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The Impact of Demand Uncertainty on Consumer Subsidies for Green Technology Adoption

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This paper studies government subsidies for green technology adoption while considering the manufacturing industry's response. Government subsidies offered directly to consumers impact the supplier's production and pricing decisions. Our analysis expands the current understanding of the price-setting newsvendor model, incorporating the external influence from the government who is now an additional player in the system. We quantify how demand uncertainty impacts the various players (government, industry and consumers) when designing policies. We further show that for convex demand functions, an increase in demand uncertainty leads to higher production quantities and lower prices, resulting in lower profits for the supplier. With this in mind, one could expect consumer surplus to increase with uncertainty. In fact, we show this is not always the case and the uncertainty impact on consumer surplus depends on the trade-off between lower prices and the possibility of under-serving customers with high valuations. We also show that when policy makers such as governments ignore demand uncertainty when designing consumer subsidies, they can significantly miss the desired adoption target level. From a coordination perspective, we demonstrate that the decentralized decisions are also optimal for a central planner managing jointly the supplier and the government. As a result, subsidies provide a coordination mechanism.

Key words: Government Subsidies, Green Technology Adoption, Newsvendor, Cost of Uncertainty, Supply Chain Coordination

1. Introduction

Recent developments in green technologies have captured the interest of the public and private sectors. For example, electric vehicles (EV) historically predate gasoline vehicles, but have only received significant interest in the last decade (see Eberle and Helmot (2010) for an overview). In the height of the economic recession, the US government passed the American Recovery and Reinvestment Act of 2009 which granted a tax credit for consumers who purchased electric vehicles. Besides boosting the US economy, this particular tax incentive was aimed at fostering further research and scale economies in the nascent electric vehicle industry. In December 2010, the all-electric car, Nissan Leaf, and the plug-in hybrid General Motors' Chevy Volt were both introduced in the US market. After a slow first year, sales started to pick up and most major car companies are now in the process of launching their own versions of electric vehicles.

More recently in 2012, Honda introduced the Fit EV model and observed low customer demand. After offering sizable leasing discounts, Honda quickly sold out in Southern California[1]. It is not uncommon to read about waitlists for Tesla's new Model S or the Fiat 500e, while other EVs are sitting unwanted in dealer parking lots. Both stories of supply shortages or oversupply have been commonly attributed to electric vehicle sales. At the root cause of both these problems is demand uncertainty. When launching a new product, it is hard to know how many units customers will request. In addition, finding the correct price point is also not a trivial task, especially with the presence of a government subsidy. In fact, understanding demand uncertainty should be a first order consideration for both manufacturers and policy-makers alike.

For the most part, the subsidy design literature in green technologies has not studied demand uncertainty (see for example, Benthem et al. (2008), Atasu et al. (2009), Lobel and Perakis (2013) and Alizamir et al. (2013)). In practice, demand uncertainty has also often been not considered. As suggested in private communication with several sponsors of the MIT Energy Initiative[2], policy makers often ignore demand uncertainty when designing consumer subsidies for green technology adoption (see a recent article in Energy Futures magazine, Stauffer 2013). The purpose of this paper is to study whether incorporating demand uncertainty in the design of subsidy programs for green technologies is important. In particular, we examine how governments should set subsidies when considering the manufacturing industry's response under demand uncertainty. We show that demand uncertainty plays a significant role in the system's welfare distribution and should not be overlooked.

Consider the following two examples of green technologies: electric vehicles and solar panels. By the end of 2013, more than $10GW$ of solar photovoltaic (PV) panels had been installed in the United States, producing an annual amount of electricity roughly equivalent to two Hoover Dams. While still an expensive generation technology, this large level of installation was only accomplished due to the support of local and federal subsidy programs, such as the SunShot Initiative. In 2011, the US energy secretary Steven Chu announced that the goal of the SunShot Initiative by 2020 is to reduce the total cost of PV systems by 75%, or an equivalent of \$1 a Watt (DOE 2012), at which point solar technology will be competitive with traditional sources of electricity generation. Even before this federal initiative, many states have been actively promoting solar technology with consumer subsidies in the form of tax rebates or renewable energy credits.

[1] Los Angeles Times, 06/05/2013, http://www.latimes.com/business/autos/la-fi-hy-autos-electric-cars-sold-out-20130605,0,1186526,full.story

[2] http://mitei.mit.edu/about/external-advisory-board

Similarly, federal subsidies were also introduced to stimulate the adoption of electric vehicles through the Recovery Act. As we previously mentioned, General Motors and Nissan have recently introduced affordable electric vehicles in the US market. GM's Chevy Volt was awarded the most fuel-efficient compact car with a gasoline engine sold in the US, as rated by the United States Environmental Protection Agency (EPA (2012)). However, the price tag of the Chevy Volt is still considered high for its category. The cumulative sales of the Chevy Volt in the US since it was launched in December 2010 until September 2013 amount to $48,218$. It is likely that the $\$7,500$ government subsidy offered to each buyer through federal tax credit played a significant role in the sales volume. The manufacturer's suggested retail price (MSRP) of GM's Chevy Volt in September 2013 was $\$39,145$ but the consumer was eligible for $\$7,500$ tax rebates so that the effective price reduced to $\$31,645$. The amount of consumer subsidies has remained constant since launch in December 2010 until the end of 2013. This seems to suggest that in order to isolate the impact of demand uncertainty without complicating the model, it is reasonable to consider a single period setting.

In this paper, we address the following questions. How should governments design green subsidies when facing an uncertain consumer market? How does the uncertain demand and subsidy policy decision affect the supplier's price (MSRP) and production quantities? Finally, what is the resulting effect on consumers? In practice, policy makers often ignore demand uncertainty and consider average values when designing consumer subsidies. This ignorance may be caused by the absence (or high cost) of reliable data, among other reasons. We are interested in understanding how the optimal subsidy levels, prices and production quantities as well as consumer surplus are affected when one explicitly considers demand uncertainty relative to the case when demand is approximated by its deterministic average value.

While the government designs subsidies to stimulate the adoption of new technologies, the manufacturing industry responds to these policies with the goal of maximizing its own profit. In this paper, we model the supplier as a price-setting newsvendor that responds optimally to the government subsidy. More specifically, the supplier adjusts its production and price (MSRP) depending on the level of consumer subsidies offered by the government to the consumer. In such industries where production is long and incurs large fixed costs, we consider the production quantities to be equivalent to the capacity investment built in the manufacturing facility. This study also helps us to expand the price-setting newsvendor model while accounting for the external influence of the government. Consequently, as our problem cannot be cast as a traditional supply-chain model, we need to develop a new approach to derive optimal policies and insights.

Another feature of the aforementioned green subsidy policy is that the government often sets an adoption target. For example, in the 2011 State of the Union, US President Barack Obama mentioned the following goal: "With more research and incentives, we can break our dependence on oil with bio-fuels and become the first country to have a million electric vehicles on the road by 2015" (DOE 2011). Another example of such adoption target has been set for solar panels in the California Solar Incentive (CSI) program, which states that: "The CSI program has a total budget of $2.167 billion between 2007 and 2016 and a goal to install approximately 1,940 MW of new solar generation capacity" (CSI 2007). Hence, in our model, we optimize the subsidy level to achieve a given adoption target level while minimizing government expenditure (or maximizing the total welfare) as we discuss in Section 3.

Motivated by the EV market, we analyze the case where both price and production level are decision variables of the supplier. We compare the optimal policies to the simple case where demand is approximated by its average value, i.e., a deterministic function. We further extend the comparisons in a continuous fashion as a function of the magnitude of demand uncertainty. In other words, we quantify the impact of demand uncertainty and characterize who bears the cost of uncertainty depending on the structure of the demand model. Finally, we study the supply-chain coordination i.e., when the government owns the supplier, and show that subsidies coordinate the overall system. More precisely, we show that the price paid by consumers as well as the production level coincide in both the centralized (where the supplier is managed/owned by the government) and the decentralized models (where supplier and government act separately).

Contributions

Given the recent growth of green technologies, supported by governmental subsidy programs, this paper explores a timely problem in supply chain management. Understanding how demand uncertainty affects subsidy costs, as well as the economic surplus of suppliers and consumers, is an important part of designing sensible subsidy programs. The main contributions of this paper are:

- *Demand uncertainty does not always benefit consumers: Nonlinearity plays a key role.*

As uncertainty increases, quantities produced increase whereas the price and the supplier's profit decrease. In general, demand uncertainty benefits consumers in terms of effective price and quantities. One might hence expect the aggregate consumer surplus to increase with uncertainty. In fact, we show this is not always true. We observe that the effect of uncertainty on consumer surplus depends on the demand form. For example, for linear demand, uncertainty increases the consumer surplus, whereas for iso-elastic demand the opposite result holds. Depending on the demand pattern, the possibility of not serving customers with high valuations can outweigh the benefit of reduced prices for the customers served.

- *By ignoring demand uncertainty, the government will under-subsidize and miss the desired adoption target.*

Through the case of the newly introduced Chevy Volt by General Motors in the US market, we measure by how much the government misses the adoption target by ignoring demand uncertainty. We show that when the supplier takes into account demand uncertainty information while the government considers only average information on demand, the resulting expected sales can be significantly below the desired target adoption level.

- *The cost of demand uncertainty is shared between the supplier and the government.*

We analyze who bears the cost of demand uncertainty between government and supplier, which we show depends on the profit margin of the product. For linear demand models, the cost of demand uncertainty shifts from the government to the supplier as the adoption target increases or the production cost decreases. The impact of uncertainty on the government also depends on the shape of the demand curve. On one hand, a linear demand model suggests that subsidy levels always increase with demand uncertainty. On the other hand, for iso-elastic demand, uncertainty will increase/decrease subsidy levels when there is high/low price elasticity. As a result, the linear demand model, which is the most common in the literature, is not sufficient to fully understand the government's share in the cost of demand uncertainty.

- *Consumer subsidies are a sufficient mechanism to coordinate the government and the supplier.*

We compare the optimal policies to the case where a central planner manages jointly the supplier and the government. We determine that the price paid by the consumers and the production levels coincide for both the decentralized and the centralized models. In other words, consumer subsidies coordinate the supply-chain in terms of price and quantities.

2. Literature Review

Our setting is related to the newsvendor problem which has been extensively studied in the literature (see, e.g., Zipkin (2000), Porteus (1990), Winston (1994) and the references therein). An interesting extension that is even more related to this research is the price-setting newsvendor (see Petruzzi and Dada (1999) and Yao et al. (2006)). More recently, Kocabıyıkoğlu and Popescu (2011) identified a new measure of demand elasticity, the elasticity of the lost sales rate, to generalize and complement assumptions commonly made in the price-setting newsvendor. Kaya and Özer (2012) provide a good survey of the literature on inventory risk sharing in a supply chain with a newsvendor-like retailer, which is closer to our framework. Nevertheless, our problem involves an additional player (the government) that interacts with the supplier's decisions and complicates the analysis and insights. Most previous works on the stochastic newsvendor problem treat the additive and multiplicative models separately (e.g., in Petruzzi and Dada (1999)) or focus exclusively on one case, with often different conclusions regarding the price of demand uncertainty. In our problem however, we show that our conclusions hold for both demand models.

In the traditional newsvendor setting, the production cost is generally seen as the variable cost of producing an extra unit from raw material to finished good. In capital-intensive industries like solar panels and electric vehicles, the per-unit cost of capacity investment in the manufacturing facility is usually much larger than the per-unit variable cost. For this reason, we define the production quantities of the supplier to be a capacity investment decision, similar to Cachon and Lariviere (1999).

In the economics literature, one can find a vast amount of papers that consider welfare implications and regulations for a monopolist (see, e.g., Train (1991)). There is also a relevant stream of literature on market equilibrium models for new product introduction (see, e.g., Huang and Sošić (2010)). However, most of these papers do not consider demand uncertainty. Nevertheless, some works on electricity peak-load pricing and capacity investments address the stochastic demand case (see Crew et al. (1995) for a review on that topic). In this context, it is usually assumed that the supplier knows the willingness to pay of customers and can therefore decline the ones with the lowest valuations in the case of a stock-out. In our application however, one cannot impose such an assumption and the demand model follows a general price dependent curve while the customers arrive randomly and are served according to a first-come-first-serve logic.

Another stream of research related to our paper considers social welfare and government subsidies in the area of vaccines (see, e.g., Arifoglu et al. (2012), Mamani et al. (2011) and Taylor and Xiao (2013)). In Arifoglu et al. (2012), the authors study the impact of yield uncertainty, in a model that represents both supply and demand, on the inefficiency in the influenza vaccine supply chain. They show that the equilibrium demand can be greater than the socially optimal demand. In Taylor and Xiao (2013), the authors assume a single supplier with stochastic demand and consider how a donor can use sales and purchase subsidies to improve the availability of vaccines.

Among papers that study the design of subsidies for green technologies, Carlsson and Johansson-Stenman (2003) examine the social benefits of electric vehicle adoption in Sweden and report a pessimistic outlook for this technology in the context of net social welfare. Avci et al. (2013) show that adoption of electric vehicles has societal and environmental benefits, as long as the electricity grid is sufficiently clean. In Benthem et al. (2008), the authors develop a model for optimizing social welfare with solar subsidy policies in California. These two papers assume non-strategic industry players. While considering the manufacturer's response, Atasu et al. (2009) study the use of a take-back subsidy and product recycling programs. In a similar way as the previous papers mentioned above, they optimize social welfare of the system assuming a known environmental impact of the product. Our work focuses on designing optimal policies to achieve a given adoption target level, which can be used to evaluate the welfare distribution in the system. In this paper, we also incorporate the strategic response of the industry into the policy making decision. Also considering an adoption level objective, Lobel and Perakis (2013) study the problem of optimizing subsidy policies for solar panels and present an empirical study of the German solar market. The paper shows evidence that the current feed-in-tariff system used in Germany might not be efficiently using the positive network externalities of early adopters. Alizamir et al. (2013) also tackle the feed-in-tariff design problem, comparing strategies for welfare maximization and adoptions targets. Finally, Ovchinnikov and Raz (2013) present a price setting newsvendor model for the case of public interest goods. The authors compare, for the case of linear demand, different government intervention mechanisms and study under what conditions the system is coordinated in terms of welfare, prices and supply quantities. On the other hand, in this paper we investigate the impact of demand uncertainty on the various players of the system for non-linear demands and model explicitly the strategic response of the manufacturer to the subsidy policy.

Numerous papers in supply-chain management focus on linear demand functions. Examples include Anand et al. (2008) and Erhun et al. (2011) and the references therein. These papers study supply chain contracts where the treatment mainly focuses on linear inverse demand curves. In this paper, we show that the impact of demand uncertainty on the optimal policies differs for some classes of non-linear demand functions relative to linear models. In particular, we observe that the effect of demand uncertainty depends on whether demand is convex (rather than linear) with respect to the price. In addition, the demand non-linearity plays a key role on the consumer surplus.

As mentioned before, our paper also contributes to the literature on supply chain coordination (see Cachon 2003 for a review). The typical supply chain setting deals with a supplier and a retailer, who act independently to maximize individual profits. Mechanisms such as rebates (Taylor 2002) or revenue-sharing (Cachon and Lariviere 2005) can coordinate the players to optimize the aggregate surplus in the supply chain. Liu and Özer (2010) examine how wholesale price, quantity flexibility or buybacks can incentivize information sharing when introducing a new product with uncertain demand. Lutze and Özer (2008) study how a supplier should share demand uncertainty risk with the retailer when there is a lead-time contract. In Granot and Yin (2005, 2007, 2008), the authors study different types of contracts in a Stackelberg framework using a price-setting newsvendor model. In particular, Granot and Yin (2008) analyze the effect of price and order postponements in a decentralized newsvendor model with multiplicative demand, wherein the manufacturer possibly offers a buyback rate. In our setting, the government and the supplier are acting independently and could perhaps adversely affect one another. Instead, we show that the subsidy mechanism is sufficient to achieve a coordinated outcome. Chick et al. (2008) and Mamani et al. (2011) have looked at supply chain coordination in government subsidies for vaccines. Nevertheless, as we discussed above the two supply chains are fairly different.

The remainder of the paper is structured as follows. In Section 3, we describe the model. In Section 4, we consider both additive and multiplicative demand with pricing (price setter model), analyze special cases and finally study the effect of demand uncertainty on consumer surplus. In Section 5, we study the supply-chain coordination and Section 6 presents some computational results. Finally, we present our conclusions in Section 7. The proofs of the different propositions and theorems together with the price taker case are relegated to the Appendix.

3. Model

We model the problem as a two-stage Stackelberg game where the government is the leader and the supplier is the follower (see Figure 1). We assume a single time period model with a unique supplier and consider a full information setting. The government decides the subsidy level r per product and the supplier follows by setting the price p and production quantities q to maximize his/her profit. The subsidy r is offered from the government directly to the end consumer. We consider a general stochastic demand function that depends on the effective price paid by consumers, $z = p - r$, and on a random variable ϵ, denoted by $D(z, \epsilon)$. Once demand is realized, the sales level is determined by the minimum of supply and demand, that is, $\min(q, D(z, \epsilon))$.

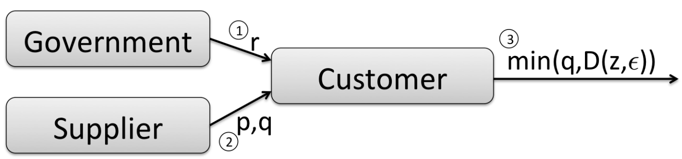

Figure 1 Order of events: 1. Subsidies; 2. Price and Quantity; 3. Sales

The selling price p can be viewed as the manufacturer's suggested retail price (MSRP) that is, the price the manufacturer recommends for retail. Additionally, in industries where production lead time is long and incurs large fixed costs, we consider the production quantities to be equivalent to the capacity investment built in the manufacturing facility.

The goal of our model is to study the overall impact of demand uncertainty. In order to isolate this effect, we consider a single period monopolist model. These modeling assumptions are reasonable approximations for the Chevy Volt, which we use in our numerical analysis. Note that since the introduction of electric vehicles, the MSRP for the Chevy Volt and the subsidy level have remained fairly stable. Consumer subsidies were posted before the introduction of these products and have remained unchanged ($\$7,500$) since it was launched in December 2010. We assume the supplier is aware of the amount of subsidy offered to consumers before starting production. The supplier modeling choice is motivated by the fact that consumer subsidies for EVs started at a time where very few competitors were present in the market and the product offerings were significantly different. The Chevy Volt is an extended-range mid-priced vehicle, while the Nissan Leaf is a cheaper all-electric alternative and the Tesla Roadster is a luxury sports car. These products are also significantly different from traditional gasoline engine vehicles so that they can be viewed as price setting firms within their own niche markets.

Given a consumer subsidy level, r, announced by the government, the supplier faces the following profit maximization problem. Note that c denotes the cost of building an additional unit of manufacturing capacity.

$$\Pi = \max_{q,p} p \cdot \mathbb{E}\big[min(q, D(z,\epsilon))\big] - c \cdot q \tag{1}$$

Denote Π as the optimal expected profit of the supplier. We consider the general case for which the supplier decides upon both the price (MSRP) and the production quantities (i.e., the supplier is a price setter). An alternative case of interest is the one for which the price is exogenously given (i.e., the supplier is a price taker and decides only production quantity). As mentioned before, we consider the early stages of the EV market as a good application of the monopolist price setting model. In contrast, the solar panel manufacturing industry is highly competitive, as the top ten companies share less than half of the US market. As a result, the solar manufacturing market is possibly better represented with a price taker model. In this paper, we treat both settings, but focus on the more complex price setter model. Due to space limitations, we relegate the price taker case to the Appendix.

We assume the government is introducing consumer subsidies, r, in order to stimulate sales to reach a given adoption target. We denote by Γ the target adoption level, which is assumed to be common knowledge. Conditional on achieving this target in expectation, the government wants to minimize the total cost of the subsidy program. Define Exp as the minimal expected subsidy expenditures, which is defined through the following optimization problem:

$$\begin{aligned} Exp = \min_{r}\; & r \cdot \mathbb{E}\big[min(q, D(z,\epsilon))\big] \\ \text{s.t. }\; & \mathbb{E}\big[min(q, D(z,\epsilon))\big] \geq \Gamma \\ & r \geq 0 \end{aligned} \tag{2}$$

In what follows, we discuss the modeling choices for the government in more detail.

Government's constraints The adoption level constraint used in (2) is motivated by real policy-making practice. For example, President Obama stated the adoption target of 1 million of electric vehicles by 2015 (see DOE 2011). More precisely, the government is interested in designing consumer subsidies so as to achieve the predetermined adoption target. An additional possibility is to incorporate a budget constraint for the government in addition to the adoption target. In various practical settings, the government may consider both requirements (see for example CSI 2007). Incorporating a budget constraint in our setting does not actually affect the optimal subsidy solution of the government problem (assuming the budget does not make the problem infeasible). In addition, one can show that there exists a one-to-one correspondence between the target adoption level and the minimum budget necessary to achieve this target. Hence, we will only solve the problem with a target adoption constraint, but the problem could be reformulated as a budget allocation problem with similar insights.

Given that actual sales are stochastic, the constraint used in our model meets the adoption target in expectation:

$$\mathbb{E}\big[min(q, D(z,\epsilon))\big] \geq \Gamma. \tag{3}$$

Our results can be extended to the case where the government aims to achieve a target adoption level with some desired probability (chance constraint) instead of an expected value constraint. Such a modeling choice will be more suitable when the government is risk-averse and is given by:

$$\mathbb{P}\Big(\big[min(q, D(z,\epsilon))\big] \geq \Gamma\Big) \geq \Delta. \tag{4}$$

Δ represents the level of conservatism of the government. For example, when $\Delta = 0.99$, the government is more conservative than when $\Delta = 0.9$. We note that the insights we gain are similar for both classes of constraints (3) and (4) and therefore in the remainder of this paper, due to space limitations, we focus on the case of an expected value constraint.

Government's objective Two common objectives for the government are to minimize expenditures or to maximize the welfare in the system. In the former, the government aims to minimize only its own expected expenditures, given by:

$$Exp = r \cdot \mathbb{E}\big[min(q, D(z,\epsilon))\big]. \tag{5}$$

Welfare can be defined as the sum of the expected supplier's profit (denoted by Π and defined in (1)) and the consumer surplus (denoted by CS) net the expected government expenditures:

$$W = \Pi + CS - Exp. \tag{6}$$

The consumer surplus is formally defined in Section 4.3 and aims to capture the consumer satisfaction. Interestingly, one can show that under some mild assumptions, both objectives are equivalent and yield the same optimal subsidy policy for the government. The result is summarized in the following Proposition.

PROPOSITION 1. *Assume that the total welfare is a concave and unimodal function of the subsidy r. Then, there exists a threshold value Γ^* such that for any given value of the target level above this threshold, i.e., $\Gamma \geq \Gamma^*$, both problems are equivalent.*

Proof. Since the welfare function is concave and unimodal, there exists a unique optimal unconstrained maximizer solution. If this unconstrained solution satisfies the adoption level target, the constrained problem is solved to optimality. However, if the target adoption level Γ is large enough, this solution is not feasible with respect to the adoption constraint. By using the non-decreasing property of the expected sales with respect to r (see Lemma 2 in the Appendix), one can see that the optimal solution of the constrained welfare maximization problem is obtained when the adoption level constraint is exactly met. Otherwise, by considering a larger subsidy level, one still satisfies the adoption constraint but does not increase the welfare. Consequently, both problems are equivalent and yield the same optimal solution for which the adoption constraint is exactly met. $\square$

In conclusion, if the value of the target level Γ is sufficiently large, both problems (minimizing expenditures in (5) and maximizing welfare in (6)) are equivalent. Note that the concavity and unimodality assumptions are satisfied for various demand models including the linear demand function. In particular, for linear demand, the threshold Γ^* can be characterized in closed form and is equal to twice the optimal production with zero subsidy and therefore satisfied in most reasonable settings. Furthermore, for smaller adoption target levels, Cohen et al. (2013) show that even for multiple products in a competitive environment, the gaps between both settings (minimizing expenditures versus maximizing welfare) are small (if not zero) so that both problems yield solutions that are close to one another. For the remainder of the paper, we assume the government objective is to minimize expenditures, while satisfying an expected adoption target, as in (2). This modeling choice was further motivated by private communications with sponsors of the MIT Energy Initiative.

With the formal definitions of the optimization problems faced by the supplier (1) and the government (2), in the next section we analyze of the optimal decisions of each party and the impact of demand uncertainty.

4. The Price Setter Model

For products such as electric vehicles, where there are only a few suppliers in the market, it is reasonable to assume that the selling price (MSRP) of the product is endogenous. In other words, p is a decision variable chosen by the supplier in addition to the production quantity q. In this case, the supplier's optimization problem can be viewed as a price setting newsvendor problem (see e.g., Petruzzi and Dada (1999)). Note though that in our problem the solution also depends on the government subsidy. In particular, both q and p are decision variables that should be optimally chosen by the supplier for each value of the subsidy r set by the government. To keep the analysis simple and to be consistent with the literature, we consider separately the cases of a stochastic demand with additive or multiplicative uncertainty. In each case, we first consider general demand functions and then specialize to linear and iso-elastic demand models that are common in the literature. Finally, we compare our results to the case where demand is approximated by a deterministic average value and draw conclusions about the cost of ignoring demand uncertainty.

In practice, companies very often ignore demand uncertainty and consider average values when taking decisions such as price and production quantities. As a result, we are interested in understanding how the optimal subsidy levels, prices and production quantities are affected when we explicitly consider demand uncertainty relative to the case when demand is just approximated by its deterministic average value. For example, the comparison may be useful to quantify the value of investing some large efforts in developing better demand forecasts.

We next present the analysis for both additive and multiplicative demand uncertainty.

4.1. Additive Noise

Define additive demand uncertainty as follows:

$$D(z,\epsilon) = y(z) + \epsilon. \tag{7}$$

Here, $y(z) = \mathbb{E}\big[D(z,\epsilon)\big]$ is a function of the effective price $z = p - r$ and represents the nominal deterministic part of demand and ϵ is a random variable with cumulative distribution function (CDF) F_ϵ.

Assumption 1. *We impose the following conditions on demand:*

- *Demand depends only on the difference between p and r denoted by z.*
- *The deterministic part of the demand function $y(z)$ is positive, twice differentiable and a decreasing function of z and hence invertible.*
- *When $p = c$ and $r = 0$ the target level cannot be achieved, i.e., $y(c) < \Gamma$.*
- *The noise ϵ is a random variable with zero mean: $\mathbb{E}[\epsilon] = 0$.*

Under Assumption 1, we characterize the solution of problems (1) and (2) sequentially. First, we solve the optimal quantity $q^*(p,r)$ and price $p^*(r)$ offered by the supplier as a function of the subsidy r. By substituting the optimal solutions of the supplier problem, we can solve the government problem defined in (2). Note that problem (2) is not necessarily convex, even for very simple instances, because the government needs to account for the supplier's best response $p^*(r)$ and $q^*(p,r)$. Nevertheless, one can still solve this using the tightness of the target adoption constraint. Because of the non-convexity of the problem, the tightness of the constraint cannot be trivially assumed. We formally prove the constraint is tight at optimality in Theorem 1. Using this result, we obtain the optimal subsidy of the stochastic problem (2), denoted by r_{sto}. The resulting optimal decisions of price and quantity are denoted by $p_{sto} = p^*(r_{sto})$ and $q_{sto} = q^*(p_{sto}, r_{sto})$. From problems (1) and (2), the optimal profit of the supplier is denoted by Π_{sto} and government expenditures by Exp_{sto}.

We consider problems (1) and (2), where demand is equal to its expected value, that is: $\mathbb{E}\big[D(z,\epsilon)\big] = y(z)$. We denote this deterministic case with the subscript "det", with optimal values: $r_{det}, p_{det}, q_{det}, z_{det}, \Pi_{det}, Exp_{det}$. We next compare these metrics in the deterministic versus stochastic case.

THEOREM 1. *Assume that the following condition is satisfied:*

$$2y'(z) + (p-c)\cdot y''(z) + \frac{c^2}{p^3}\cdot\frac{1}{f_\epsilon\Big(F_\epsilon^{-1}\Big(\frac{p-c}{p}\Big)\Big)} < 0. \tag{8}$$

The following holds:

1. *The optimal price of problem* (1) *as a function of* r *is the solution of the following non-linear equation:*

$$y(p-r) + \mathbb{E}\Big[min(F_\epsilon^{-1}\Big(\frac{p-c}{p}\Big),\epsilon)\Big] + y'(p-r)\cdot(p-c) = 0. \tag{9}$$

In addition, using the solution from (9)*, one can compute the optimal production quantity:*

$$q^*(p,r) = y(p-r) + F_\epsilon^{-1}\Big(\frac{p-c}{p}\Big). \tag{10}$$

2. *The optimal solution of the government problem is obtained when the target adoption level is exactly met.*

3. *The optimal expressions follow the following relations:*

$$z_{sto} = y^{-1}\big(\Gamma - K_\epsilon\big) \le z_{det} = y^{-1}\big(\Gamma\big)$$
$$q_{sto} = \Gamma + F_\epsilon^{-1}\Big(\frac{p_{sto}-c}{p_{sto}}\Big) - K_\epsilon \ge q_{det} = \Gamma$$

If, in addition, the function $y(z)$ *is convex:*

$$p_{sto} = c + \frac{\Gamma}{|y'(z_{sto})|} \le p_{det} = c + \frac{\Gamma}{|y'(z_{det})|}$$
$$\Pi_{sto} = \frac{\Gamma^2}{|y'(z_{sto})|} - c\cdot(q_{sto} - \Gamma) \le \Pi_{det} = \frac{\Gamma^2}{|y'(z_{det})|}$$

We define K_ϵ as:

$$K_\epsilon = \mathbb{E}\Big[min(F_\epsilon^{-1}\Big(\frac{p_{sto}-c}{p_{sto}}\Big),\epsilon)\Big]. \tag{11}$$

REMARK 1. Note that for a general function $y(p-r)$, one cannot derive a closed form solution of (9) for $p^*(r)$. This is consistent with the fact that there does not exist a closed form solution for the price-setting newsvendor. However, one can use (9) to characterize the optimal solution and even numerically compute the optimal price by using a binary search method (see more details in the Appendix). Assumption (8) guarantees the uniqueness of the optimal price as a function of the subsidies, as it implies the strict concavity of the profit function with respect to p. In case this condition does not hold, problem (1) is still numerically tractable (see Petruzzi and Dada 1999). For the remainder of this paper, we will assume condition (8) is satisfied. For the case of linear demand we discuss relation (14) which is a sufficient condition that is satisfied in many reasonable settings.

REMARK 2. The results of Theorem 1 can be generalized to describe how the optimal variables (i.e., z, q, p and Π) change as demand uncertainty increases. Instead of comparing the stochastic case to the deterministic case (i.e., where there is no demand uncertainty), one can instead consider how the optimal variables vary in terms of the magnitude of the noise (for more details, see the proof of Theorems 1 and 4 in the Appendix). In particular, the quantity that captures the effect of demand uncertainty is K_ϵ.

Since the noise ϵ has zero-mean, the quantity K_ϵ in (11) is always non-positive. In addition, when there is no noise (i.e., $\epsilon = 0$ with probability 1), $K_\epsilon = 0$ and the deterministic scenario is obtained as a special case. For any intermediate case, K_ϵ is negative and non-increasing with respect to the magnitude of the noise. For example, if the noise ϵ is uniformly distributed, the inverse CDF function can be written as a linear function of the standard deviation σ as follows:

$$F_\epsilon^{-1}\Big(\frac{p-c}{p}\Big) = \sigma\sqrt{3}\cdot\Big(2\cdot\frac{p-c}{p} - 1\Big). \tag{12}$$

Therefore, K_ϵ scales monotonically with the standard deviation for uniform demand uncertainty. In other words, all the comparisons of the optimal variables (e.g., effective price, production quantities etc) are monotonic functions of the standard deviation of the noise. For other distributions, the relationship with the standard deviation is not as simple but the key quantity is K_ϵ. As a result, one can extend our insights in a continuous fashion with respect to the magnitude of the noise. For example, the inequality of the effective price is given by: $z_{sto} = y^{-1}\Big(\Gamma - K_\epsilon\Big)$. This equation is non-increasing with respect to the magnitude of K_ϵ and is maximized when there is no noise (deterministic demand) so that $z_{det} = z_{sto} = y^{-1}\big(\Gamma\big)$. In general, as the magnitude of the noise increases, the gaps between the optimal decision variables increase (see plots of optimal decisions as functions of the standard deviation of demand uncertainty in Figure 4 of Section 6).

Remark 3. The solution of the optimal quantity q and the effective price z provide another interesting insight. Theorem 1 states that when demand is uncertain, the consumers are better off in terms of effective price and production quantities (this is true for any decreasing demand function). Furthermore, the selling price and the profit of the supplier are lower in the presence of uncertainty, assuming demand is convex. These results imply the consumers are in general better off when demand is uncertain. Nevertheless, as we will show in Section 4.3, this is not always the case when we use the aggregate consumer surplus as a metric.

In various settings, the selling price p is exogenously given (i.e., price taker model). This setting is relevant for products where the market is fairly saturated with suppliers. We present the results for the price taker model in the Appendix. We observe that for the price setter model, the results depend on the structure of the demand function unlike in the price taker case, where the results of Theorem 4 (see Appendix) are robust with respect to the type of demand.

By focusing on a few demand functions, we can provide additional insights. We will first consider the linear demand case, which is the most common in the literature. The simplicity of this demand form enables us to derive closed-form solutions and a deeper analysis of the impact of demand uncertainty. Note that the insights can be quite different for non-linear demand functions. The results presented in Theorem 1 justify the need for considering non-linear functions as well. For this reason, we later consider the iso-elastic demand case and compare it to the linear case.

Linear Demand In what follows, we quantify the effect of demand uncertainty on the subsidy level and the expected government expenditures. We can obtain such results for specific demand models, among them the linear demand model. Define the linear demand function as:

$$D(z,\epsilon) = \bar{d} - \alpha \cdot z + \epsilon, \tag{13}$$

where $\bar{d}$ and α are given positive parameters that represent the maximal market share and the price elasticity respectively. Note that for this model, a sufficient condition for assumption (8) to hold is given by:

$$\alpha > \frac{1}{2c \cdot \inf_{x} f_\epsilon(x)}.$$

For example, if the additive noise is uniformly distributed, i.e., $\epsilon \sim U[-a_2, a_2];\ a_2 > 0$, (note that since the noise is uniform with zero mean, it has to be symmetric) we obtain:

$$\alpha > \frac{a_2}{c}. \tag{14}$$

One can see that by fixing the cost c, condition in (14) is satisfied if the price elasticity α is large relative to the standard deviation of the noise. Next, we derive closed form expressions for the optimal price, production quantities, subsidies, profit and expenditures for both deterministic and stochastic demand models and compare the two settings.

Theorem 2. *The closed form expressions and comparisons for the linear demand model in* (13) *are given by:*

$$p_{sto} = c + \frac{\Gamma}{\alpha} = p_{det}$$
$$q_{sto} = \Gamma + F_{\epsilon}^{-1}\Big(\frac{p_{sto}-c}{p_{sto}}\Big) - K_{\epsilon} \geq q_{det} = \Gamma$$
$$r_{sto} = \frac{2\Gamma}{\alpha} + c - \frac{\bar{d}}{\alpha} - \frac{1}{\alpha}\cdot K_{\epsilon} \geq r_{det} = \frac{2\Gamma}{\alpha} + c - \frac{\bar{d}}{\alpha}$$
$$\Pi_{sto} = \frac{\Gamma^2}{\alpha} - c\cdot(q_{sto} - \Gamma) \leq \Pi_{det} = \frac{\Gamma^2}{\alpha}$$
$$Exp_{sto} = \Gamma\cdot r_{sto} \geq Exp_{det} = \Gamma\cdot r_{det}$$

We note that the results of Theorem 2 can be presented in a more general continuous fashion as explained in Remark 2. Surprisingly, the optimal price is the same for both the deterministic and stochastic models. In other words, the optimal selling price is not affected by demand uncertainty for linear demand. On the other hand, with increased quantities, the expected profit of the supplier is lower under demand uncertainty. At the same time, the optimal subsidy level and expenditures increase with uncertainty. Therefore, both supplier and government are worse off when demand is uncertain. Corollary 1 and the following discussion provide further intuition in how this cost of demand uncertainty is shared between the supplier and the government.

Corollary 1.

1. $q_{sto} - q_{det}$ *decreases in* c *and increases in* Γ.
2. $r_{sto} - r_{det}$ *increases in* c *and decreases in* Γ.
3. *Assume that* ϵ *has support* $[a_1, a_2]$. *Then, the optimal subsidy for the stochastic and deterministic demands relate as follows:*

$$r_{det} \leq r_{sto} \leq r_{det} + \frac{|a_1|}{\alpha}.$$

Corollary 1 can be better understood in terms of the optimal service level for stochastic demand, denoted by $\rho = \frac{p_{sto}-c}{p_{sto}}$. Note that ρ is an endogenous decision of the supplier, which is a function of the optimal price p_{sto}. For linear demand, the optimal service level can be simplified as: $\rho = \frac{\Gamma}{c\alpha+\Gamma}$. This service level is decreasing in the cost c but increasing with respect to the target adoption Γ.

On one hand, when the optimal price is significantly higher than the production cost, i.e., $p_{sto} \gg c$, the high profit margin encourages the supplier to satisfy a larger share of demand by increasing its production. As p_{sto} increases, the service level ρ increases and in the limit, when $p_{sto} \to \infty$, the service level goes to 1 and the supplier has incentives to overproduce and bear all the inventory risk. In this case, the government may set low subsidies, in fact the same as in the deterministic case, which guarantee that the average demand meets the target. On the other hand, when p_{sto} is close to c (low profit margin), or equivalently in the limit when $\rho \to 0$, the supplier has no incentives to bear any risk and produces quantities to match the lowest possible demand realization. In this case, the government will bear all the inventory risk by increasing the value of the subsidies. By assuming the worst case realization of demand uncertainty when deciding the subsidy, the government also induces a production level that meets the target without risk for the supplier.

Note that as production cost c increases, the required subsidy is larger for both stochastic and deterministic demands, meaning the average subsidy expenditure is higher. At the same time, the service level ρ decreases and, from Corollary 1, the gap between r_{sto} and r_{det} increases. This can be viewed as an increase in the cost of demand uncertainty for the government.

A similar reasoning can be applied to the target adoption level. As Γ increases, the overall cost of the subsidy program increases, as expected. Interestingly, the service level ρ also increases. From Corollary 1, the gap between q_{sto} and q_{det} widens, meaning the supplier with stochastic demand is building more inventory. At the same time, the gap between r_{sto} and r_{det} shrinks. Effectively, the burden of demand uncertainty is transferred from the government to the supplier as Γ increases. An interpretation can be that a higher target adoption will induce the product to be more profitable as it needs larger subsidies to generate enough supply and demand. This will make the supplier take on more of the inventory risk and consequently switching who bears the cost of demand uncertainty.

Corollary 1.3 shows that the government subsidy decision is bounded by the worst case demand realization normalized by the price sensitivity. In other words, it provides a guarantee on the gap between the subsidies for stochastic and deterministic demands.

In conclusion, by studying the special case of a linear demand model, we obtain the following additional insights: (i) The optimal price does not depend on demand uncertainty. (ii) The optimal subsidy set by the government increases with demand uncertainty. Consequently, the introduction of demand uncertainty decreases the effective price paid by consumers. In addition, the government will spend more when demand is uncertain. (iii) The cost of demand uncertainty is shared by the government and the supplier and depends on the profit margin (equivalently, service level) of the product. As expected, lower/higher margins mean the supplier takes less/more inventory risk. Therefore, increasing the adoption target or decreasing the manufacturing cost will shift the cost of demand uncertainty from the government to the supplier.

4.2. Multiplicative Noise

In this section, we consider a demand with a multiplicative noise (see for example, Granot and Yin (2008)). The nominal deterministic part is assumed to be a function of the effective price, denoted by $y(z)$:

$$D(z, \epsilon) = y(z) \cdot \epsilon \tag{15}$$

Assumption 2. • *Demand depends only on the difference between p and r denoted by z.*

• *The deterministic part of the demand function $y(z)$ is positive, twice differentiable and a decreasing function of z and hence invertible.*

• *When $p=c$ and $r=0$ the target level cannot be achieved, i.e., $y(c)<\Gamma$.*

• *The noise ϵ is a positive and finite random variable with mean equal to one: $\mathbb{E}[\epsilon]=1$.*

One can show that the results of Theorem 1 hold for both additive and multiplicative demand models. The proof for multiplicative noise follows a similar methodology and is not repeated due to space limitations. We next consider the iso-elastic demand case to derive additional insights on the optimal subsidy.

Iso-Elastic Demand Define the iso-elastic demand as:

$$y(z)=\bar{d}\cdot z^{-\alpha} \qquad (\alpha>0). \tag{16}$$

Note that this function $y(z)$ is convex with respect to z for any value $\alpha>0$. Therefore the results from Theorem 1 hold. Using the demand structure, we obtain the following additional results on the optimal subsidy:

Proposition 2. *For the iso-elastic demand model in* (16)*, we have:*

- *If* $\alpha>1$*:* $\quad r_{sto}\geq r_{det}$.
- *If* $0<\alpha<1$*:* $\quad r_{sto}\leq r_{det}$.
- *If* $\alpha=1$*:* $\quad r_{sto}=r_{det}=c$.

We note that the results of Proposition 2 can be presented in a continuous fashion, as explained in Remark 2. The iso-elastic model considered in the literature usually assumes that $\alpha>1$ in order to satisfy the Increasing price Elasticity (IPE) property (see, e.g., Yao et al. (2006)). This allows us to recover the same results as the linear additive demand model. These two cases show that the subsidy increases with demand uncertainty.

4.3. Consumer Surplus

In this section, we study the effect of demand uncertainty on consumers using consumer surplus as a metric. For that purpose, we compare the aggregate level of consumer surplus under stochastic and deterministic demand models. The consumer surplus is an economic measure of consumer satisfaction calculated by analyzing the difference between what consumers are willing to pay and the market price. For a general deterministic price demand curve, the consumer surplus is denoted by CS_{det} and can be computed as the area under the demand curve above the market price (see, e.g., Vives (2001)):

$$CS_{det}=\int_0^{q_{det}}\Big(D^{-1}(q)-z_{det}\Big)dq=\int_{z_{det}}^{z_{max}}D(z)dz. \tag{17}$$

We note that in our case, the market price is equal to the effective price paid by consumers $z = p - r$. Denote, $D^{-1}(q)$ as the effective price that will generate demand exactly equal to q. For an illustration, see Figure 2. Note that z_{det} and q_{det} represent the optimal effective price and production, whereas z_{max} corresponds to the value of the effective price that yields zero demand. The consumer surplus represents the surplus induced by consumers that are willing to pay more than the posted price.

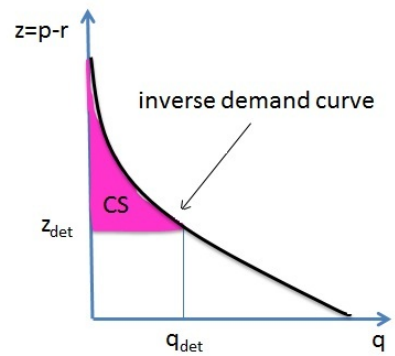

Figure 2 **Consumer surplus for deterministic demand**

When demand is uncertain however, defining the consumer surplus (denoted by CS_{sto}) is somewhat more subtle due to the possibility of a stock-out. Several papers on peak load pricing and capacity investments by a power utility under stochastic demand address partially this modeling issue (see Carlton (1986), Crew et al. (1995) and Brown and Johnson (1969)). Nevertheless, the models developed in this literature are not applicable to the price setting newsvendor. More specifically, in Brown and Johnson (1969) the authors assume that the utility power facility has access to the willingness to pay of the customers so that it can decline the ones with the lowest valuations. This assumption is not justifiable in our setting where a "first-come-first-serve" logic with random arrivals is more suitable. In Ovchinnikov and Raz (2013), the authors study a price setting newsvendor model for public goods and consider the consumer surplus for linear additive stochastic demand.

For general stochastic demand functions, the consumer surplus $CS_{sto}(\epsilon)$ is defined for each realization of demand uncertainty ϵ. If there was no supply constraint, considering the effective price and the realized demand, the total amount of potential consumer surplus is defined as: $\int_{z_{sto}}^{z_{max}(\epsilon)} D(z,\epsilon)dz$. Since customers are assumed to arrive in a first-come-first-serve manner, irrespective of their willingness to pay, some proportion of these customers will not be served due to stock-outs. The proportion of served customers is given by the ratio of actual sales over potential demand: $\frac{min(D(z_{sto},\epsilon),q_{sto})}{D(z_{sto},\epsilon)}$. Therefore, the consumer surplus can be defined as the total available surplus times the proportion of that surplus that is actually served.

$$CS_{sto}(\epsilon) = \int_{z_{sto}}^{z_{max}(\epsilon)} D(z,\epsilon)dz \cdot \frac{min\big(D(z_{sto},\epsilon), q_{sto}\big)}{D(z_{sto},\epsilon)}. \tag{18}$$

We note that in this case, the consumer surplus is a random variable that depends on the demand through the noise ϵ. Note that we are interested in comparing CS_{det} to the expected consumer surplus $\mathbb{E}_\epsilon[CS_{sto}(\epsilon)]$. For stochastic demand, (18) has a similar interpretation as its deterministic counterpart. Nevertheless, we also incorporate the possibility that a consumer who wants to buy the product does not find it available. As we will show, the effect of demand uncertainty on consumer surplus depends on the structure of the nominal demand function. In particular, we provide the results for the two special cases we have considered in the previous section and show that the effect is opposite. For the linear demand function in (13), we have:

$$CS_{det} = \int_0^{q_{det}} \Big(D^{-1}(q) - z_{det}\Big)dq = \frac{q_{det}^2}{2\alpha} = \frac{\Gamma^2}{2\alpha}. \tag{19}$$

For iso-elastic demand from (16), we obtain:

$$CS_{det} = \int_0^{q_{det}} \Big(D^{-1}(q) - z_{det}\Big)dq = \frac{\bar{d}}{\alpha - 1} \cdot \Big(\frac{\bar{d}}{\Gamma}\Big)^{\frac{1-\alpha}{\alpha}} \quad (\alpha > 1). \tag{20}$$

One can then show the following results regarding the effect of demand uncertainty on the consumer surplus for these two demand functions.

Proposition 3. *For the linear demand model in* (13)*, we have:*

$$\mathbb{E}[CS_{sto}] \geq CS_{det}. \tag{21}$$

For the iso-elastic demand model in (16) *with* $\alpha > 1$*, we have:*

$$\mathbb{E}[CS_{sto}] \leq CS_{det}. \tag{22}$$

Proposition 3 shows that under linear demand, the expected consumer surplus is larger when considering demand uncertainty, whereas it is lower for the iso-elastic model. We already have shown in Theorem 1 that the effective price is lower and that the production quantities are larger when considering demand uncertainty relative to the deterministic model. In addition, this result was valid for both models (i.e., additive and multiplicative noises for linear and non-linear demand). As a result, demand uncertainty benefits overall the consumers in terms of effective price and available quantities. With this in mind, one could expect consumer surplus to increase with uncertainty. However, when comparing the consumer surplus using equation (18) for stochastic demand, we obtain that for the iso-elastic demand, consumers are in aggregate worse-off when demand is uncertain. On one hand, demand uncertainty benefits the consumers since it lowers the effective price and increase the quantities. On the other hand, demand uncertainty introduces a stock-out probability because some of the consumers may not be able to find the product available. These two factors (effective price and stock-out probability) affect the consumer surplus in opposite ways. For iso-elastic demand, the second factor is dominant and therefore the consumer surplus is lower when demand is uncertain. In particular, the iso-elastic demand admits some consumers that are willing to pay a very large price. If these consumers experience a stock-out, it will reduce drastically the aggregate consumer surplus. For linear demand, the dominant factor is not the stock-out probability and consequently, the consumer surplus is larger when demand is uncertain. We note that this result is related to the structure of the nominal demand rather than the noise effect. For example, if we were to consider a linear demand with a multiplicative noise, we will have the same result as for the linear demand with additive noise.

At first glance one can naively infer that demand uncertainty always benefits consumers. Indeed, with greater demand uncertainty, the price paid by consumers decreases and the production quantities increase. However, under iso-elastic demand, uncertainty actually decreases the average consumer surplus. In general, the uncertainty impact on consumer surplus will depend on the trade-off between lower prices and the possibility of under-serving customers with high valuations.

5. Supply-chain Coordination

In this section, we examine how the results change in the case where the system is centrally managed. In this case, one can imagine that the government and the supplier take coordinated decisions together. The central planner needs to decide the price, the subsidy and the production quantities simultaneously. This situation may arise when the firm is owned by the government. We study the centrally managed problem as a benchmark to compare to the decentralized case developed in the previous sections. In particular, we are interested in understanding if the decentralization will have an adverse impact on either party and more importantly if it will hurt the consumers. We show in this section that this is not the case. In fact, the decentralized problem achieves the same outcome as the centralized problem and hence, government subsidies act as a coordinating mechanism as far as consumer are concerned. Supply chain coordination has been extensively studied in the literature. In particular, some of the supply-chain contracting literature (see, e.g., Cachon (2003)) discusses mechanisms that can be used to coordinate operational decisions such as price and production quantities.

Define the central planner's combined optimization problem to maximize the firm's profits minus government expenditures as follows:

$$\max_{q,p,r} \, (p-r)\cdot \mathbb{E}\big[min(q, D(z,\epsilon))\big] - c\cdot q \tag{23}$$

$$\text{s.t.} \qquad p \geq c$$
$$\mathbb{E}\big[min(q, D(z,\epsilon))\big] \geq \Gamma$$
$$r \geq 0$$

Note that in this case, we impose the additional constraint $p \geq c$ so that the selling price has to be at least larger than the cost. Indeed, for the centralized version, it is not clear that this constraint is automatically satisfied by the optimal solution as it was in the decentralized setting. Our goal is to show how the centralized solutions for q, p and r compare to their decentralized counterparts from Section 4. We consider both deterministic and stochastic demand models and focus on additive uncertainty under Assumption 1.

THEOREM 3. *The optimal effective price $z = p - r$ and production level q are the same in both the decentralized and centralized models. Therefore, consumer subsidies are a sufficient mechanism to coordinate the government and the supplier.*

Note that for problem (23), one can only solve for the effective price and not p and r separately. In particular, there are multiple optimal solutions for the centralized case and the decentralized solution happens to be one them. If the government and the supplier collude into a single entity, this does not affect the consumers in terms of effective price and production quantities. Therefore, the consumers are not affected by the coordination. This result might be surprising as one could think that the coordination will add additional information and power to the central planner as well as mitigate some of the competition effects between the supplier and the government. However, in the original decentralized problem, the government acts as a quantity coordinator in the sense that the optimal solutions in both cases are obtained by the tightness of the target adoption constraint.

6. Computational Results

In this section, we present some numerical examples that provide further insights into the results derived in Section 4. The data used in these experiments is inspired by the sales data of the first eighteen months of General Motors' Chevy Volt (between December 2010 and June 2012). The total aggregate sales was roughly equal to $3,500$ electric vehicles, the listed price (MSRP) was $\$40,280$ and the government subsidies was set to $\$7,500$. In addition, we assume a 10% profit margin so that the per-unit cost of building manufacturing capacity is $\$36,000$. For simplicity, we present here the results using a linear demand with an additive Gaussian noise. We observe that our results along with the analysis are robust with respect to the distribution of demand uncertainty. In fact, we obtain in our computational experiments the same insights for several demand distributions (including non-symmetric ones). As discussed in Section 3, the government can either minimize expenditures or maximize the total welfare. In particular, the two objectives are equivalent and give rise to the same optimal subsidy policies for any target level Γ above a certain threshold. In this case, this threshold is equal to 860 and the condition is therefore easily satisfied.

Throughout these experiments, we compute the optimal decisions for both the deterministic and the stochastic demand models by using the optimal expressions derived in Section 4.1. We first consider a fixed relatively large standard deviation $\sigma = 4,200$ (when demand is close to the sales, this is equivalent to a coefficient of variation of 1.2) and plot the optimal subsidy, production level, supplier's profit and government expenditures as a function of the target level Γ for both the deterministic and stochastic models. The plots are reported in Figure 3. We have derived in Section 4.1 a set of inequalities regarding the relations of the optimal variables for deterministic and stochastic demand models. The plots allow us to quantify the magnitude of these differences and study the impact of demand uncertainty on the optimal policies. One can see from Figure 3

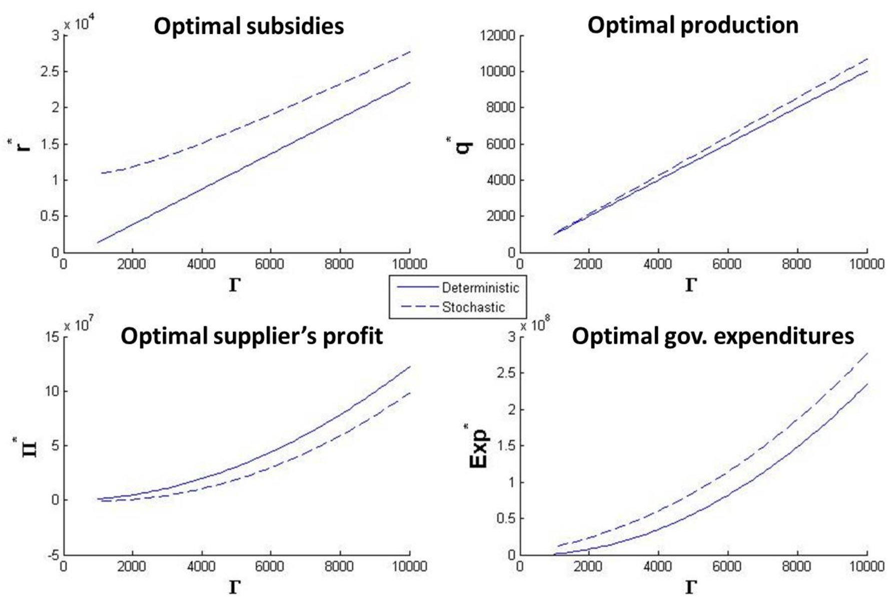

Figure 3 **Optimal values as a function of the target level**

that the optimal production levels are not strongly affected by demand uncertainty (even for large values of σ) when the target level Γ is set close to the expected sales value of $3,500$. However, the optimal value of the subsidy is almost multiplied by a factor of 2 when demand uncertainty is taken into account. In other words, when the government and the supplier consider a richer environment that accounts for demand uncertainty, the optimal subsidy nearly doubles.

This raises the following interesting question. What happens if the government ignores demand uncertainty and decides to under-subsidize by using the optimal value from the deterministic model? It is clear that in this case, since the real demand is uncertain, the expected sales will not attain the desired expected target adoption. We address this question in the remaining of this section but we first plot the subsidies and the supplier's profit as a function of the standard deviation of the noise that represents a measure of the demand uncertainty magnitude.

More precisely, we plot in Figure 4 the relative differences in subsidies (i.e., $\frac{r_{sto}-r_{det}}{r_{det}}$) as well as the supplier's profit as a function of the target level Γ (or equivalently, the expected sales) for different standard deviations of the additive noise varying from 35 to $10,500$. For $\Gamma = 3,500$, this is equivalent to a coefficient of variation varying between 0.01 and 3. As expected, one can see from Figure 4 that as the standard deviation of demand increases, the optimal subsidy is larger whereas the supplier's profit is lower. As a result, demand uncertainty benefits consumers at the expense of hurting both the government and the supplier.

Figure 4 Relative normalized differences in subsidies (a) and supplier's profit (b)

Finally, we analyze by how much the government will miss the actual target level (now Γ is fixed and equal to $3,500$) by using the optimal policy assuming demand is deterministic, r_{det}, instead of using r_{sto}. Recall that $r_{sto} \geq r_{det}$. In other words, the government assumes a simple average deterministic demand model whereas in reality demand is uncertain. In particular, this allows us to quantify the value of using a more sophisticated model that takes into account demand uncertainty instead of simply ignoring it. Note that this analysis is different from the previous comparisons in this paper, where we compared the optimal decisions as a function of demand uncertainty. Here, we assume that demand is uncertain with some given distribution but the government decides to ignore the uncertainty. To this extent, we consider two possible cases according to the modeling assumption of the supplier. First, we assume that the supplier is non-sophisticated, in the sense that he uses an average demand approximation model as well (i.e., no information on demand distribution is used). In this case, both the supplier and the government assume an average deterministic demand but in reality demand is random. Second, the supplier is more sophisticated. Namely, the supplier optimizes (over both p and q) by using a stochastic demand model together with the distribution information. The results are presented in Figure 5, where we vary the value of the coefficient of variation of the noise from 0 to 0.7. When the government and the supplier are both non-

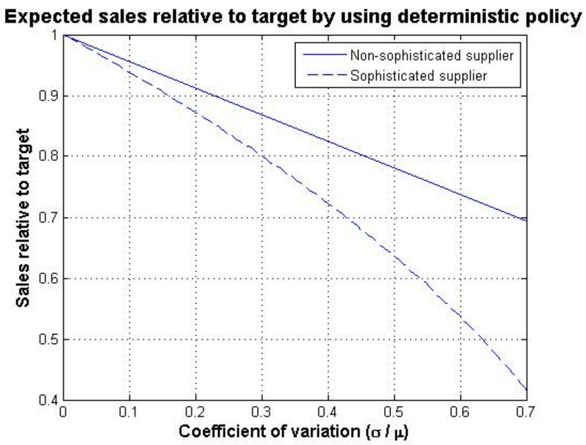

Figure 5 Expected sales relative to target

sophisticated, the government can potentially save money (by under-subsidizing) and still gets close to the target in expectation when demand uncertainty is not very large. As expected, when the supplier has more information on demand distribution (as it is usually the case), the expected sales are farther from the target and the government could miss the target level significantly. If in addition, demand uncertainty is large (i.e., coefficient of variation larger than 1), the government misses the target in both cases.

One can formalize the previous comparison analytically by quantifying the gap by which the government misses the target by ignoring demand uncertainty. In particular, let us consider the additive demand model given in (7).

PROPOSITION 4. *Consider that the government ignores demand uncertainty when designing consumer subsidies.*

1. *The government misses the adoption target (in expectation) regardless of whether the supplier is sophisticated or not.*

2. *The exact gaps are given by:*

- *For non-sophisticated supplier:*

$$\mathbb{E}\big[min(q, D(z,\epsilon))\big] = \Gamma + \mathbb{E}\big[min(0,\epsilon)\big] \leq \Gamma. \tag{24}$$

- *For sophisticated supplier, assuming a linear demand model:*

$$\mathbb{E}\big[min(q, D(z,\epsilon))\big] = \Gamma + \frac{1}{2}\cdot\mathbb{E}\Big[min(F_\epsilon^{-1}\Big(\frac{p^*-c}{p^*}\Big),\epsilon)\Big] \leq \Gamma. \tag{25}$$

3. *For low (high) profit margins, the gap is larger (smaller) when the supplier is sophisticated (non-sophisticated).*

p^* denotes the optimal price set by the supplier. Note that all the results of Proposition 4 (with the exception of equation (25)) are valid for a general demand model. Nevertheless, when the supplier is sophisticated, one needs to assume a specific model (in our case, linear) in order to compute the expected sales. As expected, the previous analysis suggests that the target adoption will be missed by a higher margin as demand becomes more uncertain. When comparing the non-sophisticated and sophisticated cases, one can see that in the former the government misses the target by $\mathbb{E}\big[min(0, \epsilon)\big]$, whereas in the latter by $\frac{1}{2} \cdot \mathbb{E}\Big[min(F_\epsilon^{-1}\Big(\frac{p^*-c}{p^*}\Big), \epsilon)\Big]$. Consequently, this difference depends not only on the distribution of the noise but also on the profit margin. Since the EV industry has rather low profit margins, the gap may be much larger when the supplier is sophisticated. Indeed, the sophisticated supplier decreases the price (relative to p_{det}). In addition, he reduces the production quantities as he is not willing to bear significant over-stock risk due to the low profit margin. As a result, the expected sales are lower and therefore the government misses the adoption target level. In conclusion, this analysis suggests that policy makers should take into account demand uncertainty when designing consumer subsidies. Indeed, by ignoring demand uncertainty, one can significantly miss the desired adoption target.

7. Conclusions

We propose a model to analyze the interaction between the government and the supplier when designing consumer subsidy policies. Subsidies are often introduced at the early adoption stages of green technologies to help them become economically viable faster. Given the high level of uncertainty in these early stages, we hope to have shed some light on how demand uncertainty affects consumer subsidy policies, as well as price and production quantity decisions from manufacturers and the end consumers.

In practice, policy makers often ignore demand uncertainty and consider only deterministic forecasts of adoption when designing subsidies. We demonstrate that uncertainty will significantly change how these programs should be designed. In particular, we show by how much the government misses the adoption target by ignoring demand volatility. Among some of our main insights, we show that the shape of the demand curve will determine who bears demand uncertainty risk. When demand is uncertain, quantities produced will be higher and the effective price for consumers will be lower. For convex demand functions prices will be lower, leading to lower industry profits.

Focusing on the linear demand model, we can derive further insights. For instance, to compensate for uncertain demand, quantities produced and subsidy levels are shifted by a function of the service level, i.e., the profitability of the product. For highly profitable products, the supplier will absorb most of the demand risk. When profit margins are smaller, the government will need to increase the subsidy amount and pay a larger share for the risk.

When evaluating the uncertainty impact on consumers, we must consider the trade-off between lower effective prices and the probability of a stock-out (unserved demand). We again show that the shape of the demand curve plays an important role. For linear demand, consumers will ultimately benefit from demand uncertainty. This is not the case, for instance, with an iso-elastic demand model, where the possibility of not serving customers with high valuations will out-weight the benefits of decreased prices.

We also compare the optimal policies to the case where a central planner manages jointly the supplier and the government and tries to optimize the entire system simultaneously. We show that the optimal effective price and production level coincide in both the decentralized and centralized models. Consequently, the subsidy mechanism is sufficient to coordinate the government and the supplier and the collusion does not hurt consumers in terms of price and quantities.

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APPENDIX

Proof of Theorem 1

Proof. 1. Equations (9) and (10) are obtained by applying the first order conditions on the objective function of problem (1) with respect to q and then to p.

2. We next prove the second claim about the fact that tha optimal solution of the government problem is obtained when the target adoption level is exactly met. Using condition (8), one can compute the optimal value of $p^*(r)$ by using a binary search algorithm (note that equation (9) is monotonic in p for any given value of r). In particular, for any given r, there exists a single value $p^*(r)$ that satisfies the optimal equation (9) and since all the involved functions are continuous, we may also conclude that $p^*(r)$ is a continuous well defined function. As a result, the objective function of the government when using the optimal policy of the supplier is also a continuous function of r. In addition, the target level cannot be attained when $r = 0$ by Assumption 1. We then conclude that the optimal solution of the government problem is obtained when the inequality target constraint is tight. In addition, one can see that the expected adoption target equation is monotonic in r so that one can solve it by applying a binary section method.

3. Finally, let us show the third part. For the deterministic demand model, we have: $q_{det} = y(z_{det}) = \Gamma$. On the other hand, when demand is stochastic, we have $\mathbb{E}\big[min(q_{sto}, D(z_{sto}, \epsilon))\big] = \Gamma$ so that we obtain: $q_{sto} \geq q_{det}$. In addition, the above expression yields: $y(z_{sto}) = \Gamma - K_\epsilon \geq \Gamma$. Therefore, we obtain: $y(z_{sto}) \geq y(z_{det})$. Since $y(z)$ is non-increasing with respect to $z = p - r$ (from Assumption 1), we may infer the following relation for the effective price: $z_{det} \geq z_{sto}$. We next compute the optimal price for the deterministic model p_{det} by differentiating the supplier's objective function with respect to p and equate it to zero (first order condition):

$$\frac{\partial}{\partial p_{det}}\big[q_{det} \cdot (p_{det} - c)\big] = y'(z_{det}) \cdot (p_{det} - c) + y(z_{det}) = 0.$$

One can see that in both models, we have obtained the same optimal equation for the price: $y(z) = -y'(z) \cdot (p - c)$. Namely, the optimal price satisfies: $p = c + \frac{\Gamma}{|y'(z)|}$. We note that the previous expression is not a closed form expression as both sides depend on the optimal price p. This is not an issue as our goal here is to compare the optimal quantities in the two models rather than deriving the closed form expressions. Assuming that the deterministic part of demand $y(z)$ is a convex function, we know that $y'(z)$ is a non-decreasing function and then: $0 > y'(z_{det}) \geq y'(z_{sto})$. We then have the following inequality for the optimal prices: $p_{sto} \leq p_{det}$. We note that the optimal subsidy in both models does not follow such a clear relation and it will actually depend on the specific demand function. We next proceed to compare the optimal supplier's profit in both models. For the deterministic demand model, the optimal profit is given by: $\Pi_{det} = q_{det} \cdot (p_{det} - c) = \frac{\Gamma^2}{|y'(z_{det})|}$. In the stochastic model, the expression of the optimal profit is given by:

$$\Pi_{sto} = p_{sto} \cdot \mathbb{E}\big[min(q_{sto}, D(z_{sto}, \epsilon))\big] - c \cdot q_{sto} = \frac{\Gamma^2}{|y'(z_{sto})|} - c \cdot (q_{sto} - \Gamma) \leq \Pi_{det}.$$

□

The Price Taker Model

Consider a firm that faces a stochastic demand D for its product and must decide upon production q before observing demand. This framework is known as the newsvendor problem and was extensively studied in the literature (see, e.g., Porteus (1990)). In this section, we consider the case where the selling price p is exogenously given (price taker setting) and we derive the solutions of problems (2) and (1). The problem is modeled as a two-stage game and can be solved sequentially by backward induction. Given a subsidy level r, the optimal solution of problem (1) may be obtained similarly to the newsvendor problem and is given by:

$$q^*(z) = F^{-1}_{D(z,\epsilon)}\Big(\frac{p-c}{p}\Big). \tag{26}$$

Here, $F^{-1}_{D(z,\epsilon)}\Big(\frac{p-c}{p}\Big)$ denotes the inverse Cumulative Distribution Function (CDF) of the random demand evaluated at the quantile $\frac{p-c}{p}$. We note however that the supplier's production $q^*(z)$ depends on the value of the effective price z through demand. We next impose the following assumption on demand.

Assumption 3. *The function $D(z,\epsilon)$ is positive, decreasing and continuous with respect to z (note that here p is fixed so that decreasing with z translates to increasing with r) for any realization of the noise ϵ. By this, we mean that the corresponding CDF is a decreasing function of z:*

$$F_{D(z_1,\epsilon)}(x) < F_{D(z_2,\epsilon)}(x);\ \forall z_1 = p - r_1 < z_2 = p - r_2$$

In addition, $\mathbb{E}\big[D(p,\epsilon)\big] < \Gamma$ for the given price p and $r = 0$. That is, the target adoption level cannot be achieved without subsidies (otherwise, the problem is not relevant).

The first part of Assumption 3 is a special case of the first-order stochastic dominance where for each value of z, $D(z,\epsilon)$ is viewed as a different random variable. Using Assumption 3, we prove formally (see details below) that the optimal subsidy of problem (2) is obtained when the target adoption level constraint is binding. As a result, one can find the optimal solution of problem (2) by using the tightness of the target adoption constraint. In addition, the optimal effective price z^* can be computed efficiently by using a bisection search method (see details below). One can then compute $q^*(z^*)$ using equation (26).

In particular, by substituting the expression for $q^*(z)$ from (26) to the government's problem, the government leads the game by solving the following optimization problem:

$$\begin{aligned} \min_{z} \quad & r \cdot \mathbb{E}\big[min(q^*(z), D(z,\epsilon))\big] \\ \text{s.t.} \quad & \mathbb{E}\big[min(q^*(z), D(z,\epsilon))\big] \geq \Gamma \\ & r \geq 0 \end{aligned} \tag{27}$$

Note that the problem above is a single variable optimization problem in the subsidy z (since p is fixed, the real decision is in fact r or equivalently $z = p - r$). Note also that we impose a non-negative constraint on the consumer subsidies since the target adoption cannot be achieved with $r = 0$. However, even for simple demand functions, this problem is not necessarily convex. In addition as the overall problem is a bi-level optimization problem (that is, the government problem takes into consideration the production quantities set by the supplier) the overall problem is hard.

We next compare the optimal decision variables when demand is stochastic relative to the case of a deterministic average approximation. We use the subscripts "sto" and "det" to denote the optimal variables for the stochastic and deterministic models respectively. For simplicity, we consider an additive uncertainty in the following form:

$$D(z,\epsilon) = y(z) + \epsilon.$$

Here, $y(z) = \mathbb{E}\big[D(z,\epsilon)\big]$ is a function of the effective price $z = p - r$ and represents the nominal deterministic part of demand whereas ϵ is a random variable with zero mean. Assumption 3 implies that $y(z)$ is decreasing with respect to z. We next present the comparison between considering explicitly demand uncertainty relative to the case where demand is just approximated by its average value. The results are summarized in the following Theorem.

Theorem 4. *Under Assumption 3, we have:*

$$\begin{aligned} z_{sto} = p - r_{sto} \leq & \; z_{det} = p - r_{det} \\ q_{sto} \geq & \quad q_{det} \\ r_{sto} \geq & \quad r_{det} \\ Exp_{sto} \geq & \quad Exp_{det} \\ \Pi_{sto} \leq & \quad \Pi_{det} \end{aligned}$$

Π and Exp denote the supplier profit and government expenditures respectively.

Proof. For the deterministic model, one can obtain that the optimal solution satisfies: $q_{det} = y(z_{det}) = \Gamma$. When demand is stochastic, we have $\mathbb{E}\big[min(q_{sto}, D(z_{sto},\epsilon))\big] = \Gamma$ so that: $q_{sto} \geq q_{det}$. In addition, the above expression yields: $y(z_{sto}) = \Gamma - \mathbb{E}\big[min(F_\epsilon^{-1}\Big(\frac{p-c}{p}\Big),\epsilon)\big] \geq \Gamma$. Therefore, we have: $y(z_{sto}) \geq y(z_{det})$. Now, since $y(z)$ is assumed to be a non-increasing function of $z = p - r$ (from Assumption 3), we may infer the following relation about the effective price: $z_{det} \geq z_{sto}$. Therefore, we also have: $r_{sto} \geq r_{det}$. The government expenditures are equal to the optimal subsidies multiplied by the expected sales. Since for both deterministic and stochastic demands, the expected sales are equal exactly to the target level Γ, we conclude that: $Exp_{sto} \geq Exp_{det}$. We next proceed to compare the optimal profit of the supplier in both models. For the deterministic model, the optimal profit is given by: $\Pi_{det} = q_{det} \cdot (p - c) = \Gamma \cdot (p - c)$. In the stochastic model, the expression of the optimal profit is given by: $\Pi_{sto} = p \cdot \mathbb{E}\big[min(q_{sto}, D(z_{sto},\epsilon))\big] - c \cdot q_{sto} = p \cdot \Gamma - c \cdot q_{sto} \leq \Pi_{det}$. □

For a general decreasing demand function in terms of the effective price $p - r$ (since in this case the price p is fixed, demand is increasing with r), uncertainty benefits the consumers in the sense that the price is lower and the produced quantities are higher. However, both the government and the supplier are worse-off when demand is uncertain. We observe that for the price taker model considered here, the results do not depend on the structure of demand as long as it satisfies Assumption 3. In other words, the results of Theorem 4 are robust with respect to the form of the demand function. The case where the noise is multiplicative also leads to the same set of results but is not presented here due to space limitations. Remember that for the price setter model presented in Section 4, the results actually do depend on the underlying demand structure.

Proof of the tightness of the constraint for the price taker model

Proof. First, we show that the objective function in problem (27) is a non-decreasing function with respect to the subsidy r. Then, since we are minimizing a non-decreasing function subject to an inequality constraint and a non-negativity constraint, the optimal solution will be obtained by the tightness of the adoption constraint. The stochastic demand function $D(z,\epsilon)$ is assumed to be a strictly decreasing and continuous function of z (see Assumption 3). By this, we mean that the corresponding CDF is a decreasing function of r: $F_{D(z_1)}(x) < F_{D(z_2)}(x);\ \forall z_1 = p - r_1 < z_2 = p - r_2$. Namely, if r increases (and therefore z decreases), the investment becomes more subsidized and hence more profitable. Consequently, demand tends to increase too and therefore the CDF decreases since it represents the following probability: $F_{D(z,\epsilon)}(x) = \mathbb{P}(D(z,\epsilon) \leq x)$. Regarding the optimal inventory level $q^*(r)$, we have obtained that:

$$q^*(z) = F^{-1}_{D(z,\epsilon)}\Big(\frac{p-c}{p}\Big). \tag{28}$$

We next show that the expression in (28) is decreasing with respect to z.

Lemma 1. *Under Assumption 3, the expression of $q^*(z)$ in (28) is an decreasing function of z.*

Proof. Let us take two different values z_1 and z_2 such that $z_1 < z_2$. As we previously explained, from Assumption 3 we have: $F_{D(z_1,\epsilon)}(x) < F_{D(z_2,\epsilon)}(x)$. By using the non-decreasing property of the CDF, we have: $F_{D(z,\epsilon)}(x_1) \geq F_{D(z,\epsilon)}(x_2);\ \forall x_1 \geq x_2$. Recall that we want to show that: $F^{-1}_{D(z_1,\epsilon)}(t) > F^{-1}_{D(z_2,\epsilon)}(t)$. Let us denote: $y_1 \triangleq F^{-1}_{D(z_1,\epsilon)}(t);\ y_2 \triangleq F^{-1}_{D(z_2,\epsilon)}(t)$. Now, let us assume by contradiction that: $y_1 \leq y_2$. Then, we have:

$$\begin{cases} F_{D(z_1,\epsilon)}(y_1) = F_{D(z_1,\epsilon)}\big(F^{-1}_{D(z_1,\epsilon)}(t)\big) = t \\ F_{D(z_2,\epsilon)}(y_2) = F_{D(z_2,\epsilon)}\big(F^{-1}_{D(z_2,\epsilon)}(t)\big) = t \end{cases}$$

However, $t = F_{D(z_2,\epsilon)}(y_2) > F_{D(z_1,\epsilon)}(y_2) \geq F_{D(z_1,\epsilon)}(y_1) = t$ and this is a contradiction. □

We next look at the expected sales, $\mathbb{E}\big[min(q^*(z), D(z,\epsilon))\big]$ as a function of z.

Lemma 2. *The function $\mathbb{E}\big[min(q^*(z), D(z,\epsilon))\big]$ is a non-increasing function with respect to z.*

Proof. Let us define the following (non-negative) random variable as a function of z: $W(z) = min(q^*(z), D(z,\epsilon))$. Then, its CDF is given by:

$$F_{W(z)}(x) = \mathbb{P}(W(z) \leq x) = \mathbb{P}\big(min(q^*(z), D(z,\epsilon)) \leq x\big) = \begin{cases} 1, & \text{if } q^*(z) \leq x \\ \mathbb{P}(D(z,\epsilon) \leq x) = F_{D(z,\epsilon)}(x), & \text{if } q^*(z) > x \end{cases}$$

Using the result from Lemma 1, we can see that $F_{W(z)}(x)$ is a non-decreasing function of z. We next compute the desired expectation using the following relation:

$$\mathbb{E}\big[min(q^*(z), D(z,\epsilon))\big] = \mathbb{E}\big[W(z)\big] = \int \big[1 - F_{W(z)}(x)\big]dx.$$

Since we have shown that $F_{W(z)}(x)$ is a non-decreasing function of z, we conclude that the integrand is non-increasing in z and the desired result follows. □

In conclusion, we have shown that the objective function is a non-increasing function of z and therefore the optimal minimizing solution is obtained when the target constraint is exactly met.

We next show that the optimal solution z_{sto} can be computed efficiently and derive the optimal equation to be solved. By using the tightness of the target constraint, we obtain:

$$\mathbb{E}\big[min(q^*(z_{sto}), D(z_{sto}, \epsilon))\big] = \Gamma. \tag{29}$$

Here, z_{sto} denotes the optimal effective price. Since the price p is fixed, the quantile $\frac{p-c}{p}$ is fixed too and therefore $F^{-1}_{D(z,\epsilon)}(\cdot)$ is only a function of z. One can further simplify equation (29) as follows:

$$q^* \cdot \big[1 - F_{D(z_{sto},\epsilon)}(q^*)\big] + \mathbb{E}[D(z_{sto}, \epsilon)|D(z_{sto}, \epsilon) \leq q^*] \cdot F_{D(z_{sto},\epsilon)}(q^*) = \Gamma.$$

Since $F_{D(z_{sto},\epsilon)}(q^*) = \frac{p-c}{p}$, we obtain: $\frac{c}{p} \cdot q^* + \frac{p-c}{p} \cdot \mathbb{E}\big[D(z_{sto}, \epsilon)|D(z_{sto}, \epsilon) \leq q^*\big] = \Gamma$. Note that: $\mathbb{E}\big[D(z_{sto}, \epsilon)|D(z_{sto}, \epsilon) \leq q^*\big] = \frac{p}{p-c} \cdot \int_0^{q^*} x \cdot f_{D(z_{sto},\epsilon)}(x)\, dx$. Here, $f_{D(z_{sto},\epsilon)}(x)$ represents the Probability Density Function (PDF) of the random demand $D(z, \epsilon)$ evaluated at $z = p - r_{sto}$. Finally, since $\frac{c}{p} \cdot q^* + \int_0^{q^*} x \cdot f_{D(z_{sto},\epsilon)}(x)\, dx = \Gamma$, using integration by parts, we obtain:

$$\begin{aligned}\int_0^{q^*} x \cdot f_{D(z_{sto},\epsilon)}(x)\, dx &= x \cdot F_{D(z_{sto},\epsilon)}(x)\Big|_0^{q^*} - \int_0^{q^*} F_{D(z_{sto},\epsilon)}(x)\, dx \\ &= q^* \cdot \frac{p-c}{p} - \int_0^{q^*} F_{D(z_{sto},\epsilon)}(x)\, dx.\end{aligned}$$

Therefore: $q^* - \int_0^{q^*} F_{D(z_{sto},\epsilon)}(x)\, dx = \Gamma$. Since $q^* = F^{-1}_{D(z_{sto},\epsilon)}\left(\frac{p-c}{p}\right)$, we obtain the following optimal equation for z_{sto}:

$$F^{-1}_{D(z_{sto},\epsilon)}\left(\frac{p-c}{p}\right) - \int_0^{F^{-1}_{D(z_{sto},\epsilon)}\left(\frac{p-c}{p}\right)} F_{D(z_{sto},\epsilon)}(x)\, dx = \Gamma. \tag{30}$$

Equation (30) is a monotonic equation in z_{sto}. Therefore, one can find the optimal effective price z_{sto} from the previous equation using a bisection search method. □

Proof of Theorem 2

Proof. For the linear demand model in (13), the optimal solution of the supplier's optimization problem has to satisfy the following first order condition:

$$\bar{d} + \alpha \cdot (r + c - 2p) + \frac{c}{p} \cdot F_\epsilon^{-1}\left(\frac{p-c}{p}\right) + \frac{p-c}{p} \cdot \mathbb{E}\left[\epsilon | \epsilon \leq F_\epsilon^{-1}\left(\frac{p-c}{p}\right)\right] = 0.$$

Note that it does not seem easy to obtain a closed form solution for $p^*(r)$. In addition, since the previous equation is not monotone one cannot use a binary search method.

Instead, one can express r as a function of p: $r = 2p - c - \frac{\bar{d}}{\alpha} + \frac{\alpha \cdot c^2}{\alpha \cdot p^2}$. We next proceed to solve the government optimization problem by using the tightness of the inequality target adoption constraint: $\mathbb{E}\big[min(q^*(p^*(r), r), D(p^*(r) - r, \epsilon))\big] = \alpha \cdot (p^*(r) - c) = \Gamma$. One very interesting conclusion from this analysis is that we have a very simple closed form expression for the optimal price, that is the same than for the deterministic case:

$$p_{sto} = p_{det} = c + \frac{\Gamma}{\alpha}. \tag{31}$$

We can at this point derive the optimal supplier's profit for both models. In the deterministic case, the profit of the supplier is given by: $\Pi_{det} = q_{det} \cdot (p_{det} - c) = \frac{\Gamma^2}{\alpha}$. In the stochastic model, the optimal profit is given by:

$\Pi_{sto} = p_{sto} \cdot \mathbb{E}\big[min(q_{sto}, D(z_{sto}, \epsilon)\big] - c \cdot q_{sto} = \frac{\Gamma^2}{\alpha} - c \cdot (q_{sto} - \Gamma) \leq \Pi_{det}$. We next derive the optimal production level for both models. In the deterministic case, we obtained that: $q_{det} = \Gamma$. For the stochastic case, after substituting all the corresponding expressions we obtain:

$$q_{sto} = \bar{d} + \alpha \cdot (r_{sto} - p_{sto}) + F_\epsilon^{-1}\Big(\frac{p_{sto} - c}{p_{sto}}\Big) = \Gamma + F_\epsilon^{-1}\Big(\frac{p_{sto} - c}{p_{sto}}\Big) - K_\epsilon \geq q_{det}.$$

We finally compare the effect of demand uncertainty on the optimal subsidy. One can show after some appropriate manipulations that the optimal subsidy for the deterministic linear demand model is given by: $r_{det} = \frac{2\Gamma}{\alpha} - \frac{\bar{d}}{\alpha} + c$. For the stochastic demand model, we have the following optimal equation: $\alpha \cdot (2p_{sto} - r_{sto} - c) - \bar{d} = K_\epsilon$. Hence, one can find the expression for the optimal subsidies as a function of the optimal price p_{sto}: $r_{sto} = 2p_{sto} - c - \frac{\bar{d}}{\alpha} - \frac{1}{\alpha} \cdot K_\epsilon$. By replacing: $p_{sto} = c + \frac{\Gamma}{\alpha}$ from (31), we obtain: $r_{sto} = \frac{2\Gamma}{\alpha} + c - \frac{\bar{d}}{\alpha} - \frac{1}{\alpha} \cdot K_\epsilon = r_{det} - \frac{1}{\alpha} \cdot K_\epsilon \geq r_{det}$. □

Proof of Corollary 1

Proof. 1. We provide the proof of Corollary 1.1 by using the facts $\frac{d\rho}{dc} \leq 0$ and $\frac{d\rho}{c\Gamma} \geq 0$. We then show that the production gap widens as the service level ρ increases. Note that $q_{sto} - q_{det} = F_\epsilon^{-1}(\rho) - K_\epsilon = E\big[max(F_\epsilon^{-1}(\rho) - \epsilon, 0)\big]$. Taking the derivative with respect to the cost c, we obtain:

$$\frac{d(q_{sto} - q_{det})}{dc} = \frac{d\big(F_\epsilon^{-1}(\rho)\big)}{d\rho} \cdot \frac{d\rho}{dc} \cdot \rho = \frac{1}{f(F_\epsilon^{-1}(\rho))} \cdot \frac{d\rho}{dc} \cdot \rho \leq 0.$$

Similarly, for the target level Γ:

$$\frac{d(q_{sto} - q_{det})}{d\Gamma} = \frac{d\big(F_\epsilon^{-1}(\rho)\big)}{d\rho} \cdot \frac{d\rho}{d\Gamma} \cdot \rho = \frac{1}{f(F_\epsilon^{-1}(\rho))} \cdot \frac{d\rho}{d\Gamma} \cdot \rho \geq 0.$$

2. To prove Corollary 1.2, we show that the subsidy gap decreases with respect to ρ. Note that $r_{sto} - r_{det} = -K_\epsilon/\alpha = -E\big[min(F_\epsilon^{-1}(\rho), \epsilon)\big]/\alpha$. Taking the derivative with respect to the cost c, we obtain:

$$\frac{d(r_{sto} - r_{det})}{dc} = -\frac{1}{\alpha}\Big[\frac{d\big(F_\epsilon^{-1}(\rho)\big)}{d\rho} \cdot \frac{d\rho}{dc} \cdot (1-\rho)\Big] = -\frac{1}{\alpha} \cdot \frac{1}{f(F_\epsilon^{-1}(\rho))} \cdot \frac{d\rho}{dc} \cdot (1-\rho) \geq 0.$$

Similarly, for the target level Γ:

$$\frac{d(r_{sto} - r_{det})}{d\Gamma} = -\frac{1}{\alpha}\Big[\frac{d\big(F_\epsilon^{-1}(\rho)\big)}{d\rho} \cdot \frac{d\rho}{d\Gamma} \cdot (1-\rho)\Big] = -\frac{1}{\alpha} \cdot \frac{1}{f(F_\epsilon^{-1}(\rho))} \cdot \frac{d\rho}{d\Gamma} \cdot (1-\rho) \leq 0.$$

3. We next present the proof of Corollary 1.3. We assume that ϵ is an additive random variable with support $[a_1, a_2]$, not necessarily with a symmetric PDF. For the linear demand model from (13), we have:

$$r_{sto} = r_{det} - \frac{1}{\alpha} \cdot K_\epsilon. \tag{32}$$

First, let us prove the first inequality by showing that the term on the right in equation (32) is non-positive for a general parameter y. We have: $\mathbb{E}\big[min(y, \epsilon)\big] = y \cdot \mathbb{P}(y \leq \epsilon) + \mathbb{E}[\epsilon | \epsilon < y] \cdot \mathbb{P}(\epsilon < y)$. Now, let us divide the analysis into two different cases according to the sign of y. If $y \leq 0$, we obtain: $\mathbb{E}\big[min(y, \epsilon)\big] = y \cdot \mathbb{P}(y \leq \epsilon) + \mathbb{P}(\epsilon < y) \cdot \mathbb{E}[\epsilon | \epsilon < y] \leq 0$. In the previous equation, both terms are non-positive. For the case where $y > 0$, we have: $\mathbb{E}\big[min(y, \epsilon)\big] < \mathbb{E}[\epsilon] = 0$. Therefore, $\mathbb{E}\big[min(F_\epsilon^{-1}\big(\frac{p-c}{p}\big), \epsilon)\big] \leq 0$, showing the first inequality: $r_{det} \leq r_{sto}$. We now show the second inequality. We know from the optimality that: $p \geq c$. Let us evaluate the expression of r_{sto} in (32) for different values of p. If $p = c$, we obtain: $F_\epsilon^{-1}\big(\frac{p-c}{p}\big) = F_\epsilon^{-1}(0) = a_1 < 0$. Then, we have: $r_{sto} = r_{det} - \frac{1}{\alpha} \cdot \mathbb{E}\big[min(a_1, \epsilon)\big] = r_{det} - \frac{a_1}{\alpha} > r_{det}$. If $p \gg c$, we obtain: $F_\epsilon^{-1}\big(\frac{p-c}{p}\big) \to F_\epsilon^{-1}(1) = a_2 > 0$. Therefore, we obtain: $r_{sto} \to r_{det} - \frac{1}{\alpha} \cdot \mathbb{E}\big[min(a_2, \epsilon)\big] = r_{det} - \frac{1}{\alpha} \cdot \mathbb{E}[\epsilon] = r_{det}$. Since r_{sto} is continuous and non-increasing in p for any $p \geq c$, the second inequality holds. □

Proof of Proposition 2

Proof. By applying a similar methodology as in the proof of Theorem 1, one can derive the following expressions (the steps are not reported for conciseness):

$$\begin{aligned} q_{det} &= \Gamma; & q_{sto} &> \Gamma \\ p_{det} &= c + \tfrac{1}{\alpha}\cdot\left(\tfrac{\bar{d}}{\Gamma}\right)^{\frac{1}{\alpha}}; & p_{sto} &= c + \frac{1}{\alpha}\cdot\left(\frac{\bar{d}}{\Gamma}\cdot K(p_{sto})\right)^{\frac{1}{\alpha}} \\ r_{det} &= c + \left(\tfrac{\bar{d}}{\Gamma}\right)^{\frac{1}{\alpha}}\cdot\left(\tfrac{1}{\alpha}-1\right); & r_{sto} &= c + \left(\frac{\bar{d}}{\Gamma}\cdot K(p_{sto})\right)^{\frac{1}{\alpha}}\cdot\left(\frac{1}{\alpha}-1\right) \end{aligned}$$

Here, $K(p_{sto}) = \mathbb{E}\big[min(F_\epsilon^{-1}\left(\frac{p_{sto}-c}{p_{sto}}\right),\epsilon)\big]$, so that the above expressions for p_{sto} and r_{sto} are not in closed form. Since $\mathbb{E}[\epsilon]=1$ and $\epsilon \geq 0$, we have $0 \leq K(p_{sto}) \leq 1$. Therefore, as expected: $p_{sto} \leq p_{det}$. Regarding the optimal subsidy, one can see that: if $\alpha > 1$: $r_{sto} \geq r_{det}$, if $0 < \alpha < 1$: $r_{sto} \leq r_{det}$ and if $\alpha = 1$: $r_{sto} = r_{det} = c$. □

Proof of Proposition 3

Proof. For the linear additive demand model presented in (13), one can compute the consumer surplus for given values of p, r and q:

$$CS_{sto}(\epsilon) = \begin{cases} \frac{D(z,\epsilon)^2}{2\alpha}; \text{ if } D(z,\epsilon) \leq q \\ \frac{D(z,\epsilon)\cdot q}{2\alpha}; \text{ if } D(z,\epsilon) > q \end{cases} = \frac{D(z,\epsilon)}{2\alpha}\cdot\big[\min(D(z,\epsilon),q)\big].$$

Therefore, we have: $CS_{sto}(\epsilon) \geq \frac{\left[\min(D(z,\epsilon),q)\right]^2}{2\alpha}$. By applying the expectation operator, we obtain:

$$\mathbb{E}\big[CS_{sto}(\epsilon)\big] \geq \frac{\mathbb{E}\Big\{\big[\min(D(z,\epsilon),q)\big]^2\Big\}}{2\alpha} \geq \frac{\Big\{\mathbb{E}\big[\min(D(z,\epsilon),q)\big]\Big\}^2}{2\alpha} = \frac{\Gamma^2}{2\alpha} = CS_{det}.$$

Where, the second inequality follows by Jensen's inequality (or the fact that the variance of any random variable is always non-negative). The last equality follows from the previous result that the target inequality constraint is tight at optimality.

We next compute the consumer surplus defined in (18) for the iso-elastic demand from (16). In particular, we observe that $z_{max}(\epsilon) = \infty$ for any value of ϵ (if we assume that ϵ is strictly positive and finite). In addition, we have: $z_{sto} = p_{sto} - r_{sto} = \left[\frac{\bar{d}}{\Gamma}\cdot K(p_{sto})\right]^{\frac{1}{\alpha}}$. Note that $K(p_{sto})$ is a deterministic constant and does not depend on the realization of the noise ϵ. In particular, one can solve the non-linear fixed point equation with respect to p_{sto} and find the corresponding value of $K(p_{sto})$: $p_{sto} = c + \frac{1}{\alpha}\cdot\left[\frac{\bar{d}}{\Gamma}\cdot\mathbb{E}\big[\min(F_\epsilon^{-1}\left(\frac{p_{sto}-c}{p_{sto}}\right),\epsilon)\big]\right]^{\frac{1}{\alpha}}$. We also have $D(z_{sto},\epsilon) = \bar{d}\cdot\left[\frac{\bar{d}}{\Gamma}\cdot K(p_{sto})\right]^{-1}\cdot\epsilon$. Therefore, when computing $CS_{sto}(\epsilon)$ for a given ϵ, since demand is multiplicative with respect to the noise, one can see that ϵ cancels out and that simplifies the calculation. We obtain: $CS_{sto}(\epsilon) = \frac{1}{\alpha-1}\cdot\left[\frac{\bar{d}}{\Gamma}\cdot K(p_{sto})\right]^{\frac{1-\alpha}{\alpha}}\cdot\left[\frac{\bar{d}}{\Gamma}\cdot K(p_{sto})\right]\cdot\min\left(D(z_{sto},\epsilon),q_{sto}\right)$. Then, by taking the expectation operator, we obtain: $\mathbb{E}[CS_{sto}(\epsilon)] = \frac{\bar{d}}{\alpha-1}\cdot\left(\frac{\bar{d}}{\Gamma}\right)^{\frac{1-\alpha}{\alpha}}\cdot(K(p_{sto}))^{\frac{1}{\alpha}} = CS_{det}\cdot\left[K(p_{sto})\right]^{\frac{1}{\alpha}}$. Here, we have used the fact that the inequality adoption constraint is tight at optimality, that is: $\mathbb{E}\big[\min\left(D(z_{sto},\epsilon),q_{sto}\right)\big] = \Gamma$. In addition, this is the only term that depends on the noise ϵ. Since we have $0 \leq K(p_{sto}) \leq 1$, one conclude that for any $\alpha > 1$: $\mathbb{E}[CS_{sto}(\epsilon)] \leq CS_{det}$. □

Proof of Theorem 3

Proof. We present first the proof for the deterministic demand model and then the one for stochastic demand. Let us first consider the unconstrained optimization problem faced by the central planner. If demand is deterministic, the objective function is given by: $J(p,r) = q(p,r)\cdot(p-r-c)$. We assume that demand is a function of the effective price (denoted by z), that is: $q(p,r) = y(p-r) = y(z)$. Next, we compute the unconstrained optimal solution denote by z^* by imposing the first order condition: $\frac{dJ(z)}{dz} = 0 \Rightarrow z^* = c - \frac{y(z^*)}{y'(z^*)}$. Although, we did not derive a closed form expression for z^*, we know that it should satisfy the above fixed point equation. We now show that any unconstrained optimal solution is infeasible for the constrained original problem since it violates the target inequality constraint:

$$q(p^*,r^*) = y(z^*) = y\Big(c - \frac{y(z^*)}{y'(z^*)}\Big) \leq y(c) < \Gamma.$$

We used the facts that demand is positive, differentiable and a decreasing function of the effective price (see Assumption 1). In addition, since we assumed that the target level cannot be achieved without subsidies, we have shown that the unconstrained optimal solution is not feasible. Therefore, we conclude that the target inequality constraint has to be tight at optimality, namely: $q(p_{det}, r_{det}) = \Gamma$. In other words, the optimal effective price and production level are the same than in the decentralized model.

We now proceed to present the proof for the case where demand is stochastic. Let us consider the constrained optimization problem faced by the central planner. We denote by J the objective function (multiplied by minus 1) and by: $\lambda_i;\ i = 1,2,3$ the corresponding KKT multipliers of the three constraints. The KKT optimality conditions are then given by:

$$\frac{\partial J}{\partial q} - \lambda_2 \cdot \mathbb{P}(q \leq D) = 0; \quad \frac{\partial J}{\partial p} - \lambda_1 - A\cdot\lambda_2 = 0; \quad \frac{\partial J}{\partial r} - \lambda_3 - B\cdot\lambda_2 = 0 \tag{33}$$

Here, A and B are given by:

$$A = \frac{\partial}{\partial p}\mathbb{E}\big[min(q, D(z,\epsilon))\big] = y'(z)\cdot F_{D(z,\epsilon)}(q)$$
$$B = \frac{\partial}{\partial r}\mathbb{E}\big[min(q, D(z,\epsilon))\big] = -y'(z)\cdot F_{D(z,\epsilon)}(q) = -A$$

If in addition the noise is additive, we have: $F_{D(z,\epsilon)}(q) = F_\epsilon(q - y(z))$. We also have:

$$\frac{\partial J}{\partial q} = c - z\cdot[1 - F_{D(z,\epsilon)}(q)]; \quad \frac{\partial J}{\partial p} = -\mathbb{E}\big[min(q, D(z,\epsilon))\big] - z\cdot A = -\frac{\partial J}{\partial r}$$

We note that the last two equations are symmetric and hence equivalent. Equivalently, the central planner decides only upon the effective price $z = p - r$ and not p and r separately. We next assume that $\lambda_1 = \lambda_3 = 0$. This corresponds (from the complementary slackness conditions) to assume that both corresponding constraints are not tight. Indeed, clearly the optimal subsidies may be assumed to be strictly positive since we assumed that when $r = 0$, the adoption constraint cannot be satisfied. We further assume that the supplier wants to achieve positive profits, so that the optimal price is strictly larger than the cost. Therefore, the KKT conditions (both stationarity and complementary slackness) can be written as follows:

$$c \ \ -(z+\lambda_2)\cdot[1 - F_\epsilon(q - y(z))] = 0 \tag{34}$$

$$- \ \ (z-\lambda_2)\cdot y'(z)\cdot F_\epsilon(q - y(z)) = \mathbb{E}\big[min(q, D(z,\epsilon))\big] \tag{35}$$

$$\lambda_2 \ \cdot\big(\Gamma - \mathbb{E}\big[min(q, D(z,\epsilon))\big]\big) = 0 \tag{36}$$

We now have two possible cases depending on the value of λ_2. Let us investigate first the case where $\lambda_2 = 0$. From equation (34), we have: $F_\epsilon(q - y(z)) = \frac{z-c}{z}$. By using equation (35), we obtain: $z = c - \frac{\mathbb{E}\left[min(q, D(z,\epsilon))\right]}{y'(z)}$. Now, we have: $\mathbb{E}\big[min(q, D(z,\epsilon))\big] = y\Big(c - \frac{\mathbb{E}\left[min(q,D(z,\epsilon))\right]}{y'(z)}\Big) + \mathbb{E}\big[min(F_\epsilon^{-1}\big(\frac{p-c}{p}\big), \epsilon)\big]$. Since we assume that the function $y(z)$ is a decreasing function of the effective price z and that both q and $D(z,\epsilon)$ are non-negative, we obtain: $\mathbb{E}\big[min(q, D(z,\epsilon))\big] < y(c) + \mathbb{E}\big[min(F_\epsilon^{-1}\big(\frac{p-c}{p}\big), \epsilon)\big] \le y(c)$. In the last step, we used the fact that $\mathbb{E}\big[min(F_\epsilon^{-1}\big(\frac{p-c}{p}\big), \epsilon)\big] \le 0$. Therefore, we conclude that: $\mathbb{E}\big[min(q, D(z,\epsilon))\big] < \Gamma$. In other words, the solution is not feasible since it violates the target inequality constraint. Hence, we must have $\lambda_2 > 0$ and the inequality constraint is tight at optimality: $\mathbb{E}\big[min(q, D(z,\epsilon))\big] = \Gamma$. Now, by using equation (34), we obtain: $F_\epsilon(q - y(z)) = \frac{z+\lambda_2-c}{z+\lambda_2}$. We then substitute the above expression in equation (35): $z = c - \lambda_2 - \frac{\Gamma}{y'(z)}$. Now, we have: $\Gamma = y(z) + \mathbb{E}\big[min(F_\epsilon^{-1}\big(\frac{z+\lambda_2-c}{z+\lambda_2}\big), \epsilon)\big]$. By expressing the previous equation in terms of the effective price, we obtain: $\Gamma = y(z) + \mathbb{E}\big[min(F_\epsilon^{-1}\big(\frac{-\frac{\Gamma}{y'(z)}}{c-\frac{\Gamma}{y'(z)}}\big), \epsilon)\big]$. Therefore, one can solve the previous equation and find the optimal effective price z. We note that this is exactly the same equation as in the decentralized case, so that the effective prices are the same. The optimal production levels are given by: $q = y(z) + F_\epsilon^{-1}\Big(\frac{-\frac{\Gamma}{y'(z)}}{c-\frac{\Gamma}{y'(z)}}\Big)$. Similarly, the equations are the same in both the decentralized and centralized models so that the optimal production levels are identical. Finally, we just need to show that $\lambda_2 > 0$ in order to complete the proof. We have:

$$\Gamma = y\Big(c - \lambda_2 - \frac{\Gamma}{y'(z)}\Big) + \mathbb{E}\big[min(F_\epsilon^{-1}\Big(\frac{-\frac{\Gamma}{y'(z)}}{c - \frac{\Gamma}{y'(z)}}\Big), \epsilon)\big] \le y\Big(c - \lambda_2 - \frac{\Gamma}{y'(z)}\Big) < y\big(c - \lambda_2\big)$$

If we assume by contradiction that $\lambda_2 < 0$, we obtain: $\Gamma < y\big(c - \lambda_2\big) < y(c)$. This is a contradiction so that: $\lambda_2 > 0$ and the proof is complete. $\square$

Proof of Proposition 4

Proof. We first consider the scenario where the supplier is non-sophisticated. In this case, the optimal decision variables are still r_{det}, q_{det} and p_{det}. However, in reality demand is uncertain and therefore the expected sales are given by:

$$\mathbb{E}\Big[min(q_{det}, D(z_{det},\epsilon))\Big] = \mathbb{E}\Big[min(q_{det}, y(z_{det}) + \epsilon)\Big] = \Gamma + \mathbb{E}\big[min(0,\epsilon)\big] \le \Gamma. \tag{37}$$

Here, we have used the fact that: $q_{det} = y(z_{det}) = \Gamma$.

Next, we assume that the supplier is sophisticated. Note that in this case, the optimal subsidies set by the government are still equal to r_{det}. In other words, the government does not have any distributional information on demand uncertainty and believes neither does the supplier. In particular, the subsidies are set such that: $y(p_{det} - r_{det}) = \Gamma$. However, the supplier is sophisticated in the sense that he uses distributional information on demand uncertainty in order to decide the optimal price and production. In a similar way as in equations (9) and (10) from Theorem 1, the optimal price when $r = r_{det}$ can be obtain as the solution of the following non-linear equation:

$$y(p - r_{det}) + \mathbb{E}\big[min(F_\epsilon^{-1}\Big(\frac{p-c}{p}\Big), \epsilon)\big] + y'(p - r_{det}) \cdot (p - c) = 0. \tag{38}$$

In addition,one can compute the optimal production level as follows:

$$q^*(p, r_{det}) = y(p - r_{det}) + F_\epsilon^{-1}\Big(\frac{p-c}{p}\Big).$$

For the linear demand model, equation (38) becomes: $\bar{d} - \alpha \cdot (2p - r_{det} - c) + \mathbb{E}\big[min(F_{\epsilon}^{-1}\big(\frac{p-c}{p}\big), \epsilon)\big] = 0$. Equivalently, the optimal price denoted by p^* follows the following relation:

$$p^* = c + \frac{\Gamma}{\alpha} + \frac{1}{2\alpha} \cdot \mathbb{E}\big[min(F_{\epsilon}^{-1}\Big(\frac{p^* - c}{p^*}\Big), \epsilon)\big].$$

Note that the optimal price when demand is deterministic is equal to $p_{set} = c + \frac{\Gamma}{\alpha}$ and therefore: $p^* \leq p_{det}$. As a result, the expected demand is given by:

$$y(p^* - r_{det}) = \bar{d} - \alpha \cdot (p^* - r_{det}) = \Gamma - \frac{1}{2} \cdot \mathbb{E}\big[min(F_{\epsilon}^{-1}\Big(\frac{p^* - c}{p^*}\Big), \epsilon)\big] \geq \Gamma.$$

We next proceed to compute the expected sales:

$$\begin{aligned}\mathbb{E}\Big[min\big(q^*, D(p^* - r_{det}, \epsilon)\big)\Big] &= \mathbb{E}\Big[min\big(y(p^* - r_{det}) + F_{\epsilon}^{-1}\Big(\frac{p-c}{p}\Big), y(p^* - r_{det}) + \epsilon\big)\Big] \\ &= \Gamma + \frac{1}{2} \cdot \mathbb{E}\Big[min(F_{\epsilon}^{-1}\Big(\frac{p^* - c}{p^*}\Big), \epsilon)\Big] \leq \Gamma.\end{aligned}$$

□