OLIVIER TOUBIA, JOHN R. HAUSER, and DUNCAN I. SIMESTER*

The authors propose and test a new "polyhedral" choice-based conjoint analysis question-design method that adapts each respondent's choice sets on the basis of previous answers by that respondent. Polyhedral "interior-point" algorithms design questions that quickly reduce the sets of partworths that are consistent with the respondent's choices. To identify domains in which individual adaptation is promising (and domains in which it is not), the authors evaluate the performance of polyhedral choice-based conjoint analysis methods with Monte Carlo experiments. They vary magnitude (response accuracy), respondent heterogeneity, estimation method, and question-design method in a 4×2^3 experiment. The estimation methods are hierarchical Bayes and analytic center. The latter is a new individual-level estimation procedure that is a by-product of polyhedral question design. The benchmarks for individual adaptation are random designs, orthogonal designs, and aggregate customization. The simulations suggest that polyhedral question design does well in many domains, particularly those in which heterogeneity and partworth magnitudes are relatively large. The authors test feasibility, test an important design criterion (choice balance), and obtain empirical data on convergence by describing an application to the design of executive education programs in which 354 Web-based respondents answered stated-choice tasks with four service profiles each.

Polyhedral Methods for Adaptive Choice-Based Conjoint Analysis

Choice-based conjoint analysis (CBC) describes a class of techniques that are among the most widely adopted market research methods. In CBC tasks, respondents are presented with two or more product profiles and asked to choose the profile that they prefer (for an example, see Figure 1). This contrasts with other conjoint tasks that ask respondents to provide preference ratings for product attributes or profiles. Because choosing a preferred product profile is often a natural task for respondents that is consistent with marketplace choice, supporters of CBC have argued that it yields more accurate responses. In comparison with estimates of marketplace demand, CBC methods have been shown to perform well (Louviere, Hensher, and Swait 2000).

Academic research investigating CBC methods has sought to improve the design of the product profiles that are shown to each respondent. This has led to efficiency improvements, thereby yielding more information from fewer responses. Because an increasing amount of market research is conducted on the Internet, new opportunities for efficiency improvements have arisen. Online processing power makes it feasible to adapt questions on the basis of prior responses. To date, the research on adaptive question design has focused on adapting questions on the basis of responses from prior respondents (i.e., aggregate customization). Efficient designs are customized through the use of parameters obtained from pretests or from managerial judgment (for examples of aggregate customization methods, see

*Olivier Toubia is a graduate student, Marketing Group (e-mail: toubia@mit.edu), John R. Hauser is Kirin Professor of Marketing (e-mail: jhauser@mit.edu), and Duncan I. Simester is a professor (e-mail: simester@mit.edu), Sloan School of Management, Massachusetts Institute of Technology (MIT). The authors gratefully acknowledge the contribution of Robert M. Freund, who proposed the use of the analytic center and the approximation of ellipsoids and gave detailed advice on the application of these methods. This research was supported by the Sloan School of Management and the Center for Innovation in Product Development at MIT. All authors contributed fully and synergistically to this article. They thank Ray Faith, Aleksas Hauser, Janine Sisk, Limor Weisberg, and Toby Woll for the visual design, programming, and project management on the executive education study. This article has benefited from presentations at the CIPD Spring Research Reviews, the Epoch Foundation Workshop, the Marketing Science Conferences (Wiesbaden, Germany; Alberta, Canada; and College Park, Maryland), MIT's "Managing Corporate Innovation," the MIT Marketing Workshop, the MIT Operations Research Seminar Series, the Marketing Science Institute's Young Scholars Conference, the New England Marketing Conference, the New York University Seminar Series, the Stanford Marketing Workshop, and the University of California, Los Angeles marketing seminar series.

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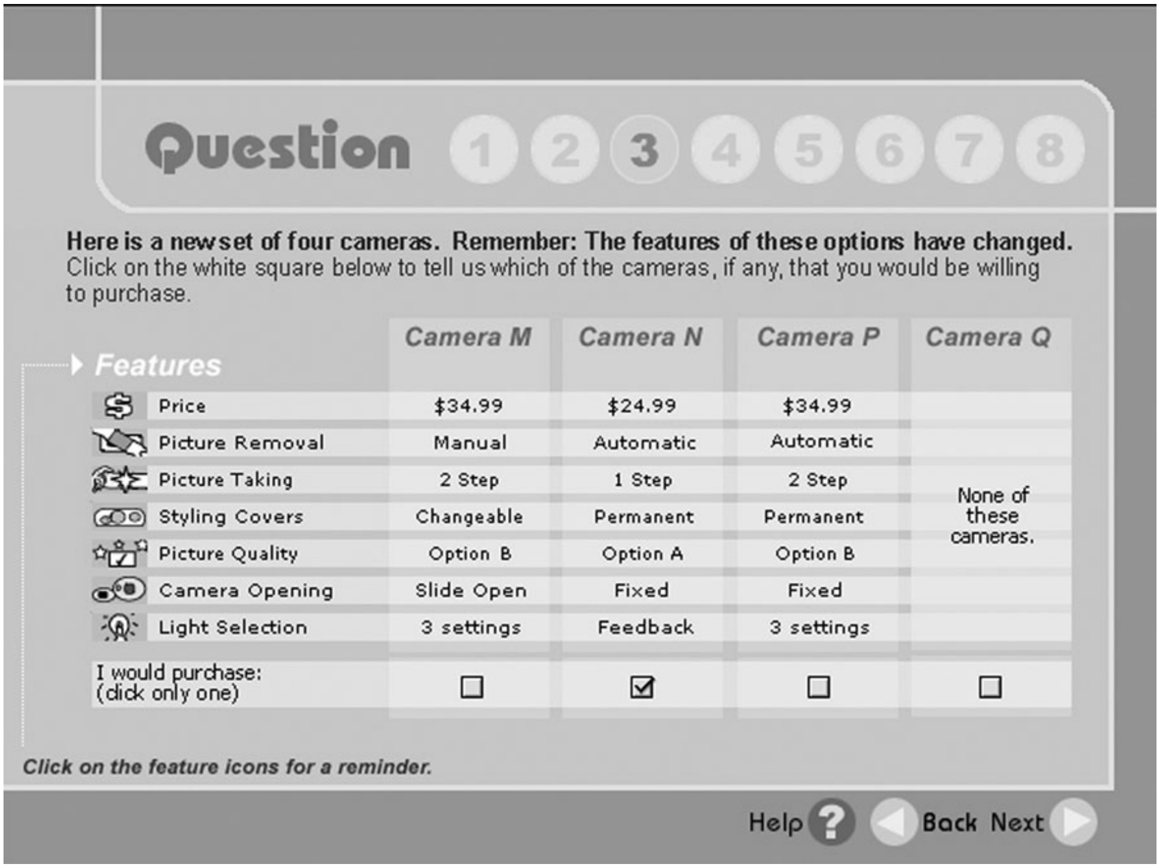
Figure 1
EXAMPLE OF A CBC TASK FOR THE REDESIGN OF POLAROID'S I-ZONE CAMERA



Arora and Huber 2001; Huber and Zwerina 1996; and Sándor and Wedel 2001).

In this study, we propose a CBC question-design method that adapts questions by using the previous answers from that respondent (i.e., individual adaptation). The design of each choice task varies according to the respondent's selection from prior choice tasks. The approach is motivated in part by the success of aggregate customization, which uses the responses from other respondents to design more efficient questions. The algorithm that we propose focuses on what is *not* known about partworths (given the respondent's answers to prior questions) and seeks to reduce quickly the sets of partworths that are consistent with the respondent's choices. To achieve this goal, we focus on four design criteria: (1) nondominance, (2) feasibility, (3) choice balance, and (4) symmetry. We also describe an analogy between the proposed algorithm and D-efficiency.

After data are collected with the adaptive questions, partworths can be estimated with standard methods (aggregate random utility or hierarchical Bayes [HB] methods). As an alternative, we propose and test an individual-level estimation method that relies on the analytic center of a feasible set of parameters.

Our proposal differs in both format and philosophy from the other individual-level adaptive conjoint analysis (ACA) methods. We focus on stated-choice data rather than ACA's metric paired comparisons, and we focus on analogies to efficient design rather than ACA's utility balance subject to orthogonality goals. Polyhedral methods are also feasible for metric-paired-comparison data (Toubia et al. 2003). However, as will become apparent, there are important differences between the metric-paired-comparison algorithm and the algorithm we propose.

The remainder of the article is organized as follows: We begin by reviewing existing CBC question-design and estimation methods. We next propose a polyhedral approach to the design of CBC questions. We then evaluate the proposed polyhedral methods using a series of Monte Carlo simulations, in which we hope to demonstrate the domains in which the proposed method shows promise (and those in which existing methods remain best). We compare performance to three question-design benchmarks, including an aggregate customization method that uses prior data from either managerial judgment or pretest respondents. Because we expect that individual adaptation is most promising when responses are accurate and/or respondents are heterogeneous, we compare the four question-design methods across a range of customer heterogeneity and response-error domains while also varying the estimation method. We then describe an empirical application of the proposed method to the design of executive education programs at a major university. We conclude with a review of the findings, limitations, and opportunities for further research.

EXISTING CBC QUESTION-DESIGN AND ESTIMATION METHODS

To date, most applications of CBC assume that each respondent answers the same set of questions or that the questions are either blocked across sets of respondents or chosen randomly. For these conditions, McFadden (1974) shows that the inverse of the covariance matrix, Σ, of the maximum likelihood estimates is proportional to

$$(1)\quad \Sigma^{-1} = R\sum_{i=1}^{q}\sum_{j=1}^{J_i}(\vec{z}_{ij} - \sum_{k=1}^{J_i}\vec{z}_{ik}P_{ik})'P_{ij}(\vec{z}_{ij} - \sum_{k=1}^{J_i}\vec{z}_{ik}P_{ik}),$$

where R is the effective number of replicates, J_i is the number of profiles in choice set i, q is the number of choice sets, $\vec{z}_{ij}$ is a binary vector that describes the jth profile in the ith choice set, and P_{ij} is the probability that the respondent chooses profile j from the ith choice set. Without loss of generality, we use binary vectors in the theoretical development to simplify notation and exposition. We use multilevel features in both the simulations and the application.

Precision can be increased by a decrease in a measure (norm) of the covariance matrix, that is, by either an increase in the number of replicates or an increase in the terms in the summations of Equation 1. Equation 1 also demonstrates that the covariance of logit-based estimates depends on the choice probabilities, which in turn depend on the partworths. In general, the experimental design that provides the most precise estimates will depend on the parameters.

Many researchers have addressed choice-set design. A common measure is D-efficiency, which attempts to reduce the geometric mean of the eigenvalues of Σ (Kuhfeld, Tobias, and Garratt 1994).[1] If $\vec{u}$ represents the vector of the partworths, the confidence region for maximum likelihood estimates ($\hat{\vec{u}}$) is an ellipsoid defined by $(\vec{u} - \hat{\vec{u}})'\Sigma^{-1}(\vec{u} - \hat{\vec{u}})$ (Greene 1993, p. 190). The lengths of the axes of this ellipsoid are given by the eigenvalues of the covariance matrix, so that a minimization of the geometric mean of the eigenvalues shrinks the confidence region around the estimates.

Because efficiency depends on the partworths, it is common to assume a priori that the stated choices are equally likely. In this article, we label such designs "orthogonal" efficient designs. Arora and Huber (2001), Huber and Zwerina (1996), and Sándor and Wedel (2001) demonstrate that efficiency can be improved with the use of data from either pretests or prior managerial judgment. These researchers improve D-efficiency by "relabeling," which permutes the levels of features across choice sets; by "swapping," which switches two feature levels among profiles in a choice set; and by "cycling," which is a combination of rotating levels of a feature and swapping them. The procedures stop when no further improvement is possible.[2] Simulations suggest that these procedures improve efficiency and thus reduce the number of respondents that are necessary. Following the literature, we label these designs "aggregate customization."

Estimation

In classical logit analysis, partworths are estimated with maximum likelihood techniques. Because it is rare that a respondent is asked to make enough choices to estimate partworth values for each respondent, the data usually are merged across respondents to estimate population-level (or segment-level) partworth values. However, managers often want estimates for each respondent. Hierarchical Bayes methods provide (posterior) estimates of partworths for individual respondents by using population-level distributions of partworths to inform individual-level estimates (Allenby and Rossi 1999; Arora, Allenby, and Ginter 1998; Johnson 1999; Lenk et al. 1996). In particular, HB methods use data from the full sample to estimate iteratively both the posterior means (and distribution) of individual-level partworths and the posterior distribution of the partworths at the population level. The HB method is based on Gibbs sampling and the Metropolis-Hastings algorithm.

Liechty, Ramaswamy, and Cohen (2001) demonstrate the effectiveness of HB methods for choice menus, and Arora and Huber (2001) show that it is possible to improve the efficiency of HB estimates with choice sets designed on the basis of Huber–Zwerina (1996) relabeling and swapping. In other research, Andrews, Ainslie, and Currim (2002, p. 479) present evidence that HB models and finite mixture models estimated from simulated scanner-panel data "recover household-level parameter estimates and predict holdout choice about equally well except when the number of purchases per household is small."

POLYHEDRAL QUESTION-DESIGN METHODS

We extend the philosophy of customization by developing algorithms to adapt questions for each respondent. Stated choices from each respondent provide information about parameter values for the respondent that can be used to select the subsequent questions. In high dimensions (high *p*), this is a difficult dynamic optimization problem. We address this problem by making use of extremely fast algorithms based on projections in the interior of polyhedra (much of this research began with that of Karmarkar [1984]). In particular, we draw on the properties of bounding ellipsoids, discovered in theorems by Sonnevend (1985a, b) and applied by Freund (1993), Nesterov and Nemirovskii (1994), and Vaidja (1989).

We begin by illustrating the intuitive ideas in a two-dimensional space with two product profiles ($J_i = 2$) and then generalize to a larger number of dimensions and multichotomous choice (the simulations and the application are based on multichotomous choice). The axes of the space represent the partworths (utilities) associated with two different product attributes, u_1 and u_2. A point in this space has a value on each axis and is represented by a vector of the two partworths. The ultimate goal is to estimate the point in this space (or distribution of points) that best represents each respondent. The question-design goal is to focus precision toward the points that best represent each respondent. This goal is not unlike D-efficiency, which attempts to minimize the confidence region for estimated partworths. Without loss of generality, we scale all partworths in the figures to be nonnegative and bounded from above. Following conven-

[1]This is equivalent to maximization of the pth root of the determinant of Σ^{-1}. Other norms include A-efficiency, which maximizes the trace of Σ^{-1}/p, and G-efficiency, which maximizes the maximum diagonal element of Σ^{-1}.

[2]The Huber–Zwerina (1996) and Arora–Huber (2001) algorithms maximize $\det\Sigma^{-1}$ on the basis of mean partworths by assuming that the mean is known from pretest data (or managerial judgment). Sándor and Wedel (2001) also include a prior covariance matrix in their calculations. They then maximize the expectation of $\det\Sigma^{-1}$, for which the expectation is over the prior subjective beliefs.

tion, we arbitrarily set the partworth associated with the least-preferred level to zero.[3]

Suppose that we have already asked i – 1 stated-choice questions and that the hexagon (polyhedron) in Figure 2 represents the partworth vectors that are consistent with the respondent's answers. Suppose also that the ith question asks the respondent to choose between two profiles with feature levels $\vec{z}_{i1}$ and $\vec{z}_{i2}$. If there were no response errors, the respondent would select Profile 1 whenever $(z_{i11} - z_{i21})u_1 + (z_{i12} - z_{i22})u_2 \geq 0$, where z_{ijf} refers to the fth feature of z_{ij}, and u_f denotes the partworth associated with the fth feature. This inequality constraint defines a separating line or, in higher dimensions, a separating hyperplane. In the absence of response errors, if the respondent's true partworth vector is above the separating hyperplane, the respondent chooses Profile 1; if it is below, the respondent chooses Profile 2. Thus, the respondent's choice of profiles updates knowledge of which partworth vectors are consistent with the respondent's preferences, thus shrinking the feasible polyhedron.

Selecting Questions

Questions are more informative if they reduce the feasible region more rapidly. To implement this goal, we adopt four criteria. First, neither profile in the choice set should dominate the other profile. Otherwise, no information is gained when the dominating profile is chosen, and the partworth space is not reduced. Second, the separating hyperplane should intersect with and divide the feasible region derived from the first i – 1 questions. Otherwise, there could be an answer to the ith question that does not reduce the feasible region. A corollary of the criteria is that for each profile, there must be a point in the feasible region for which that profile is the preferred profile.

The ith question is more informative if, given the first i – 1 questions, the respondent is equally likely to select each of the J_i profiles. This implies that, a priori, all answers to the ith question should be approximately equally likely. This leads to the third criterion, which we label "choice balance." Choice balance shrinks the feasible region as rapidly as is feasible. For example, if the points in the feasible region are equally likely (on the basis of i – 1 questions), the predicted likelihood, π_{ij}, of choosing the jth region is proportional to the size of the region. The expected size of the region after the ith question is then proportional to $\Sigma_{j=1}^{2}\pi_{ij}$, which is minimized when $\pi_{ij} = ½$.[4] Arora and Huber (2001, p. 275) offer a further motivation for choice balance based on D-efficiency. For two product profiles, the inverse covariance matrix, Σ^{-1}, is proportional to a weighted sum of $\pi_{ij}(1 - \pi_{ij})$, which is also maximized for $\pi_{ij} = ½$.

The choice-balance criterion will hold approximately if we favor separating hyperplanes that pass through the center of the feasible polyhedron and cut the feasible region approximately in half. This is illustrated in Figure 3, Panel A, where we favor bifurcation cuts relative to "sliver" cuts that yield unequally sized regions. If the separating hyperplane is a bifurcation cut, both the nondomination and the feasibility criteria are automatically satisfied.

However, not all bifurcation cuts are equally robust. Suppose that the current feasible region is elongated, as in Figure 3, Panel B, and we must decide among many separating hyperplanes, two of which are illustrated. One cuts along the long axis and yields long, thin feasible regions, and the other cuts along the short axis and yields feasible regions that are more symmetric. The long-axis cut focuses precision where there already is high precision, whereas the short-axis cut focuses precision where there now is less precision. For this reason, we prefer short-axis cuts to make the postchoice feasible regions reasonably symmetric. We can also motivate this fourth criterion relative to D-efficiency. D-efficiency minimizes the geometric mean of the axes of the confidence ellipsoid, a criterion that tends to make the confidence ellipsoids more symmetrical.

For two profiles, we favor the four criteria of nondominance, feasibility, choice balance, and postchoice symmetry if we select profiles such that the separating hyperplanes (1) pass through the center of the feasible region and (2) are perpendicular to the longest "axis" of the feasible polyhedron, as defined by the first i – 1 stated choices. To implement these criteria, we propose the following heuristic algorithm:

Step 1. Find the center and the longest axis of the polyhedron on the basis of i – 1 questions.
Step 2. Find the two intersections between the longest axis and the boundary of the polyhedron.

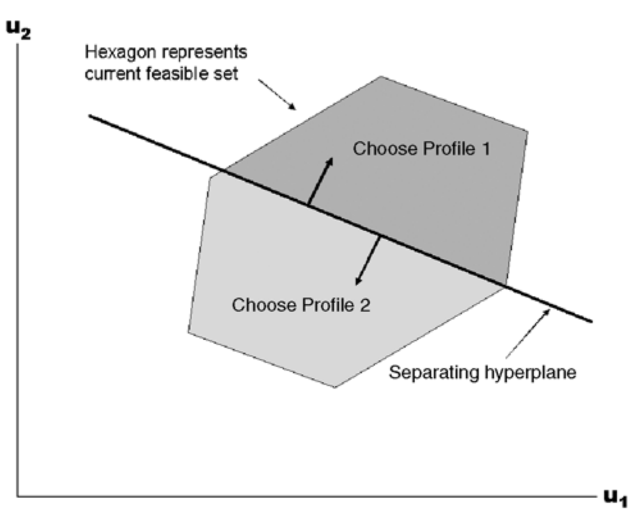
Each intersection point is defined by a partworth vector, and the difference between the vectors defines a hyperplane that is perpendicular to the longest axis. Because respondents choose from profiles rather than partworth vectors, a third step is needed to identify profiles that yield this separating hyperplane:

Step 3. Select a profile that corresponds to each intersection point such that the separating hyperplane divides the region into two approximately equal subregions.

The basic intuition remains the same when this heuristic algorithm is extended to more than two profiles ($J_i > 2$),

[3]In an application we describe subsequently, we use warm-up questions to identify the lowest level of each feature (a common solution to the issue).

Figure 2
STATED-CHOICE RESPONSES DIVIDE THE FEASIBLE REGION

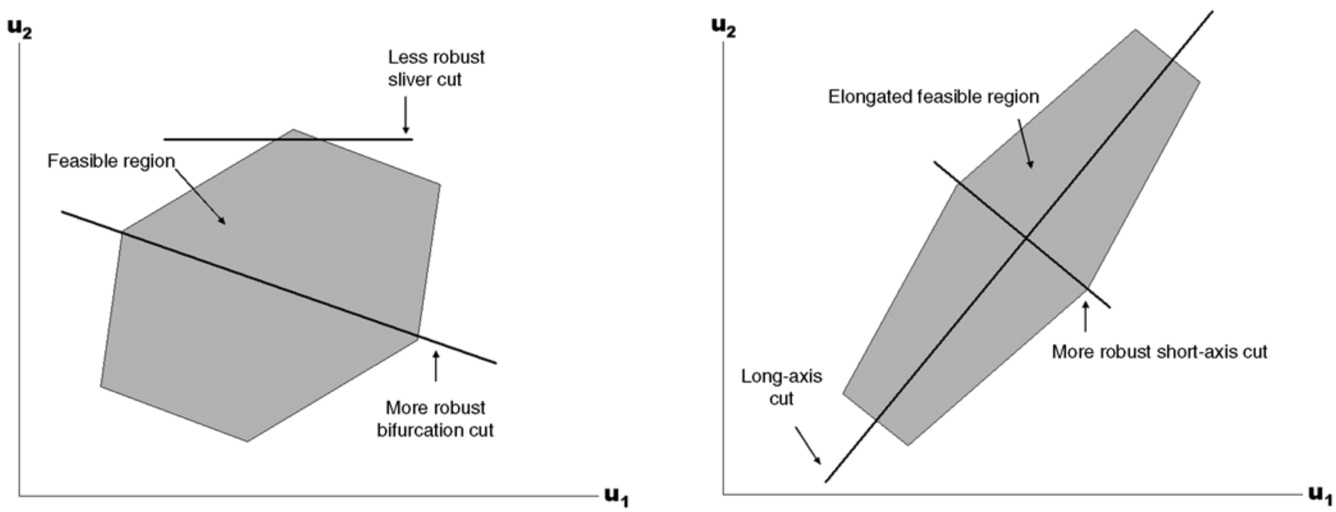


[4]For J_i profiles, equally sized regions also maximize entropy, defined as $-\Sigma_j\pi_{ij}\log\pi_{ij}$. Formally, maximum entropy is equal to the total information obtainable in a probabilistic model (Hauser 1978, p. 411, Theorem 1).

Figure 3

COMPARING CUTS AND THE RESULTING FEASIBLE REGIONS

A: Bifurcation Cuts *B: Short- Versus Long-Axis Cuts*



though the geometry becomes more difficult to visualize (some hyperplanes become oblique, but the regions remain equally probable). We address this generalization and then describe several implementation issues, including how we use utility maximization to select the profiles in Step 3.

Selecting More Than Two Profiles

In a choice task with more than two profiles, the respondent's choice defines more than one separating hyperplane. The hyperplanes that define the ith feasible region depend on the profile chosen by the respondent. For example, consider a choice task with four product profiles, labeled 1, 2, 3, and 4. If the respondent selects Profile 1, the respondent prefers Profile 1 to Profile 2, Profile 1 to Profile 3, and Profile 1 to Profile 4. This defines three separating hyperplanes: The resulting polyhedron of feasible partworths is the intersection of the associated regions and the prior feasible polyhedron. In general, J_i profiles yield $J_i(J_i - 1)/2$ possible hyperplanes. For each of the J_i choices available to the respondent, $J_i - 1$ hyperplanes contribute to the definition of the new polyhedron. The full set of hyperplanes and their association with stated choices define a set of J_i convex regions, one associated with each answer to the stated-choice question.

We extend Steps 1 to 3 as follows: Rather than find the longest axis, we find the $(J_i/2)$ longest axes and identify the J_i points at which the $(J_i/2)$ longest axes intersect the polyhedron. If J_i is odd, we select randomly among the vectors that intersect the $(J_i/2)$th longest axis. We associate profiles with each of the J_i partworth vectors by solving the respondent's maximization problem for each vector (as we describe next). Our solution to the maximization problem ensures that the hyperplanes pass (approximately) through the center of the polyhedron. The approximation arises because we design the profiles from a discrete attribute space. They would pass exactly through the analytic center if the attribute space were continuous.

It is easy to show that such hyperplanes divide the feasible region into J_i collectively exhaustive and mutually exclusive convex subregions of approximately equal size (except for the regions' "indifference" borders, which have zero measure). Nondominance and feasibility remain satisfied, and the resulting regions tend toward symmetry. Because the separating hyperplanes are defined by the profiles associated with the partworth vectors (Step 3), not the partworth vectors themselves (Step 2), some of the hyperplanes do not line up with the axes. For $J_i > 2$, the stated properties remain approximately satisfied on the basis of "wedges" formed by the $J_i - 1$ hyperplanes. We subsequently examine the effectiveness of the proposed heuristic for $J_i = 4$. Simulations examine overall accuracy, and an empirical test examines whether feasibility and choice balance are achieved for real respondents.

Implementation

Implementation of this heuristic raises challenges. Although it is easy to visualize (and implement) the heuristic with two profiles in two dimensions, practical CBC problems require implementation with J_i profiles in large p-dimensional spaces with p-dimensional polyhedra and $(p - 1)$-dimensional hyperplane cuts. Furthermore, the algorithm should run sufficiently fast so that there is little noticeable delay between questions.

The first challenge is finding the center of the current polyhedron and the $J_i/2$ longest axes (Step 1). If the longest axis of a polyhedron is defined as the longest line segment in the polyhedron, it is necessary to enumerate all vertices of the polyhedron and to compute the distances between the vertices. Unfortunately, for large p, this problem is computationally intractable (Gritzmann and Klee 1993); its solution would lead to lengthy delays between questions for each respondent. Furthermore, this definition of the longest axes of a polyhedron may not capture the intuitive concepts that we used to motivate the algorithm.

Instead, we turn to Sonnevend's (1985a, b) theorems, which state that the shape of polyhedra can be approximated with bounding ellipsoids centered at the "analytic center" of the polyhedron. The analytic center is the point that maximizes the geometric mean of the distances to the boundaries.

Freund (1993) provides efficient algorithms to find the analytic centers of polyhedra. When the analytic center has been found, Sonnevend's results provide analytic expressions for the ellipsoids. The axes of ellipsoids are well defined and capture the intuitive concepts in the algorithm. The longest axes are found with straightforward eigenvalue computations, for which there are many efficient algorithms. With well-defined axes, it is simple to find the partworth vectors on the boundaries of the feasible set that intersect the axes (Step 2). We provide technical details in the Appendix.

To implement Step 3, we must define the respondent's utility maximization problem. We do so in an analogy to economic theory. For each of the J_i utility vectors on the boundary of the polyhedron, we obtain the jth profile, $\vec{z}_{ij}$, by solving

$$\text{(OPT1)}\ \max \vec{z}_{ij}\vec{u}_{ij},\ \text{subject to } \vec{z}_{ij}\vec{c} \le M,\ \text{elements of } \vec{z}_{ij} \in \{0, 1\},$$

where $\vec{u}_{ij}$ is the utility vector chosen in Step 2, $\vec{c}$ are costs of the features, and M is a budget constraint. We implement (approximate) choice balance by setting $\vec{c}$ equal to the analytic center of the feasible polyhedron ($\bar{u}_{i-1}$) computed after the first i – 1 questions. At optimality, the constraint in OPT1 will be approximately binding, which implies that $\vec{z}_{ij}\bar{u}_{i-1} \cong \vec{z}_{ik}\bar{u}_{i-1} \cong M$ for all k ≠ j. (This may be approximate because of the integrality constraints in OPT1 [elements of $\vec{z}_{ij} \in \{0, 1\}$].) This ensures that the $J_i(J_i - 1)/2$ separating hyperplanes pass (approximately) through the analytic center. The binding constraints are all bifurcations, which tend to make the regions approximately equal in size. Finally, we also know that the solution to OPT1 ensures that each separating hyperplane passes through the feasible polyhedron because each profile is preferred at the utility vector to which it corresponds.

Solving OPT1 for profile selection (Step 3) is a knapsack problem that is well studied and for which efficient algorithms exist. We randomly draw the arbitrary constant M from a compact set (up to m times) until all profiles in a stated-choice task are distinct. If the profiles are not distinct, we use those that are distinct. If none of the profiles are distinct, we ask no further questions (in practice, this is a rare occurrence in both simulation and empirical situations).

OPT1 also illustrates the relationship between choice balance and utility balance, a criterion in aggregate customization. In our algorithm, the J_i profiles are chosen to be equally likely on the basis of data from the first i – 1 questions. In addition, for the partworths at the analytic center of the feasible region, the utilities of all profiles are approximately equal. However, utility balance only holds at the analytic center, not throughout the feasible region. Thus, although, a priori, the profiles are equally likely to be chosen, it is rare that the respondent is indifferent among the profiles. Thus, choice balance is unlikely to lead to respondent fatigue, and we observed none in our empirical application.

We illustrate the algorithm for J_i = 4 with the two-dimensional example in Figure 4. We begin with the current polyhedron of feasible partworth vectors (Figure 4, Panel A). We then use Freund's (1993) algorithm to find the analytic center of the polyhedron, as illustrated by the black dot in Figure 4, Panel A. We next use Sonnevend's (1985a, b) formulas to find the equation of the approximating ellipsoid and to obtain the $J_i/2$ longest axes (Figure 4, Panel B), which correspond to the $J_i/2$ smallest eigenvalues of the matrix that defines the ellipsoid. We then identify J_i target partworth vectors by finding the intersections of the $J_i/2$ axes with the boundaries of the current polyhedron (Figure 4, Panel C). Finally, for each target utility vector, we solve OPT1 to identify J_i product profiles. The J_i product profiles each imply J_i – 1 hyperplanes (illustrated for Profile 1 in Figure 4, Panel D). A respondent's choice of Profile 1 implies a new smaller polyhedron defined by the separating hyperplanes. As drawn in Figure 4, Panel D, one of the hyperplanes is redundant, which is less likely in higher dimensions. Were we to draw all $J_i(J_i - 1)/2 = 6$ hyperplanes, they would divide the polyhedron into mutually exclusive and collectively exhaustive convex regions of approximately equal size. We continue for q questions or until OPT1 no longer yields distinct profiles.

Incorporating Managerial Constraints and Other Prior Information

Previous research suggests that prior constraints enhance estimation (Johnson 1999; Srinivasan and Shocker 1973). For example, self-explicated data might constrain the rank order of partworth values across features. Such constraints are easy to incorporate and shrink the feasible polyhedron. Most conjoint analysis studies use multilevel features, some of which are ordinal scaled (e.g., picture quality). For example, if u_{fm} and u_{fh} are the medium and high levels of feature f, we add the constraint $u_{fm} \le u_{fh}$ to the feasible polyhedron.[5] We similarly incorporate information from managerial priors or pretest studies.

Response Errors

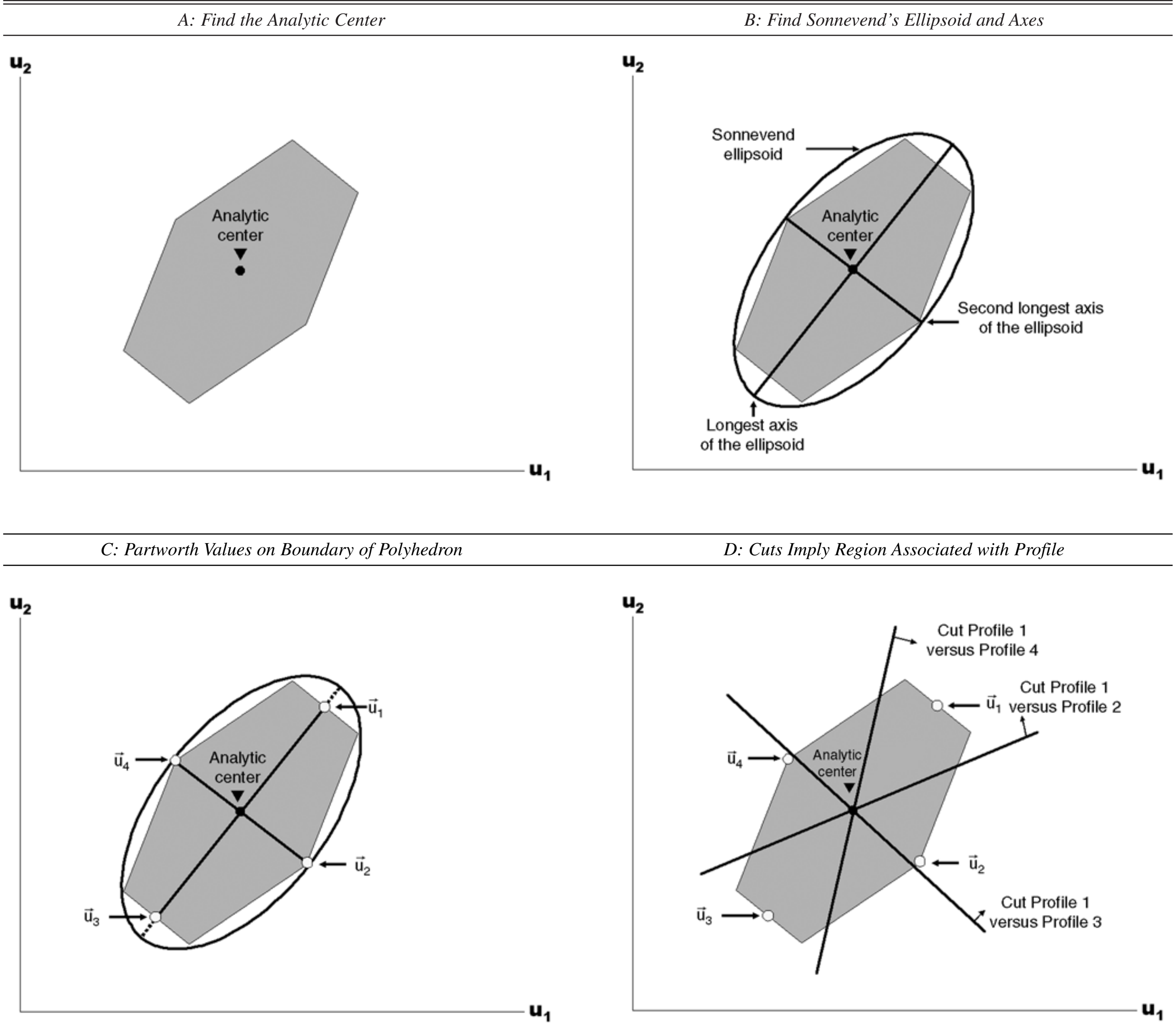
In real questionnaires, there are likely response errors in stated choices. When there are response errors, the separating hyperplanes are approximations rather than deterministic cuts. For this and other reasons, we distinguish question selection and estimation. The algorithm we propose is a question-selection algorithm. After we collect the data, we can estimate the respondents' partworths with most established methods, which address response error formally. For example, polyhedral questions can be used with classical random-utility models or HB estimation. It remains an empirical question as to whether response errors counteract the potential gains in question selection due to individual-level adaptation. Although we hypothesize that the criteria of choice balance and symmetry lead to robust stated-choice questions, we also hypothesize that individual-level adaptation works better when response errors are smaller. We examine these issues in the next section.

Analytic Center (AC) Estimation

The analytic center of the ith feasible polyhedron provides a natural summary of the information in the first i stated-choice responses. This summary measure is a good working estimate of the respondent's partworth vector. It is a natural by-product of the question-selection algorithm and is available as soon as each respondent completes the ith stated-choice question. Such estimates might also be used as starting values in HB estimation, as estimates in classical Bayes updating, and as priors for aggregate customization.

[5]In the theoretical derivation, we used binary features without loss of generality for notational simplicity. An ordinal multilevel feature constraint is mathematically equivalent to a constraint that links two binary features.

Figure 4
BOUNDING ELLIPSOIDS AND THE ANALYTIC CENTER OF THE POLYHEDRA



The AC estimates also provide a means to test the ability of the polyhedral algorithm to implement the feasibility and choice-balance criteria. Specifically, if the ith AC estimate is used to forecast choices for q > i choice sets, it should predict 100% of the first i choices (feasibility) and $(1/J_i)$ percent of the last q – i choices (choice balance). When J_i does not vary with i, the internal predictive percentage should approximately equal $[i + (1/J_i)(q - i)]/q$. We examine this statistic in the empirical application in a subsequent section.

The AC estimate can be given a statistical interpretation if we assume that the probability of a feasible point is proportional to its distance to the boundary of the feasible polyhedron. In this case, the analytic center maximizes the likelihood of the point (geometric mean of the distances to the boundary).

Analytic center estimates each respondent's partworth vectors on the basis of data from only that respondent. This advantage is also a disadvantage because, unlike HB estimation, AC estimation does not use information from other respondents. This suggests an opportunity to improve the accuracy of AC estimates through the use of data from the population distribution of partworths. Although the full development of such an analytic center algorithm is beyond the scope of this article, we can test its potential by using the (known) population distribution as a Bayesian prior to update AC estimates. We hypothesize that AC estimates are less accurate than HB estimates when the respondents are homogeneous but that this disadvantage can be offset with the development of an AC–Bayesian hybrid.

Incorporating Null Profiles

Many researchers prefer to include a null profile as an additional profile in the choice set (as in Figure 1). Polyhedral concepts generalize readily to include null profiles. If the null profile is selected from choice set i, we add the following constraints: $\vec{z}_{ij}\vec{u} \leq \vec{z}_k^*\vec{u} \quad \forall j, k \neq i$, where $\vec{z}_k^*$ denotes the profile chosen from choice set k (given that the null profile was not chosen in choice set k). Intuitively, the constraints recognize that if the null profile is selected in one choice set, all of the alternatives in that choice set have a lower utility than do the profiles selected in other choice sets (excluding other choice sets in which the null was chosen). We also can expand the parameter set to include the partworth of an outside option and write the appropriate constraints. After incorporation of the constraints, the question-design heuristic (and AC estimation) proceed as we previously described. We leave practical implementation, Monte Carlo testing, and empirical applications with null profiles to further research.

Metric-Paired-Comparison Questions

Polyhedral methods are also feasible for metric-paired-comparison questions. In particular, Toubia and colleagues (2003) propose a polyhedral method for metric-paired-comparison data that also uses Sonnevend's (1985a, b) ellipsoids. However, metric-pair and the CBC formats present fundamentally different challenges that result in different polyhedral algorithms. For example, each metric-pair question defines equality constraints, which reduce the dimensionality of the feasible polyhedron. In the CBC algorithm, the *inequality* constraints do not reduce the dimensionality of the feasible polyhedron. Furthermore, the metric-pairs polyhedron becomes empty after sufficient questions. The metric-pairs algorithm must revert to an alternative question-selection method and the metric-pairs AC algorithm must address infeasibility. In the CBC algorithm, the polyhedron always remains feasible. Moreover, because the metric-pairs algorithm identifies the partial profiles directly, the utility-maximization knapsack algorithm is new to the CBC algorithm.

Summary

Polyhedral (ellipsoid) algorithms provide a means to adapt stated-choice questions for each respondent on the basis of the respondent's answers to the first i – 1 questions. The algorithms are based on the intuitive criteria of nondominance, feasibility, choice balance, and symmetry, and they represent an individual-level analogy to D-efficiency. Specifically, the polyhedral algorithm focuses questions on what is *not* known about the partworth vectors, and it does so by seeking a small feasible region. Choice balance, symmetry, and the shrinking ellipsoid regions provide analogies to D-efficiency, which seeks questions to minimize the confidence ellipsoid for maximum likelihood estimates.

Although both polyhedral question design and aggregate customization are compatible with most estimation methods, including AC estimation, the two methods represent a key trade-off. Polyhedral question design adapts questions for each respondent but may be sensitive to response errors. Aggregate customization uses the same design for all respondents but is based on prior statistical estimates that take response errors into account. This leads us to hypothesize that polyhedral methods have their greatest advantages over existing methods (question design and/or estimation) when responses are more accurate and/or when respondents' partworths are more heterogeneous. We next examine individual-level adaptation and AC estimation with Monte Carlo experiments.

MONTE CARLO EXPERIMENTS

We use Monte Carlo experiments to investigate whether polyhedral methods show sufficient promise to justify further development and to identify the empirical domains in which the potential is greatest. Monte Carlo experiments are widely used to evaluate conjoint analysis methods, including studies of interactions, robustness, continuity, attribute correlation, segmentation, new estimation methods, and new data-collection methods. In particular, they have proved particularly useful in the first tests of aggregate customization and in establishing domains in which aggregate customization is preferred to orthogonal designs. Monte Carlo experiments offer several advantages for an *initial* test of new methods. First, as with any heuristic, computational feasibility needs to be established. Second, Monte Carlo experiments enable the exploration of many domains and the control of the parameters that define those domains. Third, other researchers can readily replicate and extend Monte Carlo experiments, thereby facilitating further exploration and development. Finally, Monte Carlo experiments enable the control of the "true" partworth values, which are unobserved in studies with actual consumers.

However, Monte Carlo experiments are but the first step in a stream of research. Assumptions must be made about characteristics that are not varied, and the assumptions represent limitations. In this article, we explore domains that vary in terms of respondent heterogeneity, response accuracy (magnitude), estimation method, and question-design method. This establishes a 4×2^3 experimental design. We encourage subsequent researchers to vary other characteristics of the experiments.

Structure of the Simulations

For consistency with prior simulations, we adopt the basic simulation structure of Arora and Huber (2001), who varied response accuracy, heterogeneity, and question-design method in a 2^3 experiment that used HB estimation. Huber and Zwerina (1996) previously used the same structure to vary response accuracy and question design with classical estimation, and more recently Sándor and Wedel (2001) used a similar structure to compare the impact of prior beliefs.

The Huber–Zwerina (1996) and Arora–Huber (2001) algorithms were aggregate customization methods based on relabeling and swapping. The algorithms work best for stated-choice problems in which relabeling and swapping are well defined. We expand Arora and Huber's design to include four levels of four features for four profiles, which ensures that complete aggregate customization and orthogonal designs are possible. Sándor and Wedel (2001) include cycling, though they note that cycling is less important in designs in which the number of profiles equals the number of feature levels.[6]

Within a feature, Arora and Huber (2001) choose partworths symmetrically with expected magnitudes of $-\bar{\beta}$, 0, and $+\bar{\beta}$. They vary response accuracy by varying $\bar{\beta}$. Larger

[6]In the designs we use, the efficiency of Sándor and Wedel's (2001) algorithm is approximately equal to the efficiency of Huber and Zwerina's (1996) algorithm.

$\bar{\beta}$s imply higher response accuracy because the variance of the Gumbel distribution, which defines the logit model, is inversely proportional to the squared magnitude of the partworths (Ben-Akiva and Lerman 1985, p. 105, Property 3). For four levels we retain the symmetric design with magnitudes of $-\bar{\beta}$, $-\frac{1}{3}\bar{\beta}$, $\frac{1}{3}\bar{\beta}$, and $\bar{\beta}$. Arora and Huber model heterogeneity by allowing partworths to vary among respondents according to normal distributions with variance, σ^2_{β}. They specify a coefficient of heterogeneity as the ratio of the variance to the mean. Specifically, they manipulate low response accuracy with $\bar{\beta}$ = .5 and high response accuracy with $\bar{\beta}$ = 1.5. They manipulate high heterogeneity with $\sigma^2_{\beta}/\bar{\beta}$ = 2.0 and low heterogeneity with $\sigma^2_{\beta}/\bar{\beta}$ = .5. Given these values, they draw each respondent's partworths from a normal distribution with a diagonal covariance matrix. Each respondent then answers the stated-choice questions, and probabilities are determined by a logit model based on the respondent's partworths. Arora and Huber compare question selection using root mean square error (RMSE). For comparability, we adopt the same criterion and report three more intuitive metrics.

We selected magnitudes and heterogeneity that represent the *range* of average partworths and heterogeneity that might be found empirically. Although we could find no meta-analyses for the values, we had data available to us from a proprietary CBC application (D-efficient design, HB estimation) in the software market. The study included data from approximately 1200 home consumers and more than 600 business customers. In both data sets, $\bar{\beta}$ ranged from approximately –3.0 to +2.4. We chose our high magnitude (3.0) from this study, recognizing that other studies might have even higher magnitudes. For example, Louviere, Hensher, and Swait (2000) report stated-choice estimates (logit analysis) in the range of 3.0 and higher.

After selecting $\bar{\beta}$ for high magnitudes, we set the low magnitude $\bar{\beta}$ to the level Arora and Huber (2001) chose. In the empirical data, the estimated variances ranged from .1 to 6.9, and the heterogeneity coefficient varied from .3 to 3.6.[7] To approximate this range and to provide symmetry with the magnitude coefficient, we manipulated high heterogeneity with a coefficient of three times the mean. Following Arora and Huber, we manipulated low heterogeneity as half the mean. We believe that these values are representative of those that might be obtained in practice. Recall that as a first test of polyhedral methods, we seek to identify domains that can occur in practice and for which polyhedral methods show promise. More important, these levels illustrate the directional differences among methods and thus provide insight for further development.

Experimental Design

In addition to manipulating magnitude (two levels) and heterogeneity (two levels), we manipulated estimation method (two levels) and question-design method (four levels). The estimation methods we used are HB and AC estimation. The former is well established and incorporates information from other respondents in each individual estimate; the latter is the only feasible method, of which we are aware, that provides individual-level estimates using information only from that respondent. The question-design methods are random, orthogonal designs with equally likely priors, aggregate customization (Arora and Huber 2001), and polyhedral methods. To simulate aggregate customization, we assumed that the pretest data were obtained without cost, and on the basis of this data, we applied the Arora–Huber algorithm. Specifically, we simulated an orthogonal "pretest" that used the same number of respondents as in the actual study. For the orthogonal design, we adopted the Arora–Huber methods as detailed by Huber and Zwerina (1996, pp. 310–12).

We set q = 16 so that orthogonal designs, relabeling, and swapping are well defined. Exploratory simulations suggest that the estimates become more accurate as we increase the number of questions, but the relative comparisons of question design and estimation for q = 8 and q = 24 provide similar qualitative insights.[8] For each combination of question-design method, estimation method, heterogeneity level, and magnitude level, we simulated 1000 respondents.[9]

Practical Implementation Issues

To implement the polyhedral algorithm, we made two implementation decisions: First, we randomly drew M up to thirty times (m = 30) for the simulations. We believe that the accuracy of the method is relatively insensitive to this decision. Second, because the polyhedron is symmetrical before the first question, we selected the first question by randomly choosing from among the axes.

Other decisions may yield greater (or lesser) accuracy; thus, the performance of the polyhedral methods tested in this article should be considered a lower bound on what is possible with further improvement. For example, further research might use aggregate customization to select the first question. We describe all polyhedral optimization, question selection, and estimation algorithms in the Appendix, and they are implemented in Matlab code. The Web-based application we describe subsequently uses PERL and HTML for Web page presentation.[10]

Comparative Results of the Monte Carlo Experiments

Table 1 reports four metrics that describe the simulation results in a format similar to that of Arora and Huber (2001): RMSE, the percentage of respondents for whom each question-design method has the lowest RMSE, the hit rate, and the average correlation between the true and estimated partworths. The hit rate measures the percentage of times each method predicts the most-preferred profile and is based on 1000 sets of holdout profiles. Tables 2 and 3 summarize the best question-design method and the best estimation method, respectively, for each metric in each domain. The

[7]There was also an outlier with a mean of .021 and a variance of .188, implying a heterogeneity coefficient of 9.0. Such cases are possible, but they are less likely to represent typical empirical situations. Researchers who prefer a unitless metric for heterogeneity can rescale using the standard deviation rather than the variance. For our two-level manipulation, directional differences are the same.

[8]The estimates at q = 16 are approximately 25% more accurate than those at q = 8, and the estimates at q = 24 are approximately 12% more accurate than those at q = 16.

[9]The simulations are based on ten sets of 100 respondents (details are available from the authors). To reduce unnecessary variance, we used the same true partworths for each of the 1000 respondents for each question-design method.

[10]All code (and the orthogonal design) is available at http://mitsloan.mit.edu/vc and is open source.

Table 1
MONTE CARLO SIMULATION RESULTS

			RMSE		Percentage Best		Hit Rates		Correlations	
Magnitude	*Heterogeneity*	*Question Design*	*HB*	*AC*	*HB*	*AC*	*HB*	*AC*	*HB*	*AC*
Low	High	Random	.892*	1.116	26.9*	17.2	.538	.508	.656*	.506
		Orthogonal	.904	.950	21.7	22.1	.540	.526	.651*	.569
		Customized	.883*	.993	27.6*	26.9	.548*	.539*	.660*	.553
		Polyhedral	.928	.880*	23.8*	33.8*	.527	.538*	.632	.609*
Low	Low	Random	1.027	1.206	21.9	16.5	.421	.403	.589	.446
		Orthogonal	.964*	1.012*	29.6*	28.9	.438*	.425	.629*	.535*
		Customized	1.018	1.086	28.7*	37.9*	.423	.436*	.589	.523*
		Polyhedral	1.033	1.103	19.6	16.7	.418	.403	.587	.509
High	High	Random	.595	.812	23.5	15.0	.627*	.584	.813	.666
		Orthogonal	.815	.871	4.3	5.8	.590	.581	.715	.646
		Customized	.611	.891	31.9	10.0	.630*	.581	.802	.626
		Polyhedral	.570*	.542*	40.3*	69.2*	.626*	.636*	.819*	.760*
High	Low	Random	.446	.856	31.8	16.3	.750	.632	.903	.676
		Orthogonal	.692	.824	2.8	11.7	.668	.626	.804	.680
		Customized	.769	1.012	18.8	10.1	.612	.558	.698	.570
		Polyhedral	.418*	.571*	46.6*	61.9*	.761*	.704*	.912*	.798*

*Best or not significantly different from best at $p < .05$.

Notes: Lower values of RMSE reflect increased accuracy, and higher values on percentage best, hit rates, and correlations reflect increased accuracy. Text in italic bold indicates the best question-design method for each estimation method within an experimental domain (and any other methods that are not statistically different from the best method).

Table 2
COMPARISON SUMMARY FOR QUESTION DESIGN

Magnitude	*Heterogeneity*	*RMSE*	*Percentage Best*	*Hit Rates*	*Correlations*
Low	High	Random Customized Polyhedral	Polyhedral	Customized	Random Orthogonal Customized
Low	Low	Orthogonal	Orthogonal Customized	Orthogonal Customized	Orthogonal
High	High	Polyhedral	Polyhedral	Polyhedral	Polyhedral
High	Low	Polyhedral	Polyhedral	Polyhedral	Polyhedral

Table 3
COMPARISON SUMMARY FOR ESTIMATION

Magnitude	*Heterogeneity*	*RMSE*	*Percentage Best*	*Hit Rates*	*Correlations*
Low	High	AC HB	AC	HB	HB
Low	Low	HB	AC HB	AC HB	HB
High	High	AC	AC	AC	HB
High	Low	HB	HB	HB	HB

entries in Tables 2 and 3 correspond to the best overall method (question design × estimation) for each domain.

For comparability between estimation methods, we first normalized the partworths to a constant scale. Specifically, for each respondent, we normalized both the true partworths and the estimated partworths so that their absolute values summed to the number of parameters and their values summed to zero for each feature. In this manner, the RMSEs can be interpreted as a percentage of the mean partworths. Within an estimation method, subject to statistical confidence, this scaling does not change the relative comparisons among question-design methods, and it has the additional advantage of making the results roughly comparable in units for the different manipulations of magnitude (response accuracy) and heterogeneity. This scaling addresses two issues. First, AC estimation is unique to a positive linear transformation and thus focuses on the *relative* values of the partworths, as many managerial applications require. Second, unscaled logit analyses confound the magnitude of the stochasticity of observed choice behavior with the magnitude of the partworths. Our scaling enables us to focus on the relative partworths. For volumetric forecasts, we recommend the methods that Louviere, Hensher, and Swait (2000) propose and validate; these methods are well documented and reliable and have proved appropriate for matching disparate scales.

Question-Design Methods

As we hypothesized, polyhedral question design performs well in the high-magnitude (high-response-accuracy) domains. For these domains, polyhedral question design is best, or tied for best, for all metrics and for both estimation methods. These domains favor individual-level adaptation because the design of subsequent questions is based on more-accurate information from previous answers. When magnitudes are low, the greater response error works against individual adaptation, and polyhedral question design does not do as well.

The impact of heterogeneity is more complex. We expect that individual-level adaptation performs well when respondents are heterogeneous. When both magnitudes and heterogeneity are high, conditions favor polyhedral question design, and it performs well. In this domain, the accuracy of the data enables individual-level adaptation to be efficient. However, for low magnitudes, the disadvantages of high response errors appear to offset the need for customization (high heterogeneity). In this domain, polyhedral question design is best only for AC estimation, perhaps because the two polyhedral methods are complementary: Polyhedral question design shrinks the feasible region rapidly, thereby making AC estimation more accurate. We expect low heterogeneity to reduce the need for customization and low response accuracy (low magnitude) to work against deterministic customization. In this domain, as we predicted, polyhedral methods do not perform as well as orthogonal and aggregate customized designs.

Perhaps a surprise in Table 1 is the strong performance of random designs compared with orthogonal designs, especially for HB estimation and when either magnitudes or heterogeneity are high. We believe that two phenomena explain this relative performance. First, orthogonal designs are optimal only when the partworths are zero. For higher magnitudes, orthogonal designs are farther from optimal (Arora and Huber 2001; Huber and Zwerina 1996). For example, when we compute D-errors for orthogonal and random designs in the high-magnitude domains, the D-errors are higher for orthogonal questions than for random questions. Second, HB uses interrespondent information effectively. As Sándor and Wedel (2003) illustrate, multiple designs are more likely to contribute incremental information about the population. The accuracy of random designs compared with that of fixed, orthogonal designs is consistent with simulations of differentiated designs as proposed by Sándor and Wedel (2003).

Finally, we note three aspects of Table 1. First, Table 1 replicates the Arora–Huber (2001) simulations when the domains and metric are matched: Arora and Huber use HB methods and report RMSE. In the Arora–Huber simulations, aggregate customization is superior to orthogonal designs when magnitudes are high and when heterogeneity is high. Second, there are ties in Table 1, especially for the low-magnitude and high-heterogeneity domain, perhaps because high heterogeneity favors customization whereas low magnitudes make customization more sensitive to errors. Even when we increase the sample size to 1500 (for RMSE), we are unable to break the ties. In this domain, for most practical problems, question design appears less critical. Third, performance varies slightly by metric, especially when there are ties and especially for low magnitudes.

Estimation Methods

The findings in Table 1 also facilitate the comparison of estimation methods. We already noted that AC performs well when matched with polyhedral questions for high heterogeneity. In most other domains (and for most metrics), HB is more accurate. In theory, the advantages of HB derive from several properties, one of which is the use of population-level data to moderate individual-level estimates (shrinkage). We expect that this is particularly beneficial when the population is homogeneous, because the population-level data provide more information about individual preferences in these domains. This is consistent with Table 1 and may help explain why the relative performance of AC improves when the population is more heterogeneous.

Before we reject AC estimation for the domains in which HB is superior, we investigate the theoretical potential to improve AC by replacing the AC estimate with a convex combination of the AC estimate and the population mean (shrinkage). If the population mean ($\bar{\beta}$), its variance (σ_{β}^2), and the accuracy of AC (RMSE) are known, classical Bayes updating provides a formula with which to implement shrinkage. To test shrinkage, we used the known population mean, its variance, and the RMSE from Table 1. With these values, we computed a single parameter for each domain, α, with which we weighed the population mean. Using these theoretical αs, a convex combination of AC and the population mean provides the best overall estimate in all four domains: RMSEs of .863, .871, .510, and .403, respectively, and the last three are significantly best at the .05 level. In practice, α is unknown, so it must be estimated from the data. Optimal estimation is beyond the scope of this article, but surrogates, such as use of either HB or classical logit analysis to estimate α, should perform well. We conclude that AC shows sufficient promise to justify further development.

Summary of Monte Carlo Experiments

We summarize the results of the Monte Carlo experiments as follows:

- Polyhedral question design shows the most promise when magnitudes are high (response errors are low).
- If magnitudes are low (response errors are high), it may be best not to customize designs; fixed orthogonal questions appear to be more accurate than polyhedral or customized methods.
- The HB estimation method performs well in all domains.
- The AC estimation method performs well when matched with polyhedral question design and when heterogeneity is high.
- The AC estimation method shows sufficient promise to justify further development. Preliminary analysis suggests that a modified Bayesian AC estimate is particularly promising if an optimal means can be found to estimate a single parameter, α.

As for many new technologies, we hope that polyhedral question design and AC estimation will improve further with use, experimentation, and evolution (Christensen 1998).

APPLICATION TO THE DESIGN OF EXECUTIVE EDUCATION PROGRAMS

Polyhedral methods for CBC have been implemented in at least one empirical application. We describe this application briefly as evidence that it is feasible to implement the proposed methods with actual respondents. Furthermore, the data provide empirical estimates of magnitude and heterogeneity, enable us to test the feasibility and choice-balance criteria, and provide insight on the convergence of AC and HB estimates. Because the managerial environment required that we implement a single question-design method, we could not compare question-design methods in this first proof-of-concept application.

Feature Design and Sample Selection for the Polyhedral CBC Study

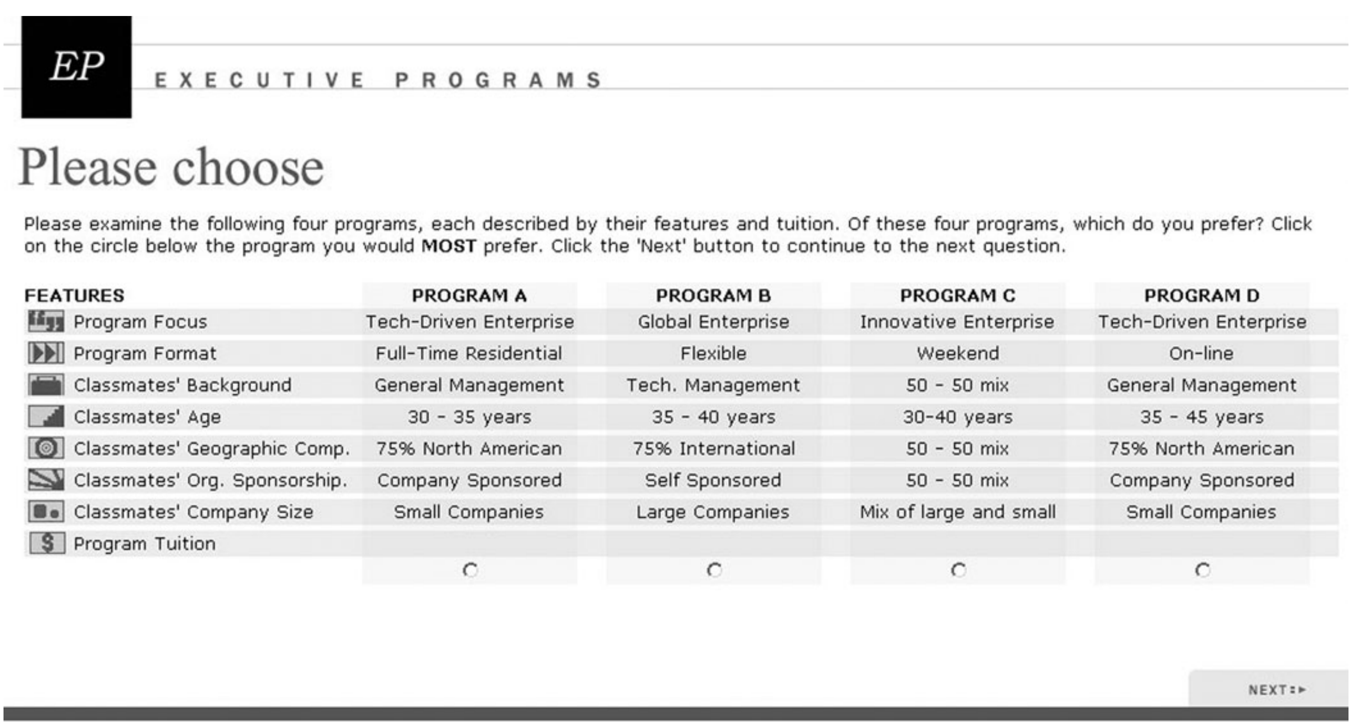
The application supported the design of new executive education programs for a business school at a major university. The school was the leader in 12-month executive advanced-degree programs and had a 50-year track record that has produced alumni who include chief executive officers, prime ministers, and a secretary general of the United Nations. However, the demand for 12-month programs was shrinking because it has become increasingly difficult for executives to be away from their companies for 12 months. The senior leadership of the school was considering a radical redesign of its programs. As part of the effort, the leadership sought input from potential students. On the basis of two qualitative studies and detailed internal discussions, the senior leadership of the school identified eight program features that were to be tested with conjoint analysis. The features included program focus (three levels), format (four levels), class composition (three levels of interest focus, four age categories, and three types of geographic focus), sponsorship strategy (three levels), company focus (three levels), and tuition (three levels). This $4^2 \times 3^6$ design is relatively large for CBC applications (e.g., Huber 1997; Orme 1999) but proved feasible with professional Web design. We provide an example screenshot in Figure 5 (the university logo and tuition levels are redacted). Before respondents answered the stated-choice questions, they reviewed detailed descriptions of the levels of each feature and could access the descriptions at any time by clicking the feature's logo.

After wording and layout were refined in pretests, potential respondents were obtained from the Graduate Management Admissions Council through its Graduate Management Admissions Search Service. Potential respondents were selected on the basis of their age, geographic location, educational goals, and Graduate Management Admission Test (commonly referred to as "GMAT") scores. Random samples were chosen from three strata: respondents who lived within (1) driving distance of the university, (2) a short airplane flight, and (3) a moderate airplane flight. Respondents were invited to participate by e-mail from the director of executive education. As an incentive, respondents were entered in a lottery in which they had a one-in-ten chance of receiving a university-logo gift worth approximately $100.

Pretests confirmed that respondents could comfortably answer 12 stated-choice questions (recall that respondents were experienced executives who received minimal response incentives). Of the respondents who began the CBC section of the survey, 95% completed the section. The overall response rate was within ranges that are expected for both

Figure 5

EXAMPLE OF WEB-BASED STATED-CHOICE TASK FOR EXECUTIVE EDUCATION STUDY


EP

EXECUTIVE PROGRAMS

Please choose

Please examine the following four programs, each described by their features and tuition. Of these four programs, which do you prefer? Click on the circle below the program you would **MOST** prefer. Click the 'Next' button to continue to the next question.

FEATURES	PROGRAM A	PROGRAM B	PROGRAM C	PROGRAM D
Program Focus	Tech-Driven Enterprise	Global Enterprise	Innovative Enterprise	Tech-Driven Enterprise
Program Format	Full-Time Residential	Flexible	Weekend	On-line
Classmates' Background	General Management	Tech. Management	50 - 50 mix	General Management
Classmates' Age	30 - 35 years	35 - 40 years	30-40 years	35 - 45 years
Classmates' Geographic Comp.	75% North American	75% International	50 - 50 mix	75% North American
Classmates' Org. Sponsorship.	Company Sponsored	Self Sponsored	50 - 50 mix	Company Sponsored
Classmates' Company Size	Small Companies	Large Companies	Mix of large and small	Small Companies
Program Tuition				

NEXT

proprietary and academic Web-based studies (Couper 2000; De Angelis 2001; Dillman et al. 2001; Sheehan 2001).[11] We found no significant differences between the partworths estimated for early responders and those estimated for later responders. The committee judged the results intuitive, but enlightening, and adequate for managerial decision making. On the basis of the conjoint study results and internal discussions, the school redesigned and retargeted its 12-month programs with a focus both on global leadership and on innovation and entrepreneurship. Beginning with the class of 2004, it will add a new, flexible format to its traditional offerings.

Technical Results

The data provide an opportunity to examine several technical issues. We estimated the magnitude and heterogeneity parameters using HB methods. We obtained estimates of magnitude ($\underline{\beta}$) that ranged from 1.0 to 3.4, with an average of 1.5. The heterogeneity coefficient ranged from .9 to 3.2, with an average of 1.9. The observed magnitude and the observed heterogeneity coefficients span the ranges addressed in the Monte Carlo simulations.

Hit rates are more complex. By design, polyhedral questions select the choice sets that provide maximum information about the feasible set of partworths. If the AC estimates remain feasible through the 12th question, they obtain an internal hit rate of 100% (by design). They remained feasible, and we achieved this internal hit rate. This hit rate is not guaranteed for HB estimates, which nonetheless perform quite well, with internal hit rates of 94%. As we described previously, we can examine internal consistency by comparing the hit rates based on AC estimates from the first i questions with their theoretical value, $[i + \frac{1}{4}(12 - i)]/12$. The fit is almost perfect (adjusted $R^2 = .9973$), suggesting that polyhedral question design was able to achieve excellent choice balance for the respondents in this study. Because the internal hit rates are not guaranteed for HB, we plot the hit rates for both estimation methods in Figure 6, Panel A. On this metric, HB is more concave than AC; it performs better for low i but not as well for high i.

To gain further insight, we adopt an evaluation method Kamakura and Wedel (1995, p. 316) use to examine how rapidly estimates converge to their final values. Following their structure, we compute the convergence rates as a function of the number of stated-choice tasks (i) by using scaled RMSE to maintain consistency with both Arora and Huber (2001) and the Monte Carlo simulations. From the third question onward, AC and HB are quite close, achieving roughly equal convergence. The HB method does not perform as well for i = 1 and 2, most likely because individual-level variation (in a population with moderately high heterogeneity) counterbalances the benefit to HB of the population-level data.

On the basis of this initial application, we conclude that adaptive polyhedral choice-based questions are practical and achieve both feasibility and choice balance. The AC estimation method appears to be comparable to the HB estimation method and deserves further study, perhaps with the Bayesian hybrids we suggested previously.

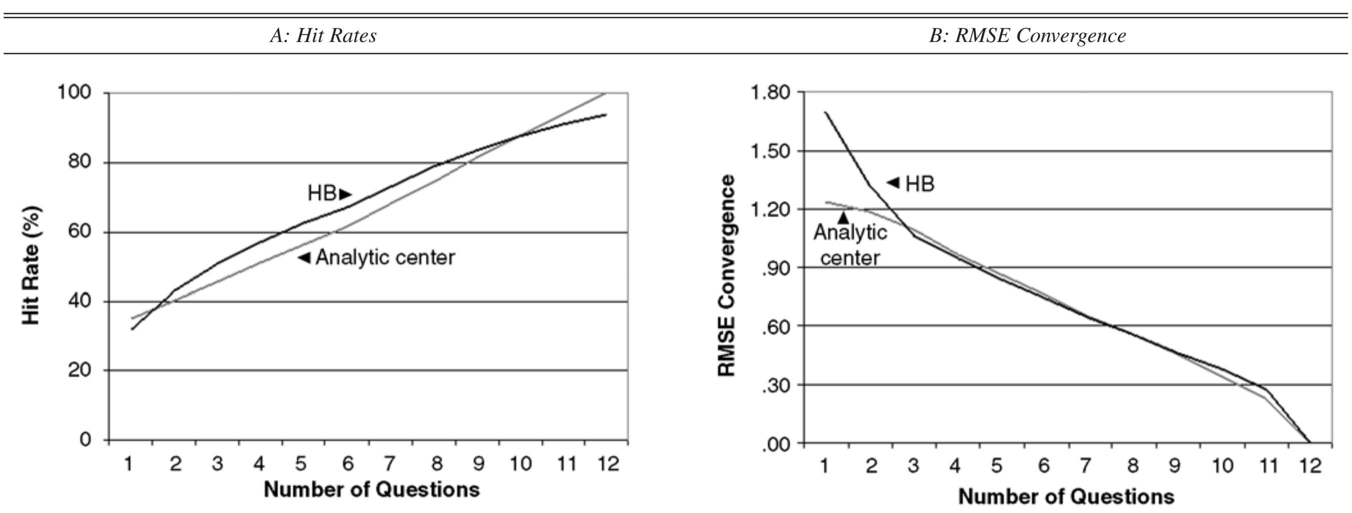
CONCLUSIONS AND RESEARCH OPPORTUNITIES

Research on stated-choice question design suggests that careful selection of choice sets has the potential to increase accuracy and to reduce costs because it requires fewer respondents, fewer questions, or both. This is particularly true in CBC, because the most efficient design depends on the true partworth values. In this article, we explore whether the success of aggregate customization can be extended to individual-level adaptive question design. We propose heuristics for designing profiles for each choice set. We then rely on new developments in dynamic optimization to

[11]Couper (2000, p. 384) estimates a 10% response rate for open-invitation studies and a 20%–25% response rate for studies with prerecruited respondents. De Angelis (2001) reports click-through rates of 3%–8% from an e-mail list of eight million records. In a study designed to optimize response rates, Dillman and colleagues (2001) compare mail, telephone, and Web-based surveys. They obtain a response rate of 13% for the Web-based survey. Sheehan (2001) reviews all published studies cited in Academic Search Elite, Expanded Academic Index, ArticleFirst, Lexis-Nexis, Psychlit, Sociological Abstracts, ABI-Inform, and ERIC and finds that response rates are dropping at 4% per year. Sheehan's data suggest an average response rate for 2002 of 15.5%. The response rate in the executive education study was 16%.

Figure 6

EMPIRICAL HIT RATES AND RMSE CONVERGENCE FOR EXECUTIVE EDUCATION STUDY



implement the heuristics. As a first test, we seek to identify whether the proposed methods show promise in at least some domains. It appears that such domains exist. As with many proposed methods, we do not expect polyhedral methods to dominate in all domains, and indeed they do not. However, we hope that by identifying promising domains we can inspire other researchers to explore hybrid methods and/or to improve the heuristics.

Although polyhedral methods are feasible empirically and show promise, many challenges remain. For example, we might allow fuzzy constraints for the polyhedra. Such constraints might provide greater robustness at the expense of precision. Future simulations might explore other domains, including nondiagonal covariance structures, probit-based random-utility models, mixtures of distributions, and finite mixture models. Recently, ter Hofstede, Kim, and Wedel (2002) demonstrated that self-explicated data can improve HB estimation for full-profile conjoint analysis. Polyhedral estimation handles such data readily; hybrids might be explored that incorporate both self-explicated and stated-choice data. Future developments in dynamic optimization might enable polyhedral algorithms that look several steps ahead. We close by recognizing research on other optimization algorithms for conjoint analysis. Evgeniou, Boussios, and Zacharia (2003) propose support vector machines to balance complexity of interactions with fit. They are currently exploring hybrids based on support vector machines and polyhedral methods.

APPENDIX
MATHEMATICS OF POLYHEDRAL METHODS FOR CBC

We designed this appendix to be self-contained. In their metric-pairs algorithm, Toubia and colleagues (2003) present related math programming that is involved in finding an interior point, the analytic center, and the Sonnevend (1985a, b) ellipsoid in detail. We include the modified math programming formulations here for completeness. We caution readers that there are important differences between the stated-choice formulations as we detail in this article and the metric-pair formulations that Toubia and colleagues present. The stated-choice algorithm and the knapsack problem do not arise in the metric-pairs setting.

Definitions and Assumptions

It is helpful to begin with several definitions:

u_f = the fth parameter of the respondent's partworth function, where $u_f \geq 0$ is the high level of the fth feature (we assume that there are binary features without loss of generality) and $\sum_{f=1}^{p} u_f = 100$;
p = the number of (binary) features;
$\vec{u}$ = the $p \times 1$ vector of parameters;
r = the number of externally imposed constraints, of which $r' \leq r$ are inequality constraints;
$\vec{z}_{ij}$ = the $1 \times p$ vector that describes the jth profile in the ith choice set, where j = 1 indexes the respondent's choice from each set; and
X = the $q(J - 1) \times p$ matrix of $\vec{x}_{ij} = \vec{z}_{i1} - \vec{z}_{ij}$, for i = 1 to q and for j = 2 to J (to simplify notation, we drop the i subscript from J_i).

We incorporate inequality constraints by adding slack variables. For example, if there are multiple levels and $u_m \leq u_h$, then $u_h = u_m + v_{hm}$, and $v_{hm} \geq 0$. If there are no errors, the respondent's choices would imply $X\vec{u} \geq \vec{0}$, where $\vec{0}$ is a vector of zeros. We add slack variables and augment $\vec{u}$ such that $X\vec{u} = \vec{0}$. We incorporate the additional constraints by augmenting these equations so that $\vec{u}$ and X include r′ additional slack variables and r additional equations. This forms a polyhedron $P_{CBC} = \{\vec{u} \in \Re^{p + q(J-1) + r'} | X\vec{u} = \vec{a},\ \vec{u} \geq \vec{0}\}$, where $\vec{a}$ contains nonzero elements because of the external constraints. We begin by assuming that P_{CBC} is nonempty, that X is full rank, and that no j exists such that $u_f = 0$ for all $\vec{u}$ in P_{CBC}.

Interior-Point Math Program

To find a feasible interior point, we solve the following linear program (see Freund, Roundy, and Todd 1985):

$$\text{(A1)} \qquad \max_{\vec{u},\vec{y},\theta} \sum_{f=1}^{p+q(J-1)+r'} y_f, \quad \text{subject to } X\vec{u} = \theta\vec{a},\ \theta \geq 1,$$

$$\vec{u} \geq \vec{y} \geq \vec{0}, \text{ and } \vec{y} \leq \vec{e},$$

where $\vec{e}$ is a vector of ones. Let $(\vec{u}^*, \vec{y}^*, \theta^*)$ denote a solution. If $\vec{y}^* > \vec{0}$, then $\theta^{*-1}\vec{u}^*$ is an interior point of P_{CBC}. If $y_f^* = 0$, then $u_f = 0$ for all $\vec{u} \in P_{CBC}$. If the linear program is infeasible, P_{CBC} is empty.

Analytic Center Math Program

We solve the following math program:

$$\text{(A2)} \quad \max \sum_{f=1}^{p+q(J-1)+r'} \ln(u_f), \quad \text{subject to } X\vec{u} = \vec{a}, \text{ and } \vec{u} > \vec{0}.$$

We solve the program using an algorithm developed by Freund (1993), which begins with the feasible point $\vec{u}^0$ that was found previously. At each iteration, we set $\vec{u}^{t+1} = \vec{u}^t + \alpha^t \vec{d}^t$, where we find $\vec{d}^t$ using the following quadratic approximation of the objective function:

$$\text{(A3)} \qquad \sum_{f=1}^{p+q(J-1)+r'} \ln(u_f + d_f^t) \approx \sum_{f=1}^{p+q(J-1)+r'} \ln(u_f) + \sum_{f=1}^{p+q(J-1)+r'} \left(\frac{d_f^t}{u_f} - \frac{{d_f^t}^2}{2u_f^2}\right).$$

If U^t is a diagonal matrix of the $u^t{}_f$'s, then $\vec{d}^t$ solves

$$\text{(A4)} \quad \max \vec{e}^T(U^t)^{-1}\vec{d} - \left(\frac{1}{2}\right)\vec{d}^T(U^t)^{-2}\vec{d}, \quad \text{subject to } X\vec{d} = \vec{0}.$$

Using the Karush-Kuhn-Tucker conditions, we determine that $\vec{d}^t = \vec{u}^t - (U^t)^2X^T[X(U^t)^2X^T]^{-1}\vec{a}$. If $\|(U^t)^{-1}\vec{d}^t\| < .25$, then $\vec{u}^t$ is already close to optimal, and we set $\alpha^t = 1$. Otherwise, we find the optimal α^t with a line search. The program continues to convergence at $\vec{\bar{u}}$.

If P_{CBC} is empty, we employ the error modeling procedure that Toubia and colleagues (2003) present. However, note that P_{CBC} will not be empty if CBC questions are chosen with the polyhedral algorithm. If X is not full rank, $X(U^t)^2X^T$ might not invert. There are two practical solutions: (1) Select questions such that X is full rank, or (2) make X full rank by removing redundant rows (see Toubia et al. 2003). If, when searching for feasibility, we identify some f's for which $u_f = 0$ for all $\vec{u} \in P_{CBC}$, we can find the analytic center of the remaining polyhedron by removing those f's and by setting $u_f = 0$ for those indexes.

The Longest Axes of the Sonnevend Ellipsoid

If $\vec{\bar{u}}$ is the analytic center and $\bar{U}$ is the corresponding diagonal matrix, Sonnevend (1985a, b) demonstrates that $E \subseteq P_{CBC} \subseteq E_{p + q(J-1) + r'}$, where $E = \{\vec{u} \mid X\vec{u} = \vec{a}, \sqrt{[(\vec{u} - \vec{\bar{u}})^T \bar{U}^{-2} (\vec{u} - \vec{\bar{u}})]^{1/2}} \le 1\}$ and $E_{p + q(J-1) + r'}$ is constructed proportional to E by replacing 1 with $[p + q(J - 1) + r']$. Because we are interested only in the direction of the longest axes of the ellipsoids, we can work with the simpler of the proportional ellipsoids, E. Let $\vec{g} = \vec{u} - \vec{\bar{u}}$, such that the longest axis is a solution to

$$\text{(A5)} \qquad \max \vec{g}^T\vec{g}, \quad \text{subject to } \vec{g}^T\bar{U}^{-2}\vec{g} \le 1, \text{ and } X\vec{g} = \vec{0}.$$

Under the Karush-Kuhn-Tucker conditions, the solution to this problem is the eigenvector of the matrix, $[\bar{U}^{-2} - X^T(XX^T)^{-1}X\bar{U}^{-2}]$, associated with its smallest positive eigenvalue. The direction of the next longest axis is given by the eigenvector associated with the second smallest eigenvalue, and so on.

Profile Selection for Target Partworth Values

We select the values of the $\vec{u}_{ij}$'s for the subsequent question (i = q + 1) on the basis of the longest axes. Each axis provides two target values. For odd J, we randomly select from target values derived from the [(J + 1)/2]th eigenvector. To find the extreme estimates of the parameters, $\vec{u}_{ij}$, we solve for the points at which $\vec{u}_{i1} = \vec{\bar{u}} + \alpha_1\vec{g}_1$, $\vec{u}_{i2} = \vec{\bar{u}} - \alpha_2\vec{g}_1$, $\vec{u}_{i3} = \vec{\bar{u}} + \alpha_3\vec{g}_2$, and $\vec{u}_{i4} = \vec{\bar{u}} - \alpha_4\vec{g}_2$ intersect P_{CBC} (the generalization to $J \ne 4$ is straightforward). For each α, we increase α until the first constraint in P_{CBC} is violated. To find the profiles in the choice set, we select feature costs, $\vec{c}$, and a budget, M, as researcher-determined parameters. Without such constraints, the best profile is trivially the profile in which all features are set to their high levels. Subject to this budget constraint, we solve the following knapsack problem with dynamic programming:

$$\text{(OPT1)} \quad \max \vec{z}_{ij}\vec{u}_{ij}, \text{ subject to } \vec{z}_{ij}\vec{c} \le M, \text{ elements of } \vec{z}_{ij} \in \{0, 1\}.$$

For multilevel features, we impose constraints on OPT1, such that only one level of each feature is chosen. In the algorithms we have implemented to date, we have set $\vec{c} = \vec{\bar{u}}$ and drawn M from a uniform distribution on [0, 50], redrawing M (up to 30 times) until all four profiles are distinct. If distinct profiles cannot be identified, it is likely that P_{CBC} has shrunk sufficiently for the managerial problem. For null profiles, constraints should be extended accordingly, as we describe in text.

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