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Sensor-Driven Condition-Based Generator Maintenance Scheduling Part 2: Incorporating Operations

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***Abstract*—A framework for sensor driven condition based generator maintenance scheduling was proposed in Part 1 [1]. In Part 2, we extend the previous model by incorporating the unit commitment and dispatch into the optimal maintenance scheduling problem. We reformulate this extended maintenance scheduling problem as a two-stage mixed integer program. We use this reformulation to construct an algorithm that obtains the global optimal solution to the proposed generator maintenance problem. Finally, we test and analyze the proposed model through extensive experiments conducted on IEEE-118 bus system. For every experiment, we present a benchmark analysis against the maintenance models used in current industry practice and power systems literature. Experimental results indicate that the proposed maintenance schedules provide considerable improvements in both cost and reliability.**

***Index Terms*—Condition based maintenance, generator maintenance scheduling, two-stage mixed-integer optimization**

NOMENCLATURE

Decision Variables:

$\nu_{t,i,k} \in \{0,1\}$ $\nu_{t,i,k} = 1$ iff the k^{th} maintenance of generator i starts at maintenance epoch t. If a certain maintenance k is not scheduled, then $\nu_{t,i,k} = \nu_{t,i,k_\ell}$ for all $t \in \mathcal{T}$, where k_ℓ is the last scheduled maintenance.

$z_{t,i,k} \in \{0,1\}$ $z_{t,i,k} = 1$ iff the duration between the start of the k^{th} and the $(k-1)^{th}$ maintenances of generator i is t maintenance epochs.

$z^o_{i,k} \in \{0,1\}$ $z^o_{i,k} = 0$ iff the k^{th} maintenance is scheduled for generator i within the planning horizon.

$x^t_{s,i} \in \{0,1\}$ $x^t_{s,i} = 1$ iff generator i is committed in hour s within maintenance epoch t.

$\pi^{U,t}_{s,i} \in \{0,1\}$ $\pi^{U,t}_{s,i} = 1$ iff generator i starts up in hour s within maintenance epoch t.

$\pi^{D,t}_{s,i} \in \{0,1\}$ $\pi^{D,t}_{s,i} = 1$ iff generator i shuts down in hour s within maintenance epoch t.

$y^t_{s,i} \in \mathbb{R}^n_+$ Generation output of generator i in hour s within maintenance epoch t.

$\psi^{DC,t}_{s,p} \in \mathbb{R}^n_+$ Demand curtailment in hour s within maintenance epoch t at demand bus p.

$\psi^{TL,t}_{s,\ell} \in \mathbb{R}^n_+$ Transmission line slack variable in hour s within maintenance epoch t at line ℓ.

Sets:

$\mathcal{V}$ Set of loads.

$\mathcal{G}$ Set of generators.

$\mathcal{K}_i$ Set of possible maintenances for generator i.

$\mathcal{L}$ Set of transmission lines.

$\mathcal{S}$ Set of hours within one maintenance epoch.

$\mathcal{T}$ Set of maintenance epochs within the planning horizon.

Constants:

$B^t_{s,i}$ Generation cost of generator i in hour s within maintenance epoch t.

$C^{d,i}_{t^o_i,t}$ Cost of maintenance for a partially degraded generator i, when the maintenance is scheduled to t maintenance epoch after the time of observation t^o_i.

$C^{n,i}_t$ Cost of maintenance for a new generator i, when the age of the generator at the time of its maintenance is t maintenance epochs.

H Planning horizon in maintenance epochs.

L Maximum number of generators that can be under maintenance simultaneously.

M_i Maximum number of maintenances to be scheduled for generator i within the planning horizon.

P_{DC} Penalty cost for unit unsatisfied demand.

P_{TL} Penalty cost for unit overload on a transmission line.

R_i Remaining time required for maintenance of generator i at the start of the planning horizon.

$U^{U,t}_{s,i}$ Start-up cost of generator i in hour s within maintenance epoch t.

$U^{D,t}_{s,i}$ Shut-down cost of generator i in hour s within maintenance epoch t.

$V^t_{s,i}$ No-load cost of generator i in hour s within maintenance epoch t.

Y Maintenance duration in maintenance epochs.

ζ^d_i Period within which at least one maintenance should be scheduled to start for degraded generator i.

ζ^n Period within which at least one maintenance should be scheduled to start for a new generator.

I. INTRODUCTION

In this paper, we expand on the adaptive predictive generator maintenance model introduced in [1] by incorporating unit commitment and dispatch decisions. We present a solution methodology to solve this extended scheduling problem in large cases. Finally, we run a series of experiments to present the performance of the proposed model. The results indicate that the use of adaptive predictive model provides considerable

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improvements in both cost and reliability as identified by the IEEE task force [2].

The paper is organized as follows. In Section II we propose a new adaptive predictive maintenance model (APMII) that considers unit commitment and dispatch decisions. In Section III, we reformulate APMII as a two-stage mixed integer problem, and also introduce its relaxation. In Section IV, we propose a new reformulation of the APMII model, which has a relaxed subproblem structure but the objective is augmented so that it exactly recovers the true cost of the APMII model. In Section V, we propose an exact algorithm to solve a reformulation of this problem, which is particularly useful for solving large-scale cases of the APMII model. In Section VI, we present an experimental framework that uses a degradation database to study a number of test cases. We show the effectiveness of our model through extensive comparative studies. In section VII, we conclude this paper with some closing remarks.

II. Adaptive Predictive Maintenance Problem II

In this section, we expand our analysis to consider generation commitment and dispatch in the optimal maintenance scheduling problem. The key balance in APMII is between explicit and implicit costs of maintenance. We continue leveraging the results of the predictive analytics to ensure an adaptive characterization of the costs of maintenance, but at the same time, we now consider the impact of maintenance on operations, such as the overall production cost and network feasibility. The main intuition behind APMII model is that, in most practical applications, it would be preferable to deviate from the pure maintenance optimal policy (APMI policy presented in [1]) in an effort to decrease the unit commitment and dispatch cost. Utility companies put great emphasis on the forecasted demand while deciding on the maintenance schedules. APMII provides a model that considers the consequences of every maintenance action on the operational side, while benefiting from the adaptive predictive estimates on the generator failure risks. The maintenance variables are identical as in AMPI, besides we also have commitment variable $\boldsymbol{x}$ and dispatch variable $\boldsymbol{y}$ as defined in the beginning of the paper.

A. Objective Function

The objective is to minimize the maintenance and operations cost:

$$\xi_m \left[\sum_{i \in \mathcal{G}} \sum_{t=R_i+1}^{H} C^{d,i}_{t_i^o, t-R_i} \cdot z_{t,i,1} + \sum_{i \in \mathcal{G}} \sum_{t=Y+1}^{H} \sum_{k=2}^{M_i} C^{n,i}_{t-Y} \cdot z_{t,i,k} \right] + \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{G}} \sum_{s \in \mathcal{S}} \left(V^t_{s,i} \cdot x^t_{s,i} + U^{U,t}_{s,i} \cdot \pi^{U,t}_{s,i} + U^{D,t}_{s,i} \cdot \pi^{D,t}_{s,i} + B^t_{s,i} \cdot y^t_{s,i} \right) + \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \left(\sum_{p \in \mathcal{D}} \left(P_{DC} \cdot \psi^{DC,t}_{s,p} \right) + \sum_{\ell \in \mathcal{L}} \left(P_{TL} \cdot \psi^{TL,t}_{s,\ell} \right) \right), \quad (1)$$

where ξ_m is the maintenance criticality coefficient. The objective function (1) consists of two components: dynamic maintenance cost (the first line) and operational cost including UC, dispatch, and penalty costs (the second and third lines). For the explanation on the dynamic maintenance cost in the first line of the objective function, we refer the reader to Section V-B in [1]. The remaining cost factors are typical in UC literature, e.g. [3], [4]

B. Constraints

The cost (1) is minimized subject to some of the constraints defined in [1]. More specifically, APMII is subject to:

1) *Maintenance time limits:* This set refers to the restrictions on the time of the first maintenance, and the time between consecutive maintenances, i.e. (7)-(8) in [1].
2) *Maintenance coordination:* These constraints i) impose logical restrictions such as maintenance durations, ii) allow flexible number of maintenances within the planning horizon, and iii) ensure a mapping between the time of maintenance, and the age of the generator at the time of maintenance, i.e. (9)-(17) in [1].
3) *Maintenance capacity:* This set of constraints ensure that the number of ongoing maintenances at any time t does not exceed a prespecified limit L, i.e. (18) in [1].

We consider two additional sets of constraints for coupling of maintenance and operations, and unit commitment (UC).

1) ***Coupling of maintenance and operations**:*

- In cases where a certain generator is under maintenance at the start of the planning horizon, the corresponding commitment variable $\boldsymbol{x}$ is set to zero.

$$x^t_{s,i} = 0 \quad \forall i \in \mathcal{G} \text{ and } R_i > 0 \quad \forall s \in \mathcal{S}, t \in \{1, \ldots, R_i\}. \quad (2)$$

- In the following set of constraints, we couple the maintenance decision variable $\boldsymbol{\nu}$ with the commitment variable $\boldsymbol{x}$. Constraint (3) ensures that if a unit is under maintenance during maintenance epoch t, it cannot be committed in any of the hours within that epoch. The rational of (3) is similar to that of (18) in [1].

$$x^t_{s,i} \leq 1 - \sum_{k \in \mathcal{K}_i} \sum_{e=0}^{Y-1} \nu_{t-e,i,k} \quad (3a)$$
$$\forall i \in \mathcal{G},\ t \in \{1, \ldots, H - \zeta^n\},\ s \in \mathcal{S}$$

$$x^t_{s,i} \leq 1 - \sum_{k \in \mathcal{K}_i} \sum_{e \in \mathcal{J}^1(t)} \nu_{t-e,i,k} + \sum_{e \in \mathcal{J}^2(t)} \nu_{t-e,i,M_i} \quad (3b)$$
$$\forall i \in \mathcal{G},\ t \in \{H - \zeta^n + 1, \ldots, H - \zeta^n + Y - 1\},\ s \in \mathcal{S}$$

$$x^t_{s,i} \leq 1 - \sum_{e=0}^{Y-1} \nu_{t-e,i,k} \quad (3c)$$
$$\forall i \in \mathcal{G},\ t \in \{H - \zeta^n + Y, \ldots, H\},\ s \in \mathcal{S},$$

where the sets $\mathcal{J}^1(t) = \{t - H + \zeta^n, \ldots, Y - 1\}$ and $\mathcal{J}^2(t) = \{0, \ldots, t - H + \zeta^n - 1\}$.

2) ***Unit commitment** :*

- The UC problem includes constraints on i) commitment status: such as minimum up/down, and start-up/shutdown, ii) dispatch level: such as energy balance, transmission limit and ramping, iii) commitment coupling: such as minimum and maximum dispatch levels for each generator based on the commitment status. In its compact form, we represent this set of constraints as follows:

$$\boldsymbol{F x}+\boldsymbol{G y} \leq \boldsymbol{\ell} \tag{4}$$

where $\boldsymbol{x}$ includes the generator commitment, start-up, and shut-down variables, and $\boldsymbol{y}$ includes generation dispatch, demand curtailment, and line slack variables.

C. APMII Model

In summary, the APMII model is given as:

$$\begin{aligned}
(APMII) \quad \min_{\boldsymbol{z},\boldsymbol{\nu},\boldsymbol{x},\boldsymbol{y}} \quad & (1) \\
\text{s.t.} \quad & \{(7)\text{-}(18) \text{ from Part I: } [1]\}, \\
& (2)-(4) \\
& \{\boldsymbol{z},\boldsymbol{\nu}\} \in \mathcal{F}^m, \\
& \boldsymbol{x} \in \{0,1\}^{3|\mathcal{G}|\times H\times|\mathcal{S}|}, \boldsymbol{y} \in \mathbb{R}_+^{(|\mathcal{G}|+J)\times H\times|\mathcal{S}|},
\end{aligned}$$

where $J=|\mathcal{V}|+|\mathcal{L}|$. It turns out that we can relax the $z_{t,i,k}$ variables to be continuous and still obtain a binary optimal solution for both APMI and APMII, as shown in the following lemma.

Lemma 1. *If the binary variables $z_{t,i,k}$'s in (APMII) are relaxed to be continuous, then the relaxed problem still has a binary optimal solution, which is thus optimal for (APMII). The same statement also holds for APMI introduced in [1].*

Proof. See Appendix B. □

III. Two-Stage Reformulation of APMII

The APMII model has a natural two-stage structure, namely the first stage makes the maintenance decision, while the second stage deals with the UC problem based on the maintenance decision.

A. APMII Reformulation

The APMII model can be written in the following compact form:

$$\begin{aligned}
\min_{\boldsymbol{z},\boldsymbol{\nu},\boldsymbol{x},\boldsymbol{y}} \quad & \boldsymbol{c}^\top \boldsymbol{z} + \boldsymbol{v}^\top \boldsymbol{x} + \boldsymbol{b}^\top \boldsymbol{y} && \text{(5a)} \\
\text{s.t.} \quad & \boldsymbol{A z} + \boldsymbol{K \nu} \leq \boldsymbol{g} && \text{(5b)} \\
& \boldsymbol{B \nu} + \boldsymbol{E x} \leq \boldsymbol{h} && \text{(5c)} \\
& \boldsymbol{F x} + \boldsymbol{G y} \leq \boldsymbol{\ell} && \text{(5d)}
\end{aligned}$$

$$\{\boldsymbol{z},\boldsymbol{\nu}\} \in \mathcal{F}^m, \boldsymbol{x} \in \{0,1\}^{3|\mathcal{G}|\times H\times|\mathcal{S}|}, \boldsymbol{y} \in \mathbb{R}_+^{(|\mathcal{G}|+J)\times H\times|\mathcal{S}|}.$$

where $\boldsymbol{z},\boldsymbol{\nu}$ are the maintenance variables, $\boldsymbol{x}$ is the generator commitment, start-up, and shut-down variables, and $\boldsymbol{y}$ includes generation dispatch, demand curtailment, and line slack variables. Here, $\dim \boldsymbol{g} = 8\cdot\sum_{i\in\mathcal{G}} M_i \cdot |\mathcal{G}| + H\cdot|\mathcal{G}| + H - 4\cdot|\mathcal{G}|$, $\dim \boldsymbol{h} = H\cdot|\mathcal{S}|\cdot|\mathcal{G}|$, and $\dim \boldsymbol{\ell} = H\cdot|\mathcal{S}| + 4\cdot H\cdot|\mathcal{S}|\cdot|\mathcal{G}| + 2\cdot H\cdot|\mathcal{S}|\cdot|\mathcal{L}|$.

In this formulation, the objective function is identical to (1). The constraint $(5b)$ corresponds to maintenance decisions, such as the maintenance labor capacity constraints and the interaction between different maintenance variables, namely constraints (7)-(18) in [1]. Constraint $(5c)$ couples the maintenance and the unit commitment variables, so that a generating unit is not committed, if a maintenance activity is still being conducted on that particular unit. They correspond to the constraints in (3) and (2).

The APMII model (5) can be decomposed into a two-stage program, where the maintenance problem resides in the first stage, and UC given maintenance decisions constitutes the second-stage problem as follows,

$$\begin{aligned}
\min_{\boldsymbol{z},\boldsymbol{\nu}} \quad & \boldsymbol{c}^\top \boldsymbol{z} + q(\boldsymbol{\nu}) && \text{(6a)} \\
\text{s.t.} \quad & \boldsymbol{A z} + \boldsymbol{K \nu} \leq \boldsymbol{g} && \text{(6b)} \\
& \{\boldsymbol{z},\boldsymbol{\nu}\} \in \mathcal{F}^m.
\end{aligned}$$

where $q(\boldsymbol{\nu})$ denotes the UC problem given maintenance decision $\boldsymbol{\nu}$. We consider minimum up/down, and ramping constraints within the hours of the same maintenance epoch (e.g. a week). In this way, once the maintenance decision $\boldsymbol{\nu}$ is fixed, the unit commitment decisions for any maintenance epoch $t\in\mathcal{T}$, namely $\{\boldsymbol{x}^t,\boldsymbol{y}^t\}$, become independent. Thus the subproblem $q(\boldsymbol{\nu})$ can be further decomposed into different maintenance epochs, $q(\boldsymbol{\nu}) = \sum_{t=1}^N q^t(\boldsymbol{\nu})$ where $q^t(\boldsymbol{\nu})$ is given by:

$$\begin{aligned}
q^t(\boldsymbol{\nu}) = \min_{\boldsymbol{x}^t,\boldsymbol{y}^t} \quad & (\boldsymbol{v}^t)^\top \boldsymbol{x}^t + (\boldsymbol{b}^t)^\top \boldsymbol{y}^t && \text{(7a)} \\
\text{s.t:} \quad & \boldsymbol{E}^t \boldsymbol{x}^t \leq \boldsymbol{h}^t - \boldsymbol{B}^t \boldsymbol{\nu}, && \text{(7b)} \\
& \boldsymbol{F}^t \boldsymbol{x}^t + \boldsymbol{G}^t \boldsymbol{y}^t \leq \boldsymbol{\ell}^t && \text{(7c)} \\
& \boldsymbol{x}^t \in \{0,1\}^{3|\mathcal{G}|\times|\mathcal{S}|}, \boldsymbol{y}^t \in \mathbb{R}^{(|\mathcal{G}|+J)\times|\mathcal{S}|}.
\end{aligned}$$

Even in large cases, it is not computationally expensive to solve $q^t(\boldsymbol{\nu})$ for a given $\boldsymbol{\nu}$.

B. Relaxation for APMII (R-APMII)

We next define a relaxation for the APMII problem, namely R-APMII, by relaxing the binary UC variables to continuous variables so that $\boldsymbol{x}^t \in [0,1]^{3|\mathcal{G}|\times|\mathcal{S}|} \forall t\in\mathcal{T}$. This new model can be decomposed into a master maintenance problem and a linear relaxation of the UC subproblem in a similar manner. We denote the objective of the relaxed UC subproblem in R-APMII as $\tilde{q}(\boldsymbol{\nu})$, and its cost for any maintenance epoch t as $\tilde{q}^t(\boldsymbol{\nu})$. That is, $\tilde{q}(\boldsymbol{\nu}) = \sum_{t\in\mathcal{T}} \tilde{q}^t(\boldsymbol{\nu})$. Then, R-APMII can be represented as follows:

$$\begin{aligned}
\min_{\boldsymbol{z},\boldsymbol{\nu}} \quad & \boldsymbol{c}^\top \boldsymbol{z} + \sum_{t\in\mathcal{T}} \tilde{q}^t(\boldsymbol{\nu}) && \text{(8a)} \\
\text{s.t.} \quad & \boldsymbol{A z} + \boldsymbol{K \nu} \leq \boldsymbol{g} && \text{(8b)} \\
& \boldsymbol{z},\boldsymbol{\nu} \in \mathcal{F}^m.
\end{aligned}$$

We note that this new formulation provides a lower bound for the APMII problem. It is considerably easier to solve through Benders' Decomposition, since the subproblems are non-integer. However, there is a need to link the solution of this relaxed formulation to the APMII problem.

IV. Alternative Formulation (AF) for APMII

In this section, we construct an alternative formulation (AF) that can recover the true cost of the APMII problem with a subproblem structure identical to the relaxed model R-APMII.

We start with the observation that the interaction between the maintenance and the unit commitment variables can be completely characterized through generator's maintenance status. That is, for any maintenance epoch t, the UC cost can be determined if we know which generators have an ongoing maintenance. For the sake of clarity, we define an additional variable m_i^t that takes the value $m_i^t = 1$ if generator i is undergoing maintenance at maintenance epoch t, and $m_i^t = 0$ otherwise. We note that m_i^t is uniquely determined by the maintenance variables $\boldsymbol{\nu}$. In particular, generator i would have an ongoing maintenance at maintenance epoch t, if its maintenance has started during $\{t-Y+1,\ldots,t\}$, in other words, if $\sum_{k\in\mathcal{K}_i}\sum_{e=0}^{Y-1}\nu_{t-e,i,k}\geq 1$. The idea is that to find the cost for a certain maintenance status $\hat{\boldsymbol{m}}$, we can solve the relaxed model, R-APMII, and then add the difference $\sum_{t\in\mathcal{T}} q^t(\hat{\boldsymbol{m}}^t) - \tilde{q}^t(\hat{\boldsymbol{m}}^t)$ back to the objective cost of the relaxed model R-APMII. In this way, the cost of the true model APMII is recovered.

We assume for the time being that we can enumerate all possible maintenance statuses for every $t\in\mathcal{T}$. This complete set is denoted by $\bar{\Omega} := \{\bar{\Omega}^1,\ldots,\bar{\Omega}^H\}$ with cardinality $H\times 2^{|\mathcal{G}|}$. We let $\hat{\boldsymbol{m}}_h^t$ denote one of these statuses at t with a corresponding status index $h\in\bar{\Omega}^t$. In what follows, we show how we can i) create an additional variable and constraint to check if the maintenance solution $\boldsymbol{\nu}$ corresponds to the particular maintenance status $\hat{\boldsymbol{m}}_h^t$, and ii) recover the true UC cost when the maintenance solution ν implies $\hat{\boldsymbol{m}}_h^t$.

We start with the first objective. For generator status h at t, we define a binary variable η_h^t subject to:

$$\eta_h^t \geq \left(\sum_{i\in\mathbb{K}(\hat{\boldsymbol{m}}_h^t)} m_i^t - \sum_{i\in\mathbb{F}(\hat{\boldsymbol{m}}_h^t)} m_i^t - \left|\mathbb{K}(\hat{\boldsymbol{m}}_h^t)\right| + 1\right), \quad (9)$$

where the index set $\mathbb{K}(\hat{\boldsymbol{m}}_h^t) := \{i|\hat{m}_{h,i}^t = 1\}$, and $\mathbb{F}(\hat{\boldsymbol{m}}_h^t) := \{i|\hat{m}_{h,i}^t = 0\}$. This constraint ensures that the binary variable $\eta_h^t \geq 1$ when $\boldsymbol{m}^t = \hat{\boldsymbol{m}}_h^t$, and η_h^t is not bounded otherwise. This claim holds since: i) when $\boldsymbol{m}^t = \hat{\boldsymbol{m}}_h^t$, the right hand side equals 1, ii) otherwise, if there is at least one i where $m_i^t \neq \hat{m}_{h,i}^t$, then the right hand side becomes less than or equal to zero. Constraint (9) is presented for the sake of clarity. In reality, we need to link the solution $\boldsymbol{\nu}$ to $\hat{\boldsymbol{m}}_h^t$. We note again that given the maintenance start variables $\boldsymbol{\nu}$, maintenance status variables $\boldsymbol{m}$ can be obtained in a straight forward way. We also note that if the k-th maintenance is not scheduled for generator i, then $\nu_{t,i,k} = \nu_{t,i,k-1}\ \forall t\in\mathcal{T}$. In order to eliminate double-counting of maintenance instances, the constraint (9) can be constructed using $\boldsymbol{\nu}$ as follows:

$$\eta_h^t \geq \sum_{i\in\mathcal{G}}\left(R_{h,i}^t\cdot\left(\sum_{k\in\mathcal{K}_i}\sum_{e=0}^{Y-1}\nu_{t-e,i,k}\right) - U_{h,i}^t\right) + 1 \quad (10a)$$
$$\forall t\in\{1,\ldots,H-\zeta^n\}$$

$$\eta_h^t \geq \sum_{i\in\mathcal{G}} R_{h,i}^t\cdot\left(\sum_{k\in\mathcal{K}_i}\sum_{e\in\mathcal{J}^1(t)}\nu_{t-e,i,k} + \sum_{e\in\mathcal{J}^2(t)}\nu_{t-e,i,M_i}\right) - \sum_{i\in\mathcal{G}} U_{h,i}^t + 1 \quad (10b)$$
$$\forall t\in\{H-\zeta^n+1,\ldots,H-\zeta^n+Y-1\}$$

$$\eta_h^t \geq \sum_{i\in\mathcal{G}}\left(R_{h,i}^t\cdot\sum_{e=0}^{Y-1}\nu_{t-e,i,M_i} - U_{h,i}^t\right) + 1 \quad (10c)$$
$$\forall t\in\{H-\zeta^n+Y,\ldots,H\},$$

where $R_{h,i}^t = 1, U_{h,i}^t = 1$ if $\hat{m}_{h,i}^t = 1$. Otherwise, $R_{h,i}^t = -1, U_{h,i}^t = 0$. Note that the term with $R_{h,i}^t$ corresponds to the difference of summations in (9). The second term with $U_{h,i}^t$ provides the cardinality of the set in (9). $\mathcal{J}^1(t)$ and $\mathcal{J}^2(t)$ are defined similarly in [1, Eq.(18b)-(18c)]. We denote the constraint (10) for maintenance epoch t and the binary variable η_h^t in its compact form as: $(\boldsymbol{r}_h^t)^\top\boldsymbol{\nu} + \eta_h^t \geq u_h^t$.

Define the cost e_h^t associated with the h-th maintenance status $\hat{\boldsymbol{m}}_h^t$ as the difference between the true and relaxed costs of the UC subproblem at time t, namely,

$$e_h^t = q^t(\hat{\boldsymbol{m}}_h^t) - \tilde{q}^t(\hat{\boldsymbol{m}}_h^t). \quad (11)$$

where $q^t(\hat{\boldsymbol{m}}_h^t)$ is the solution $q^t(\nu)$ in (7) for any $\boldsymbol{\nu}$ that implies $\hat{\boldsymbol{m}}_h^t$.

We can repeat this process for all $h\in\Omega_h^t$. Then, the following holds for any $\boldsymbol{\nu}$:

$$q^t(\boldsymbol{\nu}) = \tilde{q}^t(\boldsymbol{\nu}) + \min_{\boldsymbol{\eta}}\left\{\sum_{h\in\bar{\Omega}^t}\eta_h^t e_h^t : \eta_h^t \geq u_h^t - (\boldsymbol{r}_h^t)^\top\boldsymbol{\nu}, \forall h\in\bar{\Omega}^t\right\}. \quad (12)$$

In fact, the solution for this problem is clear, that is, only for one $h\in\bar{\Omega}^t$, $u_h^t - (\boldsymbol{r}_h^t)^\top\boldsymbol{\nu} = 1$ is true. We denote this term by h^*. Then: $q^t(\boldsymbol{\nu}) = \tilde{q}^t(\boldsymbol{\nu}) + e_{h^*}^t$.

We next use this observation to reformulate the APMII problem, by replacing $q(\boldsymbol{\nu})$ with its equivalent in (12). The following AF problem can attain the optimal objective cost and maintenance decisions of APMII:

AF Problem:

$$\min_{\boldsymbol{z},\boldsymbol{\nu},\boldsymbol{\eta}} \quad \boldsymbol{c}^\top\boldsymbol{z} + \boldsymbol{e}^\top\boldsymbol{\eta} + \sum_{t\in\mathcal{T}}\tilde{q}^t(\boldsymbol{\nu}) \quad (13a)$$
$$\text{s.t.} \quad \boldsymbol{A}\boldsymbol{z} + \boldsymbol{K}\boldsymbol{\nu} \leq \boldsymbol{g} \quad (13b)$$
$$(\boldsymbol{r}_h^t)^\top\boldsymbol{\nu} + \eta_h^t \geq u_h^t \qquad \forall t\in\mathcal{T}, \forall h\in\bar{\Omega}^t \quad (13c)$$
$$\boldsymbol{z},\boldsymbol{\nu}\in\mathcal{F}^m, \boldsymbol{\eta}\in\{0,1\}^{H\times 2^{|\mathcal{G}|}}. \quad (13d)$$

In the following, we develop an iterative algorithm to solve this AF problem.

V. Solution Algorithm for APMII

APMII is a computationally expensive problem to solve. Therefore, it is important to design an efficient algorithm to solve large-scale APMII models. In this section, we present an exact solution algorithm that uses the special structure of APMII to intelligently reconstruct the elements of (13), in an attempt to find the optimal solution for APMII. We have two observations to motivate the algorithm at this point: i) the cost $\tilde{q}^t(\boldsymbol{\nu})$ can be recovered through Benders' decomposition, ii) more importantly, it would be sufficient to incorporate subset of maintenance statuses $\Omega \subseteq \bar{\Omega}$ in the AF problem to recover the true UC cost. This set is typically small due to the following properties of the maintenance problem:

1) The minimizer of the maintenance cost term $\boldsymbol{c}^\top \boldsymbol{z}$ is an important factor for determining the time of maintenance. The APMII's optimal solution typically do not schedule maintenance very far from this minimizer.
2) The difference between the total cost of the relaxed formulation R-APMII and the APMII problem is small. Therefore, when one considers the maintenance cost and relaxed UC cost, it suffices to check only a number of different points before the true costs from these points reside below the lower bounds of conducting maintenance in other time epochs.

In line with these claims, for a given set of generator maintenance statuses $\Omega := \cup_{t\in\mathcal{T}}\{\Omega^t : \Omega^t \subseteq \bar{\Omega}^t\}$ and a set of Benders' optimality cuts $\mathcal{BD} := \cup_{t\in\mathcal{T}}\{\mathcal{BD}^t\}$, a restricted master problem $\mathbf{RMP}(\Omega, \mathcal{BD})$ can be presented as follows:

$\mathbf{RMP}(\Omega, \mathcal{BD})$ Problem:

$$\min_{\boldsymbol{z},\boldsymbol{\nu},\boldsymbol{\eta},\boldsymbol{\varphi}} \quad \boldsymbol{c}^\top \boldsymbol{z} + \boldsymbol{e}^\top \boldsymbol{\eta} + \sum_{t\in\mathcal{T}} \varphi^t \tag{14a}$$

$$\text{s.t.} \quad \boldsymbol{A}\boldsymbol{z} + \boldsymbol{K}\boldsymbol{\nu} \le \boldsymbol{g} \tag{14b}$$

$$(\boldsymbol{r}_h^t)^\top \boldsymbol{\nu} + \eta_h^t \ge u_h^t \qquad \forall t\in\mathcal{T}, \forall h \in \Omega^t \tag{14c}$$

$$(\boldsymbol{\alpha}_k^t)^\top(\boldsymbol{h}_k^t - \boldsymbol{B}_k^t \boldsymbol{\nu}_k) + (\boldsymbol{\beta}_k^t)^\top \boldsymbol{\ell}_k^t \le \varphi^t \qquad \forall t\in\mathcal{T}, \forall k \in \mathcal{BD}^t \tag{14d}$$

$$\boldsymbol{z}, \boldsymbol{\nu} \in \mathcal{F}^m, \boldsymbol{\eta} \in \{0,1\}^{|\Omega|}.$$

The Benders' optimality constraints (14d) will be discussed in the algorithm description. When $\Omega^t = \Omega_c^t$ for all t, and the set $\mathcal{BD}$ ensures Benders' convergence so that the optimal $\varphi^{t*} = \tilde{q}^t(\nu^*)$ for all t, the optimal maintenance decisions $\{\boldsymbol{z}^*, \boldsymbol{\nu}^*\}$, and the objective total cost becomes identical in APMII, AF, and RMP. This simple observation comes from Eq. (12). As we noted previously, only a subset of these generator availability vectors may be needed to recover the optimal cost and maintenance decisions $\{\boldsymbol{z}^*, \boldsymbol{\nu}^*\}$ for APMII. This observation provides a claim parallel to the findings of [5].

Due to the two-stage nature of this problem, we propose a two-level algorithm to solve APMII. In the upper level, the algorithm solves the restricted master problem $\mathbf{RMP}(\Omega, \mathcal{BD})$ iteratively to generate Benders' optimality cuts for every maintenance epoch. The Benders' optimality cuts are appended to the set $\mathcal{BD}$, and the algorithm repeats the Benders' process until convergence. Then the current solution is used to generate variables and constraints as in Eq. (10) in order to recover the true cost of APMII. We append the maintenance scenario of the current solution to the set Ω, then check for convergence in terms of true cost recovery. Repeat the process if cost convergence criteria is violated; otherwise, terminate. Flowchart of the algorithm is illustrated in Figure 1 and the method is formally presented in Algorithm 1. The following theorem proves the convergence of the algorithm.

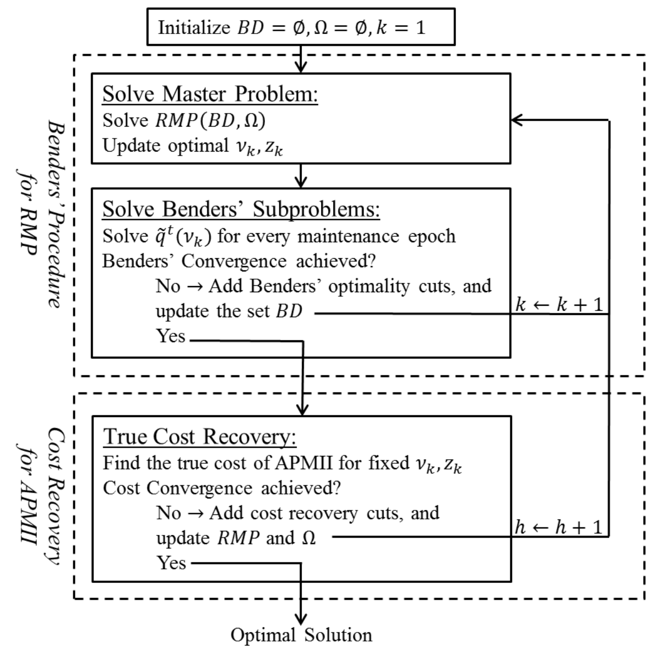

Fig. 1. Flowchart of the proposed algorithm for solving APMII.

Theorem 1. *Algorithm 1 with tolerances ϵ^b and ϵ^c terminates in a finite number of steps, and returns an ϵ-optimal maintenance solution $\{\boldsymbol{z}^*, \boldsymbol{\nu}^*\}$, i.e. $\rho^* \le \rho(\boldsymbol{z}^*, \boldsymbol{\nu}^*) \le \rho^*(1+\epsilon)$, where ρ^* is the optimal cost of APMII, $\rho(\boldsymbol{z}^*, \boldsymbol{\nu}^*) = \boldsymbol{c}^\top \boldsymbol{z}^* + q(\boldsymbol{\nu}^*)$ in (6), and $\epsilon = (1+\epsilon^b)(1+\epsilon^c) - 1$.*

Proof: See Appendix A. ∎

Remark: Note that the current form of the algorithm has slack variables in the demand balance and line flow constraints in the unit commitment subproblem, so it remains feasible for any maintenance decision $\boldsymbol{\nu}$. We can also remove these slack variables and incorporate Benders' feasibility cuts to $\mathbf{RMP}(\Omega, \mathcal{BD})$.

VI. Experiments

In this section we present the experimental implementation and results for APMII. We first present a description of the experimental procedure. In what follows, we provide a convergence analysis for the solution of APMII using the algorithm introduced in Section V. We then briefly introduce the experimental procedure, and use this procedure to conduct two comparative studies on APMII. The first study considers the basic case, where we assume that handling a failed generator takes the same amount of time as conducting preventive maintenance. The second study considers a more realistic case where the failure interruption takes twice as long as a planned maintenance interruption. We will illustrate the effectiveness of our approach in each study.

A. Experiment Implementation

In all of our analyses, we use the IEEE 118-bus system. The system has 54 generators, 118 buses, and 186 transmission lines. We obtain the age of generators at the start of the experiments by running the generators for a warming period. We set

Algorithm 1: Solution Algorithm for APMII

Let $\mathcal{BD} \leftarrow \emptyset$, $\Omega \leftarrow \emptyset$, $k \leftarrow 0$, and $h \leftarrow 0$.
Denote the tolerance levels of Benders' decomposition and total cost as $\epsilon^b \geq 0$ and $\epsilon^c \geq 0$, respectively. Define the corresponding convergence flags as Benders' decomposition convergence (BDC), and total cost convergence (TCC). Let $\epsilon \leftarrow (1+\epsilon^b)(1+\epsilon^c)-1$, $BDC \leftarrow 0$, and $TCC \leftarrow 0$.
while $TCC = 0$ **do**
  $h \leftarrow h+1$
  /* Start Benders' for current RMP */
  while $BDC = 0$ **do**
    $k \leftarrow k+1$
    Solve $\mathbf{RMP}(\Omega, \mathcal{BD})$. Denote its optimal solution as $\{\boldsymbol{z}_k, \boldsymbol{\nu}_k, \boldsymbol{\eta}_k, \boldsymbol{\varphi}_k\}$ and optimal cost as ρ_k^*.
    for $t \in \mathcal{T}$ **do**
      Solve the dual of $\tilde{q}^t(\boldsymbol{\nu}_k)$:

$$\begin{aligned} \tilde{q}^t(\boldsymbol{\nu}_k) = \max_{\boldsymbol{\alpha}^t, \boldsymbol{\beta}^t} \quad & (\boldsymbol{\alpha}^t)^\top(\boldsymbol{h}^t - \boldsymbol{B}^t\boldsymbol{\nu}_k) + (\boldsymbol{\beta}^t)^\top \boldsymbol{\ell}^t \\ \text{s.t.} \quad & (\boldsymbol{E}^t)^\top \boldsymbol{\alpha}^t + (\boldsymbol{F}^t)^\top \boldsymbol{\beta}^t \leq \boldsymbol{v}^t \\ & (\boldsymbol{G}^t)^\top \boldsymbol{\alpha}^t \leq \boldsymbol{b}^t \\ & \boldsymbol{\alpha}^t \leq \boldsymbol{0}, \boldsymbol{\beta}^t \leq \boldsymbol{0} \end{aligned}$$

      Denote optimal solution as $\{\boldsymbol{\alpha}_k^t, \boldsymbol{\beta}_k^t\}$.
    end
    if $\sum_{t\in\mathcal{T}} \tilde{q}^t(\boldsymbol{\nu}_k) > (1+\epsilon^b) \cdot \sum_{t\in\mathcal{T}} \varphi_k^t$ **then**
      for $t \in \mathcal{T}$ **do**
        if $\tilde{q}^t(\boldsymbol{\nu}_k) > \varphi_k^t$ **then**
          Generate a Benders' optimality cut $(\boldsymbol{\alpha}_k^t)^\top(\boldsymbol{h}_k^t - \boldsymbol{B}_k^t\boldsymbol{\nu}) + (\boldsymbol{\beta}_k^t)^\top \boldsymbol{\ell}_k^t \leq \varphi^t$. Add this cut to the list $\mathcal{BD}^t$.
        end
      end
    else
      $BDC \leftarrow 1$
    end
  end
  /* End Benders' for current RMP */
  Execute TCR($\mathbf{RMP}(\cdot), \boldsymbol{\nu}_k, \rho_k^*, (\tilde{q}^t(\boldsymbol{\nu}_k))_{\forall t}, \epsilon^c, \Omega, h$)
  /* Run the TCR procedure */
end
$\boldsymbol{z}^* \leftarrow \boldsymbol{z}_k$ and $\boldsymbol{\nu}^* \leftarrow \boldsymbol{\nu}_k$.
Output: Maintenance solution $\{\boldsymbol{z}^*, \boldsymbol{\nu}^*\}$.

the maintenance decisions weekly, and operational decisions hourly. The decisions are subject to all the constraints outlined in Section II-B. Planning horizon for each problem is set at 110 weeks. We set the preventive maintenance cost $c_p = 200,000$ and the failure cost $c_f = 800,000$. In all our experiments, we use Gurobi 5.6.0 [6].

In our experimental studies, we use a degradation analysis procedure similar to [1]. More specifically, we take the vibrational data from rolling element bearings as representative of generator degradation. To model the degradation of the bearings, we use the exponential degradation function with Brownian error. We refer the reader to [1] for details on the

Procedure True Cost Recovery(TCR) for APMII

Input: $\mathbf{RMP}(\cdot), \boldsymbol{\nu}_k, \rho_k^*, (\tilde{q}^t(\boldsymbol{\nu}_k))_{\forall t}, \epsilon^c, \Omega, h$
$\delta^h \leftarrow 0$.
for $t \in \mathcal{T}$ **do**
  Find the generator statuses at t corresponding to the solution $\boldsymbol{\nu}_k$, namely $\hat{\boldsymbol{m}}^t$.
  if $\hat{\boldsymbol{m}}^t$ *is not contained in the list* Ω^t **then**
    $\Omega^t \leftarrow \Omega^t \cup \{\hat{\boldsymbol{m}}^t\}$. Solve $q^t(\boldsymbol{\nu}_k)$ in model (7).
    $e_h^t \leftarrow q^t(\boldsymbol{\nu}_k) - \tilde{q}^t(\boldsymbol{\nu}_k)$, and $\delta^h \leftarrow \delta^h + e_h^t$.
    Add variable η_h^t, cut $(\boldsymbol{r}_h^t)^\top \boldsymbol{\nu} + \eta_h^t \geq u_h^t$ and objective cost $\eta_h^t e_h^t$ to $\mathbf{RMP}(\cdot)$.
  end
end
if $\rho_k^* + \delta^h \leq \rho_k^*(1+\epsilon^c)$ **then** /* If current RMP cost is sufficiently close to its corresponding true cost */
  $TCC \leftarrow 1$, $BDC \leftarrow 1$
else
  $TCC \leftarrow 0$, $BDC \leftarrow 0$
end
Output: $\mathbf{RMP}(\cdot), \delta^h, \Omega, BDC, TCC$

estimation of the prior estimates, and the real-time Bayesian updates of the degradation parameters.

In order to test the effectiveness of APMII, we design an experimental framework consisting of two main modules: i) optimization module, and ii) execution module. In the optimization module, given dynamic maintenance costs and remaining maintenance downtimes for each generator, we solve APMII. Then in the execution module, we fix the maintenance schedule during the freeze time, and execute the chain of events during a freeze period. Experimental implementation for the APMII is similar to that of APMI, except that in the implementation for APMII, for every time period, we determine which generators are available (not failed, or undergoing maintenance), and solve a unit commitment problem with the available generators. This allows us to calculate the resulting operational costs for each week. We let the freeze period $\tau_R = 8$ weeks, and solve the maintenance problems in a rolling horizon fashion to cover a period of 48 weeks.

In order to ensure a fair comparison, we repeat this implementation spanning 48 weeks, using generator with different ages. We take the average of these experiments to obtain any of the metrics we present.

B. Experiment Results

1) Convergence Analysis: We first highlight the convergence performance of our algorithm using one of the instances of APMII used in our case studies. Direct solution of this problem with the state-of-the-art MIP solver such as Gurobi proves to be problematic with the solver quickly running into memory problems on our computer with 8GB RAM. However, we can solve this problem to 0.3% optimality gap using the proposed algorithm in 20 Benders' iterations ($k = 20$) and 15 cost recovery iterations ($h = 15$). The total running time is 121 minutes.

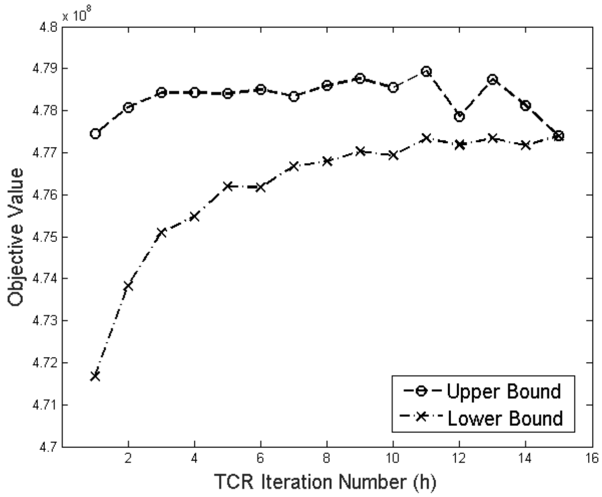

Fig. 2. Convergence Analysis for APMII on IEEE-118 Bus System.

Figure 2 shows the cost recovery iterations. The dashed line denote the dual solution of the RMP problem for current Benders' iteration k, namely $\boldsymbol{c}^\top \boldsymbol{z}_k + \boldsymbol{e}^\top \boldsymbol{\eta}_k + \sum_{t\in\mathcal{T}} \tilde{q}^t(\boldsymbol{\nu}_k)$. This provides a valid lower bound as indicated in the proof of Theorem 1. We also calculate the upper bound ($\rho_k^* + \delta^h$ in TCR Procedure, line 10). We note when $\epsilon^b = 0$, the lower bound is monotonically increasing, while the corresponding feasible solution ($\rho_k^* + \delta^h$) does not exhibit a monotone behavior.

In our experiments, we enforce a higher optimality gap of 1%, and the proposed algorithm converges in at most 4 iterations, within 35 minutes.

2) Comparative Study on APMII: In this section, we consider the fleet maintenance scheduling of conventional generators. For these generators, the effect of any outage on the operational costs is significant. In this comparative study, we set the generator maintenance downtime ratio to be $1 : 1$, meaning that conducting preventive maintenance takes the same amount of time as handling a failure. We use the algorithm presented in section IV to solve APMII with optimality gap of 1%.

We show the superiority of the maintenance scheduling of APMII by comparing it with the periodic, RBM and APMI models. The periodic model has the same modifications imposed on APMII. As a result, the periodic model conducts maintenance at a maintenance epoch i between the 66^{th} and 69^{th} weeks with the objective of minimizing the total operational cost. Therefore, the periodic model for this study is a cost minimization problem. For the RBM case, we use the exact optimization model of APMII, however the cost function for this scenario is derived using a Weibull distribution. We first derive a Weibull estimate using the failure times from the rotating machinery application $F_W(t)$, and then condition this distribution on the time of survival to estimate the remaining life distribution and the associated maintenance costs. We also evaluate the performance of the APMI model in this study. To find the resulting schedule, we first implement the APMI model during the freeze period. Based on the fixed schedule, we find the resulting operational (UC) costs. We continue this process in a rolling horizon fashion.

Table I presents the reliability and cost metrics for the four policies considered in the comparative study. We first compare APMII with the periodic model and RBM. We note that RBM remains a conservative policy in comparison to the periodic model, since it schedules more preventive maintenances (24.9 v.s. 23.9), incurs less number of unexpected failures (12.3 v.s. 13.7) and sacrifices more lifetime (1019.6 v.s. 943.6 weeks).

APMII, on the other hand, benefits from the additional sensor data to learn more about the ongoing degradation in the generators. Consequently, APMII decreases the number of unexpected failures by 86.9% compared to the periodic model, and by 85.4% compared to the RBM model. Considering the total useful life unused among the 54 generators, we see that APMII provides significant improvements, i.e., unused life of APMII is only 31.9% of the periodic model, and 29.5% of RBM, respectively.

We note that periodic and RBM policies have comparable maintenance costs, with periodic policy incurring an additional maintenance cost of \$0.9M on average. APMII incurs a smaller maintenance cost that constitutes 42.31% of the periodic, and 44.94% of the RBM policy. The periodic maintenance policy results in significantly higher operational costs. This is because the periodic policy enforces a maintenance window that limits the flexibility of the maintenance policy to adapt to the demand profile.

The operational costs of the RBM and the APMII shows an interesting pattern. This pattern reflects the trade-off between the minimization of the operations cost, and the maintenance cost. RBM and APMII uses the same problem structure, however, since the remaining life estimates of RBM is not as accurate as APMII, the dynamic cost function of RBM is more flat. This, in turn, allows more flexibility for RBM to further minimize the operational cost, at the expense of increased risks of unexpected failures. We note that the flat dynamic maintenance cost function of RBM generates a slightly lower operational cost, but increases the maintenance cost so much that the total cost of RBM exceeds that of APMII.

Another interesting pattern can be recognized between APMI and APMII. We note that APMI minimizes the maintenance cost without considering the impact on operational costs. APMII, on the other hand, optimizes the maintenance schedule to minimize the total cost, thus deviating from the optimal maintenance cost (provided by APMI), to ensure more gains from the operational cost. This makes APMI marginally more reliable, yet significantly more expensive than APMII.

In terms of the total cost, we see that the RBM policy performs better than the periodic policy. This is due to the considerable difference in the operational costs of the periodic policy and the other policies. But APMII achieves the smallest total cost among four policies with savings of \$12.6M, \$7.9M, and \$1.2M compared to the periodic, RBM, and APMI, respectively.

3) Comparative Study on APMII with Realistic Failure Recovery Times: In the previous section, we assumed that conducting a preventive maintenance takes the same amount of time as handling an unexpected failure. In reality, when a generator fails, maintenance practitioners need significantly more time to put the generator back online, since: i) an

TABLE I
BENCHMARK FOR APMII - MAINTENANCE DOWNTIME RATIO 1:1

	Periodic	RBM	APMI	APMII
# Preventive	23.9	24.9	26.6	26.1
# Failures	13.7	12.3	1.5	1.8
# Total Outages	37.6	37.2	28.1	27.9
Unused Life (wks)	943.6	1019.6	309.5	300.7
Maintenance Cost	\$15.74 M	\$ 14.82 M	\$6.52 M	\$6.66 M
Operations Cost	\$ 188.19 M	\$ 184.35 M	\$185.98 M	\$184.62 M
Total Cost	\$ 203.92 M	\$ 199.17 M	\$192,50 M	\$191.28 M

TABLE II
BENCHMARK FOR APMII - MAINTENANCE DOWNTIME RATIO 1:2

	Periodic	RBM	APMI	APMII
# Preventive	24.0	25.3	26.6	25.7
# Failures	13.7	12.2	1.5	1.9
# Total Outages	37.7	37.5	28.1	27.6
Unused Life (wks)	950.1	1012.9	309.4	295.6
Maintenance Cost	\$15.76 M	\$14.82 M	\$6.52 M	\$6.66 M
Operations Cost	\$191.24 M	\$186.54 M	\$ 186.09 M	\$185.08 M
Total Cost	\$207.00 M	\$201.36 M	\$ 192.61 M	\$191,74 M

unexpected failure can cause other subcomponents to fail as well, increasing the scope of inspection and maintenance, ii) full inventory of the needed maintenance equipment and crew would not be ready to start the maintenance immediately. To model this realistic scenario, we set the failure recovery time twice as long as a preventive maintenance duration, thus using maintenance downtime ratio $1:2$.

Table II presents the reliability and cost metrics in this scenario. Reliability results are comparable to the previous section. We note that since APMI and APMII incurs less number of failures, they are effected only minimally by the set of changes introduced in this section. However, we observe a significant effect of these changes on the operational costs of the periodic and RBM policies. Introducing realistic failure recovery increases the operational cost of APMII by \$0.46M, while the periodic and RBM policies incur an additional operational cost of \$3.1M and \$2.2M, compared to the corresponding numbers in Table I, respectively. This effect is due to the significant number of failures experienced by the periodic and RBM policies.

Table II also shows that APMII provides significant savings on operational cost and total cost. In particular, the operational cost of APMII is \$6.2M, \$1.5M, and \$1.0M lower than the that of the periodic, RBM, and APMI policies, respectively. Correspondingly, the total cost of APMII is \$15.3M, \$9.6M, and \$0.9M lower.

4) Discussion on the Results: The results show that the proposed framework has significant advantages in terms of maintenance and operational costs and system reliability over the traditional approaches. More specifically, comparing to the best performance of the periodic and RBM policies, Table II shows the following advantages of our approach:

- *APMI/II significantly reduce the number of unexpected failures:* In all our experiments, we observe that our models provide significant improvements in terms of the unexpected failures. Comparing to the best among the periodic and RBM policies, APMI and APMII only have 12.3% and 15.6% of the unexpected failures, respectively.
- *APMI/II extend the equipment lifetime:* Using the additional sensor observations allow our policy to utilize more of the generator lifetime. This is because our approach can reason through predictive analytics when a maintenance might not be necessary. We observe that the unused lifetimes of APMI and APMII are 32.6% and 31.1% of the best among the periodic and RBM policies, respectively.
- *APMI/II require less outages:* Compared to the benchmarks, our approach always required less interruptions to the generator's dispatch schedule, i.e. the total outages of APMI and APMII are 74.9% and 73.6% of the best among the periodic and RBM policies, respectively.
- *APMI/II significantly reduce the maintenance costs:* Our approach incurs less than 44.9% of the maintenance costs associated with the periodic and RBM policies.
- *APMII significantly reduce the total cost:* In terms of the total cost, APMII outperforms all three other models, with savings of \$15.3M, \$9.6M, and \$0.9M comparing to the periodic, RBM, and APMI policies.

VII. CONCLUSION

In this paper, we present an extended model on the unified framework that links low-level performance and condition monitoring data with high-level operational and maintenance decisions for generators. The operational decisions identify the optimal commitment and dispatch profiles that satisfy the demand and network feasibility requirements. Maintenance decisions focus on arriving at an optimal fleet-level sensor-driven schedule that accounts for optimal asset-specific schedules driven by the condition monitoring data. We provide an effective solution algorithm to solve large instances of APMII, and show the effectiveness of our approach. To conduct the computational studies, we implement an experimental framework that integrates the dynamic information obtained from sensor measurements and predictive analytics with the proposed maintenance scheduling module. Extensive computational experiments are conducted on this platform. In particular, real-world degradation data collected from sensor measurements of rotating bearings are used in the experiments. The experiments compare the proposed sensor-driven condition-based generation maintenance approach (APMII) with the traditional periodic and reliability-based approaches, and the APMI model introduced in [1].

In this two paper study, we provided an integration of the sensor driven predictive analytics into the generator maintenance problem. We presented the set of changes triggered by low-level sensor observation, all the way to the high-level fleet maintenance scheduling. We provided two novel maintenance models driven by this sensor information. We developed an algorithm that capitalizes on the specific structure of the maintenance model we proposed. To test the effectiveness of our approach, we modeled a number of benchmarks, and developed an experimental framework that benefits from degradation data from real-life rotating machinery application. We presented the advantages of our approach in a number of different settings.

We hope that the sensor-driven optimization framework presented herein can be used as a basis for a new paradigm for generation maintenance in power systems.

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APPENDIX A
PROOF OF LEMMA 1

The APMII model (6) can be represented as follows:

$$\min_{\boldsymbol{z}^o,\boldsymbol{\nu}} q(\boldsymbol{\nu}) + \sum_{i\in\mathcal{G}}\sum_{k\in\mathcal{K}_i}\min_{\boldsymbol{z}_{:,i,k}}\left\{\boldsymbol{c}_{i,k}^\top \boldsymbol{z}_{:,i,k} | \boldsymbol{z}_{:,i,k}\in\mathcal{P}_k^i(\boldsymbol{z}^o,\boldsymbol{\nu})\right\},$$

where, given feasible $\boldsymbol{z}^o,\boldsymbol{\nu}$, the APMII's constraints $\mathcal{P}_k^i(\boldsymbol{z}^o,\boldsymbol{\nu})$ over $\boldsymbol{z}_{:,i,k}$'s are decoupled for each i,k. We want to show that for any fixed $\{\boldsymbol{z}^o,\boldsymbol{\nu}\}$, if we relax $\boldsymbol{z}_{:,i,k}$ to be in $[0,1]$, the relaxed problem still has a binary optimal solution in $\boldsymbol{z}_{:,i,k}$. For $k=1$, $\boldsymbol{z}_{:,i,1}=\boldsymbol{\nu}_{:,i,1}$ has to be binary due to (14) in [1]. In the following, we focus on $k\geq 2$. For any $i\in\mathcal{G}, k\geq 2$, if $z_{i,k}^o=1$, then $z_{t,i,k}=0\ \forall t$ by (10) in [1]. If $z_{i,k}^o=0$, then $\sum_{t\in\mathcal{T}} z_{t,i,k}=1$ by (10) in [1] and $\sum_{t\in\mathcal{T}} t\, z_{t,i,k}=\sum_{t\in\mathcal{T}} t\,\nu_{t,i,k}-\sum_{t\in\mathcal{T}} t\,\nu_{t,i,k-1}=: b_{ik}$ by (15) in [1], which, together with constraint (12) in [1], ensures that b_{ik} is a nonnegative integer. Denote the k-th maintenance cost of generator i at time t as $\phi_{i,k}(t)=C_t^{n,i}$, which is convex in t given by the definition of the dynamic maintenance cost functions in [1]. Since $\sum_{t\in\mathcal{T}} z_{t,i,k}=1$ and $z_{t,i,k}\geq 0$, then for any $z_{t,i,k}$ feasible for the relaxed problem, the Jensen's inequality suggests

$$\begin{aligned}&\phi_{i,k}(b_{ik})=\phi_{i,k}(z_{1,i,k}+2z_{2,i,k}+\cdots+Hz_{H,i,k})\\ &\leq z_{1,i,k}\phi_{i,k}(1)+z_{2,i,k}\phi_{i,k}(2)+\cdots+z_{H,i,k}\phi_{i,k}(H)\\ &=\boldsymbol{c}_{i,k}^\top\boldsymbol{z}_{:,i,k}.\end{aligned}$$

Thus, $q(\boldsymbol{\nu})+\sum_{i\in\mathcal{G}}\boldsymbol{c}_{i,1}^\top\boldsymbol{z}_{:,i,1}+\sum_{i,k\geq 2}\phi_{i,k}(b_{ik})$ is a lower bound to the optimal cost of the relaxed problem for the fixed $\boldsymbol{z}^o,\boldsymbol{\nu}$. In fact, this lower bound can be achieved by the solution $\boldsymbol{z}_{t,i,k}=1$ if $t=b_{ik}$ and 0 otherwise for $k\geq 2$. This binary solution together with $\boldsymbol{z}_{:,i,1}=\boldsymbol{\nu}_{:,i,1}\ \forall i\in\mathcal{G}$ is feasible for the relaxed problem, therefore also optimal for APMII.

Lastly, we let $q(\nu)\leftarrow 0$, and APMII reduces to AMPI. Thus the lemma also applies to AMPI. This completes the proof.

APPENDIX B
PROOF OF THEOREM 1

Finite convergence: In each iteration h of Algorithm 1, the Benders' decomposition with tolerance ϵ_b terminates in finite steps; then if the condition in line 10 of TCR is true, Algorithm 1 terminates. Otherwise, TCR augments the set Ω by at least one different maintenance status. Since the number of all possible statuses is $H2^{|\mathcal{G}|}$, TCR is executed at most $H2^{|\mathcal{G}|}$ number of times, at which point Ω becomes $\bar{\Omega}$ and $\rho_k^*+\delta^h=\rho_k^*$, thus terminating the algorithm.

ϵ-Optimality: (i) We first prove that $\rho_k^*\leq\rho^*$, where ρ_k^* is the optimal cost of the final RMP solved before Algorithm 1 terminates. We note that for any feasible maintenance solution $\{\boldsymbol{z},\boldsymbol{\nu}\}$, we have

$$\boldsymbol{c}^\top\boldsymbol{z}+\min_{\boldsymbol{\eta},\boldsymbol{\varphi}}\left\{\boldsymbol{e}^\top\boldsymbol{\eta}+\sum_{t\in\mathcal{T}}\varphi^t | s.t: (14c),(14d)\right\}\leq\boldsymbol{c}^\top\boldsymbol{z}+q(\boldsymbol{\nu}).$$

Let $\{\boldsymbol{z}',\boldsymbol{\nu}'\}$ be the optimal solution of APMII, i.e: $\rho^*=\rho(\boldsymbol{z}',\boldsymbol{\nu}')$. Then we have

$$\begin{aligned}&\rho_k^*=\min_{\boldsymbol{z},\boldsymbol{\nu}\in\mathcal{I}}\left\{\boldsymbol{c}^\top\boldsymbol{z}+\min_{\boldsymbol{\eta},\boldsymbol{\varphi}}\left\{\boldsymbol{e}^\top\boldsymbol{\eta}+\sum_{t\in\mathcal{T}}\varphi^t | s.t: (14c),(14d)\right\}\right\}\\ &\leq\boldsymbol{c}^\top\boldsymbol{z}'+\min_{\boldsymbol{\eta},\boldsymbol{\varphi}}\left\{\boldsymbol{e}^\top\boldsymbol{\eta}+\sum_{t\in\mathcal{T}}\varphi^t | s.t: (14c),(14d)\right\}\\ &\leq\boldsymbol{c}^\top\boldsymbol{z}'+q(\boldsymbol{\nu}')=\rho^*,\end{aligned}$$

where $\mathcal{I}$ denotes the feasible set for problem (6).

(ii) Next, we claim that $\rho_k^*\leq\rho(\boldsymbol{z}^*,\boldsymbol{\nu}^*)\leq\rho_k^*(1+\epsilon)$. The first inequality holds because $\rho_k^*\leq\rho^*$ and $\rho^*\leq\rho(\boldsymbol{z}^*,\boldsymbol{\nu}^*)$.

If the condition in Algorithm 1, line 12, does not hold, i.e. if $\sum_{t\in\mathcal{T}}\tilde{q}^t(\boldsymbol{\nu}_k)\leq(1+\epsilon^b)\cdot\sum_{t\in\mathcal{T}}\phi_k^t$, then:

$$\rho_k^*\leq\boldsymbol{c}^\top\boldsymbol{z}_k+\boldsymbol{e}^\top\boldsymbol{\eta}_k+\sum_{t\in\mathcal{T}}\tilde{q}^t(\boldsymbol{\nu}_k)+\delta^h=\rho(\boldsymbol{z}^*,\boldsymbol{\nu}^*) \tag{17a}$$

$$\leq(1+\epsilon^b)\left(\boldsymbol{c}^\top\boldsymbol{z}_k+\boldsymbol{e}^\top\boldsymbol{\eta}_k+\sum_{t\in\mathcal{T}}\phi_k^t+\delta^h\right), \tag{17b}$$

where (17b) $=(1+\epsilon^b)(\rho_k^*+\delta^h)$. Since $\rho_k^*+\delta^h\leq\rho_k^*(1+\epsilon^c)$:

$$\rho_k^*\leq\text{(17b)}\leq(1+\epsilon^b)(1+\epsilon^c)\ \rho_k^*=(1+\epsilon)\rho_k^*.$$

Using (i) and (ii), we have $\rho^*\leq\rho(\boldsymbol{z}^*,\boldsymbol{\nu}^*)\leq(1+\epsilon)\rho^*$. This concludes the proof.

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