Operating Leverage and Hedging: A Tale of Two Production Costs for Asset Pricing *

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September 2020
Preliminary Version

Abstract

We investigate the joint asset pricing effects of variable costs and fixed costs in a firm's production process. While the latter such as SG&A expenses create an operating leverage effect, the variable costs allow firms to hedge against aggregate profitability shocks. Taking into account both types of production costs explains the empirical patterns in the cross-section asset returns in portfolios sorted by the gross profitability and operating leverage. Our model reconciles the seemingly contradictory phenomena that higher productivity firms earn lower returns (İmrohoroğlu and Tüzel (2014)), whereas more profitable, often more productive, firms earn higher returns (Novy-Marx (2013)). It also offers a novel explanation for the negative idiosyncratic volatility premium (Ang, Hodrick, Xing, and Zhang (2006)) based on production costs.

JEL Classifications: G12, E44

Keywords: Operating leverage, operating hedging, variable costs, fixed costs, cross-sectional asset pricing

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1 Introduction

The operation in a firm's production process affects its exposure to aggregate risks. For majority of firms, their production expenses can be broadly categorized as fixed costs and variable costs, depending upon their variability with outputs. While Sales, General and Administrative (SG&A) expenses are relatively stable in the production process, the inputs that are directly related to the production of outputs such as raw materials, intermediate inputs, services, among others (i.e., COGS as classified in Compustat) strongly covary with outputs. Indeed, by aggregating firm data from Compustat, we find that the elasticity of aggregate COGS with respect to the aggregate sales revenue is greater than one (1.05). In contrast, the elasticity of SG&A is significantly lower than one (0.39) (see Table 1), leading to different cyclicality between variable and fixed costs.

[Insert Table 1 Here]

The difference in the cyclicality of these inputs has very different asset pricing implications. The presence of fixed cost creates an operating leverage effect which has been argued to affect a firm's risk premium. This channel has been extensively studied and used to explain the well-documented value premium (e.g., Carlson, Fisher, and Giammarino (2004), Zhang (2005)), total factor productivity (TFP) premium (İmrohoroğlu and Tüzel (2014)), and labor share premium (Donangelo, Gourio, Kehrig, and Palacios (2018)). On the other hand, variable costs, which account for up to 70 percent of total production costs, create an operating hedging effect. Their procyclicality reduces the elasticity of gross profits to aggregate revenue to only 0.87 in the data. Kogan, Li, and Zhang (2019) first document this effect and show that the cross-sectional difference in the strength of operating hedging is important for the gross profitability premium (Novy-Marx (2013)).

Despite the economic importance of operating leverage and operating hedging, no existing study has investigated both effects in a unified framework. Conceptually, the operating leverage effect can be attenuated by the presence of variable costs, while incorporating fixed costs can potentially weaken the operating hedging effect. After all, it is the combined effects of different components of production inputs that determine their overall effect on asset pricing. More important, these inputs are endogenously chosen in a firm's value maximization problem, which further complicates their asset pricing implications in such a setting. Our study fills this void of the asset pricing literature.

We introduce a nested constant elasticity of substitution (CES) production function with three types of inputs: physical capital (such as PPE), fixed inputs (e.g., SG&A), and variable inputs (e.g., COGS). Following the literature on production functions, we first nest physical capital and fixed inputs and then nest this combined input with variable inputs.[1] Using this approach, we allow the elasticity of substitution to differ among the production inputs. In addition to an aggregate profitability shock that impacts all firms in the economy, we also introduce different types of firm-specific shocks affecting the efficiency of fixed and variable inputs. With firms optimally choosing the amount of fixed and variable inputs, our setup incorporates both the operating leverage and operating hedging effects endogenously.

Our model has two immediate predictions. First, the operating hedging effect from variable costs exists no matter if there are fixed costs. When we compare the exposure of gross profits to the aggregate profitability shock with the exposure of outputs, the hedging effect is present as long as 1) the price of variable inputs is elastic to with respect to aggregate profitability shock, and 2) the physical capital and variable inputs are complements in the production function. Both conditions have been confirmed in Kogan, Li, and Zhang (2019). Second, under two empirically verified conditions: 1) the elasticity of substitution between physical capital and fixed inputs is less than one; and 2) the price of fixed inputs is "sticky" and does not show strong procyclicality, the effect from fixed costs on the riskiness of a firm depends on the firm's gross margin. When a firm's gross margin is high, fixed costs raise the exposure of operating profits to the aggregate profitability shock relative to gross profits, giving rise to an operating leverage effect. When a firm's gross margin is sufficiently low, the operating leverage effect is dominated by the operating hedging from variable inputs, so that fixed costs even lower the firm's risk premium. Our results therefore call for caution in the existing studies that examine the operating leverage effect while not considering variable inputs.

Calibrating the model with parameter values consistent with empirical estimates, we have the following main findings on the cross-sectional asset returns from the numerical solution and model simulations. First, our model generates a positive relation between gross profitability and stock returns, and the gross profitability premium is substantially stronger among firms with higher operating leverage. This matches the observed relation in the data for 5-by-5 portfolios double sorted by gross profitability and a measure of operating leverage (defined as the ratio of operating profit to gross profit), as shown in Figure 1. Consistent with the economic channel in Kogan, Li, and Zhang (2019), this premium originates from the cross-sectional heterogeneity in the strength of operating hedging from variable inputs. Less profitable firms experience higher operating hedging than more profitable firms, leading to more profitable firms having a larger exposure to the aggregate profitability shock than

[1] This structure has been confirmed as a good approximation of the production behavior in several studies. See, for example, Carlstrom and Fuerst (2006), Bodenstein, Erceg, and Guerrieri (2011), and Kemfert (1998). Another advantage of this specification is that accounting variables including gross margin and operating leverage naturally emerge from the first order conditions of firm's optimization problem.

less profitable firms. For a moderate level of fixed cost, the operating leverage effect further raises the gross profitability premium. Second, our model predicts an operating leverage premium whose sign depends on firm's gross margin (and gross profitability). For firms with high gross margin, the relation between operating leverage and risk premium is positive, consistent with the existing literature on the asset pricing implications of operating leverage effect. However, when gross margin is sufficiently low, the operating leverage premium becomes negative, which is also confirmed by the pattern in the average realized returns in Figure 1.

[Insert Figure 1 Here]

Our model reconciles the seemingly puzzling coexistence of a positive gross profitability premium and a negative total factor productivity (TFP) premium. Estimating the TFP as the firm-level Solow residuals, İmrohoroğlu and Tüzel (2014) document that high TFP firms earn lower returns than low TFP firms. They interpret this negative TFP premium and attribute it to the operating leverage effect. However, this finding seems at odds with the positive gross profitability premium, because high TFP firms are also more profitable. Our framework offers a resolution to this puzzle. In the model, a firm's gross profitability is mostly driven by the idiosyncratic shock that affects the productivity of variable inputs, but a firm's choice of fixed inputs is affected by the idiosyncratic shocks on both fixed and variable inputs. When one projects firms' gross profits onto physical capital and fixed inputs, the estimated residual, i.e., TFP, mostly captures the idiosyncratic productivity of fixed inputs. In other words, a firm's gross profitability and TFP contain information about different sources of firm-level uncertainties. While a positive shock to the variable input productivity raises a firm's risk premium due to the operating hedging effect, a positive shock to the fixed input productivity reduces the risk premium from the operating leverage effect. As a result, both premiums emerge in the same framework.

Our model also offers a novel explanation for the negative relation between stock excess return and idiosyncratic volatility (see Ang, Hodrick, Xing, and Zhang (2006)). In our model, firms with high idiosyncratic volatility have low productivity to the variable inputs and low gross margin, and the associated operating hedging effect lowers their risk premiums. In the meanwhile, their lower productivity raises their sensitivity to the idiosyncratic productivity shocks due to the operating leverage effect. The joint effects of operating hedging and operating leverage give rise to the negative relation between idiosyncratic volatility and systematic risk. Empirically, we find the gross profitability premium and the operating leverage premium together explain about half of the time series variation in the idiosyncratic volatility premium. Controlling for these two premiums, the idiosyncratic volatility premium is reduced by almost 80% and no longer statistically significant.

Our paper is closely related to the literature on the effects of operating leverage and operating hedging on asset pricing. Majority of existing studies focus on operating leverage. For instance, Zhang (2005) and Carlson, Fisher, and Giammarino (2004) show how operating leverage can generate a value spread in a neoclassical model of firm investment. Novy-Marx (2010) proposes an empirical measure of operating leverage and documents its positive predictive power for cross-sectional stock returns. A recent strand of related literature focuses on the effects of labor costs on stock return, emphasizing wage rigidity as a source of operating leverage. For instance, Danthine and Donaldson (2002) show that wage rigidity can induce a strong labor leverage and improve the performance of asset pricing models with production to better match aggregate market volatility and equity premium. Favilukis and Lin (2015) examine the quantitative effect of wage rigidity and labor leverage on both the equity premium and the value premium. Donangelo, Gourio, Kehrig, and Palacios (2018) document that firms with high labor shares have higher expected returns than firms with low labor shares. In a new direction of exploration deviating from the operating leverage, Kogan, Li, and Zhang (2019) uncover the importance of variable inputs in lowering firm's risk premium, originating from an operating hedging effect. They demonstrate that operating hedging is important in understanding the gross profitability premium in Novy-Marx (2013). Existing literature however only separately explored the operating leverage and the operating hedging effect. To the best of our knowledge, we are the first to examine their joint effects on asset pricing.

The paper proceeds as follows. In Section 2, we develop a production-based economic model to study how the interaction of fixed and variable costs affects a firm's risk premium. We pay special attention to the conditions for the existence of the operating hedging and operating leverage effects. In Section 3, we discuss the data sources, variable construction, and model calibration. We study the model's quantitative implications for the gross profitability premium and operating leverage premium, as well as its additional implications including the negative TFP premium and idiosyncratic volatility premium in Section 4. We conclude in Section 5.

2 The Economic Model

Our economy is populated by a large number of profit-maximizing firms. Each firm produces its output (Y) using three inputs: physical capital (K), fixed inputs (A), and variable inputs (M). Physical capital includes properties, plants, and equipments. Examples of fixed costs include sales, general and administrative expenses such as CEO compensation. Variable inputs include all inputs directly used in a firm's production process such as materials, intermediate goods and services, typically reflected in the costs of goods sold (COGS). We assume a constant elasticities of substitution (CES) production. Following the literature on production functions from multiple inputs, we adopt a nested specification by first combining physical capital (K) and fixed inputs (A) to obtain integrated inputs (V) with a constant elasticity of substitution ρ between K and A. We then combine integrated inputs (V) and variable inputs (M) with a constant elasticity of substitution of θ. All firms in the economy are subject to the aggregate profitability shock X.

Specifically, firm i's production function at time t is given by

$$Y_{it} = \left(\left\{ \left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \right\}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} X_t, \tag{1}$$

where U_{it} and Z_{it} represent idiosyncratic productivity shocks to the fixed inputs and variable inputs for firm i, respectively. Let V_{it} denote firm i's integrated inputs by combining physical capital K and fixed inputs A, that is,

$$V_{it} = \left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}. \tag{2}$$

Firm i's output Y_{it} can then be expressed as

$$Y_{it} = \left[V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} X_t. \tag{3}$$

Firms in our economy own physical capital. So firm i aims to maximize its operating profit OP_{it} by choosing variable inputs M_{it} and fixed inputs A_{it}. That is

$$OP_{it} = \max_{\{M_{it}, A_{it}\}} \{Y_{it} - P_M M_{it} - P_A A_{it}\} \tag{4}$$

where P_M and P_A are the prices of variable and fixed inputs, respectively.

The first order conditions are given by

$$\frac{\partial OP_{it}}{\partial M_{it}} \Rightarrow P_M = \left[V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{\theta-1}} Z_{it}^{\frac{\theta-1}{\theta}} M_{it}^{-\frac{1}{\theta}} X_t, \tag{5}$$

$$\frac{\partial OP_{it}}{\partial A_{it}} \Rightarrow P_A = \left[V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{\theta-1}} \left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}} \right]^{\frac{\theta-\rho}{(\rho-1)\theta}} U_{it}^{\frac{\rho-1}{\rho}} A_{it}^{-\frac{1}{\rho}} X_t, \tag{6}$$

and the variable input share ($\frac{P_M M_{it}}{Y_{it}}$) and fixed input share ($\frac{P_A A_{it}}{Y_{it}}$) are

$$\frac{P_M M_{it}}{Y_{it}} = \frac{(Z_{it}M_{it})^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} = \left(\frac{X_t Z_{it}}{P_M}\right)^{\theta-1}, \tag{7}$$

$$\frac{P_A A_{it}}{Y_{it}} = \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{(U_{it}A_{it})^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}}. \tag{8}$$

Note that firm's gross margin (GM) and operating leverage (OL) are related to these two input shares via:

$$GM_{it} = 1 - \frac{P_M M_{it}}{Y_{it}} = 1 - \left(\frac{X_t Z_{it}}{P_M}\right)^{\theta-1}, \tag{9}$$

$$OL_{it} = \frac{P_A A_{it}}{Y_{it} - P_M M_{it}} = \frac{P_A A_{it}}{Y_{it}} \times \frac{1}{GM_{it}} = \frac{P_A A_{it}}{Y_{it}} \times \frac{1}{1 - \frac{P_M M_{it}}{Y_{it}}}. \tag{10}$$

where OL is a flow-based operating leverage measure, defined as the fixed cost divided by gross profit. All else equal, a higher variable input share is associated with lower gross margin. Holding gross margin constant, firms with higher fixed input share have higher operating leverage. Furthermore, the second equality in equation (9) shows that the cross-sectional heterogeneity in gross margin only originates from variable input productivity shock Z. In contrast, both variable input productivity shock Z and fixed input productivity shock U can affect operating leverage (OL).

Plugging equations (7) and (8) into equation (4), we can show that firm i's gross profit GP_{it} and operating profit OP_{it} are given by

$$GP_{it} = Y_{it} - P_M M_{it} = Y_{it} \cdot \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}}, \tag{11}$$

$$OP_{it} = Y_{it} \cdot \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{K_{it}^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}}. \tag{12}$$

The difference between the exposures of gross profits and outputs to the aggregate profitability shock measures the operating hedging effect in Kogan, Li, and Zhang (2019). In the appendix, we show that

$$\frac{\partial \log GP_{it}}{\partial \log X_t} - \frac{\partial \log Y_{it}}{\partial \log X_t} = (\theta - 1)\left(\frac{\partial \log P_M}{\partial \log X_t} - 1\right)\left(\frac{Z_{it}M_{it}}{V_{it}}\right)^{\frac{\theta-1}{\theta}}. \tag{13}$$

This equation indicates that as long as $\theta < 1$ and $\frac{\partial \log P_M}{\partial \log X_t} > 1$, which is empirically confirmed in Kogan, Li, and Zhang (2019), the variable input always reduces the firm's risk exposure. In other words, the operating hedging effect exists regardless if there is fixed inputs in the production function. Furthermore, since $\left(\frac{Z_{it}M_{it}}{V_{it}}\right)^{\frac{\theta-1}{\theta}} = \frac{1}{GM_{it}} - 1$, the strength of operating hedging decreases with gross margin and Z. This result is consistent with the explanation in Kogan, Li, and Zhang (2019) for the gross profitability premium and indicates that the operating hedging drives the profitability premium.

The difference between the exposures of operating profits and gross profits to the aggregate profitability shock captures the operating leverage effect associated with fixed inputs. In the Appendix, we show that

$$\begin{aligned}\frac{\partial \log OP_{it}}{\partial \log X_t} - \frac{\partial \log GP_{it}}{\partial \log X_t} &= (1-\rho)\left(\frac{U_{it}A_{it}}{K_{it}}\right)^{\frac{\rho-1}{\rho}}\left[\left(1 - \frac{\partial \log P_A}{\partial \log X_t}\right) + \left(\frac{Z_{it}M_{it}}{V_{it}}\right)^{\frac{\theta-1}{\theta}}\left(1 - \frac{\partial \log P_M}{\partial \log X_t}\right)\right] \\ &= (1-\rho)\frac{OL_{it}}{1-OL_{it}}\left(\frac{1 - \frac{\partial \log P_M}{\partial \log X_t}}{GM_{it}} + \frac{\partial \log P_M}{\partial \log X_t} - \frac{\partial \log P_A}{\partial \log X_t}\right). \end{aligned} \tag{14}$$

In the special case where there is no variable inputs, i.e., $\left(\frac{Z_{it}M_{it}}{V_{it}}\right)^{\frac{\theta-1}{\theta}} = 0$, when $\rho < 1$ and $\frac{\partial \log P_A}{\partial \log X_t} < 1$, a condition which we verify in the empirical analysis, fixed cost always raises the risk premium of a firm. This is the channel emphasized by Donangelo, Gourio, Kehrig, and Palacios (2018) in explaining the relation between the risk premium and firm's labor leverage. In general, the effect of fixed inputs on the firm's risk exposure depends on $\left(\frac{Z_{it}M_{it}}{V_{it}}\right)^{\frac{\theta-1}{\theta}}$ and hence Z_{it}. For firms with high Z, the first term in the square bracket dominates, so the difference in betas between operating profits, $\frac{\partial \log OP_{it}}{\partial \log X_t}$, and gross profits, $\frac{\partial \log GP_{it}}{\partial \log X_t}$, is positive, corresponding to an operating leverage effect. However, for firms with sufficiently low Z, the second term in the square bracket dominates, the difference in betas between operating profits and gross profits becomes negative. In such cases, fixed costs such as SG&A expenses reduce the firm's risk exposure.

A firm's overall exposure to the aggregate profitability shock combines the effects of variable inputs and fixed inputs. Plugging the expression of Y_{it} from equation (3) and the expression of V_{it} from equation (2) into equation (12), we arrive at a firm's operating profit exposure to the aggregate profitability shock (denoted as β) as follows

$$\beta \equiv \frac{\partial \log OP_{it}}{\partial \log X_t} = \frac{\partial \log P_A}{\partial \log X_t} + \frac{1}{1 - OL_{it}} \left(\frac{1 - \frac{\partial \log P_M}{\partial \log X_t}}{GM_{it}} + \frac{\partial \log P_M}{\partial \log X_t} - \frac{\partial \log P_A}{\partial \log X_t} \right). \tag{15}$$

In our model, β is also the exposure of firm value to the aggregate profitability shock. Eq. 15 has the following implications. First, when the variable input price is strongly procyclical, i.e., $\frac{\partial \log P_M}{\partial \log X_t} > 1$, a firm's beta to the aggregate profitability shock (β) increases in firm's gross margin (GM), holding the firm's operating leverage (OL) constant. Therefore, high profitability firms have higher exposure to the aggregate profitability shock at a given level of operating leverage. This generates a gross profitability premium. Second, the relation between firm exposure to the aggregate profitability shock and operating leverage is more complex and can be increasing or decreasing depending upon the firm's gross margin. For firms with high gross margin, their exposure to the aggregate profitability shock increases in the firm's operating leverage (the term in the parentheses of equation (15) is positive). When firm's gross margin is low (the term in the parentheses of equation (15) becomes negative), firm value exposure to the aggregate profitability shock decreases in the firm's operating leverage.

3 Data and Calibration

In this section, we first describe the sources of data and variable definitions used in our empirical analyses. We then estimate the two elasticities of substitution in the production function in Section 3.2. Lastly, we describe the model calibration in Section 3.3.

3.1 Data and variable definitions

The data used in our analyses come from several sources. Stock return data are from the Center for Research in Security Prices (CRSP) database, and the firm-level accounting data are from the Compustat annual database. We only include stocks with share code (CRSP item SHRCD) of 10 or 11, and exchange code (CRSP item EXCHCD) of 1, 2, or 3. We also exclude firms in the financial industry (SIC between 6000 and 6999) and utility industry (SIC between 4950 and 4999). Our benchmark sample is from July 1963 to December 2016.

Following Novy-Marx (2013), we define gross profitability (GP/A) as the ratio of gross profits (Compustat data item GP) to total asset (Compustat data item AT). Gross margin (GM) measures sales revenue a company retains after incurring the direct costs associated with producing the goods it sells and the services it provides, and is defined as the ratio of gross profits (Compsutat data item GP) to revenues (Compustat item REVT). We measure a firm's operating leverage (OL) as its selling, general, and administrative expenses (Compustat item XSGA) divided by gross profits (Compustat item GP). There are two major differences between our measure and the measure that has been used in Novy-Marx (2010). First, we differentiate cost of goods sold (Compustat item COGS) and SG&A expenses. As discussed in the introduction, these two types of costs have different cyclicality with respect to outputs. Thus, they should be treated differentially in studying their implications for asset prices. The concept of operating leverage is more appropriate for the operating costs that are relatively "sticky" such as SG&A, which is the numerator of our measure. Second, our OL definition is flow-based, and its denominator is gross profit (the item right above SG&A in firm income statement). Again, this choice of denominator is more consistent with the convention that operating leverage is associated with fixed costs driving up the riskiness of cash flows. In contrast, the denominator in the OL definition from Novy-Marx (2010) is total asset.

Table 2 reports the elasticities of operating profits with respect to sales and gross profits for the 10 portfolios sorted on OL. These elasticities are estimated using firm-level Fama-MacBeth cross-sectional regressions within each OL decile. Table 2 shows that the both sales elasticity and gross profit elasticity of operating profits increase monotonically from low to high OL decile, confirming the validity of this flow-based operating leverage measure. In the low OL decile, a 1% increase in revenues (gross profits) is associated with 1.18% (1.12%) increase in operating profits. In contrast, the associated increase in operating profits is more than 4 times larger, at 4.81% and 4.33%, in the high OL decile. Therefore, the SG&A expense substantially raises the volatility and cyclically of operating profits with respect to sales and gross profit fluctuations.

[Insert Table 2 Here]

3.2 Elasticities of substitution among production inputs

As detailed in the appendix, the capital productivity $\frac{Y_{it}}{K_{it}}$ can be written as

$$\log \frac{Y_{it}}{K_{it}} = \frac{\rho}{\rho - 1} \log \left(\frac{Y_{it} - P_M M_{it}}{Y_{it} - P_M M_{it} - P_A A_{it}} \right) + \frac{\theta}{\theta - 1} \log \left(\frac{Y_{it}}{Y_{it} - P_M M_{it}} \right) + \log X_t. \quad (16)$$

Both sides of equation (16) are observable except for the aggregate profitability shock (X), so we can use the cross-sectional relation between variables on both sides of the equation to estimate the elasticity of substitution between K and A (i.e., ρ) and the elasticity of substitution between V and M (i.e., θ). However, because the slope coefficients on the right-hand-side of equation (16) are nonlinear functions of ρ and θ, we rearrange equation (16) into two equations and estimate ρ and θ separately to faciliate the estimation and inference of the distributions for these elasticity parameters.

Specifically, equation (16) can be rewritten as

$$\log\left(\frac{Y_{it}-P_M M_{it}-P_A A_{it}}{Y_{it}-P_M M_{it}}\right)=(1-\rho)\log\left(\frac{Y_{it}-P_M M_{it}-P_A A_{it}}{K_{it}}\right) - \frac{1-\rho}{1-\theta}\log\left(\frac{Y_{it}-P_M M_{it}}{Y_{it}}\right)+(\rho-1)\log X_t, \tag{17}$$

which can be used to directly estimate ρ. Alternatively, equation (16) can be rewritten as

$$\log\left(\frac{Y_{it}-P_M M_{it}}{Y_{it}}\right)=(1-\theta)\log\left(\frac{Y_{it}-P_M M_{it}-P_A A_{it}}{K_{it}}\right) - \frac{1-\theta}{1-\rho}\log\left(\frac{Y_{it}-P_M M_{it}-P_A A_{it}}{Y_{it}-P_M M_{it}}\right)+(\theta-1)\log X_t, \tag{18}$$

and it can be used to estimate θ.

We estimate ρ and θ in two ways. The most straightforward approach is to run cross-sectional regressions on all firms, but this simple procedure ignores any industry heterogeneity. As an alternative approach, we estimate the elasticities of substitution separately for each industry, and then take the average of the industry estimates as our estimates for ρ and θ, respectively. The industry classification we use is based on the GDP by industry account from Bureau of Economic Analysis (BEA). Merging with the firm-level accounting data Compustat, we end up with 14 industries in total.

Table 3 reports the estimated elasticity of substitution coefficients ρ and θ. In Panel A, the estimated elasticity of substitution between physical capital and fixed inputs ρ is 0.37 when all firm observations are used in the estimation. At the same time, the estimated elasticity of substitution between the combined inputs V and variable inputs M, θ, is 0.68. There is a variation in the estimated elasticities across industries. Panel B shows that ρ is low in manufacturing and retail trade industries, with an estimated ρ of about 0.25. The industry with the highest ρ is "Financial activities", whose ρ is about 0.92. On the other hand, the estimated θ ranges between 0.43 for the manufacturing industry to 1.03 for "Financial activities". The average elasticity of substitution ρ and θ are 0.48 and 0.71, respectively, across industries, which are slightly higher but close to the estimates based on all firms. Overall, these elasticities of substitution estimates suggest that there are more flexibility in variable inputs than other inputs in firm production.

[Insert Table 3 Here]

Along with the fact that variable input price is highly procyclical ($\frac{\partial \log P_M}{\partial \log X_t} > 1$) and fixed inputs price is relatively sticky ($\frac{\partial \log P_A}{\partial \log X_t} < 1$), ρ and θ between 0 and 1 are the two necessary conditions for our model to have operating leverage and operating hedging effects. In addition, the smaller elasticity of substitution between physical capital (K) and organization inputs (A) relative to that between the combined input (V) and variable inputs (M) suggests that the combinatory use of physical capital and fixed inputs is less flexible than the variable inputs usage.

3.3 Model calibration

In this subsection, we describe the model calibration. Table 4 reports the parameter values in our benchmark calibration at the annual frequency.

[Insert Table 4 Here]

We set the elasticities of substitution between physical capital and fixed inputs, ρ, to 0.47, and between the combined input and variable inputs, θ, to 0.74, respectively. These values are consistent with the empirical estimates from the previous subsection. We assume input prices P_M and P_A to have a constant elasticity with respect to the aggregate profitability shock and specify them as

$$\log P_M = \log P_M^0 + P_M^1 \log X, \tag{19}$$

$$\log P_A = \log P_A^0 + P_A^1 \log X, \tag{20}$$

where P_j^0, $j = A,\ M$, captures the level of input prices and P_j^1, $j = A,\ M$, measures their elasticities with respect to X. We set P_A^1 to 0.45 and P_M^1 to 1.39 to match the empirically estimated elasticity of aggregate SG&A and COGS with respect to the aggregate revenue, taking into account the ability of the model to match intended variable moments. The aggregate profitability shock is assumed to take three values, x_{min}, $(x_{max} + x_{min})/2$, and x_{max}, with equal probability. The parameters x_{min}, x_{max}, along with P_A^0, and P_M^0, jointly determine the level and volatility of aggregate GP/A, the average SG&A-to-revenue ratio, and the average COGS-to-revenue ratio. We set these parameters to be 1.91 and 1.93 for P_A^0, and 0.26 and 0.44 for P_M^0, respectively. The firm-level productivity shocks to fixed inputs (u) and variable inputs (z) are drawn from normal distributions $\mathrm{N}(\mu_u, \sigma_u^2)$, $\mathrm{N}(\mu_z, \sigma_z^2)$, and we set their means and standard deviations to match the cross-sectional distribution of gross profitability, operating leverage, and gross margin as close as possible. Finally, we choose the risk premium for the aggregate profitability shock λ to match the equity premium.

4 Results and Discussions

We solve the firm's value maximization problem, i.e., equation (4), numerically. In Section 4.1, we show the firm's optimal policies on production inputs, profitability, operating leverage, and value function. We discuss the model's asset pricing implications using portfolio sorts in Section 4.2. We simulate 2,000 firms at each level of aggregate profitability shock, and use the model-implied expected return ($\beta \times \lambda$) to measure average return.

4.1 Value and policy functions

Figure 2 plots the the firm's optimal fixed input (A) and variable input (M), gross profitability (GP/A), operating leverage (OL), gross margin (GM), and operating profitability (OP/A), against the firm-level productivity of fixed inputs (U) and variable inputs (Z).

[Insert Figure 2 Here]

The top left and top middle panels of Figure 2 show that the firm's optimal fixed inputs and variable inputs both increase with the productivity of variable inputs (Z). However, the relation between the firm's optimal production inputs and the fixed input productivity (U) is more complex. While there is always a positive relation between the variable inputs (M) and the fixed input productivity (U), the relation between the optimal fixed inputs (A) and the fixed input productivity (U) depends upon the level of the variable input productivity Z. When the variable input productivity Z is low, the optimal fixed input A increases with fixed input productivity (U). At high level of the variable input productivity, the fixed inputs A decreases in the fixed input productivity U. More generally, the relation between the fixed inputs A and the fixed input productivity U can be non-monotonic.

The top right and bottom left panels of Figure 2 plots how a firm's gross profitability (GP/A) and operating leverage (OL), respectively, vary with the variable input productivity Z and the fixed input productivity U. While firm gross profitability is mostly driven by the idiosyncratic variable input productivity Z, a firm's operating leverage is affected by both its variable input productivity Z and fixed input productivity U. Firms with both low variable input and fixed input productivities have high operating leverage. The bottom middle panel of Figure 2 confirms equation (9) that a firm's gross margin only depends on its productivity on variable inputs (Z). Therefore, under the benchmark calibration, gross profitability and gross margin are strongly correlated. The bottom right panel plots the operating profitability (the firm value in our economy) against these two idiosyncratic productivities. Despite a similar pattern to that of the gross profitability (top right panel), we find operating profitability (OP/A) shows stronger relation to the fixed input productivity (U) than gross profitability (GP/A).

An important question for asset pricing is how the risk premium varies across firms. Given the focus of our study, we are particularly interested in the relation of a firm's risk premium to its gross profitability and operating leverage. The top panel of Figure 3 shows the relation of the firm's aggregate profitability shock exposure to the fixed input productivity U and the variable input productivity Z. We find that the firm's exposure to the aggregate profitability shock monotonically increases in its variable input productivity Z. In the meantime, the firm's aggregate profitability shock exposure increases in the fixed input productivity U when the variable input productivity Z is low, but the relation reserves when the firm's variable input productivity Z is high.

[Insert Figure 3 Here]

More important, when we plot the firm's aggregate profitability shock exposure against the firm's gross profitability (GP/A) and operating leverage (OL) in the bottom panel of Figure 3, the following patterns emerge. First, the firm's risk exposure to the aggregate profitability shock increases in firm's gross profitability at all levels of the firm's operating leverage. Therefore, our model predicts a positive gross profitability premium. In contrast, the relation between risk exposure and operating leverage depends on gross profitability. Specifically, the firm's risk exposure decreases in firm's operating exposure at low level of firm's profitability, and only slightly increases at high level of firm's profitability, which is consistent with equation (15). The patterns in the risk premiums along these two dimensions also match those for the average returns from Figure 1. We test these predictions using characteristic-sorted portfolios in the next section.

4.2 Portfolio sorts

4.2.1 Gross profitability premium

In this subsection, we examine the relation between the gross profitability premium and the operating hedging effect. We sort stocks into decile portfolios based on their gross profitability (GP/A), and report their characteristics and average returns. Table 5 presents our findings. Panel A is based on the empirical data and Panel B is from the simulated data. In the empirical data, we observe a large cross-sectional dispersion in the gross profitability. The average GP/A is 0.11 for low profitability firms, as compared to 0.91 for high profitability firms. Our model reproduces this large dispersion, and the average GP/A increases from

0.05 for low profitability firms to 0.66 for high profitability firms based on the simulated data from the model.

Our model also generates a positive correlation between the gross margin (GM) and the gross profitability (GP/A). The difference in the gross margin between high and low profitability stocks is 0.2 in the model, very close to the difference of 0.17 observed in the empirical data. Equation (9) and Figure 2 show that both GM and GP/A are mostly driven by the idiosyncratic productivity on variable inputs z, which we confirm in Panel B of Table 5. While the gross profitability increases monotonically in z, the idiosyncratic productivity on fixed inputs u is hump-shaped across portfolios of different gross profitability.

The last row of each panel reports the average excess returns of gross profitability portfolios. Consistent with the large gross profitability premium documented in the literature, our model generates a gross profitability premium of 6.12% per year. This is close to the gross profitability premium of 5.74% (t-statistic = 2.68) observed in the data. Because the exposure of the gross profitability portfolio to the aggregate profitability shock is non-monotonic in OL and monotonically increasing in GP/A, the profitability premium is driven by the variable input hedging effect, and not by the operating leverage effect.

[Insert Table 5, Here]

4.2.2 Operating leverage premium

In this subsection, we examine the implication of a firm's operating leverage for its risk premium. Table 6 reports the results for decile portfolios sorted on operating leverage (OL). The difference in OL between low OL and high OL portfolios is 0.48, which is smaller than 1.38 in the empirical data. This divergence may reflect firm's dynamic considerations in reality. In our static model, a firm would not choose a large operating cost so that its operating income becomes negative. In dynamic models with fixed inputs accumulation, however, firms can trade off current operating profits for future operating profits to maximize firm value. In such models, operating leverage can be great than one.

[Insert Table 6 Here]

In our model, gross margin (GM) decreases from low to high OL portfolios. In the data, the difference in gross margin between high OL and low OL portfolios is also negative, but the overall pattern of gross margin in OL is non-monotonic and exhibits a hump-shape. Both idiosyncratic productivity u and z have large effects on the operating leverage in the model. Across OL decile portfolios, the average z decreases from 3.17 to 1.91, and the average u decreases from 2.02 to 1.51. More important, our model replicates the hump-shaped relation between average returns and the operating leverage. In the data, the average return increases from 5.36% in low OL decile to 8.96% in decile 7, and then falls to 0.5% in decile 10, giving rise to an OL premium of −4.85%. In our model, the average return increases from 7.42% to 7.9% initially and then decreases to 4.86% in decile 10, so our model generates an OL premium of −2.56%.

4.2.3 Double sorts on gross profitability and operating leverage

To demonstrate how the interaction between gross profitability and operating leverage affects risk premium, we double sort stocks into 5-by-5 portfolios based on their GP/A and OL. Table 7 reports the average excess returns for these double-sorted portfolios as well as the long-short portfolios in each dimension. We observe the following two interesting patterns in the data, as reported in Panel A. First, the sign of OL premium depends on the gross profitability. At low levels of gross profitability, the average return is 4.63% for low OL stocks and −3.1% for high OL stocks. The low operating stocks thus earn 7.73% higher return than that for high operating leverage stocks. However, this relation is reversed at high levels of gross profitability. The average return for low OL stocks is 7.88% and 10.74% for high OL stocks. High operating leverage stocks thus earn 2.86% higher average return than that for low operating leverage stocks. Second, the gross profitability premium is stronger among high OL stocks. In low levels of operating leverage, the gross profitability premium is only 3.25% per year and statistically insignificant from zero. In contrast, among stocks with high operating leverage, the gross profitability premium is 13.84% per year.

[Insert Table 7 Here]

Our production-based model replicates these patterns. Panel B of Table 7 shows that although the economical magnitudes are smaller, the average OL premium is initially negative at −1.36% per year among low GP/A stocks, but becomes positive at 0.78% per year among high GP/A stocks. This change in signs of the OL premium confirms the prediction in Eq. 10 and Eq. 15 that the effect of operating leverage on risk premium varies with the gross profitability and gross margin. Unlike the literature focusing exclusively on how operating leverage increases risk premium, we find that the procyclical variable inputs can change this relation, especially at low levels of profitability. On the other hand, the GP/A premium is 3.26% among low OL stocks, much smaller than 5.4% among high OL stocks.

4.2.4 Firm-level TFP premium

İmrohoroğlu and Tüzel (2014) document a negative firm-level total factor productivity (TFP) premium. Estimating the firm-level TFP as the Solow residuals from the cross-sectional relation between value-added, capital stock, and labor inputs, they find that stocks with low TFP earn higher average returns than stocks with high TFP. They attribute this firm-level TFP premium to firm operating leverage (e.g., Carlson, Fisher, and Giammarino (2004) and Zhang (2005)). Compared with firms with high TFP, firms with low TFP have higher operating leverage, thus higher risk and earn higher expected returns. However, this finding seems to be at odds with the positive gross profitability premium in Novy-Marx (2013). This is because more productive firms have higher profitability.

We empirically confirm the finding by İmrohoroğlu and Tüzel (2014) in Panel A of Table 8. The average returns of low TFP stocks is 6.51% and the average returns for high TFP stocks is 5.23%. The return difference is −1.28% albeit statistically insignificant from zero in our sample. In terms of characteristics, high TFP stocks indeed have higher gross profitability and higher gross margin than low TFP stocks. The average GP/A is 0.22 for low GP/A stocks, as compared with 0.35 for high GP/A stocks. On the other hand, TFP and operating leverage are negatively correlated, with the average OL almost doubled among stocks with low TFP (OL=0.82) than stocks with high TFP (OL=0.43), which is in line with the operating leverage interpretation in İmrohoroğlu and Tüzel (2014).

[Insert Table 8 Here]

Our model qualitatively reproduces these results. Since physical capital is fixed in the model, we estimate the model counterpart of the firm-level TFP. This is accomplished by running cross-sectional regressions of the logarithm of gross profits (i.e., value added) on the logarithm of fixed costs. As reported in Panel B of Table 8, the average firm-level TFP premium in the simulated data is −1.69% per year. Our model is thus capable of generating the coexistence of a positive gross profitability premium and a negative TFP premium. Portfolio characteristics provide hints on the underlying mechanism for the reconciliation of these two premiums. While the variable input productivity z modestly increases with TFP, giving rise to a positive correlation between GP/A (and GM) and TFP, TFP sorts create a large cross-sectional dispersion in the fixed input productivity u. In our benchmark calibration, the premium on u is negative due to the operating leverage effect, so the model predicts a negative TFP premium. Taken together, although GP/A and TFP are positively correlated in our model, their premiums originate from different sources of firm-level productivity shocks. While GP/A mostly captures the variable inputs productivity z, TFP is mainly driven by the fixed input productivity u.

4.2.5 Idiosyncratic volatility premium

Another widely studied return anomaly is the idiosyncratic volatility (IVOL) premium (see Ang, Hodrick, Xing, and Zhang (2006)). Ang, et. al. (2006) compute the idiosyncratic volatility using daily stock returns in the previous month controlling for standard factors including the market, the value premium factor, and the size premium factor (Fama and French (1992)). They report a negative relation between stock excess returns and idiosyncratic volatility of these stocks. We replicate their results in Panel A of Table 9 in the sample period between July 1963 and December 2016. The average return of low IVOL stocks is 6.09%, as compared with −2.70% in high IVOL stocks. The difference is more than 8% per year and statistically significant at the 5% level. High IVOL stocks have low average gross margin of 0.28 but high operating leverage of 0.75. In contrast, low IVOL stocks have a higher average gross margin of 0.34 and a lower operating leverage of 0.5.

[Insert Table 9 Here]

Panel B of Table 9 reports the results from our model. We compute a firm's IVOL as $\sqrt{\beta_z^2\sigma_z^2 + \beta_u^2\sigma_u^2}$, where β_z and β_u are the exposures of firm value to z and u, respectively. Panel B shows that our model reproduces a negative and sizable IVOL return spread. Consistent with the pattern in the empirical data, stocks with low IVOL have a high average return of 7.6% per year, while high IVOL stocks have a low average return of 2.29%. In addition, IVOL is positively correlated with operating leverage but negatively correlated with gross margin. Both findings are consistent with the empirical evidence in the data. Examining the pattern of z and u across IVOL portfolios, we find that the IVOL premium is mostly driven by high IVOL stocks having lower idiosyncratic variable input productivity z than low IVOL stocks. A low z is associated with a stronger operating hedging effect with respect to the aggregate profitability shock and hence a higher risk premium. In the meanwhile, a low z is also related to a greater operating leverage effect with respect to the idiosyncratic productivity shocks. The joint effects of operating hedging and operating leverage give rise to the negative relation between idiosyncratic risk (IVOL) and systematic risk.

To further examine the plausibility of the above mechanism in the empirical data, we run the factor spanning tests and examine the explanatory power of the gross profitability premium and operating leverage premium on the idiosyncratic volatility premium. Our production-based model predicts that the idiosyncratic volatility premium should have a negative exposure to the gross profitability premium and a positive exposure to the operating leverage premium. More important, the abnormal return should disappear after controlling for the gross profitability premium and operating leverage premium. Table 10 reports the test results. Specification (2) is for the univariate time series regression of IVOL premium on GP/A premium. We observe a strong negative coefficient on the GP/A premium (−0.79) with a t-statistic of −12.14. In addition, controlling for the GP/A premium reduces the magnitude IVOL premium from 8.82% per year (Table 9) to 4.2% per year, and the GP/A premium alone explains 52% of the IVOL premium. In Specification (3), we run spanning test of the IVOL premium on the OL premium. The coefficient of the OL premium is 0.75 with a t-statistic of 19.51, and the OL premium explains 38% of the IVOL premium. Specification (4) includes both GP/A premium and OL premium. These two premiums together explain almost half of the time series variation in the IVOL premium ($R^2 = 48.8\%$), and the abnormal return of the IVOL premium further shrinks to −1.87% per year and becomes statistically insignificant. The spanning tests therefore provide compelling evidence for our economic mechanism for the idiosyncratic volatility premium. This result is noteworthy because idiosyncratic volatility is based on stock return data, whereas the gross profitability and operating leverage are constructed from accounting data from financial statements.

[Insert Table 10 Here]

5 Conclusion

We introduce both fixed inputs and variable inputs into a nested production function to study the joint effects of two different types of production inputs on asset pricing. The former is "sticky" in firm operation, leading to an operating leverage effect. The latter shows strong procyclicality and thus creates an operating hedging. We find that operating hedging due to variable inputs reduces firms' exposure to aggregate profitability shocks, leading to a lower risk premium. The effect of operating leverage however depends on firm's gross margin. When gross margin is high, operating leverage increases the risk premium of a firm, a channel that has been widely studied in the literature. However, when gross margin is sufficiently low, operating leverage is negatively related to risk premium. Our results therefore call for cautions of the studies that examine the operating leverage effect while ignoring the operating hedging.

We examine the asset pricing implication of our model using portfolio sorts. Both in the data and in the model, we find a strong positive gross profitability premium and a hump-shaped operating leverage premium. In the portfolios double sorted on gross profitability and operating leverage, we find operating leverage premium changes sign from negative to positive as we increase gross profitability. In the meanwhile, the gross profitability premium is significantly stronger among high operating leverage stocks. Our model reconciles the coexistence of the positive profitability premium and negative TFP premium, two seemingly contradictory phenomena in the cross-sectional stock returns. We also offer a novel explanation for the negative relation between idiosyncratic volatility and future stock returns based on the joint effect of operating hedging and leverage from different production costs. The results from the factor spanning tests provide strong support for this explanation.

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Appendix

First order conditions

Specifically, firm i's production function is given by

$$Y_{it} = \left(\left\{ \left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \right\}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} X_t \tag{A.1}$$

Where U_{it} and Z_{it} represent idiosyncratic productivity shocks to the fixed inputs and variable inputs, respectively. Let V_{it} to denote the integrate capital by combining physical capital K and fixed inputs A, we have

$$V_{it} = \left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \tag{A.2}$$

Firm i's output Y_{it} can then be expressed as

$$Y_{it} = \left[V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} X_t \tag{A.3}$$

Firm i maximizes its operating profit OP_{it} by choosing fixed inputs A_{it} and variable inputs M_{it}. That is

$$OP_{it} = \max_{\{M_{it}, A_{it}\}} \{Y_{it} - P_M M_{it} - P_A A_{it}\} \tag{A.4}$$

The first order conditions are given by

$$\frac{\partial OP_{it}}{\partial M_{it}} = \left[V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}-1} X_t Z_{it}^{\frac{\theta-1}{\theta}} M_{it}^{\frac{\theta-1}{\theta}-1} - P_M = 0$$

$$\Rightarrow P_M = \left[V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{\theta-1}} Z_{it}^{\frac{\theta-1}{\theta}} M_{it}^{-\frac{1}{\theta}} X_t \tag{A.5}$$

$$\frac{\partial OP_{it}}{\partial A_{it}} = \left[V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}-1} X_t V_{it}^{\frac{\theta-1}{\theta}-1} \left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}-1} U_{it}^{\frac{\rho-1}{\rho}} A_{it}^{\frac{\rho-1}{\rho}-1} - P_A = 0$$

$$\Rightarrow P_A = \left[V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right]^{\frac{1}{\theta-1}} \left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}} \right]^{\frac{\theta-\rho}{(\rho-1)\theta}} U_{it}^{\frac{\rho-1}{\rho}} A_{it}^{-\frac{1}{\rho}} X_t \tag{A.6}$$

Capital productivity $\dfrac{Y_{it}}{K_{it}}$

Multiplying both sides of equation (A.5) by $\dfrac{M_{it}}{Y_{it}}$ yields

$$\frac{P_M M_{it}}{Y_{it}} = \frac{(Z_{it}M_{it})^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} = \frac{\left(\frac{Z_{it}M_{it}}{V_{it}}\right)^{\frac{\theta-1}{\theta}}}{1+\left(\frac{Z_{it}M_{it}}{V_{it}}\right)^{\frac{\theta-1}{\theta}}} \tag{A.7}$$

Multiplying both sides of equation (A.6) by $\dfrac{A_{it}}{Y_{it}}$ yields

$$\begin{aligned}\frac{P_A A_{it}}{Y_{it}} &= \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{(U_{it}A_{it})^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}} \\ &= \frac{1}{1+\left(\frac{Z_{it}M_{it}}{V_{it}}\right)^{\frac{\theta-1}{\theta}}} \cdot \frac{\left(\frac{U_{it}A_{it}}{K_{it}}\right)^{\frac{\rho-1}{\rho}}}{1+\left(\frac{U_{it}A_{it}}{K_{it}}\right)^{\frac{\rho-1}{\rho}}}\end{aligned} \tag{A.8}$$

From equation (A.7) we have

$$\left(\frac{Z_{it}M_{it}}{V_{it}}\right)^{\frac{\theta-1}{\theta}} = \frac{P_M M_{it}}{Y_{it} - P_M M_{it}} \tag{A.9}$$

Plugging equation (A.9) into equation (A.8) gives

$$\left(\frac{U_{it}A_{it}}{K_{it}}\right)^{\frac{\rho-1}{\rho}} = \frac{P_A A_{it}}{Y_{it} - P_M M_{it} - P_A A_{it}} \tag{A.10}$$

Equation (A.1) implies that the capital productivity $\dfrac{Y_{it}}{K_{it}}$ is

$$\begin{aligned}\frac{Y_{it}}{K_{it}} &= \left\{\left[1+\left(\frac{U_{it}A_{it}}{K_{it}}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\cdot\frac{\theta-1}{\theta}} + \left(\frac{Z_{it}M_{it}}{K_{it}}\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}} X_t \\ &= \left\{\left[1+\left(\frac{U_{it}A_{it}}{K_{it}}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\cdot\frac{\theta-1}{\theta}} + \left(\frac{Z_{it}M_{it}}{V_{it}}\right)^{\frac{\theta-1}{\theta}}\left(\frac{V_{it}}{K_{it}}\right)^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}} X_t\end{aligned} \tag{A.11}$$

Since equation (A.2) can also be expressed in per unit of capital term, that is,

$$\frac{V_{it}}{K_{it}} = \left[1+\left(\frac{U_{it}A_{it}}{K_{it}}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{A.12}$$

Plugging equation (A.12) into equation (A.11) gives

$$\begin{aligned}
\frac{Y_{it}}{K_{it}} &= \left\{ \left[1 + \left(\frac{U_{it}A_{it}}{K_{it}} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}\cdot\frac{\theta-1}{\theta}} + \left(\frac{Z_{it}M_{it}}{V_{it}} \right)^{\frac{\theta-1}{\theta}} \left[1 + \left(\frac{U_{it}A_{it}}{K_{it}} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}\cdot\frac{\theta-1}{\theta}} \right\}^{\frac{\theta}{\theta-1}} X_t \\
&= \left\{ \left[1 + \left(\frac{U_{it}A_{it}}{K_{it}} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}\cdot\frac{\theta-1}{\theta}} \left[1 + \left(\frac{Z_{it}M_{it}}{V_{it}} \right)^{\frac{\theta-1}{\theta}} \right] \right\}^{\frac{\theta}{\theta-1}} X_t \\
&= \left[1 + \left(\frac{U_{it}A_{it}}{K_{it}} \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \left[1 + \left(\frac{Z_{it}M_{it}}{V_{it}} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} X_t
\end{aligned} \tag{A.13}$$

Plugging equations (A.9) and (A.10) into equation (A.13) gives

$$\frac{Y_{it}}{K_{it}} = \left(\frac{Y_{it} - P_M M_{it}}{Y_{it} - P_M M_{it} - P_A A_{it}} \right)^{\frac{\rho}{\rho-1}} \left(\frac{Y_{it}}{Y_{it} - P_M M_{it}} \right)^{\frac{\theta}{\theta-1}} X_t \tag{A.14}$$

Exposure of firm inputs to aggregate profitability shock

The production function is augmented by three inputs K_{it}, A_{it}, and M_{it}. K_{it} is fixed in the model, so we have

$$\frac{\partial \log K_{it}}{\partial \log X_{it}} = 0 \tag{A.15}$$

$\frac{\partial \log A_{it}}{\partial \log X_t}$ and $\frac{\partial \log M_{it}}{\partial \log X_t}$ can be solved from taking partial derivative of the logarithm of both sides of equations (A.5) and (A.6), that is,

$$\frac{\partial \log P_M}{\partial \log X_t} = \frac{1}{\theta - 1} \frac{\partial \log \left(V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right)}{\partial \log X_t} - \frac{1}{\theta} \frac{\partial \log M_{it}}{\partial \log X_t} + 1 \tag{A.16}$$

$$\frac{\partial \log P_A}{\partial \log X_t} = \frac{1}{\theta - 1} \frac{\partial \log \left(V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right)}{\partial \log X_t} + \frac{\theta - \rho}{(\rho - 1)\theta} \frac{\partial \log \left(K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}} \right)}{\partial \log X_t} - \frac{1}{\rho} \frac{\partial \log A_{it}}{\partial \log X_t} + 1 \tag{A.17}$$

We have that

$$\frac{\partial \log \left(K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}} \right)}{\partial \log X_t} = \frac{\frac{\rho-1}{\rho} U_{it}^{\frac{\rho-1}{\rho}} A_{it}^{\frac{\rho-1}{\rho}-1} \frac{\partial A_{it}}{\partial \log X_t}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}}$$

$$= \frac{\rho-1}{\rho} \cdot \frac{(U_{it}A_{it})^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}} \cdot \frac{\partial \log A_{it}}{\partial \log X_t} \tag{A.18}$$

and that

$$\frac{\partial \log \left(V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right)}{\partial \log X_t} = \frac{\frac{\theta-1}{\theta} V_{it}^{\frac{\theta-1}{\theta}-1} \frac{\partial V_{it}}{\partial \log X_t} + \frac{\theta-1}{\theta} Z_{it}^{\frac{\theta-1}{\theta}} M_{it}^{\frac{\theta-1}{\theta}-1} \frac{\partial M_{it}}{\partial \log X_t}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}}$$

$$= \frac{\theta-1}{\theta} \left[\frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{\partial \log V_{it}}{\partial \log X_t} + \frac{(Z_{it}M_{it})^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{\partial \log M_{it}}{\partial \log X_t} \right] \tag{A.19}$$

Further note that

$$\frac{\partial \log V_{it}}{\partial \log X_t} = \frac{\left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}-1} U_{it}^{\frac{\rho-1}{\rho}} A_{it}^{\frac{\rho-1}{\rho}-1} \frac{\partial A_{it}}{\partial \log X_t}}{\left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}}$$

$$= \frac{(U_{it}A_{it})^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}} \cdot \frac{\partial \log A_{it}}{\partial \log X_t} \tag{A.20}$$

Bringing back equation (A.20) to equation (A.19) gives

$$\frac{\partial \log \left(V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}} \right)}{\partial \log X_t} = \frac{\theta-1}{\theta} \left[\frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{(U_{it}A_{it})^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}} \cdot \frac{\partial \log A_{it}}{\partial \log X_t} \right.$$
$$\left. + \frac{(Z_{it}M_{it})^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{\partial \log M_{it}}{\partial \log X_t} \right] \tag{A.21}$$

Plugging equations (A.18) and (A.21) into the equation system (A.16) and (A.17) yields the following equation system

$$\frac{\partial \log P_M}{\partial \log X_t} - 1 = \frac{1}{\theta} \cdot \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{(U_{it}A_{it})^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}} \cdot \frac{\partial \log A_{it}}{\partial \log X_t} - \frac{1}{\theta} \cdot \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{\partial \log M_{it}}{\partial \log X_t} \tag{A.22}$$

$$\frac{\partial \log P_M}{\partial \log X_t} - \frac{\partial \log P_A}{\partial \log X_t} = \left[\frac{1}{\rho} \cdot \frac{K_{it}^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}} + \frac{1}{\theta} \cdot \frac{(U_{it}A_{it})^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}}\right] \frac{\partial \log A_{it}}{\partial \log X_t} - \frac{1}{\theta} \cdot \frac{\partial \log M_{it}}{\partial \log X_t} \tag{A.23}$$

Considering equations (A.7) and (A.8), we can simplify notations in equations (A.22) and (A.23) by introducing the following expressions for the gross profit margin GM_{it} and the firm operating leverage OL_{it}, respectively,

$$\frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} = \frac{Y_{it} - P_M M_{it}}{Y_{it}} = GM_{it} \tag{A.24}$$

$$\frac{(U_{it}A_{it})^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}} = \frac{P_A A_{it}}{Y_{it} - P_M M_{it}} = OL_{it} \tag{A.25}$$

Let $\beta_M = \frac{\partial \log P_M}{\partial \log X_t}$ and $\beta_A = \frac{\partial \log P_A}{\partial \log X_t}$ represent the variable input price elasticity and the fixed input price elasticity to the aggregate profitability shock, respectively. The solution to the equation system (A.22) and (A.23) can be written as

$$\begin{aligned}\frac{\partial \log A_{it}}{\partial \log X_t} &= \frac{\rho\left[\left(1 - \frac{\partial \log P_M}{\partial \log X_t}\right) - \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}}\left(\frac{\partial \log P_A}{\partial \log X_t} - \frac{\partial \log P_M}{\partial \log X_t}\right)\right]}{\frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{K_{it}^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}}} \\ &= \frac{\rho[(1-\beta_M) - GM_{it}(\beta_A - \beta_M)]}{GM_{it}(1 - OL_{it})}\end{aligned} \tag{A.26}$$

$$\begin{aligned}\frac{\partial \log M_{it}}{\partial \log X_t} &= \frac{\frac{\rho(U_{it}A_{it})^{\frac{\rho-1}{\rho}} + \theta K_{it}^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}}\left(1 - \frac{\partial \log P_M}{\partial \log X_t}\right) - \rho \cdot \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{(U_{it}A_{it})^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}}\left(\frac{\partial \log P_A}{\partial \log X_t} - \frac{\partial \log P_M}{\partial \log X_t}\right)}{\frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{K_{it}^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}}} \\ &= \frac{[\rho OL_{it} + \theta(1 - OL_{it})](1-\beta_M) - \rho GM_{it} OL_{it}(\beta_A - \beta_M)}{GM_{it}(1 - OL_{it})}\end{aligned} \tag{A.27}$$

Therefore, we have equations (A.15), (A.26), and (A.27) to be the exposures of physical inputs

K_{it}, fixed inputs A_{it}, and variable inputs M_{it}, respectively, to the aggregate profitability shock.

Exposure of operating profit to aggregate profitability shock

Plugging equations (A.7) and (A.8) into equation (A.4), operating profit OP_{it} can be written as

$$\begin{aligned} OP_{it} &= \max_{\{M_{it},A_{it}\}} \{Y_{it} - P_M M_{it} - P_A A_{it}\} \\ &= \max_{\{M_{it},A_{it}\}} \left\{ Y_{it}\left(1 - \frac{P_M M_{it}}{Y_{it}} - \frac{P_A A_{it}}{Y_{it}}\right)\right\} \\ &= Y_{it}\left[1 - \frac{(Z_{it}M_{it})^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} - \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{(U_{it}A_{it})^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}}\right] \\ &= Y_{it}\left[\frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} - \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{(U_{it}A_{it})^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}}\right] \\ &= Y_{it} \cdot \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{K_{it}^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}} \end{aligned} \tag{A.28}$$

Plugging the expression of Y_{it} from equation (A.3) and the expression of V_{it} from equation (A.2) into equation (A.28) gives

$$\begin{aligned} OP_{it} &= \frac{\left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}\cdot\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}} \cdot \frac{K_{it}^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}} \cdot \left[V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} X_t \\ &= \left[V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}\right]^{\frac{1}{\theta-1}} \left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}\right]^{\frac{\theta-\rho}{(\rho-1)\theta}} K_{it}^{\frac{\rho-1}{\rho}} X_t \end{aligned} \tag{A.29}$$

With equation (A.6) of P_A, we can further simplify equation (A.29) as

$$\begin{aligned} OP_{it} &= \underbrace{\left\{\left[V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it}M_{it})^{\frac{\theta-1}{\theta}}\right]^{\frac{1}{\theta-1}} \left[K_{it}^{\frac{\rho-1}{\rho}} + (U_{it}A_{it})^{\frac{\rho-1}{\rho}}\right]^{\frac{\theta-\rho}{(\rho-1)\theta}} U_{it}^{\frac{\rho-1}{\rho}} A_{it}^{-\frac{1}{\rho}} X_t\right\}}_{=P_A} U_{it}^{\frac{1-\rho}{\rho}} A_{it}^{\frac{1}{\rho}} K_{it}^{\frac{\rho-1}{\rho}} \\ &= P_A A_{it}^{\frac{1}{\rho}} K_{it}^{\frac{\rho-1}{\rho}} U_{it}^{\frac{1-\rho}{\rho}} \end{aligned} \tag{A.30}$$

Taking partial derivative of the logarithm of both sides of equation (A.30) with respect to $\log X_t$ yields

$$\frac{\partial \log OP_{it}}{\partial \log X_t} = \frac{\partial \log P_A}{\partial \log X_t} + \frac{1}{\rho} \cdot \frac{\partial \log A_{it}}{\partial \log X_t} + \frac{\rho-1}{\rho} \cdot \frac{\partial \log K_{it}}{\partial \log X_t} \tag{A.31}$$

Plugging equations (A.15) and (A.26) into equation (A.31), we arrive at a firm's operating profit exposure to the aggregate profitability shock as follows

$$\frac{\partial \log OP_{it}}{\partial \log X_t} = \beta_A + \frac{1}{1 - OL_{it}} \left(\frac{1 - \beta_M}{GM_{it}} + \beta_M - \beta_A \right) \tag{A.32}$$

Conditions for operating hedging

We can get the following expression for gross profit GP_{it} from equations (A.7) and (A.24),

$$GP_{it} = Y_{it} - P_M M_{it} = Y_{it} GM_{it} = Y_{it} \cdot \frac{V_{it}^{\frac{\theta-1}{\theta}}}{V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it} M_{it})^{\frac{\theta-1}{\theta}}} \tag{A.33}$$

Rearranging accounting variables to the left-hand-side of equation (A.33) and taking partial derivative of the logarithm of both sides of the equation with respect to $\log X_t$ yields

$$\frac{\partial \log GP_{it}}{\partial \log X_{it}} - \frac{\partial \log Y_{it}}{\partial \log X_{it}} = \frac{\theta - 1}{\theta} \cdot \frac{\partial \log V_{it}}{\partial \log X_{it}} - \frac{\partial \log \left(V_{it}^{\frac{\theta-1}{\theta}} + (Z_{it} M_{it})^{\frac{\theta-1}{\theta}} \right)}{\partial \log X_t} \tag{A.34}$$

Plugging equations (A.20), (A.21), (A.24), (A.26), and (A.27) to equation (A.34), we have

$$\begin{aligned} \frac{\partial \log GP_{it}}{\partial \log X_t} - \frac{\partial \log Y_{it}}{\partial \log X_t} &= (\theta - 1) \left(\frac{\partial \log P_M}{\partial \log X_t} - 1 \right) \left(\frac{Z_{it} M_{it}}{V_{it}} \right)^{\frac{\theta-1}{\theta}} \\ &= (\theta - 1)(\beta_M - 1) \frac{1 - GM_{it}}{GM_{it}} \end{aligned} \tag{A.35}$$

Conditions for operating leverage

Plugging equation (A.33) into equation (A.28) gives the following expression for operating profit OP_{it},

$$OP_{it} = GP_{it} \cdot \frac{K_{it}^{\frac{\rho-1}{\rho}}}{K_{it}^{\frac{\rho-1}{\rho}} + (U_{it} A_{it})^{\frac{\rho-1}{\rho}}} \tag{A.36}$$

Rearranging accounting variables to the left-hand-side of equation (A.36) and taking partial derivative of the logarithm of both sides of the equation with respect to $\log X_t$ yields

$$\frac{\partial \log OP_{it}}{\partial \log X_t} - \frac{\partial \log GP_{it}}{\partial \log X_t} = \frac{\rho - 1}{\rho} \cdot \frac{\partial \log K_{it}}{\partial \log X_t} - \frac{\partial \log \left(K_{it}^{\frac{\rho-1}{\rho}} + (U_{it} A_{it})^{\frac{\rho-1}{\rho}} \right)}{\partial \log X_t} \tag{A.37}$$

Plugging equations (A.15), (A.18), (A.24), (A.25)and (A.26) into equation (A.37), we have

$$\begin{aligned}\frac{\partial \log OP_{it}}{\partial \log X_t} - \frac{\partial \log GP_{it}}{\partial \log X_t} &= (1-\rho)\left(\frac{U_{it}A_{it}}{K_{it}}\right)^{\frac{\rho-1}{\rho}}\left[\left(1-\frac{\partial \log P_A}{\partial \log X_t}\right)+\left(\frac{Z_{it}M_{it}}{V_{it}}\right)^{\frac{\theta-1}{\theta}}\left(1-\frac{\partial \log P_M}{\partial \log X_t}\right)\right] \\ &= (1-\rho)\frac{OL_{it}}{1-OL_{it}}\left(\frac{1-\beta_M}{GM_{it}}+\beta_M-\beta_A\right) \qquad \text{(A.38)}\end{aligned}$$

Table 1: Cyclicality of gross profits, COGS, and SG&A

This table reports the results of time series regressions in which annual growth rate of aggregate gross profitability ($\Delta\log$GP), aggregate cost of good sold ($\Delta\log$COGS), and aggregate selling, general, and administrative expenses ($\Delta\log$XSGA) are regressed on the annual growth aggregate revenue ($\Delta\log$REVT). All growth rates are adjusted for inflation. The sample period is from 1963 to 2016.

	$\Delta\log$GP	$\Delta\log$COGS	$\Delta\log$XSGA
Intercept	0.70	-0.28	2.89
	(1.60)	(-1.39)	(6.74)
$\Delta\log$REVT	0.87	1.05	0.39
	(14.14)	(36.70)	(6.45)
R^2	0.797	0.964	0.449

Table 2: Elasticity of operating profits across OL decile portfolios

This table reports firm-level elasticities of operating profits with respect to revenues (β_{REVT}(OP)) and with respect to gross profits (β_{GP}(OP)) across decile portfolios sorted by operating leverage (OL). For each decile portfolio, we estimate β_{REVT}(OP) by running Fama-MacBeth regressions $\Delta\log\text{OP}_{it} = a_t + b_t \times \Delta\log\text{REVT}_{it}$ and report the time series average of b_t, Similarly for β_{GP}(OP). All growth rates are adjusted for inflation. The sample period is from 1963 to 2016.

OL port	Lo	2	3	4	5	6	7	8	9	Hi
β_{REVT}(OP)	1.18	1.24	1.27	1.34	1.46	1.54	1.70	2.67	4.63	4.81
	(6.48)	(4.82)	(5.23)	(4.99)	(5.06)	(5.25)	(4.94)	(5.36)	(9.90)	(7.56)
β_{GP}(OP)	1.12	1.25	1.35	1.42	1.52	1.60	1.80	2.67	4.25	4.33
	(25.77)	(12.47)	(10.74)	(7.07)	(6.24)	(7.24)	(5.67)	(6.11)	(9.20)	(9.18)

Table 3: Estimates of elasticities of substitution

This table reports the estimates and standard errors of elasticity of substitution (ρ) between physical capital (K) and fixed inputs (A) and elasticity of substitution (θ) between K-A-integrated inputs (V) and variable inputs (M). The estimates of ρ and θ are obtained from the following Fama-MacBeth regressions:

$$\log\left(\frac{OP_{it}}{GP_{it}}\right) = (1-\rho_t)\log\left(\frac{OP_{it}}{AT_{it}}\right) - \frac{1-\rho_t}{1-\theta_t}\log\left(\frac{GP_{it}}{REVT_{it}}\right) + \epsilon_{it}$$

$$\log\left(\frac{GP_{it}}{REVT_{it}}\right) = (1-\theta_t)\log\left(\frac{OP_{it}}{AT_{it}}\right) - \frac{1-\theta_t}{1-\rho_t}\log\left(\frac{OP_{it}}{GP_{it}}\right) + \nu_{it}$$

In Panel A, ρ and θ are estimated using all firms from 1963 to 2016. In Panel B, ρ and θ are estimated separately for all firms within each of the 14 industries from 1974 to 2016.

Panel A: Firm-level estimates: All firms

	Estimates	Std.Err
ρ (for K and A)	0.369	0.012
θ (for V and M)	0.679	0.011

Panel B: Firm-level estimates: Within industry

14 industries	ρ	Std.Err	θ	Std.Err
Agriculture, forestry, fishing and hunting	0.773	0.074	0.617	0.056
Leisure and hospitality	0.452	0.021	0.826	0.020
Construction	0.522	0.025	0.841	0.032
Education and health services	0.276	0.014	0.515	0.018
Financial activities	0.918	0.013	1.037	0.014
Information	0.298	0.014	0.727	0.013
Manufacturing	0.254	0.006	0.432	0.008
Mining, quarrying, and oil and gas extraction	0.524	0.017	0.738	0.025
Other Services (except public administration)	0.640	0.075	0.671	0.043
Professional and business services	0.272	0.013	0.676	0.015
Retail trade	0.254	0.011	0.625	0.021
Transportation and warehousing	0.591	0.031	1.005	0.028
Utilities	0.633	0.074	0.732	0.101
Wholesale trade	0.290	0.011	0.512	0.026
Industry average	0.478	0.029	0.711	0.030

Table 4: Parameter values in model calibration

This table reports the parameter values used in model calibration at the annual frequency.

Symbol	Parameter description	Value
ρ	Elasticity of substitution b/w physical capital (K) and fixed inputs (A)	0.47
θ	Elasticity of substitution b/w K-A bundle and variable inputs (M)	0.74
x_{min}	The minimum value of aggregate profitability shock	1.91
x_{max}	The maximum value of aggregate profitability shock	1.93
μ_z	Mean of firm-level variable input productivity	2.45
σ_z	Standard deviation of firm-level variable input productivity	0.91
μ_u	Mean of firm-level fixed input productivity	1.45
σ_u	Standard deviation of firm-level fixed input productivity	0.42
P_M^0	Level of price of variable inputs	0.44
P_M^1	Elasticity of variable input price w.r.t. aggregate profitability shock	1.39
P_A^0	Level of price of fixed inputs	0.26
P_A^1	Elasticity of fixed input price w.r.t. aggregate profitability shock	0.45
λ	Risk premium of aggregate profitability shocks	0.09

Table 5: Gross profitability decile portfolios

This table reports the characteristics and average annualized value-weighted excess returns of decile portfolios sorted by gross profitability (GP/A) in the data (Panel A) and in the model (Panel B). Gross profitability is defined as the ratio of gross profits (Compustat items REVT minus COGS) to total asset (Compustat item AT). The characteristics include gross profitability (GP/A) and gross margin (GM). Panel B also reports the variable input productivity (z) and the fixed input productivity (u). The sample period in the empirical data is from July 1963 to December 2016. The model is simulated at three levels of aggregate profitability shock (x), with 2,000 firms at each level.

Panel A: Data

	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
GP/A	0.11	0.19	0.25	0.30	0.35	0.41	0.47	0.54	0.67	0.91	0.81
GM	0.21	0.25	0.24	0.29	0.32	0.35	0.38	0.39	0.42	0.38	0.17
Ret-Rf	2.67	3.68	5.27	7.54	5.49	6.08	6.00	5.79	8.90	8.41	5.74
t-stat	(0.87)	(1.49)	(2.19)	(3.14)	(2.30)	(2.38)	(2.45)	(2.40)	(3.98)	(3.55)	(2.68)

Panel B: Model

	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
GP/A	0.05	0.11	0.16	0.22	0.29	0.37	0.46	0.53	0.59	0.66	0.61
GM	0.38	0.43	0.46	0.48	0.51	0.53	0.55	0.58	0.58	0.59	0.20
z	1.74	2.09	2.27	2.44	2.64	2.81	3.00	3.18	3.26	3.28	1.53
u	1.80	1.50	1.50	1.52	1.49	1.54	1.51	1.39	1.50	1.87	0.07
Ret-Rf	1.51	5.36	6.24	6.75	7.24	7.44	7.76	8.10	7.99	7.63	6.12

Table 6: Operating leverage decile portfolios

This table reports the characteristics and average annualized value-weighted excess returns of decile portfolios sorted by operating leverage (OL) in the data (Panel A) and in the model (Panel B). OL is defined as selling, general ,and administrative expenses (Compustat data item SG&A) divided by gross profits (Compustat items REVT minus COGS). The characteristics include operating leverage (OL) and gross margin (GM). Panel B also reports the variable input productivity (z) and the fixed input productivity (u). The sample period in the empirical data is from July 1963 to December 2016. The model is simulated at three levels of aggregate profitability shock (x), with 2,000 firms at each level.

Panel A: Data

	Lo	2	3	4	5	6	7	8	9	Hi	Hi- Lo
OL	0.25	0.40	0.50	0.57	0.63	0.69	0.75	0.82	0.95	1.63	1.38
GM	0.26	0.29	0.34	0.37	0.33	0.32	0.32	0.32	0.30	0.21	-0.05
Ret-Rf	5.36	5.89	5.87	6.46	7.02	7.91	8.96	8.48	4.97	0.50	-4.85
t-stat	(2.26)	(2.57)	(2.55)	(2.92)	(2.90)	(3.20)	(3.23)	(2.56)	(1.31)	(0.12)	(-1.52)

Panel B: Model

	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
OL	0.29	0.34	0.38	0.42	0.45	0.50	0.54	0.60	0.68	0.77	0.48
GM	0.58	0.57	0.56	0.56	0.55	0.53	0.51	0.48	0.45	0.42	-0.16
z	3.17	3.10	3.02	2.95	2.89	2.71	2.55	2.31	2.11	1.91	-1.25
u	2.02	1.75	1.64	1.53	1.42	1.42	1.40	1.46	1.48	1.51	-0.51
Ret-Rf	7.42	7.60	7.67	7.78	7.90	7.75	7.61	6.90	6.27	4.86	-2.56

Table 7: Gross profitability and operating leverage double sorts

This table reports average annualized value-weighted excess returns of 5-by-5 portfolios double sorted on gross profitability (GP/A) and operating leverage (OL) in the data (Panel A) and in the model (Panel B). The sample period in the empirical data is from July 1963 to December 2016. The model is simulated at three levels of aggregate profitability shock (x), with 2,000 firms at each level.

Panel A: Excess return: Data

	Lo	2	OL	3	Hi	Hi-Lo	t-stat
Lo	4.63	2.75	4.32	3.37	-3.10	-7.73	(-2.23)
2	6.14	5.47	7.07	7.88	7.15	1.01	(0.28)
GP/A	5.89	6.58	6.97	7.54	4.43	-1.46	(-0.45)
3	6.27	4.34	8.20	6.57	5.86	-0.41	(-0.15)
Hi	7.88	9.00	8.80	13.11	10.74	2.86	(0.80)
Hi-Lo	3.25	6.26	4.49	9.74	13.84		
t-stat	(1.41)	(3.12)	(1.96)	(3.63)	(4.30)		

Panel B: Excess return: Model

	Lo	2	OL	3	Hi	Hi-Lo
Lo	4.18	4.20	5.03	4.82	2.81	-1.36
2	5.78	6.25	6.50	7.02	8.21	2.43
GP/A	6.61	7.03	7.35	7.78	8.59	1.97
3	7.11	7.45	8.07	8.43	8.99	1.88
Hi	7.43	7.64	7.85	8.01	8.21	0.78
Hi-Lo	3.26	3.44	2.81	3.19	5.40	

Table 8: TFP decile portfolios

This table reports the characteristics and average annualized value-weighted excess returns of decile portfolios sorted by total factor productivity (TFP) in the data (Panel A) and in the model (Panel B). Firm-level TFP data is from Selale Tuzel's website. In the model, we measure TFP as the residual from regression of logarithm of gross profits (GP) onto logarithm of fixed input cost ($P_A A$). The characteristics include TFP, gross profitability (GP/A), operating leverage (OL), and gross margin (GM). Panel B also reports the variable input productivity (z) and the fixed input productivity (u). The sample period in the empirical data is from July 1963 to December 2015. The ending year fo 2015 is restricted by the data availability of firm-level TFP. The model is simulated at three levels of aggregate profitability shock (x), with 2,000 firms at each level.

Panel A: Portfolio characteristics and excess returns: Data

	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
TFP	-0.90	-0.60	-0.48	-0.40	-0.33	-0.26	-0.19	-0.11	0.01	0.28	1.18
GP/A	0.22	0.28	0.31	0.32	0.34	0.35	0.37	0.38	0.35	0.35	0.13
OL	0.82	0.69	0.67	0.64	0.63	0.61	0.60	0.59	0.53	0.43	-0.39
GM	0.22	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.35	0.39	0.16
Ret-Rf	6.51	6.04	8.18	8.86	7.24	7.00	7.97	6.44	6.11	5.23	-1.28
t-stat	(1.94)	(1.97)	(2.80)	(3.30)	(2.75)	(2.84)	(3.34)	(2.86)	(2.76)	(2.32)	(-0.58)

Panel B: Portfolio characteristics and excess returns: Model

	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
TFP	-0.31	-0.22	-0.15	-0.09	-0.04	0.02	0.08	0.13	0.21	0.37	0.68
GP/A	0.28	0.29	0.28	0.30	0.31	0.35	0.28	0.34	0.45	0.56	0.29
OL	0.58	0.53	0.50	0.48	0.45	0.42	0.41	0.39	0.35	0.29	-0.29
GM	0.54	0.54	0.53	0.53	0.53	0.54	0.54	0.54	0.55	0.56	0.02
z	2.78	2.69	2.60	2.62	2.59	2.65	2.38	2.53	2.85	3.04	0.26
u	0.94	1.16	1.32	1.44	1.55	1.64	1.75	1.86	1.90	2.09	1.15
Ret-Rf	8.95	8.32	7.94	7.69	7.51	7.50	7.38	7.29	7.27	7.26	-1.69

Table 9: Idiosyncratic volatility decile portfolios

This table reports the characteristics and average annualized value-weighted excess returns of decile portfolios sorted by idiosyncratic volatility (IVOL) in the data (Panel A) and in the model (Panel B). Following Ang, Hodrick, Xing, and Zhang (2006), we estimate IVOL as (annualized) volatility of the Fama and French (1992) 3-factor model residuals using daily stock returns during the previous month. In the model, we compute IVOL as $\sqrt{\beta_z^2\sigma_z^2 + \beta_u^2\sigma_u^2}$, where β_z and β_u are firm's exposures to z and u. The characteristics include idiosyncratic volatility (IVOL), gross profitability (GP/A), operating leverage (OL), and gross margin (GM). Panel B also reports the variable input productivity (z) and the fixed input productivity (u). The sample period in the empirical data is from July 1963 to December 2016. The model is simulated at three levels of aggregate profitability shock (x), with 2,000 firms at each level.

Panel A: Portfolio characteristics and excess returns: Data

	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
IVOL	0.13	0.19	0.24	0.29	0.34	0.40	0.47	0.56	0.71	1.09	0.96
GP/A	0.33	0.34	0.34	0.34	0.34	0.34	0.34	0.33	0.33	0.32	-0.01
OL	0.50	0.54	0.56	0.57	0.58	0.60	0.63	0.66	0.69	0.75	0.25
GM	0.34	0.31	0.30	0.30	0.30	0.29	0.29	0.28	0.28	0.28	-0.06
Ret-Rf	6.09	6.72	7.58	7.01	7.88	6.82	4.60	3.75	-0.59	-2.70	-8.82
t-stat	(3.30)	(3.12)	(3.12)	(2.55)	(2.59)	(2.10)	(1.27)	(0.95)	(-0.14)	(-0.59)	(-2.26)

Panel B: Portfolio characteristics and excess returns: Model

	Lo	2	3	4	5	6	7	8	9	Hi	Hi-Lo
IVOL	0.96	1.08	1.19	1.32	1.52	1.82	2.24	2.91	4.08	7.08	6.12
GP/A	0.66	0.59	0.52	0.45	0.38	0.29	0.22	0.16	0.11	0.05	-0.60
OL	0.30	0.36	0.40	0.42	0.45	0.49	0.54	0.60	0.69	0.76	0.46
GM	0.59	0.58	0.57	0.56	0.54	0.51	0.49	0.46	0.44	0.39	-0.19
z	3.27	3.23	3.14	3.00	2.84	2.64	2.45	2.27	2.12	1.79	-1.48
u	1.90	1.59	1.49	1.50	1.48	1.50	1.51	1.50	1.45	1.71	-0.18
Ret-Rf	7.60	7.87	7.94	7.79	7.64	7.27	6.89	6.38	5.82	2.29	-5.32

Table 10: Idiosyncratic volatility premium: empirical spanning tests

This table reports the results from the factor spanning test of the idiosyncratic volatility premium using the gross profitability premium and the operating leverage premium. Each premium is defined as the long-short portfolio returns in the decile portfolios sorted by the corresponding firm characteristic. We run time series regressions of idiosyncratic volatility premium on a constant in Specification (1), on the gross profitability premium (GP/A Prm.) in Specification (2), on the operating leverage premium (OL Prm.) in Specification (3), and on both GP/A premium and OL premium in Specification (4). Newey-West t-statistics reported in parentheses control for autocorrelation and heteroskedasticity. The data is monthly from July 1963 to December 2016.

Specification	(1)	(2)	(3)	(4)
α	8.82	-4.20	-5.00	-1.87
	(-2.26)	(-1.19)	(-1.63)	(-0.67)
GP/A Prm.		-0.79		-0.61
		(-12.14)		(-11.44)
OL Prm.			0.75	0.68
			(19.51)	(18.95)
R^2	0.7%	19.1%	38.0%	48.8%

Figure 1: Average returns of portfolios double sorted on GP/A and OL

This figure plots average monthly excess returns of 5-by-5 double sorted on gross profitability (GP/A) and operating leverage (OL). The sample period is from July 1963 to December 2016.

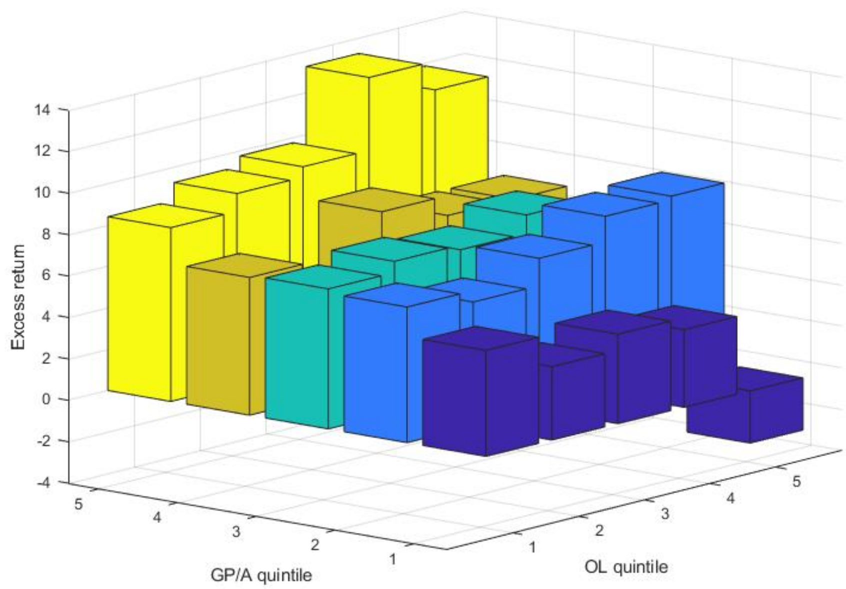

Figure 2: Value and policy functions

This figure plots the optimal policies for fixed input (A) and variable input (M), gross profitability (GP/A), operating leverage (OL), gross margin (GM), and operating profitability (OP/A), against fixed input productivity (u) and variable input productivity (z).

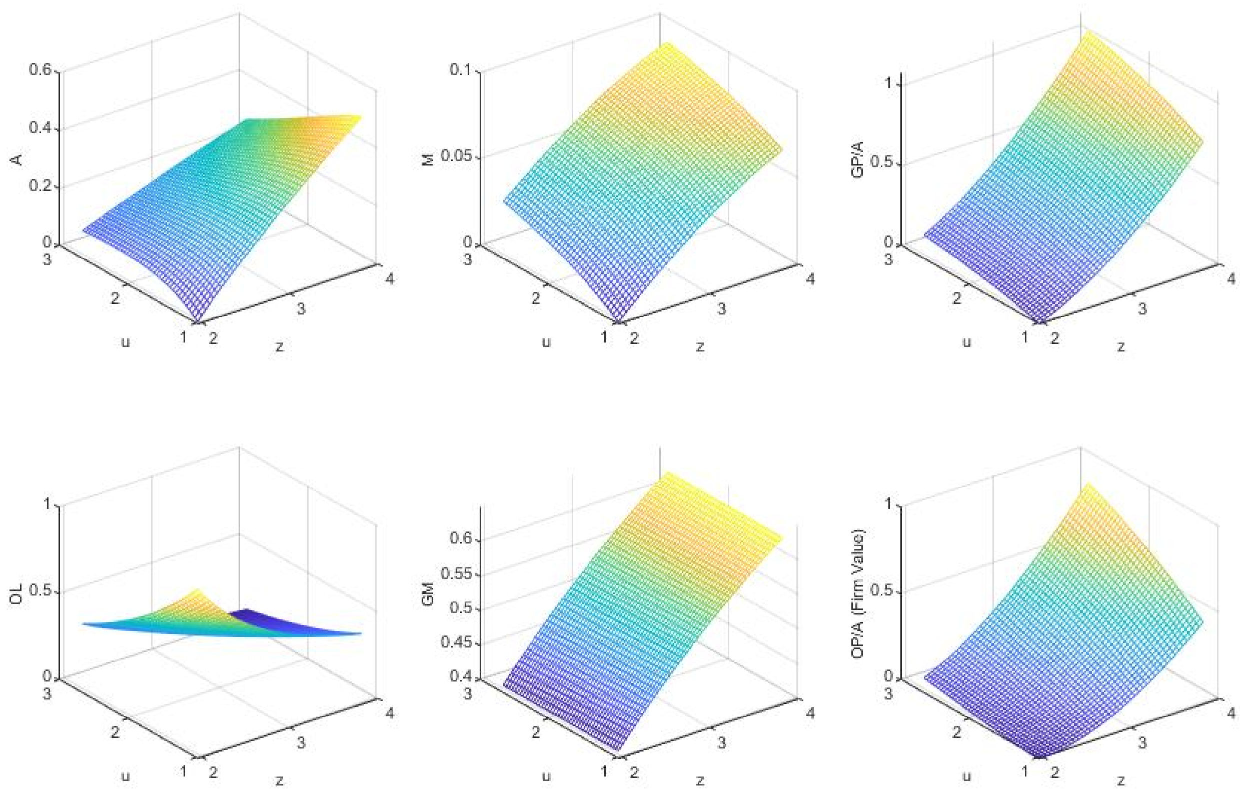

Figure 3: Risk exposures

This figure plots firm's exposure to the aggregate profitability shock (beta) against the fixed input productivity (u) and the variable input productivity (z) in Panel A, and against gross profitability (GP/A) and operating leverage (OL) in Panel B.