CONSUMER DURABLES: ACTUAL BUDGETS COMPARED TO VALUE PRIORITY MODEL - PRELIMINARY RESULTS AND MANAGERIAL IMPLICATIONS

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ABSTRACT

This paper compares actual consumer budget priorities to predictions from the value priority model. The actual consumer budgets are obtained via personal interviews in which consumers are asked to indicate which durable goods they plan to purchase in 1983, in 1984, and in 1985 and to prioritize their purchases within each budget year. The value priority model hypothesizes that consumers will prioritize goods according to "value", that is, "utility" per dollar.

In order to operationalize the model empirically, we estimate the "utility" of each item via a linear program (LP) that combines disparate data types into a convergent estimation procedure. The data types we use for estimation are (1) reservation price, the price at which a good leaves the budget; (2) purchase probability, the consumer's estimate of the likelihood he (she, or they) will actually purchase the good; (3) lottery order, an ordering of goods without regard to price; and (4) comparison lottery prizes, choices among combinations of goods. These utilities, when divided by price, allow a prediction of the priority order in the consumer's budget. The actual rank order budget priorities are compared to the estimated values to test the model.

The full data set contains three budgets (1983, 1984, 1985) for each of 170 consumers. This paper reports on the preliminary analysis of 23 budgets. Early results indicate that (a) the value-priority model is a reasonable descriptor/predictor of consumer durable purchasing behavior, (b) that convergent LP estimation is feasible and leads to new insights on data collection, and (c) that the best measure of utility varies by person, but overall, purchase probabilities appear to be the best data with which to estimate "utility" for the value-priority model.

PERSPECTIVE

In 1982, we published a model of consumer purchasing behavior for durable products, that is, for products that last many years, are expensive to purchase, and come in discrete units (Hauser and Urban 1982). In that paper, we attempted to merge ideas from economic theory and information processing theory to develop a model which was feasible to estimate and which would provide the basis for prelaunch forecasting of the sales of new consumer durable products.

In 1983, we undertook a major empirical study (with the cooperation of an American automobile manufacturer). The study provided the data necessary to estimate the parameters of the model and to test the model's predictions. That data collection is documented in Hauser, Roberts and Urban (1983).

This paper introduces the estimation procedures and provides a preliminary empirical test of the model. We believe both the estimation procedure and the empirical test are important contributions.

The estimation procedure uses linear programming (LP) to minimize a weighted sum of estimation errors. In this way, we can use multiple data sources, each based on a different aspect of the theory, to estimate the key forecasting parameters, consumer "utilities". By placing different weights on different types of error, we can place more, or less, emphasis on each data source. Unlike traditional methods, e.g., structural equations (LISREL) and path analysis, convergent LP estimation does not simply utilize comparable multiple measures linked in a nomological network. Instead, each measure is linked by theory through different equation systems to the construct of interest, which in this case is consumer "utility". Convergent LP estimation allows us to use disparate measures (e.g., probability scales and paired comparisons) directly in a unified estimation procedure. After estimating the utilities, predictive tests are carried out to test the adequacy of the estimators.

The preliminary empirical test of the model is important because of the growing scientific and managerial interest in consumer durable goods buying behavior. Scientific interest is strong because durable goods purchases must weigh inter-category comparisons (e.g., auto versus personal computers), budget effects, and multiple period decision making. Managerial interest is strong because understanding the effects of recessionary family budget constraints and of differential inflation across goods is critical for established products. In managing new durable goods, attention is high because new product development launch budgets for durables are large, (e.g., automobiles, may be as high as one billion dollars) and because most key strategic decisions must be made prior to new product introduction.

By comparing our model's predictions to actual consumer budgets, we provide some initial and evolutionary evidence toward a marketing theory of consumer durable purchasing.

THE BASIC MODEL

We begin by presenting the single period consumer model. We indicate briefly extensions to multiple periods including borrowing, savings, depreciation, operating costs, trade-ins, and interproduct complementarity. Details are in Hauser and Urban (1982).

The consumer is faced with the following problem. He is asked to allocate a fixed budget among n durable goods, and among all non-durable goods. Let g_j for $j = 1, 2, \ldots, n$ be the amount of good j he selects. Remember, g_j is discrete, that is, an integral number of goods. If we represent his budget by K, the amount he spends on non-durable goods by y, the prices of the durable goods by $p_1, p_2, p_3, \ldots, p_n$, and his utility function by $U(\cdot, \cdot, \ldots)$, then the mathematical problem he is asked to solve is

$$\text{maximize} \quad U(g_1, g_2, g_3, \ldots, g_n, y) \qquad \text{(P1)}$$

$$\text{subject to:} \quad \sum_{j=1}^{n} p_j g_j + y \leq K$$

This is the standard microeconomic consumer behavior model. Depending upon the functional form of the utility function, the solution to problem P1 can involve complex non-linearities and discretization effects. Exact solution of P1 may be difficult even for advanced mathematical programming computer algorithms.

It is unlikely that consumers solve P1 in its full complexity in everyday decision making. In fact, there are a variety of scientific literatures that suggest otherwise. Some example citations include new economic theory (Heiner, 1983), information processing theory (Sternthal and Craig, 1982), mathematical psychology (Tversky and Kahneman, 1974), social psychology (Johnson and Tversky, 1983), and marketing science (Shugan, 1980).

We show in Hauser and Urban (1982) that a very simple consumer decision rule will, in most realistic cases, lead to an allocation of the budget giving a value of utility very close to the maximun attainable utility. We call this rule the value priority algorithm.

Value Priority Algorithm

Suppose that the consumer can assign to each good a marginal utility, u_j, that represents the amount of utility he obtains from possessing that durable good.[1] If the consumer considers more

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[1]For now, assume that u_j does not depend on the other items in the budget. We can relax this assumption later.

than one unit of the durable good, we assign values u_{j1}, u_{j2}, . . ., etc. to the first, second, etc. units of good j with the usual assumption that $u_{j1} > u_{j2}$, etc.

In the value priority algorithm, the consumer ranks all goods according to their "value", that is, utility per dollar as measured by u_j/p_j. The consumer then chooses the highest value goods as long as their value is above some cutoff, λ, which represents the value of spending an additional dollar on non-durable goods. That is, $\lambda = \partial u(\cdot, y)/\partial y$ evaluated at the budget constraint.[2]

For example, suppose the consumer is considering a bedroom set, a dishwasher, a videotape recorder, an automobile, a personal computer, home improvements, and a desk. He would consider the pleasure and usefulness, i.e., utility, he would obtain from owning the best choice from each, consider the price of the best choice, and rank them according to value as shown below.

Bedroom Set
Dishwasher
Video Tape Recorder
← Budget Constraint (λ)
Automobile
Personal Computer
Home Improvements
Desk

He would first choose the bedroom set (and some non-durables with value up to $u_{bedroom}/p_{bedroom}$), then the dishwasher, then the video tape recorder. At this point, he would find that the three durables (plus the corresponding non-durables) would exhaust his budget. If he were to borrow, or otherwise obtain additional funds, the next durable item he would purchase would be an automobile.

Because this algorithm is so simple and because it so often leads to an optimal or near optimal solution of problem P1, we posit that it will be a good representation of consumer purchasing behavior. Whether it indeed does represent behavior and whether the utilities

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[2]Mathematically, λ will be a complex function of the other utilities. We show in Hauser and Urban (1982) that there exists a simple allocation scheme which iteratively allocates the budget to durable goods and non-durable goods according to "value" and which leads the consumer implicitly to the appropriate value of λ.

and the budget cutoff can be estimated from real data are empirical questions that this paper begins to address.

Extensions

Consumer problem P1 is a simple one period allocation problem, but the idea extends readily to many realistic issues. For example, in a multiperiod problem with borrowing (savings) and depreciation, the "value" becomes the depreciated time stream of utility divided by the price in "current" dollars. The budget constraints for each period are also related via the interest rate.

Operating costs become an addition to the price, discounted over time; replacement (trade-ins) are modeled by computing net utility gain and net price; and complementarity is approximated by first order dependence on higher valued purchases. For details and equations see Hauser and Urban (1982).

DATA

The value priority model is formulated at the level of the individual consumer, thus, we need individual level data with which to test the model. In March, 1983, we were given the opportunity to obtain the necessary data.

An American automobile manufacturer planned to introduce a new automobile in Spring 1984 and, among other things, wanted to know with which durable products the automobile would compete. This automobile was a luxury model for upscale consumers and competition from vacations, second homes, pools, boats, and college tuition was a management concern.

Budget Task

To obtain budget information, we gave consumers a deck of cards in which each card represented a potential purchase. For example, these cards included college tuition, vacations, home improvements, major clothing purchases, landscaping, cameras and accessories, furniture, home fuel savings devices, dishwashers, color televisions, stereo systems, jewelry, etc. After an extensive pretest, we were able to identify 52 items that accounted for most purchases. (Consumers were given blank cards for additional purchases.)

Consumers first sorted these cards according to whether they (A) now owned the durable, (B) would consider purchasing it in the next three years, or (C) would not consider purchasing it in the next three years. Consumers next considered pile A, "currently own", and removed those items they would either replace or supplement by buying an additional unit. Finally, they selected from pile B, "would consider", and from the replacement/additional pile, those items for which they would specifically budget and plan. These items are now their budgetable durable goods.

Consumers then allocated these items to the years 1983, 1984, and 1985 and ordered the items according to priority within each year. This rank order of items becomes our measure of their budget allocation. We estimate utilities with other data, described below, and attempt to forecast the measured rank order buying priorities.

The task was administered with trained and experienced personal interviewers. It took approximately 50 minutes and was the opening part of a larger, two-hour interview in which respondents were paid $25 for their time. The 170 respondents were chosen at random from the community, but in proportion to previous purchases of automobiles similar to the automobile of interest. For 12 percent of the interviews, both husband and wife participated in making a joint budget allocation.

Since our theory and the data are at the level of the individual consumer, this data should be sufficient for an initial test of the theory. However, the specific durables and the magnitude of the budget are not generalizable to the U.S. population because our sample was weighted towards potential luxury car buyers.

EXPLANATORY MEASURES

Obtaining utility measures that can be used to infer value among product categories is a difficult task. Almost every utility measurement procedure of which we are familiar, including conjoint analysis, preference regression, logit analysis, expectancy values, and von Neumann-Morgenstern assessment, measures utility within a product category. In a series of pre-test measurements in 1981 and 1982, we tried over a dozen different methods including directly scaled (0 - 100 scale) points on "utility" and on "value", constant sum paired comparisons among items, and constant sum allocations among all items. We found four measures that appeared feasible and provided meaningful tasks to the consumer. These four measures were included in our interviews.

None of the four measures were explicit measures of utility. However, for each consumer measure, we use the value priority hypothesis to infer relationships among utilities. Details are given in the estimation section below. The measures were:

Reservation Price. The consumer was asked to specify the minimum price at which he, she, or they would no longer purchase the durable.

Purchase Probability. The consumer was asked to estimate the probability that he, she, or they would actually purchase the durable in the period of interest (0 to 10 "Juster" scale).

Lottery Order. The consumer was asked to imagine that he, she, or they had won a lottery and would be allowed to select a prize. They were then to rank the durables allocated to each

year in the order corresponding to the order in which he, she, or they would choose a prize in the lottery. Note that this ordering will usually be different than the budget allocation ordering because price is not to be considered in this task.

Combination Lottery Prizes. The consumer was again told that he, she, or they had won a lottery, but this time the task was to choose among two pairs of prizes. For example, the consumer(s) might be asked to choose among receiving either (a) the first and fourth ranked prize, or (b) the second and third ranked prize. Consumers were asked up to eight such pairs or combinations for each budget year.

Example Respondent

Table 1 lists the actual data obtained from one respondent. This respondent, a 30 year old, married woman with three children and a $35,000 per year family income, has six durable goods in her 1985 budget. For example, she expects to purchase a $5,000 automobile with probability .70. This durable good is ranked first in the lottery prize question and has a reservation price of $10,000. If price were not an issue she would rather have the automobile plus a freezer than paid tuition plus a vacation.

For each respondent, there are three tables such as Table 1, one for each year.

TABLE 1
DATA FROM EXAMPLE RESPONDENT

DURABLE	PRICE	RESERVATION PRICE	PURCHASE PROB.	LOTTERY ORDER
Automobile	$5,000	$10,000	.70	1
Furniture	2,000	4,000	.60	2
Tuition	2,000	5,000	.99	3
Movie Camera	500	1,000	.60	4
Vacation	1,000	1,500	.70	5
Freezer	300	500	.50	6

COMBINATION LOTTERY PRIZES

(1)	Automobile, Freezer	>	Tuition, Vacation
(2)	Automobile, Vacation	>	Tuition, Camera
(3)	Tuition, Vacation	>	Furniture, Freezer
(4)	Tuition, Freezer	>	Camera, Vacation
(5)	Freezer, Vacation	>	Camera
(6)	Tuition	>	Camera, Freezer
(7)	Tuition, Freezer	>	Furniture

CONVERGENT LINEAR PROGRAMMING ESTIMATION

Each of the measures in Table 1 provides information about utility

values, but none of the data is a direct measure of utility. For example, the purchase probability might be a non-linear function of utility and of λ while the lottery order and combination lottery prizes provide only rank order information about utility.

Because two data types, lottery orders and combination lottery prizes, are rank order relationships and because the other data types are continuous (and non-linear) traditional methods based on continuous, linear relationships (structural equation, path analysis, simultaneous equations, etc.) are not appropriate for our purposes. Fortunately, linear programming (LP) does provide a means to incorporte all four data types in a single convergent estimation procedure.

The idea behind convergent LP estimation is quite simple. Each datum implies a relationship either among various utility values or among a utility value and the datum. The relationship varies by data type. Our goal is to select utility values such that all relationships are satisfied. However, in the presence of measurement error and approximation error, it is unlikely that we will be able to satisfy all relationships simultaneously. Thus, for each datum, say a lottery prize answer, we will be able only to satisfy approximately the relationship. The amount by which we cannot satisfy the relationship we call "error". Thus, we choose utility values to minimize a weighted sum of errors where the weights (chosen by the analyst) allow us to put different emphasis on different data types.

This minimization of errors can be accomplished with a linear program. The objective function is the weighted sum of errors and the constraints are the relationships implied by each datum. In general terms this is (LP1):[3]

$$\begin{aligned} \text{minimize} \quad & W_1 * (\text{errors based on reservation price answers}) \\ & + W_2 * (\text{errors based on purchase probability answers}) \\ & + W_3 * (\text{errors based on lottery order answers}) \\ & + W_3 * (\text{errors based on combination lottery prize answers}) \end{aligned}$$

subject to relationships implied by the value priority model. We will now illustrate the specific mathematical relationships.

Reservation Price Relationships

The reservation price is the price at which the durable good leaves the budget. Thus, if r_j and u_j are the reservation price and utility of the j^{th} item, then the value priority model implies:

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[3]It is useful to distinguish between the mathematical program, P1, which is the consumer's budget problem and the linear program, LP1, which is the analyst's estimation problem.

$$u_j/r_j = \lambda \qquad (1)$$

because at the reservation price, the j^{th} item just falls below the budget cuttoff, λ.

To include equation (1) as a relationship in an LP, we define "errors based on reservation price answers" as the absolute value of the difference between u_j/r_j and λ, that is, $|u_j/r_j - \lambda|$. In linear programming mathematics, this becomes

$$\text{errors based on reservation price answers} = e^+_{rj} + e^-_{rj} \qquad (2)$$

where the constaint relationships are,

$$(u_j/r_j) - e^+_{rj} + e^-_{rj} = \lambda \qquad (3)$$

$$u_j,\ e^+_{rj},\ e^-_{rj} \geq 0.$$

Equations (2) and (3) are the standard LP formulation for minimizing absolute error, e.g., Gass (1979, p. 320). If values for u_j and λ are estimated and u_j/r_j exceeds λ, only e^+_{rj} will take on a positive value because minimization of equation (2) in LP1 forces e^-_{rj} to zero. If λ exceeds u_j/r_j, only e^-_{rj} will be positive.

Since the LP seeks to minimize $e^+_{rj} + e^-_{rj}$ and since it can simultaneously set u_j and λ, one trivial solution is to set all variables equal to zero. We avoid this problem by recognizing that utility, and hence λ, are ratio scales and thus unique to a positive constant. Thus, we can set one utility value, or λ, arbitrarily. In our formulations we set $\lambda = 1$, thus scaling everything in terms of dollars.

Purchase Probability Relationships

The purchase probability is the consumer's estimate of the probability that the durable good will actually be purchased in the budget period. It is based on the utility and price of the durable good but also upon unobserved events that make the purchase more or less favorable. If these unobserved events represent observation error, then, according to the value priority model, the probability of purchasing good j is given by:

$$L_j = \text{Prob}\{u_j/p_j + \text{error} \geq \lambda\} \qquad (4)$$

That is, the likelihood of purchase (L_j) is the probability that the value (u_j/p_j) is greater than the budget constraint (λ) after adjusting for error.

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[3]It is useful to distinguish between the mathematical program, P1, which is the consumer's budget problem and the linear program, LP1, which is the analyst's estimation problem.

If we assume that the observation error is distributed with a double exponential probability distribution, then equation (4) becomes the logit model shown in equation (5) where ß is a parameter to be estimated.

$$L_j = \frac{\exp\{ß\, u_j/p_j\}}{\exp\{ß(u_j/p_j)\} + \exp\{ß\lambda\}} \tag{5}$$

For derivation, see McFadden (1974). Equation (5) can be linearized by dividing through $(1-L_j)$ and taking logarithms.

Finally, we again use the standard LP formulation for minimizing absolute error to obtain the objective function and constraint relationships for purchase probability. For the criterion function in LP1:

$$\text{errors based on purchase probability answers} = e^+_{\ell j} + e^-_{\ell j} \tag{6}$$

and the associated constraint is

$$(u_j/p_j) - (ß^{-1})\{\log[L_j/(1-L_j)]/p_j\} - e^+_{\ell j} + e^-_{\ell j} = \lambda \tag{7}$$

$$u_j,\ ß^{-1},\ e^+_{\ell j},\ e^-_{\ell j} \geq 0$$

In these equations, L_j and p_j are observed and u_j, ß, $e^+_{\ell j}$ and $e^-_{\ell j}$ are variables. As before, we establish the scale by setting $\lambda = 1$.

Lottery Orders

The lottery order is a rank order of the durable goods according to their usefulness or desireability to the consumer. As such, they imply rank orders on the magnitude of the utilities. For example, if u_1 is the utility of the first ranked durable, u_2 is the utility of the second ranked durable, etc., then the lottery orders imply:

$$u_1 > u_2 \tag{8}$$

$$u_2 > u_3$$

$$\text{etc.}$$

The reader will notice that this data and the constraints implied by equations (8) are similar to the LP conjoint analysis algorithm LINMAP as proposed by Srinivasan and Shocker (1973). The only difference is that we are interested in the utilities of alternative durable goods whereas Srinivasan and Shocker were interested in the utilities of factorial combinations of product characteristics.

The scaling of the errors varies by the type of relationship, but this is easily reflected in the weights chosen by the analyst.[4] Repeated estimates can be made with alternate weights.

PREDICTIVE TESTS

The data on reservation prices, purchase probabilities, lottery orders, and combination lottery prizes gives us the ability to estimate the utilities of the durable goods in the budget. If the value priority algorithm is a reasonable model of consumer purchasing behavior, then the rank order of "value", that is, estimated utility divided by price, should provide an estimate of the consumer's rank order buying priorities. We thus formulate a predictive test by comparing the estimated utilities (divided by price) to the consumers' budget priorities.

We illustrate the predictive tests with an example.

Example Predictive Test

Consider the data in table 1 and suppose we place equal weight on each data type, that is $W_1 = W_2 = W_3 = W_4$. Applying convergent LP estimation provides the estimates of utility shown in the second column of Table 2. Dividing by price gives the estimates in the third column of Table 2. Notice that the estimated utilities would predict that this consumer would rank 'tuition' as her first budget priority (value = 5.1), a movie camera as her second budget priority (value = 2.5), . . ., and a freezer as her last budget priority (value = 1.0).

We now compare the budget priority predicted by the estimated utilities to that actually observed. Remember, the observed budget priorities were not used in the estimation, thus, the comparison in Table 2 is a test of predictive ability, rather than of data fitting ability. Comparing rank orders implied by the data in the third column of Table 2 to the fourth column we see that the predictions are reasonable but not perfect.

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[4] One can assure that the scales are commensurate by multiplying through by r_j in equation (3) or p_j in equation (7).

TABLE 3.
VARYING WEIGHTS ON TYPES OF INPUT DATA

	SPEARMAN CORRELATION
EQUAL WEIGHTS TO ALL FOUR TYPES	.87
RESERVATION PRICE WEIGHTED HEAVILY*	.31
PURCHASE PROBABILITY WEIGHTED HEAVILY	.82
LOTTERY ORDER WEIGHTED HEAVILY	.82
PAIRED LOTTERY PRIZES WEIGHTED HEAVILY	.87

*"Weighted heavily" means the relevant weight is 100 times more than others.

Figure 1 reports the Spearman correlations of the predicted and actual budgets when the "utilities" are estimated placing equal weights on the data sources. Overall, the value priority algorithm does well. For over one-third of the budgets, correlations are .50 or better and the majority of the correlations are positive. Significance levels are complex (because many ties are possible) and vary by budget (the number of items in each budget varies). Thus, there is no single overall critical value that can be applied to Figure 1.

FIGURE 1
VARIATION ACROSS INDIVIDUALS
Equal Weighting of Res. Price, Prob., Lotteries, and Pairs

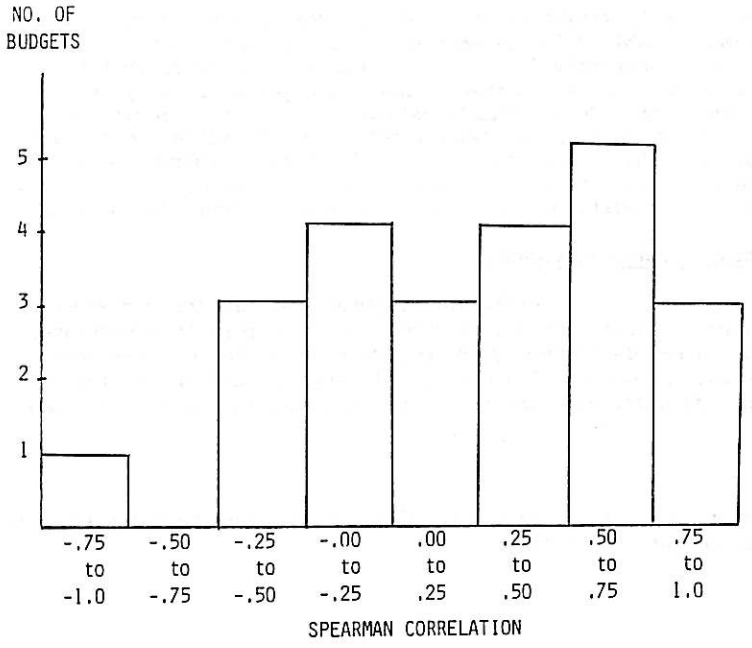
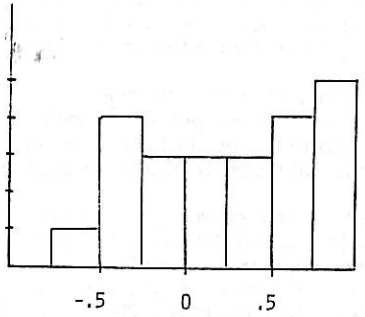
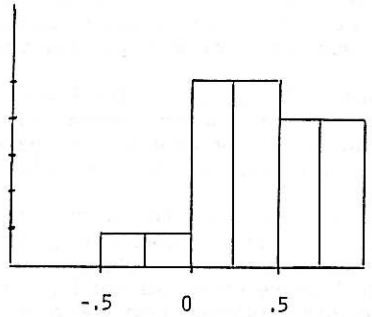


FIGURE 2

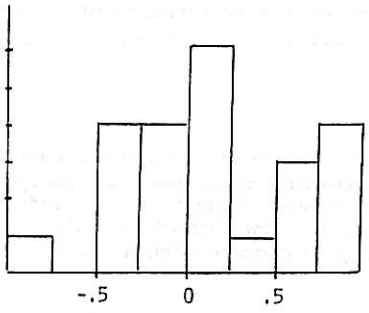
PREDICTIVE RESULTS EMPHASIZING DIFFERENT DATA SOURCES



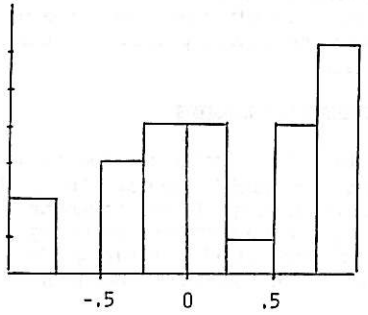
(a) RESERVATION PRICE



(b) PURCHASE PROBABILITIES



(c) LOTTERY ORDERS



(d) PAIRED LOTTERY PRIZES

If we assume that the observation error is distributed with a double exponential probability distribution, then equation (4) becomes the logit model shown in equation (5) where ß is a parameter to be estimated.

$$L_j = \frac{\exp\{ß\, u_j/p_j\}}{\exp\{ß(u_j/p_j)\} + \exp\{ß\lambda\}} \tag{5}$$

For derivation, see McFadden (1974). Equation (5) can be linearized by dividing through $(1-L_j)$ and taking logarithms.

Finally, we again use the standard LP formulation for minimizing absolute error to obtain the objective function and constraint relationships for purchase probability. For the criterion function in LP1:

$$\text{errors based on purchase probability answers} = e^+_{\ell j} + e^-_{\ell j} \tag{6}$$

and the associated constraint is

$$(u_j/p_j) - (ß^{-1})\{\log[L_j/(1-L_j)]/p_j\} - e^+_{\ell j} + e^-_{\ell j} = \lambda \tag{7}$$

$$u_j,\ ß^{-1},\ e^+_{\ell j},\ e^-_{\ell j} \geq 0$$

In these equations, L_j and p_j are observed and u_j, ß, $e^+_{\ell j}$ and $e^-_{\ell j}$ are variables. As before, we establish the scale by setting $\lambda = 1$.

Lottery Orders

The lottery order is a rank order of the durable goods according to their usefulness or desireability to the consumer. As such, they imply rank orders on the magnitude of the utilities. For example, if u_1 is the utility of the first ranked durable, u_2 is the utility of the second ranked durable, etc., then the lottery orders imply:

$$u_1 > u_2 \tag{8}$$

$$u_2 > u_3$$

$$\text{etc.}$$

The reader will notice that this data and the constraints implied by equations (8) are similar to the LP conjoint analysis algorithm LINMAP as proposed by Srinivasan and Shocker (1973). The only difference is that we are interested in the utilities of alternative durable goods whereas Srinivasan and Shocker were interested in the utilities of factorial combinations of product characteristics.

Following similar methods, we count errors only when the inequality relationships are violated. That is,

$$\text{lottery order error} = (1-\delta_{jk})e^{+}_{ojk} + (\delta_{jk})e^{-}_{ojk} \tag{9}$$

$$u_j - u_k - e^{+}_{ojk} + e^{-}_{ojk} = 0 \tag{10}$$

$$u_j,\ u_k,\ e^{+}_{ojk},\ e^{-}_{ojk} \geq 0$$

where

$$\delta_{jk} = \begin{cases} 1 \text{ if } j \text{ is preferred to } k \\ 0 \text{ if } k \text{ is preferred to } j \end{cases}$$

In equations (9) and (10), the (0, 1) variable, δ_{jk}, is the "answer" to the lottery order question which tells us which product is preferred as a prize in the lottery.

Unlike Srinivasan and Shocker (1973), we need not worry about the scaling of the utilities because their scaling is already established by the constraints associated with the reservation price and purchase probability data.

Combination Lottery Prizes

The combination lottery prize questions imply rank order relationships among pairs of utilities. For example, if the combination of goods 1 and 4 are preferred to the combination of goods 2 and 3, then

$$u_1 + u_4 \geq u_2 + u_3 \tag{11}$$

Objective functions for the paired etc. comparison lottery error,

$$(1-\delta_m)\ e^{+}_{cm} + (\delta_m)\ e^{-}_{cm} \tag{12}$$

and constraints similar to (9) and (10) can be established for each combination lottery question, m. For ease of exposition, we do not repeat them here.

Summary

The estimation LP is now to minimize the weighted sum of errors, given by equations (LP1), (2), (6), (9), and (12) subject to the constraints of (3), (7), (10), and the mathematical formulation of (11). For example, for the six durable goods in Table 1, there are six reservation price relationships, six probability relationships, five lottery order relationships, and seven combination lottery prize relationships totalling 24 constraints and 24 independent errors in the objective function.

The scaling of the errors varies by the type of relationship, but this is easily reflected in the weights chosen by the analyst.[4] Repeated estimates can be made with alternate weights.

PREDICTIVE TESTS

The data on reservation prices, purchase probabilities, lottery orders, and combination lottery prizes gives us the ability to estimate the utilities of the durable goods in the budget. If the value priority algorithm is a reasonable model of consumer purchasing behavior, then the rank order of "value", that is, estimated utility divided by price, should provide an estimate of the consumer's rank order buying priorities. We thus formulate a predictive test by comparing the estimated utilities (divided by price) to the consumers' budget priorities.

We illustrate the predictive tests with an example.

Example Predictive Test

Consider the data in table 1 and suppose we place equal weight on each data type, that is $W_1 = W_2 = W_3 = W_4$. Applying convergent LP estimation provides the estimates of utility shown in the second column of Table 2. Dividing by price gives the estimates in the third column of Table 2. Notice that the estimated utilities would predict that this consumer would rank 'tuition' as her first budget priority (value = 5.1), a movie camera as her second budget priority (value = 2.5), . . ., and a freezer as her last budget priority (value = 1.0).

We now compare the budget priority predicted by the estimated utilities to that actually observed. Remember, the observed budget priorities were not used in the estimation, thus, the comparison in Table 2 is a test of predictive ability, rather than of data fitting ability. Comparing rank orders implied by the data in the third column of Table 2 to the fourth column we see that the predictions are reasonable but not perfect.

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[4] One can assure that the scales are commensurate by multiplying through by r_j in equation (3) or p_j in equation (7).

TABLE 2
EXAMPLE PREDICTIVE TEST

DURABLE	ESTIMATED UTILITY	UTILITY ÷ PRICE (000's)	ACTUAL BUDGET PRIORITY ORDER
Automobile	10.00	2.0	4
Furniture	4.00	2.0	3
Tuition	10.27	5.1	2
Movie Camera	1.22	2.5	1
Vacation	1.50	1.5	6
Freezer	0.30	1.0	5

CORRELATION OF ESTIMATE WITH BUDGET PRIORITY

Spearman ρ = .87

Kendall τ = .69

Tuition and the movie camera are predicted and observed to be the top two items, but estimated "value" predicts tuition as the top priority while the consumer feels that the movie camera is her top priority. Overall, the Spearman rank order correlation of the predicted rank from utility per dollar (column 3) and the actual rank (column 4) is .87, while the Kendall rank order correlation is .69

However, equally weighting of the data types is not the only choice. For example, Table 3 indicates the results we obtained by using each data source separately.[5] For this consumer, it appears that the purchase probabilities, lottery orders, and paired lottery prizes each, alone, provide reasonable estimates of budget priorities; however, in this case, reservation prices do not appear to be as good data as the other measures. In fact, if we drop reservation prices and use equal weights on the other three data sources, we get a higher rank order correlation, .93, than if we use all four data sources.

Variation Across Individuals

At this point in time, efficient computer software for convergent LP estimation is still being developed. In developing this software, we selected a sample of twenty-three budgets with which to test our estimation procedure. The software development and the analysis of the full data set are expected to be completed by the end of 1984.

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[5]We report only the Spearman correlation for ease of exposition. Results are similar with Kendall's τ.

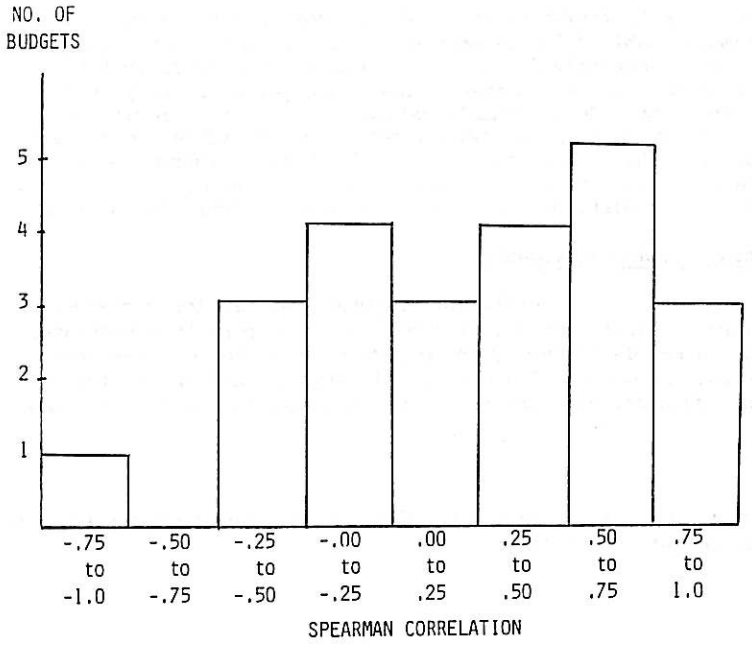
TABLE 3.
VARYING WEIGHTS ON TYPES OF INPUT DATA

	SPEARMAN CORRELATION
EQUAL WEIGHTS TO ALL FOUR TYPES	.87
RESERVATION PRICE WEIGHTED HEAVILY*	.31
PURCHASE PROBABILITY WEIGHTED HEAVILY	.82
LOTTERY ORDER WEIGHTED HEAVILY	.82
PAIRED LOTTERY PRIZES WEIGHTED HEAVILY	.87

*"Weighted heavily" means the relevant weight is 100 times more than others.

Figure 1 reports the Spearman correlations of the predicted and actual budgets when the "utilities" are estimated placing equal weights on the data sources. Overall, the value priority algorithm does well. For over one-third of the budgets, correlations are .50 or better and the majority of the correlations are positive. Significance levels are complex (because many ties are possible) and vary by budget (the number of items in each budget varies). Thus, there is no single overall critical value that can be applied to Figure 1.

FIGURE 1
VARIATION ACROSS INDIVIDUALS
Equal Weighting of Res. Price, Prob., Lotteries, and Pairs



Variation Across Alternative Weightings of Data Types

By varying W_1, W_2, W_3, and W_4 in LP1, we can place differential emphasis on the four data types. For example, if we make W_1 much larger than W_2, W_3, and W_4, we emphasize reservation prices as the primary data source. The results of emphasizing reservation prices are shown in Figure 2a. Overall, reservation prices do about as well as equal weighting of all data sources. For some people, they do very well (correlation = .75 to 1.0).

Figure 2 also gives the results for emphasizing data on purchase probabilities, lottery orders, and combination lottery prizes.[6] From this initial small subsample, it appears that probabilities do best overall; however, we hesitate to generalize until the full sample has been analyzed.

Finally, we can ask the question: "What if we had known a priori which data source was best for each consumer?" If we had known data quality a priori, we would have chosen to emphasize the best data type in our estimation and we would have selected the weights accordingly. Figure 3 displays the results we would have obtained from the best weighting. In this case, 20 of the 23 comparisons,[7] would have had positive correlations and 15 of the 23 comparisons would have had correlations of .50 or better. This is higher than the separate or equally weighted values in figure 2.

Figure 3 is more like a test of the model's fit to the data than a true predictive test because we used the dependent measure to select the best weighting of data types. But even interpreted as a fit measure, it does suggest strongly that the value priority algorithm is a reasonable explanation of observed budget priorities and the best utility measure may vary by individual.

MANAGERIAL INSIGHTS

The value priority algorithm is a model of how consumers allocate their budgets to durable goods. It is also interesting to review some insights on which durable goods are given priority in consumer budgets. This information is valuable to managers planning strategies for new automobile models because it indicates which durable goods are likely to compete with automobiles for a share of the consumers' budgets.

The first summary measure is the average lottery prize rank (utility rank not considering price) of a durable when it is in the budget. As indicated in Table 4, the automobile is alive and well as a durable good. On average it was the second highest ranked good with an average rank of 1.33. Houses were the highest ranked and ranks 3 through 6 had something to do with recreation.

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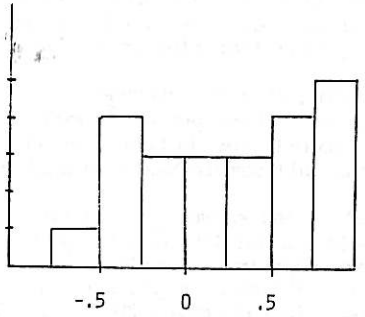
[6] To avoid trival solutions to LP1, we require that either W_1 or W_2 is non-zero.

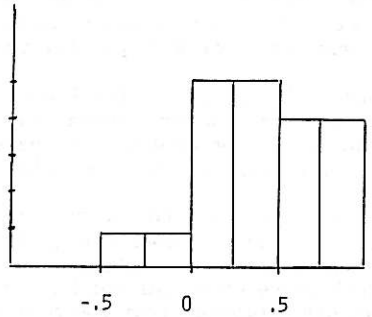
[7] Three budgets have negative correlations in Figure 3 vs. two in Figure 2b because, to date, we have analyzed only 20 of the 23 budgets for the weighting emphasizing purchase probabilities.

FIGURE 2

PREDICTIVE RESULTS EMPHASIZING DIFFERENT DATA SOURCES



(a) RESERVATION PRICE



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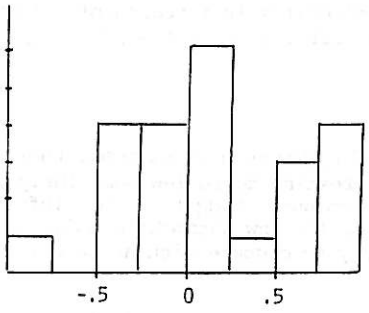
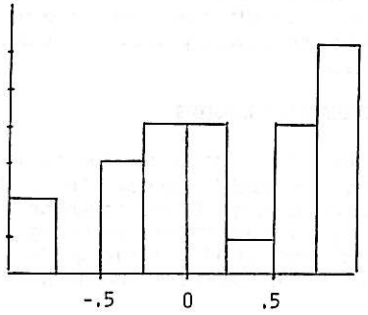


(c) LOTTERY ORDERS



(d) PAIRED LOTTERY PRIZES

FIGURE 3
"PREDICTIVE" TESTS WITH BEST WEIGHTINGS OF DATA TYPES

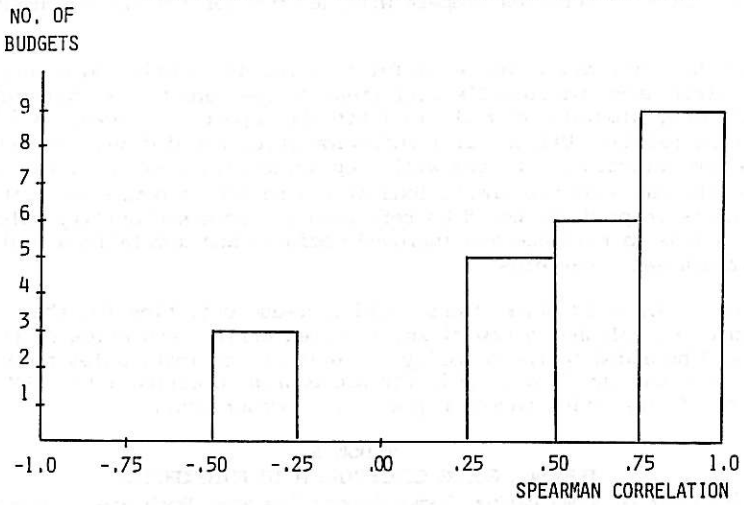


TABLE 4.
WHAT IS VALUED MOST BY CONSUMERS

DURABLE GOOD	AVERAGE RANK OF UTILITY
House	1.03
Automobile	1.33
Summer House	1.46
Pool	1.69
Boat	1.79
Vacation	2.00

Table 4 gives utility rank. But, the value priority algorithm suggests tha value, i.e., utility/dollar, is the appropriate comparison measure. Table lists those items that were ranked over automobiles when both an automobile and that item was in the three year (1983, 84, 85) budget plan. For exampl when both an automobile and 1983 school tuition were in a budget plan, 1983

school tuition was ranked over an automobile 96.4% of the time. Table 5 suggests that many durables compete with automobiles for the consumers' dollars.

The value priority model can be useful in forecasting existing product class sales. After each individual's utilities, prices, and budget constraint (λ) are estimated, simulations would indicate the impact of recessions (changing budget constraint). Differential inflation rates would change prices - perhaps home electronics prices will drop while cars rise and these changes could be the basis of new simulations of the shifts in buying priorities. Improvements in products would be reflected in increased utility values (e.g., cars with less maintenance and improved comfort) and simulations would estimate new sales patterns.

New products in an existing class could increase utilities for that class. If the product establishes a new class, measured utility and price of the new good would be added to the priority of buying to estimate sales potential (see Hauser, Roberts, and Urban, 1983, for procedures to estimate utility and diffusion of innovation based on pre-market measurement).

TABLE 5
DURABLE GOODS COMPETING WITH AUTOMOBILES
(Percent of Time Ranked Above Automobile When Both are in Budget)

	DURABLE	PERCENT		DURABLE	PERCENT
1.	School Tuition 1983	96.4	13.	Dishwasher	63.2
2.	Vacation 1983	92.8	14.	Color Television	59.1
3.	Home Improvement (Minor)	84.0	15.	Stereo System	57.9
4.	Major Clothing	78.8	16.	Jewelry	55.6
5.	Landscaping	77.8	17.	House	53.3
6.	School Tuition 1984	76.7	18.	Oven	50.0
7.	Gifts/Donations	76.0	19.	Movie/Video Camera	50.0
8.	Cameras and Accessories	70.6	20.	Video Tape Recorder	46.9
9.	Furniture	68.0	21.	Refrigerator/Freezer	46.2
10.	Home Fuel Savings Device	67.7	22.	School Tuition 1985	45.0
11.	Home Improvement (Major)	67.3	23.	Home Computer	44.7
12.	Vacation 1985	64.2	24.	Vacation 1985	37.0

SUMMARY AND FUTURE DIRECTIONS

This paper describes the first preliminary test of the value priority algorithm as a model of consumer durable goods purchasing behavior. Subject to confirmation on the full data set we feel:

- the value-priority model appears to be a reasonable descriptor/predictor of planned durable goods purchases;
- convergent LP estimation is feasible and can be used to combine disparate data sources;

- consumers vary in their ability to answer specific question types, but convergent LP estimation can be used to explore this phenomenon and identify the best question type for each consumer; and,

- value priorities provide useful strategic information for durable goods manufacturers.

Based on our experience to date, we believe that the value-priority model and convergent LP estimation provide a number of opportunities for scientifically interesting and managerially relevant research. Among our research priorities are:

- analysis with the full data set;

- a priori identification of which data to rely upon for each individual consumer;

- extension of the empirical analysis to the multi-period theory with interest, borrowing, and depreciation;

- validation via followup interviews to determine what goods the interviewed consumers actually purchased in 1983; and,

- incorporation of the value-priority analyses in the full prelaunch forecasting system for new consumer durables.

We encourage other researchers to join us by taking up the challenge of analyzing consumers' budget allocations.[8]

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[8]This research was sponsored by a grant from General Motors (Buick Motor Division) to M.I.T. Special thanks to John Dabels, General Director of Marketing and Sales Planning, for his support, insights, and guidance throughout this project.

We are indebted to Paul G. Schiodtz of M.I.T. who has developed the convergent LP estimation software and who has been instrumental in the data analysis. John Roberts, Lisa Tener, Janny Leung, Young Sohn, Fareena Sultan, Gayle Shlea, Andy Cjazka, Tom Rose, Lori Curtis, Jan Woznick, and Barbara Gaston provided valuable assistance, creative insights, and hard work throughout the project, mini-test, and full-scale data collection.

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