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TECHNICAL Note

EXISTENCE AND UNIQUENESS OF PRICE EQUILIBRIA IN DEFENDER

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This note provides a formal proof that for the Defender model, with uniformly distributed tastes, the Nash price equilibria exist and, for a given set of positions, that they are unique. We also show that the profit function is well-behaved—continuous and quasi-concave—and that the maximum occurs on the concave portion of the curve.
(Pricing; Positioning)

The Defender consumer model is an analytic model that represents profits as a function of decisions by firms with respect to price and positioning. For detailed equations see Hauser and Shugan (1983) or Hauser (this issue). We assume that each firm has one brand and that $n > 1$ brands are not located at identical positions. Finally, we assume that each consumer has a finite reservation price beyond which no firm can expect sales. Let c be the identical marginal costs with $c > 0$.

We first establish that the Defender profit function is well behaved and then prove existence and uniqueness.

LEMMA 1. *The Defender profit function is continuous and quasi-concave in price. It is strictly quasi-concave on the range of positive profits.*

COROLLARY 1. *The Defender profit function is concave in price between marginal costs and its only inflection point and the maximum occurs in this concave range.*

LEMMA 2. *Let G be the Jacobian of the pseudo-gradient of the Defender profit function, that is, the relevant second derivative conditions, then G is negative quasi-definite whenever prices are below the inflection point and profits are positive.* (*This set is connected.*)

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THEOREM. *A Nash equilibrium to the Defender price game with uniformly distributed tastes exists and is unique.*

PROOF. Existence follows from Lemma 1 and Friedman's (1977, p. 153) Theorem 7.1. That is, (1) the number of brands is finite; (2) the strategy is compact because prices are bounded by costs and the reservation price; (3) the profit function is continuous and bounded from above by $(p_i - c)$; and (4) it is strictly quasi-concave for positive profits.

For uniqueness we recognize that in equilibrium no brand would consider a price above the inflection point since maximum profits are always at prices below that point. Uniqueness then follows from Corollary 1, Lemma 2, and Rosen's (1965) Theorems 4 and 6.

The proofs to the lemmas and corollary are available upon request from the authors.[1]

[1] This paper was received in February 1987 and has been with the authors 1 month for 1 revision.

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