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CONSUMPTION OVER THE LIFE CYCLE

BY PIERRE-OLIVIER GOURINCHAS AND JONATHAN A. PARKER¹

Abstract. This paper estimates a structural model of optimal life-cycle consumption expenditures in the presence of realistic labor income uncertainty. We employ synthetic cohort techniques and Consumer Expenditure Survey data to construct average age-profiles of consumption and income over the working lives of typical households across different education and occupation groups. The model fits the profiles quite well. In addition to providing reasonable estimates of the discount rate and risk aversion, we find that consumer behavior changes strikingly over the lifecycle. Young consumers behave as buy-stock agents. Around age 40, the typical household starts accumulating liquid assets for retirement and its behavior mimics more closely that of a certainty equivalent consumer. Our methodology provides a natural decomposition of saving and wealth into its precautionary and life-cycle components.

1. Introduction

This paper estimates a dynamic stochastic model of the life-cycle saving behavior of households. We focus on estimation of structural preference parameters and upon characterizing optimal behavior when households face exogenous, stochastic, labor income processes. We are motivated by two observations.

First, better methodology, data, and creative use of natural experiments are leading to significant evidence against consumption smoothing as an accurate description of household-level behavior. Despite generally poor-quality data on household consumption, recent tests often find that consumption responds to predictable changes in income. At lower frequencies, there is some evidence that consumption tracks expected and unexpected income changes across groups of households.²

Second, recent theoretical work has improved our understanding of optimal household consumption and saving behavior under uncertainty (Ayagari (1994), Zeldes (1989), Deaton (1991), Carroll (1997)). Usual log-linearized Euler equations can provide a poor approximation to marginal utility smoothing as well as unreliable estimates of the preference parameters in uncertain environments (Ludvigson and Paxson (1997)). Uncertainty can generate a positive correlation between expected

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$$S_1 @ J_1 S_{13} \cdot Q_1$$

Wkh wudqvlwru| vkrfnvX₁/ duh lqghshqghqwo| dqg lghqwlfdoo| glvwulexwhg/ wdnh wkh ydoxh 3 zlw sure0 delolw| 3 s ? 4>dqg duh rwxhuzlvh orj0qrupdoo| glvwulexwhgX₁q Q 3> ² 1 Wkh orj ri wkh shupdqhqw frpsrqhqw ri lqfrph/ oq S₁/ hyroyhv dv d udqgrp zdon zlw djh vshfl f guliw oq|₁=Wkh vkrfn wr wkh shupdqhqw frpsrqhqw ri lqfrph/ Q₁/ lv lqghshqghqwo| dqg lghqwlfdoo| orj0qrupdoo| glvwulexwhg/ @ Q 3> ² = Wkxv oq \ | PD +4,>lqfrph lv d vhuldoo| fruhodwhg surfhv/ zlw erwk shupdqhqw dqg wudqvlwru| vkrfnv/ dqg d srvlwlyh suredelolw| ri }hur lqfrph lq hyhu| shulrg[†]

Wzr srlqvw duh zruwk qrwlj1 Iluvw/ lqarydwlrqv wr wkh shupdqhqw frpsrqhqw ri lqfrph duh rqo| dv shupdqhqw dv wkh uhpdlqlqj ohqjwk ri wkh zrunlj olih= doo vkrfnv duh xowlpdwho| wudqvlwru| dv frqvxphuv uhwluh dqg glh1 Dv d frqvtxhqfh/ wkh surshqvlw| wr frqvxph rxw ri cshupdqhqw* vkrfnv zloo ydu| zlw djh/ d srlqw hpskdvl}hg e| Fodulgd +4<<4,1 Wklv surshuw| krogv wuxh iru wkh FHT00FK dovr1 Vhfrqg/ lq wklv vhwxs frqvxphuv zloo fkrvh qhyhu wr eruurz djdlqvw ixwxuh oderu lqfrph/ d srlqw vkrzq e| Vfkfhkwpdq +4<:9,1 Wklv iroorzv iurp +d, wkhuh ehqj d vwulfwo| srvlwlyh suredelolw| wkdw oderu lqfrph zloo eh duelwudulo| forvh wr }hur iru wkh uhvw ri wkh zrunlj olih dqg +e, wkh Lqgdg frqglwlrq oq f x +, @ 4 1 Wr vhh wklv/ vxssrvh wkh krxvhkrog zhuh wr eruurz lq wkh qh{w wr odvw zrunlj shulrg1 Wkhq/ zlw vwulfwo| srvlwlyh suredelolw| lw zrxog eh ohiw zlwkrxw dq| zhdowk lq wkh odvw zrunlj shulrg1 Wkh krxvhkrog zrxog wkhq kdyh dq lq qlwh h{shfwhg pdujlqdo xwlolw|1 Edfnzdug lqgxfwlrq lpsolhv wkdw lw zloo qhyhu eh rswlpdo wr eruurz lq dq| shulrg1

Wkh suhfdxwlrqdu| prwlyh dfww dv dhoi0lpsrvhgoltxlglw| frqvwudlqw1 Lw lv lpsruwdqw wr qrwh wkdw
 wkly krogv wuxh hyhq zkhq/ wkh suredelolw| ri vwulfwo| }hur lqfrph/ lv vhw wr }hur1 Wkh nh| khuh
 lv wkdw wkh lqfrph surfhvv kdv d }hur orzhu erxqg1 Wkly dqg wkh uhtxluhphqw wkdw wkh frqvxphu
 glhv zlwkrxw ghew doprvw vxuho/ lpsrvh d qdwxudo eruurzlj olplw htxdo wr 3 +vhh D|djdul +4<<7,,1
 Zlwk d vwulfwo| srvlwlyh orzhu erxqg rq lqfrph/ wkh frqvxphu frxog eruurz rgo| xs wr wkh suhvhqw
 glvfrxqwhg ydoxh ri fhuwdlq ixwxuh lqfrph1

Jrlqj iurp wkh prgho wr wkh gdwd/ zh qhhg wr pdnh irxu dvvxpswlrqv1 Iluvw/ lq rughu wr vroyh wkh
 frqvxphu*v sureohp dv vwdwhg/ zh qhhg wr vshfli| erwk wkh qdwxuh ri xqfhuwdlqw| gxulqj uhwluhphqw
 dqg d ehtxhvw ixqfwrq1 Zkloh wkhuh kdyh ehq jrrg dwwhpswv dw prghoolqj frqvxphu ehkdylru diwhu
 uhwluhphqw/zh ihho wkdw zh nqrz wr olwwoh derxw wkh irup wkdw xqfhuwdlqw| wdnhv diwhu uhwluhphqw
 wr xv xru phwkrgrorj| dqg gudz lqihuhqfhv iurp srw0uhwluhphqw ehkdylru1 Xqfhuwdlqw| dulvhv
 iurp gl huhqw vrxufhv ~ h1j1 phglfdo h{shqvhv/ wkh wlpqj ri ghdkw dqg vhw uhwxudw|dwhu0ylyrv
 ehtxhvvv duh dovr olno| wr eh lpsruwdqw1 Dowkrxjk wkhvh vrxufhv ri xqfhuwdlqw| duh dovr suhvhqw
 wr vrph h{whqw lq wkh odvw zrunlqj |hdv/ oderu lqfrph xqfhuwdlqw| lv wkh grplqdqw vrxufh ri
 xqfhuwdlqw| zkhq |rxqj1 Ixuwkhu/ kljk txdolw| lqirupdwlrq rq krxvhkrog dvvhw krogqjv/ wrjhwkhu
 zlw frqvxpswlrq dqg lqfrph/ lv qrw dydlodeoh1 Jlyhq wkdw lqyhvwphqw lqfrph/ vrfldo vhfzulw|
 dqg shqvlrqv uhsuhvhqw wkh pdlq vrxufhv ri lqfrph gxulqj uhwluhphqw/ lw lv fxuuhqwo| gl fxow wr
 hvwdeolvk frqvxpswlrq sdwwhuqv dv d ixqfwrq ri wrwdo zhdowk1 Odvwo| hyhq zlw d surshu wuhdwphqw
 ri uhwluhphqw lvvxhv/ rqh zrxog kdyh wr pdnh d jxhvv derxw wkh ehtxhvw ixqfwrq1 Lqvwddg/ zh pdnh
 xv ri Ehoopdq*v rswlpdolw| sulqflsoh/ dqg wuxqfduh xru sureohp dw wkh gdwh ri uhwluhphqw1

Vhfrqg/ zh dvvxph wkdw djh yduldwlrqv lq +j |, duh frpprq dfurvv krxvhkrogv ri wkh vdph djh w
 ghwhuplqlwlf/ dqg frph iurp fkdqjvh lq idplo| vl}h/ vr wkdw wkh hyroxwlrq ri wkh frqvxphu sureohp
 fdq eh fdswxuhg e| d vlqjoh vwdwh yduldwlrq1 Wkly kdv wkh dgghg dgydqwdjh wkdw zh fdq dgmxvw dqg
 uhsruw frqvxpswlrq dqg lqfrph sur ohv wkdw pdlqwdlq d frqwdqw h hfwylyh idplo| vl}h dfurvv djhv1

Wklug/ wkh prgho lpsrvhv d vlqjoh yhklfoh iru suhfdxwlrqdu| dqg uhwluhphqw zhdowk dffxpodwlrq/
 vlqfh wkhuh lv rgo| rqh dvvhw1 Lq sudfwlfh/ pxfk ri qhw zruwk dw uhwluhphqw lv dffxpodwhg lq wkh
 irup ri looltxlg zhdowk/ rgo| dydlodeoh diwhu uhwluhphqw1 Wkly vxjjhvvw wkdw wkh uhohydqw prgho ri
 frqvxpswlrq ehkdylru vkrxog lqfrusrudwh dq dgglwlrqdo dvvhw wkdw ltxlgodqg dffhvvleoh rgo|
 diwhu uhwluhphqw1 Krzhyhu/ wkly zrxog vxevwdqwdoo| frpsoldwh wkh sureohp e| lqwurgxflqj dqrwku

frqwuro yduldeoh +krz pxfk wr vdyh lq oltxlg yhuvxv looltxlg dvvhwv, dqg vwdwh yduldeoh +looltxlg dvvhwv,1 Lq rughu wr nhhs rxu hwwlpdwlrq surfhgxuh wudfwdeoh/ zh lqvwhdg dvvkwvlygdwv koltwlygdwv dffxpxodwhv h{rjhqrxvo/ fdqqrw eh eruurzHg djdlqvw/ dqg wkdw looltxlg zhdowk lq wkh uvw |hdu ri uhwluhphqw $K_{An} > lv$ sursruwlrqdo wr wkh odvw shupdqhqw frpsrqhqw ri lqfrph $K_{An} \cdot kS_{An} @ kS_{A1}$ Wkhvh dvvxpswlrqv holplqdwh erwk looltxlg dvvhwv dv d vwdwh yduldeoh ri wkh surjudp/ dqg frqwulexwlrqv wr looltxlg dvvhwv dv d frqwuro yduldeoh1

Ilqdoo/ zh srwxodwh d uhwluhphqw ydoxh ixqfwlrq zklfk vxppdul}hv wkh frqvxpheu*v sureohp dw uhwluhphqw wlp1 Gh qlqj fdvk rq kdqg $|n \cdot dv$ wrwdo oltxlg qdqfido uhvrxfhv $|n \cdot @ U \{ | F |, \cdot \setminus |n \cdot @ Z |n \cdot \setminus |n \cdot >$ zh fkrv d ixqfwlrqdo irup wkdw pdlqwdlqv wkh wudfwdelolw ri wkh sureohp dqg lv h{leoh hqrxjk wr doorz urexvwqhvv fkhfnv=

$$Y_{An} \cdot \{ | An \cdot > K_{An} \cdot > \} An \cdot , @ y \{ | An \cdot , \{ | An \cdot \cdot K_{An} \cdot , \cdot^{34} >$$

iru vrph frqvwdqw = Xqghu wkh FUUD dvvxpswlrq/ wklv ixqfwlrqdo irup lv h{dfwo| fruuhfw li wlp ri ghdkw dqg2ru dvvhw uhwxuqv duh wkh rqo| vrxfhv ri xqfhuwdlqwl diwhu uhwluhphqw +Phuwraq +4<:4,,1 Xqghu wklv dvvxpswlrq/ frqvpswlrq lv olqhdv lq wrwdo zhdowk dw uhwluhphqw1 Vdqf prw krxvh0 krogv kdyh odujh dprxqvw ri looltxlg zhdowk lq Vrfido Vhfxulw| dqg shqvlrqv dw uhwluhphqw/ d olqhdv dssur{lpdwlrq iru wkh wuxh frqvpswlrq uxoh lv olhho| wr eh d jrrg dssur{lpdwlrq1

Gh qlqj wkh ydoxh ixqfwlrq iru wkh krxvhkrog sureohp dw wlp dv $Y >rxu$ sureohp ehfrphv=

$$Y \{ | > S > \} , @ \quad \text{\%} \begin{matrix} A \\ | \end{matrix} \quad \& \quad \text{pd} \{ \begin{matrix} c \\ c \end{matrix} \} \quad H \quad |^3 \quad y \{ | , \frac{F \cdot^{34}}{4} \cdot \quad An \cdot^3 \quad y \{ | An \cdot , \{ | An \cdot \cdot kS_{An} \cdot , \cdot^{34} \\ v=w= |h \cdot @ U \{ | F |, \cdot \setminus |n \cdot > \quad [An \cdot \quad 3 > dqg \quad jlyhq \quad +6,1$$

zkhuh lqfrph lv gh qhg dv glvsrvdeoh lqfrph/ qhw ri Vrfido Vhfxulw| wd{hv dqg vdyldj lq looltxlg dvvhwv1 Wkh dvvxpswlrq wkdw looltxlg zhdowk fdqqrw eh eruurzHg djdlqvw lpsrvhv wkh eruurzlgj frqvwdlqwk wkdw oltxlg zhdowk pxvw eh zhdno| srvlwlyh dw uhwluhphqw1

5151Vroyldj iru Rswlpdo Frqvxpheu Ehkdylru1 Wkh vhwxs ri wkh sureohp frpelqhg zlwkw rxu sduwlfxodu fkrlfh ri uhwluhphqw ydoxh ixqfwlrq pdnhv wkh sureohp krprjhqrxv ri ghjuhh +4 , lq wkh shupdqhqw frpsrqhqw ri lqfrph S_1 Zh zulwh wkh rswlpdo frqvpswlrq uxoh dv d ixqfwlrq

•f

DP-Y Q 7 \$ c -l.-

ri djh/ w dqg wkh udwlr ri fdvk rq kdqg wr wkh shupdqhqw frpsrqhqw ri lqfrph/ { | [| @ \$ 1 Wkh exgjhw frqvwudlqw ehfrphv=

+7,
$$\{ |n \cdot @ f | f |, \frac{U}{J |n \cdot Q |n \cdot} \cdot X |n \cdot > 4 \text{ w } W =$$

zkhuh orzhufdvh ohwwhuv duh qrupdol}hg e| wkh shupdqhqw frpsrqhqw ri lqfrph1 Wkh iroorzlqj Hxohu htxdwlrq krogv iruw ? W

+8,
$$x \# | \# |, @ U H \frac{y \# |n \cdot}{y \# |}, x \# |n \cdot \# |n \cdot, J |n \cdot Q |n \cdot, >$$

zkhuh f | \# |, uhsuhvhqvw wkh rswlpdo frqvxpswlrq uxoh dw wkh qrupdol}hg, / dv d ixqfwlrq ri qrupdol}hg fdvk rq kdqg { | 1 Lq wkh odvw zrunlqj shulrg/ wkh Hxohu htxdwlrq ehfrphv=

$$x \# A \# A, @ pd \{ x \# A, > U \frac{y \# A_n \cdot}{y \# A,} x \# A_n \cdot \# A_n \cdot, ,$$

vlqfh wkh krxvhkrog fdqqrw eruurz djdlqw looltxlq zhdowk $K_{A_n \cdot}$ dqg $S_{A_n \cdot} @ S_A =$

Jlyhq rxu uhwluphqw ydoxh ixqfwlrq/ rswlpdo frqvxpswlrq lv olqhdv lq wrwdo zhdowk dw uhwluphqw=

$$F_{A_n \cdot} @ \cdot \# A_n \cdot \cdot K_{A_n \cdot},$$

zkhuh \cdot uhsuhvhqvw wkh pdujlqdo surshqvlw| wr frqvxph rxw ri zhdowk qrupdol}hg frqvxpswlrq/ $f_{A_n \cdot} > lv$ wkhq olqhdv lq qrupdol}hg fdvk rq kdqg $f_{A_n \cdot} =$

+9,
$$f_{A_n \cdot} @ f \cdot \cdot \{ A_n \cdot >$$

zlwk $f \cdot \cdot k1$

Wkh vroxwlrq wr wkh frqvxphu sureohp frqvlvww ri d vhw ri frqvxpswlrq uxoh $f | \# |, j \cdot \$ | \$ A 1$ Wkh frqvxpswlrq uxoh lq wkh odvw zrunlqj shulrg lv wkh vroxwlrq wr wkh deryh Hxohu htxdwlrq iru doo ydoxhv ri fdvk rq kdqg/ zkhuh zh uhsodfh $f_{A_n \cdot}$ xvlqj htxdwlrq +9,1 Vroylqj uhfxuvlyhol jhqhudwhv frqvxpswlrq ixqfwlrqv $f_A \# A, > ==> f \cdot, 1$ D frpsohwh ghvfulswlrq ri wkh vroxwlrq phwkrq lv surylghg lq Dsshqgl{ D1

5161Fkdudfwhul}dwlrq ri Lqglylgxdo Fr qvxpswlrq Ehkdylru1 Ilijxuh 4 sdqho D vkrzv frq0
 vxpswlrq uxohv dw ydulrxv djhv/ iru d w|slfdo krxvhkrog zrunlqj iurp djh 58 wr 98 dqg uhwlulqj
 wkhuhdiwhu zlwkw d frqvxpswlrq uxoh dw djh 99 fkdudfwhul}hg e|=@ 3334> . @ 33:4=Wkhvh
 sdudphwhu ydoxhv duh wkrvh ri rxu edvholqh hvwlpdwhv1dqg] zhuh frqvwdqw/ wkh qlwh krul}rq
 sureohp zrxog frqyhujh wr wkh lq qlwh krul}rq rqh/ dv zh pryh ixuwkhu dzd| iurp uhwlulphqw1
 Frqvxpswlrq lv dozd|v srvlwlyh/ lqfuhdvlqj/ dqg frqfdyh lq fdvk rq kdqg1 Hduo| lq olih/ krxvhkrogv
 h{klelw cex hu vwrfn* ehkdylru= iru orz ohyho ri fdvk rq kdqg/ w|slfdoo| ohvv wkdw wkh shupdqhw
 frpsrqhw ri wkhlu lqfrph + { 4,/ krxvhkrogv frqvxph prvw/ exw qhyhu doo/ ri wkhlu qdqfido
 zhdowk/ dqg pryh wr wkh qh{w shulrg zlwkw d yhu| orz ohyho ri oltxlq dvvhwv1 Dw kljk ohyhov ri fdvk
 rq kdqg/ wkh suhfdxwlrqdu| prwlyh lv vpdoo dqg krxvhkrogv frqvxph pruh wkdw wkh lqfrph wkh|
 h{shfw wr uhfhlyh +zklfk htxdov 4, dqg vr uxq grzq wkhlu dvvhwv1 Dv zh glvfxvv odwhu/ rxu edvholqh
 uhwlulphqw frqvxpswlrq uxoh lpsolhv hlwkhu olwwoh looltxlq zhdowk ru d orz surshqvlw| wr frqvxph
 iurp lw1 Wkxv dv krxvhkrogv djh/ wkh| pxvw vdyh iru uhwlulphqw= frqvxpswlrq uxohv ghfolqh dqg
 krxvhkrogv dffpxodwh vljq| fdqw dprxqvv ri oltxlq zhdowk1

Ilijxuh 4 sdqho E uhsruwv frqvxpswlrq uxohv iru d vlplodu krxvhkrog zlwkw wkh dowhuqdwlyh uhwluloh
 phqw frqvxpswlrq uxoh f @ 33<7> . @ 33::71*^S Wkh kljkhu ydoxh ri f lpsolhv hlwkhu odujhu
 looltxlq zhdowk dw uhwlulphqw ru d odujhu surshqvlw| wr frqvxph iurp lw1 Wkh wzr krxvhkrogv kdyh
 yhu| vlplodu frqvxpswlrq uxohv hduo| lq olih/ iru wkh vdph uhdvraq wkdw frqyhujhqfh rffxuv lq wkh
 vwdwlrqdu| sureohp1 Erwk krxvhkrogv ehkdylru dv ex hu vwrfn krxvhkrogv lq wkhlu |rxwk1 Odwhu lq
 olih/ wkh krxvhkrog lq sdqho E grhv qrw qhng wr dffpxodwh dv pxfk oltxlq zhdowk vlqfh lw fdq uho|
 pruh rq looltxlq zhdowk wr pdlqwdlq frqvxpswlrq lq uhwlulphqw1 Wkxv wkh frqvxpswlrq uxohv ghfolqh
 ohvv dv wkh krxvhkrog djfiv1

Zkdw jhqhudo frqglwlrqv duh qhfhvvdul wr jhqhudwh ex hu0vwrfn ehkdylruB Lq d vwdwlrqdu| lq qlwh0
 krul}rq prgho/ ex hu vwrfn ehkdylru dulvhv zkhq djhqvv duh vx flhqwo| lpsdwlhqw qrw wr dffpxodwh
 zhdowk zlwkrxw erxqgv1 Irupdoo| wkh pdsslqj iurp fxuuhqw zhdowk wr h{shfwng zhdowk lq wkh qh{w
 shulrg pxvw eh d frqwudefwlrq1 Lq wklv fdvh/ Fduuroo +4<<<, dqg Ghdwraq +4<<4, suryh wkdw d vroxwlrq
 wr wkh frqvxphu sureohp h{lvwv li=

+,

U H +JQ ,³⁴ ? 4=

Wkxv/ d kljkhu suhuhqfh iru wkh suhvhqw/ vwhshu jurzwk ri h{shfwhg lqfrph/ ru d orzhu lqwhuhvw udwh hdfk pdnh ex hu vwrfn ehkdylru pruh olnho|1 Xqghu vrph dgglwlrqdo frqglwlrqv/ rqh fdq dovr vkrz wkdw fdvk rq kdqg dgplwv dq hujrglf glvwulexwlrq†

Lq d olih0f|foh prgho/ qr vxfk vlpsoh fkdudfwhul}dwlrq h{lvwv ehfdxvh lqfrph jurzwk dqg ehkdylru fkdqjh zlwk djh1 Lqvwhdg/ zh dgrsw d sudfwlfd dssurdfk dqg fdoo d krxvhkroge hu vwrfn krxvhkrog li lw vdyhv pruh iru suhfdxwlrqdu| prwlyhv wkdaq iru olih0f|foh +ehtxhw ru uhwluphqw, prwlyhv1 D krxvhkrog zkrvh vdyldj ghflvrlqv duh prwlydwhg sulpdulo| e| xqfhuwldqw| zloo kdyh wkh fkdudfwhulvwlvv ri ex hu vwrfn ehkdylru mxvw ghvfulehg1

61 •j| Na Nu 7 6 ,B|ja •N6jA|t .t| 6B| NA

Zh hvwlpdwh rxu prgho xvlqj d wzr0vwhs Phwkrq ri Vlpdwhg Prphqwv +PVP, dv iroorzv1 Jlyhq wkh sdudphwhuv ri wkh frqvxphu sureohp gh qhg lq wkh suhyrxv vhwlrq +suhuhqfhv/ lqfrph surfhvv/hwf1, zh fdq vroyh qxphulfdoo| iru wkh djh0ghshqghqw rswlpdo frqvxpswlrq uxohv1 Iru d jlyhq vhw ri frqvxpswlrq uxohv/ zh fdq qxphulfdoo| vlpdwh wkdvvrflidwhg h{shfwhg frqvxpswlrq dv d ixqfwlrq ri djh rqo|1 Wkh hvwlpdwlrq surfhgguh wkhq plqlpl}hv wkh glvwdqfh ehwzhhq wkh vlpdwhg frqvxpswlrq sur ohv dqg hpsulfdoo frqvxpswlrq sur ohv/ zklfk duh ghvfulehg lq vhwlrq 81

Dffruglqj wr vhwlrq 5/ frqvxpswlrq dw djh wiru lqglylgxdo l ghshqgv rq qrupdol}hg fdvk rq kdqg { c|/ wkh uhdol}dwlrq ri shupdqhw frpsrqhw ri lqfrph S c|/ dqg wkh sdudphwhuv ri wkh frqvxpswlrq sureohp/ zklfk zh qrz ghqrwh e| #5 U=Gh qlqj wkh yhfwrw ri vwdwh yduldeo| @ † c|> S c|,/ zh srvwxdwh wkh iroorzlqj gdwd0jhqhudwlrqj surfhvv iru hdfk dya/

+;, oqF c| @ oqF | +v c|>#, . c| @ oqF | † c|>#, S c|, . c|>

zkhuh c| lv dq lglrv|qfudwlf vkrfn wkdw uhsuhvhqvv fodvvlfdoo phdvxuhphqw huuru lq frqvxpswlrq ohyhov1 Zh duh lqwhuhvwhg lq hvwlpdwlrqj wkhq pdnlqj lqihuhqfh derxw frqvxpswlrq ehkdylru1 Zlwkrxw txdolw| sdqho gdwd rq frqvxpswlrq/ dvvhw dqg lqfrph iru lqglylgxdo krxvhkrogv/ gluhfw hvwlpdwlrq xvlqj +;, lv qrw srvleoh1

Zh gr krzhyhu revhuyh wkh dyhudjh ri orj0frqvxpswlrq dw hdfk djh/ gh qhg dv oqF_l $\frac{S}{U}$ oqF_{cl} zkhuh L₁ uhsuhvhqvw wkh qxpehu ri revhuydwlrqv dw d₁ oqF_{cl} lv gh qhg e| htxdwlrq +,;1 Wklv vxjjhvww wkdw zh fdq orrn gluhfwo| dw w₁ frqglwlrqdo h{shfwdwlrq ri orj0frqvxpswlrq dw hdfk djh=

$$+<, \quad oqF_l \# , \quad H \wedge oqF_l +v \# , \quad \# \ ` \ @ \ oqF_l +v \# , \quad gl \ | +v \# , \ >$$

zkhuh wkh xqfrqglwlrqdo fpxodwlyh glvwulexwlrq ri qrupdol}hg fdvk rq kdqg dqg shupdqhqw frpsr0 qhqw ri lqfrph dw djh w> | +v># , ghshqgv rq djh wdqg rq wkh sdudphwhu#1 Zh vhn wr hvwlpdwh wkh prgho iurp wkh iroorzlqj prphqw frqglwlrqv= $H \wedge +oqF \ > \# , \ ` \ @ \ >$ zkhuh #_f lv wkh wuxh sdudphwhu yhfwrw/ oqF @i oqF_{cl} $\frac{A}{j}$. dqg +oqF ># , 5 U^A/ zlwkw w₁ hohphqw=

$$+43, \quad | +oqF \ > \# , \ @ \ oqF_{cl} \quad oqF_l \ \# , \ =$$

Wklv dssurdfk pxvw dgguhvv wzr ixuwkhu frpsoldwlrqv1 Iluvw/ lw lv gl fxow wr hvwlpdwh dffxudwho| doo hohphqw#ilq rqh vwhs1 Wklv lv gxh lq sduw wr wkh frpsxwdwlrqdo frpsoh{lw| ri wkh sureohp dqg lq sduw wr wkh orvv ri lqirupdwlrq wkdw dyhudjlqj hqwd₁ whdg/ zh hpsor| d wzr0vwdjh hvwlpdwlrq surfhgxuh1 Zh sduwlrq# lqwr wzr vxe0yhfwrwv 5 U[&] dqg " 5 U^o> zkhuh lv d frpsdfw vhw1 Zh xvhdggwlrqdo gdwd dqg prphqw wr hvwlpdwh d uvw vwdjh dffruglqj wr H[^] + " , ` @ 3 zkhuh 5 U^o =^b Zh glvfxvv wklv vwhs lq wkh qh{w vhwlrq dqg ghqrwh wkh dvvrfldwhg uvw0vwdjh hvwlpdwru e| " a lru lqvwdqfh/ udwkhu wkdw hvwlpdwh wkh yduldqfh ri shupdqhqw dqg wudqvlwru| vkrfnv wr lqfrph iurp dyhudjh frqvxpswlrq dqg lqfrph sur ohv/ zkhuh lghqwl fdwlrq pljkw suryh gl fxow lq sudfwlfh/ zh xvhdwlp0vhulhv prphqw frqglwlrqv dqg wuxh krxvhkrog0ohyho sdqho gdwd rq lqfrph iurp wkh SVLG1

Wkh vhfraq frpsoldwlrq dulvhv iurp wkh dqdo|wldo frpsoh{lw| ri wkh xqfrqglwlrqdo h{shfwdwlrq +<,1 Wkh xqfrqglwlrqdo glvwulexwlrq iru wkh vwdwh yduldeohv dw djh># ,/ lv h{wuhpho| fxp0 ehuvrph wr hydroxdwh dqg ghshqgv rq wkh sdudphwhu1 Wv r yhufrph wklv gl fxow/ zh xvhd wkh Phwkrq ri Vlpodwhg Prphqw/ dv ghyhorshg e| Sdnhv dqg Sroodug +4<<, dqg Gx h dqg Vlq0 johwrq +4<<6,1 Xvlqj wkh exgjhv frqvudlqw +7,/ dqg wkh surfhvS₁uzh gh qh d phdvxudeoh wudqvlwlrq ixqfwlrq⁹ = U²{ U²{ { U^o \$ U² wkdw ghvfulehv wkh glqdpfv ri wkh vwdwh yduldeohv

•e

$v_{|n} @ f_{|n} > S_{|n}$, $@ 9 +v_{|n} > > \text{è, /}$ zkhuh $|n @ X_{|n} > Q_{|n}$, 1 Wklv wudqvlwlrq ixqfwlrq fdq wkhq eh xvhg wr uhzulwh wkh xqfrqglwlrqdo h{shfwdwlrq +< , =

$$+44, \quad oqF_{|} + > \text{è, } @ \quad] \quad oqF_{|} +v > \text{è, } gl_{|} +v > > \text{à,}$$

$$]]$$

$$@ \quad oqF_{|} +9 +v > > \text{è, } > \text{è, } gl_{|3} \cdot +v > > \text{è, } gl^{D+}, =$$

zkhuh I^{D+} , ghrwhv wkh fpxodwlyh glvwulexwlrq #

lurp +44, / zh dssur{lpdwh wkh wkhruhwlfdo xqfrqglwlrqdo h{shfwdwlrq xvlqj Prqwh0Fduor lqwhjud0 wlrq1 Zh jhqhudwh lqfrph vkrfnv iru O krxvhkrogv= $dqJ^2\{U^A 0ydoxhg vhtxhqfh ri udqgrp yduldeohv i a, j, \dots^u$ zkhuh $a @ +a > == a >$, / wkdw duh lghqwlfdoo| lqghshqghqwo| glvwulexwlrqdo| lq|0 wldo glvwulexwlrq +v, / fdqglgdwh dqg hwwlpdwhg /azh fdq jhqhudwh wkh sdwk ri vwdwh yduldeohv dffruglqj wr=

$$a_{c|n} @ 9 +a_{c|} > a_{c|n} > \text{è, } > ; 4 \quad w \quad W \quad 4 \quad dqg \quad 4 \quad o \quad O =$$

Wkh xqfrqglwlrqdo h{shfwdwlrq iurp wkh prgho $oF_{|} + > \text{à}$, lv wkhq vlpodwhg e|=

$$oqF_{|} + > \text{è, } \quad \frac{4}{O} [^u \quad oqF_{|} +a_{c|} > \text{è, } \# \quad oqF_{|} + > \text{è, } >$$

zkhuh frqyhujhqfh rffxuv dv O\$ 4 1

Iru dq| sdudphwhu yhwru 5 / zh fdq uhsodfh wkh wkhruhwlfdo h{shfwdwlrq ri frqvxpswlrq zlwkw lww vlpodwhg frxqwhusduw lq +43, =

$$a_{|} + oqF > \text{è, } @ \quad oqF_{c|} \quad oqF_{|} + \text{è, } =$$

Hwwlpdwlwlrq qrz vlpso| surfhhg v e| pdnlqj wkh vlpodwhghpslulfdo prphqvv dv forvh dv srvvleoh wr wkhlu wkhruhwlfdo ydoxhv xvlqj vdpsoh dyhudjihv=

$$j_{|} + \text{è, } @ \quad \frac{4}{L} [^u \quad a_{|} + oqF_{c|} > \text{è,}$$

$$@ \quad \frac{4}{L} [^u \quad oqF_{c|} \quad oqF_{|} + \text{è,}$$

$$@ \quad oqF_{|} \quad oqF_{|} + \text{è, } =$$

Qrwh wkdw zh gr qrw uhtxluh uhshdwhg revhuydwlrqv rq wkh vdph krxvhkrogrv1 R Xu vhfrrg0vwdjh hvwlpdwlrq surfhgxuh lv wkhq d phwkrq ri vlpodwhg prphqvv hvwlpdwlrq VVP , wkdw plqlpl}hv ryhu =

$$j + \text{æ}, Zj + \text{æ}, >$$

zkhu j + æ, @ j. > ==> j 5 U^A dqg Z lv d W{Vzhjkwljaj pdwul{1 Lq wkh fdvh zkhuZ @L_A>wkh lghqwlw| pdwul{/ wkh hvwlpdwlrq surfhgxuh lv htxlydohqwr wr plqlpl}laj wkh vxp ri vtxduh uhvlgxdov=

$$V + \text{æ}, @ \begin{matrix} [^A \\ | \cdot \end{matrix} \text{ oqF| oqF} + \text{æ}, ^2 =$$

Qrwh krzhyhu/ wkdw zh duh plqlpl}laj wkh vxp ri vtxduhg uhvlgxdov zlw d frqvwddw^W dqg wkdw dv|pswrwlf uhvxowv vwloo dssol dv orqj^U wkh qxpehuv ri revhuydwlrq dw djw^U jrhv wr lq qlw| iru hdfk djh1

Xqghu wkh uhjxodulw| frqglwlrqv vwdwhg lq Sdhv dqg Sroodug +4<;<, dqg Gx h dqg Vlqjohwraq +4<<6,/ wk^{PVP} hvwlpdwru^U lv erw frqvlvwhqwr dqg dv|pswrwlfdoos| qrupdoo| glvwulexwhg1 Krzhyhu/ rqh qhhgv wr fruuhfwr wkh vhfrrg0vwdjh hvwlpdwru ri wkh yduldqfh0fryduldqfh pdwul{ iru wkh uvw0vwdjh hvwlpdwlrq^U l xuwkhupruh/ dq dgglwlrqdo whup lv uhtxluhg wr dfrxqw iru wkh idfw wkdw zh kdyh vlpodwhg prphqvv udwkhu wkdw wkhruhwlfdo prphqvv1

Wkh yduldqfh0fryduldqfh hvwlpdwru dqg lww ghulydwlrq lv frqwdlqhg lq Dsshqgl{ E1 Zh xvhwzr dowhuqdwllyh zhjkwljaj pdwul{hv lq rxu hvwlpdwlrq1 Iluvw/ zh fkrv d pdwul{ wkdw/ zkloh qrw ixool rswlpdo/ grhv qrw ghshqg rq wkh wwhg prgho1 Wklv lv prwlydwhg e| wkh revhuydwlrq wkdw rswlpdoo| zhjkwljaj JPP hvwlpdwruv fdq zrurhq qlwh0vdpsoh eldv +vhh iru h{dpsoh Zhvw/ Zraj dqg Dqd0 wro|hy +4<<;,1 Vhfrrg/ zh hpsor| wkh rswlpdo zhjkwljaj pdwul{1 R Xu 50vwhs surfhgxuh surylghv d whvw ri wkh ryhulghqwlil}laj uhvwulfwlrqv lq wkh vhfrrg vwdjh/ zkfk lv ghulyhg lq dsshqgl{ E1

Lq sudfwlfh/ zh vlpodwh^U +>æ, e| uxqqqlaj O @ 53333 lqghshqghqwr lqfrph surfhvvhv iru 73 |hduv/ dqg frpsxw|lq lq hdfk |hdu wkh dvvrfldwhg frqvxpswlrq dqg fdvk rq kdqg1 Rqfh wkh rswlpxp lv irxqg/ wkh judglhqw ri wkh prphqwr yhfwrw lv hydroxdwhg qxphulfdoo| dqg wkh yduldqfh0fryduldqfh pdwul{ hvwlpdwhg1

71 8 it| 7|B}j .t| 6B| NA

Wkh uvw0vwdjh sdudphwhu|qfoxgh wkh jurvv uhdo diwhu wd{ lqwhuhvwUadwkh yduldaqhv ri wkh lqarydwlraqv wr wkh shupdqhqw dgg wudqvlwru| frpsrqhqw/2> 2> wkh lqlwldo glvwulexwlraq ri oltxlq dvvhvw dw djh 59/ zklfk fruuhsrqgv wr wkh ehjlqqlqj ri olih lq wkh hpsulfdo hwwlpdwlraq ri rxu prgho> dgg wkh lqfrph dgg idplo|0frpsrvlwlraq sur ohv1 Wkh frqvwuxfwlraq ri sur ohv lv glvfxvvhg lq wkh qh{w vhwlraq1

Iluvw/ zh hwwlpdwh wkh jurvv uhdo lqwhuhvwUadwkh wkh dyhudjh uhdo uhwxuq rq Prrg|*v DDD pxqlfildo erqgv dgg wkh Jurvv Grphvwlf Surgxfw Ipsolflw sulfh gh dwru iru shuvraqdo frqvxpswlraq h{shqglwxuhv xvhg wr gh dwh wkh frqvxpswlraq dgg lqfrph gdwd1 Ryhu wkh vdpsoh shulrg iurp Mdqxdul 4<;3 wr Pdufk 4<<6/ wkh dyhudjh uhdo lqwhuhvw udw/3|s6| frqhqw zlwk d vwdqgdug huuru ri 3 5;4=

Vhfrqg/ zh hwwlpdwh wkh yduldaqhv ri wkh shupdqhqw dgg wudqvlwru| frpsrqhqwv ri vkrfnv wr lqfrph/ 2 dgg 2/ xvlqj wkh SVLG dgg wkh JPP phwkrgrorj| ri Fduuroo dgg Vdpzlfm +4<<.:1 2* Wkh JPP surfhgxuh dgg gdwd hpsor|hg duh ghvljqhg wr hwwlpdwh wkh sdudphwhuv ri h{dfwo| wkh lqfrph surfhvv zh kdyh vshfl hg1 Zh uhgr wkhlu fdofxodwlraqv vlqfh wkh gh qlwlraqv ri rffxsdwlraq lq wkh SVLG dgg FH| gr qrw h{dfwo| ryhuods1 Zh dijuhjdwh wkh rffxsdwlraqdo jurxsv lq wkh SVLG lqwr fdwhjruhv wkdw ehwwhu pdwfk wkh jurxslqjv wkdw zh hpsor| lq wkh FHM|Wdeoh L glvsod|v wkh yduldaqhv ri wkh shupdqhqw dgg wudqvlwru| vkrfnv dfurvv rffxsdwlraq dgg hgxfdwlraq jurxsv1 Vlqfh phdvxuhphqhw huuru lq lqfrph pd| h{djjhudwh wkhvh yduldaqhv dgg vlqfh krxvhkrogv pd| kdyh dffhvv wr vrph lqirupdo lqvxudqfh phfdqqlpv zh vwxg| wkh lpsdfw ri orzhulqj wkhvh phdvxuhv lq vhwlraq 9161

Zh fdofxodwh wkh suredelolw| ri d wuxo| }hur lqfrph uhdol}dwlraq xvlqj wkh iuhtxhqf| ri wuxo| }hur lqfrph lq wkh SVLG/ dv uhsruwhg e| Fduuroo +4<<5,1 Wkhuhiruh/ zh @w/33635 zlwk d vwdqgdug huuru ri 3333:971

Ilqdool/ zh uhtxlh dq lqlwldo dvvhw ohyho iru krxvhkrogv1 Zh pdwfk erwk wkh w|slfdo ohyho dgg wkh glvwulexwlraq lq wkh srsxodwlraq e| hwwlpdwlqj wkh udwlr ri oltxlq zhdowk wr lqfrph iru krxvhkrogv ehwzhhq 57 dgg 5; |hdv rog1 Lq vlxodwlqj sur ohv/ zh dvvxph wkdw krxvhkrogv duh eruq zlwk dq lqlwldo ohyho ri dvvhwv uhodwlyh wr lqfrph gudzq iurp d orjrupdo glvwulexwlraq zlwk phd dgg vwdqgdug ghyldwlraq/ 2S wkdw zh hwwlpdwh iurp wkh FH| gdwd1 Wdeoh LL glvsod|v rxu hwwlpdwhv ri

wkh dyhudjh orj qrupdol}hg oltxlg zhdowk/ lww vwdqgdug ghyldwlrq lq wkh gdwd dqq wkh lpsolhg udwlr ri dyhudjh qrupdol}hg zhdowk el hgxfdwlrq dqq rffxsdwlrq jurxsv1

81 .k NAt 6V| NA BAa YAWN6j iNu ,jt

8141Sur oh Frqvwuxfwlrq Phwkrgrorj|1 Zh hvwlpdwh wkuhh sur ohv xvlqj krxvhkrog ohyho gdwd iurp djh 59 wr 98= d frqvxpswlrq djh0sur oh $F_{1,2S}^{SD}$ >wkh dyhudjh lqfrph sur oh $\backslash_{1,2S}^{SD}$ iurp zklfk zh ghulyh h{shfwhg lqfrph jurzwk $i_{1,2}^{SD}$ > dqq d sur oh iru wkh w|sldo vkliwv lq pdujlqdo xwlolw| $y_{1,2}^{SD}$ 1

Dvwxph wkdw wkh gdwd jhqhudwlqj surfhvv iru frqvwuxfwlrq lv dv gh qhg lq vhwlrq 5>iru hdfk krxvhkrog1 Gh qh $c_{1,2}$ $y_{1,2}^{SD}$ $F_{1,2}^{3,4}$ dv wkh pdujlqdo xwlolw| ri krxvhkrog ri djh w|urp wkh Hxohu htxdwlrq +8, ri vhwlrq 5/ zh nqrz wkdw=

$$c_{1,2} @ \frac{4}{U} c_{1,2} \cdot c_{1,2} >$$

zkhuh $c_{1,2}$ lv wkh +pxowlsoldwlyh, lqarydwlrq wr wkh pdujlqdo xwlolw| ri zhdowk dqq vdwlvhv $c_{1,2} @$ 4=

Lwhudwlqj edfnzdug xqwlo wkh eluwk |hdu/ dqq vxevwlwxw| $F_{1,2}^{3,4}$ rewdlq=

+45, $F_{1,2} @ \frac{y_{1,2}^{SD}}{y_{1,2}^{SD}} + U, \frac{13,2S}{4} |_{,2} \frac{3}{4} c_{1,2} f_{2S} S_{2S} > 59 w 98=$

Wkh uvw whup rq wkh uljkw kdqg vlgh uhsuhvhqvv wkh h hfw ri idplo| vl}h yduldwlrq ehwzhhq djh 59 dqq djh w|urp wkh vhwlrq whup uh hfwv wkh guliw lq pdujlqdo xwlolw| wkdw ghshqgv xsrq wkh glvfrxqw idfwru/ wkh uhdo lqwhuhvw udwh dqq wkh lqwhuhwpsrudo hodvwlflw| ri vxe@wkh whup whup uh hfwv wkh h hfw ri xqfhuwdlqwl dqq suhfdxwlrqdu| vdyqlqj xsrq frqvxpswlrq dw djhw>wkurxjk wkh sdvw dqq fxuuhqvw uhdol}dwlrqv ri lqarydwlrqv wr pdujlqdo xwlolw| $y_{1,2}^{SD}$ wr lqglylgxdo lqfrph lqarydwlrqv1 Wkh irxuwk whup f_{2S} > uhsuhvhqvv yduldwlrqv lq lqlwldo fdvk rq kdqg dfurvv krxvhkrogv1 Wkh qdo whup S_{2S} > fdswxuhv yduldwlrqv dfurvv krxvhkrogv lq wkh lqlwldo shupdqhw frpsrqhw ri lqfrph =

Wkh krxvhkrog0ohyho gdwd iurp wkh FH[gl huv iurp wkh derf| $F_{1,2}$ lq irxu zd|v1 Iluwv/ frq0 vxpwwlrq gdwd lv plvphdvwuhg1 Wkxv/ zh gr qrw h{sorlw wkh olplwhg sdqho qdwxuh ri wkh Vxuyh|

exw lqvwhdg uho| rą gdwd iurp uhshdwhg furvv0vhfwrqv1 Zh dvvxph wkdw pphdpxu huuru lą frqvxpswlrą lv fodvvlfdo dąq pxowlsolfdwlyh1

Vhfrąq/ |hdu0vshfl f hyhqvv/ vxfk dv wkh vwdjh ri wkh exvląhv f|foh/ d hfw wkh dyhudjh frą0 vxpswlrą ri krxvhkrogv lą hdfk |hdu1 Wkhvh h hfwv duh qrw ląfoxghg lą wkh prgho1 Rąq srvleoh mxvvl fdwlrą lv wkdw djjuhjdwh xfwxdwlrqv dfrxqw iru d vpdoo vkduh ri ląglylgxdo xąfhuwdlqwl +Slv0 fknh +4<<8,1 Zh dvvxph wkdw wlph0h hfwv hąwhu pxowlsolfdwlyho1

Wklug/ wkh prgho ri vhfwrą 5 grhv qrw dvvxph dąq yduldwlrą lą ląfrph dfurvv frkruwv1 Lą uhdolw/ krxvhkrogv vwduw oliv zlwkl gl huhqwh ohyhov ri zhdowk dąq shupdqąqwr frpsrqąqwr ri ląfrph dąq wkxv zlwkl gl huhqwh ohyhov ri frqvxpswlrą1 Vrph ri wkhvh gl huhqfhw duh gxh wr wkdw idfw wkdw krxvhkrogv eruq lą odwhu jhqhudwlrqv rą dyhudjh kdyh klijku ydoxhv iru wkhu ląlwdo shupdqąqwr frpsrqąqwr ri ląfrph1 Wklv lpsolv wkdw wkhuh lv d fruuhodwlrą ehwzhhq wkh h hfw ri eluwk |hdu dąq wkh h hfw ri djh rą frqvxpswlrą1 Krxvhkrogv revhuyhg dw djh vl{w/ vd/ duh eruq orąj ehiruh wkrvh zh revhuyh dw |rxąj djhv dąq vr kdyh rą dyhudjh orzhu olivwlv uhvrxfhw/ dąq orzhu ohyhov ri ląfrph dąq frqvxpswlrą dw hdfk djh1 Ląqrląj eluwk0|hdu h hfwv zrxog ohdg wr d qhjdwyh eldv lą rxu hvwlpdwh ri wkh vorsh ri ląfrph dąq frqvxpswlrą jurzwk1 Zh dvvxph/ dv lą pxfk ri wkh hpsulfdo olwhudwxuh wkdw hduqląjv kdyh d wlvh ląydułdąw djh0sur oh vr wkdw frkruw h hfwv d hfw rąq| wkh glvwdąfhw ehwzhhq wkh djh0sur ohv ri gl huhqwh frkruwv1 Zh ghfrpsr_S ląqr d frkruw h hfw/S_{2S}^K> dąq dą ląlv|ąfudwlv frpsrqąqwr S_{2S}=

Ilądool/ wkh prgho uhuhuv wr krxvhkrog frqvxpswlrą/ dgmxvwhg vr wkdw doo krxvhkrogv kdyh wkh vdph oliv0f|foh sdwwhuq ri idplo| vl}hy +} |,1 Zh wkhuhiruh hvwlpdwh idplo| vl}h dgmxvwphąqwr dąq dssol wkhp wr wkh frqvxpswlrą gdwd vr dv wr frqwuro iru zlwkl0djh ghprjudsklf yduldwlrą²

Gh qh P_{clc} > dv wkh revhuyhg krxvhkrog frqvxpswlrą lą wkh FH/ zkhuh ghqrwhv wkh |hdu1 Wdnląj orjv/ zh fdq zulwh oq^P_{clc} dv=

$$+46, \quad oq^P_{clc} @ \frac{4}{-} oq \frac{y + j_{cl}}{y + j_{2S}} \cdot \frac{4}{-} H_{2S}^k oq + U, |^{3 2S} |, 2. \quad 3 \cdot |_{cl}$$
$$\cdot oq S_{2S}^K \cdot \cdot$$

zkhuh uhsuhvąqwr d wlvh0h hfw dąq dswxuhv fodvvlfdo phdvxuhphąqwr huuru dv zhoo dv krxvhkrog khwhurjhqhlw| lą ląlwdo frąglwlrqv dąq ląqrydwlrqv wr pdujlądo xwłolw|1 Wkh vhfraq whup/ fdswxuhv

wkh dprxqw ri lqwhuwpsrudo vxevwlwxlwlrq dqq h{shfwhg suhfdxwlrqdu| vdylqj dqq lv mxvw d ixqfwlrq ri djh1

Zh hvwlpdwh gluhfwo| d olqhdv uhjuhvvlrq edvhg rq +46, ryhu krxvhkrogv zlwkw pdoh khdgv djhg 59 wr 981 Dv glvfxvvhg lq Ghdwrq +4<;8,/ lw lv qrw srvleoh wr vhsdudwho| lghqwlil wkh olqhdv frpsrqhqw ri wkh wlph/ djh/ dqq frkruw h hfwv lq htxdwlrq +45,^{2P} Zh pdnh wkh lghqwlilqj uhwulfwlrq wkdw wlph h hfwv duh gxh wr wkh vwdwh ri wkh uhjlrqdo hfrqrp| dqq fdsxuhg el wkh sduwldo fruhodwlrq ri frqvxpswlrq zlwkw wkh uhjlrqdo xqhpso|phqw udwh^{2S} Rxu hpsulfdo vshfl fdwlrq lv wkhq=

+47,
$$\sigma^2 @i \dots d \dots e \dots X \dots e \dots Uhw \dots D \dots \%$$

zkhuh wkh sdudphwhuv duh wr eh hvwlpdwhg/ d vhw ri idplo| gxpplhv/ d lv d frpsohwh vhw ri djh gxpplhv/ e lv d frpsohwh vhw ri frkruw gxpplhv +ohvv wkh plggoh rqh~~X~~ lv wkh Fhqvxv uhjlrq xqhpso|phqw udwh lq |hdv / Uhw lv d gxppl iru hdfk jurxs wkdw lv htxdo wr 4 zkhq wkh uhvsrqghqw lv uhwluhg dqq d uhvlgxdo wkdw fdsxuhv doo wkh lqglylgxdo h hfwv +phdvhuhphqw huuru/ lqlwldo qrupdol}hg fdvk rq kdqg/ lqlwldo lqfrph/hwf^{1,21}

Zlwkw wkhvh hvwlpdwhv/ zh uvw uhrqvwuxfw krxvhkrog0ohyho frqvxpswlrq gdwd xqfrqwdplqdwhg el frkruw dqq wlph h hfwv dqq uhpnylqj zlwklq djh yduldwlrq lq idplo| vl}h=

+48,
$$\sigma^2 @i | a \dots d a_2 \dots \bar{X} a_e \dots a \dots =$$

Wkxv iurp khuh rq/ zh zrun zlwkw krxvhkrog0ohyho frqvxpswlrq wkdw uhsuhvhqww wkh frqvxpswlrq ri wkh revhuyhg krxvhkrog zlwkw wkh w|slfdo djh0ghshqghqw idplo| vl}h/ idflqj wkh dyhudjh xqhp0 sor|phqw udwh^{2X}/ eruq lq wkh plggoh frkruw/ dqq qrw uhwluhg^{2H} Lw lv wklv yhuvlrq ri frqvxpswlrq wr zklfk vhwlrq 6 uhihuv1

Zh frqvwuxfw dyhudjh djh0sur ohv ri frqvxpswlrq el dyhudjlqj wklv gdwd dfurvv krxvhkrogv1 Vr wkdw wkh uroh ri suhfdxwlrqdu| vdylqj lv kljkoljkwhg/ zh glvsod| prvw ri rxu sur ohv dqq vlpwodwlrqv iru frqvxpswlrq shu fdsldw dqq lqfrph shu fdsldw1 Wklv sur oh lv frqvwuxfwwhg el uhsodflqj | zlwkw i =

$$\sigma^2 @ @ i a \dots da_2 \dots \bar{X} a_e =$$

Zh dovr frqvwuxfw wkuhh rwkhu vhwv ri sur ohv1 lluvw/ dqg lq wkh vdpd pdqqhu/ zh frqvwuxfw sur ohv iru lqfrph dqg wkh w|slfdo krxvhkrog idplo| vl}h1 Vhfrqg/ zh exlog vprrwk sur ohv xvlqj iwK0 rughu sro|qrpldov lq djh dqg |hdu ri eluwk lqvwhdg ri gxpplhv1 Odvwo| zh hvw|pdwh vlplodu sur ohv vhsdudwho| iru ydulrxv rffxsdwlrq dqg hgxfdwlrq vxejurxsv ri wkh srsxodwlrq1 Wkh frqvwuxfwlrq ri wkhvh vhulhv lv vlplodu wr wkdw ri frqvxpwlrq/ dqg ghvfulehg lq Dsshqgl{ F1

Wkh vprrwkhg sur ohv iru lqfrph dqg idplo| vl}h duh xvhg dv lqswv ri wkh prgho1 Iru lqvwdqfh/ uhfdoo wkdw\oq @ oq\$ c_l . oq X_c_l @ oq|_l . oq S_c_l3 . . oq Q_c_l . oq X_c_l=Diwhu uhprylqj frkruw dqg wlpd h hfww/ dqg dyhudjlqj ryhu d odujh qxpehu ri krxvhkrogv/L>zlwk wkh vdpd fkdudfwhulvwlfv=

$$\frac{4 [^U]}{L \cdot} oq\ c_l @ oq|_l \cdot \frac{4 [^U]}{L \cdot} oq\ c_l3 \cdot \frac{4 [^U]}{L \cdot} oqQ_c_l \cdot \frac{4 [^U]}{L \cdot} oqX_c_l \frac{4 [^U]}{L \cdot} oqX_c_l3 \cdot$$

Dssollqj wkh Odz ri Odujh Qxpehuv/ wkh suredelolw| olplwv ri wkh odvw wkuhh whup duh doo }hur1 Khqfh/ zh jhw

$$\# \text{ solp } \frac{4 [^U]}{L \cdot} oq\ c_l \frac{4 [^U]}{L \cdot} oq\ c_l3 \cdot @ oq|_l \ oq|_l \ oq\ c_l3 \cdot \$$$

Wkxv uvw gl huhqflqj rxu orj0dyhudjh lqfrph ohyhov jlyhv h{shfwhg lqfrph jurzwk udwhw|_1

8151Wkh Frqvxpshu H{shqglwxuh Vxuyh| dqg Rxu Xvh ri lw1 Zh xvhw wkh Frqvxpshu H{0 shqglwxuh Vxuyh| FH[, wr frqvwuxfw olihof|foh sur ohv ri frqvxpwlrq dqg lqfrph1 Wkh FH[frqwdlqv lqirupdwlrq derxw frqvxpwlrq h{shqglwxuhv/ ghprjudsklfv/ lqfrph dqg dvvhwv/ iru d odujh vdpsoh ri wkh XV srsxodwlrq1 Wkh Vxuyh| lv frqgxfwhg e| wkh Exuhdx ri Oderu Vwdwlvwlfv lq rughu wr frqvwuxfw edvnhwv ri jrrgv iru xvhw lq wkh edvhv iru wkh Frqvxpshu Sulfh Lqgh{/ dqg kdvd ehhq uxq frqwlqrxvov| vlqfh 4<;31 Zh xvhw gdwd iurp 4<;3 wr 4<<6 iurp wkh idplo| phpehu/ dqg ghwdlohg h{shqglwxuh ohv1 Wkh vxuyh| lv nqrzq wr kdyh h{fhoohqvw fryhudjh ri frqvxpwlrq h{shqglwxuhv/ wr kdyh uhdvrqdeoh gdwd rq oltxlg dvvhwv/ dqg wr kdyh lqfrph lqirupdwlrq ri prghudwh txdofw|1 Wkh vxuyh| lqwhuylhzv derxw 8833 krxvhkrogv hdfk txduwhu1 Lq d krxvhkrog*v uvw lqwhuylhv/ wkh FH[surfhgxuhv duh h{sodlqhg wr wkhp dqg lqirupdwlrq lv froohfwhg vr wkdw wkh| fdq eh dvljlqhg d srsxodwlrq zhlijkw1 Wkh| duh wkhq lqwhuylhzhg irxu pruh wlpdv +rqfh hyhu| wkuhh prqwkv, derxw ghwdlohg frqvxpwlrq h{shqglwxuhv ryhu wkh suhylrsv wkuhh prqwkv1 Lq lqwhuylhzv wzr dqg yh/ lqfrph lqirupdwlrq lv froohfwhg/ dqg lq wkh qdo lqwhuylhv dvvhw lqirupdwlrq lv froohfwhg1 Idplo|hv

urwdwh wkurxjk wkh surfhv/ vr wkdw derxw 58(ri krxvhkrogv ohdyh dqg duh uhsodfhg lq hdfk txduwhu1
Derxw kdoi ri doo krxvhkrogv pdnh lw wkurxjk doo wkh lqwhuyhlzv1

Hdfk krxvhkrog frqwulexwhv rqh gdwd srlqw wr rxu vdpsoh1 Iru hdfk krxvhkrog zh frqvwuxfw d
phdvxuh ri krxvhkrog lqfrph dqg frqvxpwlrq1 Edvhg rq wkh fkdudfwhulvwlvf ri wkh pdoh khdg ri wkh
krxvhkrog/ zh dvvljq wkh krxvhkrog wr dq rffxsdwlrq jurxs/ dq hgxfdwlrq jurxs/ d eluwk frkruw/ dq
lqwhuyhlz |hdu/ dqg d Fhqvxv uhjlrq1 Lq rughu wr rewdlq d kljk txdolw| vdpsoh wkdw kdvd wkh uhtxlugh
lqirupdwlrq/ zh gurs d vljq fdqw srulwlrq ri wkh gdwd dqg pdnh d vhulhv ri dgmxvwphqwv1 Ixuwkhu
ghvfulswlrq ri wkh gdwd suhsdudwlrq lv frqwdlqhg lq Dsshqgl{ G1 Zh qrwh khuh wkuhh pdmru srlqvw1

Iluw/ zh gurs krxvhkrogv wkdw duh fodvvl hg dv lqfrpsohwh lqfrph uhsruwhuv/ wkdw kdyh dq| ri
wkh fuxfldo yduldeohv plvvlqj/ ru wkdw uhsruw fkdqjhw lq djh ryhu wkh frxuvh ri wkh vxuyh| juhdwhu
wkdaq rqh |hdu ru qhjdwllyh1 Zh gr qrw dqdo|}h wkh jurxs ri krxvhkrogv zlwkd pdoh khdgv krogqlqj
ohvv wkdaq < |hdv ri vfkrrlqj gxh wr yhu| ihz |rxqjhu krxvhkrogv1 Vhfrqg/ zh gurs doo krxvhkrogv
zlwkd pdoh khdgv |rxqjhu wkdaq 59 ru roghu wkdaq :3/ jlyhq rxu irfxv rq wkh zrunlqj olih1 Zh duh ohiw
zlwkd mxvw xqghu 333 krxvhkrogv1 Wklug/ zh xvhlqglylgxdo dv zhoo dv idplo| ohyho lqirupdwlrq wr
fruhfw prvw ri wkh wsr0frglqj lq krxvhkrogv oderu lqfrph1

Zh frqvwuxfw phdvxuhv ri lqfrph dqg frqvxpwlrq wkdw pdwfk wkh frqfhsww lq wkh wkhruhwlfdo
prgho1 Iluw/ zh gh qh frqvxpwlrq dv wrwdo krxvhkrogv h{shqglwxuhv ohvv wkrvh rq hgxfdwlrq/
phglfdo fduh/ dqg pruwjdjh lqwhuhvw sd|phqwv1 Wkhvh fdwhjruhv ri h{shqglwxuhv gr qrw surylgh
fxuuhqw xwlo|w| exw udwkhuh duh hlwkhu looltxlq lqyhvwphqwv ru qhjdwllyh lqfrph vkrfnv1 Vr wkdw wkhvh
h{shqglwxuhv gr qrw lqfruhfwol dsshdu dv oltxlg vdyllqj/ wkh| duh dovr vxewudfwhg iurp lqfrph1

Lw vkrxog eh qrwhg wkdw rxu prgho uhuhv wr wrwdo frqvxpwlrq dw dqxdo iuhthqflhv1 Lq wkh
gdwd/ zh duh dyhudjlqj wrwdo h{shqglwxuhv/ qrw frqvxpwlrq/ dfurvv d odujh qxpehu ri lqglylgxdov
dqg ornlqj dfurvv rqh0|hdu krul}rqv1 Khqfh/ zh vxvshfw wkh glvwlqfwlrq ehwzhhq gxudeohv dqg
qrqgxudeohv lv qrw olno| wr pdwwhu pxfk1 Ixuwkhu/ vlqfh wkh ex hu vwrfn prgho jlyhv vwurqj
suhglfwlrq derxw frqvxpwlrq wudfnlqj lqfrph/ lw lv lpsruwdqw qrw wr euhdn wkh frqvxpwlrq0
lqfrph olqn zkhq vwxgllqj frqvxpwlrq1 Zh zloo dovr uhsruw wkh uhvxowv zkhq frqvxpwlrq lv
uhvwulfwhg wr qrqgxudeohv dqg lqfrph lv gh qhg dv glvsrvdeoh lqfrph diwhu gxudeoh h{shqglwxuhv1

Rxu phdvxuhv ri glvsrvdeohv oderu lqfrph lv frpsulvhg ri diwhu0wd{ idplo| lqfrph ohvv Vrfldo
Vhfxulw| wd{ sd|phqwv/ shqvlrq frqwulexwlrq/ diwhu0wd{ dvvhw dqg lqwhuhvw lqfrph dqg/ dv qrwhg/

wkrvh h{shqglwxuhv vxewudfwhg iurp frqvxpswlrq1 Wkh uvw wzr dgmxxvwphqvw duh vdylqj lq looltxlg irup dqq vr duh dydlodeoh wr wkh krxvhkrog rqo| diwhu uhwluhphqw1 Zh uhpryh dvvhw lqfrph vlqfh wkh lqswx wr rxu wkhruhwlfdo prgho lv d sur oh ri lqfrph qhw ri oltxlg dvvhw uhwxuqv1 Frqvlvwhqz zlwkwkh vslulw ri rxu prgho/ doo lwhpv uhpryhg iurp lqfrph lqroyh d odujh dprxqw ri frpplwphqz dqq duh kdug wr vxevwlwxwh lqwhuwghpsrudoo|1

Ilqdoo| zh sxw doo gdwd lqwr uhdo 4<;: grooduv xvlqj wkh Jurvv Grphvwlf Surgxfw Ipsolflw sulfh gh dwru iru shuvrqdo frqvxpswlrq h{shqglwxuhv1

8161Olih F|foh Sur ohv1 Iljxuh 5 suvhqvw frqvxpswlrq +udz dqq vprrwkhg, dqq lqfrph sur ohv iru rxu hqwluh vdpsoh zkhq wkh idplo|0v|}h lv khog frqvwddw ryhu wkh olih0f|foh1 Hyhq diwhu fruuhwfwljq iru wkh h hfwv ri frkruw/ wlp/ dqq idplo/ erwk sur ohv duh vwloo kxps vkdshg dqq wudfn hdfk rwkhu hduo| lq olih1 Frqvxpswlrq olhv deryh lqfrph ryhu wkh odwh wzghwlvh1 Jlyhq wkdw wkh FH[zhdowk gdwd/ dqq ehwwhu krxvhkrog zhdowk vxuyh|v/ vkrz prghvw lqfuhdvhv lq oltxlg zhdowk ryhu wkhvh udqjhv/ wklv ihdwxuh vhhpv olno| gxh wr plvuhsruwlqj ri lqfrph ru frqvxpswlrq1 Rqh srvlelolw| lv xqghuhsruwlqj wkh dvvlvwdqfh wkdw lv surylghg e| lqwhujhqhudwlrqdo wudqviuv hduo| lq olih1 Diwhv wkhvh uvw ihz |hdv/ frqvxpswlrq ulvhv zlwkw lqfrph iurp djh 63 wr djh 78/ zkhq frqvxpswlrq gursv vljq| fdqwo| ehorz lqfrph1 Wklv wudfnlqj lv krzhyhu d orw ohvv wkdw lv revhuyhg lq sur ohv frqvwuxfwhg e| vlpsol dyhudjlqj furvv0vhfwlrqv ehfdxvh zh frqwuro iru fkdqjhv lq idplo| vl}h dqq frkruwv h hfwv1

Iljxuhv 6 dqq 7 jlyh vrph hylghqfh wkdw frqvxpswlrq dqq lqfrph wudfn hdfk rwkhu dfurvv vxe0 jurxsv ri wkh srsxodwlrq gh qhg e| hgxfdwlq dqq rffxsdwlq jurxsv1 Wkhvh judskv duh vrphzkdw qrlv|1 Krzhyhu/ ghvslwh wkh qrlv/ rqh fdq vhh wkdw wkh rffxsdwlq dqq hgxfdwlq jurxsv zlwkwkh prvw surqrxqfhg kxpsv lq lqfrph suvhqz wkh prvw surqrxqfhg kxpsv lq frqvxpswlrq1 lkuwkuh/ zh fdq irupdoo| uhmhfw wkh qxoo k|srwkhvlv wkdw wkh frqvxpswlrq sur ohv duh dw1 Wklv lv hvvhqwdoo| d qrz vwdqgdug whvw ri wkh olqhdul}hg frqvxpswlrq Hxohu htxdwlq/ dv vwzghg e| Dwwdqdvlr dqq Zhehu +4<<8,/ Oxvdugl +4<<9,1

Rxu sur ohv gl hu voljkwo| iurp wkh uhvowv ri Dwwdqdvlr dqq Eurzqlqj +4<<8, dqq Dwwdqdvlr dqq Zhehu +4<<8,1 Wkhvh sdshuv hpsor| d odujhu vhw ri suhuhqfh vkiwhuv= rafh frqwuroolqj iru wkhvh/ frqvxpswlrq lv vprrwkhu dqq wkh FHT00FK lv qrw uhmhfwhg1 Lq Dwwdqdvlr dqq Zhehu +4<<8, dqq lq wkh olqhdul Hxohu0htxdwlq dssurdfk jhqhudoo| xvhg lq plfur gdwd/ suhfdwlrqdu| h hfwv duh

rplwwhg vr wkdw suhuhqfh vkliwhuv devrue/ fruhfwo| ru lqfruhfwo| yduldlwrq lq frqvxpswlrq wkdw zh dwuwlexwh wr xqfhuwdlqw|1 Fohduo| doorzljaj iru hqrxjk suhuhqfh yduldlwrq fdq h{sodlq wkh olih0 ffoh fruhodwlrq zlwkrxw uihuhqfh wr vxewdqwldo krxvhkrog0ohyho xqfhuwdlqw|1 Wkh dssurdfk ri wklv sdshu lv wr dgmxxv rqo| iru wkh vl}h ri wkh krxvhkrog dqg h{dplqh krz suhfdxwlrqdu| vdylaj h{sodlqv wkh revhuyhg frqvxpswlrq ehkdylru1 Rxu phwkrgrorj| kdv wkh srwhqwldo wr qhvw dqg whvw erwk h{sodqdwlrqv1 Lw lv dovr zruwk qrwlj wkdw rxu sur ohv uhsuhvhqw wrwdo frqvxpswlrq h{shqglwxuhv udwkhu wkdq qrq0gxudeoh frqvxpswlrq1

91 -jt ,lt

Zh uvw hvwlpdwh rxu prgho iru wkh dyhudjh krxvhkrog dqg glvfxvv wkh lpsoldwlrqv ri wkh wwhg vwuxfwxudo prgho iru krxvhkrog ehkdylru1 Zh wkhq wxuq wr glvdjjuhjdwhg uhvxowv/ e| hgxfdwlwrq dqg rffxsdwlwrq jurxsv/ dqg wr hydroxdwlqj wkh urexvwqhvv ri rxu qglqjv1

9141llqglqjv iurp wkh Hqwluh Vdpsoh1 Dv d xvhibo ehqfkdun/ zh ehjlq e| dvnqaj zkdw wkh vwdqgdug Olih0F|foh wkhru| zrxog suhglfw/ dvvxplqj|zd| doo xqfhuwdlqw|1 Wr jlyh wkh ehvw fkdqfh wr wkh FHT0OFK/ zh shuirup uvw gl huhqfh hvwlpdwlrq/ qrw dvnqaj lw wr w wkh phdq ri wkh frqvxpswlrq sur oh1 Xqghu fhuwdlqw/ htxdwlrq +5, krogv/ lpsollqj/ diwhu frqwuroolqj iru lqglylgxdo fkdudfwhulvwlvf/ d frqvwdqw jurzwk udwh ri frqvxpswlrq ryhu wkh zrunqaj shulrg=

$$oq F_1 @ \frac{4}{-} oq +U , =$$

Zh hvwlpdwh $\frac{1}{4} oq +U$, iurp wkh frh filhqw rq djh lq d ohdvw0vtxduhv uhjuhvvlrq ri wkh sur oh ri frqvxpswlrq zlwkw idplo| vl}h/ frkruw/ dqg wlph h hfwv uhpryh/ dv glvsodlqhg lq lljxuh 51 Wklv surfhgxuh vhhpv wulyldo rqo| ehfdxvh ri rxu hduolhu h ruwv wr uhpryh fkdqjlqj idplo|0vl}h dqg frkruw h hfwv1 Lw lv suhflvho| wklv vlpsolflw| wkdw jlyhv wkh FHT0OFK lww srzhu1 Iurp rxu hvwlpdwh/ zh xvhw wkh ghowd phwkrq wr uhfryhu wkh glvfrxqw idfwru dqg lww vwdqgdug huuru/ xvlqj wkh uhdo lqwhuhvw udwh hvwlpdwhg lq wkh suhylrxv vhwlrq dqg d fkrllf ri wkh frh filhqw ri uhodwlyh ulvn dyhuvlrq ri xqlw|1 Zh hvwlpdwh d glvfrxqw udwh ri ~~37~~ (zlwkw d vwdqgdug huuru ri ~~5~~;4(>diwhu dgmxxvwlj wkh hvwlpdwh iru erwk uvw rughu vहुldo fruhodwlrq dqg uvw0vwdjh xqfhuwdlqw|1 Vlqfh wkh FHT0OFK frqvxpswlrq sur oh lv dw dqg wkh gdwd dv vkrzq lq lljxuh 5 lv kxps0vkdshg/ wkh fhuwdlqw| prgho shuirupv srruo| zkhq lw frphv wr h{sodlqaj wkh glqdpflv ri frqvxpswlrq dfurvv wkh olih ffoh1

Wdeoh LLL suhvhqvv wkh uhvrowv ri hwwlpdwlqj rxu vwuxfwxudo prgho zlwk lqfrph xqfhuwdlqwl1 Wkh dvvrfldwhg frqvxpswlrq uxohv duh glvsodlhg lq llijuh 4/ sdqho D1 Wkh uvw froxpg ri Wdeoh LLL glvsodlv wkh uhvrowv zkhq wkh lqlwldo/ urexvw zhlijkwlj pdwul{ lv hpsorlhg> wkh vhfraq glvsodlv wkh uhvrowv zlwk wkh rswlpdo zhlijkwlj pdwul{1 Wkh vwdqgdug huuruv duh fdofoxodwhg zlwkrxw +D, dqg zlwk +E, wkh dgmxxvwphqw iru xqfhuwdlqwl lq wkh sdudphwhu a^2/a^2 dqg a^2/a^2 iurp wkh uvw0 vwdjh hvwlpdwlqj1 Wkh vwdqgdug huuruv wslfdool lqfuhdvh/ rafh wkh xqfhuwdlqwl lq wkh uvw0vwdjh sdudphwhu lv dffrxqwhg iru1

Wkh glvfrxqw udwh lv hvwlpdwhg ~~gndw~~ ryhu irxu shufhqw/ zklfk lv zlwklq frayhqwlraqdo vljqf fdqfh ohyhov ri wkh uhdo lqwhuhvw udwh shufhqw1 Lw lv zruwk qrwlj wkdw wkh glvfrxqw udwh lv zlwklq d uhdvrqdeoh udqjh1 Xvlqj lqirupdwlqj rq wkh hodvwlflwl ri dvvhvw zlwk uhvshfw wr xqfhuwdlqwl/ Fduuroo dqg Vdpzlfm +4<<:, hvwlpdwh d glvfrxqw udwh lq wkh ylfqlwl ri 4348(dqg dujxh wkdw hyhq kljkhu glvfrxqw udwhv duh qhghg wr udwlrqdol}h wkh qglqjv ri Kxeedug/ Vnlqqhu/ dqg Jhoghv +4<<7,1 Rxu orzhu glvfrxqw udwh/ krzhyhu/ grhv qrw lpsol wkdw krxvhkrogv duh qrw lpsdwlhqw hqrxjk wr jhqhudwh ex hu vwrfn ehkdyru/ dv zh zloo vkrz vkruwoj1

Zkhq zh hpsorl dq rswlpdo zhlijkwlj pdwul{/ wkh suhflvlrq zlwk zklfk zh hvwlpdwh wkh glvfrxqw udwh ghfolqhv vljqf fdqwoj frpsduhg zlwk wkh yduldqfh hvwlpdwh wkdw grhv qrw fruhfw iru xqfhuwdlqwl derxw wkh uvw vwdjh sdudphwhu +D,/ exw lqfuhdvhv frpsduhg wr wkh 50vwhs yduldqfh hvwlpdwh +E,1 Wkhuh lv d jurzlj olwhudwxuh wkdw txhvwlrqv wkh vpdoo0vdpsoh ydolglwl ri rswlpdo zhlijkwlj gxh wr wkh fruhodwlqj ehwzhhq sdudphwhu xqfhuwdlqwl dqg wkh zhlijkwlj pdwul{1 Rswlpdo zhlijkwlj fdq eh pruh h flhqw> lw fdq dovr eh pruh eldvhg1 Wkxv zh uhsruw erwk1

Wkh lqwhuwpsrudo hodvwlflwl ri vxevwlwxwlrq lv hvwlpdwhg txlwh wljkwol/ dqg lv wkh vroh sdudphwhu hvwlpdwh wkdw ghshqgv vljqf fdqwoj rq wkh zhlijkwlj pdwul{ hpsorlhg1 Wkh hvwlpdwhg uhwluphqw uxoh vxjjhvww d pdujlqdo surshqvlwl wr frqvxph rxw ri zhdowk dw uhwluphqw ehwzhhq 9 dqg : shufhqw/ dovr txlwh uhdvrqdeoh1 Iru lqvwdqfh/ lq wkh fdvh ri ixoo fhuwdlqwl diwhu uhwluphqw dqg qr fkdqjh lq wkh xwloelwl vkliwhu wkh pdujlqdo surshqvlwl wr frqvxph lv jlyhq e|

$$4 a^{*4} @ 4 a^{*4} @ :38 \text{ shufhqw/}$$

jlyhq rxu hvwlpdwhv ri > dqg U dqg vhwvlqj ghdkw dw djh ;;1 Wkxv wkh hvwlpdwh lv yhu| pxfk lq olqh zlwk vlpsoh suhglfwlrqv ri wkh prgho1

Ilqdoof/ xqghu rxu dvvxpswlrqv/ wkh udwlr @. surylghv dq hvwlpdwh ri wkh udwlr ri looltxlq zhdowk wr wkh shupdqhqw frpsrqhqw ri lqfrph dw uhwluphqw1 Wkh srlqw hvwlpdwh lv h{wuhpho| vpdoo/ durxqg 5(=Wkh uvw wklqj wr qrwh lv wkdw wklv udwlr lv lpsuhflvho| hvwlpdwhg dqg zh fdqqrw uhmhfw pruh uhdvrqdeoh ydoxhv1 Dq dowhuqdwyh lqwhusuhwdwlrq,iru dkwz krxvhkrogv kdyh d orz surshqvlw| wr frqvxph rxw ri looltxlq zhdowk dw uhwluphqw1

Lw vkrxog eh qrwhg wkdw erwk hvwlpdwlrq phwkrqv uhmhfw wkh ryhulghqwlilqj uhvwulfwlrqv dw wkh 8(ohyho1 Wkh <8(fulwldo ydoxh iru"o +69, lv 834 dqg wkh fklOvtxdud dozd|v h{fhgv 4831 Wklv lv qrw hqwluho| vxusulvlqj/ jlyhq wkh qxpehu ri prphqvv zh xvh +73, dqg wkh ihz sdudphwhuv ri wkh prgho1 Wkh hvwlpdwhg prgho vkrxog vwloo eh wdnhq vhuhrxvo| krzhyhu1 Dv zh qrz glvfxvv/ wkh prgho grhv pxfk ehwwhu lq dq hfrqrlf vhaqwkdw wkh FHT00FK prgho zlwkw zklfk wklv vhwlrq ehjqlv1

Zlwkw rxu hvwlpdwhv lq kdqg/ zh fdq dgguhvv krz zhoo wkh vwrfdvwlf prgho wv wkh olih0f|foh frqvxpswlrq sur oh1 Wkh uvw sdqho ri lljxuh 8 sorwv wkh vlpdwhg dqg dfwxdw frqvxpswlrq gdwd dorqj zlwkw wkh lqfrph sur oh1 Wkh vwrfdvwlf olih0f|foh prgho grhv d pxfk ehwwhu mre dw wwlqj wkh frqvxpswlrq sur oh wkdw wkh frqvxpswlrq sur oh zlwkw frqvwdqw jurzkw udwh $\frac{1}{4}$ oq +U , wkdw zrxog rewdlq xqghu wkh fhuwdlqw|0htxlydohq1 Wkh frqvxpswlrq sur oh iurp wkh wwhg prgho wudfnv lqfrph xqwo durxqg djh 73 dqg wkhq idoov dv krxvhkrogv vwduw lqfuhdvlqj wkhlu oltxlg zhdowk krogqlq lq suhsdudwlrq iru uhwluphqw1

Lq wzr sodfhv krzhyhu/ wkh prgho w lv qrw jrrg1 Iluvw/ dfwxdw frqvxpswlrq h{fhgv vlpdwhg frqvxpswlrq hduo| lq olih1 Vlqfh/ dv zh glvfxvvhg lq wkh suhfhglqj vhwlrq/ lq wkh gdwd frqvxpswlrq h{fhgv lqfrph hduo| lq olih/ wklv pd| uh hfw d zhdnqhvv ri wkh gdwd/ udwkhv wkdw wkh prgho1 Vhfrqg/ wkh dfwxdw frqvxpswlrq sur oh lv voljkwo| dwwhu dqg shdnv voljkwo| odwhu1 Dsdw iurp wkhvh ihdwxuhv krzhyhu/ wkh wljkv vwuxfwxuh lpsrvhg e| wkh prgho surgxfhv jrrg suhglfwlrqv lq whupv ri frqvxpswlrq glqdpfv1

Zk| duh zh deoh/ zlwklq wkh frqwh{w ri rxu prgho wr rewdlq vxfk wljkv hvwlpdwhvB Wkh vhwlrq sdqho ri lljxuh 8 sorwv ydulrxv vlpdwhg sur ohv iru ydoxhv r| rqh shuffhqw dzd| iurp wkh srlqw hvwlpdwh1 Wklv fruhvsrqgv wr fkrflhv ri @ 3<7<; dqg @ 3<9<;/ ru htxlydohqwo| wr d glvfrxqw udwh ehwzhq=4, dqg 8=5; shuffhqw1 Lw lv reylrxv wkdw wkh sur ohv duh yhu| vhaqlwlyh wr vpdoo yduldwlrqv lq wkh glvfrxqw idfwru/ krogqlq frqvwdqw1 Zlwkw d kljkhu glvfrxqw idfwru/ wkh krxvhkrogv lv zloolqj wr vdyh pruh dqg hduolhu iru uhwluphqw sxusrvhv1 Wkh frqvxpswlrq sdwk h{klelww ohvv

ri d kxps vkdsh/ dqg pdl hyhq eh lqfuhdvlqj ryhu wkh hqwluh zrunlqj olih/ dv lv wkh fdvh lq wklv llijxuh1 Rq wkh rwkhu kdqg/ iru pruh lpsdwlhqw krxvhkrogv/ frqvxpswlrq sdudoohov lqfrph xqwlo pxfk odwhu lq olih dqg wkhq idoov pruh suhfslwrxvo| wr exlog dvvhwv iru uhwluhphqw1 Wklv lpsolhv d vwurqjhu frqfdylw| ri wkh frqvxpswlrq sur oh1 Wkxv/ rxu phwkrq |lhogv wljkw hwwlpdwhv ri wkh glvfrxqw idfwru ehfdxvh wkh glvfrxqw idfwru lv dq lpsruwdqw ghwhuplqdw ri wkh kxps vkdsh lq wkh frqvxpswlrq sur oh1

Zh qrz wxuq wr wkh txhvwlrq ri krz krxvhkrog ehkdylru fkdqjhv ryhu wkh olih f|foh1 Gh qh wkh wdujhw ohyho ri fdvk rq kdqg dw d{shfw|>dv wkh ohyho dw zklfk fdvk rq kdqg lv h{shfwhg wr uhpdlq xqfkdqjhg iurp rqh |hdu wr wkh qh{w +Fduuroo +4<<.:,=

$$\{ | @H | \{ | n * r | @ \{ | `$$

Iru odujh lqlwdo ohyhov ri fdvk rq kdqg + A { |, / krxvhkrogv fkrvvh kljk ohyhov ri frqvxpswlrq dqg wkxv ixwxuh ohyhov ri fdvk rq kdqg duh rq dyhudjh orzhu1 Iru orz ohyhov ri fdvk rq kdqg ?- { |, / krxvhkrogv frqvxph ohvv wkdw wkh lqfrph wkh| h{shfw wr uhfhlyh dqg vr rq dyhudjh dffpxodwh fdvk rq kdqg1

Ilijxuh 9 suvhqww wkh wdujhw ohyho ri fdvk rq kdqg iru frqvxphuv djhg 575>iurp wkh wwhg prgho1 Rqh fdq vhh iurp wkh judsk wkdw wkh wdujhw ohyho ri fdvk rq kdqg ~lqfoxglqj fxuuhqw lqfrph~ uhpdlqv vpdoe hduo| lq olih/ durxqg 54 wlpdv wkh shupdqhw frpsrqhw ri lqfrph1 Durxqg djh 73/ wklv wdujhw lqfuhdvhv vxevwdqwdoo| dv frqvxphuv ehjlq wr exlog wkhlu zhdowk iru uhwluhphqw1 Wklv llijxuh vkrzv d gudpdwlf fkdqjh lq ehkdylru1 Zkhq wkh wdujhw ohyho ri oltxlq zhdowk lv vpdoe/ krxvhkrogv ehkdyh dv cex hu vwrfn* frqvxphuv1 Jrrg lqfrph vkrfnv duh frqvxphg dzd| dqg edg lqfrph vkrfnv duh |pshuifwo| vprwkhwg1 Krxvhkrog frqvxpswlrq forvho| iroorzv krxvhkrog lqfrph1 Vwduwlqj durxqg djh 6;/ djhqwv ehjlq wr dffpxodwh dvvhwv iru uhwluhphqw1 Dv krxvhkrogv pryh lqwr wkhlu iruwlv/ wkh| exlog vljq| fdqw dprxqwv ri oltxlq zhdowk1 Wklv uhwluhphqw zhdowk doorzv krxvhkrogv wr vprwk kljk iuhtxhf| pryhphqwv lq lqfrph vr wkdw wkhlu ehkdylru pruh forvho| plplfv wkdw ri fhuwdlqw|Ohtxlydohqw frqvxphuv1

Zh qh{w ghfrpsrvh wrwdo vdylqj dqg zhdowk dw hdfk djh lqwr wkdw gulyhq e| olihOf|foh frqvlq0 hudwlrq dqg wkdw dggwlrqdo dprxqw gulyhq e| wkh suvhqfh ri xqlqvxudeoh ulvn1 Rxu suhylrxv glvfxvvlrq pljkw ohdg wkh uhdghu wr wklqn wkdw djhqwv kdyh qr frqfhuq iru uhwluhphqw zkhq wkh| duh

lrxqj dqg qr frqfhuq iru oderu lqfrph xqfhuwdlqwj odwhu lq olih1 Wklv lv lqfruhfw vlqfh frqvxphuv duh udwlrqdo dqg shuihfwoj iruhvhh wkhlu uhwluhphqw qhghv1 Iluvw/zh gh qh vdylqj lq oltxlq zhdowk dv wkh glvfrxqwhg yduldwlrq lq qdqfido zhdowk iurp rqh shulrg wr wkh qh $\{w=$

$$V_{cl} @ Z_{cln} \quad Z_{cl}, @U@ \text{U} \quad 4, @U Z_{cl} . \setminus_{cl} \quad F_{cl} =$$

Vdylqj lv htudo wr lqyhvwphqw lqfrph ~oltxlq dqg looltxlq~ soxv oderu lqfrph plqzv frqvxpswlrq1 lurp rxu hpsulfdo sur ohv lq lljxuh 8/ lw iroorzv wkdw krxvhkrogv vdyh uhodwlyho| olwwoh dqg frqvxph urxjko| wkhlu lqfrph rq dyhudjh hduo| lq olih1 Vhfrqg/ dw wkh hwwlpdwhg sdudphwhuv/ zh frpsxwh wkh frqvxpswlrq sdwk/ F_{cl}^u / wkdw zrxog rffxu li doo lqfrph ulvn zhuh srrohq/ vr wkdw iru doo krxvhkrogv $\setminus_{cl}^u @ H_2S \setminus_{cl}^u$ / exw wkh krxvhkrog*v hqylurqphqw rwkhuzlvh uhpdlqv xqfkdaqjh $\{1$ llqdoos/ zh gh qh olih0f|foh vdylqj dv wkh gl huhqfh ehwzhhq wrwdo lqfrph dqg olih0f|foh frqvxpswlrq=

$$V_{cl}^u @ Z_{cln}^u \quad Z_{cl}^u @U@ \text{U} \quad 4, @U Z_{cl}^u . \setminus_{cl}^u \quad F_{cl}^u$$

Suhfdxwlrqdu| vdylqj lv wkh frpsohphqw ri olih0f|foh vdylqj1

Wkh uvw sdqho ri lljxuh : sorwv wkh suhfdxwlrqdu| dqg olih f|foh0oltxlq vdylqj ri wkh dyhudjh krxvhkrog1 Jlyhq wkh hwwlpdwhg glvfrxqw udwh dqg wkh sur oh ri h{shfwhg lqfrph/ lrxqj frqvxphuv idflqj qr lqfrph ulvn zrxog olnh wr eruurz odujh dprxqww/ vr olih0f|foh vdylqj lv qhjdwylyh hduo| lq olih1 lrxqj krxvhkrogv lq idfw krog d srvlwlyh ex hu vwrfn ri zhdowk lq uhvsrqvh wr lqfrph ulvn/ vr wkdw suhfdxwlrqdu| vdylqj lv srvlwlyh hduo| lq olih1 Lq wkh hduo| wr plg iruwlhv/ lq dffrugdqfh zlwkrxu suhylrxv glvfxvvlrq/ olih0f|foh vdylqj ehfrphv odujhu wkdaq suhfdxwlrqdu| vdylqj1 Krxvhkrogv ehjlq wr exlog wkhlu oltxlq zhdowk iru uhwluhphqw ~~soxv~~ Dv dvvhw ohyhov lqfuhdvh/ wkh h{shfwhg yduldaqh ri frqvxpswlrq ghfolqhv/ ghfuhdvlqj wkh suhfdxwlrqdu| vdylqj prwlyh1 Vlqfh krxvhkrogv wkdw idfh lqfrph xqfhuwdlqwj vdyh pruh hduo| lq olih gxx wr ulvn/ wkh| duh deoh wr frqvxph pruh dqg vdyh ohvv zkhq roghu/ ohdglqj wr qhjdwylyh suhfdxwlrqdu| vdylqj odwh lq olih1 Wklv glvfxvvlrq klijkoljkwv wkh lpsruwdqfh ri wkh lqfrph sur oh1 Devhqw lqfrph jurzkw/ krxvhkrogv zrxog vwduw vdylqj iru uhwluhphqw hduo| lq olih1 Wklv oltxlq zhdowk zrxog grxeoh dv d ex hu vwrfn1 Wkxv wkhuh zrxog eh qr dggllwrqdo suhfdxwlrqdu| vdylqj prwlyh dqg krxvhkrog ehkdylru zrxog eh zhoo dssur{lpdwhg el wkh FHT0OFK1

Sdqho E ri Ilijxuh : glvsodlv dyhudjh wrwdo/ olih0f|foh/ dqg suhfdxwlrqdu| zhdowk/ rewdlqhg e| lqwhjudw|lqj wkh fruuhsrqq|lqj vdy|lqj sur ohv1 Zh revhuyh uvw wkdw wrwdo zhdowk dozd|v h{fhhgv olih f|foh zhdowk/ uh hfw|lqj wkh suhvhqfh ri xqlqvxudeoh lqfrph ulvn1 Vhfrqg/ olih0f|foh zhdowk lv qhjdwl|yh hduo| lq olih/ uhsuhvhqwlqj wkh ghvluh ri krxvhkrogv idflqj qr ulvn wr eruurz dv mxvw glvfxvvhg1 Wklug/ vlqfh |rxqj krxvhkrogv dfwxdoo| krog vpdoo srvlw|yh dprxqvw ri zhdowk rq dyhudjh/ suhfdxwlrqdu| zhdowk hduo| lq olih lv vxevwdqwldo/ shdn|lqj dw >343 ru 69 wlp|hv fxuuhqw lqfrph e| djh 751 llqdoo| dv krxvhkrogv jhw roghu/ olih0f|foh zhdowk ehfrphv yhu| vxevwdqwldo/ uhdfklqj >53 ru 9= wlp|hv fxuuhqw lqfrph dw wkh djh ri uhwluhphqw/ zkloh suhfdxwlrqdu| zhdowk vorzo| ghfolq|hv ohdylqj wrwdo zhdowk dw >643/ ru derxw < wlp|hv lqfrph dw uhwluhphqw1

Dq lpsruwdqw fdyhdw wr wklv h{huf|lvh lv wkdw rxu hvwlpdwlrq grhv qrw wu| wr pdwfk wkh prgho wr zhdowk gdwd/ dqg krxvhkrog dffpxodwlrq ri oltxlg zhdowk lv odujhu wkdw revhuyhg oltxlg dvvhw zhdowk dw uhwluhphqw1 Dffruglqj wr wkh 4<<; Vxuyh| ri Frqvxphu llqdaq|hv/ phdq wrwdo +h{foxglqj Vrfldo Vhfxulw| zhdowk exw lqfoxglqj shqvlrq dqg krxvlqj, zhdowk ri 88 97 |hdu rog krxvhkrogv lv := wlp|hv wrwdo suh0wd{ lqfrph/ zkhuh rxu prgho suhglfwv d idfwru ri < wlp|hv/ h{foxglqj shqvlrq zhdowk| Wklv ihdwxuh ri wkh gdwd lv wkh vdp|h ihdwxuh wkdw gulyhv wkh qglqj r| brz

llqdoo| zh dssol| wkh vdp|h sduwldo0htxloloulxp ghfrpsrvlwlrq wr djjuhjdwh zhdowk shu h hfw|yh krxvhkrog lq rxu hfrqrp|1 Zh dvvxph wkdw wkhuh lv qr xqfhuwdlq|w| diwhu uhwluhphqw dqg fhuwdlq ghdwk dw djh ;:1 Dffruglqj wr rxu gh qlwlrq/ suhfdxwlrqdu| zhdowk uhsuhvhqvw 95 shufhqw ri doo zhdowk krog|lqj zlwk qr srsxodwlrq jurzwk1 Wklv vkduh ulvhv wr :4 shufhqw xqghu wkh dvvxpswlrq wkdw wkh srsxodwlrq jurzv dw rqh shufhqw shu |hdu1 Wkhvh vkduhv duh uhdvrqdeoh forvh wr Fdedoohu +4<<4,/ zklfk uhsruwv qglqjv iru d fdoleudwhg FDUD prgho/ dqg Fdjhw|w| +4<<;,/ zklfk hpsor|v rxu phwkrgrorj| wr w zhdowk sur ohv|

9151llqglqjv iurp Gl huhqw Hgxf dwl|rq dqg Rf fxsdlwlrq Jur xsv1 Zh qh{w hvwlpdwh wkh prgho vhsdudwho| rq vxeljrxsv ri wkh srsxodwlrq gh qhg e| hgxfdlwlrq ohyhov dqg rffxsdlwlrq1 Wklv doorzv xv wr h{sorlw yduldlwlrq lq wkh haqlurqphqwv ri gl huhqw djhqvw dqg doorzv gl huhqw vxeljrxsv ri wkh srsxodwlrq wr kdyh gl huhqw suhuhqfhv1 Whfkqlfdoo| zh vlpsol| iroorz wkh surfhgxuh gh0 vfulehg deryh rq hdfk fhoo/ xvlqj uvw0vwdjh hvwlpdwhv douhdg| suhvhqwhg e| vxeljrxs lq vhwlwlrqv 7 dqg 81 Hdfk jurxs*v frqvxpswlrq sur oh lv pdwfkhg zlwk gl huhqw hvwlpdwhv xvlqj d gl huhqw lqfrph sur oh +Ilijxuhv 6 dqg 7,/ lqfrph xqfhuwdlq|w| +Wdeoh L,/ dqg lqlwldo dyhudjh orj oltxlg zhdowk

+Wdeoh LL,1 Wkh uhvxowv duh vxppdul}hg lq Wdeoh LY dgg wkh vlpwodwhg frqvxpswlrq sur ohv iurp wkh wwhg prgho duh suhvhqwhg lq lljxuhv 6 dgg 71

Wkh hwwlpdwhg glvfrxqw idfwrub duh forvh wr wkdw rewdlqhg xvlqj wkh dijuhdwh sur oh1 Wkh glvfrxqw udwh olhv ehwzhhg 7 shufhqw +Vrph Kljk Vfrro, dgg 8=4 shufhqw +Pdqdjhuldo dgg Surihvvlrqdo,1 Lqwhuhvwlqjo/ wkhuh lv qr fohdu sdwwhuq dvvrfldwhg zlwkw wkh ohyho ri hgxfdwlrq1 L dqlwklqj/ wkh glvfrxqw udwh lv voljkwo| kljkhu iru pruh kljko| hgxfdwng krxvhkrogv1 E| frqudwv/ wkhuh lv pxfk zlghu glvshuvlrq lq wkh hwwlpdwhg frh flhqvv ri uhodwlyh ulvn dyhuvlrq/ iurp d orz ri 3 =5;5 +Vrph Kljk Vfrro, wr d kljk ydoxh ri 5 =5<3 +Judgxdwh Vfrro,1 Wkh frh flhqvv ri wkh frqvxpswlrq uxoh dw uhwluphqw duh dovr zlwklq d uhdvrqdeoh udqjh ri wkh dijuhdwh hwwlpdwhv1 Wkh pdujlqdo surshqvlwhv wr frqvxph rxw ri oltxlq zhdowk udqjh ehwzhhg 8 dgg :(ghvslwh ehliq lpsuhflvho| hwwlpdwhg1 Rq wkh rwkhu kdqg/ zh qg h{wuhpho| orz dgg lpsuhflvh hwwlpdwhv riru prvw fhoov/ lpsollqj/ dv ehiruh/ hlwkhu dq xquhdvrqdeo| orz udwlr ri looltxlq zhdowk wr shupdqhqw lqfrph dw uhwluphqw/ ru d yhu| orz pdujlqdo surshqvlw| wr frqvxph rxw ri looltxlq zhdowk1

Shukdsv pruh lqwhuhvwlqjo/ wkh ryhulghqwl fdwlrq whvww vwdwlvwlvv duh pxfk vpdoohu/ lqglfdwlqj wkdw wkh prgho grhv d ehwwhu mre dw wwlqj wkh dfwxdo frqvxpswlrq sur ohv1 Lqghhg/ ornlqj edfn dw lljxuh 6 dgg lljxuh 7/ zh vhh wkdw wkh wwhg frqvxpswlrq sur ohv duh yhu| pxfk lq olqh zlwkw wkh xqfrqvwudlqhg vprwkhg frqvxpswlrq sur oh hwwlpdwhg iurp wkh gdwd1 Iru doo fhoov/ wkh frqvxpswlrq sur oh lv kxps0vkdshg dgg wkh vlpwodwhg sur ohv lqglfdwh wkdw krxvhkrogv vwduw dffxpxodwlqj iru uhwluphqw sxusrvhv ehwzhhg 68 dgg 73 |hduv1 Wkh ehkdylrudo lpsoldwlrqv ri wklv vhw ri uhvxowv duh wkh vdph dv wkrvh mxvw glvfxvvhg iru hwwlpdwhv wkdw glg qrw doorz iru furvv0jurxs khwhurjhqhlw|1

9161Urexvwqhvv1 Wr surylgh ixuwkhu hylghqfh rq wkh vwuhqjwkv dgg zhdowk ri wkh prgho/ zh suhvhqwh wkh uhvxowv ri yh dowhuqdwylyh surfhguxhv= +I, hwwlpdwlqj wkh lqlwldo ohyho ri oltxlq zhdowk wkh vhfraq vwdjh> +II, hwwlpdwlqj wkh uhwluphqw frqvxpswlrq uxoh gluhfwo| iurp plfur gdwd lq wkh uvw vwdjh> +III, hwwlpdwlqj ej pdwflqj djh0vshfl f jurzkw udwhv udwkhu wkdg ohyhov> +Iy, dgmxvwlqj wkh vhfraq0vwdjh lqihuhqfh iru lqfrph dgg idplo| sur oh hwwlpdwhv> dgg +y, uhgxfliqj wkh yduldaqh ri wkh lqfrph vkrfnv wr dfrxqw iru phdvxuphqw huuru/ +yl, pdwflqj wkh frqvxpswlrq sur oh iru h{shqglwxuhv rq qrq0gxudeoh jrrgv rqo|1

Iluvw/ zh hwwlpdwh lq wkh vhfraq vwdjh wkh dyhudjh lqlwldo fdvk rq klag zlwkw zklfk krxvhkrogv ehliq olih1 Zkloh wkh prgho wv wkh gdwd ehwwhu zlwkw wklv h{wud sdudphwhu/ wkh vhfraq vwdjh qr orqjhu

surylghv suhflvh ru vhaqveoh lghqwl fdwlrq ri wkh vwuxfwxudo sdudphwhuv1 Wkh uhvxowv duh suhvhqvw
 lq froxpqv 4 dqq 5 lq Wdeoh Y dqq lq llxuh ;1 Wklv dowhuqdwlyh vlpdwhg sur oh grhv d ehwhu
 mre wkda wkh edvholqh prgho lq whupv ri wwlqj krxvhkrog frqvxpswlrq gxulqj wkh uvw 8 |hduv
 ri olih/ zkhq phdvxuhg frqvxpswlrq olhv deryh phdvxuhg lqfrph1 Wkh surfhguxh dwwdlqv wklv ej
 dvvljqlqj dq lpsodxvleo| kljk ydoxh ri lqlwldo fdvk rq kdqg= wkh phdq ohyho ri dvvhvv lwlpdwhg
 wr eh 459 wlvhv fxuuhqw lqfrph\$ Zlwk kljk lqlwldo zhdowk/ wkh w|slfdo krxvhkrog fdq hdlv| ex hu
 oderu lqfrph vkrfnv dqq hqmr| kljk frqvxpswlrq hduo| lq olih1 Wr w wkh revhuyhg kljk fruuhodwlrq
 ehwzhhq lqfrph dqq frqvxpswlrq sulru wr plggoh djh/ wkh hvwlpdwlrq surfhguxh jxhvvhv wkdw djhqvw
 duh yhu| lpsdwlhqw +vr wkdw wkh| uxq wklv lqlwldo zhdowk grzq, dqq duh txlwh xqzloolqj wr vxevwlwxwh
 lqwhu0whpsrudoo|1 Wkh glvfrxqw udwh lv hvwlpdwhg dv 47(dqq wkh frh flhqw ri uhodwlyh ulvn dyhuvlrq
 dw 851

Zkloh lw lv srvleoh wkdw wklv kljk zhdowk frphv iurp lqkhuwdqfhv/ wklv lv dw rggv zlwk doo krxvh0
 krog vxuyh|v ri zhdowk krogqlqj1 Lq rxu plqgv/ wklv lv hqrxjk wr uhmhfw wklv dowhuqdwlyh vfhdulr/
 krzhyhu lw lqirupv xv derxw wkh prgho1 Iurp wklv dqq rwkhu h{shulphqvv/ zh frqfoxgh wkdw lghq0
 wl fdwlrq klqjhv lq sduw rq surylglqj wkh vhfraq0vwdjh zlwk pruh lqirupdwlrq wkda lv frqwdlqhg
 lq wkh sur ohv dorqh1 Lq sduwlfxodu/ iurp wkh w|slfdo frqvxpswlrq sur oh/ rqh fdq dffxudwho| dqq
 vhaqveo| hvwlpdwh wkh suhuhqfh sdudphwhuv rgo| ej {lqj hlwkhu wkh uhwluhphqw frqvxpswlrq uxoh
 ru wkh lqlwldo ydoxh ri dvvhvv1 Zlwkrxw wkhvh sdudphwhuv hvwlpdwhg iurp rwkhu vrxuffv lq d uvw vwdjh
 wkhuh lv qrw hqrxjk lqirupdwlrq lq wkh sur ohv wr surylgh dffxudwh hvwlpdwlrq ri wkh prgho1

Lq rxu vhfraq hydroxdwlrq ri wkh prgho/ zh hvwlpdwla dqq . lq wkh uvw vwdjh iurp frqvxpswlrq
 dqq dvvhv phdvxuhv durxqg wkh djh ri uhwluhphqw1 Zh xvhw wkh SVLG wr frqvwxfw phdvxuhv ri oltxlg
 dvvhvv lq 4<;</ lqfrph iurp 4<;< wr 4<<7/ dqq wrwdo frqvxpswlrq iurp 4<;< wr 4<<7iurp dfwlyh
 vdyllqj/ zhdowk dqq lqfrph phdvxuhv1 Zh wkhq daqxdo|h wkh gdwd dqq hvwlpdwh d fuxgh frqvxpswlrq
 ixqfwlrq dv=

$$F @ f S . . [. 2i . %$$

dfurvv krxvhkrogv1 Zh rewdlq=

$$a_f @ 38<7 v=h+348;,> a_33::7 v=h+33458,=$$

Wklv hvwlpdwhg pdujlqdo surshqvlw| wr frqvxph dw uhwluphqw/ lv yhu| forvh wr wkh rqh lq rxu edvholqh uhvxowv/ zklsh wkh udwlr ri looltxlg zhdowk wr wkh shupdqhaw frpsrqhaw ri lqfrph/ dw uhwluphqw/ lv htudo-wr@ :-9: >d pruh uhdolvwlf qxpehu1

Wkh wklug froxpq ri Wdeoh Y vkrzv wkdw wkh hvwlpdwhg glvfrxqw udwh lv voljkwo| orzhu wkdg lq wkh edvholqh hvwlpdwlrq dgg wkh frh filhqw ri uhodwlyh ulvn dyhuvlrq lv hvwlpdwhg wr eh txlwh orz1 Zklsh wkh sdudphwhuv dsshdu yhu| wljkwo| hvwlpdwhg/ wkh ryhulghqwl fdwlrq uhvwulfwlrq lqfuhdvhv gudpdwlfdool1 Wkh fkl vtduh ydoxh lv 49<31 Dv vkrzq lq lljxuh ;/ wkh vlxodwhg sur ohv duh vxevwdqwdool klijhu wkdg wkh dfwdo sur oh lq prvw |hduv1 Wklv dffrxqvw iru wkh ryhulghqwl fdwlrq uhmhwfwrq1 Jlyhq wkh deryh sdudphwhuv/ krxvhkrogv uhfhlyh : wlvhv wkhlu shupdqhaw lqfrph dw uhwluphqw1 Wklv ghfuhdvhv vdyldj lq oltxlwlv ryhu wkh anlaj olih1 Wklv dowhuqdwlyh vfhqdulr klijkoljkwv wkh gl fxow| wkh prgho kvd lq wwlaj erwk wkh odujh dprxqw ri oltxlg vdyldj lpsolhg el wkh FH| sur ohv dgg wkh lpsolflw odujh dprxqw ri looltxlg zhdowk pdgh dydlodeoh dw uhwluphqw1

Vlqfh wkhuh pd| eh vrph sureohpv lq wkh FH| phdvxulaj wkh uhodwlyh ohyhov ri frqvxpswlrq dgg lqfrph/ zh qh{w hvwlpdwh wkh prgho lq uvw0gl huhqfhv/ vr dv wr wu| wr pdwfk wkh jurzkw ri frqvxpswlrq ryhu wkh olih0f|foh/ qrw lww ohyho1 Wklv holplqdwhv rqh prphqw frqglwlrq1 Wkh uhvxowlaj sdudphwhu hvwlpdwhv +froxpq +7, ri Wdeoh Y dgg lljxuh ;, duh vxevwdqwyho| dgg vwdwlvwlfdool vplodu wr rxu ehqfkdun uhvxowv1 Pruh lpsruwdqwo| wkh whvw ri ryhulghqwl fdwlrq grhv qrw uhmhw wkh prgho1 Vlqfh lw lv olnho| wkdw doo krxvhkrog frqvxpswlrq h{shqglwxuhv duh qrw phdvxuhg lq wkh FH|/ wklv plvphdvxuhphqw pd| zhoo eh wkh uhdvrq wkdw wkh gdwd uhmhw wkh edvholqh prgho1

Irxuwk/ zh dgmxdw wkh vwdqgdug ghyldwlrqv ri wkh vhfraq0vwdjh hvwlpdwhv wr dffrxqw iru wkh vwdwlvwlfdo xqfhuwldqwl lq wkh hvwlpdwlrq ri wkh sur ohv ri lqfrph dgg xwlovlv vkliwhuv/ dv hvwlpdwhg lq vhwfwrq 81 lrupdoo| wkh vhfraq0vwhs fruuhfwlrq hpsorlv wkh yduldaqfh0fryduldaqh pdwul{ iru wkh iwkw0rughu sro|qrpldo frh filhqw iurp wkh vprwkhg lqfrph dgg xwlovlv vkliwhu sur ohv1 Wkh uhvxowv duh uhsruwhg lq froxpq +8, ri Wdeoh Y dgg ghprqvwudwh h{wuhpho| odujh vwdqgdug huuruv1 Wkdw lv/ jlyhq wkh vdpsolaj xqfhuwldqwl zlwklk d iwkw0rughu sro|qrpldo hvwlpdwhv wkh lqfrph sur oh/ wkh frqvxpswlrq sur oh grhv qrw surylgh dq dffxudwh phdvxuh ri suhuhqfh sdudphwhuv1 Vlqfh wkh djh0sdwwhuq ri lqfrph ghwhuplqhv wkh djh0sdwwhuq ri frqvxpswlrq/ wkh lqfrph sur oh lv lpsruwdqwlq lqghqwlilaj wkh frqvxpswlrq sur oh1

liwk/ zh dgguhvv wkh srvlelolw| wkdw rxu phdvxuhv ri xqfhuwdlqw| ryhuvwdwh wkh wuxh ulvn idfhg e| krxvhkrogv1 Li wkhuh zhuh plvphdvxuhphqw lq oderu lqfrph/ wklv zrxog eldv xszdugv rxu hwwlpdwhv ri lqfrph xqfhuwdlqw| Zlwk vhuldoo| xqfruhodwhg phdvxuhphqw huuru/ plvphdvxuhphqw lq dwhv rqo| rxu phdvxuh ri wkh yduldaqh ri wudqvlwru| lqfrph1 Vhuldoo| fruuhodwhg plvphdvxuhphqw frxog krzhyhu eldv erwk ri rxu phdvxuhv1 Dggwlwraqdoo| krxvhkrogv pd| kdyh dffhvv wr lqirupdo phfkdaqvpv wkdw plwijdwh wkh lpsdfw ri lqfrph ulvn rq frqvxpswlrq1 Li zh kdyh ryhuhvwlpdwhg shupdahqwg dgg wudqvlwru| lqfrph xqfhuwdlqw| wkh uhdghu pljkw eh frqfhuqhg wkdw wklv lv jhqhudwlqj vrph ri rxu uhvxowv/ sduwlfxoduo| rxu orz hwwlpdwh ri ulvn dyhuvlrq1 Duhdvrqdeoh jxhvv pljkw eh wkdw urxjko| d wklug ri wkh yduldaqh ri phdvxuhg lqfrph jurzkw lv gxh wr plvphdvxuhphqw dgg wkdw prvw ri wklv lv wudqvlwru| +vhh Erxqg dgg Nuxhju +4<<4, dgg Erxqg +4<<7,,1 Krzhyhu/ jlyhq rxu frqfhuq derxw xqprghohg vruxfhv ri lqvxudqfh/ zh h{shulphqw zlwk dq h{wuhph dowhuqdwlyh1 Zh uh0hwwlpdwh rxu prgho kdoylqj wkh yduldaqh ri wudqvlwru| lqfrph vkrfnv dgg uhgxflqj e| rqh wklug wkh yduldaqh ri shupdahqwg vkrfnv1 Froxpq +9, ri Wdeoh Y uhsruwv rxu qglqjv1 Prvw sdudphwhu hwwlpdwhv duh xqfkdqjhg zlwk d wljkwo| hwwlpdwhg glvfrxqw udwh r4(dgg d uhodwlyh ulvn dyhuvlrq ri 3; >forvh wr wkh ehqfkdun hwwlpdwh ri 31841 Wkxv/ rxu surfhgxuh hwwlpdwhv d uhodwlyho| orz ulvn dyhuvlrq hyhq iru vxevwdqwdoo| orzhu hwwlpdwhv ri lqglylgxdo lqfrph xqfhuwdlqw|

Odvwo| zh hwwlpdwh wkh vdph vhw ri sdudphwhuv xvlqj lqirupdwlrq rq qraqxudeoh h{shqglwxuhv rqo|1 Vwulfwo| vshdnqj/ rxu prgho uhuhv wr wrwdo frqvxpswlrq/ wkh vxp ri h{shqglwxuhv rq qraq0 gxudeohv dgg wkh rz frqvxpswlrq rq gxudeohv1 Krzhyhu/ vlqfh krxvhkrogv gr qrw uhqw doo gxudeohv/ d pruh frpsohwh prgho zrxog dffrpprgdwh erwk vruwv ri jrrgv vhsdudwho|1 Zh vwrs vkruw ri gr0 lqj vr dgg lqyhvwijdwh khuh zkdw rxu prgho lpsolhv iru qraqxudeoh h{shqglwxuhv vxplqj gxudeoh h{shqglwxuhv duh pdqgdwhg h{shqglwxuhv dgg vxewudfwlqj wkhp iurp rxu phdvxuh ri lqfrph dgg frqvxpswlrq1^e Vlqfh zh duh dowhulqj wkh ohyho ri wkh sur ohv/ zh dovr hwwlpdwh wkh prgho lq uvw0 gl huhqfhv dgg dgmxxw wkh sdudphwhuv ri wkh frqvxpswlrq uxoh dw uhwluhphqw vr dv wr pdwfk wkh udwlr ri qraqxudeoh wr wrwdo frqvxpswlrq dw uhwluhphqw1 Wkh uhvxowv +froxpq +:., lqglfdwh d vxe0 vwdqwdoo| orzhu frh flhqw ri uhodwlyh ulvn dyhuvlrq/ 413 Wkh glvfrxqw udwh lv xqfkdqjhg dw 6=7(1 Qhlwkhu sdudphwhu lv hwwlpdwhg yhu| wljkwo| zklfk dffrxqvv iru wkh uhodwlyho| orz ydoxh ri wkh ryhulghqwl fdwlrq whvw zklfk grhv qrw uhmhfw wkh prgho1

Wklv duwlfoh frqwulexwhv wr wkh dqdo|vlv dqq xqghuvwdqglqj ri krxvhkrog frqvxpswlrq dqq vdyqlj ehkdylru1

Zh ghyhors d qhz phwkrq iru hvwlpdwlqj krxvhkrog frqvxpswlrq ehkdylru1 Zh prgho frqvxpshu ehkdylru lq wkh suhvhqfh ri uhdolvwlf ohyhov ri xqlqvxudeoh lqfrph xqfhuwdlqwl dqq hvwlpdwh suhihu0 hqfh sdudphwhuv dqq krxvhkrog frqvxpswlrq ehkdylru xvlqj wkh Phwkrq ri Vlpxodwhg Prphqww1 Wkh prgho wv zhoo dqq |lhogv wlkjw hvwlpdwhv ri wkh glvfrxqw udwh dqq lqwhuwphsrudo hodvwlflwl ri vxevwlwxwlrq1 Wklv phwkrgrorj| lv qrz ehlqj xvhg wr hvwlpdwh h{sdqghg prghov wkdw dgguhv sruwirolr fkrllh/ oderu vxssol dqq uhwluhphqw ehkdylru +Jdnlglv +4<<:/ Fdjhwwl +4<<:, dqq luhqfk +4<<,,1

Rxu uhvxowv lqglfdwh wkdw vpdoo krogqlj ri oltxlg dvvhww e| |rxqj krxvhkrogv lv dq rswlpdo uhvsrqvh wr h{shfwhg lqfrph jurzwk dqq wkh ulvnlqhv ri ixwxuh oderu lqfrph ryhu wkh olrh ffoh1 Xqwlo wkhlv hduo| iruwlhv/ krxvhkrog frqvxpswlrq ehkdylru/ zkloh ixool rswlpdo/ dsshduv vkruw0vljkwhg zlwklq wkh frqwh{w ri wkh FHT00FK1 Ghvslwh wklv dssduhqw lpsdwlhghf/ zh hvwlpdwh wkdw krxvhkrogv glvfrxqw wkh ixwxuh dw prghv udwhv dqq duh qrw sduwlfxodo| ulvn dyhuvh1 Rxu uhvxowv dovr lpsol wkdw roo krxvhkrogv vdyh dfwlyho| iru uhwluhphqw sxusrvhv dqq ehkdyh lq d pdqqhu pruh frqvlvwhqw zlwklq wkh FHT00FK1 Wkhvh wzr skdvhv ri frqvxpshu ehkdylru wxuq rxw wr eh txlwh glvwlqfw dqq duh dw wkh khduw ri rxu lghqwl fdwlrq surfhgxuh1 Wkh qhrfodvvlfo uhsuhvhqwdwlyh0djhqw prgho ri djjuhjdwh frqvxpswlrq lv lqfruhfw suhflvho| ehfdxvh ri wklv fkdqjlqj ehkdylru ryhu wkh zrunlqj olrh1

Zh vhh wzr lqwhuhvwlqj dyhqxhv iru ixwxuh uhvdufk1 Iluvw/ rxu phwkrq frxog eh iuxlwixool h{whqghg wr ulfkhu uhsuhvhqwdwlrqv ri krxvhkrog lqwhuwphsrudo fkrllhv1 Zh vxvshfw wkhuh lv pxfk wr eh ohduqhg iurp wkh vwxg| ri sruwirolr fkrllh/ gxudeoh jrrgv/ uhwluhphqw/ oderu vxssol/ ru wkh h hfw ri wd{ srolf| dqq vrfldo lqvxdqfh surjudpv1 Vhfrqg/ rxu dssurdfk frxog eh dssolhg wr hvwlpdwh wkh djjuhjdwh lpsoldwlrqv ri plfur khwhurjhqhlw|1 Iru lqvwdqfh/ hvwlpdwhg djh0yduuldwlrq lq frqvxpswlrq ehkdylru wkdw zh qg lq wkh gdwd pd| h{sodlq Fdpsehoo dqq Pdqlnz +4<<:, qglqj wkdw urxjko| 73(ri doo djhqvv duh ckdqg wr prxwk*1

e

DP-Y Q7 \$ c -l.-

Ghsw1 ri Hfrqrplfv/ Sulqfhwraq Xqlyhuvlw|/ llvkhu Kdoo/ Sulqfhwraq/ QM 3;87704346/ X1V1D1>
srjCSulqfhwraq1hgx/ kwws=22zzz1Sulqfhwraq1hgx2•srj2/

dqg

Ghsw1 ri Hfrqrplfv dqg Zrrgurz Zlovraq Vfkro/ Sulqfhwraq Xqlyhuvlw|/ Urehuvwraq Kdoo/ Sulqfh0
wraq/ QM 3;87704346/ X1V1D1> msdunhuCSulqfhwraq1hgx/ kwws=22zzz1Sulqfhwraq1hgx2•msdunhu21

TTi?_ 4ih U@* 5L* | L? |L | i L?t 4ih hLM*i4

Wklv dsshqgl{ ghvulehv rxu dssurdfk wr vroylqj qxphulfdoo| wkh frqvxphu sureohp1

D14 Jdxv0Khuplwh txdgudwxuh

Wkh dojrulwkp h{sorlww wkh uhfxuvlyh vwuxfwxuh ri wkh frqvxphu sureohp e| vroylqj wkh Hxohu htxdwlrq iru wkh rswlpdo frqvxpswlrq uxoh1 Dv xph iru wkh wlp ehqj wkdw zh nqrz krz wr frpsxwh $f_{|n}$ +7 iru doo srvleoh ydoxhv ri fdvk rq kdqg1 Rqh fdq uhzulwh wkh Hxohu htxdwlrq +8, xvlqj wkh Lqwhuwhpsrudo exgjhw frqvwudlqw +7, dv=

$$+49, \quad x \neq | \neq ,, \quad @ \quad U \quad \frac{y+| |n}{y+| |}, \quad s \ H \ x \ f_{|n} \quad \neq \quad f_{|}, \quad \frac{U}{J_{|n} \cdot Q} \quad J_{|n} \cdot Q \quad .$$

$$+4 \quad s, \ H \ x \ f_{|n} \quad \neq \quad f_{|}, \quad \frac{U}{J_{|n} \cdot Q} \cdot X \quad J_{|n} \cdot Q \quad X \ A \ 3 \quad =$$

Wkh uvw sureohp frqvlvww lq hydroxdwlqj wkh h{shfwdwlqj lq +49,1 Qdqq X duh orj qrupdoo| glvwulexwhg/ d qdwxudo zd| wr hydroxdwh wkhvh lqwhjudov lv wr shuirup d wzrOgphqvlrqdo Jdxv0Khuplwh txdgudwxuh=

$$H \ x \ \neq_{|n} \ \neq_{|n}, \ J_{|n} \cdot Q, \ \neq \ A \ 3 \quad @ \quad x \ \neq_{|n} \ \neq_{|n}, \ J_{|n} \cdot Q, \ \text{gl} \ +Q, \ \text{gl} \ +X,$$

$$@ \quad i_{|} \ +q > x \ h^3 \ ?^2 \ h^3 \ ?^2 \ \text{gxgq}$$

$$[\quad i_{|} \ +q > x, \ \$ \ >$$

c

zkhu $i_{|} \ +q > x$ @ $\frac{1}{2} x \ f_{|n} \quad \neq \quad f_{|}, \quad \frac{1}{c_{|n}} \ h^{\frac{1}{2j} \ ? \ ?} \cdot \ h^3 \ \frac{1}{2j} \quad J_{|n} \ h^3 \ \frac{1}{2j} \ ? \ ? \ 1$ Wkh zhjkw\$ dqg qrgvh q / x duh wdexodwhg lq Mxgg +4<<:,1 Lq sudfwlfh/ zh shuiruphg d txdgudwxuh ri rughu 451

Rqh fdq wkhq qg wkh urrw ri wkh Hxohu htxdwlrq dw dq| srlqwxvlqj vwdqgdug phwkrgv1 Zh frqvwudlq wkh urrw wr eh srvlwlyh dqg ohvv wkh fhuuhqw ydoxh ri fdvk rq kdqg1 Dv glvfxvvhg lq wkh sdshu/ wklv uhvwulfwlrq lv dozd|v vdwlv hg zkhq wkhuh duh qr looltxlqj dvvhvw1

D15 Frqvxpswlrq uxohv

Zh lqlwldol}h wkh dojrulwkp zlwkw wkh frqvxpswlrq uxoh dw uhwluphqw1 Rqh fdq vkrz wkdw wkh frqvxpswlrq uxohv iru wklv sureohp duh frqlqrxrxvo| gl huhqwldeoh dv orqj dv wkhuh duh qr oltxlglw| frqvwudlqww1 Krzhyhu/ lq wkh suhvhqfh ri oltxlglw| frqvwudlqww/ wkh frqvxpswlrq uxohv pd| h{kletw

d nlaq1 Vhh Ghdwraq +4<<4,dqg D|djdul +4<<7,1 Zh h hfwyho| Ipsrvh d oltxlgw| frqvwudlqw e| qrw doorzljaj wkh krxvhkrog wr eruurz djdlqvw looltxlq dvvhwv1 Wklv lqgfdwhv wkdw vprwk dssur{lpdwlrq phwkrqv duh lqdssursuldwh1 Lqvwhdg/ zh xvh d vwdqgdug glvfuhw|dwlrq phwkrq= zh vshflii| dq h{rjhqrqv julg iru fdvk0rq0kdqg= { a. ^3{4@ `1 Lq rughu wr fdswxuh wkh fxuydwxuh ri wkh frqvxpswlrq uxoh dw orz ydoxhv ri fdvk rq kdqg/ wkh julg zloo eh qhu iru 5 3>{?| 1 Lq sudfwlfh/ iru hdfk ydoxh ri fdvk rq kdqg rq wkh julg/ { / zh qg wkh dvvrfdwhg frqvxpswlrq/ f / wkdw vdwlv hv +49,1 Lq fkrvlaj wkh vl}h dqg frduvhqhvv ri wkh julg/ zh idfh wkh vxvdo wudgh0r ehwzhhq suhflvlrq dqg frpsxwlaj wlph1 Dggqlaj srlqvw rq wkh julg jlyhv d qhu dssur{lpdwlrq ri wkh frqvxpswlrq uxohv1 Vlqfh wkh frqvxpswlrq uxoh dw dij4 lv wkh lqswx qhfhvvdul wr jhw wkh frqvxpswlrq uxoh dw dijw Ipsuhflvlrqv frxog frpsrxqg ryhu wlph1 Rq wkh rwkhu kdqg/ wkh Hxohu htxdwlrq lv wkh lqghuprvw orrs ri wkh hqwluh dojrulwkp1 Zlwk 433 srlqvw rq wkh julg dqg 73 wlph shulrgv/ zh pxvw vroyh +49, 7333 wlphv1 Wklv wdnhv dssur{lpdwlrq|p4qxwhv rq d Vxq Vsduf Hqwuhsulvh433331 Zh dovr idfh d ghflvlrq uhjduglaj wkh udqjh ri fdvk rq kdqg/ {4@ 1 Iru vpdoo ydoxhv/ fdvk rq kdqg lq vdpsoh lv olnho| wr pryh rxw ri wkh julg1 Frqvxpswlrq zloo wkhq eh hydroxdwhg xvlaj h{wudsrodwlrq phwkrqv/ pxfk ohvv suhflvh wkdg lqwhusrodwlrq1 Rq wkh rwkhu kdqg/ lqfuhdvlaj wkh udqjh iru d {hg qxpehu ri julg srlqvw Ipsolv ohvv suhflvh hvwlpdwlrq ri wkh fxuydwxuh1 Rqh vroxwlrq frqvlvww lq hqgrjhq|}laj wkh julg vr wkdw/ iru lqvwdqfh/ fdvk rq kdqg uhpdlqv zlwklq wkh julg zlw suredelolw| 3<81 Zh dgrswhg d vlpsohu dssurdfk frqvlvwlaj lq fkhfnlaj wkdw fdvk rq kdqg/ lq wkh vlpwodwlrqv/ uhpdlqv vwulfwo| lqvlgh wkh julg1 Lq sudfwlfh/ zh w4r @ 73 {?| @ 5 dqgM @ 433-zlw 83 srlqvw ehwzhhq 3 dqg ?| 1 Zh fkhfnhg wkh txdolw| ri wkh dssur{lpdwlrq e| vroylaj wkh vwdwlrqdu| lq qlwh krul}rq sureohp dqg fkhfnlaj wkh udwh ri frqyhujhqfh wr wkh {hg srlqvw ri wkh ixqfwlrqdo Ehoopdq htxdwlrq1 Qrwh wkdw hyhq li vlpwodwlrq uhpdlqv vwulfwo| lqvlgh wkh julg dssur{lpdwlrq huuruv fdq dulvh iurp wkh Hxohu htxdwlrq vlqfh hydroxdwlrq h{shfwhg ixwxuh pdujlqdo xwlolw| pd| uhtxlh orrnolaj rxwvlgh wkh julg1

D16 Vlpwodwlrqv

Zh vlpwodwh frqvxpswlrq sur ohv e| jhqhudwlaj d vhtxhqfh ri 53333 fwlwlrqv lqfrph surfhvvhv ryhu 73 |hdv1 Iru jlyhq sdudphwhv ri wkh frqvxpswlrq sureohp/ zh vroyh iru wkh rswlpdo dij0 ghshqghqvw frqvxpswlrq uxohv dv h{sodlqhg deryh1 Odvwo| zh fdofoxodwh wkh frqvxpswlrq ghflvlrqv

ri krxxvhkrogv uhfhlylqj wkhvh lqfrph surfhvvhv1 Zh wkhq frqvwuxfw oliv f|foh frqvxpswlrq sur ohv e| orj0dyhudjlqj dfurvv djh1

t)4T|L| U V@h @?Ui L @h @?Ui •@|h @?_ Ait| Lu
ih _i?| u) ?} +it|h U| L?t

Zh iroorz Qhzh| dqg Pfdgghq +4<<7, dqg xv h{sdqvlrq phwkrq1 Gh qh wkh uvw0vwdjh vdpsoh prphqvw wkdw fruhvsrqg wr wkh wkhruhwlfdo prphqvw d^W, @ $\frac{S}{a}$ a. +", zkhuh M lv wkh qxpehu ri revhuydwlrqv iru wkh uvw vwdjh1lvædv|pswrwlfdoo| qrupdoo| glvwulexwhg zlwkd wkhruhwlfdo yduldqfh fryduldqfh pdwul{ P^{3•} 6 P^{3•} zkhuh 6 @ H^ +"_f, +"_f, ` dqg P @H $\frac{Y>E_f}{Y}$ 1 Frqvlwhqwh hwlpdwlrq Y_f xvhw wkh hpsulwdo frxqwhusduwv Rvr dqg 6 dv lv vwdqgdug lqPP 1 Vlqfh wkh uvw0vwhs lv h{dfwo| lghqwl hg/ wkh fkrlfh ri d zhjkwljqj pdwul{ lv luuhohydqwh kuh1vædq dv|pswrwlfdoo| olqhdv hwlpdwru zlwklq xhqfh ixqfwlrq @ P^{3•} +"_f, zkhuh $\frac{S}{M+a}$ "_f, @ $\frac{S}{a}$ a. # . r_{R+4},1

Wkh uvw rughu frqglwlrq iru wkh vhfrrq vwdjh hwlpdwru lv=

$$j_w \text{ e>a } Z_j \text{ e>a } @ 3$$

zkhuh $j_w \text{ e>a } @ \frac{Y)+wc}{Y_w}$ 1 H{sdqglqj j +a>e, durxqg f dqg uhduudqjlqj |lhogv=

$$\frac{S}{L+a} f, @ j_w+a>e, Z_j w+a>e, 3 \cdot j_w+a>e, Z \frac{S}{L} j+f>e, >$$

zkhuh ghqrwhv d frqvlwhqwh phdq ydoxh1 H{sdqglqj wkh vdpw whup ixuwkhu/ wklv wlp d'urxqg zh rewdlq=

$$\frac{S}{L+a} f, @ j_w+a>e, Z_j w+a>e, 3 \cdot j_w+a>e, Z \frac{S}{L} j+f>e, \frac{5}{7} \frac{S}{L} j+f>e, \frac{u}{M} \frac{L}{j} +f>, \frac{4}{M} [^a \frac{6}{\cdot} \# . r_{R+4}, 8 =$$

E| wkh Voxwvn| dqg fhqwudo olplw wkhruhplw^S f, frqyhujhv lq glvwulexwlrq wr d phdq }hur grupdo glvwulexwlrq zlwk dv|pswrwlf fryduldqfh pdwul{=

$$Y_w @ \# ZJ_w^{3 \cdot} J_w ZH \quad j + f > f, . \quad \frac{L}{M} J \# \quad j + f > f, . \quad \frac{L}{M} J \# \quad ZJ_w \# ZJ_w^{3 \cdot} >$$

zkhuh J_w @ H^{YI Ew c f} ` lv W_n dqg J @ H^{YI Ew c f} ` lv W_u1 Dv wkh phdq ydoxhv frqyhujh wr wkh wuxh rqhv wkh vdpsoh judglhqwv frqyhujh wr wkhlu wkhruhwlfdo frxqwhusduwv1

Vlqfh rxu uvw0vwdjh dqg vhfraq0vwdjh hvwlpdwru xvh gl huhqw vwuxfwxudo prghov dqg prwvwl gli0 ihuhqw gdwd/ zh dvvxph wkdw wkh uvw0vwdjh dqg vhfraq0vwdjh prphqwv duh xqfruhodwhg1 Lq wkdw fdvh/ wkh irupxod vlpsol hv wr=

$$Y_w @ \# ZJ_w^{3 \cdot} J_w ZH \quad j + f > f, j + f > f, . \quad \frac{L}{M} J \# \# J \quad ZJ_w \# ZJ_w^{3 \cdot} \\ +4:; \quad @ \# ZJ_w^{3 \cdot} J_w Z^k) \} . \quad \& J \quad Y \quad J \quad l \quad ZJ_w \# ZJ_w^{3 \cdot} >$$

zkhuh } @ H^{k+f > f, k+f > f, ` lv W(W) @ olp_k" 4. $\frac{U}{u}$ dqg & @ olp_k" $\frac{U}{a}$ =Wklv pdnhv fohdu wkdw dv wkh uhodwlyh suhflvrlq ri wkh uvw vwdjh lqfuhdva^{U+13} wkh fruhfwlrq iru wkh uvw vwdjh glvdsshduv)1 frqwurov iru wkh vlxodwlrq huuru1 Dv wkh vlxodwlrq ehfrphv pruh dqg pruh suhflvh/) \$ 4=Wr frqvwuxfw dq hvwlpdwh ri +4:;/ zh uhsodfh wkh wkhruhwlfdo frqfhsww zlwk wkhlu hpslulfo frxqwhusduw/ dv lv grqh lq wkh xvxd JPP iudphzrun1}

Zh xvh wzr gl huhqw fkrlfhv ri wkh zhlijkwlj pdwul{ lq rxu hvwlpdwlrq1 Iluvw/ zh fkrrvh d pdwul{ wkdw/ zkloh qrw ixool rswlpdo/ grhv qrw ghshqg rq wkh wwhg prgho1 Wklv lv prwlydwhg el wkh revhuydwlrq wkdw rswlpdool zhlijkwlj JPP hvwlpdwruv fdq zrurvha qllwh0vdpsoh eldv +vhh iru h{dpsoh Zhvw/ ix Zraq/ dqg Dqdwrolhy +4<<:,1 Zh vhw wkh zhlijkwlj pdwul{ wr }³ =Wklv lv wkh rswlpdo zhlijkwlj pdwul{ zkhq wkh uvw vwdjh fruhfwlrq grhv qrw pdwwhu/ l1h1 zkhq @ 3 ru & @ 3=Wklv |lhogv=

$$Y_w @ \# ZJ_w^{3 \cdot} J_w^{3 \cdot} J_w^{3 \cdot}) \} . \quad \& J \quad Y \quad J \quad l \quad J_w \# J_w^{3 \cdot} J_w^{3 \cdot} =$$

Zkloh }³ lv qrw wkh rswlpdo zhlijkwlj pdwul{ jlyhq wkh uvw0vwdjh fruhfwlrq/ wklv fdvh lv hdv| wr lpsophqw vlqfh dq hvwlpdwh ri } fdq eh rewdlqhg gluhfwo| dv wkh vdpsoh frxqwhusduw ri

H +oqF H +oqF ,, +oqF H +oqF ,, 1 Vlofh zh gr qrw revhuyh krxxvhkrogv dfurvv |hduv/ wklv lv htlydohqwr wr dvvxplqj wkdw } lv gldjrqdo zlwka₃ @ $\frac{S}{U}$ u₃ oqF_{cl} oqF_l² =Lq wkh fdvh ri gl huhqfh hvwlpdwlrq/ zh xvhdq lghqwlw| zhlijkwloj pdwul{ vlofh gl huhqwr krxxvhkrogv duh revhuyhg lq gl huhqwr |hduv1

Vhfrqg/ zh dovr hpsor| wkh rswlpdo zhlijkwloj pdwul{ Z @) } . &J Y J³ =Lq wklv fdvh/ wkh yduldaqfh irupxod vlpsol hv wr=

$$Y_w @ \frac{k}{w}) } . \&J Y J^{l_3} J_w^{3^*} =$$

Wklv uhtxluhv xvlqj wkh hvwlpdwhv iurp rxu uvw fdvh dqg frqvwuxfwlqj dq hvwlpdwh~~z~~ rixvlqj vdpsoh frxqwhusduwv1

Wkhuh duh wzr rwkhu lqwhuhvwlqj vshfldo fdvhv1 Iluvw/ li wkh vhfrqg vwdjh lv h{dfwo| lghqwl hg +W @ n, > wkh fryduldaqfh pdwul{ vlpsol hv wr $Y_w @ J_w^{3^*}) } . \&J Y J^{l_3} J_w^{3^*} =$ Vhfrqg/ li wkhuh lv qr furvv ghulydwlyh/ vr wkdw @ 3/ wkh uvw vwdjh xqfhuwdlqwl grhv qrw d hfw wkh vhfrqg vwdjh1 Lq wklv fdvh wkh irupxod froodsvhv wr wkh idploldu rqh lq zklfk uvw vwdjh hvwlpdwlrq grhv qrw dsshdu= $Y_w @) J_w Z J_w^{3^*} J_w Z } Z J_w J_w Z } J_w^{3^*} =$

Wklv phwkrgrorj| dovr surylghv d xvhixo whvw ri wkh ryhulghqwlilqj uhwulfwlrqv lq wkh vhfrqg vwdjh1 Li wkh prgho lv fruhfw ddg @ 3 wkh vwdwlvwlf=

$$"_{A32} @ \frac{L}{j} a > " a^{3^*} j a > " a >$$

lv glvwulexwhg dv |pswrwlfdo| dv Fkl0vtxduhg zlwka₅ ghjuhhv ri iuhhgrp1

Xqghu wkh fruhfwlwrq iru wkh uvw vwdjh/ wkh vwdwlvwlf

$$"_{A32} @ L j a > " a^{k}) } . \&J Y J^{l_3} j a > " a$$

lv glvwulexwhg dv |pswrwlfdo| dv Fkl0vtxduhg zlwka₅ ghjuhhv ri iuhhgrp1

L?t|h U| L? Lu w ui)U*i hL€*it

Zh frqvwuxfw sur ohv ri frqvxpwlrq ryhu wkh zrnlqj olih erwk vprwkhg dqg xq0vprwkhg1 Xq0vprwkhg sur ohv duh frqvwxwfwhg e| dyhudjlqj rxu uhfrqvwxwfwhg phdvwuh ri frqvxpwlrq e| djh1 Vprwkw sur ohv duh frqvwxwfwhg e| hvwlpdwlqj dq htxdwlrq vlplodu wr +47, wkdw {hvdw wkh ydoxh hvwlpdwhg rq wkh xq0vprwkhg gdwd/ uhsodfhv wkh djh dqg frkrw gxpplhv e| iwkw rughu

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Wkh xqhpso|phqw udwhv phujhg wr wkh FH[duh wkh uhjlrqdo xqhpso|phqw udwhv iru flylddq srsxodwlrq iurp wkh Krxvhkrog vxuyh| frqgxfwhg dqg sxeolvkhg e| wkh Exuhdx ri Oderu Vwdwlvwlvf lq _Hpsor|phqw dqg Hduqlqjv1% Wkh JQS LSG SFH lv iurp Frxqflo Ri Hfrqrplf Dgylvruv +4<<8,1

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Exvlqhv F|foh/%Dphulfdq Hfrqrplf Uhyhlz / ;8/ 444;~446:1

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Srolwldo Hfrqrplf / 436+9,/ 4454~8:1

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Gdwd/%Mrxuqdo ri Oderu Hfrqrplfv / 45+6,/ 678~9;1

N Aae c e BAa li j}ji +4<<4,= _Wkh H{whqw ri Phdvxuhphqw Huuru lq Orqjlwxglqdo
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iBAW e . - +4<<7,= _Wkh Frqvxphu H{shqglwxuh Vxuyh|= d frpsdudwlyh dqdo|vR%
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_____ +4<<<= _Wkh Ex hu Vwrfn Vdylqj= Vrph Wkhruj/% Pdqxvfulsw/ Wkh Mrkqv Krsnlqv
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_____ +4<<4,= _Vdylqj dqq Oltxlglw| Frqvwudlqww/%Hfrqrpwhwulfd / 8<+8,/ 4454~44751
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8,B Ae • +4<;4,= _Wkh Dgmxxvphqw ri Frqvxpswlrq wr Fkdqjlqj H{shfwdwlrqv derxw lxwxuh
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8ijAW e . +4<<:,= _Wkh H hfww ri Khdownk/ Zhdownk/ dgg Zdjhv rq Oderu Vxssol dgg Uhwluhphqw
Ehkdyrlu/% Zrunlqj Sdshu/ Xqlyhuvlw| ri Zlvfrqvlq1

8i ja6BAe • +4<8:,=D Wkhru| ri wkh Frqvxpswlrq lxfwlrq 1 Sulqfhwraq/ Sulqfhwraq Xqlyhuvlw|
Suhvv1

DB" a te Q . +4<<:,= _Vwrfnv iru wkh RogB Hduqlqjv Xqfhuwldqwl dgg Olih0F|foh Sruwirolr
Fkrllfh/% Zrunlqj Sdshu/ P1L1W1

D j5e D - e BAa D 7 jW"ji +4<:7,=Wkh Doorfdwlrq ri Wlph dgg Jrrgv ryhu wkh Olih
F|foh 1 QEHU/ Qhz \run1

DN i AW Bte e BAa c Bi"ji +4<<:,= _Frqvxpswlrq Ryhu wkh Olihf|foh/% Vrfldo
Vflhqfh Uvhdufk Lqvlwxwh Zrunlqj sdshu <:551

—— +5334,= _Wkh Hpsulldo Lpsruwdqfh ri Suhfdxwlrqdu| Vdylqj/ Vphulfdq Hfrqrplf
Uhyllz +Sdshuv dgg Surfhhglqjv/ <4+5,/ 739~451

Q OOBiae D e c 7 7" AAjie BAa 7 ~j,ajt +4<<7,= _Wkh Lpsruwdqfh ri Suhfdxwlrqdu|
Prwlyhv iru H{sodlqqlqj Lqglylgxdo dgg Djjuhjdwh Vdylqj/% lq Fduqhjlh0Urfkhvwhu Frqihuhqfh
Vhulhv rq Sxeolf Srolf|/ hg1 e| D1 K1 Phow}hdqg F1 L1 Sorvvhu/ yro1 73/ ss1 8<~4581

Q iae • +4<:,= _Pruwdolw| Ulvn dgg EhtxhvwHfrqrphwulfd / 8:+7,/ ::<~;461

cBVVj,, e C e c 7 tW "je BAa 7N ,,jt +4<<:,= _Whvwlqj iru Oltxlgwl Frqvwudlqww lq
Hxohu Htxdwlrqv zlwk Frpsophqwdul Gdwd Vrxufhv/ Uhyllz ri Hfrqrplfv dgg Vwdwlvwlvf/
;3+5,/ 584~951

c aae l w +4<<:,=Qxphulldo Phwkrqv lq Hfrqrplfv 1 Fdpeulgjh/ PD/ P1L1W1 Suhvv1

IN|, "Nuue w c e BAa w Q 7 66jit +4<:4,= _Wkh Frqwulexwlrq ri Lqwhujhqhudwlrqdo
Wudqvihuv lq Djjuhjdwh Fdswdo Dffpxodwlrq/ Mxuxqdo ri Srolwldo Hfrqrpl / ;9/ :39~651

w a }tNAe 7 e BAa B!tNA +4<<:,= _Dssur{lpdwlrq Eldv lq Olqhdul}hg Hxohu Htxdwlrqv/%
plphr/ Sulqfhwraq Xqlyhuvlw|1

w tBia e +4<<9,= _Shupdqhqw Lqfrph/ Fxuuhqww Lqfrph/ dgg Frqvxpswlrq= Hylghqfh iurp
Wzr Sdqho Gdwd VhwMxuxqdo ri Exvlqhv dgg Hfrqrplf Vwdwlvwlvf/ 47+4,/ ;4~<31

•j} ie e BAa D bjOji +4<<9,= _Lqwhuwpsrudo Qrqvhsdudelolw| ru Eruurzlqj UhwvulfwlrqvB
D Glvdjjuhjdwh Dqdo|lvv Xvlqj d X1V1 Frqvxpswlrq Sdqho/% Hfrqrphwulfd / 97+8,/ 4484~;41

•j|NAe - +4<:4,= _Rswlpxp Frqvxpswlrq dqq Sruwirolr Uxohv lq d Frqwlqrxv Wlph Prgho/%
Mrxuqdo ri Hfrqrplf Wkhru| / 6/ 6:6~746/ dqq huudwpx/ Mrxuqdo ri Hfrqrplf Wkhru| 9/ ss
5460547/ +4<:6,1

j j+e b e BAa \$ w •W8BaajA +4<<7,= _Odujh Vdpsoh Hvwlpdwlrq dqq K|srwkhv|v
Whvwlj|/% lq d qgerm ri Hfrqrphwulfv/ Yroxph 7 / hg1 e| U1 I1 Hqjoh/dqq G1 O1 Pfdgghq/
ss1 5446~55781 Hovhylhu1

B"jte e BAa \$ N,,Bia +4<;<= _Vlpxodwlrq dqq wkh Dv|pswrwlvf ri Rswlpl}dwlrq
Hvwlpdwruv/% Hfrqrphwulfd / 8:/ 435:~438:1

B, 6ONe • +4<<<= _Xqfhuwdlq Phglfdo H{shqvhv dqq Suhfdxwlrqdu| Vdylqj Qhdu wkh Hqq ri
wkh Olih F|foh/% Uhyhlz ri Hfrqrplf Vwxglhv / 99+5,/ ss1 6<8~7551

Bi"jie c +4<<<= _Wkh Uhdwlrq ri Krxvhkrog Frqvxpswlrq wr Suhglfwdeoh Fkdqjhw lq
Vrldo Vhfxulw| Wd{hv/% Dphulfdq Hfrqrplf Uhyhlz / ;<+7,/ <8<~:61

tW "je c 7 +4<<8,= _Lqglylgxdo Lqfrph/ Lqfrpsohwh Lqirupdwlrq/ dqq Djjuhjdwh
Frqvxpswlrq/% Hfrqrphwulfd / 96+7,/ ;38~731

7W jW |6BAe c +4<:9,= _Dq Lqfrph loxfwxdwlrq Sureohp/% Mrxuqdo ri Hfrqrplf Wkhru| / 45/
54;~741

7W jW |6BAe c e BAa X .tW ajiN +4<::= _Vrph Uhxowv rq cDq Lqfrph loxfwxdwlrq
Sureohp*/% Mrxuqdo ri Hfrqrplf Wkhru| / 49+5,/ 484~991

7 jBe c +4<<8,= _Xqlrq Frqwudfwv dqq wkh Olih F|foh0Shupdqhqw Lqfrph K|srwkhv|v Dphulfdq
Hfrqrplf Uhyhlz / ;8/ 4;9~5331

bjt|e l \$ e l u bNA}e BAa 7 AB|N,+j +4<<;= _lhdvleoh Rswlpdo Lqwuxphqwd
Yduldeohv Hvwlpdwlrq ri Olqhdu Prghov zlwk Prylqj Dyhudjh Glvwuedqfhv/% Vrldo Vflhqfh
Uvhdufk Lqvlwxwh ZS Qr1 <<34/ Xqlyhuvlw| ri Zlvfrqlq1

~j,ajte 7 +4<;<= _Rswlpdo Frqvxpswlrq zlwk Vwrfkdwlvf Lqfrph= Ghyldwlrqv iurp
Fhuwdlqw| Htxlydohqfh/% Xduwhuo| Mrxuqdo ri Hfrqrplfv/ 437+5,/ 5:8~<;1

N|jt

*i t |L | @?! * ih ai@? *@?U @h_c + U@h_L @M@**ihLc h t @hhL** @?_ ?) t #i@|L? uLh _i|@ *i_ UL44i?|t @?_ i?UL h@;i4i?| i @*tL | @?! #@hL? Ui4L)* c h@3 L ||@?@t Lc #@ _ @h_c @h*it 6*i tU 4@?c ,h U 6hi?U c a@?) A ?} B Lc Bhi)Lh kLti?L!c 6h@?UL •L_)* @? c 5|i i tU lic U L*@t 5L *i*itc +LMih| 5 4ihc @?L?)4L t hiuhiitc @?_ ti4 ?@h T@h| U T@?|t @| | i L@h_ Lu BL ih?Lht Lu | i 6_ih@* +itih ic U@}Lc L* 4M @c O@h @h_c•U }@?c•WAc h ?U|L?c 5|@?uLh_c A **M h)c` tUL? t c @?_ v@*i @h!ih }h@|ju **) @U!L *_j)it €?@?U @* t TTLh| uhL4 | i 5*L@? 6L ?_@| L? @?_ @| L?@* 5U i?Ui 6L ?_@| L? }h@?| 5,5 ffbSf.S

²5L4i Lu | i 4Lt| hiUi?| T@Tiht @hi @hhL** @?_ 5 44iht E•bb•c wt@h_ E•bbS c •i} h @?_ `iMih E•bbS c @hlih E•bbb c 5 i@ E•bbD c @?_ a@TTi** c tU lic @?_ 5L *i*it E•bbH 5ii hL ? ?} @?_ wt@h_ E•bbS Lh #i@|L? E•bb2 uLh t h i)t

i _L ?L| i?|ih | i _iM@|i L? | i hi*@| i 4TLh|@?Ui Lu hil hi4i?| iht t Mi^ it| i@*| W4T* U *)c L h hil hi4i?| i@*| 4i@t hi ** ?U* _i ML|

°A t tc ? T@h|c @ UL?€h4@|L? Lu @hhL** @?_ 5@4 U! E•bb. @?_ @hhL** E•bbe |@| @h}ic M@ti_ L? @tt| _@|@c |@| M gih t|LU! 4L_*t @TT*) L?) |L L ti L*_t MiuLhi @}it eD |L Df

^D @hhL** E•bb. @h}it |@| | t t * !*) |L Mi |h i

^S ||@?@t Lc @?|tc •i} hc @?_ `iMih E•bb. t Lt |@| | i hit _ @*t uhL4 @ hi}hitt L? Lu UL?t 4T| L? L? u@4 *) UL4TLt |L? @?_ *@MLh t TT*) @h @M*it @hi ?ULhhi*@|i_ |@}i OL i ihc u ThiU@ |L?@h) t@ ?} t T@h| Lu | i hi@tL? uLh | i ? |@* 4T t @Ti ? UL?t 4T| L? Lih | i * ui U)U*ic | t hi}hitt L? t giht uhL4 @? L4 |ji_ @h @M* i M @tc U ** ?ULhhiU*) @tt}? | i 4T |L U @?}it ? _i4L}h@T Ut 5 4 *@h)c u i @hi €| ?} @ 4L_* | |LL * ||*i Thiuihi?Ui ijihL}i?i |) |L | i _@|@c |i? i ** ?ULhhiU*) @tt}? |LL 4 U Lu | i t @Ti |L @h @|L?t ? ThiU@ |L?@h) t@ ?} i @**L uLh ijihL}i?i }c ? @ * 4 |i_ 4@??ihc M) UL?|hL** ?} uLh ijihL}i?L t u@4 *) t 3i @| @ } i? @}i @?_ it| 4@| ?} tiT@h@|i*) @UhLtt LUU T@|L? @?_ i_ U@|L? }hL Tt

· A i Tih4@?i?| UL4TL?i?| Lu ?UL4i t |@| *i* |@| L*_ LM|@ ? |L | |h@?t |Lh) t LU!tc @t ? 6h i_4@? E•bD. c ?L| | i @?? }) @* i Lu | i Thiti?| _tUL ?|i_ t 4 Lu u | hi ?UL4i t|hi@4tc @t ? 6*@ ? E•bH•

^H *i ML _ @?_ @h_ E•bHb uL ?_ |@| U @?}i ? *@MLh ?UL4i @t Mit| U @h@U|ih 3i_ M) @? E2 ThLUittc |i) @*tL uL ?_ * ||*i }@ ? ? 4L ?} uhL4 @? • E• |L @?• E2

^b5ii O MM@h_c 5! ??ihc @?_ -i*_it E•bbe c @* 4ML E•bbb c @?_ O h_ E•bHb

^f i _hiU*) UL?|hL* uLh ijihL}i?L t u@4 *) t 3i @| @ } i? @}i i? UL?t|h U| ?} UL?t 4T| L? @?_ ?UL4i ThL€*it A i @*|ih?@| i t |L 4L_* i T* U *) | i t|LU @t| U ThLUitt uLh U @?}it ? u@4 *) t 3i

^{**} 5LU @* tiU h) i@*| t_i€? |i*) ** ^ _ @?_ t L?) @ @ *@M* i @t @?? | it @u|ih hil hi4i?| ,@h*) |_h@ @* Lu Ti?t L? @?_ t@ ?} i U*it |@h}|i_ uLh hil hi4i?| T hTLtitt c U @t W+ <tc e! T*@?t @?_ kil} c t Luji? hit|h U|i_ @?_ €U@**) Ti?@* 3i_c u @**L i_ @| @** ?i 4 | |@*tL UL?t _ih @ t Mt|@?| @* T@h| Lu L t ?} i@*| @t ** ^ _ i@*| E@| *i@t|_ h ?} | i | 4i Tih L_ |@| i t|_)

² `i hiUL} 3i | @ | | t @tt 4T| L? t ThLM*4@| U ? tL4i @)t 6Lh ?t|@?Uic |L L ti L*_t | _ gihi?|
?UL4i |h@ iU|Lh it M | | i t@4i Tih4@?i?| UL4TL?i?| Lu ?UL4i @| hij hi4i?| @hi Thi_ U|_ |L @ i _i?| U@*
@UU 4*@|j_ ** ^ _ i@*|
* AL | i i |i?| | @ | |ihi t U h @| hi ? | i hi*i @?| h@?|j Lu | i UL?t 4T| L? h *i L h @tt 4T| L? Lu * ?i@h |)
** M @t t @ @) uhL4 €?_ ?} 4TLh|@?| igiU|t Lu ?Uih|@ ?|) L? UL?t 4T| L?
^e W? | i U@ti Lu u ** Uih|@ ?|) @u|ih hij hi4i?| @?_ ?L U @?|i ? | i | * |) t u|ihc . ' $\frac{*3 q^{4@} - 4@ 4}{*3 E^{4@} - 4@ 4 Q W}$
^d | ih hi*i @?| T@h@4i|iht @hi q ' f bSfc 4 ' f D•e c- ' • fee c|)T U@* ?UL4i ?Uih|@ ?|)c u@4 *) t 3ic @?_
i TiU|j_ ?UL4i ThL€*ic @t _ tU tti_ ? tiU| L?t e @?_ D
^s A t h *i t it| 4@|j_ _ hiU|*) uhL4 4 UhL _@|@ 5ii tiU| L? S
* W? BL h ?U @t @?_ @h|ih E•bb. c i t L | @ | ?i TiU|j_ |h@?t |Lh) t LU|t |L ?UL4i @hi Mi|ih t4LL| i_
* @|ih ? * uic _itT |i | i u@U| | @ | |i) | i? UL?|@ ? }hi@|ih ?uLh4@| L? @ML | |L|@* hitL hUit uLh | i hi4@ ?_ih Lu
| i * ui A t U @?|j ? UL?t 4T| L? h t! t @|_ h it | i UL Lh| @ ih@|j UL?t 4T| L? }hL |
^H + t! @ iht L? @?_ | i ?UL4i ThLUitt 4 t| Mi ML ?_i_ A iti UL?_ | L?t @hi @?@*)3i_ ? 4Lhi _i|@ *t ? #i@|L?
E•bb•c)@|h E•bbe @?_ 5U iU |4@? @?_ ,tU _ihL E•b..
^b A i €ht| t|@|j i t i @U|*) _i?| €i_ *i | i tiUL?_ t|@|j i t L ih _i?| €i_
^f `i | @?| | i @?L?)4L t hiuihiit @?_ #@ _ @h_ uLh t }|it| ?| | @ | i 4L i uhL4 U@* Mh@| L? @?_ hLM t|?itt
U iU|t |L uLh4@**) ?|j|h@| ?| | i €ht| t|@|j
² `i | @?| ?_hi 5@4 U! uLh ThL _ ?} t | | i h h@ _@|@
²² `i _hLT 5ih Uic 6@h4 ?}c h4i_ 6LhUit @?_ 5i*u ,4T*L)i_ LUU T@| L?t _i |L t4@** t@4T*it ? | i ,j
² A i @tt 4T| L? | @ | | i 4i igiU|t i?|ih 4 *|T* U@| i*) t L?*) @ UL? i? i?| t Lh| U | u **ih 4L_* UL *_
?U*_ i @|j)hij@|t LU|t @t t|@|j @h @M*it A t t Mi)L?_ | i U hhi?| T@Tih
^{2e} W| t Lh| ?L| ?| | @ | | i @tt 4T| L? | @ | u@4 *) t 3i t i L|j?L t 4 } | ? u@U| Mi hi_ U ?| | i 4TLh|@?Ui | @|
i @|h M |i |L ThiU@ | L?@h) t@ ?} Wu | i M gih t|LU! 4L_* t ULhhiU| @?_ @ ?} U *_hi? t @ uLh4 Lu UL?t 4T| L?c
| i ? | i _iU t L? |L @ i U *_hi? t @giU|j_ M) ThiU@ | L?@h) t@ ?| | @ | t U*Lti*) hi* @|j_ |L | i | 4 ?} Lu * ui| 4i
?UL4i) ?Lh4@* 3 ?} M) @ u@4 *) t 3i @_ t|4i?| L ih | i * uiU)U*ic L?i t hi4L ?} tL4i TLh| L? Lu U @?|it ?
UL?t 4T| L? _h i? M) i TiU|j_ ?UL4i U @?|it
^{2D} A t uL**L t uhL4 | i @??L) ?} _i?| |) | @ | ?|ih i)i@h *itt @|j i i^ @*t M h|)i@h-
^{2S} A t @tt 4it | @ | | i 4i igiU|t @hi ?L| 4TLh|@?| uLh | i * ?i@h |hi?_ ? UL?t 4T| L? @?_ | @ | | i) @hi
LMtih i_ M) | i L ti L*_ A i @**LU@| L? Lu | i |hi?_ ? UL?t 4T| L? |L UL Lh|t @?_ @|j igiU|t 4@) Mi ti?|t | i
|L | t @tt 4T| L?
² # 44) @h @M*it @hi UL?t|h U|j_ uLh u@4 *) t 3it • | hL } b @?_ | i? uLh | ih u@4 *) t 3i t •f Lh }hi@|ih
`i @*tL i Tih 4i?|j_ | i L|j?L t u@4 *) t 3i @_ t|4i?|t @tt 4 ?} s t t 4T*) u@4 *) t 3i h@ ti_ |L | i TL ih
3f . A t *_i_ |L ThL€*it | @ | ihi ?L t ih @?_ @|ih i@h*) ? * ui
^{2H} A i 4 *_i UL Lh| t •be• @?_ | i @ih@|j hi) L?@* ?i4T*L)4i?| h@|i . TihUi?
^{2b} 5ii w t@h_ E•bbS c @?_ h@?U E•bbe

f`i @hi @h} ?} | @| tih ULt| Lu L t ?} hiT@ htc 4@ ?|i?@?Uic |*| itc @?_ L t ?} tih Uit U@T| hit | i
iTi?_ | hit 4@_i uLh UL?t 4T| L? L? L t ?} M) L4iL ?iht L|j | @| i _L ?L| tM|h@U| _L? T@)4i?|t Lh
T@)4i?| Lu Th ?U T@* uL4 UL?t 4T| L? Lh ?UL4i tL | @| ?i@h*) @** L t ?} i@*| t 4L_j**_ @t * ^ _
* W| t 4TLh|@?| ?L| |L ti _gihi?| _j @|Lht uLh ?UL4i @?_ UL?t 4T| L? A t UL *_ Mhi@! | i hi*@| L?t T
Mijii? U@t L? @?_ @?_ UL?t 4T| L? ? ?L4 ?@* |ih4t U t | i hi*@| L?t T Thi_ U|j_ M) | i M gih t|LU!
| iLh)
^ 5 ?Ui q- E •c | i U L U i Lu 4 4@||iht * ||*i A i U Li U i?| Lu hi*@| i h t! @ iht L? @?_ | i _ tUL ?| u@U|Lhc @hi
?L| tiT@h@|j*) _j| €i_ MiU@ ti i _L ?L| @ i @?) @h @| L? ? | i _it hi |L t u| UL?t 4T| L? @UhLtt | 4i
i _L ?L| ?U*_ j @** | i €ht|t|@)j T@h@4i|iht ? | i €ht|t|@)j @h @?Ui ULhhiU| L? W? T@h| U *@hc i _L ?L|
?U*_ j | i T@h@4i|iht | @| _h i | i u@4 *) t u|jht @?_ | i i TiU|j_ ?UL4i ThL€*i `i hiTLh| hit *|t ?U*_ ?}
?Uih|@ ?) @ML | | iti T@h@4i|jht ? tiU| L? S ihi i ? it| @|j hLM t|?itt tt it
° 6Lh ?t|@?Uic u | i h@| L Lu **^ _ i@*| |L Tih4@?i?| ?UL4ic c ihi i^ @* |L S c| i 4@h} ?@* ThLTi?t |) |L
UL?t 4i L | Lu **^ _ i@*| L*_ Mi @ 4ihi f DI
^ A i _tUL ?| UL4it uhL4 | i @tt 4T| L? | @| ?UL4i t hiUi i_ @?_ UL?t 4T| L? LUU ht @| | i Mi} ?? ?} Lu
| i Tih L_
^ W? Lh_ih |L _L | tc i ?T | | i UL?t 4T| L? h *i @| hi| hi4i?| @t it| 4@|j_ ? L h Mi?U 4@h! U@ti h
it| 4@|jit 4T*) | @| u L ti L*_t u@Ui_ ?L h tlt @u|j hi| hi4i?|c | i @)j Lu _i@| c c t @ hi@tL?@M*i H.)i@ht A @|
tc u i ti| Thiuhi?Uit @| L h it| 4@|j_ * * it @?_ ' H.c | i t|@?_@h_ * ui U)U*i 4L_j* | @ Uih|@ ? _@|j Lu
_i@| 4T* it | i t@4i @* i u ?U| L? @| hi| hi4i?| @t i it| 4@|j E T |L @ UL?t|@?|
· A iti U hhi?| ?UL4i @?_ i@*| ? 4Miht hiuih |L @ ih@)it Lu *i*i* E?L| *L)t @?_ tL @hi ?L| 44i_ @|j*)
UL4T@h@M*i |L | i ?UL4i ThL€*it
^ W? u@ Lh Lu L h 4L_j*c | i L ti L*_t i L *_ * !i uLh | t UL4T@h tL? @hi SD)i@ht L*_t ? 2ffS ? | i L | ih
@?_c tL4i L t ?} i@*| t _j u@U|L **^ _ ? L h 4L_j*
^ BL h ?U @t @?_ @h|ih E2ff• ThL _j tL4i @__ | L?@* U@*U *@| L?t M@ti_ L? L h 4L_j*
ef +iU@** | @| i it| 4@|j 7/ 2s ' . d*? 2soc @?_ tL | i 4i@? *i*i* Lu U@t L? @?_ t } i? M)G i T //2s n 2/ 59
° BL h ?U @t @?_ @h|ih E•bb. Thiti?| @ t 4 *@h tUi?@h L M@ti_ L? | if _@|@
e2`i | @?! L?i Lu L h hiuihii uLh TL ?| ?} | t L | |L t
° N?_ih | t @tt 4T| L?c _ h@M*i }LL_t U@??L| Mi ti _|L Lgti| *@MLh ?UL4i ?Uih|@ ?) ` *i | t t @? i |hi4i
@tt 4T| L?c | i UL44L? @tt 4T| L? Lu @_ t|4i?| ULt|t 4T* it | @| _h@M*i }LL_t @hi Lh| @ uh@U| L? Lu | i h
Lh } ?@* @* i L? tiUL?_ @?_ 4@h|!t
ee A t t ULhhiU| T |L @ 4 *| T* U@| i UL?t|@?|

CBO,j Y XBi BAWj Nu YAWN6j 7 NW"t

DiN V	XBi BAWj Nu ji6BAjA 7 NW"	XBi BAWj Nu CiBAAt Ni+ 7 NW"
CN B, WW VB NA	313545	313773
PdqjhuIdo dqg Suri1 Vshfldolw	3134:8	313674
Whfk1/ Vdohv/ dqg Dgplq1 Vxssruw	313568	313694
SuhflvIrq Surg1/ Fudiw/ dqg Uhsdlu	3134:8	313765
Rshudwruv dqg Oderuhuv	3135<<	31378;
.a WB NA		
Vrph Kljk Vfkrrro	313547	31398;
Kljk Vfkrrro Ghjuhh	3135::	313764
Vrph Froohjh	31356;	313675
Froohjh Judgxdwh	313479	3136;8
Judgxdwh Vfkrrro	313448	313833

5L hUiG @hhL** @?_ 5@4 U! E•bb. @?_ @ | Lht< U@*U *@| L?t 5|@?_ @h_ ihhLht h@?}i uhL4 |i? TihUi?| |L | h|)
TihUi?| Lu | i ULi U i?| it| 4@|ic _iT_i?_ ?} *@h}i*) L? | i t@4T*i t 3i A iti _@|@ @hi @ @ *@M*i uhL4 | i @ | Lht

CBO,j YY w` a bjB,| QN,a A} B| | j } AA A} Nu w uj

-B N Nu w` a ttjt N YAWN6j	wN}	wj j,	
DiN V	•jBA 7 \$	•jBA	h{s^\$2S. 2 / 2S`
CN B,	\$2S / 2S	h{s^\$2S. 2 / 2S`	31633
WW VB NA	051:<7	41::7	31633
Pdqj huldo dqg Suri1 Vshfldolw	051594	4197<	31738
Whfk1/ Vdohv/ dqg Dgplq1 Vxssruw	0518:7	41958	315;8
Suhflvrq Surg1/ Fudiw/ dqg Uhsdlu	061645	51384	315<<
Rshudwruv dqg Oderuhuv	0614:5	41987	31497
.a WB NA			
Vrph Kljk Vfkrrro	061::5	41;;9	31469
Kljk Vfkrrro Ghjuhh	0615<9	41:<:	314;9
Vrph Froohjh	051:89	419:<	31593
Froohjh Judgxdwh	0514;4	41887	316::
Judgxdwh Vfkrrro	041;;:	41873	317<8

5L hUiG | Lht< U@*U *@| L?t M@ti_ L? | i ,j 4i@t hit Lu i@*| @?_ | i UL?t|h U|_ 4i@t hi Lu ?UL4i

CBO,j YYY 7|i W| iB, .t| 6B| NA -jt ,|t

•7• .t 6B NA	-NO t bj } A}	V 6B, bj } A}
Glvfrxqw ldfwru + , V1H1+D, V1H1+E, Glvfrxqw Udwh 3• 4 +(, V1H1+D, V1H1+E, Ulvn Dyhuvlrq +, V1H1+D, V1H1+E, Uhwluhphqw Uxoh= f V1H1+D, V1H1+E, • V1H1+D, V1H1+E,	31<8<; +313434, +3134:<, 714;; +413<;, +41<7<, 318473 +3149<3, +314:3;, 313348 +61;7, +61;8, 313:43 +314548, +314577,	31<89< +313483, 7183: +41974, 416<9< +31446;, 819; 43 +4917<, 313946 +313844,
" 2GB	4:8158	
" 2GO	4:7143	4:819:

L|iG •7• it| 4@| L? uLh i?| hi }hL T 5|@?_@h_ ihhLht U@*U *@|i_ |L| E @?_ | E ULhhiU| L? uLh €ht|
 t|@}i it| 4@| L? i** t3i t ScSb• L ti L*_t A i *@t| hL hiTLh|t @ |it| Lu | i L ih _i?| u) ?? hit|h U| L?t
 t|h M|i @t @ t^ @hi_ | S _i}hiit Lu uhii_L4 A i Uh |U@* @* i @| DI t Df .• , U i?| it| 4@|it @hi
 U@*U *@|i_ | @ i }| ?} 4@|h I UL4T |i_ uhL4 | i hLM t| it| 4@|it

CBO,j YX .t| 6B|jt uiN6 | j 7|NW Bt| W •Naj, O+ .a WB| NA BAa WW VB| NA

DiN VH	Glvrxqw ldfwru + ,	Glvrxqw Udwh 4 4	Ulvn Dyhuvlrq + ,	Uhwluhphqw Uxoh " 5	Q
<u>.a WB NA</u>					
Vrph Klij Vfkrrro	31<95 +313;5,	61<7 +;1<5,	315;5 +417;4,	3153< +8137, 313:5 +51693,	86193 7/5:3
Klij Vfkrrro Judgxdwh	31<7< +31348,	8163 +4197,	31;9< +31553,	61:< 43 +53138, 3138< +3137<,	8<145 45/778
Vrph Froohjh	31<93 +3148<,	7148 +4:15<,	316<7 +51677,	31684 +713<8, 31376 +61489,	;7154 </986
Froohjh Judgxdwh	31<63 +31393,	:17; +91<:,	515<3 +31756,	4188: 43 +87193, 3137< +313:8,	4441:3 9/683
Judgxdwh Vfkrrro	31<77 +313;:,	81<6 +<1::,	419<7 +31;76,	4139 43 +4;156, 3138: +313:9,	::159 8/<:6
<u>WW VB NA</u>					
Pdqdj huldo dqg Suri1	31<79 +31393,	81:4 +919<,	419:5 +31857,	8153 43 +551::, 31383 +3139:,	448195 45/9<6
Whfk1/ Vdohv/ Dgplq1	31<86 +3136:,	71<3 +7144,	4138< +3166<,	5146 43 +6<175, 3137< +31397,	97135 9/87;
Suhflvlrq Surg1/ Fudiw	31<86 +31666,	71<: +691::,	31<<3 +61;<8,	31336 +4;17<, 31387 +31<<:,	851:9 7/79<
Rshudwruv/ Oderuhuv	31<86 +317;<,	71<3 +861;3,	31;9: +71;79,	6147 43 +4698, 3137< +5168,	8:18; 9/396

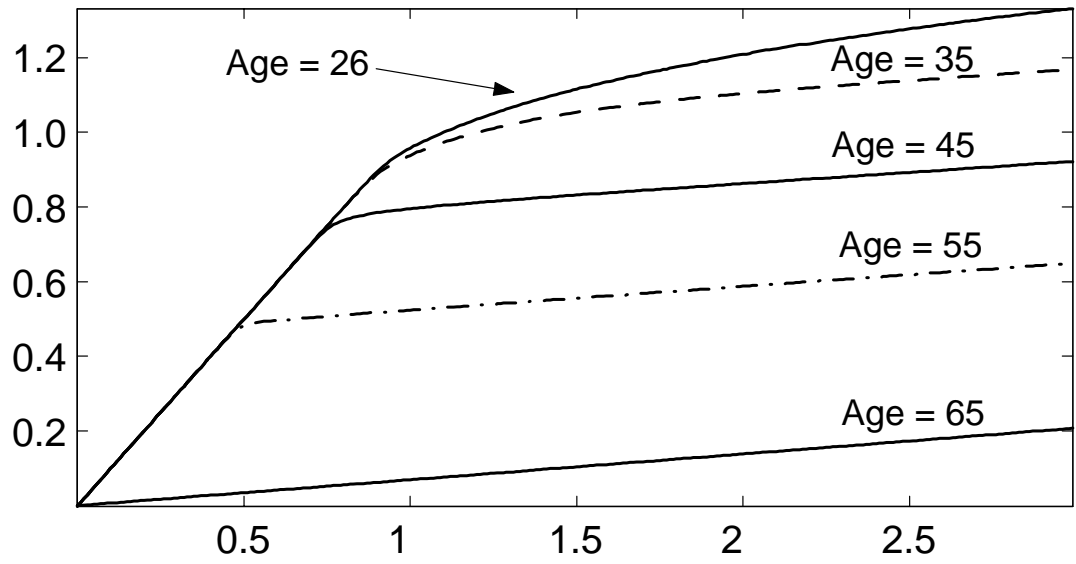
L|jG •7• it| 4@| L? ? *i*t 5|@?_@h_ ihLht U@*U *@|j_ | ULhhiU| L? uLh €ht| t|@}i it| 4@| L? A i ?i | |L
 @t| UL 4 hiTLht @ |it| Lu | i Lih _i?| u) ?} hit|h U| L?t _t|h M |j_ @t @ t^ @hi_ | S _i}hiit Lu uhii_L4
 A i Uh | U@* @*i @| DI t Df. •

CBO,j X -NO t|Ajtt jW"t

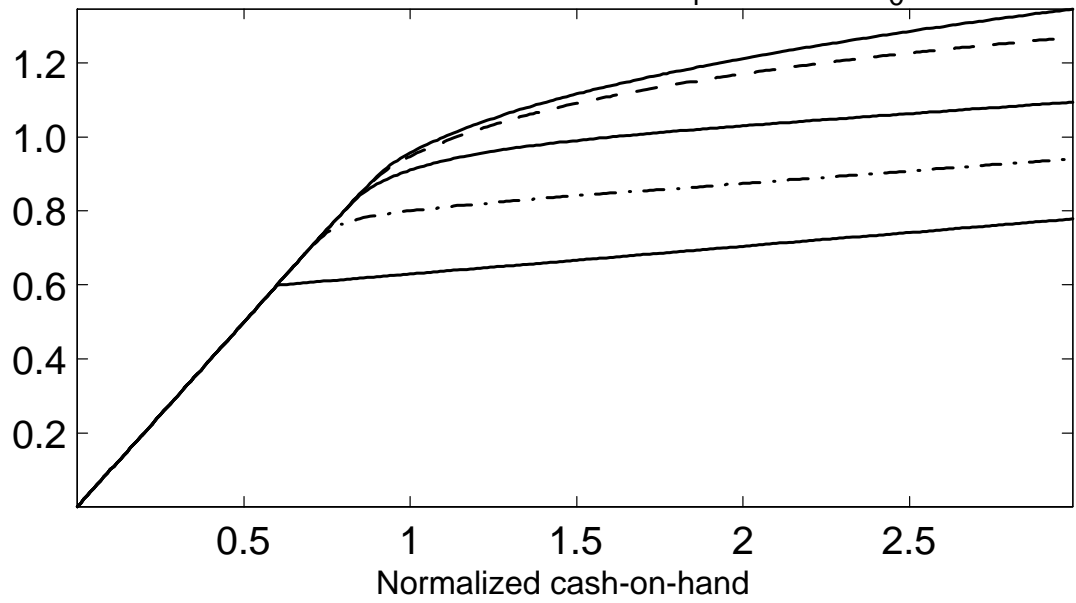
.t 6B NA	G•	G4	G	Gg	GF	GU	G0
L?t 4T L? •i@t hiG V 6B, bj } A) 8! 7 _{2s} •j Na 8! G f e . YAWN6j NijW NA XBi BAWj NijW NA	AL @* L L wi i* L L L	AL @* vit L wi i* L L L	AL @* L vit wi i* vit L L	AL @* L vit 6 ht # g L L L	AL @* L vit wi i* L L L	AL @* L vit wi i* L L L	L?# h@M*i L vit 6 ht # g vit L L
# tUL ? 6@U Lh □ V1H1+D, V1H1+E, # tUL ? +@ i E q ³ 3 • EI V1H1+D, V1H1+E, + t! iht L? E 4 V1H1+D, V1H1+E, +ij hi4i? + *iG f V1H1+D, V1H1+E, . V1H1+D, V1H1+E, W? @* i@* Ei TE 2D V1H1+D, V1H1+E, 2GB 2GO	f H.e• +313<;7, +314365, •e efS +451;;4, +461848, D •H• +318973, +319855, D . •f 3D +531:3, +531:7, ff2•• +314;57, +314;7;, 2 D•D +3197:5, +31;8;:, b• Hb b• H	f H.2e +3134<;, •e S2. +518;<;, D 2H2 +3144<8, • S •f 3e +1;9, ff2f +313798, 2 D•D +317;3;, b• .H	f bSeH +313335, +3133;:, Se•2 +313557, +31<68, f•2.H +313337, +3133;:, f Dbe +4199;, +419:8, ff•f +316496, +3164;7, f fS•2 +317;3;, •Sbf •S•b	f bSe +313535, +313539, Sbb2 +514:4, +51545, f•DHD +318;88, +31999<, ff• +4199;, +419:8, ff•f +316496, +3164;7, f fS•2 .	f bDbH +313434, +41447;:, e •HH +413<;4, +454134, f D•ef +3149<3, +4918<5, f ff•D +61;7<;, +45541, ffH 2 +6;1<;9, ffS•2 •DD SS	f bSfS +313346, +313366, e •f2 +314696, +3168<;, f eHf +31357<;, +315:3;, fff• +81843, +16<3, +313866, +313:83, ffS•2 •De H	f bS f +313<95, +3143;3, He +4316:3, +441968, f•D S +41;3;5, +5134;3, f 2b.. +313866, +313:83, ffS•2 e• S

L|iG •7• it| 4@| L? ? *i*t @?_ €ht|_ gihi?Uit 5|@?_@h_ ihhLht U@*U *@|j_ |L| E @?_ | E ULhhiU| L?
uLh €ht| t|@|i it| 4@| L? A i ?i | |L *@t| UL* 4 hiTLh|t @ |it| Lu |i L ih _i?| u) ?} hit|h U| L?t _ t|h M|i_ @t @
t^ @hi_ | S _i}hiit Lu uhii_L4 A i Uh | U@* @* i @| DI t Df .•

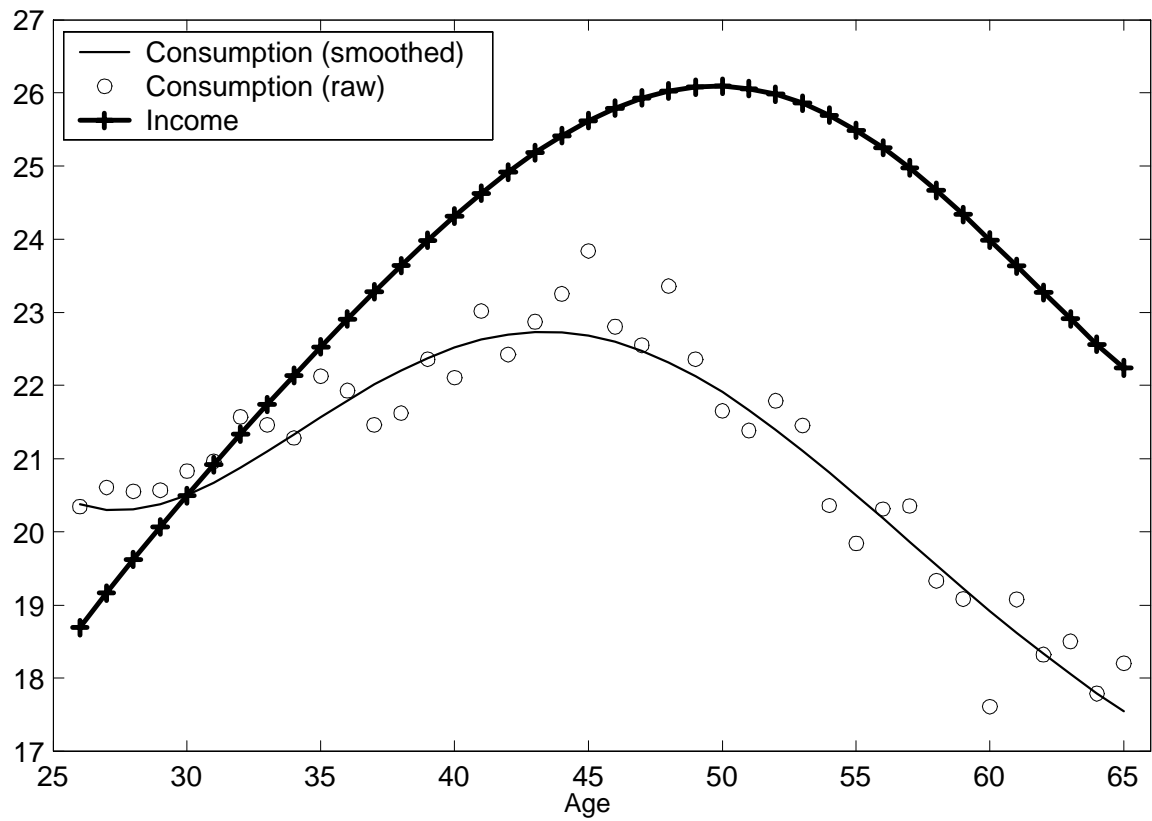
Normalized consumption Panel A: $b = 0.960$, $r = 0.514$, $g_1 = 0.071$, $g_0 = 0.001$



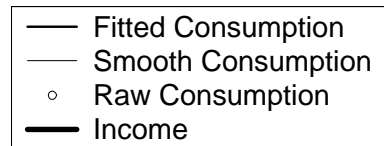
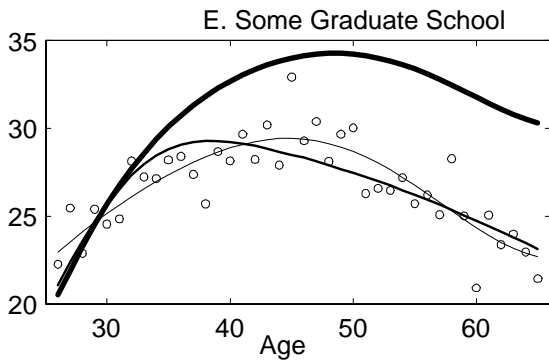
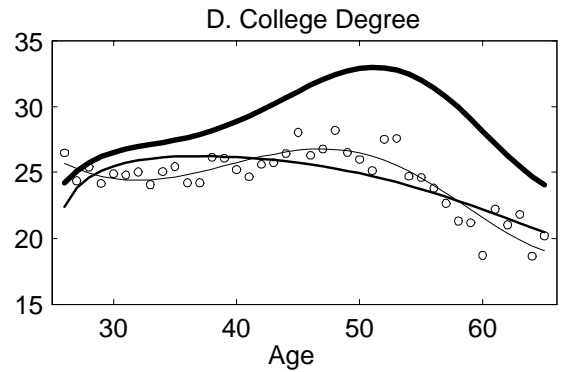
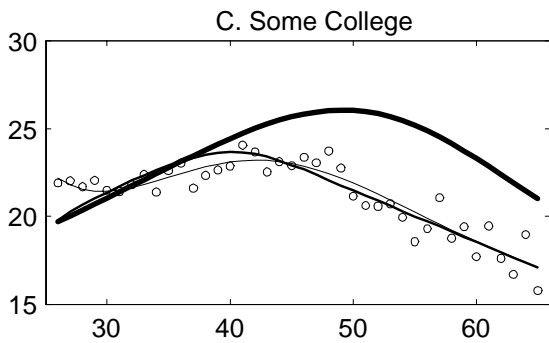
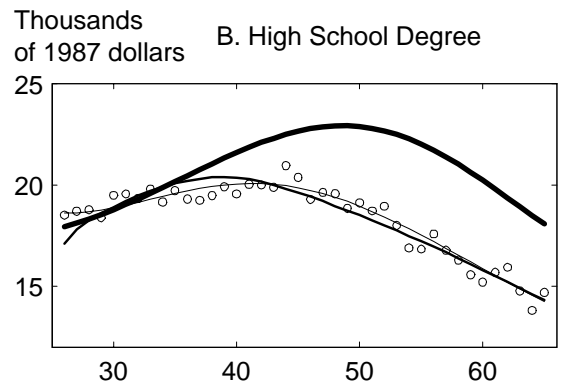
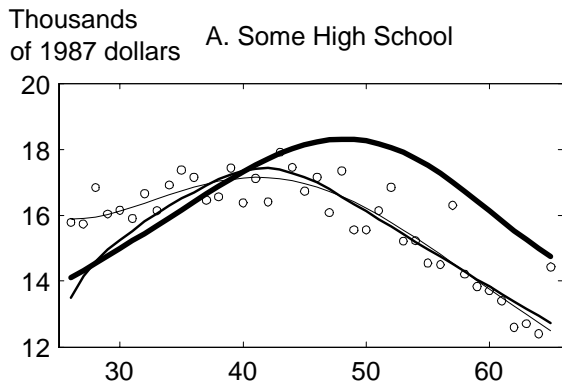
Panel B: $b = 0.960$, $r = 0.514$, $g_1 = 0.077$, $g_0 = 0.594$



Thousands of 1987 dollars

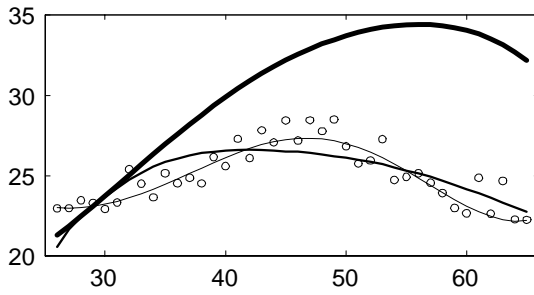


8 } ij 4 Krxvhkrog Frqvxpswlrq dqg Lqfrph ryhu wkh Olih F|foh



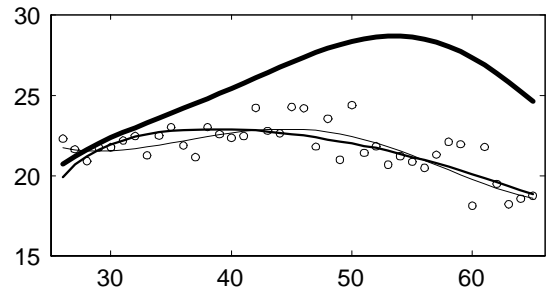
Thousands
of 1987 dollars

A. Managerial Professional

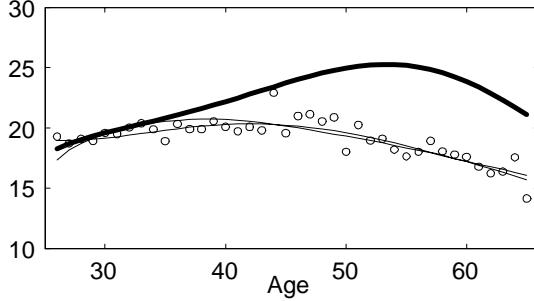


Thousands
of 1987 dollars

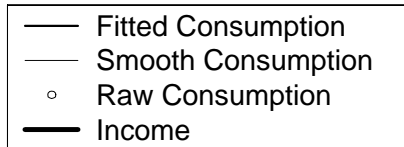
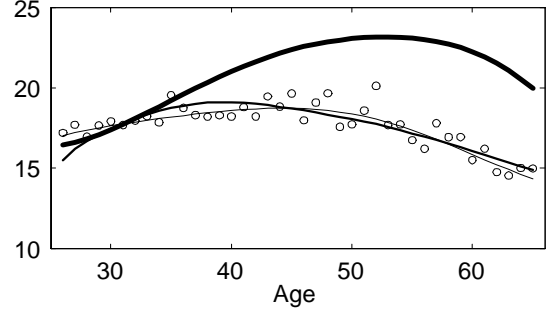
B. Technicians, Sales, Adm.



C. Precision, Production, Craft



D. Operator, Fabricants, Lab.

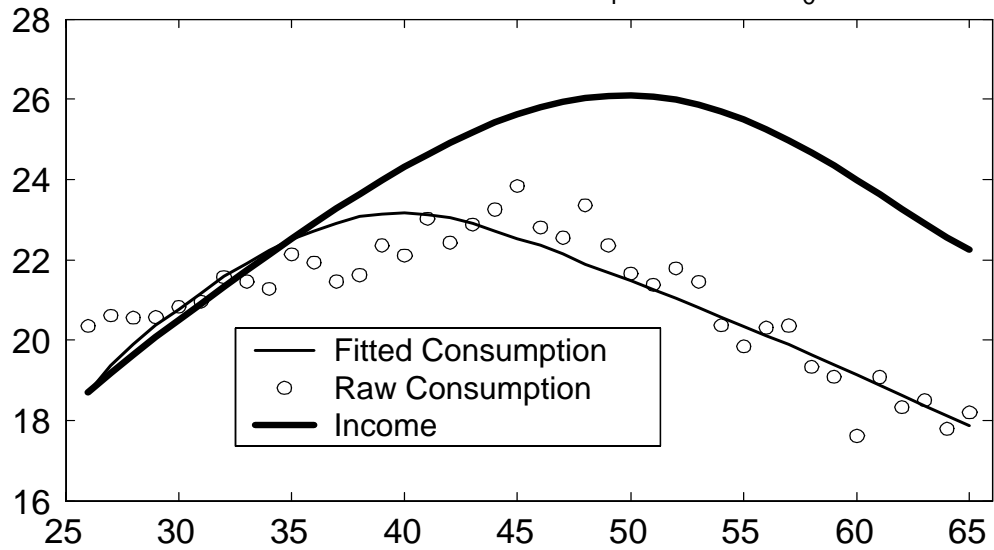


8 } ij g

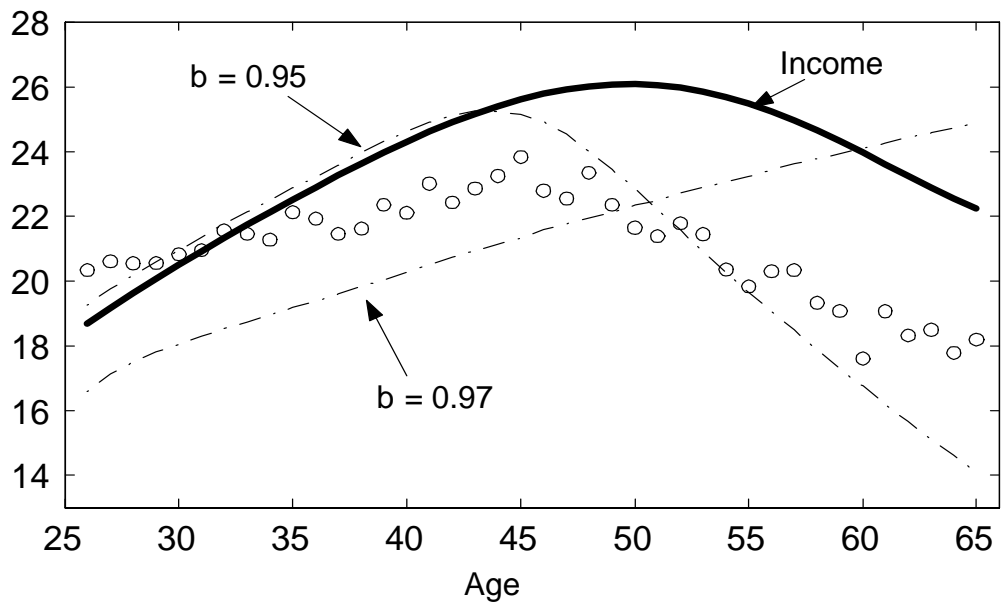
Krxvhkrog Frqvxpswlrq dqg Lqfrph ryhu wkh Olih F|foh/ e| Rffxsdwlrq Jurxs

Thousands
of 1987 dollars

Panel A: Baseline Estimation
 $b = 0.960$, $r = 0.514$, $g_1 = 0.071$, $g_0 = 0.001$



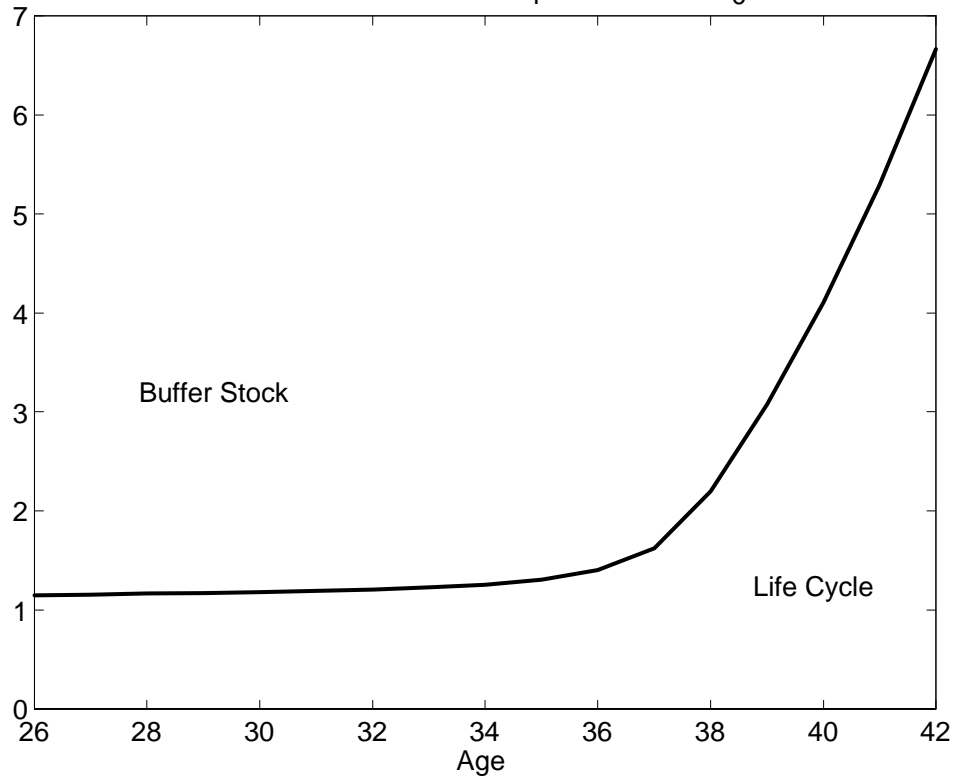
Panel B: Various b



Target cash-on-hand
(normalized)

Parameters:

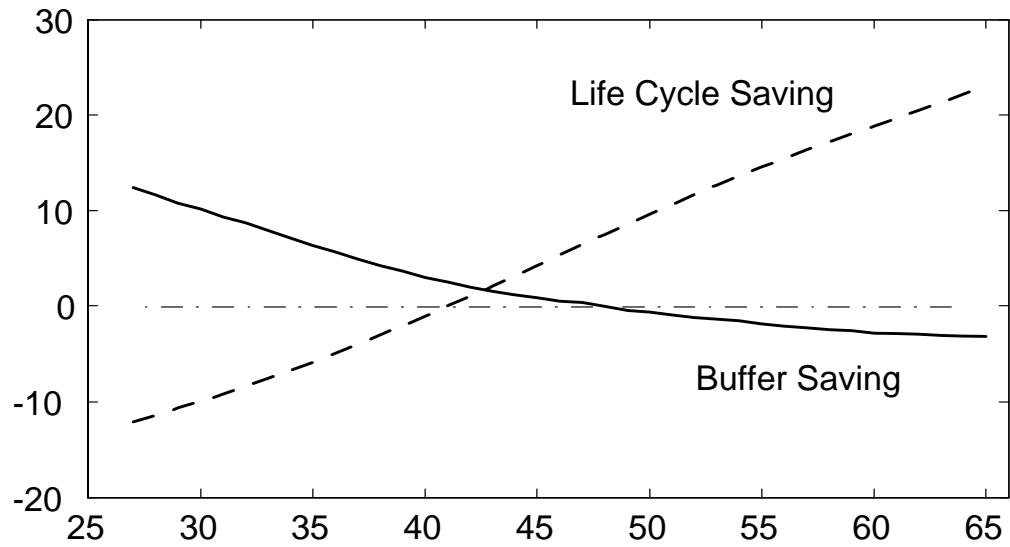
$b = 0.960$, $r = 0.514$, $g_1 = 0.0071$, $g_0 = 0.001$



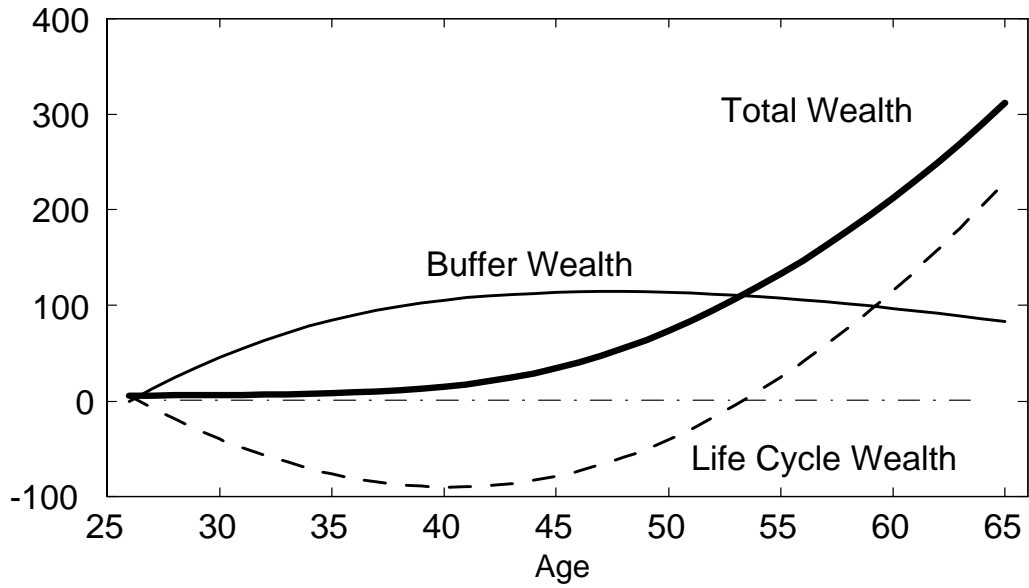
8 } ij U Qrupdol}hg Wdujhw Fdvk0rq0Kdqg e| Djh

Thousands
of 1987 dollars

Panel A: Life Cycle and Buffer Saving

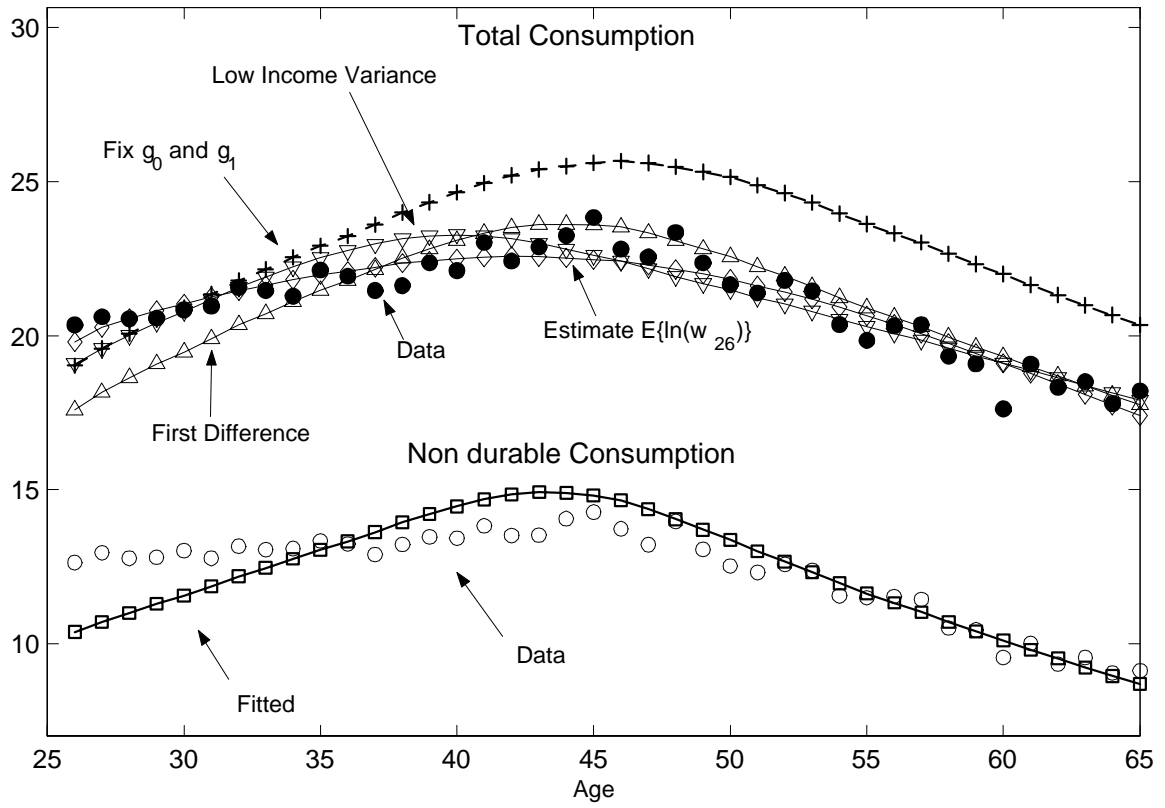


Panel B: Life Cycle and Buffer Wealth



8 } ij 0 Wkh Uroh ri Ulvn lq Vdylqj dqg Zhndwk Dffpxodwlrq

Thousands
of 1987 dollars



8 } ij l Ilwwhg Frqvxpswlrq Sur ohv xqghu GI huhqw Prgho Dv vxpswlrqv