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Richard Schmalensee

Abstract

Bolt and Tieman (2008) suggested the prevalence of profit function non-concavity may account for the widespread use of skewed pricing by two-sided platform businesses. In both the Rochet-Tirole (2003) and Armstrong (2006) models, however, skewed pricing may simply reflect substantial differences between side-specific demand functions; non-concavity is not necessary. In the Rochet-Tirole (2003) model, ubiquitous high pass-through rates, which seem implausible, are required for non-concavity to be prevalent. In the Armstrong (2006) model, non-concavity is not sufficient for skewed pricing. In both models, non-concavity is associated with strong indirect network effects; in the Armstrong (2006) model, such effects are also associated with dynamic instability.

KEYWORDS: platform, non-concavity, skewed, pricing

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Two-sided platform businesses (often labeled two-sided markets) commonly set price at or below marginal cost to one of the groups they serve, and some groups may even pay a zero price even though positive costs are incurred to serve them – see, e.g., Evans (2003) or Evans and Schmalensee (2007) for lists of examples. Thus Suarez and Cusumano (2009, p. 84) speak in generic terms of "the subsidy side" and "the money side" of such businesses. It is thus a bit surprising that the usual first-order conditions for profit maximization in standard models of multi-sided platforms do not immediately reveal why such highly skewed pricing should be the norm.

A recent paper by Bolt and Tieman (2008) shows that the second-order conditions for a maximum are violated in the Rochet-Tirole (2003) two-sided platform model if the demands of the two sides have constant elasticities.[1] In this example, the first-order conditions identify a saddle-point of the profit function, not a maximum, and profits are maximized at a corner solution, at which all potential participants on one side of the market actually participate. Bolt and Tieman suggest that this outcome may be general, so that the prevalence of highly skewed pricing in multi-sided platforms could be explained by the prevalence of such non-concave profit functions.

This note investigates the plausibility of that intriguing suggestion in the canonical two-sided platform models of Rochet and Tirole (2003) (Section 1) and Armstrong (2006) (Section 2) and, in the process, explores the association between non-concavity and strong indirect network effects in these models.[2] While Bolt and Tieman (2008) define highly skewed pricing as arising when all potential participants on one side of the market actually participate, I use here what I think is a more generally useful definition: one price is at the lower bound of the set of feasible prices – typically either zero or marginal cost. Few would disagree that pricing by network broadcast television is highly skewed, for instance, since all revenue comes from advertisers, even though some potential viewers don't in fact watch television.

Section 3 summarizes our main results and conclusions: it seems more likely that pervasive highly skewed pricing by platform reflects generally substantial differences between demand functions on the two sides of the business rather than pervasive profit function non-concavities.

[1] See Hermalin and Katz (2004) for a closely related result.

[2] Rochet and Tirole (2006) present a more general model that includes both these models as special cases and in which both access and intereractions may be priced. See Weyl (2010) for an extensive analysis of that model.

1. The Rochet-Tirole (2003) Model

In this model, the monopoly's objective function can be written as

$$\Pi(P_1, P_2) = (P_1 + P_2 - C)D_1\left(P_1\right)D_2\left(P_2\right), \tag{1}$$

where the P_i are the per-transaction prices charged to each of the two participating groups, C is the constant marginal cost of executing a transaction, and the D_i are (partial) demand functions. This model may be most directly relevant to payment cards, with the volume of transactions proportional to the product of the number of merchants accepting a particular card brand and the number of consumers carrying cards of that brand and the participation decisions of merchants and consumers assumed to depend only on the per-transaction prices they face.[3]

Assume that both demand functions in (1) exhibit declining marginal revenue. Then, using subscripts to indicate partial derivatives of the profit function, it is straightforward to show that for i = 1,2, $\Pi_{ii} < 0$ so that the point at which $\Pi_i = 0$ maximizes Π with respect to P_i, treating P_j, $j \neq i$, as constant. We can then define the functions

$$\hat{P}_i(P_j) \equiv \arg\max_{P_i} \Pi(P_i, P_j) = \arg\max_{P_i}\left[D_i\left(P_i\right)\left(P_i + P_j - C\right)\right], \quad i = 1,2,\ j \neq i. \tag{2}$$

These yield a natural summary measure of the strength of indirect network effects in this model, based on the geometric mean sensitivity of the optimal prices each side to the price on the other side:

$$T_P \equiv \left(d\hat{P}_1 \Big/ dP_2 \right)\left(d\hat{P}_2 \Big/ dP_1 \right) = \left(\Pi_{12}\right)^2 \Big/ \Pi_{11}\Pi_{22}. \tag{3}$$

But $T_P < 1$ is one of the second-order conditions for a stationary point of Π, a point at which the first-order conditions are satisfied, to be a local maximum. Thus, if the demand functions exhibit declining marginal revenue, a necessary and sufficient condition for concavity of the profit function is that average indirect network effects, as measured by T_P, not be too strong.

To be more precise, it is easy to show that the second derivatives of the profit function (1) can be written as follows, where primes indicate derivatives of the demand functions:

[3] See, for instance, Schmalensee (2002).

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(4a) $$\Pi_{ii} = -MD_1'D_2'(1/\rho_i) < 0, \quad i = 1,2, \quad \textit{and}$$

(4b) $$\Pi_{12} = \Pi_{21} = -MD_1'D_2' < 0, \quad \textit{where}$$

(4c) $$M \equiv (P_1 + P_2 - C), \quad \textit{and } \rho_i = (D_i')^2 / [2(D_i')^2 - D_iD_i''], i = 1,2.$$

That is, ρ_i is the pass-through rate on side *i*, the amount by which a monopolist with constant marginal cost and facing demand curve D_i would optimally increase price in response to a small unit increase in marginal cost (Weyl and Fabinger (2009)).

A close look at equation (2) makes it clear why pass-through rates determine the strength of the indirect network effects in this model: an exogenous increase in P_j affects the optimal value of P_i, $i \neq j$, exactly as an equivalent decrease in unit cost, C. It follows immediately from (4) that $T_P = \rho_1\rho_2$, and we have immediately

> *Proposition 1 (Weyl (2009, 2010)*: If both demand functions in the Rochet-Tirole (2003) model exhibit declining marginal revenue, a stationary point of (1) is a saddlepoint if and only if $\rho_1\rho_2 \geq 1$. If Π has a unique stationary point satisfying this condition, its maximum must occur on the boundary of the feasible set, and pricing must therefore be highly skewed.

For log-linear demand curves, $\rho > 1$, while for linear demand curves $\rho = 1/2$. Most economists seem to think that pass-through rates below one are more common than pass-through rates above one, though Weyl and Fabinger (2009) argue that there is not much evidence supporting that belief. Proposition 1 indicates that pass-through rates above one must nonetheless be pervasive if saddle-points are to be the norm in situations well-modeled by the Rochet-Tirole (2003) model.

To examine the plausibility of this outcome, suppose that pass-through rates are determined by independent draws from a uniform distribution over [0, 2], so that the average rate is above most economists' expectations. Then the probability that the product of two such draws will exceed unity is only about 0.40. While it is true that evidence on pass-through rates is scarce, it does not seem plausible that the pervasiveness of skewed pricing in two-sided markets is explained by the unusually frequent occurrence of pass-through rates above unity in those markets.

On the other hand, highly skewed pricing can easily arise when the profit function is concave when demand functions differ substantially. Suppose, for

example, that individual h in group i, i =1,2, will participate in the market under consideration if and only if $\theta_i^h \geq P_i$, where the θ_i^h are uniformly distributed between 0 and $A_i > 0$. If the number of potential participants in group i is N_i, the two demand functions can be written as

$$D_i\left(P_i\right) = N_i - b_i P_i \equiv b_i(\hat{P}_i - P_i), \quad i = 1,2, \tag{5}$$

where $b_i = N_i/A_i$ and $\hat{P}_i$ is the choke price at which demand on side i falls to zero, i = 1,2. The first-order conditions for maximizing expression (1) in this case signal a regular unconstrained optimum, but they imply $P_1 \leq 0$ if and only if

$$2\hat{P}_1 \leq \hat{P}_2 - C. \tag{6}$$

Condition (6) requires substantial differences the demand functions of the two groups. But, particularly when marginal cost is low, the required demand differences do not seem implausible for groups as different those linked by two-sided platforms in practice (though, as above, there is little or no relevant hard evidence). Consider merchants and shoppers, for instance. In any case, if condition (6) is satisfied and, as is usually the case, negative prices are infeasible, the optimum of (1) will involve highly skewed pricing with $P_1 = 0$ and $Q_1 = N_1$. We have thus proven by example

> *Proposition 2:* Non-concavity is not necessary for highly skewed pricing in the Rochet-Tirole (2003) model.

2. The Armstrong (2006) Model

In the Armstrong (2006) model, participation is priced but transactions are not, and participation, Q, is influenced via indirect network effects by participation on the other side, as well as the participation price, P. We can write the demand functions for participation by the two sides as

$$Q_i = D_i\left(P_i, Q_j\right), \quad i, j \in \{1,2\}, i \neq j. \tag{7}$$

We assume these functions are decreasing in price and non-decreasing in other-side quantity.

It is instructive to consider a natural measure of the (geometric) average strength of indirect network effects in this model:

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$$S \equiv \left(\partial D_1 \big/ \partial Q_2\right)\left(dD_2 \big/ dQ_1\right). \tag{8}$$

This quantity is a natural focus of empirical work on demand systems with network effects. It is not directly related to the concavity of the profit function in this model, however, but rather to the stability of equilibria in a family of myopic disequilibrium dynamic systems in the in the spirit of Rohlfs (1974):

$$\operatorname{sgn}\left(dQ_i \big/ dt\right) = \operatorname{sgn}\left[D_i\left(P_i, Q_j\right) - Q_i\right], \quad i, j \in \{1,2\}, i \neq j. \tag{9}$$

> *Proposition 3 (Evans and Schmalensee (2010)):* An equilibrium of (9) is stable if and only if $S < 1$ at that point. If $S \geq 1$, the equilibrium is a saddlepoint.

This result may serve to limit the set of empirically relevant profit function maxiuma. Profit maxima are necessarily equilibria of (9). Such maxima that lie the interior of the feasible set of quantities and for which $S \geq 1$ do not seem likely to be observed in practice.[4] Also, when demand functions are linear, equilibrium quantities are decreasing in prices if and only if $S < 1$; see equation (20) below.

It is generally most convenient in this model to solve the demand functions (7) for prices, so the profit function can be written as

$$\Pi\left(Q_1, Q_2\right) = Q_1\left[d_1\left(Q_1, Q_2\right) - C_1\right] + Q_2\left[d_2\left(Q_2, Q_1\right) - C_2\right]. \tag{10}$$

The C_i are the constant side-specific unit costs of participation. It is useful to recognize that the problem of maximizing (10) is formally identical to the problem of maximizing the profit of a monopolist selling complements. Note that if prices are constrained to be positive or not to be below marginal costs, the boundary of the feasible set of quantities may be affected by constraints on prices and thus may contain points at which participation is incomplete on both sides.

In general the second-order conditions for this model involve second derivatives of the d_i with respect to other-side quantities and are consequently

[4] Weyl (2010) shows that any pair of non-negative quantities can in general be sustained by what he terms *insulating tariffs*: schedules that make the price to each side conditional on the level of participation on the other side. (See also White and Weyl (2010) for a model of platform competition with insulating tariffs.) Price schedules of this sort do not seem common in practice, however, but, as noted below, prices that rise over time as participation grows may serve a similar purpose if (9) holds.

difficult to interpret.[5] Declining marginal revenues are no longer sufficient for the Π_{ii} to be negative, for instance. When those quantities are negative, however, we can define conditionally optimal quantities as above:

$$\hat{Q}_i(Q_j) \equiv \quad \arg\max_{Q_i} \Pi(Q_i, Q_j), \quad i = 1, 2,\ j \neq i. \tag{11}$$

With this definition, the third second-order condition for a stationary point of (10) to be at least a local optimum becomes a condition on the (geometric) average indirect network effect:

$$T_Q \equiv \left(d\hat{Q}_1 \Big/ dQ_2 \right)\left(d\hat{Q}_2 \Big/ dQ_1 \right) = \left(\Pi_{12}\right)^2 \Big/ \Pi_{11}\Pi_{22}. \tag{12}$$

We have immediately

> *Proposition 4:* If $\Pi_{11} < 0$ and $\Pi_{22} < 0$ at a stationary point in the Armstrong (2006) model, a necessary and sufficient condition for that point to be a local maximum is $T_Q < 1$. If $T_Q \geq 1$, the stationary point is a saddlepoint.

As in Proposition 1, if the unique stationary point is a saddlepoint, profit is maximized on the boundary of the feasible set. It is clear that the measures S and T_Q of average indirect network effects are not in general equal, since S involves first derivates only, while T_Q involves a variety of second derivatives, but it is also clear that they are not unrelated. Considering both sheds light on the sources of non-concavities and their role in this model.

In order to obtain further insight into the determinants of non-concavity and the relations between these two measures of average indirect network effects, it is useful to consider two examples. Suppose first that the D_i are log-linear:

$$D_i\left(P_i, Q_j\right) = \alpha_i P_i^{-\beta_i} Q_j^{\delta_j}, \quad i, j \in \{1, 2\}, i \neq j, \tag{13}$$

where the αs, βs, and δs are positive constants. It is immediate that $S = \delta_1\delta_2$ here. Then straightforward but tedious analysis of first- and second-order conditions yields

[5] See Theorem 4 in Weyl (2010) and its discussion in the paper's online appendix.

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> *Proposition 5:* If demands in the Armstrong (2006) model are given by (13) with $\beta_1, \beta_2 > 1$, then $\delta_i < \beta_i \Rightarrow \Pi_{ii} < 0$ at a stationary point, $i = 1,2$. If both these conditions are satisfied, then if $\delta_1, \delta_2 < 1$, then $T_Q < 1$ (and $S < 1$), while if $\delta_1, \delta_2 \geq 1$, then $T_Q \geq 1$ (and $S \geq 1$).

Thus indirect network elasticity on side i must be weaker than the corresponding price elasticity in order for $\Pi_i = 0$ to indicate a conditional maximum of Π rather than a conditional minimum. If the profit function is well-behaved in this sense and if both network elasticities are less than (greater than or equal to) one, then both T_Q and S are less than (greater than or equal to) one.

A second, somewhat more tractable example is obtained by assuming that typical individual h on side i, i =1,2, participates if and only if

$$\theta_i^h \geq P_i - \delta_i Q_j, \quad i, j \in \{1,2\}, i \neq j, \tag{14}$$

where the δ_i are positive constants. Let $N_i > 0$ be the maximum number of potential participants on side i, and assume the θ_i^h are uniformly distributed between 0 and α_i, with $\alpha_i > C_i$ $i = 1,2$. Under these assumptions the demands for participation are given by

$$D_i\left(P_i, Q_j\right) = N_i - B_i P_i + G_i Q_j, \quad i, j \in \{1,2\}, i \neq j, \tag{15a}$$

where

$$B_i = N_i / \alpha_i, \quad \textit{and} \quad G_i = \delta_i N_i / \alpha_i, \quad i = 1,2. \tag{15b}$$

In this example, $S = G_1 G_2$.

Solving equations (15) for prices yields

$$P_i = d_i\left(Q_i, Q_j\right) = \alpha_i - b_i Q_i + \delta_i Q_j, \quad i, j \in \{1,2\}, i \neq j, \tag{16a}$$

where

$$b_i = 1/B_i = \alpha_i / N_i, \quad i = 1,2. \tag{16b}$$

The firm's profit function can then be written as

$$\Pi = Q_1\left(\alpha_1 - b_1Q_1 + \delta_1Q_2 - C_1\right) + Q_2\left(\alpha_2 - b_2Q_2 + \delta_2Q_1 - C_2\right), \tag{17}$$

Using subscripts to indicate partial differentiation as above, the first-order conditions for maximization of Π are

$$\Pi_i = \left(\alpha_i - C_i\right) + \left(\delta_1 + \delta_2\right)Q_j - 2b_iQ_i = 0, \quad i, j \in \{1,2\}, i \neq j, \tag{18}$$

and the second-order necessary conditions for (18) to yield a maximum are

$$\Pi_{ii} = -2b_i < 0, \quad i = 1,2, \tag{19a}$$

and

$$T_Q \equiv \frac{\left(\Pi_{12}\right)^2}{\Pi_{11}\Pi_{22}} = \frac{\left(\delta_1 + \delta_2\right)^2}{4b_1b_2} < 1. \tag{19b}$$

Condition (19b) says that the arithmetic mean of the cross-quantity effects on price must be less than the geometric mean of the own-quantity effects.[6] Using (16b) to re-write this condition, we can re-write this condition in a form that is easier to compare with $S = G_1G_2$ and obtain

Proposition 6: The point satisfying (18) maximizes Π if and only if

$$\sqrt{T_Q} = \frac{1}{2}\left[G_1\sqrt{\frac{B_2}{B_1}} + G_2\sqrt{\frac{B_1}{B_2}}\right] < 1$$

If $T_Q \geq 1$, that point is a saddlepoint, and Π is maximized on the boundary of the feasible set of quantities. $T_Q < 1$ is sufficient for $S < 1$. It is also necessary for $S < 1$ if and only if the two demand functions are identical.

Once again, both the concavity of the profit function and stability of the general dynamic adjustment process are associated with limits on the importance of indirect network effects.

[6] For an interpretation of this condition in terms of profits with and without two-sidedness, see the discussion of Proposition 4 in the online appendix to Weyl (2010).

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To see if non-concavity of the profit function is *necessary* for highly skewed pricing, let us assume concavity and solve (15) for both quantities as functions of both prices, to obtain

$$Q_i = \frac{N_i + G_i N_j - B_i P_i - G_i B_j P_j}{1 - S}, \quad i, j \in \{1, 2\}, i \neq j. \tag{20}$$

If the profit function is concave, Proposition 6 implies S < 1, so these demand functions are well-behaved. Suppose, to avoid clutter, that $C_1 = C_2 = 0$. Then there is highly skewed pricing with concavity if (a) $T_Q < 1$ and (b) $\Pi_1 \leq 0$ when $\Pi_2 = 0$ and $P_1 = 0$, as long as the solution involves non-negative quantities. The second condition (b) can be written as

$$N_1\left[2B_2 - G_2\left(G_1B_2 - G_2B_1\right)\right] + N_2\left(G_1B_2 - G_2B_1\right) \leq 0. \tag{21}$$

It is immediate (and not surprising) that $G_2 > 0$ is necessary for this condition to be satisfied. Substituting the optimal value of P_2 when $P_1 = 0$ then establishes that $Q_1 \leq N_1$ if and only if $D_1 = 0$, which in turn implies $Q_1 = N_1$. It is then easy to find numerical values that satisfy (19b) and (21) with $D_1 = 0$. One example is $N_1 = N_2 = 100$, $B_1 = 20$, $B_2 = 10$, and $G_2 = 0.8$. Thus non-concavity is not necessary for highly skewed pricing in this example.

To see whether non-concavity is *sufficient* for highly skewed pricing in this example, suppose demand and cost functions on the two sides are identical and $T_Q > 1$. Dropping subscripts on cost and demand parameters, if $Q_1 = Q_2 = Q$, equation (17) becomes

$$\Pi = 2Q\left[\left(\alpha - C\right) + Q\left(\delta - b\right)\right]. \tag{22}$$

Since $T_Q > 1$ implies $\delta > b$, this expression is convex in Q and profits are either maximized at $Q = 0$ or $Q = N$. If

$$\left(a - C\right) + N\left(\delta - b\right) > 0, \tag{23}$$

the maximum occurs at $Q = N$. It is straightforward to show that if (23) is satisfied and if, say, $Q_2 = N$, then profits are maximized by setting $Q_1 = N$ also. Thus even though the profit function is maximized on the boundary of the feasible set of quantities, pricing is symmetric, not skewed:

$$P_1^* = P_2^* = \alpha + N(\delta - b). \tag{24}$$

We have thus completed the proof by example of

> *Proposition 7:* Non-concavity is neither necessary nor sufficient for highly skewed pricing in the Armstrong (2006) model.

When the profit function is non-concave and the demand functions are identical in this example, $S > 1$, profit maxima in the interior of the feasible set are necessarily unstable equilibria of dynamic processes satisfying (9) and thus, as argued just below Proposition 3, unlikely to be observed. But this is not an *interior* equilibrium in the set of feasible quantities. Again in the spirit of Rohlfs (1974), as long as

$$P_i < \alpha_i + \delta Q_j - bQ_i, \quad i, j \in \{1,2\}, i \neq j, \tag{25}$$

$D_i > Q_i$, i = 1,2, and, by (9), both quantities are increasing. Once the point $Q_1 = Q_2 = N$, is reached, it can be maintained by setting prices just below the level given by (24), so that all participants have strictly positive surplus.

3. Conclusions

In the Rochet-Tirole (2003) model, Section 1 showed by example that non-concavity is not necessary for highly skewed pricing. Such pricing can arise from substantial differences in the demand functions on the two sides of the market. Following Weyl (2009, 2010), high pass-through rates are sufficient for non-concavity and skewed pricing, but it seems implausible that high pass-through rates are in fact ubiquitous in platform markets.

In the more complex model of Armstrong (2006), Section 2 showed that strong network effects are associated both with non-concave profit functions and with instability of a broad class of disequilibrium adjustment processes. In this model, non-concavity was shown by example to be neither necessary nor sufficient for highly skewed pricing.

In short, the analysis here strongly suggests that highly skewed pricing by two-sided platforms is not prevalent because profit is generally maximized at corner solutions for these businesses, but rather because the demand characteristics of the two groups involved generally differ substantially.

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