

Appendix to Martin and Pindyck,
 “Averting Catastrophes: The Strange Economics of Scylla and Charybdis”
American Economic Review, October 2015

WTP to Avoid A Catastrophe that Can Occur Only Once

If nothing is done to avert a catastrophic event that can occur only once, and reduces consumption by a random fraction ϕ if it occurs, welfare is

$$V_0 = \mathbb{E} \int_0^\infty \frac{1}{1-\eta} C_t^{1-\eta} e^{-\delta t} dt = \frac{1}{1-\eta} \mathbb{E} \left[\int_0^\tau e^{-\rho t} dt + \int_\tau^\infty e^{-\phi(1-\eta)-\rho t} dt \right],$$

where $\mathbb{E}$ denotes the expectation over τ and ϕ . As before, WTP is defined as the maximum percentage of consumption, now and throughout the future, that society would give up to eliminate the possibility of the catastrophe. Define $\rho \equiv \delta + g(\eta - 1)$. If society gives up a fraction w of consumption to avert this catastrophe, net welfare is

$$V_1 = (1-w)^{1-\eta} \int_0^\infty \frac{1}{1-\eta} e^{-\rho t} dt.$$

WTP is then the value w^* that equates V_0 and V_1 .

To obtain the WTP for eliminating the event, note that welfare if no action is taken is:

$$\begin{aligned} V_0 &= \frac{1}{1-\eta} \mathbb{E} \left[\int_0^T e^{-\rho t} dt + e^{-\phi(1-\eta)} \int_T^\infty e^{-\rho t} dt \right] \\ &= \frac{1}{\rho(1-\eta)} \mathbb{E} [1 + e^{-\rho T} (e^{-\phi(1-\eta)} - 1)] \\ &= \frac{1}{\rho(1-\eta)} \left[1 + \frac{\lambda}{\lambda + \rho} (\mathbb{E} e^{-\phi(1-\eta)} - 1) \right] \\ &= \frac{1}{\rho(1-\eta)} \left[1 + \frac{\lambda}{\lambda + \rho} \frac{\eta - 1}{\beta - \eta + 1} \right] \end{aligned} \tag{1}$$

Here we have used the assumption that $z = e^{-\phi}$ follows a power distribution. If the event is eliminated, welfare net of the fraction w of consumption sacrificed is

$$V_1 = \frac{(1-w)^{1-\eta}}{\rho(1-\eta)} \tag{2}$$

Comparing (1) and (2), the WTP to eliminate the event is:

$$w^* = 1 - \left[1 + \frac{\lambda}{\lambda + \rho} \frac{\eta - 1}{\beta - \eta + 1} \right]^{\frac{1}{1-\eta}}$$

From this equation, we see that (i) w^* is an increasing function of the mean arrival rate λ ; (ii) w^* is an increasing function of the expected impact $\mathbb{E}(\phi)$, and thus a decreasing function of the distribution parameter β ; and (iii) w^* is a decreasing function of both the rate of time preference δ and the growth rate g . We would expect w^* to be higher for an event that is expected to occur sooner and have a larger expected impact, and lower if either the rate of time preference or the consumption growth rate is higher. The dependence on η is ambiguous. Given the growth rate g , a higher value of η implies a lower marginal utility of future consumption, and thus a lower WTP to avoid a drop in consumption. On the other hand it also implies a greater sensitivity to uncertainty over future consumption.

As mentioned above, for expected utility to be finite, we need $\beta > \eta - 1$. It is easy to see that as η is increased, w^* approaches 1 as η approaches $\beta + 1$. The reason is that the risk-adjusted remaining fraction of consumption is $\mathbb{E}((1 - \phi)^{1-\eta}) = \beta/(\beta - \eta + 1)$. In risk-adjusted terms, the possibility of a high- ϕ outcome weighs heavily on expected future welfare, and thus on the WTP.

A few numbers: Suppose $\beta = 2$ so the expected loss is $\mathbb{E}\phi = .33$, $\lambda = .05$ so the expected arrival time is $\mathbb{E}T = 1/\lambda = 20$ years, $\delta = g = .02$, and $\eta = 2$. Then $w^* = 0.22$. If instead $\delta = 0$, then $w^* = 0.26$. If $\delta = .02$ but we increase η to 2.5, w^* increases sharply, to 0.60.

It is useful to compare the WTP to avoid this “once-only” event with the WTP when the event can occur multiple times. As shown in Martin and Pindyck (2015), in the latter case the WTP is

$$w_m^* = 1 - \left[1 - \frac{\lambda(\eta - 1)}{\rho(\beta - \eta + 1)} \right]^{\frac{1}{\eta-1}}.$$

(The subscript m is added to emphasize that the event can occur multiple times.) Whether the event can occur only once or repeatedly: (i) w^* is increasing in the mean arrival rate λ ; (ii) w^* is increasing in the expected impact $\mathbb{E}(1 - \phi)$, and thus a decreasing function of the distribution parameter β ; and (iii) w^* is a decreasing function of both the rate of time preference δ and the growth rate g . And as expected, $w_m^* > w^*$ for all $\eta > 1, \beta > \eta - 1, \lambda > 0$. Some comparisons: (1) If $\eta = 2, g = \delta = .02$ so $\rho = .04, \lambda = .02$ and $\beta = 3$ (so $\mathbb{E}(1 - \phi) = .75$), then $w^* = .143$ and $w_m^* = .250$. (2) If instead $\lambda = .04$, then $w^* = .200$ and $w_m^* = .500$. (3) If $\lambda = .04$ but $\beta = 2.1$, then $w^* = .313$ and $w_m^* = .910$. In the last example, β is just above

the limit (2.0) at which expected utility becomes unbounded.

References

Martin, Ian W.R., and Robert S. Pindyck. 2015. “Averting Catastrophes: The Strange Economics of Scylla and Charybdis.” *American Economic Review*, 105(10): 2947–2985.