The Design and Price of Information
Dirk Bergemann, Alessandro Bonatti, and Alex Smolin
Online Appendix A: Many States and Actions

Example 1 illustrates the construction of the optimal experiment with many actions and two types (Lemma 4).

Example 1 (Noncongruent Types) *Consider uniform match values ($u_i = 1$ for all $i = 1,2,3$) and two types, $\theta^L = (1/10, 1/10, 8/10)$ and $\theta^H = (4/10, 3/10, 3/10)$. These types are noncongruent: without additional information, θ^H would choose action a_1 and θ^L would choose a_3. The likelihood ratios θ_i^L/θ_i^H are $(1/4, 1/3, 8/3)$. This implies $i_b = 2$, whereas $i_s \in \{1,2\}$ depending on the prior probability γ of the high type. For $\gamma \in [0, 1/4]$ and $\gamma \in [1/4, 1/3]$, the high type obtains positive rents. Furthermore, for $\gamma \geq 1/4$, the partially informative experiment $E(\theta^L)$ involves dropping signal s_1. The optimal experiment $E(\theta^L)$ as a function of γ is given by*

$E(\theta^L)$	s_1	s_2	s_3
ω_1	1	0	0
ω_2	0	1	0
ω_3	0	0	1
	if $\gamma < 1/4$,		

$E(\theta^L)$	s_1	s_2	s_3
ω_1	0	1/4	3/4
ω_2	0	1	0
ω_3	0	0	1
	if $\gamma \in [1/4, 1/3]$,		

$E(\theta^L)$	s_1	s_2	s_3
ω_1	0	1/4	3/4
ω_2	0	1/2	1/2
ω_3	0	0	1
	if $\gamma > 1/3$.		

Example 2 illustrates how congruent, but not strongly congruent beliefs, allow for surplus extraction. In the example, the two types deem state ω_2 the most likely. Thus the types are congruent but not strongly congruent, as they disagree on the relative likelihood of states ω_1 and ω_3.

Example 2 (Congruent Priors) *Consider uniform match values ($u_i = 1$ for all $i = 1,2,3$) and two types, $\theta^L = (5/10, 1/10, 4/10)$ and $\theta^H = (4/10, 3/10, 3/10)$. Because $i_b = 1$, the optimal experiment $E(\theta^L)$ as a function of γ is given by*

$E(\theta^L)$	s_1	s_2	s_3
ω_1	1	0	0
ω_2	0	1	0
ω_3	0	0	1
	if $\gamma \leq 1/3$,		

$E(\theta^L)$	s_1	s_2	s_3
ω_1	1	0	0
ω_2	0	1/2	1/2
ω_3	0	0	1
	if $\gamma > 1/3$,		

and the high type obtains positive rents only if $\gamma < 1/3$.

Example 3 shows that the relaxed approach is not valid with many types. In the example below with three types, no experiment $E(\theta^1)$ can lead both types θ^2 and θ^3 to follow the action recommended by every signal. Thus, the profits in the relaxed problem are strictly greater than those in original problem.

Example 3 (Many Types and Actions) *Consider uniform match values ($u_i = 1$ for all $i = 1, 2, 3$) and three types, $\theta^1 = (1/6, 1/6, 4/6)$, $\theta^2 = (1/2, 1/2, 0)$, and $\theta^3 = (1/2, 0, 1/2)$, which are all equally likely. In the* relaxed problem*, the monopolist sells the fully informative experiment to types θ^2 and θ^3. Type θ^1 is offered the partially informative experiment*

$E(\theta^1)$	s_1	s_2	s_3
ω_1	1/2	0	1/2
ω_2	0	1	0
ω_3	0	0	1

,

and the seller's revenues are equal to $5/12$. However, if type θ^2 purchased experiment $E(\theta^1)$, he would choose action a_1 when observing signal s_3. In the solution to the full problem*, which we can construct by a guess-and-verify approach, the optimal experiment $E(\theta^1)$ consists of*

$E(\theta^1)$	s_1	s_2	s_3
ω_1	1/2	0	1/2
ω_2	1/2	1/2	0
ω_3	0	0	1

, (46)

which yields revenues of $1/3$, i.e., revenues are strictly lower in the relaxed program.

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Online Appendix B: More Actions than States

We consider a setting with two types, and we relax the assumption of matching state-action payoffs. In particular, we consider the following example with two types, two states, and three actions. The data buyer's payoff is given by

$u(\omega, a)$	a_1	a_2	a_3
ω_1	1	0	4/5
ω_2	0	1	4/5

Thus, action a_i is the optimal action in state ω_i, but action a_3 provides a lower bound on the payoffs that is uniform across states—an insurance action. Let the two types be given by $\theta^L = (1/10, 9/10)$ and $\theta^H = (6/10, 4/10)$. As the results of Lemma 1 do not rely on matching payoffs, we know type θ^H receives full information, $E(\theta^H) = \overline{E}$, his incentive constraint binds, and the participation constraint of type θ^L binds.

For the case $\gamma \triangleq \Pr\left[\theta = \theta^H\right] = 3/4$, an optimal menu contains the experiment

$E(\theta^L)$	s_1	s_2	s_3
ω_1	1/3	2/15	8/15
ω_2	0	4/5	1/5

(47)

at a price $t(\theta^L) = 1/25$ and the experiment $\overline{E}$ at a price $t(\theta^H) = 1/5$. The optimal menu is illustrated in Figure 10.

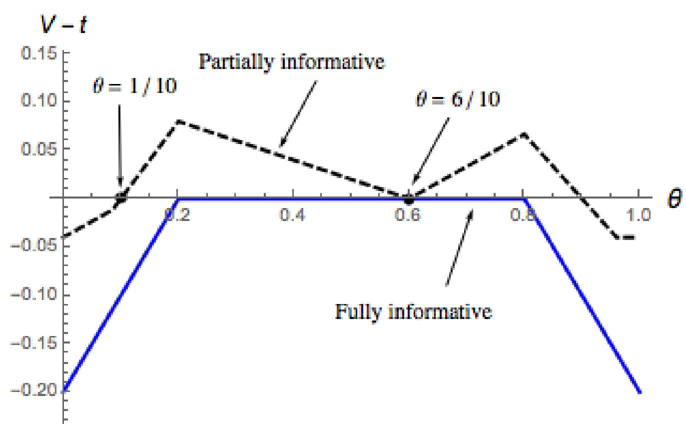

Figure 10: Optimal Menu

This menu has the following notable properties: *(i)* the seller extracts all the surplus from both types; *(ii)* type θ^L follows the recommendation of every signal in $E(\theta^L)$; *(iii)* type θ^H, if purchasing experiment $E(\theta^L)$, is indifferent between action a_2 and action a_3 when observing signal s_2 as well as between a_1 and a_3 when observing signal s_3; and *(iv)* the optimal profits are strictly lower than those in the relaxed problem.

Indeed, ignoring the off-path obedience constraints, the optimal menu is discriminatory, and the seller extracts all the rents by offering the experiment

$$\begin{array}{c|cc} E(\theta^L) & s_1 & s_2 \\ \hline \omega_1 & 2/5 & 3/5 \\ \omega_2 & 0 & 1 \end{array} \tag{48}$$

at a price $t(\theta^L) = 1/25$ and the experiment $\overline{E}$ at a price $t(\theta^H) = 2/5$.

In the full problem, the seller cannot turn experiment $E(\theta^L)$ in (47) into the *more informative* one in (48). If she did, buyer type θ^H would choose action a_3 after deviating and observing signal s_2.[27] In other words, the seller extracts the surplus, but at the cost of additional distortion—notably, there is no "1" entry in (47).

Interestingly, action a_3 may not be induced in an optimal menu yet still restrict the seller. Indeed, if we modify the above example by setting $\gamma = 2/3$, an optimal menu can be calculated to contain the experiment

$$\begin{array}{c|cc} E(\theta^L) & s_1 & s_2 \\ \hline \omega_1 & 5/6 & 1/6 \\ \omega_2 & 0 & 1 \end{array} \tag{49}$$

at a price $t(\theta^L) = 1/12$ and the experiment $\overline{E}$ at a price of $t(\theta^H) = 11/60 < 1/5 = V\left(\theta^H, \overline{E}\right)$. As before, type θ^H is indifferent between a_2 and a_3 after deviating to $E(\theta^L)$ and observing s_2. Contrary to our earlier examples, the high type makes positive rents despite the seller's discriminatory menu offering.

To emphasize, action a_3 is not chosen in an optimal menu by either type, yet it looms large and prevents the seller from extracting the full surplus. In particular, the seller would like to reduce π_{11} in order to relax the high type's incentive constraint and increase t^H. However, by doing so she would induce type θ^H to choose action a_3 after s_2. This means that the high type's *marginal benefit* from a probability shift from π_{11} to π_{12} is $\theta_1^H(-1 + 4/5) = -3/25$, while the corresponding *marginal* change in price t^L is $-\theta_1^L = -1/10$. Therefore, t^H can only increase at rate $1/50$, which is not profitable for the seller when the fraction of high types

[27] Similarly, one can show that type θ^H must be indifferent after signal s_3 in experiment (47). The argument here is by contradiction: if he strictly preferred action a_3, the seller could make the experiment more valuable for θ^L without changing its value for θ^H; and if he strictly preferred a_1, the seller could rearrange all signals and relax the incentive-compatibility constraint.

is $\gamma = 2/3$. If both types were instead required to follow the signals' recommendations, the high type's misreporting value would change at rate $-\theta_1^H = -3/5$, allowing the seller to increase t^H at the profitable rate of $1/2$. The kink in the "exchange rate" when the high type is indifferent among several actions prevents the seller from making the modification.

To reinforce the point, maintain the assumption $\gamma = 2/3$ but exclude action a_3 from the set of available actions. The optimal menu is again given by the experiment in (48), which is now *less informative* than (49), but allows the seller to extract all the rents.

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Online Appendix C: Sequential Design

We show that sequential design of experiments can increase the seller's revenues in our leading binary-type example. We focus on the simplest instance of a dynamic protocol, whereby the seller first releases a free informative experiment to the buyer and then, without observing the realized signal, offers a menu of (experiment, price) pairs from which to choose.

Let $\Omega = \{\omega_1, \omega_2\}$, $A = \{a_1, a_2\}$, and assume uniform match values, i.e.,

$$u(\omega_i, a_j) = \mathbb{I}_{[i=j]}.$$

Consider two equally likely types with interim beliefs $\theta^L = 1/8$ and $\theta^H = 1/4$,respectively, where $\theta \triangleq \Pr[\omega_1]$.

Because the two types are *congruent*, an optimal static mechanism (Proposition 3) contains only the fully informative experiment. In the current example, the seller is indifferent between charging prices $t = 1/8$ and $t = 1/4$. In either case, the monopoly profits are

$$\pi^*_{\text{static}} = 1/8.$$

Consider the following sequential scheme. First, the seller reveals an outcome of the following experiment E_0 at no cost to the buyer

E_0	s_1	s_2
ω_1	1	0
ω_2	1/3	2/3

After observing signal s_2, the buyer is convinced that the state is $\omega = \omega_2$, which confirms his prior, and does not buy further information. After realization s_1, however, the buyer's beliefs are updated to

$$\theta^L(s_1) = 3/10, \quad \theta^H(s_1) = 1/2.$$

At this point, the seller offers the fully informative experiment at a price $\bar{t} = 3/10$.

The key observation is that signal s_1 under experiment E_0 is more likely be realized for the high type $\theta^H = 1/4$ than for the low type $\theta^L = 1/8$. In particular, the signal distribution is given by

$$\Pr\left[s_1 \mid \theta^L\right] = \frac{5}{12}, \; \Pr\left[s_1 \mid \theta^H\right] = \frac{1}{2}.$$

As a consequence, the monopolist's profit is given by

$$\pi^*_{\text{dyn}} \triangleq \bar{t}\left(\gamma \Pr\left[s_1 \mid \theta^H\right] + (1-\gamma)\Pr\left[s_1 \mid \theta^L\right]\right) = 11/80.$$

Thus, the sequential sale outperforms the static sale in this example, i.e.,

$$\pi^*_{\text{dyn}} = 11/80 > 1/8 = \pi^*_{\text{static}}.$$

Taking a step back, it is clear that the seller would ideally like to condition payments on the realized states. In this case, she could charge a payment of 1 upon realization of state ω_1, which is the state less likely for either type. Both types would accept such a contract, and the seller achieves the first-best profits. As we do not allow for the payments to be made contingent on the realization of the state, a sequential mechanism essential represents a costly instrument to (partially) circumvent this restriction.

In essence, the proposed sequential scheme charges a constant price $\bar{t} = 3/10$ upon realization of signal s_1. Because the signal is correlated with the state under experiment E_0, it occurs more frequently for the higher type, allowing the seller to effectively price discriminate without ever giving the buyer a choice of experiment.

Finally, note that the seller could do better within the simple class of mechanisms that initially release a free experiment, followed by a menu.

Intuitively, as the correlation between state and signal s_1 becomes more precise (i.e., as s_1 becomes more informative), the seller's ability to condition payments on states improves. Ultimately, however, the seller must balance the ability to correlate payments with the willingness to pay for supplemental information after observing signal s_1 (e.g., the signal cannot be arbitrarily precise).

To formalize the intuition, consider offering free experiments of the following form

$E(x)$	s_1	s_2
ω_1	1	0
ω_2	$1-x$	x

These experiments lead to posterior beliefs

$$\begin{aligned}
\theta^L(x) &\triangleq \Pr\left[s_1 \mid \theta^L\right] = \frac{1}{8-7x}, \\
\theta^H(x) &\triangleq \Pr\left[s_1 \mid \theta^H\right] = \frac{1}{4-3x}.
\end{aligned}$$

These beliefs satisfy the condition $1/2 = \gamma \geq \theta^L(x)/\theta^H(x)$ for all x. Therefore, after releasing experiment $E(x)$ the seller optimally offers the fully informative experiment $\bar{E}$ at a price

$$\bar{t}(x) = \min\left\{\theta^L(x), 1 - \theta^H(x)\right\}.$$

Finally, a straightforward calculation reveals that the seller's profits are maximized by choosing x such that $\theta^L(x) < 1/2 < \theta^H(x)$. In particular, it is optimal for the seller to induce the two types to have identical willingness to pay for the full information, i.e.,

$$\theta^L(x^*) = 1 - \theta^H(x^*).$$

The optimal experiment has

$$x^* = 1 - 1/\sqrt{21} \approx 0.781,$$

which is larger than $x = 2/3$, as used in the initial example, and yields profits $\pi^* = (7 + 2\sqrt{21})/112 \approx 0.144$ that exceed $11/80$, as computed above.