Appendix to Martin and Pindyck,

"Averting Catastrophes: The Strange Economics of Scylla and Charybdis"

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WTP to Avoid A Catastrophe that Can Occur Only Once

If nothing is done to avert a catastrophic event that can occur only once, and reduces consumption by a random fraction ϕ if it occurs, welfare is

$$V_0 = \mathbb{E}\int_0^\infty \frac{1}{1-\eta} C_t^{1-\eta} e^{-\delta t} dt = \frac{1}{1-\eta}\,\mathbb{E}\left[\int_0^\tau e^{-\rho t} dt + \int_\tau^\infty e^{-\phi(1-\eta)-\rho t} dt\right] ,$$

where $\mathbb{E}$ denotes the expectation over τ and ϕ. As before, WTP is defined as the maximum percentage of consumption, now and throughout the future, that society would give up to eliminate the possibility of the catastrophe. Define $\rho \equiv \delta + g(\eta - 1)$. If society gives up a fraction w of consumption to avert this catastrophe, net welfare is

$$V_1 = (1-w)^{1-\eta} \int_0^\infty \frac{1}{1-\eta} e^{-\rho t} dt.$$

WTP is then the value w^* that equates V_0 and V_1.

To obtain the WTP for eliminating the event, note that welfare if no action is taken is:

$$\begin{aligned} V_0 &= \frac{1}{1-\eta}\,\mathbb{E}\left[\int_0^T e^{-\rho t} dt + e^{-\phi(1-\eta)} \int_T^\infty e^{-\rho t} dt\right] \\ &= \frac{1}{\rho(1-\eta)}\,\mathbb{E}\left[1 + e^{-\rho T}(e^{-\phi(1-\eta)} - 1)\right] \\ &= \frac{1}{\rho(1-\eta)}\left[1 + \frac{\lambda}{\lambda+\rho}\left(\mathbb{E}\, e^{-\phi(1-\eta)} - 1\right)\right] \\ &= \frac{1}{\rho(1-\eta)}\left[1 + \frac{\lambda}{\lambda+\rho}\frac{\eta-1}{\beta-\eta+1}\right] \qquad (1) \end{aligned}$$

Here we have used the assumption that $z = e^{-\phi}$ follows a power distribution. If the event is eliminated, welfare net of the fraction w of consumption sacrificed is

$$V_1 = \frac{(1-w)^{1-\eta}}{\rho(1-\eta)} \qquad (2)$$

Comparing (1) and (2), the WTP to eliminate the event is:

$$w^* = 1 - \left[1 + \frac{\lambda}{\lambda+\rho}\frac{\eta-1}{\beta-\eta+1}\right]^{\frac{1}{1-\eta}}$$

From this equation, we see that (i) w^* is an increasing function of the mean arrival rate λ; (ii) w^* is an increasing function of the expected impact $\mathbb{E}(\phi)$, and thus a decreasing function of the distribution parameter β; and (iii) w^* is a decreasing function of both the rate of time preference δ and the growth rate g. We would expect w^* to be higher for an event that is expected to occur sooner and have a larger expected impact, and lower if either the rate of time preference or the consumption growth rate is higher. The dependence on η is ambiguous. Given the growth rate g, a higher value of η implies a lower marginal utility of future consumption, and thus a lower WTP to avoid a drop in consumption. On the other hand it also implies a greater sensitivity to uncertainty over future consumption.

As mentioned above, for expected utility to be finite, we need $\beta > \eta - 1$. It is easy to see that as η is increased, w^* approaches 1 as η approaches $\beta + 1$. The reason is that the risk-adjusted remaining fraction of consumption is $\mathbb{E}((1-\phi)^{1-\eta}) = \beta/(\beta - \eta + 1)$. In risk-adjusted terms, the possibility of a high-ϕ outcome weighs heavily on expected future welfare, and thus on the WTP.

A few numbers: Suppose $\beta = 2$ so the expected loss is $\mathbb{E}\,\phi = .33$, $\lambda = .05$ so the expected arrival time is $\mathbb{E}\,T = 1/\lambda = 20$ years, $\delta = g = .02$, and $\eta = 2$. Then $w^* = 0.22$. If instead $\delta = 0$, then $w^* = 0.26$. If $\delta = .02$ but we increase η to 2.5, w^* increases sharply, to 0.60.

It is useful to compare the WTP to avoid this "once-only" event with the WTP when the event can occur multiple times. As shown in Martin and Pindyck (2015), in the latter case the WTP is

$$w_m^* = 1 - \left[1 - \frac{\lambda(\eta-1)}{\rho(\beta-\eta+1)}\right]^{\frac{1}{\eta-1}}.$$

(The subscript m is added to emphasize that the event can occur multiple times.) Whether the event can occur only once or repeatedly: (i) w^* is increasing in the mean arrival rate λ; (ii) w^* is increasing in the expected impact $\mathbb{E}(1-\phi)$, and thus a decreasing function of the distribution parameter β; and (iii) w^* is a decreasing function of both the rate of time preference δ and the growth rate g. And as expected, $w_m^* > w^*$ for all $\eta > 1, \beta > \eta - 1, \lambda > 0$. Some comparisons: (1) If $\eta = 2$, $g = \delta = .02$ so $\rho = .04$, $\lambda = .02$ and $\beta = 3$ (so $\mathbb{E}(1-\phi) = .75$), then $w^* = .143$ and $w_m^* = .250$. (2) If instead $\lambda = .04$, then $w^* = .200$ and $w_m^* = .500$. (3) If $\lambda = .04$ but $\beta = 2.1$, then $w^* = .313$ and $w_m^* = .910$. In the last example, β is just above the limit (2.0) at which expected utility becomes unbounded.

References

Martin, Ian W.R., and Robert S. Pindyck. 2015. "Averting Catastrophes: The Strange Economics of Scylla and Charybdis." *American Economic Review*, 105(10): 2947–2985.