# PROFIT MAXIMIZING PERCEPTUAL POSITIONS: AN INTEGRATED THEORY FOR THE SELECTION OF PRODUCT FEATURES AND PRICE*

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An important component of marketing strategy is to "position" a product in perceptual space. But to realize a perceptual position we must model the link from physical characteristics to perceptual dimensions and we must use this model to maximize profit. This paper integrates psychological theories on how consumers process information with economic models and psychometric measurement, and it develops a theory for selecting physical features and price to achieve a profit maximizing perceptual position.

The theory begins with a Lancaster-like transformation from goods space to characteristics space and investigates the implications of a mapping to a third space, perceptual space. We show that all products efficient in perceptual space must be efficient in characteristics space but not conversely. But consumers vary in their preference and the way they perceive products. Thus we introduce distributional components to the theory and derive both geometric and analytic methods to incorporate this consumer heterogeneity. Next, we investigate costs and derive a perceptual expansion path along which a profit maximizing position must exist. Conjoint analysis and quantal choice models provide the measurements to implement the theory.

The theory is illustrated with a hypothetical example from the analgesics market and some of the potential psychometric measurements are illustrated with an empirical application in the communications market.

(MARKETING; MARKETING—NEW PRODUCTS; MARKETING—PRICING)

## 1. Perspective

The processing of information by consumers is important from the firm's perspective. A common marketing strategy is to identify the perceptual dimensions (e.g., efficacy and gentleness of analgesic products) that define a market and then "position" the product against these dimensions so that the product is likely to be preferred over competitive products by a target group of consumers. (See review by Shocker and Srinivasan [43].) An important strategic issue is how to realize this "positioning" by selecting the appropriate physical features or product ingredients. Furthermore, understanding the intermediate links in the behavioral model of information processing (physical features → perceptions → preferences) is important to advertising strategy and other components in the marketing mix.

The problem of selecting physical features is also an important research question in economics, psychology, and marketing science. In economics, traditional models of the consumer assume that the consumer has perfect information about product characteristics (physical features) and uses a rational "utility maximization" procedure to select the goods (products) that will be consumed. But empirical evidence questions these assumptions. Work in psychology by Brunswick [7] and Hammond [18] suggests that proximal cues such as physical features are moderated by distal cues such as consumer

perceptions of the products. In marketing, there are good models for measuring perceptions and estimating the link from perceptions to preference and choice. See Shocker and Srinivasan [43] and Hauser and Urban [22]. A model which investigates the mapping from physical features to perceptions would complete the linkage and provide a valuable managerial tool.

In this paper we explicitly address the mapping from physical features to perceptual dimensions. Our perspective is that of a firm entering a market and maximizing short term profit. We do not explicitly consider the entry of competitive firms although a strong positioning by the entering firm often preempts "second in/better positioning" strategies by competitive firms. Once we better understand consumer response and how a profit maximizing firm should react. later research can use these results to construct market equilibrium models.

We begin with a brief review of the literature upon which we base our theory.

## 2.  Existing Literature

We base our theories and models on selected works in three areas of research: economic models, information processing, and psychometrics. This section reviews that research that is most relevant to this paper. More exhaustive reviews are contained in the references.

### Economic Theory of Consumer Behavior

Since our perspective is that of selecting appropriate product characteristics the relevant economic model of consumer behavior is the model developed by Lancaster [26], [27]. Lancaster begins with traditional analysis in which consumer choice is made with respect to goods and transforms the decision into a choice with respect to characteristics. Figure 1 illustrates the essence of this model for three goods and two characteristics.

Traditional analysis begins in goods space where the budget hyperplane and the coordinate axes form a convex polytope that represents a feasible region. For example, $g_1$ is the amount of good 1 that can be purchased with the consumer's budget. The consumer then selects that combination of goods that achieves the highest utility. Geometrically this selection is given by the point of tangency (edge or vertex in degenerate solutions) between the budget hyperplane and the indifference surfaces. (Assume for simplicity that goods are divisible. This will be discussed later.)



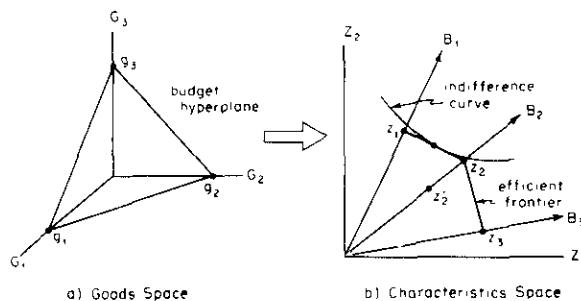a) Goods Space                    b) Characteristics Space

FIGURE 1.   Lancaster's Economic Model of Consumer Behavior.

Goods space is assumed to be transferable into characteristics space by a linear technology matrix, $B$, which indicates the amount of characteristics, $Z$, each good contains. I.e., $z = Bg$ where $z \in Z$, $g \in G$, $Z$ is the characteristics space and $G$ is the goods space. For example, the goods might include analgesics such as Anacin, Bayer, Bufferin, Excedrin, and Tylenol. The characteristics might include ingredients such as aspirin, acetaminophen, caffeine, and buffering agents (aluminum glycinate and magnesium carbonate) as well as pill size and weight. Discrete characteristics such as color complicate the model but techniques exist to handle discrete characteristics. (See Lancaster [27, pp. 108–110].) The matrix, $B$, then quantifies how much of each characteristic is contained in each separate good. For a discussion of the implications and limitations of the linearity assumption, $z = Bg$, see Lancaster [27, chapter 6], Ladd and Zober [29], [30], and Ratchford [39].

Lancaster shows that the feasible region in goods space becomes a convex polytope in characteristics space given by all convex combinations of the origin and the images of $g_1$, $g_2$, and $g_3$. The image for the $j$th product, $z_j$, is the amount of characteristics attainable if the consumer spends his entire budget for the product category on product $j$ and is given by $z_j = (k/p_j)B_j$ where $k$ is the budget, $p_j$ is the price of product $j$, and $B_j$ is the $j$th column of $B$. If preferences are monotonic in $Z_1$ and $Z_2$ and the budget is spent, the consumer will always do better by purchasing a combination of products corresponding to a point on the northeast boundary of the feasible set. This boundary is called the efficient frontier and is shown by the heavy line in Figure 1.

The extreme points, $z_j$, move in and out along vectors, $B_j$, formed by the characteristics ratios of each product. For example, if the price of product 2 is increased the consumer can buy less of product 2 for his budget. Given a sufficiently large price increase $z_2$ will move in to the point $z_2'$. If this happens the efficient frontier will connect $z_1$ and $z_3$; $z_2'$ will no longer be efficient. Note that the image of a point on the budget is not necessarily on the efficient frontier (e.g., $z_2'$), but it is easy to show that every point on the efficient frontier is the image of a point on the budget hyperplane. This is important for later analysis.

To analytically obtain those goods combinations whose images are on the frontier, we solve the "canonical linear program" given by "minimize $pg$ subject to $g \geqslant 0$, $z^* = Bg$ and $z^*$ on the efficient frontier." ($p$ is the vector of prices.) For derivations see Lancaster [27, pp. 36–37].

In the case of divisible goods the consumer will purchase that combination of products represented by the point of tangency between the efficient frontier and the indifference surfaces. For example, the consumer in Figure 1 would purchase an equal combination of products 1 and 2. (This phenomena has been proposed as one explanation of brand switching. See Blin and Dodson [6].) For indivisible goods the choice is the extreme point on the highest indifference surface; the concept of efficiency applies to the extreme points only, not their linear combinations and depends on the indifference curves. For simplicity we deal with divisible goods. The extension to indivisible goods, discussed in Lancaster [28], follows a similar conceptual framework, especially for linear indifference curves.

As presented above, the economic model applies to the consumer's entire portfolio of purchases, but it can be applied to any product category in which appropriate conditions are satisfied for the technology matrix and the preference function (Lancaster [27, Chapter 8]).

A sufficient condition for the technology matrix is that it be completely separable into submatrices such that no good outside the product category of interest shares characteristics with goods in the product category. However, such a condition may severely restrict the applicability of the model. The analysis also applies to a product category if (1) the cost of obtaining a characteristic shared with goods outside the product category is the same for all category goods (Lancaster [27, p. 137]) or (2) if the maximum contribution of the product category to a characteristic shared with goods outside the product category is small compared to the contribution to that characteristic from noncategory goods (Lancaster [27, p. 139]). These conditions are more likely to be satisfied than complete separability and may be useful in defining product categories.

The appropriate conditions on preferences for applying the model to a product category are (1) that the consumer can allocate a category budget without considering all tradeoffs within other categories, and (2) that tradeoffs among category characteristics do not depend on noncategory characteristics. Condition 1 is defined on the utility function for goods, condition 2 is defined on the utility function for characteristics. Necessary and sufficient conditions for (1), which is known as decentralizability, are that the goods utility function be weakly separable[1] with respect to the product category (Blackorby, Primont, and Russell [5]). For discussion with respect to utility trees see Strotz [48] and Gorman [16]. Condition 2 is know as preferential independence and implies a weakly separable utility function with respect to the category characteristics.

It is interesting to note that the economic conditions on the technology matrix and on preferences are all a form of separability and are thus related to the theoretical and empirical models used to identify market structure. For example, see Day, Shocker, and Srivastava [8], Tversky and Sattath [52], Urban, Johnson, and Brudnik [54].

*Information Processing*

There are many detailed models of how consumers receive, process, and retain information. See Sternthal and Craig [46] and Bettman [4]. From our perspective we are most concerned with the model in Figure 2 based on the "lens" model developed by Brunswick [7]. In this model the basic input is cues in the environment including physical characteristics ($Z$) of the product, such as the amount of aspirin and buffering agents in Bufferin. These are filtered through each consumer's cognitive process and moderated by psychosocial cues, such as advertising and peer pressure. Because of imperfect information and because of simplifications in information processing and decision rules, consumers abstract these characteristics into a relatively few dimensions such as "efficacy" and "gentleness." Because these dimensions respre-
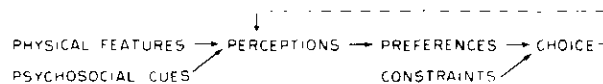


FIGURE 2.  Simplified Lens Model of Consumer Decision Making.

[1]For example, if $g$ is the goods vector for the product category and $\bar{g}$ is the goods vector for all other goods, then a utility function, $U(g, \bar{g})$ is weakly separable if there exist functions, $u, \bar{u}, \hat{U}$, such that $U(g, \bar{g}) = \hat{U}[u(g), \bar{u}(\bar{g})]$ and $\hat{U}$ is strictly increasing in its arguments.

sent how consumers perceive and interpret the product's characteristics we call these dimensions perceptual dimensions. (Let $Y$ be the set of perceptual dimensions.) Based on these perceptual dimensions the consumer forms preferences and makes his choice subject to situational constraints such as time and money budgets and product availability. Once the consumer tries the new product he updates his perceptions and may readjust his preferences and choice behavior.

The perception, preference, choice component of this model has been used for a variety of product categories, predicts preference and choice quite well, and has been supported by correlational analysis. (See Hammond [18] and Shocker and Srinivasan [34]. Tybout and Hauser [53] present correlational analyses and statistical tests of necessary conditions which are consistent with (do not reject) the full model.) Two empirical facts will be important for later analyses: (1) the number of perceptual dimensions tends to be relatively few (usually 2, 3, or 4) and almost always fewer than the number of physical cues and (2) due to measurement error and/or individual differences there is heterogeneity in measured perceptions.

*Psychometrics*

A key element in a practical application of our model is the measurement of the functional mapping of the physical features, $Z$, into the space of perceptual dimensions, $Y$. To operationalize our model we must be able to measure elements, $y$, of the set $Y$ and estimate the parameters of the mapping $F : Z \rightarrow Y$.

There are at least three methods currently used to identify perceptual dimensions: non-metric scaling (Green and Rao [14]), discriminant analysis (Pessemier [38], Johnson [24]), and factor analysis (Urban [54]). Nonmetric scaling asks consumers to evaluate the similarity of stimuli in their choice set and identifies the dimensions that best reproduce those similarity judgments. Discriminant and factor analyses are statistical procedures which identify structure within a large number (15–30) of scales such as "Excedrin relieves headaches fast." While these measurement methods do not guarantee that the underlying constructs, perceptions, are measured, many applications have strong face validity and predict preference and choice quite well. For example, in a number of studies the preference and choice models based on factor analysis dimensions have predicted roughly 50–80 percent of the consumers correctly [19]–[22], [34], [45], [53], [54], [57]. However, as we show later, theoretical considerations require that the perceptual dimensions be ratio-scaled in order to be used in our model. Thus, if perceptions are identified by traditional techniques, they must be rescaled before they are used in our model. Measurement techniques such as graded paired comparisons (Scheffe [41]) or constant sum paired comparisons (Torgerson [50]) can theoretically provide the necessary ratio properties. Hauser and Shugan [20] provide statistical tests to identify whether the ratio properties are achieved.

There are at least two ways to estimate the mapping $F$: conjoint analysis and experimental variation. In general, conjoint analysis (Tversky [51]) is a procedure for estimating a mapping from one set of variables into a single variable that is at least ordinally scaled. Specific forms of conjoint analysis have been widely used in marketing to estimate mapping from physical features or perceptions to preference. (See review by Green and Srinivasan [15].) Recently conjoint analysis has been modified to estimate the mapping from physical features to perceptions. See Green and DeSarbo [13], Neslin [34], and Urban and Neslin [57]. Basically each consumer is presented with

a series of real or artificial stimuli in which the physical features are systematically varied and he is asked to rank order (or intensity scale) the stimuli with respect to the dependent measure (preference or a perceptual dimension). The parameters of the functional mapping are then estimated with regular regression, monotonic regression, or linear programming. Estimated mappings have held up well under predictive tests. For example, Wind and Spitz [58] report 86% correct prediction on holdout samples of stimuli; Parker and Srinivasan [35] report 88% correct prediction on cross validation tests; Wittink and Montgomery [59] report 63% correct prediction of behavior; and Urban and Neslin [57] persuaded managment to act on their suggestions and results were within predicted ranges.

Experimental variation is a form of conjoint analysis in which the physical features are systematically varied and each consumer is exposed to just one combination of physical features. In such experiments the dependent variable is the measured perceptual dimension and the treatments are the levels of objective physical features to which each group of consumers is exposed.

Whichever technique is used, the dependent measure must be ratio-scaled for use in our model. If conjoint analysis is used then $F$ can be theoretically estimated for each consumer. In experimental variation, an aggregate $F$ is estimated and care must be taken to ensure that the aggregate $F$ represents the consumer population. The researcher may wish to test homogeneity and, if necessary, redo the analysis within homogeneous segments. (See Wittink and Montgomery [59].)

*Discussion*

Lancaster's model provides a mechansim for dealing with physical characteristics and mapping them into preferences when the utility function is defined directly on physical characteristics. But information processing suggests that the model be extended to handle an intermediary construct, consumer perceptions of products. Furthermore, by adding the intermediate construct we are able to provide managers with diagnostic information in the form of perceptual maps which can be used in product positioning. Since it is potentially feasible for the parameters of the mapping from characteristics space into perceptual space to be estimated by psychometrics, it is reasonable to consider the implications of such an intermediary mapping.

First, there will be some implications from introducing the perceptual space, $Y$-space, into the analysis. But in applications the analysis is complicated still further. Unlike objective physical characteristics which are the same for all consumers and can, in theory, be objectively measured, perceptions can vary by individual and are always subject to measurement error. These effects must be incorporated in the model.

Since our focus is the firm's perspective we must add a normative component which indicates how a firm can proceed to maximize profit given measures of $F$, $B$, and consumer preference and budget structures. While profit maximization can be handled by a general math program (Shocker and Srinivasan [42], Gavish, Horsky, and Srikanth [11], and Zufryden [60]) the consumer theory developed here provides structure and response models that have the potential for improving managerial control and understanding of the optimization.

## 3.  Theory

Our theory deals with the general case of heterogeneous perceptions and preferences, but for ease of exposition we introduce the heterogeneity in stages. We begin

with the case of homogeneous perception and discuss the implications of the $F$ mapping. We then consider the more realistic case of heterogeneous perceptions but for simplicity restrict the analysis to homogeneous preferences. This assumption is then relaxed (heterogeneous preferences) and production costs are added to the analysis (perceptual realization path). The final step in the theory development is the structure of profit maximization. Throughout the discussion we focus on physical features. The *theory* applies equally well if psychosocial cues, such as advertising spending and advertising copy, are added to the model as managerial inputs ($Z$). For example, it is conceptually possible to experimentally establish a causal link from alternative copy themes to perceptual positions. In fact, if alternative copy themes are equally expensive then only the best theme will be efficient. However, estimation of the $F$ mapping may be more difficult and expensive for psychosocial cues. Furthermore, if psychosocial cues are explicitly modeled, consideration must also be given to interactions among the psychosocial cues and the physical characteristics. (Interactions increase the dimensionality of the $Z$-space and hence the difficulty of using optimization algorithms.)

We purposively deal with the forward linkage in the Lens model, characteristics $\rightarrow$ perceptions $\rightarrow$ preference, since our focus is on product design and realization. The behavioral feedback loop (dotted line in Figure 2) applies when the product is being tested and repeat purchases are observed.

### Homogeneous Perceptions

If perceptions are homogeneous then the mapping, $F$, does not vary by individual and $y = Fz$ for all consumers. (We have written the mapping as linear since nonlinear mappings, e.g., Stevens [47], can be approximated with piecewise linear functions. We illustrate this approach in our empirical example.) If the mapping is homogeneous then $y = Fz = FBg$ and Lancaster's model can be used directly if perceptions are ratio-scaled. Ratio-scaling is important since we divide by price to form dimensions such as efficacy per dollar. This has been suggested earlier by Ratchford [39], although he did not deal explicitly with the scaling of perceptions. Nonetheless, some insight can be gained by considering the $F$ mapping directly.

Assuming for a moment that perceptions are ratio-scaled, the analysis in $Y$-space (Figure 3) looks much like the analysis in $Z$-space. For example, $Y_1$ might be "efficacy per dollar" and $Y_2$ might be "gentleness per dollar." The point, $y_1$, might be the amount of "efficacy per dollar" and "gentleness per dollar" obtainable by purchasing Tylenol, $y_3$ might correspond to Bayer, and $y_2$ might be a potential new product.

The managerial goal is then to move the firm's product (say $y_2$) up to and beyond the efficient frontier formed by existing products. The movement (dotted line) will depend upon $F$ and the preference and cost structure. Note that the movement is not necessarily along $FB_2$ since new ingredients could change the perceptual ratio.
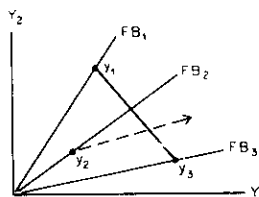


FIGURE 3. Analysis in Perceptual Space.

For example, Figure 4a is a traditional perceptual map of the analgesics category. Note that Tylenol. Bufferin. and Excedrin appear to be well perceived while the private label brand appears to be poorly perceived. Establishing a zero-point and rescaling by price we get the perceptual map in Figure 4b.[2] For example, Bufferin costs roughly 1.8¢ per tablet while the private label brand costs roughly .9¢ per tablet. Notice that Bayer (1.2¢ per table) and the private label brand are now efficient in Figure 4b while Bufferin is, on average, inefficient. (Bayer was the market leader at the time of the analysis (Media Decisions [33]).) A brand like Anacin (1.8¢ per tablet) could become efficient by lowering its price or changing its ingredients.[3]



a) Traditional Perceptual Map        b) Price-scaled Perceptual Map
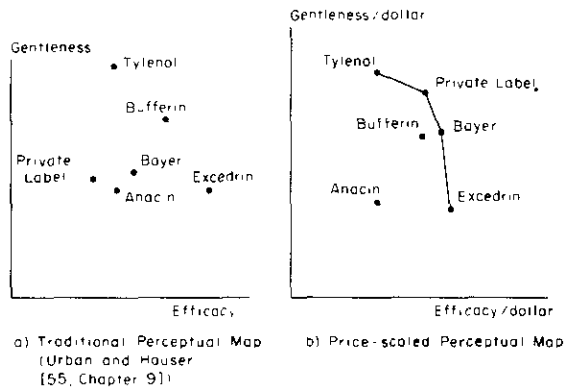   (Urban and Hauser
   [55, Chapter 9])

FIGURE 4.    Illustrative Perceptual Maps.

As in Figure 4, the identified dimensionality of perceptual space (in this case 2 dimensions) is usually less than the dimensionality of characteristics space (in this case more than 8 dimensions). Such reduced dimensionality has important consumer behavior implications.

THEOREM 1.    *For nondegenerate cases the rational consumer will choose at most as many products as there are perceptual dimensions.*

PROOF.    The rational consumer will choose a point on the efficient frontier in $Y$-space. But nondegenerate facets of the efficient frontier will be defined by at most as many points as there are dimensions in $Y$-space. Thus the rational consumer will choose a point that is a convex combination of at most as many products as there are dimensions.

Of course more products may be considered in degenerate cases when more than $N$ points define a facet in $N$-dimensional space. For example, a degenerate case would occur in Figure 3 if $y_2$ were on the line connecting $y_1$ and $y_3$.

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[2] Figure 4a is an actual map but since the zero-point in Figure 4b is arbitrary, Figure 4b must be considered hypothetical.

[3] In categories such as analgesics price can play a dual role. It can appear both in the denominator scaling the perceptual dimensions and in the $F$-mapping as an objective characteristic which affects perceptions of perceived benefit. The model still applies but we can not exploit homothedicity in the psychometric measurement. (See §4.) In Figure 4b we assume all of these brands indeed compete in the same market (branch of the utility tree).

Assuming the motive for brand switching is to obtain an optimal combination of perceived benefits, theorem 1 implies that brand switching (Bass [2]) should be among approximately as many products as there are perceptual dimensions (approximate because of degenerate solutions). This is also consistent with evidence by Urban [54] and Silk and Urban [44] that consumers only consider a small number of potential products (an average of 3 out of 18 in the case of analgesics).

THEOREM 2. *The rational consumer will always choose a product that is efficient in physical characteristics even though he simplified the evaluation process through perceptions.*

PROOF. By definition the rational consumer will choose an efficient point in $Y$-space. But because the feasible region in $Z$-space is a polytope and because the mapping is linear the feasible region in $Y$-space is also a polytope defined by all convex combinations of the images of the extreme points in $Z$-space. Thus every extreme point in $Y$-space is an extreme point in $Z$-space and every point on the efficient frontier in $Y$-space is on the efficient frontier in $Z$-space.

THEOREM 3. *If the consumer evaluates products in perceptual space then any consumer analysis that does not consider the perceptual mapping, F, could identify "consumer-optimal" combinations of product that are not efficient (in Y-space).*

PROOF (BY COUNTEREXAMPLE). Suppose that Tylenol is based on one ingredient (acetaminophen), that Bayer is based on another ingredient (aspirin) and that the new product is based on a new ingredient. If these are the only analgesics then all will be efficient in characteristics space independent of price. If the new ingredient produces finite efficacy and gentleness then there exists a sufficiently large price that forces the new product to be inefficient (as shown in Figure 3). Thus any convex combination involving the new product is efficient in characteristics space but not in perceptual space.

It is interesting to note that even if all extreme points in $Z$-space are extreme points in $Y$-space there may exist combinations of products that are efficient in $Z$-space that are not efficient in a lower dimensional $Y$-space because the lower dimensionality of $Y$-space will rule out all convex combinations (except degenerate combinations) involving more products than there are dimensions in $Y$-space. Furthermore, since the above counterexample can hold for three or even four perceptual dimensions, theorem 3 is not dependent on there being fewer dimensions in $Y$-space than in $Z$-space.

Following Lancaster [27] we can define a canonical linear program that identifies efficient goods combinations. It is "minimize $pg$ subject to $g \geqslant 0$, $y^* = FBg$, and $y^*$ on the efficient frontier in $Y$-space." The implications are similar to those for $Z$-space.

The final point that we must discuss is the scaling of perceptions. Following Lancaster's model the product positions in $Y$-space, $y_j$, are given by $y_j = (k/p_j)FB_j$. Since we handle price by moving $y_j$ in and out along $FB_j$ we implicitly assume that the measured perceptual positions can be ratio-scaled. In general, they are not. Common measures such as factor scores and discriminant scores are at most intervally scaled because they are estimated as composites of non ratio variables, but no zero-point is defined. Many non metric scaling algorithms also produce intervally scaled measures but again the axes are arbitrary.

To make full use of the theory, we must develop psychometric scaling procedures to give ratio-scaled perceptions. There is no conceptual problem with ratio-scaling

perceptions although we know of no published attempts to do so empirically. Nonetheless, ratio-scaled preference measurement (constant sum paired comparisons) is now well developed and has been shown to correctly predict approximately 80% of consumer preferences across a number of product categories (Silk and Urban [44], Hauser and Shugan [20]). Furthermore, Neslin [34] has shown empircally that intensity measurement (graded paired comparisons) is well adapted to perception measurement.

To establish the feasibility of ratio-scaling perceptions we developed and tested a number of alternative survey formats. The formats that were most successful emphasized value and asked consumers to respond with a direct ratio to indicate their perceptions of the value of $Y$. For example, in a small pretest, consumers who respond to the format in Table 1 provided data which passed the Hauser-Shugan ratio tests at the 0.05 level. (We used a design requiring 18 paired comparisons per dimension for scaling the perceptions of the six analgesics brands.) Clearly further development is necessary to simplify this measurement and apply it in a large empirical case study.

TABLE 1

*One Potential Question Format to Ratio-Scale Perceptions*

We would like you to state how much value of gentleness you get from the different pain-relievers. We will ask you to compare the pain-relievers pair-wise and indicate each time how much more gentle you think one pain-reliever is per dollar than the other.

For example:

*Value of Gentleness*

Excedrin at $1.80                  $\boxed{1\frac{1}{2} \ / \ 1}$                  Bayer at $1.20

In the above example, the opinion was that Excedrin has $1\frac{1}{2}$ times as much value of gentleness as Bayer, indicating that Excedrin was $1\frac{1}{2}$ times a better buy as far as gentleness is concerned.

If you had felt that a certain amount of Excedrin at $1.80 was as gentle per dollar as the same amount of Bayer at $1.20 you would have put a one (1) in both sides, $\boxed{1/1}$. If you felt that Bayer at $1.20 had twice as much value of gentleness as Excedrein at $1.80, you would have filled out $\boxed{1/2}$.

Remember, there are no right or wrong answers. We are interested in your opinion and your opinion is what is important to us.

| Bayer at $1.20 | $\boxed{\ \ /\ \ }$ | Bufferin at $1.80 |
|---|---|---|
| Anacin at $2.40 | $\boxed{\ \ /\ \ }$ | Tylenol at $2.40 |
| Excedrin at $2.40 | $\boxed{\ \ /\ \ }$ | Bufferin at $2.40 |

## Heterogeneous and/or Uncertain Perceptions (Homogeneous Preferences)

Under many circumstances it is reasonable to approximate preferences with a linear function of the perceptual dimensions. For example, Green and Devita [12] show that linear functions approximate well nonlinear preference models. Einhorn and Klein-

muntz [10] present information processing evidence that justifies a linear approximation. One advantage of a linear utility function is that the consumer will select an extreme point of the efficient frontier except when the indifference curve is parallel to the efficient frontier. Thus, the consumer will usually choose a single product rather than a convex combination. This linearity assumption can later be relaxed with piece-wise linear functions. Note that this linearity assumption does not require that preferences be linear in characteristics if we use the piece-wise linear approximation suggested for the $F$-mapping. ($Z \rightarrow Y$ is nonlinear hence $Z \rightarrow$ preference is nonlinear even if $Y \rightarrow$ preference is linear. Nonlinearity in characteristics is illustrated in our empirical example.)

Linear preference functions can then be represented geometrically as unit vectors in $Y$-space. We call these unit vectors "ideal" vectors. The relative importance of the perceptual dimensions are proportional to the directional cosines, $\theta$, between the coordinate axes and the preference vector. For example, if "efficacy per dollar" and "gentleness per dollar" were equally important to the consumer then the preference vector would be along the 45° line with the directional cosines, $\theta_1$ and $\theta_2$, both equal to 0.707. In this case $\theta = (0.707, 0.707)$. See Figure 5.

We model the perceptual positions of each product, $y_j$, as normal random variables with means $\bar{y}_j$ and covariance matrix $\Sigma_j$. (If the distribution of perceptions is not unimodal then we can segment the population and deal with each segment separately. This presents no conceptual problem if profit is maximized over all segments simultaneously, although it may pose practical problems.) The variance can be due to both heterogeneous perceptions ($F$ varies by individual) and/or measurement error. Geometrically we represent the perceptual positions as ellipsoids in perceptual space where the width of the ellipsoids corresponds to $\Sigma_j$. For diagonal $\Sigma_j$ the axes of the ellipsoids are parallel to the coordinate areas. See Figure 5.
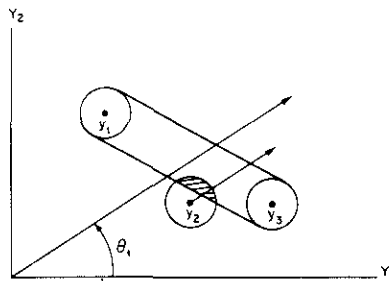


FIGURE 5. Geometric Representations of Heterogeneous Unimodal Perceptions.

The efficient frontier is now a random variable derived from the random vector $\{y_1, y_2, \ldots, y_J\}$. That is, the efficient frontier is a set of facets for each individual but we model these facets as random variables distributed across the population. Intuitively it helps to visualize this distribution as having most of its density in the region connecting the ellipsoids of the products farthest to the northeast.

As a product, say $y_2$, in Figure 5, moves out to the northeast; it is more likely to be efficient. Although the probabilistic computations developed below are more complex, a good geometric visualization of the likelihood that $y_2$ is efficient is given by the area

of overlap between $v_2$'s ellipsoid and the convex region formed by connecting the ellipsoids of $y_1$ and $y_3$. This is the shaded region in Figure 5. The more $y_2$ moves out parallel to the ideal vector, the more likely it is to be chosen.

The concept of the efficient frontier as a random variable provides a hypothesis why some products, such as the hypothetical case of Bufferin in Figure 4, are, on average, inefficient, yet have nonzero market shares. Thus the model can be parsimonious (few perceptual dimensions) yet provide a consistent interpretation of empirical observations.

To compute the probability that a product will be preferred we define $\zeta_j = \theta y_j$. So defined, $\xi_j$ represents the preference value for product $j$. The probability that product $j$ is chosen, $L_j(\theta)$, is equal to the probability that $\xi_j$ is greater than $\xi_l$ for all $l \neq j$. That is

$$L_j(\theta) = \text{Prob}(\xi_j \geq \xi_l \text{ for all } l). \tag{1}$$

Since the $y_j$ are normal, $\theta y_j$ is a sum of normal random variables and hence $\xi_j$ is a normal random variable with mean, $\theta \bar{y}_j$, and variance, $\theta \Sigma_j \theta'$.

Let $\psi_{jl}$ be the covariance matrix relating $y_j$ and $y_l$ and let $\Psi$ be a $J \times J$ matrix with elements $\theta \psi_{jl} \theta'$. Note that $\psi_{jj} \equiv \Sigma_j$. Define $M$ such that $M[\xi_1, \xi_2, \ldots, \xi_J]' = [\xi_1 - \xi_2, \xi_1 - \xi_3, \ldots, \xi_1 - \xi_J]'$ then:

$$L_1(\theta) = \int_0^\infty \cdots \int_0^\infty \phi_{J-1}(x \mid M\theta\underline{Y}, M\Psi M') \, dx \tag{2}$$

where $\phi_{J-1}(x \mid m, s)$ is a $J-1$ variate normal density with mean $m$ and covariance $s$.

We recognize equation 2 as a variation of the well known multinominal probit model. For example, for $J = 2$, $L_1(\theta)$ is given by (Domencich and McFadden [9, p. 68]):

$$L_1(\theta) = \phi\left[ \theta(\bar{y}_1 - \bar{y}_2) / \sqrt{\theta(\psi_{11} + \psi_{22} - 2\psi_{12})\theta'} \right] \tag{3}$$

where $\phi$ is the cumulative standard normal density. For $J > 2$ equation 3 has no convenient closed form although computational algorithms exist for reasonable sized problems. (The 1980 version of "CHOMP" on the CDC-6600 can handle 2000 observations, 20 alternatives, and 14 parameters.) See McFadden [32] and Albright, Lerman, and Manski [1] for practical maximum-likelihood techniques to estimate $\theta$. Note that unlike McFadden [32, equation 7], our uncertainty comes from heterogeneity in the $y_j$'s rather than heterogeneity in $\theta$.

The identification of consumer heterogeneity in $Y$-space with a quantal choice problem is an important result for our purposes. (This is also consistent with earlier suggestions by Ratchford [40].) The algorithms from quantal choice analysis provide a practical means to estimate $\theta$ and to associate market shares with points in the perceptual space ($Y$-space).

### Heterogeneous Perceptions and Preferences

For heterogeneous preferences $\theta$ can be treated as a probability distribution across the population. The probability that product $j$ is chosen, $L_j$, is given by

$$L_j = \int_\theta L_j(\theta) f(\theta) \, d\theta \tag{4}$$

where $f(\theta)$ is a probability density function for $\theta$. (4) is conceptually simple but computationally difficult.

Alternatively, we can approximate (4) if we assume that both $\theta$ and $F$ are random variables such that the composite random variable. $\theta F$, is normally distributed. Since $z_j$ is a physical characteristic, and hence not a random variable, the preference value. $\xi_j = \theta F z_j$, can be represented by a normal distribution with *unknown* variance-covariance matrix, $\Psi$. This is the general case that is addressed in most quantal choice analysis. See review by McFadden [32]. Here we use the probit model with $F z_j$ as explanatory variables to find a representative $\theta$. In other words, to get $\theta$ we use the same empirical model that we used in (2), however, we are careful to interpret the resulting equal probability contours (based on $\theta \bar{F} z_j$) as approximations to the true isodemand contours. See McFadden [32] for the discussion of aggregation issues. For unimodal distributions of perceptions we deal with a map of *average* perceptions by using the probit model to associate a market share with every positioning of the new product. (The reasonableness of this approximation remains on empirical question.)

For problems that are too large or complex for currently available multinominal probit algorithms we can approximate the estimates of $\theta$ and the market shares with the multinominal logit model (McFadden [31]). For example, the comparison table in Domencich and McFadden [9, p. 57] shows that logit is often a good approximation to probit. For a list of widely available multinominal logit programs, see McFadden [32].

*Summary of Consumer Model*

The theory developed above can be quite general incorporating heterogeneous and intercorrelated perceptions and heterogeneous preferences as well as non-linear mappings from characteristics to perceptions and perceptions to preferences. Furthermore, with reasonable approximations (linear in parameters perception to preference mapping, piece-wise linear characteristics to perceptions mapping) the theory can be made operational with standard econometric quantal choice models.

The conceptual model extends economic theory. The addition of the intermediate construct, perceptions, to the economic model makes that model more consistent with psychological theory and can provide additional managerial insight. For example, consider the counterexample used to prove Theorem 3. If the manager considers only characteristics space, he would believe that any price would make his new product efficient, but when he considers perceptual space he can determine a maximum allowable price, $p_2^*$, such that his new product is efficient only if the price, $p_2$, is less than $p_2^*$. Furthermore the addition of heterogeneity of perceptions (Figure 5) introduces the concept of heterogeneity in efficiency and makes the model consistent with the empirically observed effect that some products can be inefficient on average, but still have nonzero market share. The good empirical fit of logit choice models, based on perceptions (50–80 percent correct prediction) is consistent with this hypothesis [19], [23], [53]. Future experiments may test this implication further.

The conceptual model also adds insight to marketing practice. Figure 3 suggests a new interpretation of perceptual maps. The explicit incorporation of price suggests the need for ratio-scaled psychometric measurement. The simple concept of value, e.g., efficacy per dollar, is an interesting interpretation of consumer behavior that has not been fully exploited by traditional consumer models defined on a nonnormalized perceptual space.

We have provided two derivations, analytic and geometric. The analytic models make full use of disaggregate data and provide consistent estimates of market shares. The geometric models give intuitive insight for managerial interpretation. For homogeneous preferences the probit derivation allows us to use an average perceptual map.

Lines perpendicular to the $\theta$-vector are then isodemand curves. For heterogeneous preferences these linear isodemand curves provide approximations, although we must use equation 4 for more exact estimates.

For the remainder of our analyses we use the average map to illustrate our theory recognizing that more exact market shares may be computed with (4).

*Perceptual Realization Path*

Perceptual maps such as Figure 4b suggest to the firm how to position a new product relative to the market, but profit maximization must also consider the costs of achieving alternative perceptual positions. In particular, with any given characteristics combination, $z_j$, there will be an associated cost, $C(z_j)$. Since characteristics combination, $z_j$, corresponds to a position in perceptual space, $\bar{y}_j = \bar{F}z_j$, we can associate the cost, $C(z_j)$, with the position, $\bar{y}_j$. For example, if we know the costs of acetaminophen, aspirin, caffeine, and salicylamide (the ingredients of Excedrin) as well as production and advertising costs we can calculate the cost per tablet to produce a position in characteristics space. Since this position corresponds to a position in perceptual space (the image under $\bar{F}$), we know the cost of that position.

In general there is more than one characteristics combination corresponding to a perceptual position. We are interested only in the least cost characteristics combination. We call these least cost characteristics combinations, cost effective.

Mathematically, let $\zeta_c(p_j) = \{ z \mid C(z_j) = c, p_j \}$ be the set of equal cost characteristics combinations.[4] Note the explicit dependence of the set on price, $p_j$. The image, $\bar{F}\zeta_c(p_j)$, of $\zeta_c(p_j)$ under the mapping $\bar{F}$ traces an isocost contour in perceptual space. A point $\bar{y}_j^c \in \bar{F}\zeta_c(p_j)$ is then cost effective is there is no other point, $\bar{y}_j'$, such that $\bar{y}_j' = \bar{F}z_j'$, $C(z_j') \leqslant C(z_j)$, $\bar{y}_j'$ weakly dominates $\bar{y}_j$ along all dimensions, and $\bar{y}_j'$ strongly dominates $\bar{y}_j$ along at least one dimension. Geometrically, the cost effective portion of $\bar{F}\zeta_c(p_j)$ will be the northeast portion of $\bar{F}\zeta_c(p_j)$ that does not cross lower equal-cost contours. It is interesting to note that price changes simply rescale $\bar{y}_j$, e.g., if $p_j' = 2p_j$ then $\bar{y}_j' = (1/2) \bar{y}_j$. Thus, the isocost contours in $Y$-space are homothetic with respect to price, i.e., price changes simply rescale the isocost contours. Of course if price also acts as an objective characteristic affecting perceptions, the relationship is not necessarily homothetic.

For example, suppose for simplicity that there are just three ingredients of interest in the analgesics example:[5] an active ingredient $A$; a buffering agent, $B$; and a new ingredient, $I$. Suppose the new ingredient is due to a new technology that makes it slightly stronger than the active ingredient yet gentler to the stomach. Suppose the efficacy per dollar, $E$, and gentleness per dollar, $G$, are given by $E = (A + 1.5\, I)/p_j$ and $G = (B + 0.5\, I)/p_j$. Suppose that the cost, $C$, is given by $C = (A^2 + B^2 + I^2)/6$. Thus, the isocost contours are spheres in characteristics space and become ellipses in perceptual space. The equation of the cost effective isocost contour, $C = p_j^2(5E^2 - 6EG + 13G^2)/84$, is determined with Lagrange multipliers by minimizing cost subject to a fixed perceptual position.

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[4] Lancaster [28] calls the northeast boundary of this set the "product differentiation curve (PDC)." He assumes that PDC's are homothetic with respect to $c$, but we do not need that assumption for our analysis.

[5] We caution the reader that in the real analgesics market psychosocial cues, as well as physical characteristics, play a large role in product positioning. Any practical application to such image-laden products would naturally have to address the impact of psychosocial cues. (See §5.) Furthermore, a practical application would address the dynamic pricing that results from the extensive promotional deals that occur in the analgesics category.

Given an isocost contour the firm should position on the highest isodemand curve as defined by $\theta$ or $f(\theta)$. As shown in Figure 6a, this position will be given by the point of tangency between the cost-effective portion of $\bar{F}\zeta_c(p)$ and the isodemand curves. If the cost-effective portion of $\bar{F}\zeta_c(p)$ is not concave choose the highest isodemand curve intersecting $\bar{F}\zeta_c(p)$. Together these points trace out a *perceptual expansion path* for the firm. Since market shares are given by the isodemand curves, the *maximum profit position will be along the perceptual expansion path*. The optimal characteristics combinations are then the minimum cost inverse image of the perceptual expansion path.



a) Structure of Expansion Path

b) Example Expansion Path and Optimal New Product Position
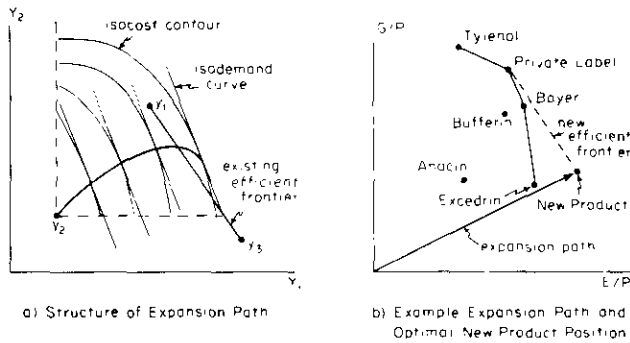
FIGURE 6. Perceptual Expansion Path.

For example, suppose that the preference function, $\theta Y$, is given by $\theta Y = E + G$. To determine the expansion path we minimize cost subject to constant demand. The optimal solution, determined by Lagrange multipliers, is a straight line, $E = 2G$. This is shown in Figure 6b. The optimal characteristics combination is given by $A = B$ and $I = 2A$. Thus, the new product should stress the new ingredient, but not use it exclusively. It is interesting to note that any finite amount of $I$ would make the analgesic efficient in characteristics space, but we need a minimum ratio of $I$ to $p_I$ before the analgesic is efficient in perceptual space.

## Profit Maximization

The analysis leading to Figure 6 reduces the profit maximization problem from multiple dimensions (characteristics plus price) to two dimensions (expansion path and price). Let $x_p \in E_p(Y)$ be a point on the expansion path and let $\pi(x_p, p) = [p - C(x_p)]$ $S(x_p)V$ be the profit associated with $x_p$, where $C(x_p)$ is given by the minimum cost inverse image of $x_p$ and $S(x_p)$ is the share associated with $x_p$. Here, for simplicity, we have assumed that market volume, $V$, is fixed, although this may be relaxed by including frequency of purchase in the probit model, or by obtaining explicit measures of consumers' implicit category budgets. We then obtain the maximum profit, $\pi^*$, by a joint optimization over $x_p$ and $p$.

Without loss generality, we can view this estimation as a recursive process. Fix $p$ and let $\pi_p^* = $ maximum $\pi(x_p, p)$ subject to $x_p \in E_p(Y)$. Let $x_p^*$ be the position associated with $\pi_p^*$. By the previous section the profit maximum for a given price must be on $E_p(Y)$, thus, $\pi_p^* \geqslant \pi(y, p)$ for all $y \in Y$. Now choose the price to maximize $\pi_p^*$. If $\pi^* = $ maximum $\pi_p^*$, and since $\pi_p^* \geqslant \pi(y, p)$ for all $y \in Y$, then, $\pi^* \geqslant \pi_p^* \geqslant \pi(y, p)$.

Thus $\pi^*$ is the profit maximum and the associated $(x_p^*, p^*)$ is the optimal position and price.

Although the maximization can proceed either directly or through the recursive process, we prefer the recursive approach since it gives the manager a set of combinations $(x_p^*, \pi_p^*, p)$ that allow him to consider other criteria in addition to profit. For example, if he has a target position, then the model tells him the best characteristics and price combination for that position and the associated profit. This use of the model is extremely useful if the manager is positioning his product to preempt competitive entry. Alternatively he can set the optimal characteristics combination for a given price. This use of the model may be appropriate if price is regulated.

For example, suppose that $S = (\theta Y - 6)/6$ for $6 \leqslant \theta Y \leqslant 12$, $S = 0$ for $\theta Y \leqslant 6$, and $S = 1$ for $\theta Y \geqslant 12$. Since $A = B = \frac{1}{2}I$ along the expansion path $C^* = I^2/4$ and $(\theta Y)^* = 3I/p$. Profit, $\pi(x_p, p)$, is given by

$$\pi(x_p, p) = (p - I^2/4) \cdot (1/2p - 1) \cdot V. \tag{5}$$

Using the recursive optimization we optimize first with respect to the ingredients which gives us

$$I^* = 2\left(p + \sqrt{p^2 + 3p}\right)/3 \quad \text{and} \tag{6}$$

$$\pi_p^* = 2V\left[p^2 - 9p + (3 + p)\sqrt{p^2 + 3p}\right]/27.$$

Finally, we determine the price that maximizes $\pi_p^*$ in the range $6 \leqslant \theta Y \leqslant 12$. The optimal price $p^* = 1/8$, the optimal ingredients, $I^* = \frac{1}{2}$, $A^* = \frac{1}{4}$, $B^* = \frac{1}{4}$, and the optimal profit, $\pi^* = V/16$. The optimal perceptual position, $E^* = 8$ and $G^* = 4$ is shown in Figure 6b. The new product is efficient in the new market. Furthermore, it is likely to draw share from those products, Excedrin and Bayer, that are made inefficient (on average) by its entry.

This completes the theoretical development illustrated with a numerical example. We now address the measurement issues and illustrate one measurement approach with an empirical example.

## 4.  Measurement: Empirical Example

In §3 we attempted to develop a theory that could be implemented in practical situations. We built on existing measurement techniques whenever possible and, when development of new measurement technology was necessary, we limited ourselves to techniques that we felt were feasible to develop.

In order for our theory to be feasible we must be able to measure $\bar{F}$, $\theta$, and $C(z)$. We measure $\bar{F}$ for a fixed price when the isocost contours and the expansion paths are homothetic with respect to price. That is, if we establish a ratio-scaled $\bar{F}_p$ for a given price, $p$, then for a new price, $p'$, $\bar{F}_{p'}$ is given by $\bar{F}_{p'} = (p/p')\bar{F}_p$. Of course, to enhance validity researchers may wish to measure $\bar{F}$ for a series of prices and check the ratio properties. Measurement of $\theta$ is well developed through quantal choice model estimation techniques. Determining $C(z)$ is an accounting function which, although may be difficult, is certainly feasible.

Rather than discuss measurement in general we refer the reader to Krantz, Luce, Suppes, and Tversky [25]. Instead we present an empirical example drawn from the communications market. This example highlights some of the measurement issues and suggests one set of practical solutions specific to the managerial problems faced in that particular new product development. We hope this example will facilitate discussion among marketing scientists, economists, and psychometricians to develop more and better techniques to obtain $\bar{F}$, $\theta$, and $C(z)$.

Our goal in the case study was to select the physical characteristics of a new telecommunications device, Narrow Band Video Telephone (NBVT), so that it would compete effectively with telephone and personal visit. (NBVT is a device that transmits still pictures over ordinary telephone lines.) Since NBVT was still in research and development, $C(z)$ was based on managerial estimates. Price was not yet a major issue since the research and development (R&D) team was interested in developing a product that was perceived well relative to telephone and personal visit. Thus we were able to use traditional techniques (nonratio) to obtain $\bar{F}$. Future analysis for profit maximization (once R&D is complete) will require ratio-scaled dependent measures such as suggested in Table 1.

*Consumer Task*

An earlier study (Hauser and Shugan [20]) had established: (1) that the perceptual dimensions in the communications market under study were effectiveness and ease of use and (2) that NBVT was clearly inefficient with respect to telephone and personal visit. We elected to sample from the same population used in the earlier study, practicing managers in the Chicago area, and to use the same perceptual dimensions. The sample was 144 consumers of which 54% or a total or 78 consumers returned completed questionnaires. This analysis sample is sufficient to illustrate the feasibility of the measurements.

Each consumer was asked to complete a twelve-page questionnaire in which he(she) indicated usage characteristics and evaluated personal visit, telephone, NBVT, and two other technologies (closed circuit television and facsimile transfer devices). This replicated the earlier study. The consumer was then asked to consider NBVT profiles that varied with respect to resolution, accessibility, hard copy availability, and transmission time and to rank order and intensity scale a fractional factorial of these profiles on (1) preference, (2) effectiveness, and (3) ease of use. Each consumer was then asked to evaluate personal visit, telephone, and the two alternative technologies on the same scales used to evaluate the NBVT profiles. The initial ordering of the profiles and the ordering of the tasks were randomized to minimize order bias. This profile evaluation is one of the standard conjoint measurement consumer tasks. See review by Green and Srinivasan [15].

*Estimation*

We estimate $\bar{F}$ with analysis of variance where the dependent measure is scaled effectiveness (or ease of use) and the treatments are the profile combinations of the four characteristics. I.e., the data are pooled so that the data matrix contains each treatment combination for each respondent. The equations, shown in Table 2, columns 1 and 2, are significant at the 0.01 level. Coefficients are normalized for ease of presentation. Notice that we have approximated the nonlinear mapping for transmission time with a piece-wise linear mapping.

TABLE 2

*Estimation Results*

| | Perception ($\bar{F}$) | | Preference | |
| --- | --- | --- | --- | --- |
| | Effec-<br>tiveness | Ease<br>of Use | $\theta\bar{F}$ | Direct<br>Estimates |
| Resolution | | | | |
| Equal to home TV | . | – | . | |
| Four times home TV | – 0.04* | – 0.07* | – 0.05* | – 0.08* |
| Accessibility | | | | |
| 30 minutes notice | – | . | – | . |
| Every Office has one | 0.13 | 0.36 | 0.21 | 0.23 |
| Hard Copy | | | | |
| None available | . | – | | – |
| Hard Copy available | 0.45 | 0.27 | 0.38 | 0.35 |
| Transmission Time | | | | |
| 30 seconds | . | – | – | |
| 20 seconds | 0.11 | 0.09 | 0.10 | 0.03 |
| 10 seconds | 0.38 | 0.31 | 0.35 | 0.35 |

* All estimates except those starred are significant at the 0.01 level.

- Indicates base value. Estimates are normalized for ease of presentation. All equations are significant at the 0.01 level.

We estimate $\theta$ with the logit approximation. The dependent measure is first preference among personal visit, telephone, NBVT, closed circuit television, and facsimile device. The explanatory variables are the measured perceptions of effectiveness and ease of use. The resulting model (estimated across respondents) is significant at the 0.01 level yielding scale adjusted estimates of $\theta_1 = 0.86$ for effectiveness and $\theta_2 = 0.50$ for ease of use.

Finally we check the internal consistency of the composite mapping, $\theta\bar{F}$, by comparing it to direct estimates of preference relative to the physical characteristics. These direct estimates are obtained from analysis of variance with scaled preference as the dependent measure and the profile combinations as treatments. As shown in Table 1, columns 3 and 4, the alternative estimates are similar. Note that the composite mapping, $\theta\bar{F}$, is nonlinear in a physical characteristic (transmission time) even though the mapping from perceptions to preference is linear.

*Cost*

The technology now available is the base level NBVT: available on 30 minutes notice, no hard copy, resolution equal to a home television, and a 30 second transmission time. Cost estimates, $C(z)$, reflect our estimates of the added production costs should the technology be developed to improve NBVT. These are: +25% to improve resolution, +100% to improve accessibility, +40% to add hard copy, +15% to decrease transmission time to 20 seconds, and +30% to decrease it to 10 seconds. These would be updated and the model analysis redone as improved cost estimates are obtained from the R & D laboratory.

*Expansion Path*

Using the psychometric models of $\overline{F}$ in Table 2 and the above cost information we draw the isocost contours. The points $A$ through $K$ correspond to different combinations of characteristics. Since improved resolution is insignificant in Table 2, we have no evidence that a product with improved resolution will be cost effective. Thus all products in Figure 7 correspond to resolution equal to that of a home television. Using the estimates of $\theta$ we draw the isodemand contours. The expansion path is determined by the intersections of the isocost and isodemand contours. As shown in Figure 7 only product $K$ (increased accessibility, hard copy, and 10 second transmission time) is efficient although product $J$ (30 minutes notice, hard copy, and 10 second transmission time) is nearly efficient. Because it is significantly less expensive, product $J$ may be the best initial strategy for research and development.

In this application profit was not the primary issue since the managerial decision was to identify what physical characteristics to stress in research and development. Once the products are feasible and the actual costs are known, the model can be used to optimize potential profit. Price changes rescale all NBVT profiles proportionally once a zero-point is established. The expansion path is unchanged if the relative preferences, $\theta$, remain unchanged. Cost updates may shift the expansion path but not the perceptual positions of products $A$ through $K$.
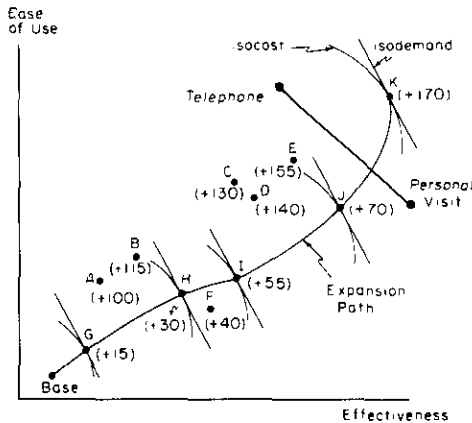


FIGURE 7. Expansion Path for Narrow Band Video Telephone (Numbers in Parentheses Indicate Estimated Incremental Costs).

## 5. Future Directions

Although our research has addressed many issues, the theory also raises many important issues for future analysis.

*Economic Issues*

Lancaster [28] deals with non-divisible goods (e.g., purchase of an automobile). A related concept of efficiency still applies on the lattice points. Price scaling is still important because the consumer implicitly is trading off the utility of all goods

forsaken by purchasing a more expensive good against the characteristics of that good. In perceptual space even divisible goods raise philosophical issues. E.g., is the gentleness of Excedrin and Tylenol additive? Our model deals with this issue through the use of a linear preference function on perceptions. (Except for degenerate cases only an extreme point is chosen if the preference mapping is linear.) Extensions to nondivisible goods when preference functions are nonlinear must face this philosophical issue, although a stochastic interpretation may provide part of the answer.

We have used separability of the utility function to develop a budget hyperplane for product categories. What are the implications for product categories defined by non-separable utility functions such as recursive utility functions (Blackorby, Primont, and Russell [5])? We have approximated nonlinear $F$-mappings with piece-wise linear mappings and have used linear preference curves in perceptual space. What are the implications of these approximations? See discussion by Pekelman and Sen [36], [37]. In general, costs, $C(z)$, are volume dependent. Many concepts of our theory apply to this case but the computational problems in obtaining volume dependent isocost curves may require improved optimization algorithms.

Our model provides a link to interesting extensions. By positing the effects and cost of a new ingredient one can obtain the potential benefits (shadow price) of its technological development. By coupling the model with equilibrium theory one can consider competitive entry and market equilibrium.

Lancaster's model is controversial. We address many of the issues raised by critics and we provide a link to marketing theory and psychometrics. For further discussion of this controversy see commentaries by Ladd and Zober [29], [30], Haines [17], Ratchford [39], [40], and Taylor [49] as well as Lancaster [27, Chapter 6].

*Marketing Issues*

Our model is based on deductive economic theory which posits that consumers tradeoff perceived benefits and price through a concept of value (benefits/price) and that this value is derived in part because higher prices paid for one product mean that less benefits can be purchased with the remaining portion of the consumer's budget. Do consumers act as if they process information in this way or should we consider price simply as another attribute? The deductive theory is compelling but this question remains an empirical issue. Ratchford [40, equation 6] does present some evidence that these two views of behavior may be mathematically equivalent.

Related to the adequacy of the descriptive consumer model is the issue of the external validity of the optimization model. In our empirical example the managerial recommendation (which was followed) was to not launch the product, but rather to return the product to research and development. It remains an empirical question to assess and document the accuracy (or lack thereof) of the descriptive model in forecasting profit for the optimal characteristics and price combination. We are optimistic since predictive tests of the psychometric components of the measurement [9], [15], [22], [32], [35], [43], [44], [57]–[59] and of the Lens model [18], [21], [53] have been encouraging.

Our model incorporates heterogeneity in perceptions and preferences and stochasticity due to achieving a tradeoff among perceived benefits. Perhaps the model can be extended to include other explanations of stochasticity observed at the individual level (Bass, Pessemier, and Lehmann [2]).

Figure 2 indicates that constraints as well as preference affect choice. The consumer's budget is one such constraint. In some categories there may be other constraints such as awareness and availability. Our model treats these constraints as random error in the probit formulation. Alternatively they can be added post hoc to scale down demand (Urban and Hauser [55, Chapter 11]). Research is now underway to model these phenomena more explicitly.

*Psychometric Issues*

In our model profit maximization (selection of price and product features) depends on obtaining a ratio-scaled mapping, $\bar{F}$. Table 1 suggests it is feasible to obtain a ratio-scaled dependent measure and Table 2 suggests that it is feasible to obtain an interval-scaled mapping from features to perceptions. Research is necessary to refine these consumer tasks to obtain better data to estimate a ratio-scaled $\bar{F}$.

Finally, although the managerial emphasis in our applications to date has been on the selection of physical features and price. *theoretically* the model can be extended to psychosocial cues. When the effect of psychosocial cues is small relative that of the physical characteristics (as is the case in many consumer frequently purchased products), the psychosocial cues are controllable actions in $Z$-space and must therefore be independent variables in the $\bar{F}$-mapping. For example. in the communications example we might systematically vary the advertising copy used in the concept statements. Advertising copy might then be treated as a main effect in the analysis of variance. Advertising intensity might be handled through an advertising response curve relating dollars spent to position obtained. While the incorporation of psychosocial cues may require the development of new measurement methodology. we do not feel the problems are insurmountable. In fact. the theoretic challenge of incorporating psychosocial cues and trading them off against physical product modification may provide new insight on how psychosocial cues do influence consumer choice.

## 6. Conclusions

This paper has attempted to integrate ideas and practice from economic and marketing science to develop a theory to select the ingredient-price combination for a profit maximizing perceptual position. To economic science the theory adds a realistic behavioral component of a characteristics to perceptual mapping, explicitly considers consumer heterogeneity, and extends the consumer model to profit maximization. To marketing science the theory adds a new interpretation of perceptual maps (ratio-scaling by price), explicitly incorporates the concepts of efficiency and cost-effectiveness, and addresses the managerial problem of where to position and price a new product given cost-of-ingredients information. Finally, we explicitly consider implementation by discussing the relationship of the theory to techniques which estimate the preference structure (probit analysis) and the characteristics to perceptions mapping (conjoint measurement).

The analgesics example provides a simplified illustration of how the model might be used for profit maximization. The communications example illustrates some of the measurement techniques. The model is now being used for the design of industrial products (a supplier of hospital equipment) and services (transportation). Achieving a

profit maximizing perceptual position appears to be of managerial significance and our theory suggests a practical model that can address this issue.[6]

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