

# Preference Learning and Demand Forecast <sup>\*</sup>

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## Abstract

Understanding consumer preferences is important for new product management, but is famously challenging in the absence of actual sales data. Stated-preferences data are relatively cheap but less reliable, whereas revealed-preferences data based on actual choices are reliable but expensive to obtain prior to product launch. We develop a cost-effective solution. We argue that people do not automatically know their preferences, but can make an effort to acquire such knowledge when given sufficient incentives. The method we develop regulates people's preference-learning incentives using a single parameter, *realization probability*, meaning the probability with which an individual has to actually purchase the product she says she is willing to buy. We derive a theoretical relationship between realization probability and elicited preferences. This allows us to forecast demand in real purchase settings using inexpensive choice data with small to moderate realization probabilities. Data from a large-scale field experiment support the theory, and demonstrate the predictive validity and cost-effectiveness of the proposed method.

**Keywords:** preference elicitation, demand forecasting, incentive alignment, choice experiment, field experiment, external validity.

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# 1 Introduction

Each year, more than 30,000 new consumer products are brought into the market (Olenski 2017). Accurately forecasting market demand of a product is essential for its design, distribution, promotion, and pricing strategies. More broadly, demand forecasting affects a range of managerial decisions such as manufacturing, R&D investment, and market entry. However, demand forecasting is also famously difficult for new products, due to the lack of historical sales data which would otherwise reveal valuable information about consumer preferences (see Ben-Akiva et al. (2019) for a recent survey).

One of the most direct, and in a sense heroic, solutions to this problem is to create actual sales data in experimental test markets prior to full-scale launch (e.g., Silk and Urban 1978, Urban and Katz 1983). Derived from real purchase environments, the resulting demand forecast tends to have high external validity. However, test-market data are costly to obtain. Even in the 1970s, the cost could surpass one million U.S. dollars for each test (Silk and Urban 1978). Besides high operational overhead, firms incur opportunity costs of selling actual products at suboptimal prices in the test market – by definition, a firm will probably not know the optimal price to charge before it is able to forecast demand.<sup>1</sup> In addition, it may be challenging for some firms to provide a sufficient number of new products prior to launch, which limits the test market’s power of statistic inference.

A different approach to demand forecasting, opposite to test markets in terms of cost, is to rely on consumers’ stated-preference data. Consumers answer questions about their preferences or participate in choice experiments without actual consequences of purchase. Various methods have been developed and refined. For example, contingent valuation methods estimate people’s willingness-to-pay for public goods (e.g., Mitchell and Carson 1989), and choice-based conjoint analysis measures consumers’ tradeoffs among multi-attribute products (see Hauser and Rao 2004 and Rao 2014 for an overview). Stated-preference data can be obtained at relatively low costs because no actual transaction is needed, but their ability to predict market demand has

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<sup>1</sup>The company we collaborate with for the field experiment presented in this paper indicated that management had refrained from running test markets for this reason.

been questioned. In fact, hypothetical contingent valuation and hypothetical choice experiments are both found to overestimate product valuation (Diamond and Hausman 1994, Cummings et al. 1995, Wertenbroch and Skiera 2002, Miller et al. 2011). A primary reason is participants’ lack of incentive to provide accurate statements of preferences in a non-market setting (Camerer and Hogarth 1999, Ding 2007).

A stream of research tries to overcome the hypothetical bias of stated-preference data while avoiding the full cost of test markets. A well-known approach is called “incentive alignment” (e.g., Becker et al. 1964, Ding 2007, Toubia et al. 2012). The idea is to incentivize truth-telling by making participants partially responsible for the consequences of their choices. In its simplest yet representative form, an incentive-aligned choice task appears as follows.

The price of [a product] is  $p$ . If you state you are willing to buy the product at this price, with probability  $r$ , you will actually pay this price and purchase the product.

Are you willing to buy?

The probability  $r$  is called the “realization probability” in the literature. In theory, incentive alignment induces truth-telling for any positive  $r$ . Mathematically, denoting the participant’s product valuation as  $v$ ,  $\text{sign}(v - p) = r \times \text{sign}(v - p)$  for any  $r > 0$ .<sup>2</sup> At the same time, incentive alignment should be less costly than test markets for any  $r < 1$ . To achieve the same sample size of choice data, one only expects to sell a fraction of the number of actual products that would otherwise be required in a test market. Despite its theoretical soundness, empirical performance of incentive alignment is mixed. It often outperforms its hypothetical counterpart (e.g., Ding et al. 2005, Ding 2007), but its accuracy in forecasting demand in real purchase settings is still questionable (e.g., Kaas and Ruprecht 2006, Miller et al. 2011).

In a particularly illuminating paper, Yang et al. (2018) show that, contrary to the theoretical premise of incentive alignment, its empirical performance relies on the realization probability chosen for the choice experiment. This paper predicts using the bounded rationality literature,

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<sup>2</sup>A related incentive-aligned mechanism is developed by Becker et al. (1964), often called the BDM mechanism. Under the BDM, a participant must purchase a product if a randomly drawn price is less than or equal to her stated product valuation.

and shows using eye-tracking experiments, that respondents pay more attention to the choice as realization probability increases. Moreover, the paper predicts using the psychological distance literature, and shows experimentally, that respondents become more price sensitive under higher realization probabilities. These findings suggest that incentive alignment, at least in its traditional form, may not guarantee external validity.

In our paper, we study the external validity of preference elicitation from the “information acquisition” perspective. We emphasize that the validity of elicited preference data depends on two factors: participants’ incentives to truthfully state their preferences, and to diligently learn their preferences. Traditional incentive alignment methods have focused on truth-telling, whereas “truth-learning” may be equally important in some contexts. Participants may need to spend an inspection cost to understand product specifications, a search cost to evaluate alternative options, or a cognitive cost to imagine their potential use of the product (e.g., Shugan 1980, Wernerfelt 1994, Villas-Boas 2009, Kuksov and Villas-Boas 2010, Wathieu and Bertini 2007, Guo and Zhang 2012, Huang and Bronnenberg 2018).<sup>3</sup>

Indeed, there is abundant evidence from behavioral research showing that human beings do not always know their preferences; instead, they often construct their preferences during decision making and, in particular, incur a cost to identify their preferences from an inherent “master list” (Payne et al. 1993, Lichtenstein and Slovic 2006, Simonson 2008). We posit that the preferences consumers evoke and manifest through their choices in any environment depend on their preference-learning efforts, and consumers’ incentives to engage in these efforts depend on the stake of their choices in this environment.<sup>4</sup> As such, traditional incentive alignment methods may fail to predict actual demand because participants fail to think through their

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<sup>3</sup>There is a growing theory literature built upon the notion of costly learning of preferences. In a recent paper, Kleinberg et al. (2018) show that costly valuation learning renders the popular increasing-price auction ineffective. The idea of endogenous effort as a choice mediator is also related to Hauser et al. (1993) and Yang et al. (2015), who revisit bounded rationality from the lens of decision cost, and to Chassang et al. (2012), who study the design of randomized controlled experiments from the principal-agent perspective. More generally, the paper is related to the “rational inattention” literature, which interprets seemingly irrational behavior in light of costly information acquisition (e.g., Caplin and Dean 2015).

<sup>4</sup>Consistent with this view, neuroeconomics research finds that, when humans choose among consumer goods, brain activation is stronger and more widespread in the real choice condition than in the hypothetical condition (Camerer and Mobbs 2017).

preferences as carefully as they would have in actual choice settings.

Based on the idea of endogenous preference learning, we develop a method called “augmented incentive alignment” (abbreviated as AIA thereafter) to accurately forecast new product demand without having to actually launch the product in test markets. To facilitate comparison, we focus on the canonical application of incentive alignment, as described in the aforementioned choice task. In this setting, participants’ stake of choices and therefore preference-learning effort incentives are shaped by one parameter – the realization probability of their choices. Intuitively, if a participant knows that her product choice is unlikely to be realized, she will have little incentive to uncover her true product valuation and will make her choice based on her prior belief. On the contrary, if a participant knows that her product choice is for real, she will want to think about how much she truly values the product and make her choice based on her true valuation. As a result, there exists a microfounded relationship between realization probability and manifested demand. Our proposed AIA method thus proceeds in two steps: first, estimate this relationship using less costly (than test markets) incentive-aligned choice data under realization probabilities that are smaller than one; second, use the estimation results to forecast product demand in actual purchase settings where realization probability equals one.

We formalize the mechanism of the AIA method with a theory model, in which consumers decide whether they are willing to purchase a product for a given price and a given realization probability. The model predicts that manifested price sensitivity increases with realization probability. To understand the intuition, imagine that the product had been offered for free. Agreeing to buy the product would have been a no-brainer. Now, suppose the price rises gradually. As the price approaches a consumer’s prior valuation for the product, she will have a greater incentive to zoom in and think carefully about her true need for the product, and the only change this thinking brings to her decision is to not buy the product. A higher realization probability increases the gravity of the purchase decision and amplifies this negative effect of price on demand. The same intuition applies to the mirror case of a price cut from a prohibitively high level. Therefore, it will appear as if consumers are more price sensitive under higher realization probabilities.

We run a large-scale field experiment to test the preference-learning theory and to evaluate the AIA method. We choose the field, as opposed to the lab, in order to minimize factors that may affect external validity other than realization probability (e.g., Simester 2017). In particular, even if choice is realized for certain, people may still choose differently in the lab than in an actual purchase setting due to differences in the decision environment. We conduct the choice experiment in the field in an effort to address this potential discrepancy.

We collaborate with a mobile platform for fantasy soccer games. The new product is a new game package that may enhance user performance. We experiment with four realization probabilities: 0,  $1/30$ ,  $1/2$ , and 1. The 0-probability condition is designed to capture the effect of stated-preference approaches, the two conditions with interim probabilities  $1/30$  and  $1/2$  correspond to incentive alignment, whereas the 1-probability condition mirrors the actual purchase setting. In addition, for each realization probability, we vary prices to measure the corresponding demand curve. We randomly assign prices and realization probabilities across users exposed to the experiment.

The experiment result supports the theory prediction – consumers are indeed more price sensitive under higher realization probabilities. We rule out a number of competing explanations of this effect using data from a post-choice survey. Moreover, we obtain measures of consumers’ preference-learning effort. We find that effort does increase with realization probability, consistent with the preference-learning mechanism underlying the theory prediction. These findings echo the conclusions of Yang et al. (2018).

Having validated its theory foundation, we empirically evaluate the AIA method, using choice data from incentive alignment. More specifically, we estimate a model of consumer preference learning and product choice using data from the subsample of interim realization probabilities ( $1/30$  and  $1/2$  in the field experiment). We then use the parameter estimates to forecast product demand in real purchase settings and compare the forecast against the holdout sample where realization probability equals 1. The AIA method performs remarkably well. Compared with the optimal profit the seller could have made with perfect knowledge of actual demand, the AIA demand forecast leads to a profit that only misses the optimal profit by

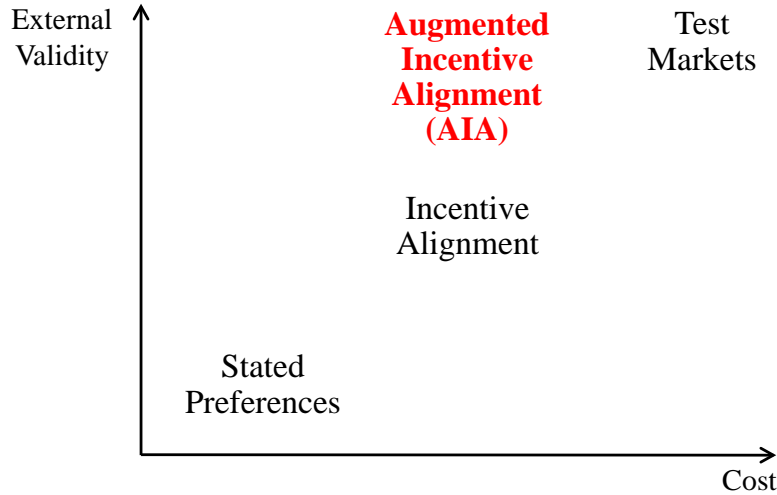
0.57%. To put this number in context, the profit loss is about 23% when realization probability is 1/2, 50% when realization probability is 1/30, and as high as 90% when the choice task is hypothetical – stated preferences overpredict demand and recommend a prohibitively high price in this setting. Notably, simple extrapolation of data from incentive alignment to actual purchase settings yields a profit loss of 7%. This suggests that the external validity of the AIA method hinges on its ability to capture the preference-learning mechanism. Finally, we find that, compared with test markets, the AIA method significantly reduces the cost of data on various measures of cost.

Conceptually, this paper contributes to the growing literature that emphasizes preferences as endogenous manifestations as opposed to endowed primitives. We develop a parsimonious theory of how preference learning shapes manifested demand. We document supporting evidence of this theory. We find evidence that the demand curve, which serves as the foundation of various managerial decisions, is not a passive object of measurement but an active response to the preference elicitation method.

Practically, the idea of endogenous preference learning allows us to develop a theory-based, cost-effective demand forecasting method that helps resolve the cost-validity conundrum of existing preference elicitation methods. The method only requires incentive-aligned choice data with small to moderate realization probabilities, yet it is able to accurately forecast demand in actual purchase settings. Figure 1 summarizes the contribution of this paper in relation to existing preference elicitation methods.

The rest of the paper proceeds as follows. We continue in Section 2 with a theory model to illustrate the preference-learning mechanism, to formulate predictions, and to lay the foundation for the AIA method. We then present the field experiment in Section 3 and discuss evidence of the theory in Section 4. In Section 5, we develop and evaluate the AIA method. We conclude in Section 6 with discussions of future research.

Figure 1: Preference Elicitation Methods – Cost-Validity Tradeoff



## 2 Theory Model

In this section, we use a simple model to illustrate the preference-learning mechanism and its effect on manifested demand. Consider a firm that offers a new product. The product’s true value is potentially heterogeneous across consumers, following a distribution unknown to both the firm and consumers. (If the distribution is known, the firm can derive the demand curve without going through the demand forecasting exercise.) We use preference and valuation interchangeably in this setting.

Consider a representative consumer. The consumer does not know her true product valuation  $v$  but maintains a prior belief about it. The mean of her prior belief is  $\mu$ , which can be decomposed as  $\mu = v + e$ , where the perception error  $e$  follows a distribution  $g(\cdot)$ . The consumer knows  $g(\cdot)$ . We make no functional-form assumptions about  $g(\cdot)$  except that it has a mean of zero and has positive support everywhere over  $(-\infty, \infty)$ . The zero-mean assumption is justifiable because, if it does not hold, the consumer will know that her prior belief is systematically biased and will rationally “debias” her belief accordingly. The assumption of positive support everywhere guarantees that Propositions 1 and 2 hold strictly. If this assumption is relaxed, Propositions 1 and 2 will still hold at least weakly (see the Appendix for proof). An example of  $g(\cdot)$  is the familiar normal distribution around the mean of zero.



The consumer can make a preference-learning effort to uncover her true valuation of the product. To capture this process in a simple way, we assume that, if the consumer devotes effort  $t \in [0, 1]$ , she will learn the true value of  $v$  with probability  $t$ , and her knowledge of her product valuation stays at her prior belief  $\mu$  with probability  $1 - t$ . As an example of a choice context this formulation captures, imagine the new product is a camera specialized in taking beach photos. To learn how much value this camera truly generates for her, a consumer can make an effort to predict whether she will take a beach vacation in the near future. Alternatively, we can model the preference-learning effort as smoothly reducing the consumer's uncertainty about her true product valuation. The qualitative insight of the theory model remains the same.

Effort is costly. We follow the common assumption of convex cost functions and, for the ease of presentation, write the cost of preference-learning effort  $t$  as  $ct^2/2$ , where  $c > 0$ . We assume that the consumer is risk neutral, enjoys a true purchase utility of  $U = v - p$ , and has a reservation utility of zero. As such, the consumer will purchase the product if and only if her expected value of  $v$ , given her knowledge of her preference, is no less than product price  $p$ .

The sequence of actions unfolds as follows. The consumer observes realization probability  $r$  and product price  $p$ . She is informed that if she chooses "willing to buy," with probability  $r$  she will have to actually pay  $p$  and receive the product, and with probability  $1 - r$  she will pay nothing and will not receive the product. If she chooses "not willing to buy," no transaction will happen. Based on the values of  $r$  and  $p$ , the consumer chooses her level of preference-learning effort,  $t$ . The consumer then decides whether to choose "willing to buy" based on the outcome of her preference-learning effort. If she is willing to buy, with probability  $r$  she will pay price  $p$  and receive the product as promised.

We first derive the optimal preference-learning effort of this representative consumer. The consumer chooses effort  $t$  to maximize her expected net utility:

$$\mathbb{E}U(t; r, p, \mu) = r \left[ t \int_{-\infty}^{\mu-p} (\mu - e - p)g(e)de + (1 - t)(\mu - p)^+ \right] - \frac{1}{2}ct^2. \quad (1)$$

Equation (1) highlights the effect of realization probability – the consumer makes a lump

sum effort to learn her preference, yet the return to this effort is scaled by realization probability  $r$ . Meanwhile, Equation (1) captures the information value of the preference-learning effort – the consumer’s chance of learning her true valuation increases with  $t$ , so does her ability to make a better decision based on knowledge of her true valuation.

The first-order condition of  $\partial EU(t; r, p, \mu)/\partial t = 0$  yields the consumer’s optimal level of preference-learning effort:

$$t^*(r, p; \mu) = \frac{r}{c} \left[ \int_{-\infty}^{\mu-p} (\mu - e - p)g(e)de - (\mu - p)^+ \right]. \quad (2)$$

The second-order condition is trivially satisfied.

We prove the following results.

**Proposition 1** *The consumer’s optimal preference-learning effort increases with realization probability, and decreases with the distance between price and her prior belief of her product valuation. A greater realization probability amplifies the latter effect. That is,*

$$\frac{\partial t^*(r, p; \mu)}{\partial r} > 0, \quad \frac{\partial t^*(r, p; \mu)}{\partial |p - \mu|} < 0, \quad \frac{\partial^2 t^*(r, p; \mu)}{\partial r \partial |p - \mu|} < 0. \quad (3)$$

Proof: see the Appendix.

The first result is straightforward. At one extreme, where realization probability equals 0, choices are hypothetical with no impact on consumer utility, and the consumer has no incentive to learn her product valuation via costly effort.<sup>5</sup> When realization probability increases, the consumer has more incentive to make an effort to learn her preferences. At the other extreme, where realization probability equals 1, the consumer makes the same preference-learning effort as in real purchase decisions.

The remaining results are more subtle yet still intuitive. When price is extremely low (or high), the consumer may trivially decide to buy (or not buy) regardless of her true valuation, which makes it unnecessary to make an effort to learn her preference. When price is closer to a

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<sup>5</sup>When choices are hypothetical, consumers may choose randomly or be pro-social towards the researcher and choose truthfully based on her prior belief. Identifying the exact process is outside the scope of this paper.

consumer’s prior valuation, making a purchase decision based on the prior belief alone is more likely to lead to a mistake, and the consumer will want to invest more effort to discern her true valuation. A greater realization probability amplifies this effect because the consequence of a wrong purchase decision is more severe when purchase is more likely to be realized.

Based on the consumer’s optimal choice of preference-learning effort, we can derive her *manifested demand* of the product, defined as the expected probability for a consumer of prior belief  $\mu$  to choose “willing to buy” given realization probability  $r$  and price  $p$ :

$$D(r, p; \mu) = \int \left[ t^*(r, p; \mu) \mathbf{1}(\mu - e \geq p) + (1 - t^*(r, p; \mu)) \mathbf{1}(\mu \geq p) \right] g(e) de, \quad (4)$$

where  $t^*(r, p; \mu)$  is given by Equation (2).

We emphasize the notion of manifested demand, as opposed to estimated demand, to highlight the theoretical effect of preference learning on consumer choice. In other words, even if consumers are behaving truthfully given all they know about their product valuation and even if there is no empirical error in estimation, manifested demand may still differ from actual demand if consumers fail to learn their preferences as diligently as they would have in actual purchase environments. This notion is consistent with the view of Yang et al. (2018).

A key result of interest is the effect of realization probability on manifested demand. We prove the following proposition.

**Proposition 2** *Manifested consumer price sensitivity increases with realization probability whenever it is well-defined. That is,*

$$\frac{\partial^2 D(r, p; \mu)}{\partial r \partial p} < 0 \quad (5)$$

*whenever  $\partial D(r, p; \mu)/\partial p$  exists.*

Proof: see the Appendix.

To understand the intuition, imagine that a consumer is offered a trivially low price. The consumer can safely choose to buy without bothering to learn her true preference. Now imagine a small price increase. According to Proposition 1, such a price increase (from a trivially low

level) will induce the consumer to deliberate more on her true preference, especially so under greater realization probabilities. The only change to consumer choice (of trivially deciding to buy) this extra deliberation brings is a decision to not buy after learning the true preference, as if the consumer has become more price sensitive than what the standard demand-reducing effect of higher prices would indicate. A greater realization probability further amplifies this effect, because product choice is more consequential if it is more likely to be real. Similarly, imagine a small price cut from a trivially high level. Due to the preference-learning mechanism, the consumer will deliberate more and will respond to the price cut more than what the standard price effect would indicate, especially so under greater realization probabilities. Therefore, the consumer’s manifested price sensitivity increases with realization probability.

A remark on this paper’s theoretical relationship with Yang et al. (2018) is in order. Drawing on the bounded rationality literature, Yang et al. (2018) successfully predict that respondents will process choice-relevant information more carefully under higher realization probabilities. Our Proposition 1 can be seen as formalizing this prediction with a model of preference learning. Yang et al. (2018) also build on the psychological distance literature to successfully predict greater price sensitivity under higher realization probabilities. Our Proposition 2 shows that preference learning alone can predict this result, which provides a parsimonious way to understand the relationship between realization probability, preference-learning effort, and price sensitivity in a unified framework.

To recap, using a simple theory model, we demonstrate how a higher realization probability induces a consumer to invest more preference-learning effort and in turn manifest greater price sensitivity. In what follows, we test the theory and evaluate the AIA method derived from the theory.

### **3 Field Experiment**

We use data from a field experiment to validate the theory prediction and mechanism, and to evaluate the AIA method. As discussed earlier, we choose the field experiment approach to

minimize threats to external validity. This allows us to focus on realization probability as a determinant of the external validity of various preference elicitation methods.

We collaborate with a top mobile platform of fantasy soccer games in China. Founded in 2013, the platform currently hosts 80,000 daily active users, generating 2 million US dollars in monthly revenue. In the game, each user manages a soccer team with the goal to win as many times as possible. A team’s likelihood of winning depends on the number of high-quality players it enlists. The new product we sell in the field experiment is a “lucky player package” that consists of six high-quality players. This player package had never been sold on the game platform prior to the experiment.<sup>6</sup>

The design of the field experiment consists of two orthogonal dimensions of exogenous variation. First, we exogenously vary realization probability to identify its causal impact on manifested demand. We set four realization probabilities: 0, 1/30, 1/2, and 1. The 0-probability condition is designed to replicate the stated-preferences method, the 1-probability condition captures the actual purchase setting, whereas the interim realization probability conditions mirror the incentive alignment approach. We assign two interim realization conditions because the AIA method needs at least two realization probability levels for empirical identification and we choose only two for a conservative test of the method’s predictive power. In terms of specific values of interim realization probabilities, 1/2 is a natural choice to observe the effect of a moderate realization probability. For a small realization probability, we choose 1/30 because a minimum sample size of 30 has been suggested in the literature for statistical inference (Pett 1997). In future applications of the AIA method using this minimum number of 30 participants per condition, the realization probability of 1/30 can be implemented as one out of the 30 participants getting to buy the product for real, which makes the experiment appear more trustworthy than using a smaller realization probability.

Second, we exogenously vary price to identify its causal impact on manifested demand for any given realization probability. We set five price levels, measured as 1600, 2000, 2400, 2800,

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<sup>6</sup>In this game, most users can only play against the computer. Only when users advance to very high levels can they have the chance to play against other users. Thus the network effect of obtaining high-quality soccer players is negligible.

or 3200 “diamonds.” Diamond is the currency of the game. Users need to pay real money to obtain diamonds. The exchange rate is about 1 US dollar for 100 diamonds. We discuss with the company to make sure this price range is reasonable and at the same time the gap between prices is large enough to elicit different purchase rates at different prices.

The five price levels, orthogonally combined with the four realization probabilities, lead to 20 conditions for the experiment. Once a user enters the experiment, she is randomly assigned to one of the 20 conditions.

More specifically, once a user enters an experimental condition, she is presented a screen of the choice task. (Figure OA1 in the Online Appendix shows the screen for the 1/30-probability condition.) On this screen, the user is informed that she has a chance to purchase a lucky player package at price  $p$  and is asked to choose between “willing to buy” and “not willing to buy.” For the 0-probability condition, the user is informed that this is a hypothetical survey and no actual transaction will take place. For the 1-probability condition, the user is told that she will receive the package if she chooses “willing to buy.” For the interim probability conditions ( $r \in \{1/30, 1/2\}$ ), the user is told that if she chooses “willing to buy,” a lottery will be drawn and there is probability  $r$  that she will actually receive the player package and will be charged price  $p$  automatically. If the user chooses “not willing to buy” or does not win the lottery, she will not receive the player package and will not be charged. Users can click on the player package icon and see the set of players contained therein (Figure OA2 of the Online Appendix). They can also click on each player and see what skills the player has. After making the purchase decision, the user will be directed to a follow-up survey, which is designed to obtain auxiliary data for further tests of the theory.

The experiment took place from 12AM, December 2, 2016 to 12PM, December 4, 2016. We randomly selected half of the platform’s Android servers, and all users on these servers automatically entered the experiment once they accessed the game during the period of the experiment. We chose a short time window and a fraction of users for the experiment to limit communications among users about the potentially different experimental treatments they were

receiving.<sup>7</sup>

A total of 5,420 users entered the experiment, 271 assigned to each condition. Among these users, 3,832 (70.7%) completed the choice task. Among those who completed the choice task, 2,984 (77.87%) filled out the survey. Table 1 reports the number of users assigned to each of the four probability conditions and each of the five price conditions, and the number that completed the choice task or the survey. Table OA1 of the Online Appendix further breaks down these numbers into the 20 conditions. We notice higher completion rates in the 0-probability condition. However, reassuringly, for all conditions with positive realization probabilities, completing the choice task and completing the survey are statistically independent of the assigned realization probability ( $\chi^2(2) = 1.519$ ,  $p = 0.468$  for choice;  $\chi^2(2) = 4.234$ ,  $p = 0.120$  for survey).<sup>8</sup> For all users who entered the experiment, completing the choice task and completing the survey are statistically independent of the assigned price ( $\chi^2(4) = 1.217$ ,  $p = 0.857$  for choice;  $\chi^2(4) = 3.836$ ,  $p = 0.429$  for survey).

For each user who completed the choice task, we collect data on her characteristics at the time of the experiment, including the number of diamonds the user has (Diamonds) and the VIP level of the user (VIP-Level). The VIP level is an integer between 0 and 15, and is determined by how much money the user has spent in the game. Table 2 presents the summary statistics of user characteristics. Table OA1 of the Online Appendix further breaks down the mean values of Diamonds and VIP-Level for each of the 20 conditions.

As a balance check, we perform ANOVA analysis of observable user characteristics across conditions for all users who completed the choice task. The interactions between Diamonds and realization probability ( $F(3, 3828) = 2.00$ ,  $p = 0.112$ ), between Diamonds and price ( $F(4, 3827) = 1.07$ ,  $p = 0.368$ ), between VIP-Level and realization probability ( $F(3, 3828) = 0.16$ ,  $p = 0.926$ ), and between VIP-Level and price ( $F(4, 3827) = 0.43$ ,  $p = 0.789$ ) are all insignificant. These results suggest that, based on the two observed characteristics, participants

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<sup>7</sup>We monitored the online forum of this mobile game for the period of the experiment. We did not find discussions of this player package.

<sup>8</sup>As we will discuss later, we exclude the 0-probability condition from the AIA model estimation, so that the different participation rate in this condition does not affect the AIA model.

Table 1: Number of Users by Realization Probability and by Price

Realization Probability	# Entered Experiment	# Completed Choice	# Completed Survey
0	1355	1095	882
1/30	1355	920	708
1/2	1355	922	723
1	1355	895	671
Sum	5420	3832	2984

Price (Diamonds)	# Entered Experiment	# Completed Choice	# Completed Survey
1600	1084	774	599
2000	1084	757	575
2400	1084	764	589
2800	1084	761	603
3200	1084	776	618
Sum	5420	3832	2984

Notes. Diamond is the currency of the game. Users need to pay real money to obtain diamonds, at an exchange rate of about 1 US dollar for 100 diamonds.

Table 2: Summary Statistics of User Characteristics

	Mean	SD	Median	Min	Max	<i>N</i>
Diamonds	3134.44	5498.09	1614.00	0	150969	3832
VIP-Level	3.00	3.10	2.00	0	15	3832

Notes. The sample consists of all users who completed the choice task.

in the choice task are balanced across treatment conditions.

## 4 Evidence of the Preference-Learning Theory

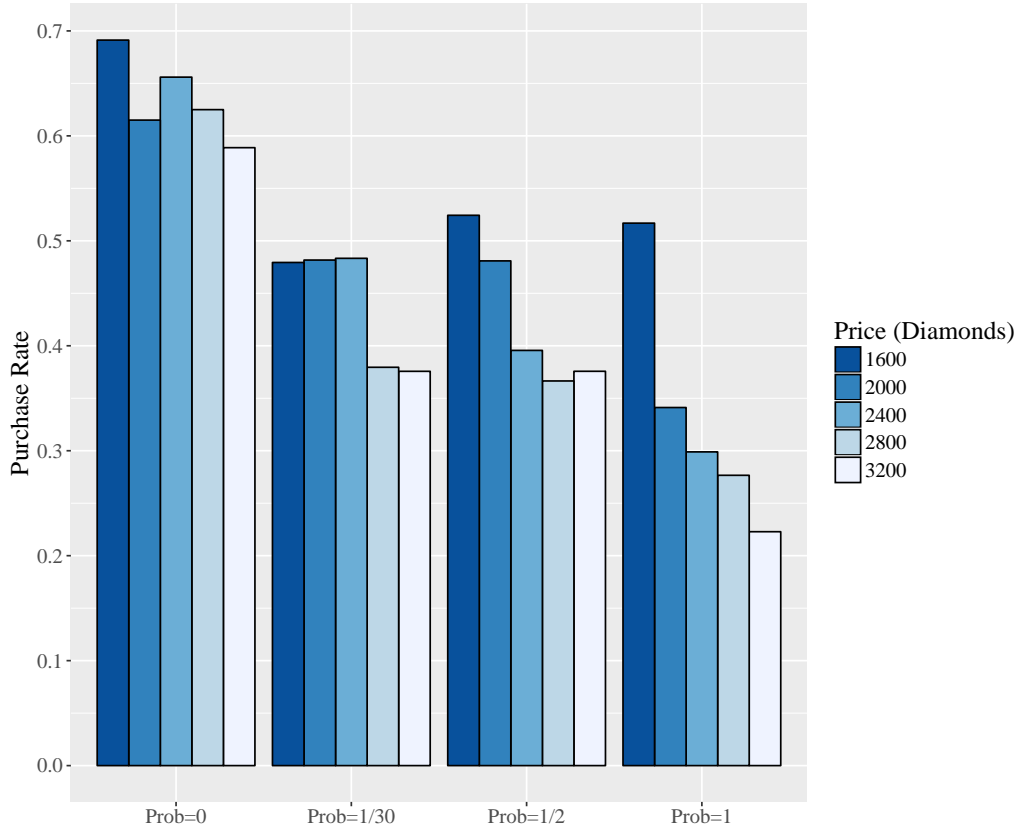
In this section, we present evidence of the preference-learning theory, in terms of both prediction and mechanism, using data from the field experiment.

We first examine aggregate demand, defined as the proportion of users who choose “willing to buy” out of those who completed the choice task in each condition. Figure 2 shows how aggregate demand changes with price under each realization probability. We see a pattern –



as realization probability increases, demand decreases faster with price; in addition, the overall level of demand decreases.

Figure 2: Realization Probability and Manifested Demand



Notes. Purchase rate is the fraction of users who choose “willing to buy” out of those who completed the choice task in each experimental condition. Prob means realization probability.

To verify these observations statistically, we fit a logistic demand curve for each realization probability condition by regressing individual-level purchase decisions on price. The dependent variable Purchase equals 1 if the user chooses “willing to buy” and 0 if the user chooses “not willing to buy.” For the ease of presentation, we normalize the five price levels to 4, 5, 6, 7, 8 respectively in this regression and subsequent analysis. Table 3 presents the estimated price coefficients and intercepts of the demand curves. The price coefficient decreases with realization probability, consistent with the prediction of the theory.

We further examine how individual-level purchase decisions are jointly influenced by price and realization probability. We estimate a logistic model of purchase decisions pooling data

Table 3: Manifested Demand Curves by Realization Probability

	(Prob=0)	(Prob=1/30)	(Prob=1/2)	(Prob=1)
	Purchase	Purchase	Purchase	Purchase
Price	-0.0851*	-0.124***	-0.170***	-0.303***
	(0.0442)	(0.0466)	(0.0480)	(0.0532)
Constant	1.067***	0.503*	0.729**	1.089***
	(0.276)	(0.285)	(0.293)	(0.316)
$N$	1095	920	922	895
Pseudo $R^2$	0.003	0.006	0.010	0.031

Notes. Logistic regression. Prob means realization probability. Dependent variable is the Purchase dummy variable. Prices are normalized to {4, 5, 6, 7, 8}. Standard errors in parentheses.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

from all realization probability conditions. Following Balli and Sørensen (2013), we normalize the mean values of price and realization probability to zero in this model, so that the magnitude of the main effects and interaction effect can be visualized more transparently. Columns (1)-(3) of Table 4 present the estimation results. Column (1) shows that individual users' purchase likelihood decreases with price, as expected. Purchase likelihood also decreases with realization probability, consistent with the prediction of Proposition 2 – if demand declines faster with price under higher realization probabilities, it is not surprising that demand is lower for higher realization probabilities at a given price. The result echoes findings from the literature that hypothetical preference elicitation tends to overestimate demand (e.g., Diamond and Hausman 1994, Cummings et al. 1995).

An alternative explanation to the negative effect of realization probability on manifested demand is that a smaller realization probability induces consumers to perceive the product as being more precious and of higher quality. In the post-choice survey (see the Online Appendix for details), we ask users whether they think the opportunity to buy this player package is rare. The answer could be yes, indifferent, or no (coded as 1, 2, and 3, respectively). We find that perceived rarity is not significantly correlated with realization probability (Corr = 0.0100,  $p = 0.646$ ) for positive realization probabilities. We also ask users to rate how they perceive the quality of this player package on a 5-point scale. The rating is not significantly correlated with

realization probability either ( $\text{Corr} = 0.0078$ ,  $p = 0.721$ ) for positive realization probabilities. These results help mitigate the alternative explanation of rarity to some degree.

Table 4: Manifested Price Sensitivity Increases with Realization Probability

	(1)	(2)	(3)	(4)
	Purchase (Logistic)	Purchase (Logistic)	Purchase (Logistic)	Purchase (Linear)
Price	-0.155*** (0.0236)	-0.161*** (0.0237)	-0.157*** (0.0239)	-0.0360*** (0.00554)
Realization Probability	-0.944*** (0.0836)	-0.960*** (0.0848)	-0.965*** (0.0848)	-0.224*** (0.0189)
Price $\times$ Realization Probability		-0.209*** (0.0607)	-0.196*** (0.0610)	-0.0390*** (0.0134)
Log-Diamonds			0.0939*** (0.0219)	0.0217*** (0.00499)
VIP-Level			-0.0690*** (0.0113)	-0.0159*** (0.00255)
Constant	-0.138*** (0.0332)	-0.144*** (0.0333)	-0.605*** (0.154)	0.362*** (0.0352)
$N$	3832	3832	3832	3832
Pseudo/adj. $R^2$	0.033	0.035	0.044	0.058

Notes. Logistic regression for Columns (1)-(3). OLS regression for Column (4). Dependent variable is the Purchase dummy variable. Prices are normalized to  $\{4, 5, 6, 7, 8\}$ . Following Balli and Sørensen (2013), we further normalize the mean values of Price and Realization Probability to zero to facilitate interpretation. Standard errors in parentheses.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

For a direct test of Proposition 2, we add the interaction term of price and realization probability to the aforementioned regression of individual purchase decisions on these two factors. As column (2) of Table 4 shows, this interaction term has a significantly negative coefficient. In column (3), we further control for user characteristics, namely, Diamonds and VIP-Level. Since Diamonds is a highly right-skewed variable, we transform it into a new variable  $\text{Log-Diamonds} = \log(\text{Diamonds} + 1)$  and will use this new variable in subsequent analysis. We find that having more diamonds and having lower VIP levels are associated with higher purchase rates. All other coefficients remain stable. Furthermore, because interaction terms in nonlinear models may not be straightforward to interpret (Greene 2010), we estimate the linear counterpart of Column (3). As Column (4) shows, the conclusion is robust – users are more

price sensitive under higher realization probabilities, consistent with Proposition 2.

So far, data support the predicted effect of realization probability on manifested demand. Next we examine whether this effect is indeed driven by the preference-learning mechanism we propose. To this end, we need a measure of users’ preference-learning effort. Measuring individuals’ effort engagement in choice tasks is difficult (Bettman et al. 1990). We approach this problem using different proxies of preference-learning effort.

For a first proxy of preference-learning effort, we draw upon the classic measure of decision effort as decision time (Wilcox 1993). We record decision time as the number of seconds it takes from the point the user first arrives at the choice task page to the point she makes a choice. Table 5 reports the summary statistics. It turns out the decision time variable is right-skewed with some extremely large values. Therefore, we also examine a log transformation of this variable, Log-Decision Time, which is calculated as  $\log(\text{Decision Time} + 1)$ .<sup>9</sup>

Admittedly, decision time may not be an accurate measure of preference-learning effort, as some users may think quickly but diligently. Therefore, we supplement the mechanism test with another proxy of preference-learning effort, leveraging the unique context of the field experiment. Recall that users can click on the player package to acquire information about the players contained therein. If a user has carefully thought about her valuation of the player package, arguably, she should know its content. Therefore, in the post-choice survey, we ask each user to answer “which of the following soccer players was **not** included in the player package” (see the Online Appendix for details). The corresponding measure of effort equals 1 if the user provides the correct answer (there is only one correct answer), and 0 if the user gives the wrong answer or chooses “I don’t know.”

As a direct mechanism test, we regress these three measures of preference-learning effort on realization probability, price, and their interaction term. We again normalize the mean values of realization probability and price to zero following Balli and Sørensen (2013). Table 6 presents the result. For all three measures, users’ preference-learning effort increases with real-

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<sup>9</sup>All users in the sample did complete the choice task. Therefore, we choose not to simply remove users with extremely long decision time from the data.

Table 5: Summary Statistics of Preference-Learning Effort Measures

	Mean	Std Dev	Median	Min	Max	$N$
Decision Time (seconds)	1630.87	4892.93	7.08	0.56	23999.17	3832
Log-Decision Time (seconds)	3.07	2.56	2.09	0.44	10.09	3832
Correct Answer (binary)	0.55	0.50	1.00	0	1	2984

Notes. Decision Time is recorded for all users who completed the choice task. Log-Decision Time is calculated as  $\log(\text{Decision Time} + 1)$ . Correct Answer is recorded for all users who completed the survey.

ization probability, consistent with Proposition 1. The effects of price and its interaction with realization probability are largely insignificant. One possible explanation is that Proposition 1 offers ambiguous predictions regarding these two effects; their signs depend on how each user’s assigned price compares with her prior belief of her product valuation.

Table 6: Preference-Learning Effort Increases with Realization Probability

	(1)	(2)	(3)	(4)	(5)	(6)
	Decision Time	Decision Time	Log-Decision Time	Log-Decision Time	Correct Answer	Correct Answer
Realization Probability	916.4*** (204.4)	916.3*** (204.5)	0.281*** (0.106)	0.281*** (0.106)	0.0549** (0.0227)	0.0537** (0.0227)
Price	-52.62 (54.75)	-52.54 (54.93)	-0.0372 (0.0293)	-0.0371 (0.0293)	-0.00314 (0.00639)	-0.00276 (0.00638)
Price $\times$ Realization Prob.		15.34 (141.4)		0.0148 (0.0739)		0.0358** (0.0158)
Constant	1630.9*** (78.83)	1630.8*** (78.84)	3.070*** (0.0413)	3.070*** (0.0413)	0.553*** (0.00910)	0.553*** (0.00909)
$N$	3832	3832	3832	3832	2984	2984
Adj. $R^2$	0.005	0.005	0.002	0.002	0.001	0.003

Notes. OLS regression. Prices are normalized to  $\{4, 5, 6, 7, 8\}$ . Following Balli and Sørensen (2013), we further normalize the mean values of Price and Realization Probability to zero to facilitate interpretation. Standard errors in parentheses.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

In summary, data from the field experiment support the theory in both its prediction (Proposition 2) and its underlying mechanism (Proposition 1). These results are consistent with Yang et al. (2018)’s finding that consumers’ price sensitivity increases with realization probability, although we do not evoke the psychological distance explanation. In fact, our preference-learning explanation is consistent with Yang et al. (2018)’s finding that consumers’ attention to the choice task increases with realization probability. Built on these findings, in the following sec-

tion, we develop and evaluate a method to forecast demand with low-cost choice experiment data.

## 5 AIA Demand Forecasting Method

In this section, we develop the AIA demand forecasting method and evaluate its performance using data from the field experiment. The core of the method is an AIA model of consumer product choice based on the preference-learning mechanism developed in the theory section. We estimate the AIA model drawing on choice data from the incentive alignment conditions (i.e., the 1/2-probability and 1/30-probability conditions), leaving data from the actual purchase condition (i.e., the 1-probability condition) as the holdout sample. We then use the model estimates to forecast demand in actual purchase settings (i.e., settings where realization probability equals 1), and compare the forecast with actual demand in the holdout sample. To assess the value of having a theory-based model, we also compare the AIA forecast with simple extrapolation of data from incentive alignment conditions to real purchase settings. Finally, we compare the AIA method with the test-market approach on cost of data.

### 5.1 AIA Model of Consumer Product Choice

The AIA model of consumer product choice captures the behavioral process described in the theory section but operationalizes it to match the empirical context. For a conservative evaluation of the AIA method, we strive to keep the model parsimonious.

We operationalize product valuation following the established multi-attribute linear utility framework (e.g., Roberts and Urban 1988). Let user  $i$ 's true valuation of the product be

$$v_i = b_0 + b_1 \text{Log-Diamonds}_i + b_2 \text{VIP-Level}_i + \epsilon_{vi}, \quad (6)$$

where  $\epsilon_{vi}$  represents the unobserved heterogeneity in users' true product valuation, which follows a normal distribution  $N(0, \sigma_v^2)$ . Recall that  $\text{Log-Diamonds}_i = \log(\text{Diamonds}_i + 1)$ , where

Diamonds<sub>*i*</sub> is the number of diamonds user *i* has at the time of the experiment. VIP-Level<sub>*i*</sub> denotes the VIP level of user *i* at the time of the experiment, which is determined by how much this user has spent in the game. For the ease of interpretation, we scale both Log-Diamonds<sub>*i*</sub> and VIP-Level<sub>*i*</sub> to [0, 1] by dividing each variable by its maximum value. We conjecture that a user with more diamonds at hand is likely to have a higher willingness-to-pay for the product. The sign of VIP-Level is *a priori* ambiguous. A user who has spent a lot may be more likely to spend on the new product out of habit or ability, or less likely to spend because she has already recruited enough players she wanted for her team.

User *i*'s prior belief about her product valuation follows the normal distribution  $N(v_i, \sigma_{0i}^2)$ , where the prior uncertainty term  $\sigma_{0i}$  is operationalized as

$$\sigma_{0i} = \exp(a_0 + a_1 \text{VIP-Level}_i). \quad (7)$$

We use the exponential function here to guarantee that  $\sigma_{0i}$  is positive. We expect VIP-Level to have a negative coefficient because, other things being equal, more spending arguably means greater experience with the game and hence less uncertainty about product valuation. As such, the estimated sign of VIP-Level helps assess the face validity of the preference-learning mechanism.

Knowing her prior mean valuation of the product  $\mu_i$  and her prior uncertainty  $\sigma_{0i}$ , user *i* can derive her optimal level of effort in the same way as in the theory model:

$$t_i^* = \min \left\{ \frac{r_i}{c_i} (\mathbb{E}[(v_i - p_i)^+] - (\mu_i - p_i)^+), 1 \right\}, \quad (8)$$

where the expectation is taken over consumer *i*'s prior belief  $v_i \sim N(\mu_i, \sigma_{0i}^2)$ .  $p_i$  and  $r_i$  are the price and realization probability randomly assigned to user *i* in the experiment. We restrict effort  $t_i^*$  to be no larger than 1 because it is defined as the probability that the consumer will learn her true valuation (see Section 2). As we will discuss later, estimated effort levels are well below 1, which reduces the concern that capping effort levels affects the estimation results. We

further operationalize user  $i$ 's effort cost  $c_i$  as

$$c_i = \exp(c_0 + c_1 \epsilon_{ci}), \quad (9)$$

where  $\epsilon_{ci} \sim N(0, 1)$ . The exponential transformation again guarantees that effort cost is positive. The  $\epsilon_{ci}$  term allows effort cost to be heterogeneous among users.

Given her effort level  $t_i^*$ , with probability  $t_i^*$ , user  $i$  learns her true product valuation  $v_i$  and buys the product if  $v_i \geq p_i$ . With probability  $1 - t_i^*$ , user  $i$  retains her prior belief and buys if  $\mu_i \geq p_i$ . We make the common assumption that users have a response error when making purchase decisions, and that the response error follows the i.i.d. standard Type I extreme value distribution. It follows that user  $i$ 's probability of choosing "willing to buy," encoded as  $\text{Buy}_i = 1$ , is given by the standard logit formula:

$$\Pr(\text{Buy}_i = 1) = t_i^* \frac{\exp(v_i - p_i)}{1 + \exp(v_i - p_i)} + (1 - t_i^*) \frac{\exp(\mu_i - p_i)}{1 + \exp(\mu_i - p_i)}. \quad (10)$$

The log-likelihood function of the observed purchase decision data is

$$LL = \sum_{i=1}^N \left[ \mathbf{1}(\text{Buy}_i = 1) \log \Pr(\text{Buy}_i = 1) + \mathbf{1}(\text{Buy}_i = 0) \log (1 - \Pr(\text{Buy}_i = 1)) \right], \quad (11)$$

where  $N$  is the number of users who completed the choice task.

The above formulation of the log-likelihood function does not rely on actual data on consumer effort choices. Instead, it calculates effort choices based on model parameters following the process described in the theory model. We do have proxies of effort from the field experiment. We could in theory incorporate these measures to derive additional moments for the estimation. However, for a fair evaluation of the AIA method, we deliberately avoid relying on effort data for model calibration. We would like the AIA method to perform well (in particular, outperform incentive alignment) not because it uses more data, but because it uses the same incentive alignment data better. In addition, being able to perform well in the absence of effort measures lowers the data requirement and broadens the applicability of the AIA method.



## 5.2 Estimation Procedure

The AIA model is estimated using the simulated maximum-likelihood estimation approach (Train 2009). For a given set of parameter values, we calculate the purchase probability of each user averaged over a large number of pre-simulated random draws, and then calculate the log-likelihood by summing up the log-likelihood of each user. The estimated parameter values are found by maximizing the simulated log-likelihood. The standard error is estimated using the inverse of Hessian matrix at the estimated parameter values. We present the detailed estimation procedure in the Online Appendix.

As discussed, we use data from incentive alignment conditions, where realization probability equals 1/30 or 1/2, to estimate the model parameters. We leave the 1-probability condition as the holdout sample to assess the predictive validity of the AIA method. We do not use data from the 0-probability condition in estimation for two reasons. First, our theory does not predict how consumers will choose in the hypothetical setting. Thus we need to make further assumptions to interpret choice data from this condition. For instance, we could estimate an additional parameter that captures consumers' tendency to act on their true beliefs when indifferent. The identification of this parameter, however, still has to rely on information from the 1/30-probability and 1/2-probability conditions. Second, we include the 0-probability condition in the field experiment to assess how stated preferences perform compared with other preference elicitation methods within the same empirical context. Application of the AIA method, however, does not require data from the 0-probability condition. We exclude this condition from the estimation to keep the AIA method "lean" in terms of data requirement.

## 5.3 Identification

The parameters we need to estimate are the constant and coefficients in users' true valuation equation  $(b_0, b_1, b_2)$ , prior uncertainty equation  $(a_0, a_1)$ , and effort cost equation  $(c_0, c_1)$ , as well as the standard deviation of users' unobserved heterogeneity in true valuation  $(\sigma_v)$ .  $b_0$  is identified from the overall level of demand.  $b_1$  and  $b_2$  are identified from users' variations

in observable characteristics (i.e., Log-Diamonds and VIP-Level) and in purchase decisions.  $(a_0, a_1)$  and  $(c_0, c_1)$  together determine users' optimal preference-learning effort and, in turn, their manifested demand.  $(a_0, a_1)$  can be separately identified from  $(c_0, c_1)$  because, according to Equation (8), even if effort cost is held constant, variations in price help reveal the effect of prior uncertainty (via the  $\mathbb{E}$  operator) on optimal effort and thus manifested demand.  $a_1$  is further identified from how VIP level moderates this effect. Finally, since every user only makes one purchase decision in our data, unobserved heterogeneity  $\sigma_v$  is identified from the part of heterogeneity in product valuation (as revealed in product choices) that cannot be captured by observable user characteristics.

## 5.4 Estimation Results

Table 7 reports the parameter estimates and their standard errors. Users' true valuation of the product, not surprisingly, increases with the amount of currency they own in the game ( $b_1 > 0$ ,  $p < 0.01$ ). Users' true valuation of the product also decreases with the VIP level ( $b_2 < 0$ ,  $p = 0.03$ ). As discussed before, one explanation is that users with higher VIP levels tend to have spent more in the game and, as a result, are more likely to have staffed their teams with high-quality players already, so that the new player package is of less value to them. In addition to these observed variations, there is unobserved heterogeneity in users' true valuation ( $\sigma_v > 0$ ,  $p = 0.09$ ). The magnitude of this unobserved heterogeneity is nontrivial given that Log-Diamonds and VIP-Level are both normalized to  $[0, 1]$  for estimation. Moving on, there is significant prior uncertainty ( $a_0 > 0$ ,  $p < 0.001$ ), which means preference learning is indeed relevant in this empirical context. Meanwhile, users with higher VIP levels are more certain about their valuation of the product ( $a_1 < 0$ ,  $p < 0.001$ ), which adds to the face validity of the preference-learning theory. Finally, the effort cost parameter  $c_0$  is positive and significant ( $p < 0.001$ ), but the heterogeneity term  $c_1$  is not significantly different from zero. These results suggest that preference learning is costly, and similarly costly to all users in this field experiment.

Table 7: Estimation Results of the AIA Model

Variable		Parameter	Estimate	S.E.
True Valuation	Constant	$b_0$	-2.342	2.943
	Log-Diamonds	$b_1$	12.479***	4.756
	VIP-Level	$b_2$	-8.062**	3.788
	Unobserved Heterogeneity Magnitude	$\sigma_v$	0.857*	0.505
Prior Uncertainty	Constant	$a_0$	4.822***	0.619
	VIP-Level	$a_1$	-3.100***	0.902
Effort Cost	Constant	$c_0$	3.245***	0.571
	Heterogeneity Magnitude	$c_1$	3.839e-6	0.169
$N$			1842	
Log-Likelihood			-1238.68	

Notes. The sample for estimation consists of conditions in which realization probability equals 1/30 or 1/2. Prices are normalized to {4, 5, 6, 7, 8}. Log-Diamonds and VIP-Level are normalized to [0, 1].

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

To put the estimation results in context, we calculate each user’s optimal preference-learning effort ( $t_i^*$ ) based on the parameter estimates. Table 8 presents the mean and standard deviation of estimated effort by realization probability. Estimated effort does increase with realization probability. It equals zero in the 0-probability condition by definition. In the actual purchase condition with realization probability equal to 1, users on average spend an effort of 0.452 out of a normalized range of 0 to 1. The fact that estimated effort largely lies in the interior of the 0-to-1 interval suggests that, reassuringly, model estimation is not driven by corner solutions in users’ effort choices.

Table 8: Estimated Preference-Learning Effort

Condition	Estimated Effort Level ( $t_i^*$ )	
	Mean	S.D.
Realization Probability=0	0	0
Realization Probability=1/30	0.017	0.008
Realization Probability=1/2	0.254	0.117
Realization Probability=1	0.452	0.183

Notes. Effort level is normalized between 0 and 1. Effort equals 0 in the 0-probability condition by definition.

In addition, based on the estimation results, we calculate users’ mean valuation of the player

package as 1384 diamonds. Recall that the lowest price offered in the field experiment is 1600 diamonds, which is significantly higher than users' mean valuation (S.E. = 14.7,  $p < 0.001$ ). As an auxiliary test of the AIA model's face validity, in the post-choice survey we ask users to rate how they perceive the price of the product on a scale from 1 (very low) to 5 (very high) (see the Online Appendix for details). Indeed, the answers confirm that users view the price as being relatively high; the mean answer is 3.99, significantly higher than the neutral level of 3 ( $t = 52.84, p < 0.001$ ).

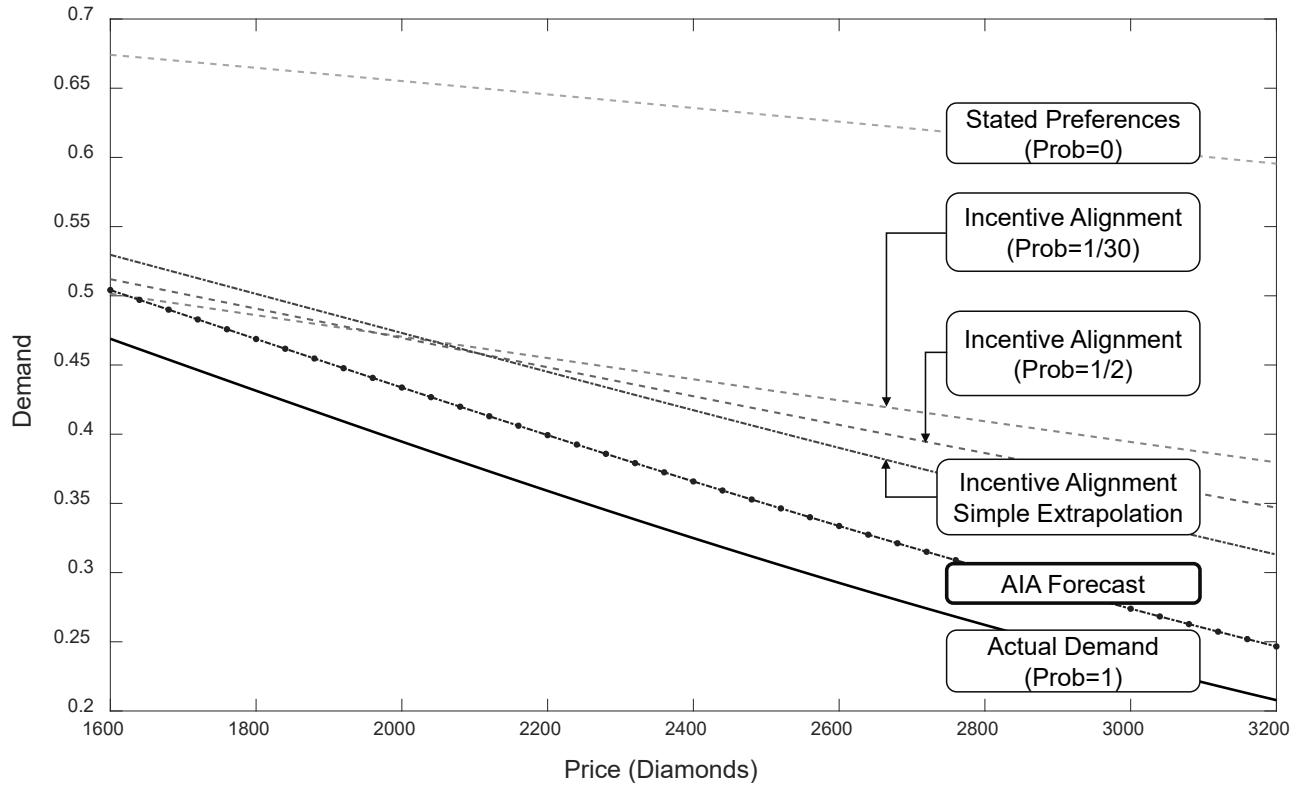
## 5.5 Forecasting Demand in Real Purchase Settings

Based on the parameter estimates, we simulate the purchase decision of each user in the AIA model for the counterfactual case of realization probability equal to 1 (see the Online Appendix for details). The simulation results form the AIA forecast of demand in real purchase settings. We compare the forecast against actual demand in the holdout sample, that is, in the 1-probability condition we have set aside. To put the forecast in context, we also compare it with manifested demand in the other three realization probability conditions. For the ease of visualization, we fit a logistic demand curve for each preference elicitation method.

Figure 3 visualizes the comparison. Consistent with prior findings from the literature, the stated-preferences approach (i.e., the 0-probability condition) performs poorly; compared with actual demand, it overestimates demand considerably and it underestimates the degree of price sensitivity. Incentive alignment (i.e., the 1/30-probability and 1/2-probability conditions) improves forecast accuracy, especially if realization probability is higher (1/2 as opposed to 1/30). The AIA forecast generates a demand curve the closest to the actual demand curve of the holdout sample.

A natural question at this point is whether one can forecast demand as accurately using simple extrapolation methods instead of the more-complex AIA model. One can use data from incentive alignment (i.e., the two interim probability conditions) and extrapolate to the case where realization probability equals 1. To answer this question, we estimate an individual-level

Figure 3: Preference Elicitation Methods – A Comparison of Demand Forecast



Notes. Logistic demand curves. Prob means realization probability.

logistic regression model of purchase decisions as a function of price, realization probability (1/30 or 1/2), their interaction terms, as well as observed user characteristics (Log-Diamonds and VIP-Level). The estimates then allow for extrapolation of purchase decisions to the case of realization probability being 1.

We plot the fitted demand curve, labeled “incentive alignment simple extrapolation,” in Figure 3. This fitted demand curve is closer to actual demand than the demand curves manifested in the two incentive alignment conditions. However, simple extrapolation performs notably worse than the AIA forecast (formal test to follow). This is true although simple extrapolation uses exactly the same data as the AIA forecast. The AIA forecast performs better here because it uses the data in a better way by imposing a theoretically sound and empirically validated behavioral process.

We formally quantify and compare the predictive validity of the preference elicitation methods presented in Figure 3. The first column of Table 9 reports the estimated logistic price

coefficient of each method. Stated preferences perform the worst, with estimated price coefficient 72% lower in absolute value than in actual purchase settings. Incentive alignment with realization probabilities of 1/30 and 1/2 perform progressively better, but still produce noticeable forecast errors. The forecast error reduces to around 25% for simple extrapolation of incentive alignment, and is only 6.6% for the AIA forecast.

Table 9: Preference Elicitation Methods – A Comparison of Predictive Validity

Preference Elicitation Method	Price Coefficient	Likelihood Ratio (vs. Actual)	Optimal Price	Profit Loss
Stated Preferences (Prob=0)	-0.0851	198.59***	\$74.85	90.48%
Incentive Alignment (Prob=1/30)	-0.1243	29.37***	\$45.24	49.85%
Incentive Alignment (Prob=1/2)	-0.1702	22.15***	\$34.65	22.61%
Incentive Alignment Simple Extrapolation	-0.2262	16.39***	\$27.87	6.95%
AIA Forecast	-0.2833	3.18	\$22.92	0.57%
Actual Demand (Prob=1)	-0.3034	0	\$21.09	0

Notes. Calculations are based on logistic demand. Prob means realization probability.

\*  $p < 0.10$ , i.e.,  $LR > q\chi^2(0.9, 2) = 4.6052$ ;

\*\*  $p < 0.05$ , i.e.,  $LR > q\chi^2(0.95, 2) = 5.9915$ ;

\*\*\*  $p < 0.01$ , i.e.,  $LR > q\chi^2(0.99, 2) = 9.2103$ .

Besides price sensitivity, Figure 3 suggests that different preference elicitation methods predict different levels of demand. We perform a likelihood ratio (LR) test to determine the overall fit of forecast demand with actual demand. For each preference elicitation method  $k \in \{\text{Prob} = 0, \text{Prob} = 1/30, \text{Prob} = 1/2, \text{Extrapolation}, \text{AIA}\}$ , its likelihood ratio is calculated as  $LR_k = -2[LL_{\text{Pooled}} - (LL_{\text{Actual}} + LL_k)]$ , where  $LL$  represents the log-likelihood of a logistic demand curve based on observed purchases or simulated purchase probabilities. The likelihood ratio follows a chi-square distribution with degrees of freedom equal to the difference in the number of free parameters, which is 2 in our case. The second column of Table 9 reports the likelihood ratio of each method relative to actual demand. We cannot reject the null hypothesis that the AIA forecast coincides with actual demand, whereas all the other methods significantly deviate from actual demand at the  $p < 0.01$  level.

To illustrate the practical value of the AIA method, we calculate the optimal price implied by the actual demand curve and by the various preference elicitation methods, respectively. We

write the fitted logistic purchase rate as  $\exp(\alpha_0 + \alpha_1 p) / [1 + \exp(\alpha_0 + \alpha_1 p)]$ . We also assume the marginal cost of production is zero, which is a reasonable assumption considering the digital nature of the product featured in the field experiment. It follows that the profit-maximizing price  $p^*$  solves  $1 + \alpha_1 p + \exp(\alpha_0 + \alpha_1 p) = 0$ . The third column of Table 9 presents the optimal price implied by the coefficients of each demand curve.

Furthermore, substituting the optimal price recommended by each method into the actual demand curve, we can calculate the expected total profit if that price is charged in actual purchase environments. Comparing the expected profit to the optimal profit in the actual demand condition, we obtain the percentage profit loss associated with each method. The last column of Table 9 presents the results. By recommending an excessively high price, stated preferences lead to an approximately 90% profit loss in this particular empirical setting. Incentive alignment performs better. Simple extrapolation of incentive alignment data introduces further improvement, reducing the profit loss to about 7%. However, the AIA method takes predictive accuracy to yet another level. It cuts the profit loss to 0.57%, which is less than one tenth of the loss under simple extrapolation.

To summarize, the AIA method performs well. It forecasts actual demand significantly better than stated preferences and incentive alignment. Moreover, it forecasts actual demand significantly better than simple extrapolation of incentive alignment data to real purchase settings. We have strived to keep the AIA model parsimonious for this first test of its predictive validity. The AIA method may perform even better if we enrich the model by, for instance, introducing more forms of consumer heterogeneity.

## 5.6 Cost of Preference Elicitation

Having examined the predictive validity of the AIA method, it will be worthwhile to discuss its cost. We have argued that the AIA method relies on lower-cost data than the test-market approach, which is conceptually equivalent to incentive alignment with realization probability equal to 1 (Figure 1). In this section, we quantify the cost savings of the AIA method compared

with test markets. To facilitate comparison, we abstract away from the operational overhead of obtaining consumer choice data, which is arguably higher for test markets. We focus on the variable cost of data, which we measure in three ways.

The first cost measure is the expected number of actual products required to achieve a given sample size of consumer choice data. The number of products matters because it can be costly and even infeasible for a firm to provide many products before launch. For instance, the number of products required can impose a serious constraint for new, physical products, or for small firms that are relying on demand forecast to raise venture funding. For each preference elicitation method, we compute this cost measure as the sum of expected demand under different price levels multiplied by the realization probability associated with this elicitation method.

The second cost measure is the amount of budget provided to participants of the choice task. If participants face liquidity concerns, a common solution in the literature and in practice is to endow them with a budget. In incentive-aligned choice tasks, a participant whose lottery succeeds will be provided a certain amount of money (denoted as  $B$ ) that is enough to buy the product (i.e.,  $B$  is greater than the maximum price in the experiment). If the participant has chosen “willing to buy” at price  $p$ , she will receive the product and keep the remaining money of  $B - p$ ; if she has chosen “not willing to buy” at price  $p$ , she will retain the entire budget  $B$ . The expected cost of endowing participants with this budget equals  $B$  multiplied by the expected number of lottery winners, and is thus proportional to realization probability. This budgeting cost can be prohibitive if the product is expensive and if realization probabilities are high.

The third cost measure is more “theoretical.” By definition, to calibrate the demand curve by varying prices in an experiment, the firm must sell the product at multiple price levels, some if not all of which will be suboptimal. We thus define this data cost as the opportunity cost of selling the product at suboptimal prices for the purpose of experimentation. More specifically, for all products sold in the experiment, we calculate this opportunity cost as the additional amount of profit the firm could have expected to earn had it sold this product with perfect information of demand in real purchase settings. Intuitively, the opportunity cost should



increase with realization probability as more products will be sold for real at suboptimal prices.

To summarize, by using incentive alignment data with less-than-one realization probabilities, the AIA method is likely to save costs compared with test markets. We quantify the cost comparison based on data from the field experiment. Note that the first two aspects of cost are not a concern in our field experiment – the product is virtual with zero marginal cost of production, and liquidity is not a problem because users are able to pay for the product automatically using diamonds banked in their accounts. Nevertheless, for completeness, we draw on data from the field experiment to illustrate the cost-effectiveness of the AIA method on all three cost measures.

Indeed, we find that the AIA method dramatically reduces the cost of data on all three measures. Compared with test markets, the AIA method requires 34.5% of the number of products, 26.7% of participant budget, and -116.3% of opportunity cost of selling. The opportunity cost even turns out negative because, compared with actual purchase settings, participants are less price sensitive and more willing to buy at high prices in interim-probability conditions. As such, the AIA method ends up generating even more profits in the field experiment than the firm would have earned with perfect knowledge of demand in real purchase environments. Whether this will happen in other applications of the AIA method depends on the specific choices of prices, realization probabilities, and the shape of manifested demand under these realization probabilities. However, we expect the cost-saving feature of the AIA method compared with test markets to be generally applicable.

Finally, note that the three cost measures are calculated based on the field experiment which assigns an equal number of participants to the 1/30-probability and 1/2-probability conditions. In future applications of the AIA method, one may be able to cut costs further by optimizing the allocation of sample size across probability conditions, and by optimizing the choice of realization probabilities.

## 6 Concluding Remarks

In this paper, we advocate the view that humans do not automatically know their preferences, but can learn their preferences through costly effort when given the proper incentive. In the context of preference elicitation, we argue that the relationship between price and demand, which forms the basis of many economic and managerial decisions, is not exogenously given as often assumed, but is an endogenous function of the elicitation method. In other words, preference is a manifestation of what people are willing to uncover.

To fix ideas, we focus on the effect of realization probability on manifested demand. Commonly used in choice experiments, realization probability refers to the probability with which a participant’s stated product choice is realized as an actual transaction. Our theory model predicts that manifested price sensitivity increases with realization probability. We find supporting evidence of this prediction and of the preference-learning mechanism from a large-scale field experiment on a mobile game platform. These findings allow us to develop an augmented incentive alignment (AIA) method that accurately forecasts real demand from inexpensive choice experiment data of small to moderate realization probabilities.

There are a number of ways to extend this research. We have chosen realization probabilities somewhat arbitrarily for a first test of the AIA method. As mentioned in the previous section, it will be meaningful to investigate the optimal choice of realization probabilities, as well as the size of each probability condition, especially if the cost of experimentation is a concern. To provide a clean proof of concept, we have also kept the choice task simple. A valuable extension is to study preference learning about multi-attribute products. More broadly, the findings of the paper are relevant to incentive compatibility design in choice experiments. While inducing truth-telling has been the focus of many research efforts to date, our results suggest that the notion of truth-finding also deserves attention.

The paper has several limitations worth addressing in future research.<sup>10</sup> First, we present a simple model that abstracts away from established behavioral decision theories such as prospect

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<sup>10</sup>We thank an anonymous reviewer for pointing out these limitations.

theory. Consumers' preference-learning incentives may depend on how actual price compares with their reference price, and how loss averse these consumers are. Second, although our model allows for varying degrees of prior valuation uncertainty, for radically new products, consumers may have difficulty forming prior beliefs, and the price itself may impose an anchoring effect on consumers' perception of product value. Third, our model focuses on risk-neutral purchase decisions without income constraints, and may not generalize to situations where products are difficult to liquidate or are substantially expensive. Last but not least, the product featured in the field experiment is a virtual package on a mobile game platform. It will be important to examine the performance of the AIA method in other contexts with different products, including physical products and products that need to be purchased with real currencies.

We would like to conclude by emphasizing one implication of our findings – that even microfounded models are not necessarily immune to the critique of Lucas (1976). To the extent that individual consumer price sensitivity is endogenous to the preference elicitation method, it will be worthwhile to ask if what have been commonly accepted as “deep preference parameters” are always policy-invariant. In fact, the reason our method is able to forecast well out-of-sample is that it allows consumer preferences to change under different policies (i.e., different realization probabilities), whereas its underlying decision process remains policy-invariant. Correspondingly, our modeling approach is structural in the sense of Marschak (1953) and the Cowles Commission, which has been pursued by many others, most notably Heckman (e.g., Heckman and Vytlačil 2007). Our findings echo their view that policy invariance is an important driver of externality validity.

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# Appendix

## A.1 Proof of Proposition 1

**Proof.** First, consider the case of  $\mu < p$ . Rearranging terms yields

$$t^*(r, p; \mu) = \frac{r}{c} \int_{-\infty}^{\mu-p} (\mu - e - p)g(e)de. \quad (\text{A1})$$

Taking derivatives then yields

$$\frac{\partial t^*(r, p; \mu)}{r} = \frac{1}{c} \int_{-\infty}^{\mu-p} (\mu - e - p)g(e)de, \quad (\text{A2})$$

$$\frac{\partial t^*(r, p; \mu)}{\partial |p - \mu|} = -\frac{r}{c} \int_{-\infty}^{\mu-p} g(e)de, \quad (\text{A3})$$

$$\frac{\partial^2 t^*(r, p; \mu)}{\partial r \partial |p - \mu|} = -\frac{1}{c} \int_{-\infty}^{\mu-p} g(e)de. \quad (\text{A4})$$

(A2)  $\geq 0$ , (A3)  $\leq 0$ , (A4)  $\leq 0$ , where the inequality holds strictly if  $g(\cdot)$  has positive support everywhere over  $(-\infty, \infty)$ .

Second, consider the remaining case of  $\mu \geq p$ . Note that  $\mu - p = \int_{-\infty}^{\infty} (\mu - p - e)g(e)de$  because  $\int_{-\infty}^{\infty} g(e)de = 1$  by definition and  $\int_{-\infty}^{\infty} eg(e)de = 0$  by assumption. Rearranging terms yields

$$t^*(r, p; \mu) = \frac{r}{c} \int_{\mu-p}^{\infty} (e - \mu + p)g(e)de. \quad (\text{A5})$$

Taking derivatives then yields

$$\frac{\partial t^*(r, p; \mu)}{r} = \frac{1}{c} \int_{\mu-p}^{\infty} (e - \mu + p)g(e)de, \quad (\text{A6})$$

$$\frac{\partial t^*(r, p; \mu)}{\partial |p - \mu|} = -\frac{r}{c} \int_{\mu-p}^{\infty} g(e)de, \quad (\text{A7})$$

$$\frac{\partial^2 t^*(r, p; \mu)}{\partial r \partial |p - \mu|} = -\frac{1}{c} \int_{\mu-p}^{\infty} g(e)de. \quad (\text{A8})$$

(A6)  $\geq 0$ , (A7)  $\leq 0$ , (A8)  $\leq 0$ , where the inequality holds strictly if  $g(\cdot)$  has positive support everywhere over  $(-\infty, \infty)$ . ■



## A.2 Proof of Proposition 2

**Proof.** First, consider the case of  $\mu < p$ . Rearranging terms yields

$$\begin{aligned} D(r, p; \mu) &= \int t^*(r, p; \mu) \mathbf{1}(\mu - e \geq p) g(e) de \\ &= t^*(r, p; \mu) \int_{-\infty}^{\mu-p} g(e) de. \end{aligned} \quad (\text{A9})$$

Taking derivatives yields

$$\frac{\partial D(r, p; \mu)}{\partial p} = \frac{\partial t^*(r, p; \mu)}{\partial p} \int_{-\infty}^{\mu-p} g(e) de - t^*(r, p; \mu) g(\mu - p), \quad (\text{A10})$$

and that

$$\frac{\partial^2 D(r, p; \mu)}{\partial r \partial p} = \frac{\partial^2 t^*(r, p; \mu)}{\partial r \partial p} \int_{-\infty}^{\mu-p} g(e) de - \frac{\partial t^*(r, p; \mu)}{\partial r} g(\mu - p). \quad (\text{A11})$$

Recall from Proposition 1 that  $\partial^2 t^*(r, p; \mu) / \partial r \partial p \leq 0$  when  $\mu < p$  and that  $\partial t^*(r, p; \mu) / \partial r \geq 0$ , where the inequality holds strictly if  $g(\cdot)$  has positive support everywhere over  $(-\infty, \infty)$ . It follows that (A11)  $\leq 0$ , where the inequality holds strictly if  $g(\cdot)$  has positive support everywhere over  $(-\infty, \infty)$ .

Second, consider the case of  $\mu > p$ . (Note that  $D(r, p; \mu)$  may not be continuous at  $\mu = p$ , in which case  $\partial D(r, p; \mu) / \partial p$  does not exist.) Rearranging terms yields

$$\begin{aligned} D(r, p; \mu) &= \int \left[ t^*(r, p; \mu) \mathbf{1}(\mu - e \geq p) + (1 - t^*(r, p; \mu)) \right] g(e) de \\ &= 1 - t^*(r, p; \mu) \int_{\mu-p}^{\infty} g(e) de. \end{aligned} \quad (\text{A12})$$

Taking derivatives yields

$$\frac{\partial D(r, p; \mu)}{\partial p} = -\frac{\partial t^*(r, p; \mu)}{\partial p} \int_{\mu-p}^{\infty} g(e) de - t^*(r, p; \mu) g(\mu - p), \quad (\text{A13})$$

and that

$$\frac{\partial^2 D(r, p; \mu)}{\partial r \partial p} = -\frac{\partial^2 t^*(r, p; \mu)}{\partial r \partial p} \int_{\mu-p}^{\infty} g(e) de - \frac{\partial t^*(r, p; \mu)}{\partial r} g(\mu - p). \quad (\text{A14})$$

Recall from Proposition 1 that  $\partial^2 t^*(r, p; \mu)/\partial r \partial p \geq 0$  when  $\mu > p$  and that  $\partial t^*(r, p; \mu)/\partial r \geq 0$ , where the inequality holds strictly if  $g(\cdot)$  has positive support everywhere over  $(-\infty, \infty)$ . It follows that (A14)  $\leq 0$ , where the inequality holds strictly if  $g(\cdot)$  has positive support everywhere over  $(-\infty, \infty)$ . ■