A Note on "Violations of Regularity and the Similarity Hypothesis by Adding Asymmetrically Dominated Alternatives to the Choice Set"

by

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Perspective

The science of choice theory advances in many ways, figure 1 represents but one of those ways. A number of papers at this conference have begun with observed phenomena and have abstracted from these phenomena essential properties or axioms. Batsell (1981) observed that item similarity within a choice set affects choice probabilities and he postulated the equation: 'choice probabilities are proportional to a logistic transformation of a weighted sum of item utility and item similarities.' When such axioms are stated mathematically, we can derive powerful implications from the axioms. For example, Hauser and Tversky (1981) develop a set of theorems addressing the interrelationship among agendas and choice probabilities. Such theorems or models imply normative applications such as Urban, Johnson, and Brudnick's (1981) hierarchical choice model which is used to identify structure within a market and ultimately to suggest the best new product opportunity. The normative applications push the theoretical models to their limits and, in many cases, develop or suggest new theory. Thus the research cycle begins again. Other applications link disparate research cycles leading to new insights and a combined research thrust. For example, Green, Goldberg, and Mahajan (1981) combine conjoint analysis and Markov models.

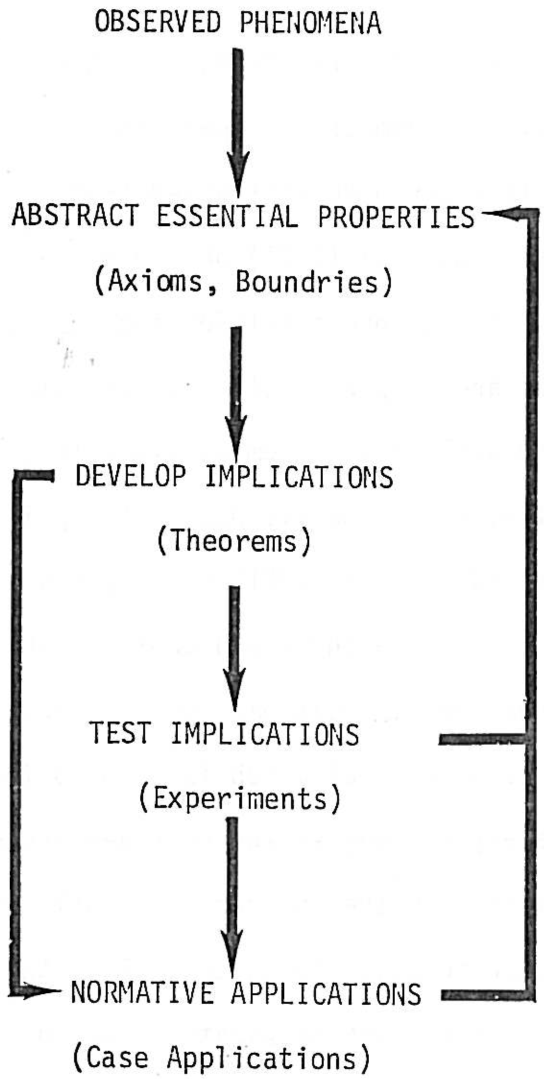

Figure 1: Research Perspective

The Huber, Payne, and Puto paper, which is reviewed in this note, represents an alternative research cycle. They begin with a fundamental choice axiom, regularity, that has been shown to be a reasonable description of observed phenomena in a wide range of situations. They then push this axiom to its limits in search of conditions under which the axiom no longer holds. In doing so, they advance choice theory in two ways. First, they identify the boundries of the axiom and hence the domain under which it holds and the domain under which it does not hold. Such results make us more comfortable with the axiom within its domain simply because its boundries are clearly identified.

Secondly, their research suggests new directions for theory development. If a domain is found under which the axiom fails, we naturally ask the question "Why?". The search for the answer can lead to new understanding, perhaps serendipitous results, and in some cases a unified choice theory that encompasses the old theory and explains the new phenomena. The latter, which we have not yet achieved, would be the psychological analog to the "quark" theory of physics in which the properties of elementary particles are explained by combinations of only four quarks: 'up', 'down', 'strangeness', and 'charm'. (Note: Even here high energy physicists continue the search for new quarks.)

Regularity Axiom

The regularity axiom says that we can not increase the probability of choosing an object by increasing the choice set. Formally, if x is a choice object, A and B are sets of choice objects, and if $P(x|A)$ is the probability an individual chooses object x from set A, then the regularity axiom states that for $x \in A \subseteq B$,

$$P(x|A) \geq P(x|B) \quad (1)$$

Since the authors' experiment includes three objects labeled target (t), competitor (c), and decoy (d), I will restate the axiom for their case,

$$P(t|\{c,t\}) \geq P(t|\{c,d,t\}) \quad (2)$$

In other words, regularity suggests that the authors can not add a decoy to the choice set to increase the probability that the target is chosen. If they can do so, they have identified a domain where regularity fails.

Of course we already know some domains where regularity may fail. (1) If we allow more than one object to be chosen the complementarity among objects can increase the probability of the target. For example, suppose t is a pair of pants, c is a two-piece suit, and d is a sports jacket color coordinated to t. (2) If we allow the choice rule to be an explicit function of an object's relationship to the choice set. For example, suppose the choice rule is 'choose the second largest object' where the measure, $m(\cdot)$, of largeness satisfies $m(d) > m(t) > m(c)$. The Huber/Payne/Puto condition suggests another domain.

Asymmetric Dominance

The authors suggest asymmetric dominance violates regularity where asymmetric dominance means t dominates d on all attributes but c does not dominate d or t. Mathematically:

$$P(d|\{d,t\}) = 0,\ 0 < P(d|\{c,d\}) < 1 \text{ and}$$

$$0 < P(t|\{c,t\}) < 1 \qquad (3)$$

The authors did a number of experiments with asymmetric dominance. One representative experiment takes the following form for six packs of beer:

	Price	Quality
t	$1.80	50
c	$2.60	70
d	$1.80	40

First we examine their condition under existing choice rules recognizing that their aspect structure forms the following preference tree.

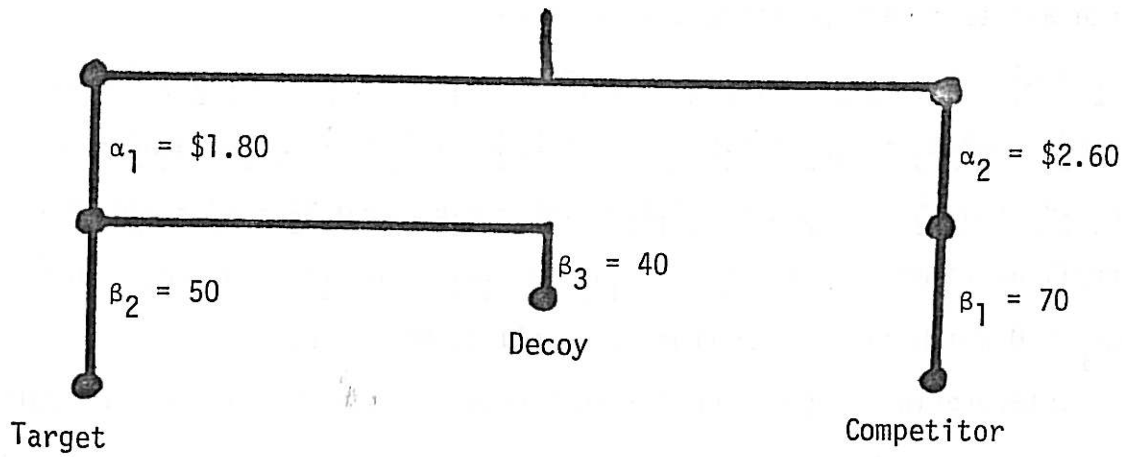

Figure 2: Preference Tree Representation of Experiment

We know that existing choice rules, elimination-by-aspects (EBA), hierarchical-elimination model (HEM), and hierarchical balance model (HBM) all satisfy regularity. Nonetheless, it is instructive to use figure 2 to compute the probabilities in equation 2 for these choice models. Since Hauser and Tversky show that all three models are equivalent on a preference tree we use HEM to compute the necessary quantities. That is:

$$P(t|\{c,t\}) = \frac{(\alpha_1+\beta_2)}{(\alpha_1+\beta_2) + (\alpha_2+\beta_1)} \qquad (4)$$

$$P(t|\{c,d,t\}) = \frac{(\alpha_1+\beta_2+\beta_3)}{(\alpha_1+\beta_2+\beta_3) + (\alpha_2+\beta_1)} \cdot \frac{\beta_2}{(\beta_2+\beta_3)} \qquad (5)$$

Proposition 1. If choice probabilities are given by equations 4 and 5, then regularity is not violated.

Proof: Regularity holds if

$$\frac{\alpha_1+\beta_2}{\alpha_1+\alpha_2+\beta_1+\beta_2} > \frac{\alpha_1\beta_2+\beta_2^2+\beta_2\beta_3}{\alpha_1\beta_2+\alpha_1\beta_3+\alpha_2\beta_2+\alpha_2\beta_3+\beta_1\beta_2+\beta_1\beta_3+\beta_2^2+2\beta_2\beta_3+\beta_3^2}$$

Since all terms are positive this is true if

$$\alpha_1^2\beta_2 + \alpha_1^2\beta_3 + \alpha_1\alpha_2\beta_2 + \alpha_1\alpha_2\beta_3 + \alpha_1\beta_1\beta_2 + \alpha_1\beta_1\beta_3 + \alpha_1\beta_2^2 + 3\alpha_1\beta_2\beta_3 + \alpha_1\beta_3^2 + \alpha_1\beta_2^2 + \alpha_2\beta_2^2 + \alpha_2\beta_2\beta_3 + \beta_1\beta_2^2 + \beta_1\beta_2\beta_3 + \beta_2^3 + 2\beta_2^2\beta_3 + \beta_2\beta_3^2 - \alpha_1^2\beta_2 - \alpha_1\beta_2^2 - \alpha_1\beta_2\beta_3 - \alpha_1\alpha_2\beta_2 - \alpha_2\beta_2^2 - \alpha_2\beta_2\beta_3 - \alpha_1\beta_1\beta_2 - \beta_1\beta_2^2 - \beta_1\beta_2\beta_3 - \alpha_1\beta_2^2 - \beta_2^3 - \beta_2^2\beta_3 > 0$$

Cancelling terms yields $\alpha_1^2\beta_3 + \alpha_1\alpha_2\beta_3 + \alpha_1\beta_1\beta_2 + \alpha_1\beta_1\beta_3 + \alpha_1\beta_2\beta_3 + \alpha_1\beta_3^2 + 2\beta_2^2\beta_3 > 0$ which is clearly true since all terms are positive.

Interpreting proposition 1 intuitively, we see that while the addition of β_3 to the left branch makes that branch more probable (first term in equation 5), the gain is more than offset by the loss due to the splitting of that probability among d and t (second term in equation 5). The *structure* of the model is such that this must be true no matter how large β_3 is.

Proposition 1 suggests that since the authors' experiments violate regularity using the conditions of equation 3, then they have indeed identified a domain requiring that existing models be modified.

Authors' Proposed Explanation

The authors propose three possible explanations for the failure of regularity: tournament, satisficing, and popularity. The first two explanations suggest modifications to the choice rule; the third explanation requires that we assume the decoy modifies our perceptions of the target. The third rule is less parsimonious than the two choice rules because it requires a construct, perceptual modification, in addition to choice theory. On this basis I prefer to find an explanation within choice theory and only add the perceptual modification construct if such an explanation can not be found. If such an explanation can be found, then we can design an experiment to compare its predictions to that of perceptual modification. However, this is a personal preference and does not imply a criticism of the authors.

Let us begin with the tournament explanation. The idea is quite simple. An individual first picks two objects, say c and d, compares them, and then compares the winner, say d, to the remaining object, in this case t. Mathematically the tournament hypothesis implies:

$$P(t|\{c,d,t\}) = p_1 [P(d|\{c,d\})\ P(t|\{d,t\}) + [1 - P(d|\{c,d\}]\ P(t|\{c,t\})] + p_2 [P(t|\{c,t\})\ P(t|\{d,t\})] + p_3 [P(t|\{d,t\})\ P(t|\{c,t\})] \quad (6)$$

where $0 < p_1, p_2, p_3 < 1$ and $p_1 + p_2 + p_3 = 1$.

Proposition 2. If the tournament explanation and asymmetric dominance, hold, then regularity is violated.

Proof. Since $P(t|\{d,t\}) = 1$ by equation 3, equation 6 becomes $P(t|\{c,d,t\}) = p_1P(d|\{c,d\})\ [1 - P(t|\{c,t\})] + p_1P(t|\{c,t\}) + p_2P(t|\{c,t\}) + p_3P(t|\{c,t\})$. Recognizing $p_1+p_2+p_3= 1$ yields $P(t|\{c,d,t\}) = P(t|\{c,t\}) + p_1 P(d|\{c,d\})[1-P(t|\{c,t\})]$. Since the second term is positive by equation 3, we have the result $P(t|\{c,d,t\}) > P(t|\{c,t\})$ which violates regularity.

Based on proposition 2 we see that a tournament explanation alone can order the authors' basic experiment.

We now examine their second explanation: satisfacing. As presented, satisficing is a rule by which an individual randomly chooses a pair of products then picks the best item. The authors' rationalization is time pressure, perhaps lack of desire for the effort involved in making an optimal decision, or perhaps an attempt to avoid the worst possible item. According to satisficing:

$$P(t|\{c,d,t\}) = q_2P(t|\{c,t\}) + q_3P(t|\{d,t\}) \quad (7)$$

where $0 < q_2, q_3 < 1$ and $q_2 + q_3 < 1$.

Propo sition 3. Satisficing for asymmetric dominance violates regularity if and only if $P(t|\{t,c\}) < q_3/(1-q_2)$.

Result 1 is as predicted but perceptual modification does not indicate why the strategies do not work in every case. Result 2 is interesting and informative but not explained by perceptual modification. Results 3 and 4 are inconsistent with a (first order) perceptual modification explanation.

The authors do suggest that confusion may make the combined strategy more difficult for the individual to evaluate. But a "confusion" explanation makes perceptual modification even less parsimonious.

Summary. Based on this short examination of the authors' explanations, we see that:

(a) A tournament always implies regularity is violated, but regularity is violated in only 18 of the authors' 24 experiments. (75%)

(b) Satisficing implies violations iff $P(t|\{c,t\}) < 1/2$. This is not supported by the data.

(c) Perceptual modification orders the basic experiment but it alone does not fully order the relative effects of the range and frequency inductions.

Discussion

None of the authors' proposed explanations, when taken alone, can fully explain the data. A tournament explanation orders the main effect, but there is not yet an explanation of why a tournament should hold on some occasions and not on others. Similarly a perceptual modification explanation orders the main effect, but does not adequately order the relative effects of range and frequency strategies.

This ambiguity is not a criticism of the authors' experiment but rather praise. They have been successful in accomplishing a test of the boundaries of the regularity axiom. At the most basic level we now suspect that regularity is weak under the domain of asymmetric dominance. This alone is a valuable contribution.

More importantly the authors have provided a sufficiently complex experiment with which to examine three different explanations of why asymmetric dominance violates reularity. Satisficing, as defined by the authors, is rejected. Tournament and perceptual modification explanations remain viable hypotheses once *their* boundries are known. We now know they do not apply for all cases of asymmetric dominance.

The authors' paper illustrates the valuable interplay between empirical experiments and mathematical theory. The authors have now challenged a fundamental axiom of choice theory. The next step is a parsimonious theory that

(1) Encompasses the old theory when asymmetric dominance does not hold,
(2) Explains why asymmetric dominance should violate regularity,
(3) Orders the relative effects of the range, frequency, and range/ frequency inductions, and
(4) Predicts the magnitudes of the effects.

Such a theory may be devloped by the authors or by another researcher motivated by their data. It is through such interplay that choice theory advances.

Closing Observations

(1) In 1980, Shugan developed a theory of the "cost of thinking." In his paper Shugan suggests that there is a cost to evaluating objects and that individuals seek to minimize this cost. Shugan proposes a measure of this thinking cost based on the number of comparisons that must be made to distinguish an object from another. He uses a tournament argument to develop an upper bound on this thinking cost when more than two objects are involved.

Shugan's theory is parsimonious and reduces to existing choice theory when thinking cost is small compared to the value of making a "correct" choice. The tournament aspect of Shugan's theory suggests that asymmetric dominance will have an effect when thinking cost is large. Furthermore, the magnitude of the effect will depend upon the magnitude of the thinking cost.

Thus one next step is to examine the authors' experiments in light of Shugan's theory.

(2) Proposition 3 suggests that the condition $P(t|\{c,t\}) > 1/2$ may be relevant. Indeed, in exhibit 5 asymmetric dominance is stronger for $P(t|\{c,t\}) > 1/2$. If we examine the frequency induction in exhibit 5 this effect is even more pronounced. Under the frequency induction, regularity is violated 5 out of 5 times for $P(t|\{c,t\}) \geq .44$ and 0 out of 3 times for $P(t|\{c,t\}) < .44$. For the mixed induction, range/frequency, we get mixed results but regularity still seems related to $P(t|\{c,t\})$. Such data begs an explantion.

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