost structure, Neven (1986) shows that firms avoid the center of the market even if the taste distribution peaks there. DePalma et al. (1985), citing marketing positioning models as motivation, show that consumer heterogeneity also can drive firms from the center of the market.

In this paper I temporarily suppress the effects of nonuniform cost and tastes to isolate the effect of competition. In particular, I assume that all firms face the same marginal costs and that for these marginal costs firms can position on a quarter circle. That is, isocost curves are given by ('ease of use')^2 + ('power')^2 = k^2.

Thus, unlike the general Defender model, positioning decisions in this paper are reduced to a single numeraire, the ratio of the two dimensions. This ratio corresponds to tan Θ_1 in Figure 1. However, when prices vary, the positions in "per dollar" space are not restricted to a quarter circle. In fact, some brands can dominate others. (Geometrically, in the polar coordinates of per-dollar perceptual space, "position" defines the angle and price defines the radius.)

I also assume that tastes are uniformly distributed on the interval 0° to 90°, hence equation (1) becomes $m_j = (\alpha_{jj+} - \alpha_{jj-})/90°$. While such assumptions abstract reality in many markets,[3] these assumptions allow us to study the effects of competition on position and/or price against a background of uniformity. Nonuniform costs and tastes then reinforce or mitigate our results depending on the specifics of the market.

To analyze mature markets I make an additional assumption about the options open to managers. I assume (1) the firms are already in the market and (2) their current positions are "sticky" in the sense that their rank order along the horizontal axis does not change. For example, if IBM is now viewed as the 'powerful' personal computer and Apple (Macintosh) is now viewed as the 'easy to use' personal computer, this assumption implies that Apple can reposition toward more 'power', but consumers may not believe a positioning of more 'power' than IBM. Such a restriction may be due to a firm's technical expertise, its previous advertising, inertia in consumer perceptions, or perhaps, perceptions that an easy to use computer cannot be powerful. My experience suggests that such an assumption is realistic in many markets. It also is a less restrictive assumption than that made in many papers in spatial economics. For example, Eaton and Wooders (1985), Prescott and Visscher (1977), and Lane (1980) assume brands do not move once they have chosen a position. In another example, Novshek (1980) modifies a "zero conjectures" equilibrium to effectively rule out leapfrogging. However, the "sticky" assumption will not apply to all markets.

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[3] For example, proprietary Defender studies suggest uniformly distributed tastes are reasonable in the plastic bags market but not in the wine cooler market or the bulk laxative market.

Finally, we "fix" an earlier problem with Defender by assuming a reservation price for the product category. In general, reservation prices can vary by consumer and can be complex functions of income and of other product categories, cf. Hauser and Urban (1986). For analytical simplicity, we consider a common reservation price, but posit that results do not change as long as the distribution of reservation prices among consumers is well-behaved.

All subsequent propositions and results are qualified by the assumptions of this section. That is,

(1) Consumer tastes are uniformly distributed.

(2) For a fixed price, brand positions are restricted to a quarter circle. (This effectively reduces positioning decisions to a decision on the relative trade-off among the two dimensions.)

(3) The mature brands are in the market. They do not exit and new brands do not enter.

(4) Positions are "sticky" in the sense that brands do not leapfrog on either dimension. (That is, the rank order of relative trade-offs among dimensions is fixed.)

Relationship to Hotelling's Model and to Lane's Model

Defender is similar to, but different than, models by Hotelling (1929) and Lane (1980). Hotelling assumes consumers' tastes are uniformly distributed on the interval $[0, l]$ and that consumers choose the brand that minimizes price plus the distance (or a function of the distance) from their tastes to the brand's location. In other words, Hotelling's model is similar to a one-dimensional ideal point model.

DePalma et al. (1985), Graitson (1982), and others have criticized the one-dimensional nature of this model and have argued for more realistic extensions to two dimensions. Economides (1986b) analyzes two dimensions, but restricts two brands to be located symmetrically on an axis through the center of the market. D'Aspremont et al. (1979), Economides (1986a), and Neven (1986) all illustrate how results on the Hotelling model are extremely sensitive to the external assumption of how distance from the ideal relates to utility.

In this paper, our "quarter circle" assumption restricts firm decisions to a single positioning decision, but we retain the more complex, and hopefully more realistic, two-dimensional representation of consumer reaction. For example, in Figure 1, ROCON's market share is a complex function[4] of the positions and prices of all three brands. That is, the market share (sales) as a function of position is derived from more primitive assumptions of consumer behavior. It is these primitive assumptions that lead ultimately to predictions of firm behavior.

Lane represents brands in a two-dimensional space. In fact, a logarithmic transformation of Lane's utility in physical characteristics space, as per Steven's (1946) power law, gives linear trade-offs in perceptual space. The key difference between the models is the treatment of price. In Lane's model, the logarithm of income minus price is treated as a third dimension, while Defender uses per-dollar attributes. This difference can have a major impact. For example, for one set of typical prices and positions[5] Defender gives a price elasticity of 4.4 while Lane's model gives a price elasticity of 0.001. Hauser and Urban (1986) show that such modeling differences can reverse a consumer's budget priorities. Hauser and Gaskin (1984) give an empirical example

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[4] Market share of ROCON $= \arctan\{[\cos \Theta_2 - (p_2/p_3) \cos \Theta_3]/[\sin \Theta_3 - (p_2/p_3) \sin \Theta_2]\} - \arctan\{[\cos \Theta_1 - (p_1/p_2) \cos \Theta_2]/[\sin \Theta_2 - (p_1/p_2) \sin \Theta_1]\}$. Only when $p_1 = p_2 = p_3$ does this reduce to a Hotelling-like equation.

[5] Positions for products 1, 2, and 3, respectably, are (10, 1), (7, 7), and (1, 10). Prices all equal 1.0 and income is 1000.

where the difference in treating price gives a 50% difference in predicted market share. The reader can verify other technical differences between the models such as the parameter of the taste distribution and product restrictions (a line in Lane's model). Suffice it to say that the models are different and results for one model may or may not apply to the other.

Finally, Lane concentrates on entry and deterrence. He assumes that brands enter one at a time in a known order and that brands cannot reposition after they enter. These assumptions allow Lane to avoid indeterminacies with respect to which brand gets which position in an equilibrium.

3. Positioning with Constant Prices

We now consider firms' behaviors. We begin by examining positioning decisions with price held constant (§3) and then examine pricing behavior with positioning held constant (§4). In §5, we combine the analyses by considering the "game" where firms choose positioning strategies with full foresight on the price equilibrium. For ease of exposition I state the results for three brands as illustrated by Figure 1. §6 indicates how the results extend to two brands and to more than three brands.

The first set of analyses is illustrated in Figure 2. I assume that prices, p_j, are equal and constant, that is, $p_1 = p_2 = p_3$. Of course, prices exceed costs so that the firms earn positive profits if they obtain positive market share. In this formulation it is convenient to represent brand positions by polar coordinates rather than rectangular coordinates, that is, by Θ_j, the angle of a ray connecting brand j to the origin (see Figure 2), and by r_j, the distance (radius) from the origin. If a firm spends k_j units on product characteristics, advertising, etc., the best it can do is to position such that ('ease of use')^2 + ('power')^2 = k_j^2. That is, $r_j = k_j/p_j$.

Interpreting the model behind Figure 2, it is easy to show that firms will have incentives to reposition toward the center of the market. This is true even though consumer tastes are distributed throughout the market. In other words, if firms make the naive assumption that competitors will not counter repositioning by price competi-
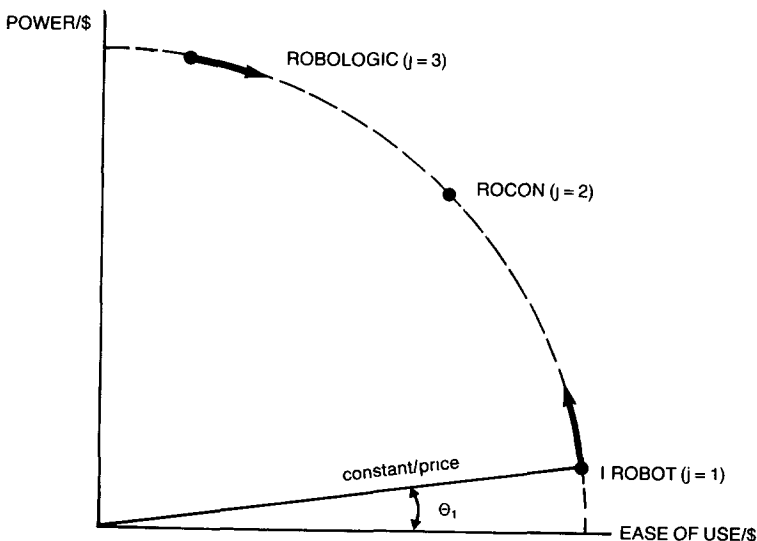

FIGURE 2. Example Analysis of Positioning Decisions When Price and Cost Are Fixed and Symmetric. (The firm's decision is to choose the relative tradeoffs as represented by Θ_1.)
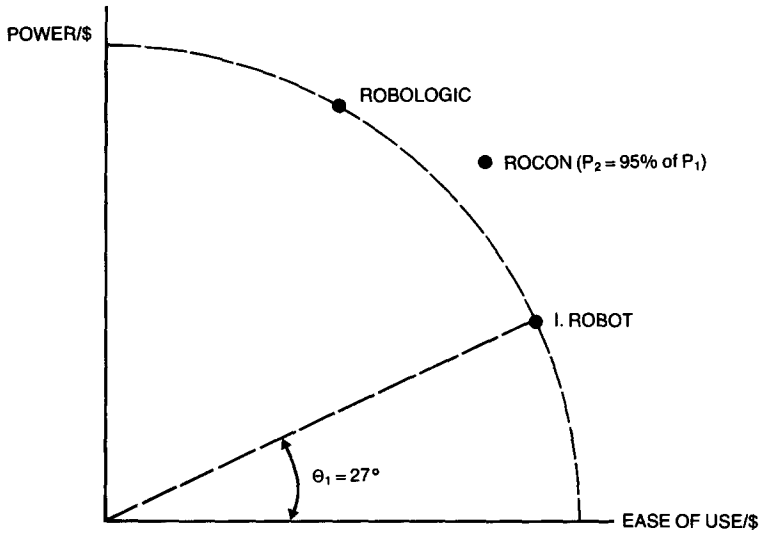
FIGURE 3. Example Equilibrium Where the Price Advantage of the Center Brand, ROCON, Balances the Central Tendencies of the Extreme Brands, ROBOLOGIC and I.ROBOT.

tion, then firms will naively position toward one another.[6] The proof is straightforward (see appendix). I state the result as a proposition to contrast it with later, more interesting, results.

PROPOSITION 1. *With symmetric and constant prices (and based on the assumptions of §2),*

(a) *profits of the extreme brands increase as they move toward the "center," that is, the central brand.*

(b) *the central brand has no incentive to move as long as it must remain the central brand, i.e., as long as* $\Theta_1 < \Theta_2 < \Theta_3$.

The central tendency in Proposition 1 is similar in spirit to Hotelling's (1929) principle of minimum differentiation which is derived without the assumption of constant prices. However, in that model price competition becomes fierce as firms approach the center of the market—so fierce that price equilibria no longer exist (d'Aspremont et al. 1979). Indeed, depending on the utility function, price competition may force firms away from one another (d'Aspremont et al. 1979; Economides 1986a). We now modify Proposition 1 by considering limited price differentiation.

Suppose that the central brand, $j = 2$, has a price advantage, that is, p_2 is less than both p_1 and p_3. See Figure 3 for an example. (Such price differences might result if the central brand entered first and now enjoys a cost advantage due to experience. The reason for the price advantage is unimportant for our analyses, we seek only to understand its implications.)

We consider now Nash equilibria,[7] that is, equilibria where each player has no incentive to move unilaterally. In particular, the equilibria we seek is a set of positions,

[6] If we were to relax the "sticky" assumption of §2, firms would jockey for inside and outside position as they approached the center of the market. See Lerner and Singer (1937, p. 178) or Prescott and Visscher (1975, pp. 387–388) for discussion of such jockeying within the context of spatial economics. Since we next relax the assumption of constant price, I will not elaborate such behavior here.

[7] To obtain an equilibrium we must specify firms' expectations, that is the expectations by each firm of its influence on the other firms. See Pindyck (1985). For our analyses we make the usual assumption, that is, firm j sets its position, Θ_j, under the belief that $d\Theta_k/d\Theta_j = 0$ for $k \neq j$. This assumption is also referred to as zero conjectural variation. For analyses with nonzero price conjectures, see Miller (1986). He shows that our conclusions are robust with respect to this conjecture.
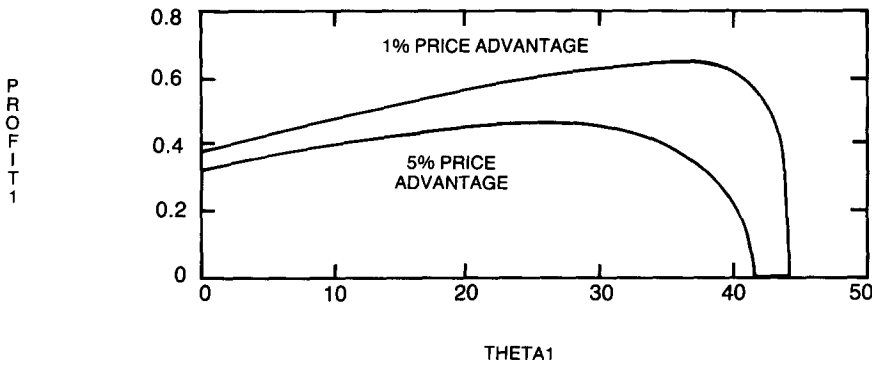
FIGURE 4. Plot of Profit (of I.ROBOT) as a Function of Its Position (Θ_1) with ROCON Fixed at $\Theta_1 = 45°$ and with a Price Advantage to ROCON ($p_2 < p_1$).

$\{\Theta_1^*, \Theta_2^*, \Theta_3^*\}$, such that if products 1 and 2 "play" Θ_1^* and Θ_2^*, it is optimal for product 3 to position at Θ_3^*, if products 2 and 3 "play" Θ_2^* and Θ_3^*, it is optimal for product 1 to position at Θ_1^*, and if products 1 and 3 "play" Θ_1^* and Θ_3^*, it is optimal for product 2 to position at Θ_2^*.

PROPOSITION 2. *With constant prices, with the assumptions of §2, and with $p_2 < p_1$ and p_3, the Nash equilibria for positions satisfy $\cos(\Theta_2 - \Theta_1) = p_2/p_1$ and $\cos(\Theta_3 - \Theta_2) = p_2/p_3$.*

PROOF. See appendix.

Proposition 2 illustrates how a price advantage can offset the central tendency of the extreme brands. For example, in Figure 3, ROCON has a 5% price advantage. I.ROBOT (and ROBOLOGIC) move toward the center of the market, ROCON, but do not reposition all the way towards the center because, as they approach ROCON, its price advantage counterbalances the advantage of being in the center of the market. In Figure 3 an equilibrium is reached at $\Theta_1^* = 27°$, $\Theta_2^* = 45°$, and $\Theta_3^* = 63°$.

To see this another way consider Figure 4. Figure 4 plots the profit of an extreme brand (I.ROBOT) as a function of position for a 1% and a 5% price advantage. Profit begins to increase as I.ROBOT moves toward the center. However, as I.ROBOT gets too close, 37° for a 1% advantage and 27° for a 5% advantage, it becomes less competitive until at 45° I.ROBOT is dominated by ROCON and earns zero market share and zero profit.

Proposition 2 also gives us insight on whether or not the extreme products will find it profitable to compete. Notice that if $\Theta_2 = 45°$ and p_2/p_1 is less than $\cos 45°$ (=0.707), the equilibrium does not exist for any feasible position because the central brand dominates. Thus, if the center brand has a sufficient price advantage, the extreme brands will choose not to enter.[8]

Finally, we note that Proposition 2 is a static result in the sense that firms with sufficient foresight could simply choose the Nash positions as their product formulations.[9] However, we can also give the result a more realistic managerial interpretation by assuming firms (1) begin at some positions, $\Theta_1 < \Theta_2 < \Theta_3$, not equal to the Nash positions, $\{\Theta_1^*, \Theta_2^*, \Theta_3^*\}$, (2) reposition in the direction of increasing profitability, (3) recompute profitability and reposition, and (4) continue doing so until the market reaches equilibrium. That equilibrium would be the equilibrium in Proposition 2.

Proposition 2 demonstrates that price competition will influence a positioning equi-

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[8] The presence of fixed costs would decrease further incentives to enter.

[9] As long as the order of brands along the horizontal axis is assumed "sticky" as discussed in §2.

librium. But it is still rather naive for firms to assume that prices will remain constant in the face of active repositioning.

4. Price Equilibria with Brand Positions Fixed

We now look at competition from another perspective. Suppose that firms have an established product formulation and advertising strategy, i.e., position, but can adjust price freely in response to competition. Such a market is illustrated in Figure 5. Here the three brands are in the market at positions Θ_1, Θ_2, and Θ_3. They cannot reposition but they can change their prices. Since a price change in "per-dollar" perceptual space simply moves brands in or out along rays connecting the products to the origin, price competition can be summarized by movement as shown by the heavy arrows in Figure 5.

We again seek Nash equilibria, but this time in prices. That is, for a given set of positions we seek equilibrium prices, $\{p_1^*, p_2^*, p_3^*\}$, such that no firm has the incentive to change its price unilaterally. For the Defender model with finite reservation prices, such equilibria can be shown to exist and to be unique (Hauser and Wernerfelt 1988).[10] However, no one has yet obtained an analytic solution for the Defender price equilibria. Fortunately, numerical solutions are readily obtainable. (Numerical solutions are also necessary for a two-dimensional Hotelling model and for Lane's model. See Eaton and Lipsey 1975 and Lane 1980, respectively.)

The condition for a Nash equilibrium, $\{p_1^*, p_2^*, p_3^*\}$, is that, for each brand j, the profit, π_j, is maximized for the Nash price, p_j^*, conditional on the other products being offered at their Nash prices. To obtain a numerical solution we search for those prices

[10] Finite reservation prices assure the strategy space is compact. We then show that the Defender profit function is quasi-concave and that considerations of undercutting preserve quasi-concavity. Existence follows from Friedman's (1977, p. 153) fixed point existence theorem. To prove uniqueness we show the profit function is concave up to its first (and only) inflection point and that the maximum must occur on the concave portion. We then show that the Jacobian of the pseudo-gradient is negative quasi-definite when strategies are restricted to the concave portion of the profit function. Uniqueness follows from Rosen's (1965) Theorems 4 and 6.
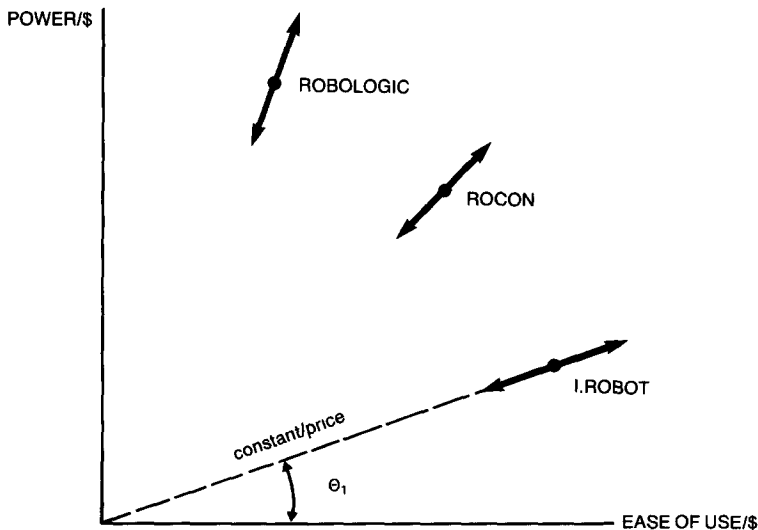

FIGURE 5.   Price Equilibria: When Relative Positions (Θ_1) Are Fixed Firms Can Adjust Prices. Movement in per-dollar space is indicated by heavy arrows.

that maximize profits, that is, those prices that solve simultaneously the following three equations:

$$\partial \pi_j(p_1^*, p_2^*, p_3^*)/\partial p_j = 0 \qquad \text{for} \qquad j = 1, 2, 3. \qquad (2)$$

The proof in Hauser and Wernerfelt (1988) assures that a solution to equation (2) exists and is unique, even when undercutting is considered. Nonetheless, I sought to verify this numerically as described below.

To solve equations (2) numerically, I used a gradient search method in which, at each step, I computed the derivatives in equations (2) and modified the prices in the direction of increasing profit as suggested by the derivatives. I continued until the prices stabilized. To verify the Nash solutions, I explored a number of alternative starting prices. In all cases, with reasonable step sizes, the solution converged rapidly to a maximum and appeared to be a unique global solution. (For example, low starting prices increased to the Nash equilibrium prices and high starting prices decreased to the same Nash equilibrium prices.) Finally, I checked incentives to undercut (dominate) by comparing profits from undercutting with the Nash solutions.

As before, the Nash solution can be viewed as a static game. However, as before, the numerical solution method suggests an interesting managerial interpretation of iterative price adjustment until the market stabilizes.

To illustrate these three-brand equilibria, I computed equilibria for Θ_2 near the center (42° through 45°) and for $\Theta_1 = 1°$ through ($\Theta_2 - 1°$). I also sampled other cases for a total of over 200 three-brand numerical solutions. An additional 600 cases for two, four, and five brands are discussed in §6.

The numerical solutions for the lower extreme brand, $j = 1$, and for the central brand, $j = 2$, are shown in Figure 6 as a function of the position of brand 1. (In Figure 6, Θ_2
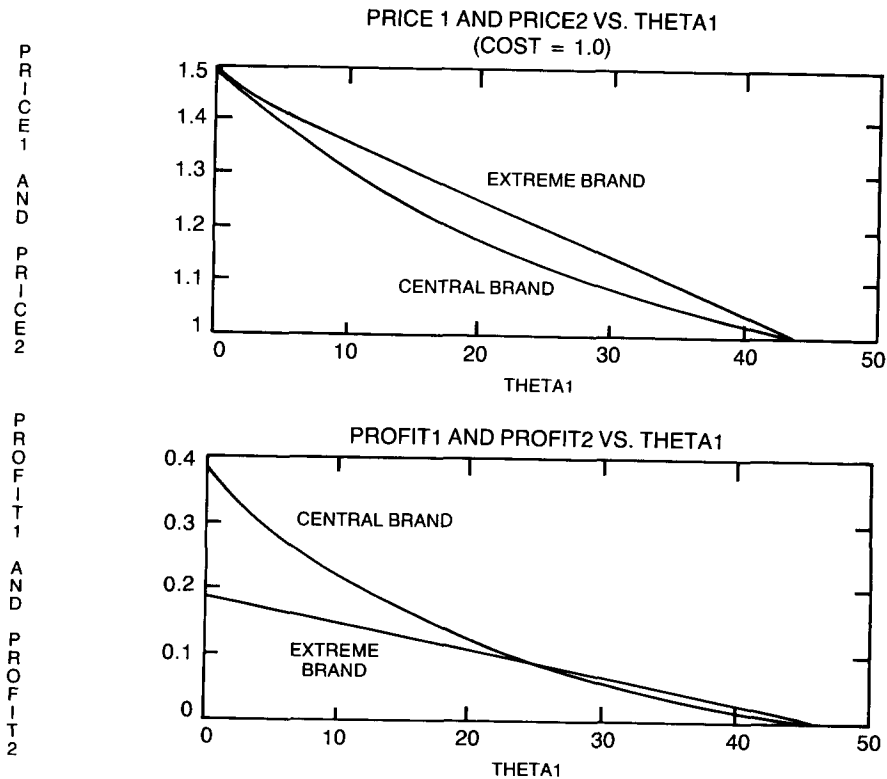

FIGURE 6. Plots of Equilibrium Prices and Profits for Three-Brand Markets as a Function of the Relative Positions of the Extreme Brands, e.g., Θ_1. The central brand stays at $\Theta_2 = 45°$.

= 45°.) Because $\Theta_3 = 90° - \Theta_1$, the profit and price of the upper extreme product equal those of the lower extreme product. The equilibrium solutions for profits and prices appear to be smooth and continuous. Furthermore, they are clearly maximized for differentiated products, that is, for $\Theta_1 = 0°$.

Similar plots are obtained for Θ_2 less than 45° (and by symmetry for Θ_2 greater than 45°) and in all cases the equilibrium profits of the central brand were maximized in the center of the market.

Interpreting Figure 6 it is clear what is happening. When the extreme brands ($j = 1$ and 3) are positioned closer to the central brand ($j = 2$), the market is more competitive and the equilibrium price is driven down by this competition. This in turn drives down profits. In the limit of undifferentiated brands the equilibrium price is driven to cost (price = 1.0 in Figure 6) and profits are driven to zero. On the other hand, when the brands are positioned away from one another they create, in essence, "local" monopolies and are able to extract more reasonable profits. Thus, as in Proposition 2, price and positioning are clearly related. We address that issue next.

5. Position and Price Strategy

Suppose now that each firm in the market is more sophisticated. That is, each firm makes a decision on position with full knowledge of the resulting price equilibria. In other words, through experience, marketing science models, or simply prescience, each firm can predict the end result of price competition for any positioning decision and will make that positioning decision accordingly.

Such a two-stage decision process is quite reasonable if product formulation changes are more "sticky" and difficult to make than price decisions. Such is the case with new products (cf. Urban and Hauser 1980) where positioning decisions are a key decision in new product design while price is chosen (or market driven) based on that position. Such two-stage decisions are also common in the economics literature. See Hotelling (1929), Prescott and Visscher (1977), d'Aspremont et al. (1979), Economides (1986a), Shaked and Sutton (1982), Neven (1986), Shilony (1981), de Palma et al. (1985), Kreps and Spence (1984), Eaton and Wooders (1985), and Lane (1980).

We again seek Nash solutions and, in addition, what are known as subgame perfect solutions (Kreps and Spence 1984). A subgame perfect solution simply requires that if we enter the "game" at any point, for example, after a first round decision, it is still optimal for each player to carry out their strategies. In other words, we rule out unrealistic bluffs and threats. For more discussion see the above references.

In our case the criterion of subgame perfect simply requires that once positioning decisions are made, the resulting pricing decisions are the Nash equilibrium solutions illustrated in Figure 6. Faced with Figure 6, the managerial positioning decision is clear, choose those positions to maximize profit in Figure 6. That is, choose $\Theta_1 = 0°$. (It is clear that $\Theta_1 = 90° - \Theta_3$ by symmetry.) For three brands numerical analysis quickly verifies that no brand has the incentive to unilaterally depart from these positions.

Notice in Figure 6 that the profits of the central brand ($j = 2$) exceed those of the extreme brands ($j = 1, 3$). Thus, as long as we accept the mature market assumption that brand positions are sufficiently "sticky" to rule out leapfrogging, the maximum profit positions in Figure 6 are indeed global Nash solutions. On the other hand, if we wish to consider entry, then there is an indeterminacy as to which brand is the center brand. Other authors have addressed this indeterminacy by assuming the order of entry is known (Lane 1980, Prescott and Visscher 1977) or by allowing mixed strategies (Gal-or 1982).

We formalize the numerical results as follows:

Result 1.   Based on the assumptions of §2, in the two-stage "game" where three firms are in the market and make positioning decisions with full knowledge of the price

equilibria that will result, the optimal (subgame perfect Nash) decisions imply maximum differentiation to $\Theta_1 = 0°$, $\Theta_2 = 45°$, and $\Theta_3 = 90°$.

In other words, when firms realize that the competitive equilibrium price depends upon their positioning decisions, they realize
- movement toward minimum differentiation implies fierce price competition, and
- movement toward maximum differentiation implies the ability to maintain "local" monopolies and resulting "monopoly" profits.

As a result they, independently and of their own volition, choose to offer differentiated products.

Such a result is intuitively pleasing. It suggests that there is valid economic reasoning to support market segmentation; it is consistent with results obtained in simpler models such as d'Aspremont et al.'s (1979) modification of Hotelling's model, Shaked and Sutton's (1982) unidimensional model of product quality, and Economides (1984, 1986a) analysis of Hotelling's problem with finite reservation prices and/or more general cost functions; and it is consistent with Lane's (1980) simulations of a model related to 'Defender.' Furthermore, it extends Hauser and Shugan's (1983) analyses to a more complete equilibrium analysis.

More importantly, Result 1 provides a bridge between the spatial economics literature and the marketing science literature. Because the consumer model of §2 is empirically based and managerially relevant and because it is readily extendable to a full array of marketing mix variables (cf. Hauser 1986a), the above propositions and results enhance the likely external validity of a principle of differentiation.

Finally, comparison of Proposition 1 and Result 1 indicates why it is important for firms to understand the strategic interdependency of their decisions. If, as in Proposition 1, firms naively assume that position does not impact price, they will seek the center of the market and suffer price wars. On the other hand, if they realize that price competition will occur and understand its equilibrium, firms will seek to segment the market and move away from the center.

6. Two Brands, More than Three Brands

For ease of exposition, all propositions and results in §§3 through 5 were stated for three brands in the market. Conceptually, similar results apply for two brands, four brands, and five brands.

Two Brands

Proposition 1a holds for two brands; both brands have the incentive to move toward the center if prices remain constant. (Since there is no central brand, Propositions 1b and 2 are not relevant for two brands.)

If positions remain constant the Nash price equilibrium exists and is unique for all interior positions ($\Theta_1 \neq 0°$ and $\Theta_2 \neq 90°$). Furthermore, the equilibrium prices and profits increase as the brands differentiate. However, as Θ_1 approaches $0°$ (with $\Theta_2 = 90° - \Theta_1$), equilibrium prices and profits become large as both brands exploit their local monopolies. The assumption of finite reservation prices becomes necessary for the equilibrium to exist.

Four Brands, Five Brands

Proposition 1 holds; with constant prices there is an incentive for brands to reposition toward the center of the market. If interior brands have price advantages, such price advantages offset the central tendency, hence propositions conceptually similar to Proposition 2 can be proven for four or five brands. However, the statements of the propositions must be made carefully.

Based on systematic numerical solutions of over 600 combinations of brand positions, we obtain the following results for the two-stage game: (1) The equilibrium prices and profits decrease as the number of brands increases, and (2) total category profits (the sum of the individual brand profits) increase as the brands differentiate. Thus, there is still incentive for differentiation (market segmentation).

However, unlike the two- and three-brand case, maximum differentiation is not a Nash equilibrium. For example when brand positions, $\{\Theta_1, \Theta_2, \Theta_3, \Theta_4\}$, are $\{0°, 30°, 60°, 90°\}$, market profits are maximized and all brands have excellent profits, but all brands have unilateral incentives to move toward the center. Such unilateral moves encourage repositioning by the other brands and a positioning war begins.[11]

Thus, as brand interrelationships become more complex we encounter a prisoner's dilemma type problem where cooperation pays but is not guaranteed in a static game. Perhaps extensions to dynamic games and supergames (Kreps and Spence 1984), to games of incomplete information (ibid.), to sequential entry (Prescott and Visscher 1977; Lane 1980), or to the study of cooperation (Axelrod 1984) could address this dilemma.

For example, one argument might begin with the realization that firms must play the game repeatedly, thus any repositioning moves this period will invite competitive reaction next period and in all subsequent periods. Suppose that firms reach maximum differentiation. Then, each firm could realize that a unilateral move will trigger a positioning war and, hence, each firm could have an incentive to avoid that war and stay at the maximum differentiation positions of $\{0°, 30°, 60°, 90°\}$. This is, of course, an informal argument. Formalization is nontrivial and is left to future research.[12]

One interesting implication of the four and five brand analyses concerns market segmentation. For three brands market segmentation (differentiation) is the result of market forces. If a stabilizing mechanism is added to the analysis this interpretation holds as well for four or five brands. Because one stabilizing mechanism is implicit cooperation (e.g. Axelrod 1984), a hypothesis worth testing is that market segmentation is a means to maintain the maximum differentiation cooperative solution. See Schmalensee (1978) for a related hypothesis based on a different model and context.

7. Discussion and Future Directions

Competitive pricing and positioning strategies are important decisions for marketing managers. This paper has illustrated how such decisions are interrelated with one another and with potential competitive reaction. In doing so, this paper has addressed some of the issues left unanswered in earlier analyses by Hauser and Shugan (1983). But as in the earlier paper, the answers raise new and interesting questions.

For example, in order to illustrate the interaction of price and positioning decisions, I made a number of assumptions in §2. Some of these, e.g., uniformly distributed tastes and constant symmetric costs, were in the tradition of comparative statics to hold "all

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[11] It is possible that such a "war" will stabilize to a Nash point, but despite extensive search I have been unable to find such a noncooperative equilibrium. For example, when you restrict brands to have equal shares at equal prices, i.e. $\{\Theta, 45° - \Theta, 45° + \Theta, 90° - \Theta\}$, a maximum exists at $\Theta \approx 7.5°$, but each brand has unilateral incentives to reposition. In this case brands 1 and 2 move toward one another rather than toward the center. Another hypothesis is that brands pair as in Eaton and Lipsey's (1975) analysis of the Hotelling problem. When I tried paired brand configurations, such as $\{22°, 23°, 67°, 68°\}$, the market moved toward maximum differentiation, $\{0°, 30°, 60°, 90°\}$, but did *not* stabilize once maximum differentiation was reached. I have also tried a logit-like extension similar to de Palma et al. (1980), but that extension exhibited similar problems. I suspect an oscillating form of behavior and posit that Nash conditions alone are not sufficient for a stable equilibrium with four brands.

[12] For example, because no static equilibrium may exist in the price war, formal arguments are difficult. However, the lack of an equilibrium in no way diminishes the incentives to avoid a price war.

else" constant. Intuitively, cost and taste asymmetries clearly affect positioning decisions and are worthwhile theoretical extensions.

Indeed, in any real market there may be forces such as nonuniform costs and tastes, threats of competitive entry, uncertainty in consumer perceptions, etc. that provide forces to mitigate *maximum* differentiation. The analyses of this paper suggest only that consumer response, as modeled by the Defender model, pushes a market toward differentiation (segmentation).

A more controversial assumption is the mature market assumption. I make this assumption based on (1) a series of Master's theses completed at MIT in which students interviewed managers at a number of firms in each of a variety of industries, (2) empirical applications of the Defender model, and (3) my own qualitative interviews with high-level managers at American, European, and Japanese firms. These qualitative data suggest that many strategic positioning decisions are made with greater focus on existing competition than on pre-emption of future entry, and that real limitations, e.g., "not as far as brand X," are perceived on repositioning. However, I do not believe that the mature market assumption applies to all markets. It should, and probably will be, challenged by future empirical research. Indeed, alternative assumptions such as Lane's (1980) and Gal-or's (1982) provide other perspectives.

I am also concerned with stability for more than three brands. Clearly, one can develop formal cooperative mechanisms to assure stability, but the real question is empirical. Do such markets stabilize, or do they remain unstable?

One dynamic story worth investigating is suggested by the qualitative data cited above. In developing markets initial entrepreneurs might enter offering brands which satisfy the greatest need, e.g., the "center" of the market. More entrepreneurs might then enter the "center," experience fierce price competition, and "learn" to differentiate.

Finally, we can extend the analyses of this paper to other marketing variables such as awareness advertising, distribution, and consumer search behavior (as modeled by evoked sets). These phenomena are common in empirical applications of the Defender model. In an example of such analyses, Schoidtz (1986) found that a three-stage game of awareness advertising, price, and position also leads to maximum differentiation in three-brand markets.[13]

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[13] This paper was received in January 1986 and has been with the author 5 months for 2 revisions.

Appendix: Proofs to Propositions 1 and 2

PROOF OF PROPOSITION 1. Let Θ_j and k_j be the angle and magnitude of the product formulation of brand j. Let p_j, c_j, m_j, π_j be the price, cost, market share, and profit of brand j. Brand j's position in per dollar space is then at rectangular coordinates $\{(k_j/p_j) \cos \Theta_j, (k_j/p_j) \sin \Theta_j\}$. Based on the 'Defender' model, the market share of brand 1 is given by $m_1 = (\alpha_{21} - \alpha_{10})/90°$ where, by definition, $\alpha_{10} = 0°$ and

$$\tan \alpha_{21} = [(k_1/p_1) \cos \Theta_1 - (k_2/p_2) \cos \Theta_2]/[(k_2/p_2) \sin \Theta_2 - (k_1/p_1) \sin \Theta_1].$$

If by assumption, $k_1 = k_2 = k_3$, then

$$\tan \alpha_{21} = [\cos \Theta_1 - R \cos \Theta_2]/[R \sin \Theta_2 - \sin \Theta_1]$$

where $R = p_1/p_2$. Profit is proportional to $(p_1 - c_1)\alpha_{21}/90°$. Since, by assumption, p_1 and c_1 do not depend upon Θ_1, profit is maximized if α_{21} is maximized. Taking derivatives via the chain rule, $\partial \alpha_{21}/\partial \Theta_1 = (1 + \tan^2 \alpha_{21})^{-1} \partial \tan \alpha_{21}/\partial \Theta_1$ which equals

$$\{(R \sin \Theta_2 - \sin \Theta_1)^2/[R \sin \Theta_2 - \sin \Theta_1)^2 + (\cos \Theta_1 - R \cos \Theta_2)^2]\}$$

$$* [\sin^2 \Theta_1 + \cos^2 \Theta_1 - R(\sin \Theta_1 \sin \Theta_2 + \cos \Theta_1 \cos \Theta_2)]/[R \sin \Theta_2 - \sin \Theta_1]^2.$$

Trigonometric substitution leaves

$$\partial \alpha_{21}/\partial \Theta_1 = [1 - R \cos (\Theta_2 - \Theta_1)]/[1 + R^2 - 2R \cos (\Theta_2 - \Theta_1)].$$

For $p_1 = p_2$, $R = 1$. For $R = 1$ and $\Theta_1 < \Theta_2$ the numerator and denominator are both positive because $\cos(\Theta_2 - \Theta_1) < 1$ for $\Theta_2 - \Theta_1 \in (0°, 90°)$. $\Theta_1 < \Theta_2$ by the assumption of mature markets (sticky positions). In fact, $\partial\alpha_{21}/\partial\Theta_1 = \frac{1}{2}$. Hence profits increase as Θ_1 increases. Similarly we show that π_3 increases as Θ_3 decreases, thus both brands 1 and 3 move toward the center brand or, if $\Theta_2 = 45°$, toward the center of the market. This completes the proof to part (a).

For part (b), it is clear that π_2 is proportional to $\alpha_{32} - \alpha_{21}$. Thus $\partial\pi_2/\partial\Theta_2$ is proportional to $\partial\alpha_{32}/\partial\Theta_2 - \partial\alpha_{21}/\partial\Theta_2$. Taking derivatives with the chain rule gives

$$\partial\alpha_{21}/\partial\Theta_2 = [R^2 - R\cos(\Theta_2 - \Theta_1)]/[1 + R^2 - 2R\cos(\Theta_2 - \Theta_1)].$$

When $R = 1$ and $\Theta_1 < \Theta_2$, this expression becomes $\partial\alpha_{21}/\partial\Theta_2 = \frac{1}{2}$. Similarly we can derive $\partial\alpha_{32}/\partial\Theta_2 = \frac{1}{2}$ when $\Theta_3 > \Theta_2$. Thus, for $R = 1$ and $\Theta_1 < \Theta_2 < \Theta_3$, $\partial\pi_2/\partial\Theta_2 = 0$ and the proposition follows. As discussed in the text, the problem becomes singular at $\Theta_1 = \Theta_2 = \Theta_3$ and firms may behave strangely.

An alternative proof to Proposition 1 uses the special conditions of $p_1 = p_2 = p_3$ and trigonometric transformations to derive $\alpha_{12} = (\Theta_1 + \Theta_2)/2$ and $\alpha_{23} = (\Theta_2 + \Theta_3)/2$. The results follow directly. We state the more general proof here to make clear the relationship to Proposition 2.

PROOF TO PROPOSITION 2.   By assumption $p_2 < p_1$ and p_3 thus $R = p_1/p_2 > 1$. Also by assumption $\Theta_1 < \Theta_2 < \Theta_3$. Hence, following the proof to Proposition 1,

$$\partial\alpha_{21}/\partial\Theta_1 = [1 - R\cos(\Theta_2 - \Theta_1)]/[1 + R^2 - 2R\cos(\Theta_2 - \Theta_1)].$$

Thus, $\partial\alpha_{21}/\partial\Theta_2 = 0$ when $\cos(\Theta_2 - \Theta_1) = R^{-1} = p_2/p_1$. (The denominator equals $R^2 - 1$ which is positive when $R > 1$.) Similarly, $\partial\alpha_{32}/\partial\Theta_3 = 0$ when $\cos(\Theta_3 - \Theta_2) = p_2/p_3$.

Also by the proof to Proposition 1, $\partial\pi_2/\partial\Theta_2$ is equal to zero if $\partial\alpha_{32}/\partial\Theta_2 = \partial\alpha_{21}/\partial\Theta_1$. That is, if

$$[R_3^2 - R_3\cos(\Theta_3 - \Theta_1)]/[1 + R_3^2 - 2R_3\cos(\Theta_3 - \Theta_2)] = [R_1^2 - R_1\cos(\Theta_2 - \Theta_1)]/[1 + R_1^2 - 2R_1\cos(\Theta_2 - \Theta_1)]$$

where $R_1 = p_1/p_2$ and $R_3 = p_3/p_2$. Substituting the Nash conditions for brands 1 and 3 gives $(R_1^2 - 1)/(R_1^2 - 1) = (R_3^2 - 1)/(R_3^2 - 1)$ which is satisfied when $p_3 > p_2$ and $p_1 > p_2$.

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