CONSUMER PREFERENCE AXIOMS: BEHAVIORAL POSTULATES FOR DESCRIBING AND PREDICTING STOCHASTIC CHOICE*

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This paper draws on econometrics, von Neumann-Morgenstern utility theory, stochastic choice theory, and consumer behavior to develop five basic axioms or postulates of stochastic choice behavior. These axioms imply the existence and uniqueness of a preference function which identifies how consumers evaluate products in terms of product attributes. The preference function produces a scalar goodness measure for each product in a consumer's choice set. These goodness measures then predict choice probabilities for each product in a consumer's choice set. The advantage of these axioms is that they extend the strengths of von Neumann-Morgenstern theory to stochastic choice and make possible the determination of consistent choice probabilities.
(MARKETING–BUYER BEHAVIOR; UTILITY/PREFERENCE–THEORY; DECISION ANALYSIS)

1. Introduction

Two important managerial problems addressed by marketing research are: (1) explaining how consumers form preferences and (2) predicting their purchase behavior. Explanatory models provide diagnostic information to managers so that they can modify demand by altering product characteristics, advertising appeals, or other aspects of the marketing strategy. Predictive models provide information to managers so that they can evaluate alternative strategies or plan production, inventory, and sales force.

Four distinct streams of research in marketing and economics have addressed aspects of these problems. Consumer behavioralists have postulated and tested models which identify the process by which consumers form preferences. The von Neumann-Morgenstern utility theoriests have axiomatically studied models to prescribe rational behavior. Both sets of models study behavior deterministically and at the level of the individual consumer. Stochastic modellers have postulated and tested models which identify the structure of a market and the distribution of preferences across the population. These models explain behavior stochastically and at the aggregate level. Finally, econometricians have postulated and estimated models based on observations of past behavior in an attempt to predict future behavior. These models explain behavior stochastically and at an intermediate level (individual choice predictions, but the same choice process for everyone).

These streams of research have often been viewed as competing, but in actuality they are complementary. Stochastic assumptions can be directly coupled with the axiomatic strengths of von Neumann-Morgenstern utility theory, the measurement strengths of consumer behavior, and the predictive strengths of econometrics to provide both explanation and prediction at the level of the individual consumer. This paper provides a common theory (definitions, axioms, and theorems) to combine these diverse disciplines and to develop a usable managerial tool which can: (1) identify the appropriate forms for preference models, (2) handle new products with uncertain attributes, (3) directly measure preference functions at the individual level, and (4) test fundamental behavioral assumptions at an understandable level.

2. Existing Literature

Consumer behavior. The multiattributed preference theories (reviewed by Wilkie and Pessemier [32]) have been devoted to models which predict preference or attitude toward a product as a weighted sum of a consumer's perceptions of the levels of the attributes describing that product. This model implies that the consumer should deterministically select the product with the highest preference value. In practice, the correlations between preference and choice are not perfect but rather range from 0.1 (Sheth and Talarzyk [28]) to 0.8 (Ryan and Bonfield [27]). Other researchers have relaxed the restrictive linear form (Green and Rao [7], Green and Devita [6], and Johnson [14]) and have used conjoint measurement (Tversky [30]) on additive, multiplicative, and pairwise interactive models to estimate consumer preference from consumers' perceptions of a product's attribute levels. Nonetheless, they too have not predicted behavior with certainty.

Utility theory. Economists have proceeded deductively from verifiable postulates about an individual's preference ordering. In particular, the von Neumann-Morgenstern [32] postulates (reformulated by Friedman and Savage [5], Marschak [22], Herstein and Milner [12], Jensen [13], and possibly others) have been particularly useful in axiomatically specifying the conditions under which a unique[1] preference scale exists. Later research identified "independence properties" which specified necessary and/or sufficient conditions under which preferences for products could be represented by parametric functions of the attribute levels. (See for example Farquhar [2], [3], Fishburn [4], Keeney [16], [17], Keeney and Raiffa [18], and Raiffa [26].) Since the parameterized functions are axiomatically derived from basic behavioral assumptions, the parameters provide explicit indications of tradeoffs, risk aversion, and interactions among the levels of the product's attributes.

Stochastic models. In 1974, Bass [1] challenged the field of consumer behavior by stating:

> Although it is heresy, in some circles, honesty compels one to question the fundamental premise that all behavior is caused. If there is *a stochastic element in the brain* which influences choice, then it is not possible, even in principle, to predict or to understand completely the choice behavior of individual consumers.

He goes on to postulate and test empirically a theory of stochastic preference and brand switching that tries to predict aggregate stochastic behavior while making no claims about a specific individual's behavior. Bass' model does not try to measure, model, or predict preference as a function of the perceived levels of product attributes. Further work is necessary to make it sensitive to decision variables, such as advertising, promotion, or product characteristics, which effect the perceived levels of product attributes.

Econometrics. Recognizing that for practical purposes it is impossible ever to specify fully a utility function, econometricians have postulated that the "true" utility function can only be partially observed. McFadden [23, 24] has operationalized this concept by postulating that the true ordinal utility of product j, u_j, consists of an observation portion, v_j, plus an error term, e_j. In other words, $u_j = v_j + e_j$. Using this error structure, McFadden derives the analytically simple logit model, $L_j = \exp(v_j)/\Sigma_l \exp(v_l)$, where l indexes the products in the choice set. For other models of this type see Luce [20], [21], Kuehn and Day [19], Pessemier [25], and Silk and Urban [29]. Although McFadden's theory allows arbitrary functions for v_j as long as the functions are linear in their parameters, most empirical applications have dealt with functions represented by weighted sums of attribute levels.

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[1] Unique to an additive and multiplicative constant.

Discussion

Examining the various approaches to understanding or predicting consumer preferences, we see a diverse set of approaches. Consumer behavior theory postulates preference models and experimentally tests them. Prescriptive utility theory ignores the prediction problem but develops powerful deductive theory to identify the appropriate preference models. Econometrics admits imperfection and statistically searches for preference models which explain as much behavior as possible.

The common goal of each of these multiattributed preference models is not to predict behavior deterministically, but rather to explain and predict as much about behavior as is possible. To do this successfully, these models should use preference functions that are as strong as is feasible but which explicitly incorporate the concept of stochastic preference. Thus, what we would like to do is combine the deductive power of prescriptive utility theory with the stochastic behavior models of econometrics and the measurement strengths of consumer behavior.

For example, we might try using a von Neumann-Morgenstern utility function in the logit model to estimate probabilities for Bass' model. Unfortunately, this simple combination may run into problems because each theory or set of theories is based on behavioral assumptions which may or may not conflict.

This paper sets forth an axiomatic structure which draws on the strengths of each theory to build a comprehensive theory for describing choice.

3. Formal Theory

Formal Definitions

Let $A = \{a_1, a_2, \ldots, a_J\}$ be a set of choice alternatives; let x_k be a performance measure, such as "quality," describing at least one alternative, $a_j \in A$. Let $X = \{X_1, X_2, \ldots, X_K\}$ be a complete set of performance measures and let $\mathbf{x}_j = \{x_{1j}, x_{2j}, \ldots, x_{Kj}\}$ be the values that the performance measures take on for a deterministic alternative a_j. Let $\mathbf{x}_{\bar{j}} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{j-1}, \mathbf{x}_{j+1}, \ldots, \mathbf{x}_J\}$. Furthermore, let $p_i(a_j \mid \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_J; \lambda_i)$ be the probability that individual i chooses alternative a_j given specific levels of the performance measures $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_J\}$, and given a vector of real-valued "preference" parameters, λ_i, for individual i.

Compaction. What we need is a real-valued function, call it a compaction function,[2] which tells us how individual i evaluates the performance measures to form his (stochastic) preferences. In particular, if we hold all other products fixed, the compaction function for a given product should produce numbers which are monotonic in the choice probabilities. Formally:

DEFINITION 1. A real-valued function, $c_j(\mathbf{x}_j, \lambda_i)$, is said to be a *compaction function* if for any fixed $\mathbf{x}_{\bar{j}}$

$$c_j(\mathbf{x}_j, \lambda_i) > c_j(\mathbf{x}'_j, \lambda_i)$$

implies

$$p_i(a_j \mid \mathbf{x}_j, \mathbf{x}_{\bar{j}}; \lambda_i) \geqslant p_i(a_j \mid \mathbf{x}'_j, \mathbf{x}_{\bar{j}}; \lambda_i)$$

and

$$c_j(\mathbf{x}_j, \lambda_i) = c_j(\mathbf{x}'_j, \lambda_i)$$

implies

$$p_i(a_j \mid \mathbf{x}_j, \mathbf{x}_{\bar{j}}; \lambda_i) = p_i(a_j \mid \mathbf{x}'_j, \mathbf{x}_{\bar{j}}; \lambda_i).$$

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[2] The word compaction has been chosen rather than utility because we are not attributing utility properties to the function. Semantically, the idea is to "compact" the K attribute measures into one scalar measure of goodness for each choice alternative.

Uniformity. An analyst tries to identify a set of performance measures which are complete. He then would hope that tradeoffs and interdependencies among those performance measures would not be alternative specific. In other words, knowing the performance measures, x_j, for alternative a_j and the preference parameters, λ_i, for individual i would be sufficient to compute individual i's compaction value, c_j, for alternative a_j. Thus a uniform compaction function has the same functional form for all alternatives (drop the j subscript on $c_j(x_j; \lambda_i)$). Formally:

DEFINITION 2. A compaction function is uniform for an alternative set, A, if

$$c_j(x_j, \lambda_i) = c(x_j, \lambda_i) \quad \text{for all } a_j \in A.$$

Notice that alternative specific *terms* can be included as performance measures as long as the *functional form* is the same for all alternatives in A.

Symmetry. Symmetry deals with the functional form of the conditional probability law. Symmetry implies that a specific value of the scalar measure of goodness has the same implications for each alternative. To better understand this, consider the new notation: $p_i(a_j \mid c_1, c_2, \ldots, c_J) =$ the probability of choosing alternative a_j given that $c_1(x_1, \lambda_i) = c_1$, $c_2(x_2, \lambda_i) = c_2$, etc. This notation is consistent by the definition of compaction. Furthermore, define $c_{\overline{jk}} = \{c_1, c_2, \ldots, c_{j-1}, c_{j+1}, \ldots, c_{k-1}, c_{k+1}, \ldots, c_J\}$, that is, $c_{\overline{jk}}$ is the set of all scalar measures of goodness (for individual i) except c_j and c_k. Thus formally:

DEFINITION 3. A compaction function (and the probability law it evokes) is said to be symmetric for an alternative set A if for all pairs of j and k, $a_j, a_k \in A$:

$$p_i(a_j \mid c_j = x, c_k = y, c_{\overline{jk}}) = p_i(a_k \mid c_j = y, c_k = x, c_{\overline{jk}}) \quad \text{and}$$

$$p_i(a_l \mid c_j = x, c_k = y, c_{\overline{jk}}) = p_i(a_l \mid c_j = y, c_k = x, c_{\overline{jk}}) \quad \text{for all } a_l \neq a_j, a_k.$$

Less formally, switching the compaction values for any $j - k$ pair switches the choice probabilities for j and k but leaves all other choice probabilities unchanged.

Stochastic Preference Defined

The definitions of compaction imply the need for a function which captures the essence of an individual's evaluation process such that the compaction values are sufficient to predict probabilities. This compaction function is parallel to a utility function except that it predicts choice probabilities rather than deterministic choice. Let us formally define stochastic preference and stochastic indifference. Note that the definition is conditioned on the alternative set.

DEFINITION 4. Let A be a set of alternatives, let a_j, a_k be elements of A, then $>_A$ is a *stochastic preference* operator on $A \times A$ if $(a_j >_A a_k)$ implies the probability of choosing a_j from A is greater than the probability of choosing a_k from A. Define stochastic indifference, written $a_j \sim a_k$, for equal probabilities and make the obvious definitions for $\gtrsim_A$, $<_A$, and $\lesssim_A$.

Definition 4 deals with stochastic preferences but deterministic alternatives. In practice, consumers rarely have perfect information about the attributes of products. Thus, we would like to consider products which are not perfectly perceived. To this end we generalize the alternative set to include alternatives with uncertain characteristics. Those familiar with utility theory will notice that the following definition is simply a von Neumann-Morgenstern standard gamble.

DEFINITION 5. A *lottery*, $L(a_j, a_k; p)$: $A \times A \times [0, 1] \to A^*$, is an alternative which has the characteristics of a_j with probability p, and the characteristics of a_k with probability $(1 - p)$. (A^* is the range of L.)

We now have the notation to present the axioms. The first three axioms are the von Neumann-Morgenstern [31] axioms restated for stochastic preference, the fourth

axiom is the choice axiom which deals with the structure of the choice set. Intuitive explanations follow the formal statements.

The Axioms

Suppose A^*, $>$, $\sim$, and L satisfy the following axioms:[3]

AXIOM 1. $>$ is a complete ordering on A^*.

(a) For any two a_j, a_k exactly one of the following holds. $a_j > a_k$, $a_j \sim a_k$, $a_j < a_k$

(b) $a_j > a_k$ and $a_k > a_l$ implies $a_j > a_l$.

(c) $a_j \sim a_k$ and $a_k \sim a_l$ implies $a_j \sim a_l$.

AXIOM 2.   Ordering and combining:

(a) $a_j \{\gtrless\} a_k$ implies $a_j \{\gtrless\} L(a_j, a_k; p)$ for all $p \in (0, 1)$.

(b) $a_j > a_k > a_l$ implies the existence of $p_1, p_2, p_3 \in (0, 1)$ such that $L(a_j, a_l; p_1) < a_k$, $L(a_j, a_l; p_2) \sim a_k$, $L(a_j, a_l; p_3) > a_k$.

AXIOM 3.   Algebra of combining:

(a) $L(a_j, a_k; p) \sim L(a_k, a_j; 1 - p)$.

(b) $L[L(a_j, a_k; p), a_k; q] \sim L(a_j, a_k; pq)$.

AXIOM 4.   Choice axiom. Let α be any finite subset of A^*, let $a_j, a_k, a_j', a_k' \in \alpha \subseteq A^*$, then $a_j \sim_{A^*} a_j'$ and $a_k \sim_{A^*} a_k'$ implies $\text{Prob}\{a_j$ from $\alpha - a_k\} = \text{Prob}\{a_j'$ from $\alpha - a_k'\}$ where $\alpha - a_k$ is the set α with the element a_k deleted.

Interpretation of the Axioms

AXIOM 1 (COMPLETE ORDERING).   (a) In utility theory this is a reasonably strong assumption, i.e., that an individual can state his preferences and that they are temporally stable. The new preference definition allows stochastic behavior, thus the new interpretation is that an individual's "average" behavior has no unmeasurable long-term trend. ($b + c$) This property is actually induced by the preference definition because $>$ and $=$ are transitive for the real numbers. It is stated explicitly to maintain a parallel with the utility axioms.

AXIOM 2 (ORDERING AND COMBINING).   (a) This states simply that if a_k is stochastically preferred to a_j, then a lottery with even a slight chance of a_k is preferred to a_j. ("Losing" the lottery gives a_j.) (b) If $a_j > a_k > a_l$, then given a lottery, $L(a_j, a_l; p_1)$, the influence of a_j can be made sufficiently small (p_1 close to 0) such that a_k is still preferred to the lottery. (Review Figure 1.) Furthermore, each individual can conceive
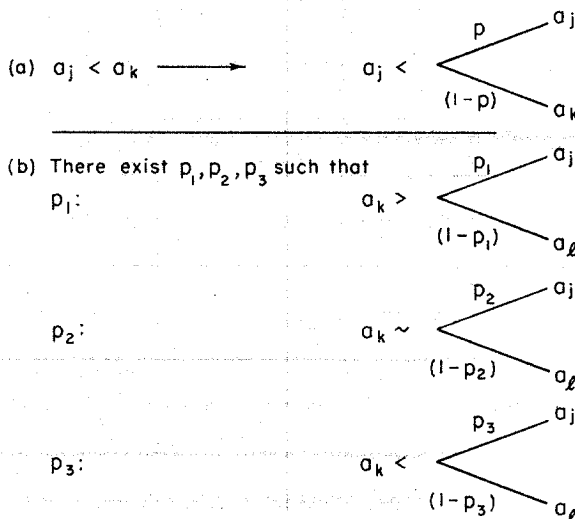

FIGURE 1.   Schematic of Ordering and Combining Axiom.

[3] The A^* subscript on $>_{A^*}$ is temporarily dropped for notational simplicity.

of some probability, p_2, which makes his stochastically indifferent between the middle alternative, a_k, and a lottery involving the extreme alternatives, a_j, and a_l. Taken together, parts (a) and (b) of this axiom imply a reasonable continuity assumption.

AXIOM 3 (ALGEBRA OF COMBINING). (a) This states simply that the lottery operation is commutative, i.e., it does not matter in which order the elements of the lottery are named. (b) This statement of associativity is perhaps the strongest assumption in the utility axioms and hence in our axioms. It states that a series of successive lotteries can be treated as an equivalent one-step lottery. In other words, it states that every individual can conceive of a complex lottery and that he will rationally react to it as if it were a simple lottery with equivalent probabilities. (Review Figure 2.)
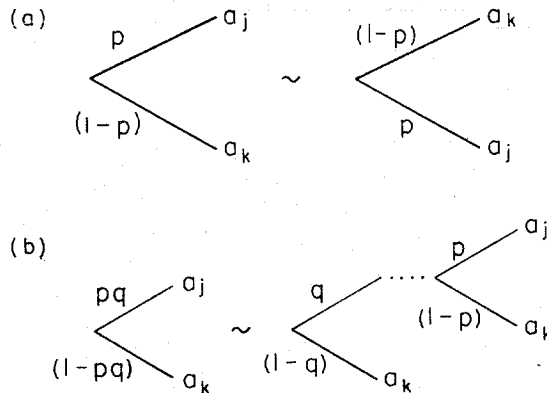

FIGURE 2. Schematic of Algebra of Combining Axiom.

AXIOM 4 (CHOICE AXIOM). This axiom states that if the probability of choosing an alternative, a_j, is equal to the probability of choosing another alternative, a_j', when all alternatives are available, then for any subset of the alternatives this equality of probabilities remains the same if some alternatives (other than a_j, a_j') are deleted from consideration. Furthermore, if a_k and a_k' are indifferent on A, then deletion of one or the other is equivalent in terms of stochastic indifference on the respective subsets. In other words, if two alternatives are equivalent on the entire choice set, then they are equivalent in their presence or their absence from any subset. This is certainly a reasonable assumption for distinct choices, but for certain types of choices, particularly hierarchical choices, it can break down.

For example, suppose a student has the following choice probabilities for health care delivery: Boston Group Practice (BGP), 0.4; private care with Dr. Jones, 0.3; private care with Dr. Smith, 0.3; and suppose these choices represent an exhaustive list. Now suppose Dr. Smith is no longer available. Will BGP still be stochastically preferred to Dr. Jones? Maybe, but perhaps the student's decision rule is to first choose between group practice and private care and then randomly select a doctor if he decides on private care. This might imply that Dr. Jones > BGP (0.6 > 0.4) after Dr. Smith departs.

This example cautions us not to blindly apply models derived from the axioms. Instead, the axioms must be verified before models are built, and if the choice process is hierarchical (sequential) it must be modeled as such. There are a number of ways to identify hierarchies in the choice set. See the stochastic models referenced earlier as well as Kalwani and Morrison [15]. Axiom 4 is needed because alternatives will be represented by sets of performance measures and compaction functions will be inferred from questions about stochastic indifference among abstract alternatives (represented by values for the performance measures). Thus, compaction functions will be determined on uncountable choice sets, $\{X_1, X_2, \ldots, X_N\}$, and applied to finite subsets, $\{a_1, a_2, \ldots, a_j\}$.

Existence and Uniqueness Theorems

The first and most significant implication of the axioms is the existence and uniqueness of a real-valued function on the expanded alternative set, A^*, which preserves (stochastic) preference and for which mathematical expectation applies. The proof of this result exactly parallels the proof for utility functions contained in the appendix of von Neumann and Morgenstern. (Let R represent the real numbers.)

THEOREM 1 (EXISTENCE). *There exists a real-valued function, c^*, on A^*, $c^* : A^* \rightarrow R$, with the following properties:*

$$a_j \left\{ \begin{matrix} > \\ \sim \\ < \end{matrix} \right\}_{A^*} a_k \Leftrightarrow c^*(a_j) \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\}_R c^*(a_k), \tag{a}$$

$$c^* \big[L(a_j, a_k; p) \big] = p c^*(a_j) + (1 - p) c^*(a_k) \tag{b}$$

where $a_j, a_k \in A^*$, $p \in [0, 1]$.

THEOREM 2 (UNIQUENESS). *The function $c^* : A^* \rightarrow R$ is unique up to a positive linear transformation.*

Empirical Use Requires Representation by Performance Measures

The existence and uniqueness of a scale function, c^*, which indicates stochastic preference over A^* is an interesting result. But the goal of a compaction function is to indicate how consumers make judgments relative to attributes that describe the products. To this end, definition 1 defined compaction in terms of an attribute set, X. For empirical use, we would like to have consumers indicate stochastic preference for abstract alternatives represented by elements of X. We would then hope that if particular vectors of attribute levels, say x_l and x_m, are realized as products, say a_l and a_m, then judgments relative to X would be valid for the expanded set of products which now includes a_l and a_m, i.e., for $\alpha = A \cup \{a_l, a_m\}$. For example, if $x_l >_X x_m$, then hopefully $a_l >_\alpha a_m$. If this is true, then the preference information captured by a compaction function could be used to understand and predict consumer response to potential products. This assumption is formalized by the following axiom:

AXIOM 5. Abstract alternatives:

(a) Consumers can indicate stochastic preference and indifference relative to X.

(b) There exists a vector-valued function, $g : A \rightarrow X \subseteq R^N$ such that $g(a_j) \neq g(a_l)$ for all $a_j \neq a_l$.

(c) If x_m is realized as a physical product, a_m, and if $x_m \left\{ \begin{matrix} > \\ \sim \\ < \end{matrix} \right\}_X x_j$ then $a_m \left\{ \begin{matrix} > \\ \sim \\ < \end{matrix} \right\}_\alpha a_j$ for all $a_j \in \alpha = A \cup \{a_m\}$ where $x_j = g(a_j)$ and $a_m \neq a_j$.

We can now construct a compaction function which has the desired properties.

THEOREM 3. *Let $A = \{a_1, a_2, \ldots, a_J\}$ be set of two or more products. Suppose there exists a complete set of performance measures, X, such that Axiom 5 holds. Suppose Axioms 1, 2, 3, and 4 hold on $A \cup B$ for all finite $B \subseteq g^{-1}(X)$. ($g^{-1}(X)$ is the inverse image of X.) Let $c : X \rightarrow R$ be a real-valued function on X such that $c(g(a_m)) = c^*(a_j)$ for all $a_j \in A \cup B$, then c is a uniform, symmetric compaction function on X.*

Summary of Formal Theory

Axioms 1 through 5 identify a set of fundamental behavioral postulates which are sufficient for the existence and uniqueness of a uniform, symmetric compaction function on a set of product attributes. The theory sets up a rigorous framework for the measurement of preference functions called compaction functions. But as indicated in §1 and reviewed by Green and Srinivasan [8], preference measurement has been quite successful in marketing. But the axiomatic approach can strengthen the applications as follows:

(1) It provides testable behavioral assumptions which, if satisfied, imply useful properties for the preference function.

(2) It provides a method to explicitly model how consumers react to risk. I.e., the compaction function is defined on lotteries as well as certain products.

(3) Since the compaction function has von Neumann-Morgenstern properties for stochastic preference, the results of prescriptive utility which derive functional forms for $c(\mathbf{x}_j; \lambda_i)$ can be used for consumer behavior.

For further discussion, see the unabridged version of this paper or see Hauser and Urban [11]. Finally, since Theorem 3 proves only sufficiency, it remains for future theory to relax these axioms and search for necessary conditions.

4. Probability of Choice

The axioms of this paper provide a formal structure for the measurement of stochastic preference, but we need an explicit probability function to predict choice. We now indicate two pragmatic solutions, each of which is theoretically incomplete, plus a general model, which is consistent with Axioms 1 through 5 but not yet practical. It remains for future research to develop a model that is both theoretically complete and practical. The full discussion is in the unabridged version.

Econometrics. The scale function, v_{ij}, in the logit model acts like a symmetric compaction function, but Axioms 2 and 3 are not guaranteed for v_{ij}. Nonetheless, it is possible that v_{ij} is a monotonic transformation of a von Neumann-Morgenstern compaction function. I.e., $v_{ij} = f[c(\mathbf{x}_j, \lambda_i)]$. A particularly simple f is the range adjusting model, $v_{ij} = \beta c(\mathbf{x}_j, \lambda_i)$. This two-step process has the advantages of measuring individual specific parameters, λ_i, and of using a derived function form for $c(\mathbf{x}_j, \lambda_i)$. Its disadvantage is that f is now arbitrary and the postulates of the two theories are not entirely compatible.

Ranked probability model. Another pragmatic model is to simply rank the compaction values for each consumer and assign a probability, p_1, to the first ranked, a probability, p_2, to the second ranked, and so on for $p_3, p_4, \ldots$. In two empirical tests (deodorants and health care plans), the predictions with this simple model were slightly superior to a range adjusting logit model. Although this is a weak test, it does indicate that the rank order effect is worth further investigation.

General probability model. Both of the above approaches are practical but sacrifice some of the power of axioms 1 through 5. It is possible to derive a model consistant with the axioms. Since the model is not yet feasible, it is presented purely to spark research.

To derive this model, we use the symmetric property of the compaction function and make an assumption that any two consumers with the same compaction values have the same probabilities of choice. Then define $rc_{i1} = \max_j \{c_{i1}, c_{i2}, \ldots, c_{iJ}\}$, rc_{i2} = second largest, etc. Let $\mathbf{rc}_i = \{rc_{i1}, rc_{i2}, \ldots, rc_{iJ}\}$. Let e_1 be the event that i chooses the product with the largest compaction value. Define $e_2, e_3, \ldots$, etc., similarly. Then applying Bayes Theorem gives:

$$p(e_j \mid \mathbf{rc}_i) = p(e_j) * \left[\frac{p(\mathbf{rc}_i \mid e_j)}{\sum_l p(e_l)p(\mathbf{rc}_i \mid e_l)} \right] \tag{1}$$

where $p(\cdot)$ implies probability. It is then a matter of bookkeeping to determine the choice probabilities from $p(e_j \mid \mathbf{rc}_i)$ for all j.

Note that $p(e_j)$ is just the ranked probability p_j, thus equation 1 is a generalization of the ranked probability model. Furthermore, since the logit model is a symmetric

probability model, (1) is also a generalization of the logit model. Finally, (1) can be used when new products are added to the choice set since in most empirical cases $p(e_{J+1})$ approaches zero. The model is estimated with parametric assumptions on $p(\mathbf{rc}_i \mid e_j)$.

5. Empirical Evidence

The theoretical strength of the von Neumann-Morgenstern compaction functions would be of purely academic interest were it not feasible to directly measure compaction functions for individual consumers. For each of 76 consumers, Hauser and Urban [10] measured the following 10 parameter compaction functions with 8 indifference questions, 5 of which were lotteries.

$$c(x_{j1}, x_{j2}, x_{j3}, x_{j4}; \lambda_i) = \sum_{k=1}^{4} \lambda_k u_k(x_{jk}) + \sum_{l>k} \sum_{k} \Lambda \lambda_k \lambda_l u_k(x_{jk}) u_l(x_{jl})$$

$$+ \sum_{m>l} \sum_{l>k} \sum_{k} \Lambda^2 \lambda_k \lambda_l \lambda_m u_k(x_{jk}) u_l(x_{jl}) u_m(x_{jm})$$

$$+ \Lambda^3 \lambda_1 \lambda_2 \lambda_3 \lambda_4 u_1(x_{j1}) u_2(x_{j2}) u_3(x_{j3}) u_4(x_{j4}) + \lambda_0$$

where

$$u_k(x_{jk}) = a_k - b_k e^{-r_k x_{jk}}.$$

The parameters $\{a_k, b_k : k = 1 \text{ to } 4\}$, λ_0, and Λ were set by scale conventions and the managerially significant "preference" parameters, $\lambda_i = \{\lambda_k, r_k : k = 1, 4\}$, were determined by eight indifference questions. (The resulting λ_k measure relative importance of attributes, the r_k measure risk aversion, and Λ, which is determined by $\sum_k \lambda_k$, measures attribute interaction.) Those readers interested in the properties of this particular function are referred to Keeney [16]. Those readers interested in the measurement, the results, and the managerial implications of this application are referred to Hauser and Urban [10].

Empirically, this assessment gave reasonable predictions as is evidenced by Table 1, which compares actual *vs.* predicted market shares for the four health care plans. Table 1 also gives the share which was predicted by a logit model estimated in the same study. It is possible that the non-linear risk averse utility functions performed slightly better than the linear logit model, because they were sensitive to the perceived risk involved in switching from existing care to a new health plan.

TABLE 1

Actual vs. Predicted Market Shares

	Existing Care	Harvard Community Health Plan	M.I.T. Health Plan	Massachusetts Health Foundation
actual share	0.34	0.11	0.42	0.13
predicted share (utility)	0.30	0.08	0.42	0.20
predicted share (logit)	0.22	0.23	0.35	0.20

6. Summary

This paper began with a definition of stochastic preference and five basic axioms about stochastic choice behavior. These axioms imply the existence and uniqueness of a "compaction" function, that is, a function which identifies how consumers evaluate

products in terms of attributes and produces real numbers which are monotonic in choice probabilities.

The measurement of such functions is important for describing and predicting choice. This paper indicates the conditions under which such functions exist, how they can be measured if they exist, and how one might use such functions to estimate choice probabilities.

Hopefully, this paper will lead to improved synergy between the theoretic rigor of von Neumann-Morgenstern utility theory and empirical experience of marketing research. This area of investigation is fertile in both theoretical and practical problems, and it deserves attention from both utility theorists and marketing researchers.

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