A Dynamic Theory of Lending Standards*

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Abstract

We develop a dynamic model of credit markets in which lending standards and the quality of potential borrowers interact. Banks privately choose lending standards: whether to pay to screen out unprofitable borrowers. Lending standards have negative externalities and are dynamic strategic complements: tighter screening worsens the borrower pool, increasing banks' incentives to screen in the future. Lending standards amplify and prolong temporary downturns, affecting lending volume, credit spreads, and default rates. We characterize constrained-optimal policy, and find it is generally implementable as a government loan insurance program. Finally, capital constraints on banks incentivize tight lending standards, leading to amplification.

JEL codes: D82, G21, G01, G10

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1 Introduction

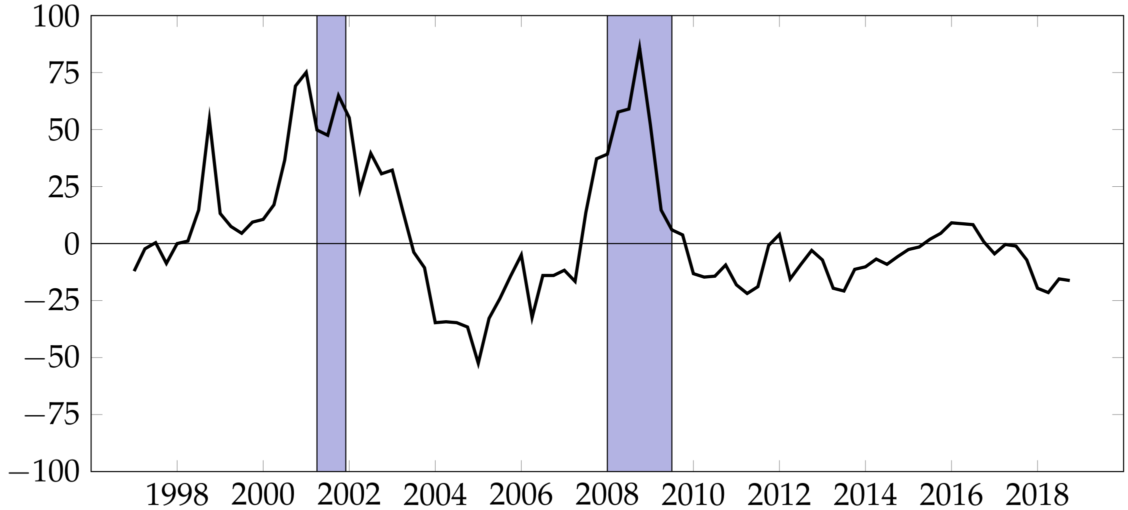
Through the allocation of external financing, lending standards play a key role in the economy, determining, for example, which entrepreneurs get initial funding, which firms grow, and which consumers buy houses. Figure 1 plots a measure of lending standards in the market for commercial and industrial (C&I) loans from banks. Lending standards, by this metric and others, are highly countercyclical, tightening in recessions and loosening in booms. For example, the recent credit boom-bust cycle was associated with relatively loose lending standards in the lending boom of the mid-2000's, when credit spreads and default rates were low, and relatively tight lending standards during the credit crunch and recession that followed, when spreads and default rates were high. Notably, the relaxation of lending standards following the crisis was slow and limited.

We develop a dynamic model of a credit market in which lending standards are endogenous, both influencing and responding to the quality of the pool of potential borrowers. We model lending standards as the extent to which banks acquire costly information about potential borrowers and condition their lending on this information. Information acquisition can take many forms, for example an interview with the applicant, verification of reported employment or income, appraisal of collateral, or an analysis of a business plan. The information collected by banks is private and borrowers whose loan applications are declined at one bank may apply at another bank in the future. This implies that lending standards are *dynamic strategic complements*: tight lending standards today imply that banks are confronted with a more adversely-selected pool in the future, which raises their incentive to impose tight lending standards.

Our dynamic model consists of competitive banks and a pool of potential borrowers, who are identical conditional on public or readily-available information (e.g. conditional on credit score). Each instant, some borrowers have the opportunity to approach banks in search of a loan to fund an investment project. Projects differ by borrower type: projects of high-quality (low-quality) borrowers have positive (negative) net present value. A bank can simply lend to a potential borrower who approaches it or it can choose to acquire costly private information about the borrower's type and condition lending on this information. To focus on the interesting dynamics of our model, we focus on the case in which, *ex ante*, potential borrowers' projects have positive expected net present value. Thus our model covers lending standards applied to borrowers or who are "prime" or have passed a preliminary evaluation.

While at any point in time the equilibrium of our model is unique, the dynamic model

Figure 1: Change in Bank Lending Standards for C&I Loans



Note: Figure shows the shows the percent of banks reporting tightening less the share reporting loosening. Banks can also report no change in standards. *Source:* Senior Loan Officer Opinion Survey on Bank Lending Practices, Board of Governors of the Federal Reserve System (2018).

exhibits multiple steady states in the single state variable, the *pool quality*, defined as the share of high-quality borrowers in the pool. In the *pooling steady state,* the pool quality is high enough that banks choose to approve loans without incurring the additional information collection costs, a normal lending standard. Low-quality borrowers are funded along with high-quality borrowers, which keeps the pool quality high. In this steady state, the volume of lending is high and loan spreads are low. Conversely, in the *screening steady state,* the pool quality is low enough that banks choose to collect costly private information on borrowers and base the lending decision on the information collected, a tight lending standard. In this steady state, the tight lending standard keeps the pool quality low, and the volume of lending is low and loan spreads are high.

Our first main result is that lending standards can lead to credit market hysteresis. Transitory shocks to market fundamentals, such as to the quality of the borrower pool, can set in motion a self-reinforcing dynamic culminating in permanent differences in lending volumes, credit spreads, and default rates. For instance, a temporary deterioration in market fundamentals, e.g. a worsening of borrowers' projects, can set in motion a self-reinforcing feedback loop between a deteriorating borrower pool quality and tighter lending standards, culminating in a *permanent* shift in the credit market equilibrium from the pooling steady state to the screening steady state. This feature of our model is consistent with the limited relaxation of lending standards following the Great Recession in Figure 1.

Our second main result is that lending standards can be inefficiently tight in which case temporary government intervention to relax lending standards can improve on private market outcomes. This result follows because tight lending standards have *negative externalities*. A bank that tightens its lending standard today increases the share of low-quality borrowers in the pool in the future which makes the credit market less efficient. As a result, the pooling steady state Pareto dominates the screening steady state, even though tighter lending standards also have social benefits by leading to less funding of low-quality (negative NPV) projects. Solving a dynamic planning problem for the socially optimal lending standards, we show that socially optimal lending standards are weakly looser than privately optimal ones at all pool qualities and strictly tighter at intermediate levels of pool quality.[1]

Thus, in response to a transitory decline in the quality of borrowers, the optimal policy response can be an intervention that ensures that banks do not tighten lending standards, consistent with the fact that governments often support high-quality credit markets in downturns. We show that the optimal policy can be implemented by a loan guarantee program. The optimal policy is temporary, and delayed policy interventions are more costly than immediate interventions so that too much delay can make it optimal to not intervene at all. Optimal policy requires collective, i.e. government, action because the pool of potential borrowers is a common resource and an individual bank cannot recover the short-term losses associated with the policy from the later increased efficiency of the competitive credit market.

While policies that target lending standards improve outcomes, they do not achieve the first-best. A first-best policy would eliminate the externality associated with screening by making any private information acquired by banks public. While credit bureaus can potentially address this externality, there are reasons why they may not fully do so in many markets (see Section 7.3) .

Our third main result is that capital constraints incentivize banks to tighten lending standards, setting in motion declines in pool quality, which can ultimately lead to hysteresis and suboptimal market outcomes as described above. When capital constraints bind, the shadow value of banks' capital increases. Thus, banks have a greater incentive to screen potential borrowers in order to lend their limited capital to the most profitable borrowers. This implies that downturns accompanied by financial crises are more likely to lead to the emergence and persistence of tight lending standards, and potentially to benefit from government policies to relax lending standards.

[1] There is also a region of low enough pool quality in which it is efficient to stop lending entirely. Because this type of lending standard does not have the externality associated with screening, the private threshold for this type of lending standard can be inefficiently high or low, depending on incidental modeling choices.

Our fourth main result is that there may be an intermediate range of pool quality in which banks restrict lending by rationing credit instead of imposing tight information-based lending standards, a situation we refer to as *slow thawing*. The logic behind this credit rationing is different from the typical credit rationing due to adverse selection (Stiglitz and Weiss 1981, Mankiw 1986). During slow thawing, lending rates fall sufficiently quickly that high-quality borrowers are indifferent between getting funded right away and waiting for their next funding opportunity. This indifference reduces the surplus from bank lending today, leading to credit rationing as some banks stop lending, which in turn reduces the speed of improvement in lending volumes and credit spreads. The speed of convergence to the pooling steady state is thus non-monotonic, and during the initial slow thawing period, the typical effects of many parameters on lending volumes and interest rates are reversed.

Related literature. Our four main contributions are about the dynamics of lending, but they build on the static models of Fishman and Parker (2015) and Bolton, Santos and Scheinkman (2016) in which the same strategic complementarity leads to multiple equilibria. In contrast, other static models focus on different aspects of lending standards. In Ruckes (2004), lenders simultaneously acquire private information about borrowers and then simultaneously quote loan rates. In that setting, lending standards can be strategic substitutes. In Dell'Ariccia and Marquez (2006) there is cream skimming by informed lenders but these lenders are endowed with their information. Hachem (2020) studies lending standards in a static model in which banks can also exert search effort to attract potential borrowers. When banks are resource constrained, banks put too much effort into searching for borrowers and too little effort into checking them upon arrival, so that lending standards are inefficiently loose.

Our model is more closely related to two recent dynamic models which are based on assumptions such that, at times, lending standards are dynamic strategic substitutes. In both Hu (2018) and Farboodi and Kondor (2020), this substitutability arises because tight lending standards raise the average quality of newly-entering borrowers.[2] In Hu (2018) economic recoveries can have interesting dynamics such as double-dip recoveries, while Farboodi and Kondor (2020) shows that credit markets can exhibit endogenous cycles in lending standards and borrower quality. In our model, the quality of new potential borrowers is exogenous and it ambiguous whether tight lending standard increase or decrease the incentive for a potential borrower to be high quality (positive NPV). On the one hand, tight lending standards reduce the probability of funding for low-quality

[2]In Farboodi and Kondor (2020), in addition, rejected borrowers go bankrupt and leave the pool of potential borrowers.

borrowers. On the other hand, tight lending standards raise interest rates, which reduces the profits net of funding costs for high-quality borrowers. Thus tight lending standards can increase or decrease the difference in expected payoff between types (see section 7.1).

A number of papers study dynamic adverse selection models without information acquisition. Daley and Green (2012, 2016) and Malherbe (2014) analyze models where current markets can break down when high-quality sellers have the incentive to wait for market prices to improve over time as the composition of sellers improves over time. In contrast during slow thawing in our model, the equilibrium *composition* of borrowers does not change, only the *speed* of lending is reduced.[3]

Finally, Our paper is also related to information acquisition and adverse selection in secondary markets. Zou (2019) analyzes a dynamic model of trade in which an agent's incentive to collect information is higher if agents in the future are expected to collect information. In Asriyan, Fuchs and Green (2017), if future market liquidity and hence prices are expected to be high (low), then prices today are high (low) and the adverse selection problem will be less (more) severe. Gorton and Pennacchi (1990), DeMarzo and Duffie (1999), and Dang, Gorton and Holmstrom (2015), among others, analyze how debt securities minimize adverse selection problems in secondary markets. While the issues of information acquisition are similar, our model is designed to address adverse selection at origination (in primary markets) and abstracts from issues of security design.

2 A Model of Lending Standards

Time is continuous and runs from 0 to infinity, $t \in [0, \infty)$. There are two sets of agents: a unit mass of borrowers and a large mass $\mathcal{J}$ of competitive banks. All agents are risk neutral with discount rate $\rho > 0$.

Borrowers. At Poisson rate $\kappa > 0$, a borrower receives an investment opportunity requiring an up-front investment of 1. Borrowers have no capital and must fund the investment externally. If the borrower raises the funds and invests at time t, the project returns a pledgeable cash flow at $t + T$ and a non-pledgeable private benefit $u > 0$ (in present value) to the borrower.

[3] Related, Zryumov (2015) and Caramp (2017) study models where bad sellers strategically enter when market prices are good. This, in and of itself, does not lead to a market shutdown (lower prices positively select entrants), but as Caramp (2017) emphasizes, the bigger presence of bad sellers can raise the likelihood of adverse selection induced market failures in the future.

There are two types of borrowers. With pledgeable cash flow D_H, type-H borrowers have positive-NPV projects, $r_H \equiv e^{-\rho T} D_H - 1 > 0$.[4] With pledgeable cash flow D_L, type-L borrowers have negative-NPV projects, $r_L \equiv e^{-\rho T} D_L - 1 < 0$. We take $r_L + u < 0$ so investing in a type-L project is not profitable even including the private benefit. Let $r^\Delta \equiv \frac{r_H - r_L}{-r_L} > 0$ equal the (normalized) *return difference* between the two types' projects. Borrowers privately observe their type.

When a project arises, the borrower chooses whether to apply to the competitive banking sector for funding to implement their project. A borrower who applies for funding is approved or denied depending on whether she satisfies the bank's lending standard. If funded, a borrower invests in her project and exits the pool to run the project. Alternatively, if a borrower does not apply for or is denied funding, she returns to the pool where, as before, at rate $\kappa > 0$ a new investment opportunity arises. Besides exiting the pool if funded, at Poisson rate $\delta > 0$, a borrower no longer receives investment opportunities and exogenously exits the pool with a zero payoff. Borrowers who exit the pool for any reason are immediately replaced by new borrowers who enter the pool as type H with exogenous probability λ and type L with probability $1 - \lambda$.[5]

The exit/entry assumption implies that the borrower pool size is constant at 1, which implies that the model has one state variable: the fraction of type-H borrowers in the pool at time t, $x_t \in [0, 1]$. This modeling makes the analysis more tractable and transparent. In Appendix B we prove that our main insights carry over to an environment with a constant inflow rather than a constant pool size.[6]

While collateral is not explicitly modeled, one can interpret the loan as a collateralized loan and u as the private benefit net of the loss of collateral (e.g. u is the net benefit of purchasing and living in a house until foreclosure; see Section 7.2). One can also re-interpret types as referring to the quality of the collateral rather than the project.[7]

Banks and lending standards. Banks make three decisions. First, banks choose whether to be *active*, entering a competitive lending market, where it may receive a borrower's

[4] We call r_H the excess return on a type-H project because $\frac{1}{T} \ln(1 + r_H) = \frac{1}{T} \ln(D_H) - \rho$.

[5] Our analysis is unchanged if we assume that borrowers whose funding applications were denied exit at a greater rate than other borrowers. This is because exits by borrowers that have not applied for a loan are irrelevant as they are replaced by a borrower of the same quality. Thus, our model is formally equivalent to one in which only denied borrowers exit at rate δ, while others do not exit.

[6] The only exception are the results in Section 3.3 on slow thawing, which are less tractable in a constant-inflow setting due to there being two state variables, the number of type-H and number of type-L borrowers in the pool.

[7] Similarly, with minor modifications, one can also re-interpret the model as capturing a secondary market in which "borrowers" are selling assets of unknown value in order to raise funds to make an investment.

loan application; or *inactive*, receiving no loan applications and making no loans. Let θ_{jt} denote the probability that bank j is active at time t. Generally all banks will be active, e.g., at steady states where x is constant. But there may be a region in the state space with equilibrium *credit rationing*, $\theta_{jt} < 1$, where banks offer fewer loans than borrowers demand. Second, active bank j chooses a *lending standard* $z_{jt} \in [0, \bar{z}]$, where $\bar{z} \in (0, 1]$ is a parameter. With lending standard z_{jt}, a type-L borrower who applies for a loan is identified as type L with probability z_{jt}, in which case her loan is denied. Otherwise, the borrower's loan is approved. A type H is never misidentified as type L, an assumption only important for tractability, as discussed in Section 7.4. A bank's cost of using lending standard z_{jt} is $\tilde{c}z_{jt}$, where $c \equiv \frac{\tilde{c}}{-r_L} > 0$ is the (normalized) marginal cost. Banks choose lending standards to maximize expected profit. Third and finally, banks choose the terms of the loan. For a borrower who meets a bank's lending standard, the bank offers to lend 1 in exchange for a promised loan payment at time $t + T$ equal to D_{jt}.

Due to symmetry and competition, it is without loss of generality to assume that all banks choose the same probability of being active, θ_t, the same lending standard z_t, and the same required loan payment D_t. With loan face value D_t, repayment is $\min\{D_t, D\}$, where D is the investment payoff, D_L or D_H depending on borrower type. Since type-L borrowers have negative-NPV investments, $D_t > D_L$ for a bank to break even in expectation. Thus, type-L borrowers always default. Nevertheless, given the private benefit u, type-L borrowers have the incentive to finance their project even though they will receive no monetary benefit. The repayment D_t is without loss of generality bounded above by D_H as any higher D_t generates no additional repayment. So, type-H borrowers will not default. Define $r_t \equiv e^{-\rho T} D_t - 1$ as the *credit spread* charged by the bank since $\rho + \frac{1}{T} \ln(1 + r_t)$ is per-period (log) return on a loan that does not default. Note that r_t lies in (r_L, r_H).

The shares of type-H and type-L borrowers in the pool are endogenously determined. For instance, the lower are past lending standards, the fewer type-L borrowers will be in the pool. Agents have common knowledge of the structural parameters of the market and the initial fraction of type-H borrowers in the pool, $x_0 \in [0, \lambda]$. Agents can infer past, current, and future x_t, the fraction of type-H borrowers in the pool.

A borrower's problem. Given a path of credit spreads $\{r_t\}$, borrowers with investment opportunities choose whether to apply for a loan at each time t. Let φ_t^H and φ_t^L denote the probability that a type-H and type-L borrower, respectively, with an investment opportunity applies for a loan—as opposed to waiting in hope of an improvement in borrowing opportunities. Let J_t^H and J_t^L denote the value functions of a type-H and type-L borrower.

The optimal strategies for the two satisfy the following Hamilton-Jacobi-Bellman equations:

$$\rho J_t^H = \max_{\varphi_t^H \in [0,1]} \kappa \theta_t \varphi_t^H \left\{ r_H - r_t + u - J_t^H \right\} + \dot{J}_t^H - \delta J_t^H \tag{1a}$$

$$\rho J_t^L = \max_{\varphi_t^L \in [0,1]} \kappa \theta_t \varphi_t^L \{ (1 - z_t)(u - J_t^L) \} + \dot{J}_t^L - \delta J_t^L, \tag{1b}$$

with transversality conditions $\lim_{t\to\infty} e^{-(\rho+\delta)t} J_t^H = \lim_{t\to\infty} e^{-(\rho+\delta)t} J_t^L = 0$ satisfied. For a type H who has an investment opportunity, is matched with a bank, and applies for financing, (1a) reflects that she will be funded, receiving a monetary payoff of $r_H - r_t$ and private benefit u. For a type-L who has an investment opportunity, is matched with a bank, and applies for a loan, (1b) reflects that with probability $1 - z_t$, she satisfies the lending standard and receives private benefit u; otherwise, she continues as a type-L borrower.[8]

With strategies $\{\varphi_t^H, \varphi_t^L\}$, there is a flow of

$$\kappa_{Ht} \equiv \kappa \varphi_t^H x_t \tag{2}$$

type-H borrowers applying for loans and a flow of

$$\kappa_{Lt} \equiv \kappa \varphi_t^L (1 - x_t) \tag{3}$$

type-L borrowers applying for loans.

A bank's problem. With a flow $\kappa_{Ht} + \kappa_{Lt}$ of loan applications it is without loss to assume there are at most a flow $\kappa_{Ht} + \kappa_{Lt}$ active banks at time t. As will be seen, there are cases where some banks are inactive in equilibrium, leaving only $\theta_t (\kappa_{Ht} + \kappa_{Lt})$ active banks, with $\theta_t \in [0, 1]$. A fraction θ_t of the flow $\kappa_{Ht} + \kappa_{Lt}$ of loan applications is then received by the $\theta_t(\kappa_{Ht} + \kappa_{Lt})$ active banks.

With flows κ_{Ht}, κ_{Lt} and credit spread r_t, an active bank's lending standard z solves

$$\Pi_t(r_t) \equiv \max_{z \in [0,\bar{z}]} \kappa_{Ht} r_t + \kappa_{Lt}(1 - z) r_L - (\kappa_{Ht} + \kappa_{Lt}) \tilde{c} z. \tag{4}$$

as type-H borrowers and a fraction $1 - z$ of type-L borrowers are funded.[9] Taking z as

[8] These equations assume that potential borrowers arrive in the pool of potential borrowers knowing their type. All of our main substantive results hold if instead borrowers do not initially know their type and only learn it either when they are rejected or when their (funded) project pays off.

[9] For each individual bank, previously screened loan applicants represent a zero mass in the pool of borrowers and can therefore be ignored.

given, Bertrand competition among banks determines r_t by

$$\Pi_t(r_t) = 0. \tag{5}$$

When (5) cannot be satisfied by any r_t, no bank finds it profitable to lend, and we set $\theta_t = 0$ (and $r_t = r_H$).

Evolution of the borrower pool. The evolution of the fraction of type-H borrowers in the pool is given by

$$\dot{x}_t = \theta_t \kappa_{Lt}(1 - z_t)\lambda - \theta_t \kappa_{Ht}(1 - \lambda) + \delta(\lambda - x_t). \tag{6}$$

The first term accounts for $\theta_t \kappa_{Lt}(1 - z_t)$ type-L borrowers who are funded and exit the pool (improving pool quality). The second term accounts for $\theta_t \kappa_{Ht}$ type-H borrowers who are funded and exit the pool (reducing pool quality). The third term accounts for exogenous borrower exit and replacement by a new potential borrower with probability λ of being type-H.

Equilibrium. Define an equilibrium as follows:

Definition 1. Given an initial share of type-H borrowers $x_0 \in [0, \lambda]$ in the pool, an *equilibrium* consists of a path of the fraction of type-H borrowers $\{x_t\}$, credit spreads $\{r_t\}$, shares of active banks $\{\theta_t\}$, borrowers' application decisions $\{\varphi_t^H, \varphi_t^L\}$, implied application flows of type-H and type-L borrowers $\{\kappa_{Ht}, \kappa_{Lt}\}$, and screening choices $\{z_t\}$ such that

- $\{\varphi_t^H, \varphi_t^L\}$ solves each borrower type's maximization problem (1) given $\{r_t, z_t, \theta_t\}$,
- $\{\kappa_{Ht}, \kappa_{Lt}\}$ are determined by (2) and (3),
- z_t solves the bank's maximization problem (4) given $\{r_t, \kappa_{Ht}, \kappa_{Lt}\}$,
- r_t is determined by the zero profit condition for banks (5) given κ_{Ht}, κ_{Lt} whenever possible; if not, $\kappa_{Ht} = 0$ (and w.o.l.o.g. $r_t = r_H$),
- $\{x_t\}$ follows the law of motion (6),
- at no time t can a bank raise its profit Π_t by being active, charging a rate $\tilde{r} < r_t$ that type-H borrowers would weakly prefer over waiting, and lending to the entire set of borrower applicants (a flow of κ_{Ht} type-H and κ_{Lt} type-L borrowers; see 2 and 3).

A *steady state (equilibrium)* is an equilibrium with $\{x_t, r_t, \theta_t, \varphi_t^H, \varphi_t^L, z_t\}$ constant over time.

To study variation in lending standards, we make the following parameter assumptions:

Assumption 1. *The cost of bank screening c is not too low or too high:*

$$1 - \lambda < c < 1 - x^s + \bar{z}^{-1} \min \left\{ x^s r^\Delta - 1, 0 \right\},$$

where $x^s = \lambda - \lambda (1 - \lambda) \frac{\bar{z}}{1 - \lambda \bar{z} + \delta \kappa^{-1}}$.

The first inequality in Assumption 1 ensures that the screening cost c is high enough that the lending standard $z = \bar{z}$ does not strictly dominate $z = 0$. The second inequality ensures that c is not so high as to rule out a steady state with $z = \bar{z}$.

With a linear screening cost, the choice of lending standard will be at a corner, $z = 0$ or $z = \bar{z}$. As we will show, the tendency will be for banks to choose too high of lending standards, $z = \bar{z}$ instead of $z = 0$. Hence we refer to $z = 0$ as a "normal" lending standard and $z = \bar{z}$ as a "tight" lending standard.

3 Equilibrium characterization

This section characterizes the dynamics of the model, starting with steady-state equilibria.

3.1 Steady-state equilibria

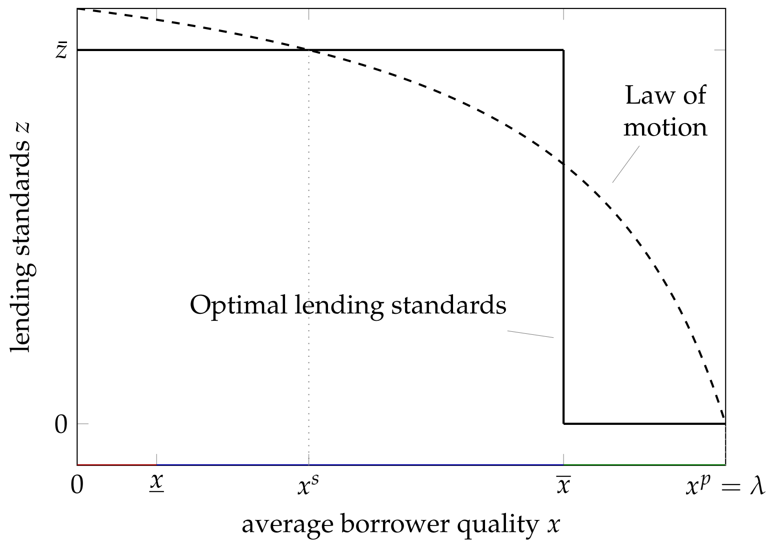
Given a constant interest rate r and lending standard z, borrowers have no incentive to wait to borrow in a steady-state equilibrium and therefore choose $\varphi^H = \varphi^L = 1$.[10] Type-$L$ borrowers never have an incentive to wait. Type-H borrowers also strictly prefer borrowing to waiting,

$$r_H - r + u - J^H > 0, \tag{7}$$

because, for any bank to be active, the loan rate needs to be weakly below the highest pledgeable payoff, $r \leq r_H$. In that case, a type-H borrower's continuation value is strictly positive, $J^H > 0$, as they cannot lose any money upon receiving funding and always receive the private benefit. Using (1a), $J^H > 0$ is equivalent to (7) in a steady state. Under these conditions, all banks are active in a steady state, $\theta = 1$.

[10] Since prices and quantities are constant, we drop the time subscripts for this subsection.

Figure 2: The two forces shaping steady-state equilibria.



Note: This figure shows two curves whose intersections yield the steady-state pool quality x and the steady-state lending standard z. The solid line represents the optimal choice of the lending standard, (9). The dashed line represents the pool quality x that is caused by any given lending standard z through the law of motion.

The steady-state quality of the pool x and the steady-state lending standard z are jointly determined by the interaction of two forces. On the one hand, the law of motion of x, (6), implies that when $\dot{x} = 0$,

$$x = \lambda - \lambda \frac{(1-\lambda)\, z}{(1-\lambda z) + \delta \kappa^{-1}}. \tag{8}$$

This equation highlights that tighter lending standards—higher z—are associated with a lower steady-state quality of the pool of borrowers x, as more type-L borrowers are rejected by banks. This effect is greater when the effects of lending standards on the pool are more persistent (low exit rate δ) or when opportunities to invest arise more frequently (high κ) and so potential investors are evaluated more frequently.

On the other hand, banks solve (4) and choose tighter lending standards precisely when the pool is more adversely selected,

$$z = \begin{cases} 0 & \text{if } x > \overline{x} \\ [0, \overline{z}] & \text{if } x = \overline{x}\,, \\ \overline{z} & \text{if } x < \overline{x} \end{cases} \quad \text{where } \overline{x} \equiv 1 - c. \tag{9}$$

The combination of equations (8) and (9) is illustrated in Figure 2. Both represent downward-sloping relations between x and z, and given Assumption 1 admit three intersections, each of which represents a steady-state equilibrium. This logic is summarized in the following proposition.

Proposition 1 (Steady state equilibria)**.** *If $\lambda r_H + (1-\lambda) r_L \geq 0$, then there exist three steady-state equilibria:*

(i) A pooling *steady state with normal lending standards $z = 0$ and $x = x^p \equiv \lambda$.*

(ii) A screening *steady state with tight lending standards $z = \bar{z}$ and $x = x^s \equiv \lambda - \lambda \frac{(1-\lambda)\bar{z}}{(1-\lambda\bar{z}) + \delta\kappa^{-1}}$.*

(iii) A mixed *steady state with $z = \frac{\lambda - \bar{x}}{\lambda - \lambda\bar{x}}\left(1 + \delta\kappa^{-1}\right) \in (0, \bar{z})$ and $x = \bar{x}$.*

If $\lambda r_H + (1-\lambda) r_L < 0$, then there exists one steady-state equilibrium: A screening *steady state with tight lending standards as above*

The root of the multiplicity in the first part of the proposition is the dynamic strategic complementarity among banks. By (9), banks respond to a lower-quality pool by tightening their lending standards; however, according to (8), tighter lending standards worsen the pool itself, creating an even bigger incentive for banks to tighten their standards in the future. This reasoning rationalizes the existence of the pooling and screening equilibria in Figure 2. A mixed steady state formally exists but will turn out to be unstable and therefore play no role in the remainder of the analysis.

In the second part of the proposition, the unconditional expected NPV of a project is negative and so no bank will lend without a tight lending standard. The dynamics here are less interesting; from any initial condition, the credit market converges to the screening steady state. Therefore, going forward, our analysis considers the case of $\lambda r_H + (1-\lambda) r_L \geq 0$.

The pooling and screening steady states have the following characteristics.

Corollary 1 (Quality of funded borrowers)**.** *Compared to the screening steady state, in the pooling steady state:*

1. *the credit spread r is lower, $r(x^p) = -\frac{r_L}{x^p}(1 - x^p) < -\frac{r_L}{x^s}\{c\bar{z} + (1-\bar{z})(1-x^s)\} = r(x^s)$*
2. *more projects are funded, $\kappa > \kappa x + \kappa(1-x)(1-\bar{z})$*
3. *the default rate is higher, $1 - x^p > \frac{(1-x^s)(1-\bar{z})}{x^s + (1-x^s)(1-\bar{z})}$*

The first point follows from the fact that the lower pool quality in the screening steady state hurts banks' profits, and therefore requires larger credit spreads for banks to break even. This is true even though banks choose tight lending standards.[11] The second point follows from the fact that screening reduces the flow of borrowers that receive funding.Key to the third point is the social benefit of screening. Borrowers exogenously exit at rate δ so a bank that rejects a type-L borrower reduces the probability that it is ever funded. If instead $\delta = 0$, then the default rate equals $1 - \lambda$ in any steady state, irrespective of the screening decision. In that case, the imposition of tight lending standards in the screening steady state exactly balances the low average project quality in the pool, yielding the same default rate.[12]

These patterns match the stylized facts about credit booms and busts we discussed in the Introduction (see e.g. Greenwood and Hanson 2013): booms feature higher lending volumes, looser lending standards, lower quality of funded borrowers, and lower credit spreads conditional on default probability. These patterns also echo previous results from the static models discussed in the introduction. The contribution of our model lies in the the dynamics, to which we now turn.

3.2 Transitional dynamics

An important factor that simplifies the steady state analysis is that banks are always active in a steady state, $\theta = 1$. This is no longer true in equilibria with dynamics. In particular, there are now two regions in which banks may choose to remain inactive. Naturally, this is the case when the pool quality x is very low, so that even the maximum loan rate does not make profits for a bank. Thus, $\Pi(r_H) = 0$ defines an $\underline{x}$ such that

$$\theta(x) = \begin{cases} 0 & \text{if } x < \underline{x} \\ [0,1] & \text{if } x = \underline{x} \end{cases}, \quad \text{where } \underline{x} \equiv \frac{1 - \bar{z} + c\bar{z}}{r^{\Delta} - \bar{z}}. \tag{10}$$

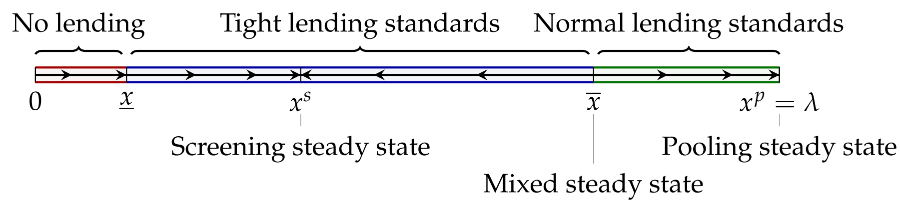
More surprising, banks may also remain inactive and so reduce total lending when lending standards are otherwise normal, $z = 0$, and x is low (for a range of x just above $\bar{x}$ defined in Proposition 2 below). Since in this region the speed of convergence to the pooling steady state is slow and increasing, we refer to this region as the "slow-thawing" region and it is described in detail in Section 3.3. Until then, we assume parameters are such that there is no such region:[13]

[11] Proposition 3 has a complete characterization of interest rate spreads.

[12] The results in Corollary 1 are robust to alternative assumptions on the dynamics of the borrower pool, e.g. assuming a constant inflow, rather than a constant pool size. See Appendix B.

[13] For the sake of exposition, this assumption is stated in terms of endogenous objects. The analytical condition is stated in the next section.

Figure 3: State space and banks' optimal strategies.



Assumption 2 (No slow thawing). *Assume that there is no slow-thawing region, that is, $\theta(x) = 1$ for all $x \geq \underline{x}$.*

Under Assumption 2, Proposition 2 completely characterizes the equilibrium transitional dynamics of x.

Proposition 2 (Transitional dynamics without slow thawing). *Suppose Assumption 2 holds and $x_0 \in [0, \lambda]$ is the initial fraction of type-H borrowers in the pool. There is a unique equilibrium, in which banks' activity policy satisfies (10) for $x \leq \underline{x}$, their lending standards are given by (9), and borrowers never wait, $\varphi_t^H = \varphi_t^L = 1$. As $t \to \infty$, the credit market converges to*

(i) the screening steady state, $x_t \to x^s$, if $x_0 < \overline{x}$.

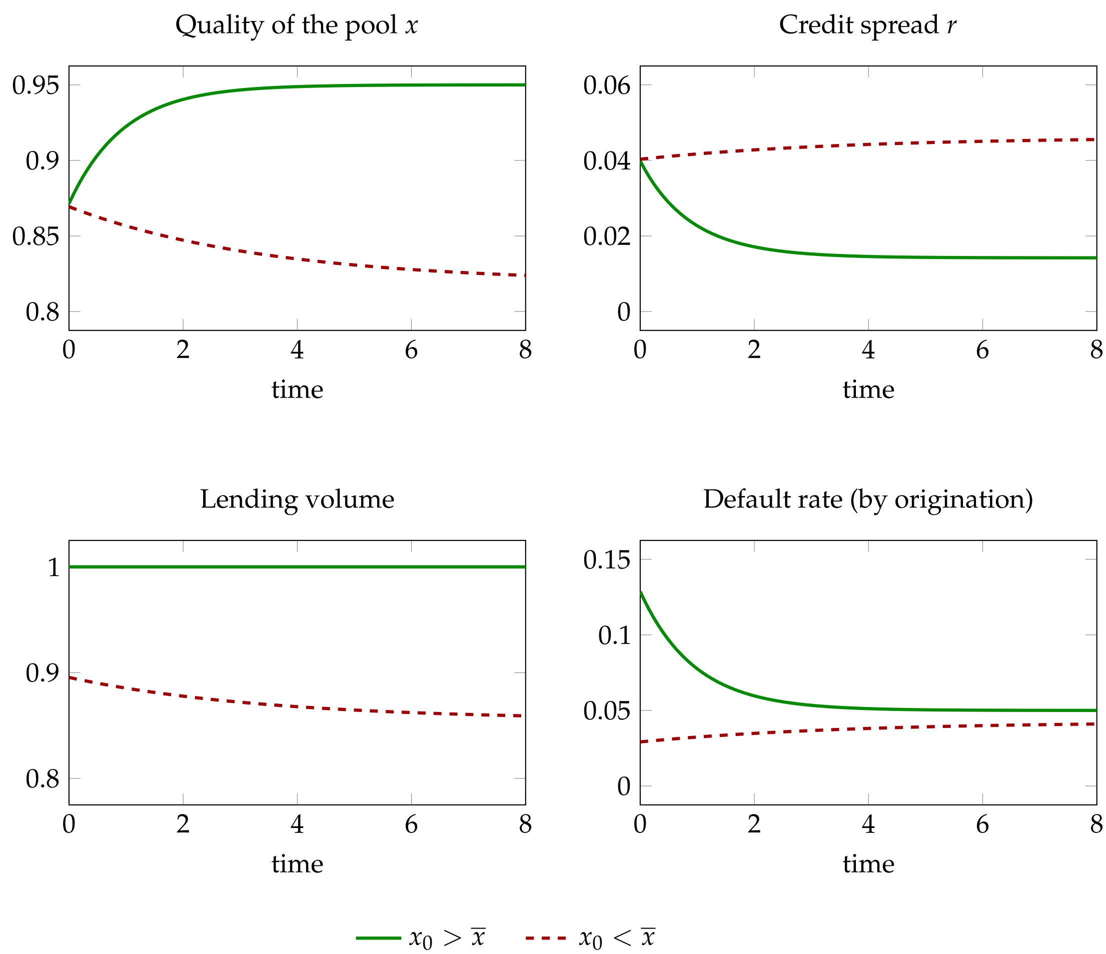
(ii) the mixed steady state, $x_t \to \overline{x}$, if $x_0 = \overline{x}$.

(iii) the pooling steady state, $x_t \to x^p$, if $x_0 > \overline{x}$.

Figure 3 illustrates the state space of the credit market and highlights the transitional dynamics in the three different regions of bank behavior: the "no lending" region for low pool qualities, where banks are inactive ($\theta_t = 0$) and pool quality improves only due to exogenous exit/replacement; the "tight lending standards" region, where banks screen borrowers $z_t = \overline{z}$ and the market approaches the screening steady state; and the "normal lending standards" region where banks choose $z_t = 0$ and the market approaches the pooling steady state.

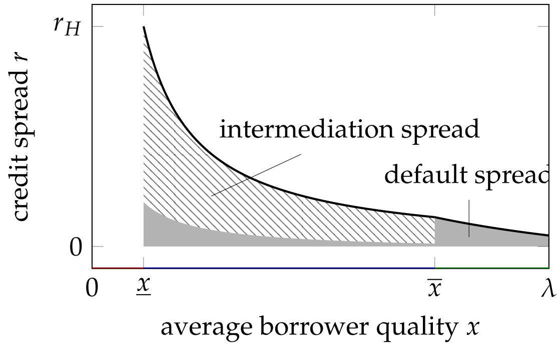
A crucial part of the diagram is at $x = \overline{x}$. This point represents a sharp boundary between the tight and normal lending standards regions and gives rise to an important model prediction, a "bifurcation" property: when x_0 lies above $\overline{x}$, the credit market converges to the pooling steady state with normal lending standards; and when x_0 lies below $\overline{x}$, the self-reinforcing nature of tight lending standards pushes the market to the screening steady state.

Figure 4: The self-reinforcing property of lending standards.



Note. This figure shows two sets of transitional dynamics in a credit market without slow thawing. Green and solid is a market starting at $x_0 = \overline{x} + \epsilon$ and therefore banks have normal lending standards; red and dashed is a market starting at $x_0 = \overline{x} - \epsilon$ and therefore banks impose tight lending standards. The parameters used for this simulation are as follows: $\rho = \delta = 0.05$, $\lambda = 0.95$, $r_L = -0.27$, $r_H = 0.30$, $\tilde{c} = 0.035$, $\kappa = 1$, $\overline{z} = 0.8$.

Figure 5: Break-even credit spread as function of pool quality x.



Note: Grey is the component of the credit spread that is due to default risk (the *default spread*). Hatched is the component of the credit spread that is due to intermediation costs (the *intermediation spread*).

The bifurcation property also comes out in Figure 4 where we simulate the credit market with two different initial values for x_0, one just above $\overline{x}$ (green, solid) and one just below $\overline{x}$ (red, dashed). As can be seen, a small difference in initial conditions leads to quite different evolutions of pool qualities x, credit spreads r, and lending volumes $\kappa_{Ht} + \kappa_{Lt}(1-z)$. The final panel of Figure 4 shows the evolution of default rates, $\kappa_{Lt}(1-z)/\left(\kappa_{Ht} + \kappa_{Lt}(1-z)\right)$, by lending cohorts. The market with the slightly *lower* initial pool quality initially has a much lower lending volume and default rate, as banks are imposing tight lending standards. Interestingly, the two markets initially have similar credit spreads. Over time, and foreshadowing our results on efficiency and optimal policy, the slightly lower initial pool quality causes convergence to a steady-state with much higher credit spreads and lower lending volume but similar default rates (see our discussion of Corollary 1).

How do credit spreads vary with pool quality x_t? Lower x_t implies higher default rates for any given lending standard, suggesting higher credit spreads. But lower x_t also can lead to tight lending standards, implying lower default rates and so lower credit spreads, but also raising the cost of lending. As the following proposition shows, the credit spread r_t is uniformly declining in pool quality x_t.

Proposition 3 (Equlibrium credit spread)**.** *The equilibrium credit spread $r_t = r(x_t)$ is decreasing in the fraction of type-H borrowers, x, and is given by*

$$r_t = r(x_t) = \begin{cases} \infty & \text{if } x_t < \underline{x} \\ (-r_L)x_t^{-1}\left\{c\overline{z} + (1-\overline{z})(1-x_t)\right\} & \text{if } \underline{x} \leq x_t < \overline{x} \\ (-r_L)x_t^{-1}\left\{1-x_t\right\} & \text{if } x_t \geq \overline{x} \end{cases}. \tag{11}$$

Using (11), we can decompose $r(x)$ into a *default spread*, $-r_L x^{-1}(1 - z(x))(1 - x) > 0$ where $z(x)$ is the optimal screening choice given x; and an *intermediation spread* $-r_L x^{-1} cz(x) \geq 0$. Figure 5 plots the credit spread $r(x)$ and these two components over the state space. The shaded areas in Figure 5 highlight that the default spread changes discretely at $x = \overline{x}$ as banks switch between tight and normal lending standards, but this change is offset by an equally large change in the intermediation spread. The spread rises significantly due to intermediation costs at lower pool qualities x. The decoupling of credit spreads and credit risk in this region of the state space provides a rationale for why, at times, credit spreads may appear to be high given the credit risk. He and Milbradt (2014) attribute such high credit spreads to low liquidity. Alternatively one might rely on risk aversion as an explanation. Here the high credit spread derives from intermediation costs.

We again emphasize that none of the results up to this point or in the next section on efficiency rely on our assumptions on the inflow of new borrowers that keep the pool size constant. Appendix B derives the same substantive results (with slightly different equations for steady-states, cutoffs, etc.) under the assumption that there is a constant inflow of new borrowers (which implies that the model has two state variables).

The monotonicity of $r(x)$ is also reflected in Figure 4, with a rising loan rate for the credit market with the lower quality of potential borrowers and a falling loan rate for the market with higher quality. A falling loan rate raises a question: would type-H borrowers have an incentive to wait for lower loan rates? The answer is yes in certain cases. There may be a reduced demand for credit even with normal lending standards. In this case, lending volume and the improvement of the borrower pool are slowed, and credit markets recover - "thaw" - much more slowly than otherwise.

3.3 Slow thawing

Say x_0 is just above $\overline{x}$ and conjecture that all banks are active, $\theta_t = 1$, and type-H borrowers do not wait to apply for loans, $\varphi_t^H = 1$. Then it is possible that the resulting increase in pool quality, x_t, over time would lead to so rapid a decline in r_t that type-H borrowers would prefer not to borrow as conjectured. Rather they would wait for lower credit spreads before applying for credit, $\varphi_t^H = 0$. Hence, in this case, our conjecture is not an equilibrium. Instead, equilibrium must exhibit a slower speed of transition. The improvement in the borrower pool and the decline in borrowing rates must occur more slowly to induce type-H borrowers to apply for loans in equilibrium. For this transition to be slower, it must be that not all banks are active, $\theta_t < 1$, which can only be the case if there are no profits to be made from making a new loan (see Definition 1). This is precisely the case when type-H

borrowers are also indifferent between waiting and applying for loans.

The following proposition establishes that these strategies are indeed an equilibrium. To keep the derivations and exposition clear, we focus on the case where the private benefit from running the project, u, is vanishingly small, $u \to 0$ (in which case $J_t^L \to 0$).

Proposition 4 (Slow thawing)**.** *There exists a threshold $\hat{x} \in (0, x^p)$, such that: (i) if $\hat{x} \leq \overline{x}$, there is no slow thawing region; if (ii) $\hat{x} > \overline{x}$, then for any $x \in [\overline{x}, \hat{x})$, a positive fraction of banks are inactive*

$$\theta(x) = \frac{(\rho + \delta)(r_H - r(x))}{-\kappa r'(x)(\lambda - x)} - \delta\kappa^{-1} < 1 \tag{12}$$

where $r(x) = -r_L x^{-1}\{1 - x\} > 0$. Type-H borrowers are indifferent and apply for loans, $\varphi_t^H = \varphi_t^L = 1$, and $\hat{x}$ is determined as the unique solution to $\theta(\hat{x}) = 1$ in $(0, x^p)$.

The intuition for the expression in (12) comes directly from the indifference condition of type-H borrowers. The HJB of a type-H borrower is given by

$$\rho J_t^H = \max_{\varphi_t^H \in [0,1]} \kappa\theta_t\varphi_t^H \left\{ r_H - r_t - J^H \right\} + \dot{J}^H - \delta J^H$$

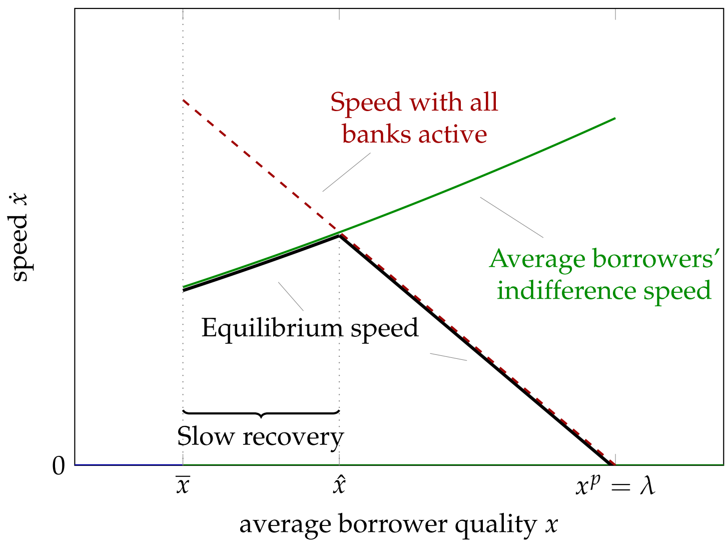
with indifference between applying for a loan or not requiring that $J^H(x) = r_H - r(x)$. Substituting this back into the HJB yields an equation for the speed $\dot{x}$ at which the pool needs to improve for type-H borrowers to be indifferent,

$$\underbrace{-r'(x)\dot{x}}_{\text{benefit of waiting}} = \underbrace{(\rho + \delta)(r_H - r(x))}_{\text{opportunity cost of waiting}} . \tag{13}$$

When is $\dot{x}$ the equilibrium speed? Precisely when θ_t is such that $\dot{x}$ satisfies the law of motion of x, (6). Together, (13) and (6) give (12).

Figure 6 schematically illustrates this logic. The green solid line represents the speed $\dot{x}$ at which type-H borrowers are indifferent between borrowing now and waiting for the pool to improve. This is an increasing line as the benefit of waiting declines the closer x is to the pooling steady state. The red dashed line represents the speed at which the pool quality improves when all banks choose to be active and all borrowers borrow. Clearly, where this line lies below the green solid line of indifference, it is also equal to the equilibrium speed, shown in black thick solid line. However, for $x < \hat{x}$, the growth of $\dot{x}$ with all banks active lies above the solid green indifference curve so that type-H borrowers would prefer to wait which cannot be an equilibrium. In this region, a fraction $1 - \theta(x)$ of banks choose to be inactive, bringing down the equilibrium speed to match the one along the green solid

Figure 6: Slowly thawing credit markets.



Note. This figure illustrates when there exists a region with "slow thawing" where credit markets recover only very slowly from a crisis. The green solid line represents the speed at which the pool quality needs to improve for type-H borrowers to be exactly indifferent between applying for loans (strictly preferred below the curve) and waiting (strictly preferred above). The red dashed line represents the speed of improvement when all banks are active. The equilibrium speed (black solid line) is the minimum of both curves.

indifference curve. This leads to a hump-shaped thawing speed: initially little lending due to the threat of type-H borrowers waiting, a period of slow thawing as lending volume and the pool quality accelerate, followed by a period of normal convergence to the steady state.[14]

Slow thawing is thus a mechanism by which lending is slow to recover after crises, based on two intuitive ideas. First, credit spreads rise during crises and come down afterwards. Second, the more financially sound (type-H) borrowers have an incentive to "wait out" crises until credit spreads come down, while low-quality borrowers are unwilling or unable to do so. These two ideas amplify each other, as banks increase credit spreads in response to a worse pool of borrowers, further incentivizing type-H borrowers to delay borrowing.

What determines how likely or how strong this period of slow thawing is? How could it be sped up? The following corollary reveals the roles of interest rates, project payoffs, and meeting frequencies.

[14]Note that Figure 6 does not show $\dot{x}$ just to the left of $\bar{x}$ because it is negative. By Proposition 3, $\dot{x} < 0$ implies $\dot{r} < 0$. With spreads decreasing over time, there is no incentive to delay and so no region of slow thawing.

Corollary 2 (Credit market recovery with normal lending standards)**.** *Fix a quality of the borrower pool $x \in (\overline{x}, x^p)$ and let $\dot{x}$ denote the speed of improvement in the pool's quality. Then in the slow thawing region:*

1. *Worse projects always slow down the recovery: $\dot{x}$ falls with lower r_L, r_H.*
2. *Increasing the rate at which borrowers apply for loans does not speed up the recovery in the slow thawing region: for $x < \hat{x}$, $\dot{x}$ does not rise with κ; for $x > \hat{x}$, $\dot{x}$ rises with κ.*
3. *More patient borrowers slow down the recovery: $\dot{x}$ falls with lower ρ if $x < \hat{x}$ (holding fixed r_L, r_H).*

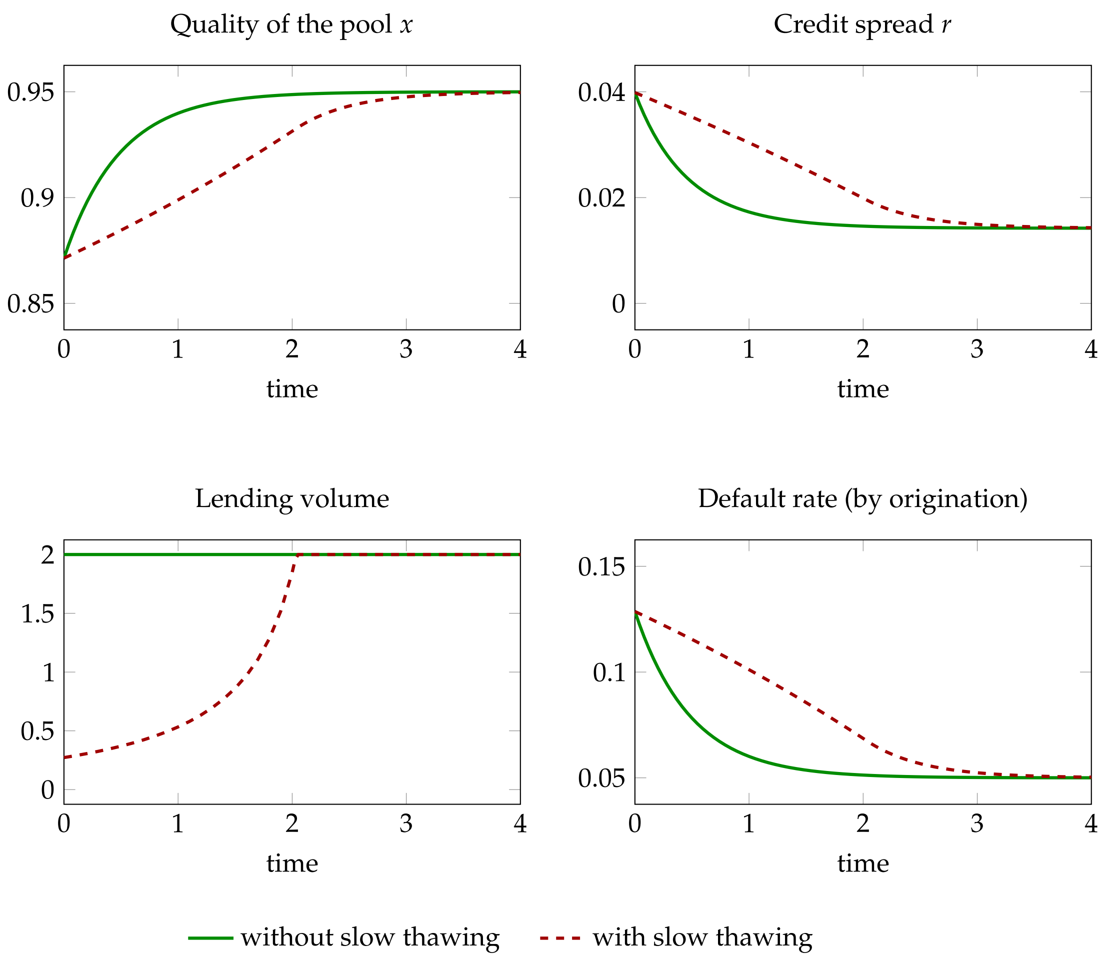
When the rate κ at which borrowers receive investment opportunities increases (part 2 of Corollary 2), the red line in Figure 6 increases. This naturally increases the speed of the recovery towards the steady state outside the slow-thawing region. Inside that region, however, it has no effect. In fact, even when $\kappa \to \infty$, the transition towards the pooling steady state is slow and entirely determined by the indifference condition (13). The reason for this is that banks are not finding it profitable to lend more, as the pool of borrowers is adversely selected, and are thus holding back lending.

Greater patience, lower ρ, makes type-H borrowers more willing to wait, shifting down the indifference curve in Figure 6 and slowing down the recovery. In practice, the patience parameter reflects how timely is the borrower's need for funding. Thus, paradoxically, when good borrowers are less desperate for funding, the recovery takes longer.

Figure 7 juxtaposes the transitional dynamics with slow thawing (dashed red line) and the transitional dynamics without slow thawing (solid green line). The latter was computed by ruling out slow thawing by assumption, imposing $\varphi_t^H = 1$, $\theta_t = 1$, and dropping equilibrium equation (7) and instead assuming that potential borrowers are myopic in the sense that when they have the opportunity to invest, they approach the competitive banking sector and accept the loan and invest rather than optimally choosing whether instead to wait for their next opportunity to borrow. As is visible in the first panel, slow thawing can greatly slow the transition back to the pooling steady state and lead to a relatively low lending volume.[15]

[15] Note that a similar region with slow thawing can also appear in the region between $\underline{x}$ and x^s and slow down the convergence to the screening steady state from the left.

Figure 7: Slowly thawing credit markets.



Note. The plots compare two transitions back to the pooling steady state. Green solid is a transition without "slow thawing", where type-H borrowers always accept current loan offers and banks do not ration credit; red dashed is a transition with slow thawing, where banks ration credit in equilibrium. The parameters used for this simulation are as follows: $\rho = \delta = 0.05$, $\lambda = 0.95$, $r_L = -0.27$, $r_H = 0.13$, $\tilde{c} = 0.035$, $\kappa = 2$, $\bar{z} = 0.8$.

4 Efficiency

Because one bank's lending standard affects future banks' borrower pools, equilibrium lending standards will, in general, not be efficient. The first-best allocation would allow the planner to fund only type-H borrowers. This section characterizes the constrained efficient allocation.

4.1 Constrained efficient policy

Our concept of constrained efficiency allows the planner to control banks' activity and screening decisions, subject to borrowers' application decisions, so as to maximize the sum of agents' utilities.[16] Throughout this section, we continue to focus on the algebraically simpler case where $u \to 0$. In this section, it is further assumed that the planner can set the path of market interest rates $\{r_t\}$, and therefore prevent type-H borrowers from waiting, i.e. there is no slow thawing. We discuss relaxing this assumption below.

The constrained efficient planning problem is given by

$$\max_{z_t \in [0,\bar{z}], \theta_t \in [0,1]} \int_0^\infty e^{-\rho t} \kappa \theta_t \left\{ x_t r_H + (1 - z_t)(1 - x_t) r_L - \tilde{c} z_t \right\} dt \tag{14}$$

subject to the law of motion of x_t, (6). The solution to this problem can be characterized as follows.

Proposition 5 (Second-best policy)**.** *There exists a threshold $\overline{x}^* \in [0, \overline{x})$ such that the planner sets:*

$$z_t = \begin{cases} \bar{z} & \text{if } x_t < \overline{x}^* \\ 0 & \text{if } x_t > \overline{x}^* \end{cases} \tag{15}$$

For any $x_t \in (\overline{x}^*, \overline{x})$, equilibrium lending standards are (second-best) inefficiently tight.
For any $\overline{x}^* > x^s$, the optimal policy for bank activity is given by

$$\theta_t = \begin{cases} 0 & \text{if } x_t < \underline{x}^* \\ 1 & \text{if } x_t > \underline{x}^* \end{cases}$$

for some $\underline{x}^* \in [0, \overline{x}^*)$.

[16]Since borrowers and banks are risk-neutral, this is without loss when transfers between agents are feasible.

Proposition 5 shows that the constrained efficient lending standard takes a similar form to the privately-optimal policy: when the pool quality is relatively high, $x > \overline{x}^*$, normal lending standards, $z = 0$, are optimal; and when $x < \overline{x}^*$, tight lending standards, $z = \bar{z}$, are optimal. But the cutoffs for the optimal policy and for the market equilibrium differ: There exists a region in the state space, $(\overline{x}^*, \overline{x})$, where equilibrium lending standards are too tight relative to the constrained-efficient outcome.

In the proof, we show that $\overline{x}^*$ is the largest x that satisfies

$$\underbrace{\frac{\rho x + \alpha^s x^s}{\rho + \alpha^s} r_H + (1 - \overline{z}) \left(1 - \frac{\rho x + \alpha^s x^s}{\rho + \alpha^s}\right) r_L - \tilde{c}\overline{z}}_{\text{Average social benefit of screening}} \geq \underbrace{\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p} r_H + \left(1 - \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}\right) r_L}_{\text{Average social benefit of pooling}} \quad (16)$$

To develop an intuition for this finding, say the current pool quality is x and banks operate normal lending standards, $z = 0$, in all periods from now on so that the credit market ultimately converges to the pooling steady state x^p. In (16), one can think of $\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}$ as the time-averaged fraction of type-H borrowers funded. The weight on current x is ρ, as with greater discounting the present becomes relatively more important; the weight on the (long-run) steady state x^p is $\alpha^p \equiv \kappa + \delta$, which is the speed at which x converges to x^p. The average social benefit of $z = 0$ is therefore the weighted average surplus from lending to each type of borrower,

$$\frac{\rho x + \alpha^p x^p}{\rho + \alpha^p} r_H + \left(1 - \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}\right) r_L.$$

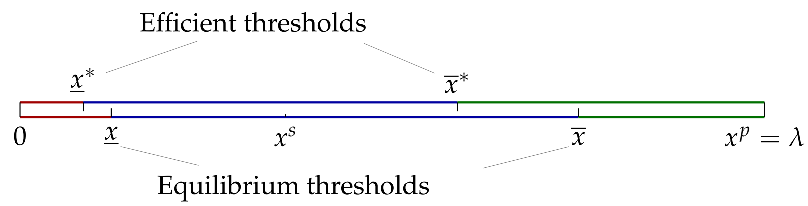
An analogous expression describes the social benefit of tight lending standards, where we additionally account for both the costs of screening and the fact that banks successfully screen out a fraction $\bar{z}$ of of low-quality borrowers. The weight on the long-run steady state here is given by $\alpha^s \equiv \kappa + \delta - \lambda \kappa \overline{z}$. This is how we obtain (16).

By contrast, the private cut-off, $\overline{x}$, is the largest value satisfying

$$\underbrace{xr + (1 - \overline{z})(1 - x) r_L - \tilde{c}\overline{z}}_{\text{Average private benefit of screening}} \geq \underbrace{xr + (1 - x) r_L}_{\text{Average private benefit of pooling}} \quad (17)$$

which is calculated using the *current* fraction x of type-H borrowers and ignores the

Figure 8: Constrained efficient vs. equilibrium lending standards.



dynamic consequences from screening and pooling. In particular, since in the relevant region it holds that

$$\frac{\rho x + \alpha^s x^s}{\rho + \alpha^s} < x < \frac{\rho x + \alpha^p x^p}{\rho + \alpha^p}$$

agents privately ignore the dynamic costs from screening relative to pooling. Therefore, $\overline{x}^* < \overline{x}$. The private and social thresholds are shown in Figure 8.[17]

The result in Proposition 5 does not imply that tight lending standards should always be prevented. In fact, tight lending standards are constrained optimal whenever the pool quality is sufficiently poor, as normal standards would involve lending to all type-L borrowers that have accumulated in the pool, and thus be very costly. This result holds despite the fact that all projects are, in expectation, positive NPV, and is an important implication of our dynamic perspective that would be lost in a static context. Despite this, the welfare in the screening steady state is always lower than welfare in the pooling steady state.

Corollary 3. *When both steady states exist (a result of Assumption 1), the screening steady state has strictly lower welfare than the pooling steady state.*

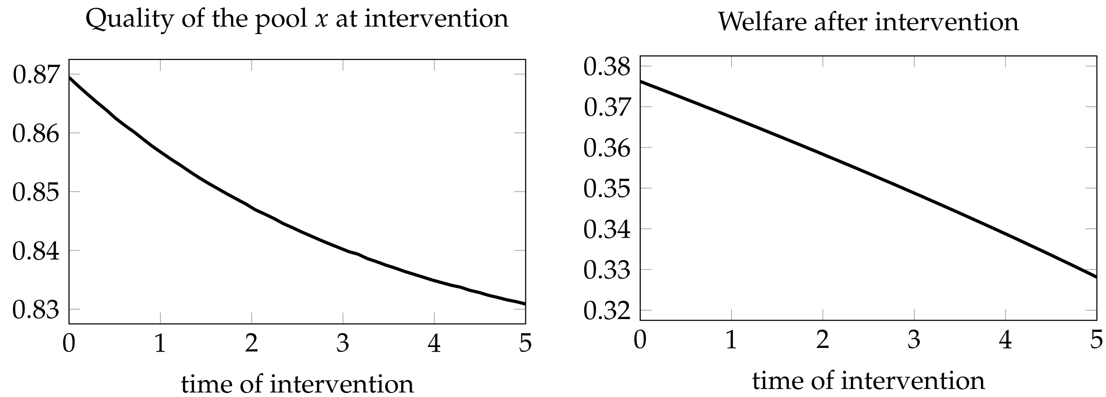
If $\delta = 0$, this result would be a simple consequence of the fact that screening potential borrowers is costly and the quality of funded borrowers is independent of the steady state (see discussion below Corollary 1). But with $\delta > 0$, screening borrowers has a social benefit because a share of them are never funded. Still, the corollary shows that welfare in the pooling steady state is higher.

There are two important practical implications from the existence of a non-empty interval $(\overline{x}^*, \overline{x})$ where the market equilibrium diverges from the constrained optimum.

1. Intervention timing matters. Figure 9 illustrates the welfare consequences of intervention in a credit market that starts at a given $x_0 \in (\overline{x}^*, \overline{x})$ for various times when an

[17] The logic that lending standards are only inefficiently tight in the region $(\overline{x}^*, \overline{x})$ follows from the linearity of the cost function. If banks' screening costs were nonlinear such that the optimal $z^*(x)$ were continuous and strictly decreasing, then lending standards could be generically too tight.

Figure 9: Early interventions dominate late ones.



Note. Figure shows how an intervention implementing normal lending standards affects a credit market that is transitioning towards the screening steady state. Horizontal axis: time at which the intervention starts (0 corresponds to the immediate, constrained efficient intervention). Parameters are as in Figure 4.

intervention starts (on the horizontal axis), requiring banks to set normal, rather than tight, lending standards. The later the time of intervention is, the lower is the quality of the borrower pool when the policy switches from screening to pooling (left panel). Later intervention times thus increase the short-run losses incurred at the start of the intervention and are therefore welfare-inferior to early interventions. In fact, after a sufficiently long time, if x_t has fallen below $\overline{x}^*$, intervening may even be welfare-dominated by not intervening at all and allowing the market to converge to the screening steady state, despite its having lower welfare than the pooling steady state.

2. Better screening technology may be detrimental to welfare. Say the cost $\tilde{c}$ of operating tight lending standards falls. While a cost reduction necessarily raises efficiency in any steady-state equilibrium, it can decrease welfare because it raises thresholds both for the market convergence to a screening equilibrium and for the efficient intervention, $\overline{x}$ and $\overline{x}^*$. Therefore, if a market is just recovering from a crisis, with x_0 just above $\overline{x}$, such a technological improvement may cause $\overline{x}$ to rise above x_0 and thereby prevent a recovery and lead to a reduction in welfare. If $\overline{x}^*$ also rises above x_0 then it is too costly for policy to mitigate this decline in welfare.

A decrease in cost $\tilde{c}$ represents an improvement in private information technology. What happens if instead public information technology (e.g. credit reporting) improves? A crude way to capture such a change is as an increase in δ, the probability that rejected borrowers die. While the exit of borrowers who have never been rejected has no effect on equilibrium as they are replaced in the pool by an equal measure of new identical borrowers, a greater exit rate of rejected borrowers does matter for equilibrium. A larger

δ increases (decreases) the speed of convergence when x is increasing (decreasing), and raises the pool quality in the screening steady state, so therefore unambiguously increases welfare. Thus, the welfare effects of improving public information are unambiguously positive, a point that leads naturally into a discussion of the first-best policy.

4.2 Implementation of the constrained optimum

Government-funded loan insurance. There are several ways a government or a regulator could implement the constrained efficient outcome, that is, normal lending standards when $x \in (\overline{x}^*, \overline{x})$. Since such an intervention entails short-run losses and the model's banking sector is competitive, either the government or type-H borrowers have to bear these losses. An example of such a policy is a government-funded loan insurance program in which the government provides an insurance benefit $b > 0$ (in present value) to be paid to a bank when a borrower defaults. This policy incentivizes banks to use normal lending standards as long as

$$\frac{b}{-r_L} > 1 - \frac{c}{1-x}.$$

This condition is satisfied for $b = 0$ in the region $x > \overline{x}$ where pooling is privately optimal. It requires nonzero insurance benefits $b = b(x) > 0$ when $x < \overline{x}$. As a function of the pool quality, $b(x)$ is decreasing in x. This means a typical intervention starting from some $x_0 < \overline{x}$ requires large insurance benefits early on, which are then phased out over time.

Mandated securitization. A second way to implement the constrained optimum would be to *require* that, whenever $x \in (\overline{x}^*, \overline{x})$, all loans made at each point in time are placed into a common pool from which each bank receives a proportionate payout as the loans mature. Such mandated securitization requires only that a loan origination is observable and contractible, not that a rejection is observable. Under this policy, no individual bank has the incentive to tighten lending standards when $x \in (\overline{x}^*, \overline{x})$ since they receive no benefits from placing a higher-quality loan into the securitized pool.

Shadow Banks and limits to implementation. In practice, policies like government-funded loan subsidies or insurance programs are rarely undertaken for the entire financial sector, but instead usually apply only to certain types of institutions, such as traditional banks but not shadow banks for example. So consider a setting where the government can affect the lending decisions of only a fraction $\eta \in [0, 1)$ of financial institutions. What is the optimal policy under such circumstances? We focus on the case without bank inactivity,

$\theta = 1$. We further assume that the share η of traditional banks always charge the same interest rates as their shadow bank competitors. This setting implies that the planner can no longer set the lending standard z_t to any number in $[0, \overline{z}]$. Instead, z_t needs to lie in $[(1-\eta)z(x_t), (1-\eta)z(x_t) + \eta\overline{z}]$, where $z(x_t)$ is the privately optimal lending standard (9).

The new constraint significantly changes optimal policy. In particular, there is now a threshold $\overline{x}^*(\eta)$ that depends on η above which the planner desires to set normal lending standards. Crucially, for low levels η, $\overline{x}^*(\eta)$ will be equal to $\overline{x}$, implying the planner prefers not to intervene at all. To see this, observe that for any $x < \overline{x}$, shadow banks apply tight lending standards, pushing the quality x down, towards the screening steady state. Even if the traditional banks are made to apply normal lending standards, it may not be possible to achieve $\dot{x} > 0$. Even if $\dot{x} > 0$ is possible, with $z = 0$ in the traditional banking sector, it make take so long to reach $\overline{x}$ that the policy is too costly to be worth implementing.. Thus, the government can lack the "firepower" to get to pooling and optimally choose not to intervene at all. In this way, a large shadow banking sector can constrain optimal government policy.[18]

5 Hysteresis

In this section we explore the implications of our model for economic fluctuations. In particular, we study a slowdown caused by a temporary decline in the inflow of type-H borrowers. We show how lending standards can tighten in such periods and propagate a temporary downturn into permanent increases in interest rate spreads and intermediation costs and declines in lending volume. Such hysteresis does not follow from shorter or smaller declines in quality, consistent with lending standards normalizing following the milder 2001 U.S. recession but not the 2008-2009 Great Recession, as displayed in Figure 1.

5.1 Downturn in fundamentals

We feed into the model a temporary decline in the size and quality of the pool of potential borrowers. More specifically, we reduce the quantity of type-H new borrowers entering the pool of potential borrowers from time 0 to time T. Our model in Section 2 involves a fixed pool size $N = 1$ and therefore needs to be amended to allow for dynamics in N.

[18] If the government intervenes for low η, it is forced to cover increasingly large losses as shadow banks impose tight lending standards, reducing the pool quality. Traditional banks then face a more adversely selected pool. This point may have been relevant for the failure of the government sponsored mortgage agencies in the U.S. in 2008.

However, as one can easily show, since the law of motion of N is exogenous in the model, the model still applies to a "normalized" version of the credit market, where all absolute quantities (volume of loans, welfare, profit, etc.) are to be thought of as normalized by N.

For the period of the lending slowdown, we reduce the inflow of new type-H borrowers into the pool by μ. During this period, only a share $(1-\mu)\lambda$ of new potential borrowers are type-H. As a result, the fraction of type-H borrowers, x, evolves according to

$$\dot{x}_t = \theta_t \kappa (1-x_t)(1-z_t)\lambda_t - \theta_t \kappa x_t (1-\lambda_t) + \delta N_t^{-1} \left(\lambda - x - \mu 1_{\{t \leq T\}} \lambda (1-x) \right) \quad (18)$$

where λ_t is the average quality of new borrowers entering into the pool,

$$\lambda_t \equiv \begin{cases} \frac{\lambda(1-\mu)}{\lambda(1-\mu)+(1-\lambda)} & t \leq T \\ \lambda & t > T \end{cases}$$

The last term in (18) represents the modified impact of the birth-death process of borrowers for pool quality. It is modified to allow for a reduction in pool quality when $\mu > 0$.

Following the slowdown, when λ_t returns to λ, the inflow of new borrowers increases to return the pool to its original size of 1, as shown by the law of motion of the total pool size N_t

$$\dot{N}_t = \delta(1-N_t) - \delta \lambda \mu 1_{\{t \leq T\}}. \quad (19)$$

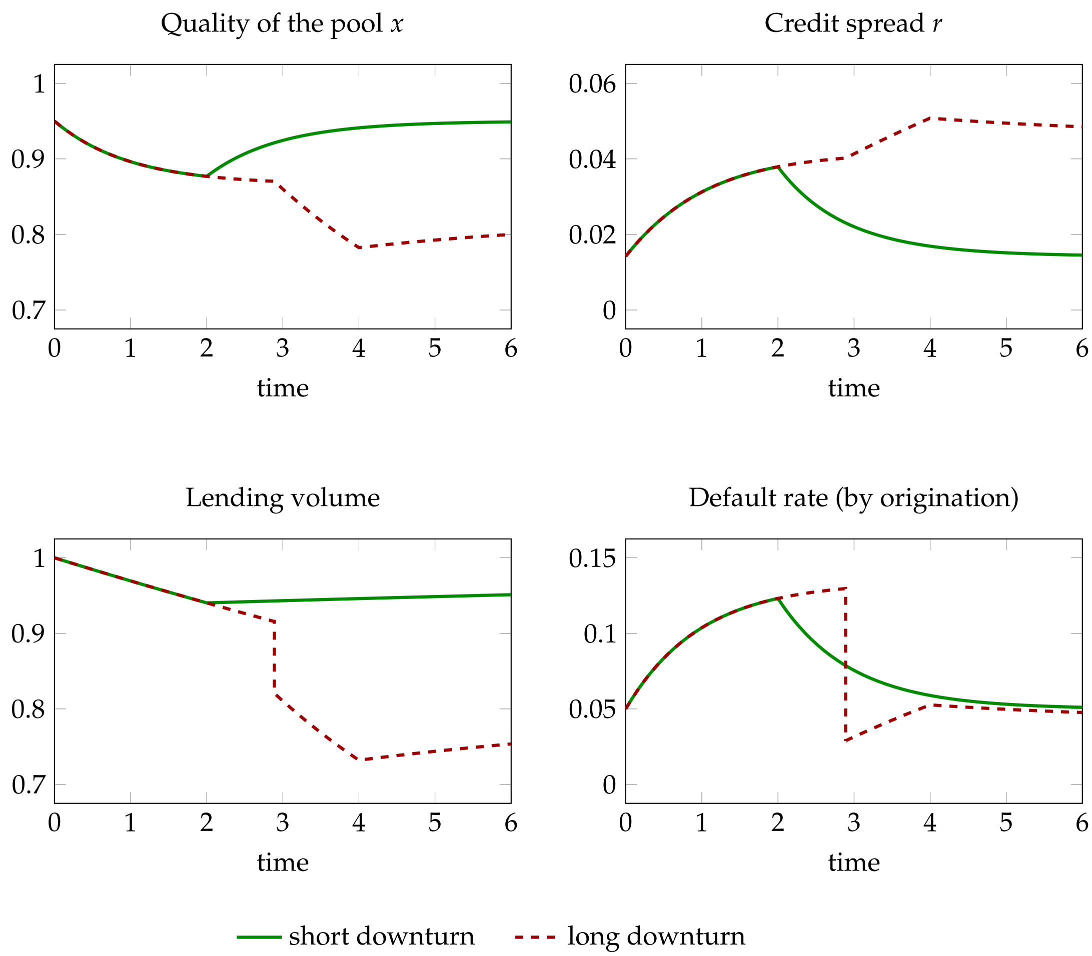
For the simulations in this section, we keep the model parameters from Figure 4 and choose $\mu = 0.66$. Under this parameterization the market has no slow thawing region.

5.2 Lending cycle and hysteresis

We simulate the response to two different lengths of the downturn, $T = 2$ period and 4 period. The results are shown in Figure 10. As the solid green line shows, the short downturn is associated with a decrease in lending volume, as fewer borrowers seek loans, and an increase in interest rate spreads, as the pool quality declines and the ex post default rate increases. When the downturn in fundamentals ends, the increase in the demand for loans and the increase in the quality of potential borrowers both lead to increased lending and decreased interest rate spreads. After a short downturn, the credit market returns to its previous equilibrium with normal lending standards.

By contrast, the longer downturn (dashed red line) is associated with a larger decline in the average quality of potential borrowers so that, roughly at $T = 3$, banks tighten lending

Figure 10: Hysteresis.



Note. This figure shows a credit market in response to a temporary reduction in the inflow of type-H borrowers: green solid (2 period long reduction), red dashed (4 period long reduction). Parameters: $\mu = 0.66$ with others as in Figure 4.

standards leading to an abrupt decline in lending. Following this tightening, pool quality deteriorates more rapidly, credit spreads increase, and the lending volume continues to contract. When the slowdown ends at $T = 4$ and more type-H borrowers start to enter the pool of potential borrowers again, the lending volume increases, but interest rate spreads remain high and lending never fully recovers.

While it is individually optimal for banks to tighten lending standards in the long downturn, it is not socially optimal. For the parameters in Figure 10, optimal policy is to maintain normal lending standards and can be implemented, for example, with a loan guarantee program as described in Section 4.2. Under optimal policy, the long downturn in Figure 10 would instead look like an extended version of the short one. With no tightening of lending standards at the end of period 3, the lending volume would continue its smooth decline during the fourth year and then slowly rise back to one following the end of the recession, like a longer version of the two-year downturn. Similarly, the rise in credit spreads and (ex post) defaults continues to slow, and starts to recover rapidly at the end of year 4. In short, the credit market deteriorates by less and recovers more quickly.

In Figure 10, we modeled the downturn as a decline in λ, and studied downturns of different lengths. However, similar dynamics would follow from a downturn in which recovery rates after loans default, r_L, worsened. In this case, banks would maintain normal lending standards for small declines in r_L, leading to no lasting damage to credit markets. In contrast, deep downturns, in which r_L falls enough to trigger tight lending standards, could trigger permanent declines in lending volume and increases in interest rate spreads due to the costs of intermediation.

We now turn to another source of tightening of lending standards and propagation of fluctuations in the real economy—capital constraints.

6 Capital constraints and lending standards

In this section, we show how regulatory or market-based limits on banks' capacity to lend, such as from balance sheet constraints, raise lending standards and propagate and amplify credit crunches through the mechanisms just analyzed.

6.1 Balance sheet constraints

We consider a stylized model of bank balance sheet constraints in which a capital constraint, $\bar{V}_t$, restricts the amount of lending proportionally by each bank so that

$$V_t \leq \overline{V}_t \tag{20}$$

where $V_t \equiv \theta_t\{\kappa_{Ht} + \kappa_{Lt}(1 - z_t)\}$ denotes total bank lending. We assume that $\bar{V}_t$ grows exogenously which implies that the bank optimization problem remains static. We also assume that $\overline{V}_t$ grows such that banks eventually become unconstrained and such that, even in the face of fluctuations in exogenous variables, once banks are unconstrained they remain unconstrained thereafter.[19] Given this last assumption, once the constraint stops binding, the equilibrium is as described in Sections 2 and 3.

While the constraint binds, each bank charges the maximum interest rate such that type-H borrowers still take loans, which we denote by $\tilde{r}$. This rate makes type-H borrowers

[19] As in Section 3.3, we work here in the limit $u \to 0$ to simplify the exposition.

indifferent as to whether to $i)$ accept a loan when given the opportunity and so receive $r_H - \tilde{r}_t$ or to instead $ii)$ return to the pool and wait for a future borrowing opportunity at a lower rate, and so receive J_t^H. In equilibrium,

$$r_t = \tilde{r}_t \tag{21}$$

and indifference implies that $J_t^H = r_H - \tilde{r}_t$. Substituting into (1a), implies $\dot{J_t}^H = (\rho + \delta) J_t^H$ so that $\tilde{r}_t$ evolves according to

$$\dot{\tilde{r}}_t = -(\rho + \delta)(r_H - \tilde{r}_t) \tag{22}$$

with the terminal condition that at a time τ', when banks become unconstrained, $\tilde{r}_{\tau'} = r(x_{\tau'})$ with $r(x)$ as in (11) in Proposition 3. Equation (21) replaces the bank zero-profit condition in our definition of equilibrium. Since (22) implies that $\tilde{r}_t$ will always lie strictly below r_H, all borrowers accept loans in equilibrium and $\kappa_{Ht} = \kappa x_t$ and $\kappa_{Lt} = \kappa(1 - x_t)$.

To understand how capital constraints affect lending standards, consider first the effect of lending standards on the profits from making one and only one loan which is given by

$$\Pi_t^{\text{single}}(z, r) \equiv \frac{x_t r + (1 - x_t)(1 - z) r_L}{x_t + (1 - x_t)(1 - z)} - \frac{\tilde{c} z}{x_t + (1 - x_t)(1 - z)} \tag{23}$$

The first term is the expected return on the loan, where $\frac{x_t}{x_t + (1 - x_t)(1 - z)}$ is the probability that the loan is made to a type-H borrower and thus that the bank earns r on the loan, and $\frac{(1 - x_t)(1 - z)}{x_t + (1 - x_t)(1 - z)}$ is the probability that the loan is made to a type-L borrower and the bank loses r_L. The second term is the expected cost of lending standards and exceeds $\tilde{c}$ because several potential borrowers may have to be checked before finding one that passes the lending standard. The bank chooses tight lending standards if $\Pi_t^{\text{single}}(\bar{z}, r) > \Pi_t^{\text{single}}(0, r)$ which collapses to the condition

$$x_t(1 - x_t)(r - r_L) > \tilde{c}. \tag{24}$$

Given the ability to make one loan, the bank prefers tight lending standards when costs of screening are low, when the information uncovered has high variance (when $x_t(1 - x_t)$ is large), and when the losses from lending to type-L borrowers are large (r_L large negative). Most importantly, the propensity to impose tight lending standards is greater the higher r. Thus binding capital constraints—which endogenously lead to higher interest rates—also incentivize tighter lending standards.

The arguments of the previous paragraph apply when the capital constraint is tight in the sense that it binds even when lending standards are tight and so the desired lending volume is low. When the capital constraint is at an intermediate level so that it would not bind if $z = \bar{z}$ but would bind if $z = 0$, banks have three possible strategies for lending standards. First, they could impose tight standards, which would generate profits

$$\kappa\left(x_t + (1 - x_t)(1 - \bar{z})\right)\Pi_t^{\text{single}}(\bar{z}, r_t).$$

Since the capital constraint does not bind when banks impose tight standards, the interest rate must equal that in the competitive economy. Thus tight lending standards only occur for $x < \bar{x}$. Second, banks can impose normal lending standards for which total profits are

$$\overline{V}_t \Pi_t^{\text{single}}(0, r_t).$$

Finally, banks can set intermediate lending standards, $z_t = \frac{1-\overline{V}_t/\kappa}{1-x_t}$, such that applying these standards to all potential borrowers leads banks to make $\overline{V}_t$ loans.[20] At this intermediate lending standard, bank profits are

$$\kappa x_t r + (\overline{V}_t - \kappa x_t) r_L - \frac{1 - \overline{V}_t/\kappa}{1 - x_t}\tilde{c}.$$

Optimal lending standards are then characterized as follows:

Proposition 6 (Optimal lending standard with capital constraints)**.**

1. (Loose constraint). *If $\overline{V}_t > \kappa$, the capital constraint does not bind: all current and future variables are identical to those in the market without constraints.*

2. (Tight constraint; Screening region). *If $\overline{V}_t < \kappa x_t + \kappa(1 - x_t)(1 - \bar{z})$, the capital constraints binds: There exists a threshold*

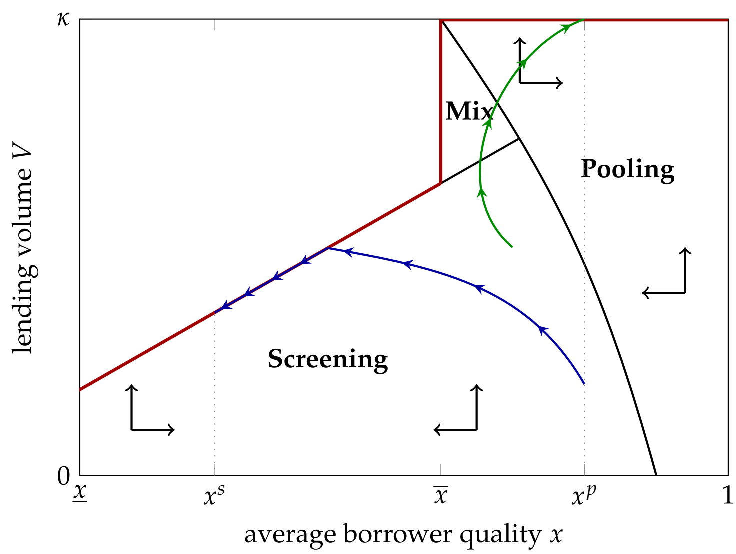
$$\bar{x}_t^C = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4\tilde{c}}{r_t - r_L}}\right) \geq \bar{x}$$

   such that banks impose tight lending standards, $z = \bar{z}$, if $x_t < \bar{x}_t^C$, and impose normal lending standards, $z = 0$, if $x_t > \bar{x}_t^C$.

3. (Intermediate constraint). *For $\kappa x_t + \kappa(1 - x_t)(1 - \bar{z}) < \overline{V}_t < \kappa$:*

[20]This strategy has the same payoff as applying the lending standard $\bar{z}$ to a share $\frac{1-V_t/\kappa}{(1-x_t)\bar{z}}$ of borrowers and lending to the remaining borrowers at $z = 0$.

Figure 11: Optimal lending standards with capital constraints



(a) (Pooling region). *For $x_t > \bar{x}_t^C$ banks lend with normal lending standards and the constraint binds*

(b) (Mixed region). *For $x_t \in \left(\bar{x}, \bar{x}_t^C\right)$ banks impose intermediate lending standards, $z_t = \frac{1-\overline{V}_t/\kappa}{1-x_t}$, and the capital constraint binds.*

(c) (Unconstrained region) *For $x < \bar{x}_t$, banks impose tight lending standards and the constraint does not bind.*

We illustrate Proposition 6 in Figure 11. The figure shows the volume of bank lending on the vertical axis and the quality of the pool of potential borrowers on the horizontal axis. The thick (red) line which increases from $\underline{x}$ to $\bar{x}$ and then rises to remain at κ from $\bar{x}$ to 1 displays the equilibrium volume of lending when the constraint does not bind (Proposition 6.1 and 6.3.c). Following Proposition 2, when the credit market is unconstrained, to the right of $\bar{x}$, the lending volume equals $V_t = \kappa$ and x_t rises towards the pooling steady state; to the left of $\bar{x}$, the economy has volume $V_t = \kappa x_t + \kappa(1 - x_t)(1 - \bar{z})$ and x_t declines towards the screening steady state.

Strictly below this thick (red) line there are three regions that the credit market can be in when capital constraints bind. In all three regions, the lending volume is always rising as capital constraints are (exogenously) relaxing. Similarly, in these constrained regions, interest rate spreads are always (weakly) declining, following (22).

In the right "pooling" region (bounded on the left by the curved solid line which intersects $x = x^p$), the pool quality is high enough that even with binding constraints, banks set normal lending standards ($z = 0$) because (24) is not met. Note that credit spreads are higher than if banks were unconstrained, but not so high that lending standards are tightened.

By contrast, within the "screening" region, x is low enough given $\bar{V}$ that lending standards are tight. For $x < \bar{x}$, lending standards are simply tight as before without lending constraints, although again, interest rate spreads are higher than without lending constraints. However, the more interesting region is the screening region where $x > \bar{x}$. Banks would impose normal lending standards if not constrained. But in this region the volume of lending is sufficiently constrained that banks impose tight lending standards. As shown in Proposition 6, this region always exists for some range of $\bar{V} < \kappa$ and $x > \bar{x}$.

Focusing on x just above $\bar{x}$, when $\bar{V}$ is not too large the credit market is in the screening region (Proposition 6.2) and banks set tight lending standards. If on the other hand, $\bar{V}$ is large enough that banks would be unconstrained if $z = \bar{z}$, then banks instead set intermediate lending standards, $z \in (0, \bar{z})$, which are either tightened or loosened over time as pool quality improves or declines at the chosen lending standards, depending on the tightness of the constraint (Proposition 6.3.b). The higher (green) convergence path illustrated in Figure 11 shows an example dynamic path for a credit market which passes through this "Mix" region.

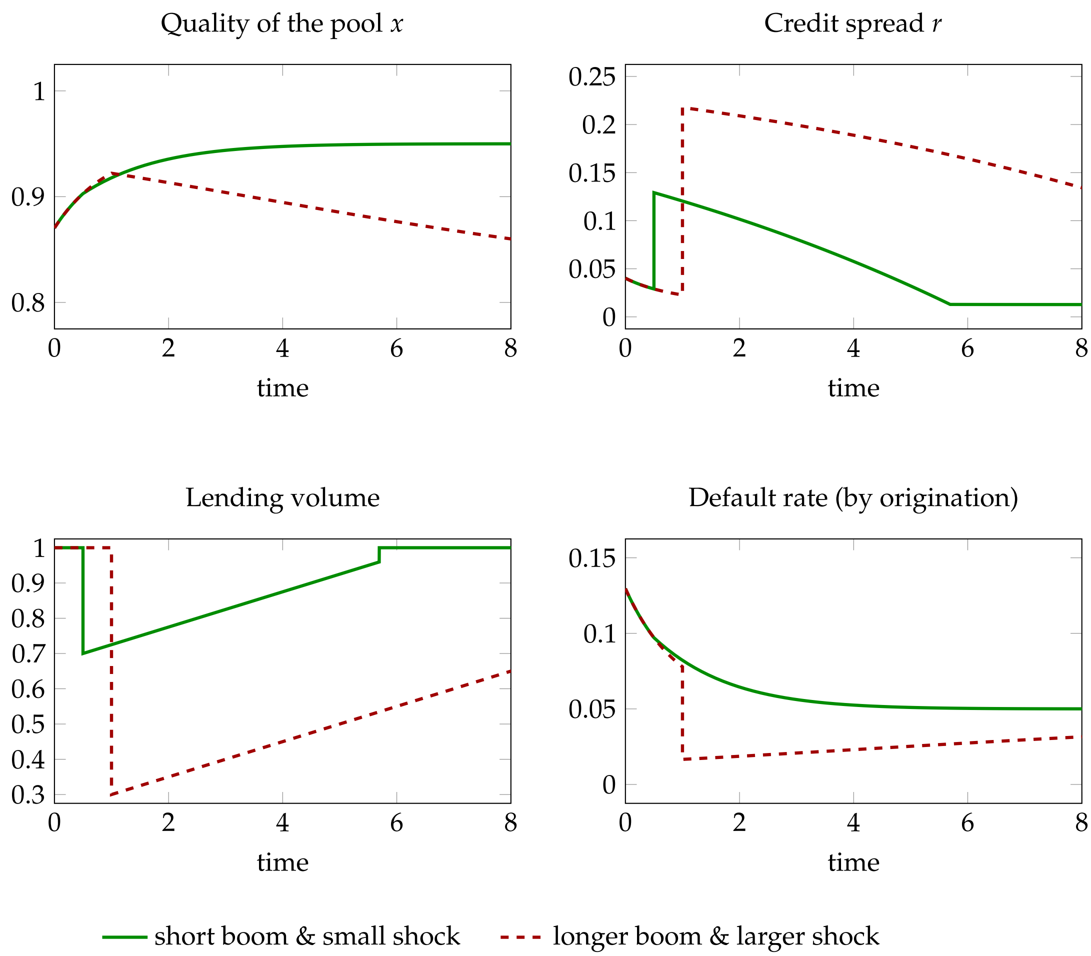
Figure 11 provides a first illustration of how the effects of balance sheet constraints can be amplified and propagated by lending standards. For a tight enough constraint on lending, banks impose tight lending standards not only for x just above $\bar{x}$ but also at much higher levels of pool quality such as that at the pooling steady state, x^p, and even higher, $x > x^p$. Thus, even when the market starts at a high pool quality at or above the pooling steady state x^p, tight balance sheet constraints can lead to tight lending standards, declines in pool quality, and ultimately to convergence to the screening steady state x^s that persists even when the constraint on lending is fully relaxed. An example of this dynamic is depicted in the lower (blue) convergence path in the Figure 11 and explored in the next subsection.

6.2 Booms and busts in lending standards and balance sheets

This subsection presents a simulation of a credit market hit by a balance sheet shock to illustrate the dynamic interaction of lending standards and balance sheet constraints. We start the economy just to the right of $\bar{x}$. Thus, unperturbed, it converges towards the pooling steady state. One may think of this convergence as a "lending boom."

After some time T, we assume that an unanticipated negative balance sheet shock hits

Figure 12: Balance sheet constraints and lending standards



This figure simulates two economies starting at $x_0 = \bar{x}$. One experiences a shock at $t = 0.5$, reducing available capital for loans to $\overline{V} = 0.7$. The other experiences a shock at $t = 1$, reducing capital to $\overline{V} = 0.3$. In both economies, capital improves with $\dot{\overline{V}} = 0.05$. The other parameters used for this simulation are as in Figure 4.

banks, reducing the volume of loans that they can make to some low level $\overline{V}_T$, after which this constraint monotonically increases at exogenous speed $\dot{\overline{V}}$. We further assume that $\overline{V}_T$ is decreasing in T. These assumptions capture the idea of an economic shock at T that causes greater one-time losses (bigger $-r_L$) for type-L borrowers which have received loans in the past. These (un-modeled) defaults would then cause banks' balance sheet constraints to bind. Further, the longer the "lending boom", i.e. the convergence towards the pooling steady state has progressed, the more type-L projects have been funded shortly before T and the tighter the balance sheet constraints are at T when the shock hits.

Figure 12 depicts the dynamics for the credit market for $T = 0.5$ and for $T = 1$. When the "short boom", $T = 0.5$, bursts, depicted by the solid (green) lines in Figure 12, balance sheet constraints bind, both reducing the volume of lending and tightening lending standards. Credit credit spreads rise, and the quality of lending rises (default rates fall). Tight lending standards cause the pool quality to start declining over time, an effect which amplifies the initial downturn in the credit market. However, because the exposure to the aggregate shock $\overline{V}_T$ is not yet that large, this amplification is temporary and goes away as the balance sheet constraint relaxes. As banks grow out of the balance sheet constraint, they start to relax lending standards, partially at first, but then more over time as the constraint relaxes further and as pool quality starts to increase. Ultimately, the economy converges to the pooling equilibrium as the balance sheet constraint on lending ceases to bind. These dynamics are those depicted in the upper (green) convergence path in Figure 11.

When the "long boom", $T = 1$, bursts, as depicted by the dashed (red) lines in Figure 12, lending standards increase as when the short boom ends, but lending volume declines further because the capital constraint $\overline{V}_T$ is tighter than following the short boom. Credit spreads increase by more than in the short boom both because it will be longer until lending is unconstrained and because when it is unconstrained, interest rates are higher because the pool quality is lower. As following the short boom, tight lending standards amplify the credit downturn, but after the long boom there is a sufficiently long period with tight lending standards that the credit crunch becomes becomes permanent. Persistent tight lending standards reduce the pool quality sufficiently such that the market converges to and remains at the screening steady state even when the balance sheet constraints have passed. As pictured in Figure 12, because lending standards are never relaxed, lending never returns to its pre-crisis level and credit spreads remain permanently high. These dynamics are those depicted in the lower (blue) convergence path in Figure 11.

These experiments emphasize that lending standards can amplify and propagate the negative effects of capital losses on credit markets. Capital constraints on lending not only lead to declines in lending and increases in spreads, but by incentivizing tighter lending standards, they further increase credit spreads, and can slow recovery or even cause self-reinforcing dynamics that perpetuate high interest rate spreads, low lending volumes, and tight lending standards.

7 Discussion of assumptions and extensions

We discuss the importance of several modeling assumptions and sketch model extensions.

7.1 Borrower effort to raise project quality

Suppose borrowers could exert effort to improve the quality of their projects. Will borrowers have a bigger incentive to exert effort when lending standards are normal or tight?

Suppose that when a borrower enters the pool, she privately chooses costly effort, where more effort increases the likelihood of being a type-H borrower. The share of type-H borrowers in the pool, x, now depends on borrower effort. Banks take x as given so a bank's problem is unchanged and thus the cutoff between normal and tight lending standards, $\bar{x}$, is unchanged. A type-H borrower with an opportunity to fund a project receives a payoff $r_H - r_t(x_t) + u$ while a type-L borrower receives an expected payoff of $(1-z)u + zJ_t^L$. With small u (and hence small J_t^L), the payoff to a type-L borrower is independent of lending standards (their payoff is always zero). Thus, in this case, the payoff to a type-H borrower, $r_H - r_t(x_t)$, determines the incentive to be type-H relative to type-L. This incentive is increasing in x through lower interest rates. So a larger x, associated with normal lending standards, is associated with a *greater* incentive to be type-H relative to type-L.

Since tight lending standards are associated with a lower incentive to be type-H, how might our results change if borrowers responded to this incentive and so were less likely to be type-H? Our strategic complementarity would be even *stronger* because whenever the pool quality x_t is low, not only are banks screening, reducing *future* pool quality, but due to higher credit spreads, borrowers exert less effort, reducing *current* pool quality. Thus, the complementarity at the core of this paper is amplified.

7.2 Screening with collateral

Our modeling assumptions imply that banks cannot use contract terms like fees or covenants to screen out type-L borrowers. Consider instead a situation in which banks use collateral requirements to screen potential borrowers. Let borrowers be endowed with a collateral asset and suppose banks require borrowers to post their collateral to receive a loan. As long as the value of the collateral exceeds the private benefits, type-L borrowers would not apply for loans if all banks require collateral. In this situation, the strategic complementarity remains, but the negative externality is eliminated.

However, in practice, several issues arise. First, when private benefits are unknown to banks, as long as the private value is greater than the collateral value for some type-L borrowers, they still apply for loans. In this case, collateral does not resolve the negative externality. Second, collateral values vary, and banks may be able to acquire costly private information about this value (e.g. appraise the collateral). In this case, the share of borrowers with good collateral would determine whether banks applied normal or tight lending standards to the collateral of borrowers, and these lending standards would act much like those in our model, exhibiting strategic complementarities and negative externalities among banks.

7.3 Credit bureaus

One reason underlying the negative externality from tight lending standards is the assumption that information on previous rejections is unobservable and non-verifiable. Does this mean our model is inapplicable to credit markets in which credit bureaus track potential borrowers?

First, note that our model applies to information above and beyond publicly available information. As we described, it applies to potential borrowers within a given credit score bracket, which summarizes the past credit information contained in the credit bureau. Second, credit bureaus typically do *not* track much of the information that lenders might investigate prior to making a loan, and that might be uncovered by tight lending standards in our model. Appendix C contains extensive details about the prevalence, coverage, and information provided by credit bureaus around the world. None of the countries that we investigated have credit bureaus that report whether credit was denied or instead turned down by the potential borrower. While many credit bureaus, like consumer bureaus in the U.S., delineate whether a credit check is *hard*, meaning associated with an application for credit, or *soft*, due to account review, marketing, or possibly hiring, only about half of the credit bureaus report the purpose of previous credit checks. Even with information on hard credit checks, lenders typically cannot tell whether a potential borrower who recently applied for a loan, applied for a mortgage, a car loan, or a credit card (again, see Appendix C for details).[21]

Finally, we note that our model may apply even in situations in which credit bureaus accurately track borrowers if lending standards are applied to collateral instead of borrowers. That is, if tight lending standards evaluate and reject on the basis of collateral (e.g. an appraisal of a house), then credit bureaus do not address the externality of tight lending

[21] Further, a past credit check without a subsequent loan does not indicate that a given borrower failed a past lending standard. The borrower may have applied for a job, or may have simply decided not to take the loan (decided not to buy that house or car, or decided to pick a different credit card). Importantly, in practice, lenders can evaluate potential borrowers before verifying their information via a credit report and leave no trail of credit checks for rejected borrowers. That is, as is common in mortgage markets for example, banks can fully apply their lending standards on the basis of information reported by the borrower and additional information gathered by the bank, and only verify information with a credit report for borrowers that pass the lending standard. Thus many applications are not recorded.

standards because they track borrowers, not the assets they wish to fund.

Given the negative externality in our model, why don't credit bureaus track rejections? Our model suggests that bureaus do not track credit rejections because it is not incentive compatible for a bank-borrower pair to report a negative evaluation or to report a rejection.[22] Statements from credit bureaus are consistent with this reasoning and suggest that credit bureaus are unable to enforce the reporting of soft information that it is not privately optimal to report (see Appendix C).

While mature credit markets in legal environments with low-cost enforcement mechanisms may track information about borrowers that mitigates the negative externality associated with tight lending standards, we conclude that this tracking appears to be insufficient to eliminate the key externality in most countries' credit markets.

7.4 Further non-essential assumptions

We simplified the analysis by assuming that the banking sector is competitive so that banks make zero profits. We conjecture that the qualitative features of the steady-states, dynamics and welfare results would remain if banks shared the surplus of a match with a given potential borrower.

Do our main results rely on our specific screening technology? If the screening technology could also mistake a type-H borrower for a type-L borrower this would change the exact formula for $\dot{x}$ but not alter our substantive points. Other changes to the screening technology are less consequential. Our model can easily incorporate a screening technology with a cost that is non-linear in the lending standard z. Concavity replicates our current results. Convexity would imply that rather than necessarily screening at a level of $\bar{z}$, banks might choose a lower level instead, equating the marginal benefit and cost of screening. Then a bank's lending standard would be more smoothly decreasing in x. There is also the possibility of more than two steady states, which would occur when the optimal lending standards line in Figure 2 decreases more smoothly and so has more intersections with the $\dot{x} = 0$ line. We assumed that screening produces a binary signal, and it would be inconsequential to instead assume a continuous signal as banks would simply choose a cutoff value for their binary lending decision. Lastly, if, when lending standards are tight,

[22] Other models suggest different reasons. For example Axelson and Makarov (2019) show the striking result that introducing a credit registry that tracks borrowers' loan application histories but not the borrowing rates offered can lead to more adverse selection and quicker market breakdown. In that model, acquiring information on a borrower is costless and the result follows from the fact that a lender who knows that a borrower's offer was rejected does not know whether the borrower was bad or whether the borrower demanded a too-low interest rate.

the probability that a given type-L borrower is funded by any bank were correlated across banks, then the dynamic strategic complementarity at the heart of our model would be stronger. This would occur because when one bank screens and rejects a potential borrower, such a correlation would make it easier for the next bank to detect that borrower as bad and so would raise the private value to screening.

Finally, our model has debt contracts. But because the model has only two borrower types, an equity contract can deliver the same payoffs to banks and borrowers of each type. With more types, our model could become significantly more complex. While the degree of complexity would depend on how well the screening technology detected different types, the extensions we have considered have all involved more state variables, which raises the possibility of non-linear dynamics that can occur in such systems.

8 Concluding remarks

We develop a dynamic theory of lending standards, based on two intuitive ideas. First, tighter lending standards lead to the rejection of unprofitable loan applications. And second, it is not costless for banks to identify unprofitable applicants, even those previously rejected by other banks. These two ideas give rise to a dynamic strategic complementarity between banks, which leads to more persistence in lending standards than in business cycles themselves. The ideas also generate negative externalities of tight lending standards, implying that lending standards are too tight for too long after negative shocks.

Which markets does our theory apply to? These principle ideas provide some guidance. The first idea is likely true for *any* kind of lending standard. The second suggests that markets in which borrowers are likely to shop for loans from multiple lenders are particularly exposed to the results in this paper. This includes markets in which borrowers have pre-existing relationships with many lenders (such as corporate lending markets) and markets in which borrowers need to roll over loans (as this raises the likelihood of looking for loans after rejection). In addition, the second idea requires lenders to rely on costly private information when applying lending standards. In contrast, markets in which borrowers have limited ability or need to approach multiple banks, or in which the outcome of lending depends purely on public information, are unlikely to be subject to the conclusions from our theory.

Our paper opens up several avenues for future research that build on our model and analysis of the negative externality associated with tight lending standards. For example, the government intervention that is analyzed effectively assumes that the negative shock that put the market on the path to the inefficient steady state is expected to never recur. But suppose it might recur. Is government intervention still optimal? Addressing this question requires a specification of the social cost of a government policy, e.g., a subsidized loan guarantee. We expect that it can be shown that if the likelihood that the shock recurs is low enough, then government intervention will be value enhancing. This is because with a low probability of a negative shock, the cost of an intervention can be amortized over a longer period of time.

Another interesting extension is to investigate the interaction of lending standards with other factors that drive bank profits and dynamic choices, such as variation in the competitiveness of the banking sector or dynamic choices of capacity to perform careful evaluation of borrowers. Similarly, we study the effect of simple, exogenous capital constraints on lending volume, and there may be interesting feedback dynamics from lending standards back into bank balance sheets and constraints. Finally, our analysis is partial equilibrium, an analysis of a single lending market. An interesting question is how and whether lending standards interact across credit markets, and how general equilibrium effects through interest rates in particular feed back into lending standards.

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A Proofs and derivations

A.1 Steady state equilibria: Proof of Proposition 1

The three pairs (x, z) mentioned in Proposition 1 are solutions to (8) and (9) if $\lambda > \overline{x}$, $x^s < \overline{x}$, and $\frac{\lambda-\overline{x}}{\lambda-\lambda\overline{x}}\left(1+\delta\kappa^{-1}\right) < \overline{z}$. The first two of these hold by Assumption 1 and the third is a straight consequence of the second.

We claim that the three pairs indeed constitute equilibria, with $\theta = 1$, $\varphi^a = \varphi^r = 1$ and with r pinned down by Proposition 3. To prove this, first note that the law of motion (6) as well as the bank's maximization problem (4) are satisfied due to (8) and (9). The zero profit condition (5) pins down the interest rate (see our proof to Proposition 3). Finally, in any steady state a type-H borrower strictly prefers a loan today, that is,

$$r_H - r + u - J^a > 0,$$

and since $r \leq r_H$ (which holds since $x^s \geq \underline{x}$ with $\underline{x}$ as in (10) due to Assumption 1) we have that $\theta = 1$ and $\varphi^a = \varphi^r = 1$.

A.2 Proof of Corollary 1

The flow of projects being funded in the pooling steady state is κ, compared to $\kappa x^s + \kappa(1 - x^s)(1-\overline{z})$ in the screening steady state. The credit spread result follows directly from Proposition 3 and the fact that $r(x)$ is strictly decreasing in x. The equilibrium default rate is given by

$$\frac{\kappa(1-x)(1-z)}{\kappa(1-x)(1-z)+\kappa x} = \left(1+\frac{x}{(1-x)(1-z)}\right)^{-1}$$

which can further be simplified to

$$(1-\lambda)\left(1+\frac{\lambda z\delta\kappa^{-1}}{(1+\delta\kappa^{-1})(1-z)}\right)^{-1}.$$

Thus, when $\delta = 0$, the equilibrium default rate is always equal to $1-\lambda$, irrespective of the steady state.

A.3 Proof of Proposition 2

Begin with $x_0 \in (\overline{x}, \lambda]$. In that case, $z = 0$ is is the optimal bank strategy (see (4)), and therefore the law of motion of x, (6), reads

$$\dot{x}_t = \kappa(1-x_t)\lambda - \kappa x_t(1-\lambda) + \delta(\lambda - x_t) = (\kappa+\delta)(\lambda - x_t) > 0$$

which is positive for any $x_t < \lambda$, implying convergence to the pooling steady state.

Next turn to $x_0 \in [\underline{x}, \overline{x})$. In that case, $z = \overline{z}$ is the optimal bank strategy (see (4)), and therefore the law of motion of x, (6), reads

$$\dot{x}_t = \kappa(1-x_t)(1-\bar{z})\lambda - \kappa x_t(1-\lambda) + \delta(\lambda - x_t) = (\delta - \kappa\bar{z}\lambda + \kappa)(x^s - x_t)$$

implying convergence to the screening steady state.

For $x_0 < \underline{x}$, note that $\theta_t = 0$ and so

$$\dot{x}_t = \delta(\lambda - x_t) > 0$$

implying that the pool quality improves until it crosses $\underline{x}$ and thereafter converges to the screening steady state.

The case of $x_0 = \overline{x}$ is straightforward as $\overline{x}$ is already a steady state.

A.4 Proof of Proposition 3

The zero profit condition (5) implies that

$$\Pi(R) = \kappa_H r + \kappa_L(1-z)r_L - (\kappa_H + \kappa_L)\tilde{c}z = 0.$$

Reformulating this we obtain

$$\kappa x r / r_L + \kappa(1-x)(1-z) + \kappa c z = 0$$

$$r = -r_L \frac{cz + (1-x)(1-z)}{x}$$

which proves Proposition 3.

A.5 Proof of Proposition 4

Define $\theta(x)$ as in (12) and define $\hat{x}$ implicitly as the unique value of $x < \lambda$ with $\theta(x) = 1$. Such a value exists since $\theta(x)$ is strictly increasing and continuous in x with $\theta(0) = -\delta\kappa^{-1} < 0$ and $\lim_{x\to\lambda}\theta(x) = \infty$.

Assume $\hat{x} > \overline{x}$. Conjecture for any $x_0 \in [\overline{x}, \hat{x})$ that the equilibrium is one with $\theta_t = \theta(x_t)$. To verify the conjecture, we need to show that type-H borrowers are indifferent between taking a loan and waiting. Assuming $u \to 0$ in (1a), this is equivalent to

$$J_t^H = r_H - r(x_t)$$

with

$$\rho J_t^H = \dot{J}_t^H - \delta J_t^H.$$

Putting the two together, we obtain (13),

$$-r'(x)\dot{x} = (\rho + \delta)(r_H - r(x)).$$

The law of motion for x with $\theta < 1$ is $\dot{x}_t = (\kappa\theta + \delta)(\lambda - x)$, which, together with (13) yields (12) and therefore confirms that type-H borrowers are, by construction, precisely indifferent.

A.6 Proof of Corollary 3

By Assumption 1, $c \geq 1 - \lambda$. Therefore, welfare in the screening steady state is bounded above,[23]

$$W^s = x^s r_H - (1 - \bar{z})(1 - x^s) r_L - \tilde{c}\bar{z} = x^s \frac{r_H}{-r_L} - (1 - \bar{z})(1 - x^s) r_L - c\bar{z}$$

$$\leq x^s \frac{r_H}{-r_L} - (1 - x^s)(1 - \bar{z}) - (1 - \lambda)\bar{z} = x^s \left(\frac{r_H}{-r_L} + 1 - \bar{z} \right) - (1 - \lambda \bar{z})$$

Welfare in the pooling steady state is $W^p = \lambda(\frac{r_H}{-r_L} + 1) - 1$. Observe that W^s increases in x^s, so W^s can only ever be above W^p if x^s is as large as possible. Clearly, given the formula for x^s, x^s is largest as a function of δ if $\delta = \infty$ where $x^s = \lambda$. In that case, we find

$$W^s \leq x^s \left(\frac{r_H}{-r_L} + 1 - \bar{z} \right) - (1 - \lambda \bar{z}) < \lambda \left(\frac{r_H}{-r_L} + 1 - \bar{z} \right) - (1 - \lambda \bar{z}) = \lambda \left(\frac{r_H}{-r_L} + 1 \right) - 1 = W^p$$

Therefore, welfare of the pooling steady state always dominates that of the screening steady state.

A.7 Proof of Proposition 5

We prove Proposition 5 in two steps. First, we determine the efficient screening policy $z^*(x)$ conditional on banks operating. Then we determine the optimal behavior for banks to operate $\theta^*(x)$.

A.7.1 Optimal screening policy $z^*(x)$

To do so, let $V(x, z)$ denote the present value of welfare if the current state of the market is x and the screening policy is z from hereafter, that is,

$$V(x, z) \equiv \frac{\rho x + \alpha^z x^z}{\rho + \alpha^z} r_H + (1 - z) \left(1 - \frac{\rho x + \alpha^z x^z}{\rho + \alpha^z} \right) r_L - \tilde{c} z. \tag{25}$$

where $\alpha^z \equiv \kappa + \delta - \lambda \kappa z$ and $x^z \equiv \lambda - \lambda \frac{(1-\lambda)z}{(1-\lambda z) + \delta \kappa^{-1}}$. Also, denote by

$$v(x, z) \equiv \rho \left\{ x r_H + (1 - z)(1 - x) r_L - \tilde{c} z \right\} \tag{26}$$

[23] We define all welfare expressions here as multiples of κ, for expositional clarity. κ multiplies both W^s and W^p equally.

the flow value of policy z at state x. Finally, we call

$$d(x,z) \equiv \kappa(1-x)(1-z)\lambda - \kappa x(1-\lambda) + \delta(\lambda - x) \tag{27}$$

the derivative of x at state x under policy z (see the law of motion in (6)). Observe that

$$\rho V(x,z) = v(x,z) + V_x(x,z)d(x,z) \tag{28}$$

as well as

$$d(x^s,\bar{z}) = 0 \;\; d(x^p,0) = 0. \tag{29}$$

We first prove the following helpful lemma.

Lemma 1. *We have:*

1. If $\lambda\kappa r^{\Delta} \geq \rho + \kappa + \delta$, pooling is strictly optimal for any state x, i.e. $z^*(x) = 0$.
2. If $\lambda\kappa r^{\Delta} < \rho + \kappa + \delta$, $V(x,z)$ has negative cross-partials, $V_{xz} < 0$.
3. If $\lambda\kappa r^{\Delta} < \rho + \kappa + \delta$ and $V(x,0) > V(x,z_1)$ for some $z_1 > 0$, then also $V(x,0) > V(x,z_2)$ for any $z_2 \in (0,z_1)$.

Proof. Assume $\lambda\kappa r^{\Delta} \geq \rho + \kappa + \delta$. Suppose pooling were not strictly optimal for every state x. First, if $d(x,z^*(x))$ is ever negative for some $x < \lambda$, there must be a steady state at some $x^0 \in [0,\lambda)$ with some $z^0 = z^*(x^0) > 0$. This cannot be optimal since

$$V(x^0,z^0) < V(x^0,0)$$

is equivalent to (after a few lines of algebra)

$$-\left(\rho + (1-\lambda)\alpha^p - \rho x^0\right)\left(r^{\Delta}\kappa\lambda - (\rho+\kappa+\delta)\right) < (\rho+\alpha^s)(\rho+\alpha^p)c$$

which is true since the left hand side is negative. Second, assume $d(x,z^*(x))$ is positive everywhere. Then, $x^p = \lambda$ is still the unique steady state. Let $\mathbf{V}(x)$ be the optimal value function. It has to hold that

$$r\mathbf{V}(x) = v(x,z^*(x)) + \mathbf{V}'(x)d(x,z^*(x)). \tag{30}$$

Rearranging,

$$\mathbf{V}'(x) = \frac{r\mathbf{V}(x) - v(x,z^*(x))}{d(x,z^*(x))} \equiv \mathbf{F}(\mathbf{V}(x),x).$$

Compare this to the ODE describing the value of pooling,

$$V_x(x,0) = \frac{rV(x,0) - v(x,0)}{d(x,0)} = \mathbf{F}^0(V(x,0),x)$$

Observe that $\mathbf{F}(V,x) > \mathbf{F}^0(V,x)$ for any x for which $z^*(x) > 0$.[24] Since $\mathbf{V}(x^p) = V(x^p, 0)$, it must be that $\mathbf{V}(x) < V(x,0)$ for some x if there is a positive measure where $z^* > 0$. This contradicts our assumption that $\mathbf{V}(x)$ is the optimal value function. Thus, pooling is optimal for every state.

Assume $\lambda\kappa r^{\Delta} < \rho + \kappa + \delta$. Simple algebra based on (25) implies that

$$V_x = \frac{\rho}{\rho + \alpha^z} r_H + (1-z)\frac{r}{\rho + \alpha^z} r_L > 0$$

and

$$V_{xz} = \rho \frac{\lambda\kappa r^{\Delta} - (\rho + \kappa + \delta)}{(\rho + \alpha^z)^2 r_L} < 0.$$

For point 3 in the lemma, fix $x \in [0, \lambda]$. Define the positive constants

$$c_0 \equiv \frac{\rho x + \lambda\kappa + \lambda\delta}{\lambda\kappa}, \quad c_1 \equiv \frac{\rho + \kappa + \delta}{\lambda\kappa}, \quad c_2 \equiv r^{\Delta}, \quad c_3 = -r_L$$

Then, after some algebra, we can write

$$\begin{aligned} V(x,z) &= (1-z)r_L - \tilde{c}z + c_3 \frac{c_0 - z}{c_1 - z}(c_2 - z) \\ &= r_L - \tilde{c}z + c_3(c_0 + c_2 - c_1) + c_3 \frac{(c_1 - c_2)(c_1 - c_0)}{c_1 - z} \end{aligned}$$

c_1 is always greater than c_0. Also, given $\lambda\kappa r^{\Delta} < \rho + \kappa + \delta$, c_1 is greater than c_2. Therefore, $V(x,z)$ is a convex function in z, and thus in particularly quasi-convex, from which the stated property follows. □

In the next lemma, we narrow down the set of optimal policies using the necessary (but not sufficient) first order conditions of (14).

Lemma 2. *Describe an efficient screening policy $z^*(x)$ by the following general form: Let $I_1, I_2, I_3 \subset [0, \lambda]$ be (possibly empty) connected intervals such that $I_1 \leq I_2 \leq I_3$, $I_1 \cup I_2 \cup I_3 = [0, \lambda]$, and*

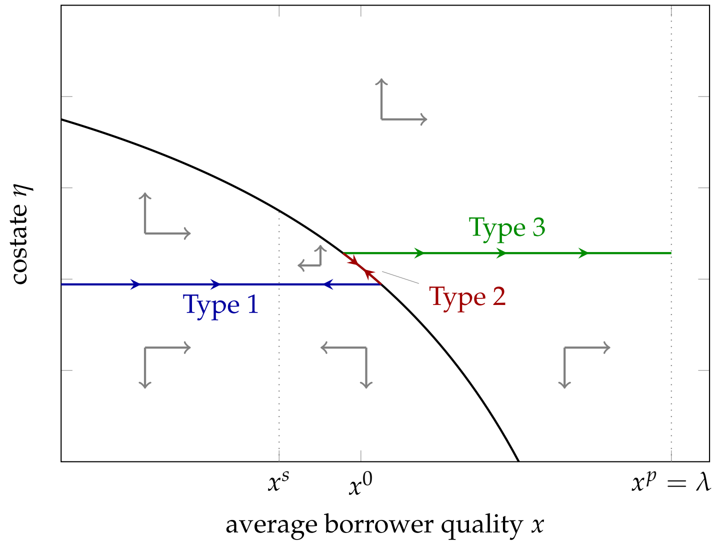
- *$z^*(x) = \bar{z}$ for $x \in I_1$*
- *$z^*(x) \in [0, \bar{z}]$ for $x \in I_2$*
- *$z^*(x) = 0$ for $x \in I_3$*

where we construct I_1 to be the largest connected interval where $z^*(x) = \bar{z}$, and similarly I_3 for $z^*(x) = 0$.

Then: If I_2 is non-empty (and thus $z^*(x)$ not bang-bang), there exists a $x^0 \in I_2$ with $d(x^0, z^*(x^0)) = 0$.

[24] Note that $\mathbf{V}'(x) > 0$ by a simple envelope argument.

Figure 13: Phase diagram for constrained efficient problem



Proof. We begin by writing down the necessary first order conditions of (14). Denoting by η the costate of x, we have the law of motion

$$\dot{\eta} = \rho\eta - \kappa \left\{ r_H - (1 - z_t) r_L \right\}$$

as well as the first order condition for z, showing that $z_t = \bar{z}$ if

$$\kappa \left\{ -(1 - x_t) r_L - \tilde{c} \right\} - \eta \left(\kappa (1 - x_t) \lambda \right) > 0 \tag{31}$$

and $z_t = 0$ if (31) hold with "$<$" inequality; with equality, z_t can be anywhere in $[0, \bar{z}]$.

Together with the law of motion of x in (6), this gives a system of two ODEs. We first note that there are three possible steady states. The two steady states for $z = 0$ and $z = \bar{z}$, as well as a third one, $z = z^0$ pinned down by $\dot{\eta} = \dot{x} = 0$ and (31) holding with equality,

$$\frac{-r_L}{\lambda} - \frac{\tilde{c}/\lambda}{1 - x^0} = \frac{\kappa}{\rho} \left\{ r_H - (1 - z^0) r_L \right\} \tag{32}$$

where

$$x^0 = \lambda \frac{1 - z^0 + \delta\kappa^{-1}}{(1 - \lambda z^0) + \delta\kappa^{-1}}. \tag{33}$$

Observe that, after substituting (33) into (32), the left hand side of (32) is increasing in z^0, while the right hand side is decreasing, so there is at most a single solution to (32).

Now consider the phase diagram in Figure 13. As can be seen, there are 3 types of candidate optimal paths. Type 1 converges to x^s, with $z = \bar{z}$ along the path and constant η; Type 3 converges to x^p, with $z = 0$ along the path and constant η; and finally type 2 converges to x^0 as in (33). Observe that the second type of paths only works if z^0, x^0 exist, solving (32) and (33).

This implies that, unless the optimal policy $z^*(x)$ is bang-bang, there has to exist a x^0 with $d(x^0, z^*(x^0)) = 0$. □

With this result in mind, we assume in the following that $\lambda\kappa r^\Delta < \rho + \kappa + \delta$ and characterize $z^*(x)$.

Lemma 3. *Assume $\lambda\kappa r^\Delta < \rho + \kappa + \delta$. The efficient screening policy $z^*(x)$ is to screen if $x < \overline{x}^*$ and to pool if $x > \overline{x}^*$, where*

$$V(\overline{x}^*, 0) = V(\overline{x}^*, \overline{z}) \tag{34}$$

as long as the solution to that equation is greater or equal to x^s. Otherwise, $\overline{x}^*$ is determined by

$$v_z(\overline{x}^*, 0) + V_x(\overline{x}^*, 0)d_z(\overline{x}^*, 0) = 0. \tag{35}$$

Proof. First, notice that $\overline{x}^*$ is indeed well-defined, in that if the solution to (34) is x^s, then (35) is also solved by x^s. Assume

$$V(x^s, 0) = V(x^s, \overline{z}).$$

Combining (28) and (29), we can rewrite $V(x^s, 0)$ and $V(x^s, \overline{z})$ and obtain

$$v(x^s, 0) + V_x(x^s, 0)d(x^s, 0) = v(x^s, \overline{z}) + V_x(x^s, \overline{z})d(x^s, \overline{z}).$$

Since $d(x^s, \overline{z}) = 0$, this can be combined into

$$v(x^s, \overline{z}) - v(x^s, 0) + V_x(x^s, 0)\left(d(x^s, \overline{z}) - d(x^s, 0)\right) = 0 \tag{36}$$

which is equivalent to (35) as v and d are linear in z. Moreover, going these steps backwards, if $\overline{x}^* < x^s$, then (36) holds with inequality and therefore

$$V(x^s, 0) > V(x^s, \overline{z}). \tag{37}$$

Now we proceed to our main argument, a proof by contradiction. We distinguish four possible cases.

Case 1: There exists $x > \overline{x}^*$ with $x \geq x^s$ where pooling is not optimal. If true, by Lemma 2, this would require there to be at least one point $x^0 \in [\overline{x}^*, \lambda)$ where the planner strictly prefers to remain at x^0 forever (by choosing strategy $z^0 \in (0, \overline{z}]$ such that $d(x^0, z^0) = 0$) over pooling. In math,

$$V(x^0, z^0) > V(x^0, 0).$$

Since V has a negative cross-partial $V_{xz} < 0$ (Lemma 1), this implies that $V(\overline{x}^*, z^0) > V(\overline{x}^*, 0)$ and $V(x^s, z^0) > V(x^s, 0)$, which, by point 3 in Lemma 1, is contradicting either (34) or (37).

Case 2: There exists $x < \overline{x}^*$ with $x \geq x^s$ where screening is not optimal. If true, by Lemma 2, this would require there to be at least one point $x^0 \in (x^s, \overline{x}^*]$ where the planner strictly prefers to remain at x^0 forever (by choosing strategy $z^0 \in [0, \overline{z})$ such that $d(x^0, z^0) = 0$) over screening. In math,

$$V(x^0, z^0) > V(x^0, \overline{z}).$$

Since V has a negative cross-partial $V_{xz} < 0$ (Lemma 1), this implies that $V(\overline{x}^*, z^0) > V(\overline{x}^*, \overline{z})$, which by point 3 in Lemma 1, contradicts (34).

Case 3: There exists $x > \overline{x}^*$ with $x \leq x^s$ where screening is optimal. If true, this would require there to be at least one point $x^0 \in [\overline{x}^*, x^s]$ where the planner strictly prefers to screen with some intensity $z^0 > 0$ in the current instant while pooling is chosen thereafter. That is,

$$v(x^0, z^0) + V_x(x^0, 0)d(x^0, z^0) > v(x^0, 0) + V_x(x^0, 0)d(x^0, 0).$$

Due to linearity of this equation, it also has to hold with $z^0 = \overline{z}$, and therefore also expressed as derivative,

$$v_z(x^0, 0) + V_x(x^0, 0)d_z(x^0, 0) > 0. \tag{38}$$

Since this is a linear equation in x^0, to be consistent with (35), it must be that (38) in fact holds for any $x^0 > \overline{x}^*$, including $x^0 = x^p = \lambda$. In that case, however, (38) simplifies to $v_z(x^p, 0) + V_x(x^p, 0)d_z(x^p, 0) > 0$, which is false, since $V_x(x, 0) > 0$, $d_z(x, 0) < 0$ and $v_z(x^p, 0) = -\kappa r_L (c - (1 - \lambda)) < 0$ by Assumption 1.

Case 4: There exists $x < \overline{x}^* \leq x^s$ where pooling is optimal. Let $\mathbf{V}(x)$ be our conjectured value function left of $\overline{x}^*$. By design, $\mathbf{V}(x)$ solves

$$\rho \mathbf{V}(x) = v(x, \overline{z}) + \mathbf{V}'(x)d(x, \overline{z})$$

where $d(x, \overline{z}) = \alpha^{\overline{z}}(x^s - x)$ and $\mathbf{V}'(x)$ solves

$$(r + \alpha^{\overline{z}})\mathbf{V}'(x) = v_x(x, \overline{z}) + \mathbf{V}''(x)d(x, \overline{z}).$$

This ODE can be solved explicitly, giving[25]

$$\mathbf{V}'(x) = \rho r_L \left(\frac{r^\Delta}{\rho + \alpha^p} - \frac{r^\Delta - \overline{z}}{\rho + \alpha^{\overline{z}}} \right) \left(\frac{x^s - x}{x^s - \overline{x}^*} \right)^{-\beta} + \rho r_L \frac{r^\Delta - \overline{z}}{\rho + \alpha^{\overline{z}}}$$

where $\beta = 1 + \frac{\rho}{\alpha^{\overline{z}}}$. The coefficient on the first term is positive, since we assumed $r^\Delta \lambda \kappa < \rho + \kappa + \delta$. Thus, $\mathbf{V}'(x)$ is bounded above by

$$\mathbf{V}'(x) \leq \mathbf{V}'(\overline{x}^*) = r(1 - R_L)\frac{r^\Delta}{\rho + \alpha^p}. \tag{39}$$

[25] Note that $v_x(x, \overline{z})$ is a constant in x.

Could it ever be that the planner prefers pooling in this region? If so, we would have an $x < \overline{x}^*$ with

$$v_z(x, 0) + \mathbf{V}'(x) d_z(x, 0) < 0$$

which due to (39) and the fact that $d_z(x, 0) = -\kappa\lambda(1 - x) < 0$ implies that

$$v_z(x, 0) + \mathbf{V}'(\overline{x}^*) d_z(x, 0) < 0.$$

Using the expressions in (26) and (27) we then see that this cannot hold as the left hand side is zero at $\overline{x}^*$ (by definition), and has a negative slope throughout,

$$v_{xz} + \mathbf{V}'(\overline{x}^*) d_{xz} = \rho r_L \left[-1 + \frac{r^\Delta \lambda \kappa}{\rho + \alpha^p} \right] < 0$$

where again we used $r^\Delta \lambda \kappa < \rho + \kappa + \delta$. This is a contradiction: there cannot be an $x < \overline{x}^*$ where pooling is optimal. □

A.7.2 Optimal bank operation policy $\theta^*(x)$

Next we focus on the optimal policy $\theta^*(x)$ for banks to operate. We prove the following result.

Lemma 4. *If it is strictly optimal to have banks operate at $\overline{x}^*$, the optimal policy describing when banks operate is bang-bang, that is,*

$$\theta^*(x) = \begin{cases} 0 & x < \underline{x}^* \\ 1 & x > \underline{x}^* \end{cases} \tag{40}$$

The threshold $\underline{x}^*$ is the supremum of all $x \in [0, \lambda]$ that solve

$$v(x, z^*(x)) + \mathbf{V}'(x) \left(\kappa \left(\lambda - x \right) - \kappa (1 - x) z^*(x) \lambda \right) < 0 \tag{41}$$

where $\mathbf{V}(x)$ is the value function associated with the optimal screening policy $z^*(x)$.

Proof. Let $\underline{x}^*$ be defined as in (41) and let $\mathbf{V}(x)$ be the value function conditional on banks operating with screening policy $z^*(x)$. If it is optimal for the planner to follow the bang-bang policy (40), then its value function for $x \geq \underline{x}^*$ is given by $\mathbf{V}(x)$, whereas for $x < \underline{x}^*$ the value function solves

$$\rho \mathbf{V}(x) = \mathbf{V}'(x) \delta(\lambda - x)$$

which can be solved to express the marginal value in state x as

$$\mathbf{V}'(x) = \mathbf{V}'(\underline{x}^*) \left(\frac{\lambda - x}{\lambda - \underline{x}^*} \right)^{-1-\rho/\delta}.$$

Observe that this is increasing in x. To prove that the bang-bang policy (40) is indeed optimal, we need to prove that

$$\max_{z\in[0,\bar{z}]} v(x,z) + \mathbf{V}'(x)d(x,z,1) \leq \max_{z\in[0,\bar{z}]} \mathbf{V}'(x)d(x,z,0) \tag{42}$$

for $x < \underline{x}^*$, where

$$d(x,z,\theta) \equiv \theta\kappa(1-x)(1-z)\lambda - \theta\kappa x(1-\lambda) + \delta(\lambda - x)$$

is the speed at which the pool improves given (x,z,θ). Simplifying (42), we obtain

$$\max_{z\in[0,\bar{z}]} v(x,z) + \mathbf{V}'(x)\left[\kappa\lambda(1-z) - \kappa x(1-z\lambda)\right] \leq 0.$$

The left hand side of this inequality has a negative cross-partial in (x,z), since $v_{xz} < 0$ and $\mathbf{V}'(x)(1-x) \propto (\lambda - x)^{-\rho/\delta}\frac{1-x}{\lambda-x}$ increases in x. Thus, given that $z = \bar{z}$ is optimal for $x = \underline{x}^*$, it is also optimal for any $x < \underline{x}^*$.

The problem then reduces from (42) to showing that for $x < \underline{x}^*$

$$F(\lambda - x) \equiv v(x,\bar{z}) + \mathbf{V}'(x)\left[\kappa\lambda(1-\bar{z}) - \kappa x(1-\bar{z}\lambda)\right] < 0. \tag{43}$$

To see this, we first show that $F(y)$ is quasi-concave (only has a single local maximum) and therefore can at most have two roots. $F(y)$ is of the form

$$F(y) = -F_0 y + F_1 y^{-\alpha-1}(y - y_0) + const$$

where $\alpha = \rho/\delta > 0$, $F_0 = \rho r_H + \rho(1-\bar{z})r_L > 0$, $F_1 = \kappa(1-\lambda\bar{z})\mathbf{V}'(\underline{x}^*)(\lambda - \underline{x}^*)^{1+\alpha} > 0$, $y_0 = \lambda - x^s > 0$. F can only ever have a single local maximum as long as these parameters are positive:

$$F'(y) = 0 \quad \Leftrightarrow \quad y^{-\alpha-2}\left[(1+\alpha)y_0 - \alpha y\right] = F_1/F_0$$

The left hand side of this equation is strictly decreasing for $y \in (0, (1+\alpha)y_0/\alpha)$ with range $(0,\infty)$ and thus admits a unique solution for any $F_1/F_0 > 0$. This establishes that $F(y)$ is quasi-concave.

Since $F(y)$ is quasi-concave, it admits at most two roots, $y_1 < y_2$, in between which $F(y)$ is positive, and negative outside of $[y_1, y_2]$. Root y_2 must correspond to $\lambda - \underline{x}^*$: if y_1 were to correspond to $\lambda - \underline{x}^*$, $\underline{x}^*$ would not be the supremum of x with $F(\lambda - x) < 0$ since for any $\epsilon > 0$ small enough, $F(\lambda - (\underline{x}^* - \epsilon)) > 0$. But if $y_2 = \lambda - \underline{x}^*$, then $F(\lambda - x) < 0$ for any $x < \underline{x}^*$, which proves (43). □

A.8 Proof of Proposition 6

There exists $\bar{x}^C \geq \bar{x}$ such that banks screen if $x < \bar{x}^C$ and $\bar{V}_t < \kappa x_t + \kappa(1-x_t)(1-\bar{z})$. Banks prefer to screen at rate $z > 0$ iff

$$\frac{x_t r_t + (1-x_t)(1-z)r_L}{1-(1-x_t)z} - \frac{\tilde{c}z}{1-(1-x_t)z} > x_t r_t + (1-x_t)r_L$$

which simplifies to

$$0 > (r_t - r_L)x_t^2 - (r_t - r_L)x + \tilde{c}.$$

The roots are

$$x_t^C = \frac{1}{2}\left[1 \pm \sqrt{1 - \frac{4\tilde{c}}{r_t - r_L}}\right] \tag{44}$$

where we denote the larger one by

$$\overline{x}_t^C = \frac{1}{2}\left[1 + \sqrt{1 - \frac{4\tilde{c}}{r_t - r_L}}\right].$$

As $\tilde{c} \to 0$, the two roots (44) converge to 0 and 1, meaning tight standards everywhere.

Proof that $\bar{x}_t^C \geq \bar{x}$. We first prove a useful property. The interest rate at $\overline{x}$ in the unconstrained equilibrium is given by

$$r(\overline{x}) = (-r_L)\frac{1-\overline{x}}{\overline{x}}.$$

With this interest rate we find a cutoff $\overline{x}^C$ of

$$\overline{x}^C(r = r(\overline{x})) = \frac{1}{2}\left[1 + \sqrt{1 - \frac{4\tilde{c}}{r(\overline{x}) - r_L}}\right] = 1 - c = \overline{x}. \tag{45}$$

Next we prove that $\bar{x}_t^C \geq \bar{x}$ more generally. We proceed by contradiction. Suppose there existed an $x_0 < \overline{x}$ and a continuous unbounded increasing function $\overline{V}_t$ starting with $\overline{V}_0 < \kappa x_0 + \kappa(1 - x_0(1-\overline{z})$ such that pooling is optimal, that is, $x_0 > \overline{x}_t^C$. Observe that since $\tilde{r}_t$ declines monotonically along the transition and x_t increases, this also implies that $x_t > \overline{x}_t^C$ at any future time t. Thus, pooling remains optimal.

Let T be the time at which banks become unconstrained. It has to be the case that $x_T > \overline{x}$. If not, we would have a contradiction since $\overline{x}_T^C < x_T$ per the discussion above, but at the same time

$$\overline{x}_T^C = \overline{x}^C(r = r(x_T)) > \overline{x}^C(r = r(\overline{x})) = \overline{x}$$

following from (45) and the fact that $r(\cdot)$ is monotone (Proposition 3). Thus, $x_T > \overline{x}$.

Now compare the path x_t that ultimately reaches x_T to a different path that reaches x_T. This second path has unconstrained banks and begins an ϵ to the right of $\overline{x}$. On its way to the pooling steady state, it crosses x_T. Observe that this second path has a slower decline in the interest rate r_t as average borrowers are not indifferent along the path, $0 > \dot{r}_t > \dot{\tilde{r}}_t$. Moreover, the transition along the second path is faster as banks are unconstrained. Together, this implies that the initial interest rate $\tilde{r}_0$ along the first path must have been greater than the initial interest rate $r(\overline{x})$ along the second path. But this means that

$$\overline{x}^C(r = \tilde{r}_0) > \overline{x}^C(r = r(\overline{x})) = \overline{x}$$

which is a contradiction to our assumptions $x_0 < \overline{x}$ and $x_0 > \overline{x}^C(r = \tilde{r}_0) = \overline{x}_0^C$.

The case $\kappa x_t + \kappa(1 - \bar{z})(1 - x_t) < \overline{V}_t < \kappa$. Consider first $x_t > \bar{x}$. If banks were to impose tight lending standards, $z_t = \bar{z}$, then they would be unconstrained and the interest rate equals the interest rate in the competitive equilibrium. At the competitive interest rate, banks prefer tight lending standards only if $x_t \leq \bar{x}$. Thus for $x_t > \bar{x}$, it can only be that $z_t < \bar{z}$ and we need only consider whether normal lending standards dominate intermediate ones. The largest z_t that does not make banks unconstrained is $z_t = \frac{1-\overline{V}_t/\kappa}{1-x_t}$.

Given our derivation in Section 6 and the fact that $\bar{z}$ did not enter (24), pooling is preferable if and only if $x_t > \overline{x}_t^C$. For $x_t < \overline{x}_t^C$, $z_t = \frac{1-\overline{V}_t/\kappa}{1-x_t}$ is the optimum. For $x_t \leq \overline{x}$, banks can screen without binding constraints. Given $x_t < x_t^C$, this is their optimal policy.

B Model with non-constant pool size

For this section, we assume that the pool size is not constant. We demonstrate that this economy gives rise to the exact same steady states and the exact same welfare predictions.[26]

B.1 Equilibria

Without a constant pool size, there are two state variables: m_H, the number of type-H borrowers in the pool and m_L, the number of type-L borrowers in the pool. The laws of motion of the state variables are given by

$$\dot{m}_H = \delta\lambda - \delta m_H - \kappa m_H \tag{46}$$

$$\dot{m}_L = \delta(1 - \lambda) - \delta m_L - \kappa(1 - z_t)m_L \tag{47}$$

The first term in both laws of motion stems from the constant inflow of $\delta\lambda$ type-H borrowers and $\delta(1 - \lambda)$ type-L borrowers. The second term captures the constant exit probability of borrowers in the pool. The final term is the flow rate of borrowers who receive a loan.

Observe that the first equation is independent of z_t. We can thus treat m_H as if it was at its steady state forever,

$$m_H = m_H^* = \frac{\delta\lambda}{\delta + \kappa} \tag{48}$$

We continue to denote the share of type-H borrowers by $x_t \equiv m_H/(m_H + m_L)$. Given (48), the law of motion of x_t can then be shown to be given by

$$\frac{\dot{x}}{x/\lambda} = \kappa(1 - x)(1 - z)\lambda - \kappa x(1 - \lambda) + \delta(\lambda - x) \tag{49}$$

[26]For simplicity, we focus on the case of active banks, $\theta_t = 1$, throughout this section.

The right hand side of (49) is identical to the one of (6) after substituting out κ_{Ht} and κ_{Lt} and setting $\theta_t = 1$. Thus, the only difference between the constant-pool-size model and the one in this section is that the speed in this one is altered by a factor x/λ, on the left hand side of (49). In particular, all results on steady states and their properties in Sections 3 carry over one-for-one to the model in this section.[27]

B.2 Welfare

The social planning problem now becomes

$$\max_{z_t \in [0,\bar{z}]} \int_0^\infty e^{-\rho t} \kappa \left\{ m_H^* r_H + (1 - z_t) m_L r_L - \tilde{c} z_t (m_L + m_H) \right\} dt$$

subject to the law of motion for m_L, (47). One can show that it has the exact same properties as the planning problem in Section 4. Relative to the privately optimal threshold $\overline{x} = 1 - c$, which corresponds to

$$\overline{m}_L = \frac{\lambda}{1 + \kappa \delta^{-1}} \frac{c}{1 - c}$$

there exists a socially optimal threshold $\overline{x}^* \equiv \frac{m_H^*}{m_H^* + \overline{m}_L^*}$ where $\overline{m}_L^*$ is determined by

$$\underbrace{(1 - \bar{z} + c\bar{z}) \frac{\rho \overline{m}_L^* + \alpha^s m_L^s}{\rho + \alpha^s} + c\bar{z} m_H^*}_{\text{Average social cost from lending to type-}L\text{ when screening}} = \underbrace{\frac{\rho \overline{m}_L^* + \alpha^p m_L^p}{\rho + \alpha^p}}_{\text{Average social cost from lending to type-}L\text{ when pooling}} \tag{50}$$

Here, we define the transition speeds for m_L under pooling and screening by $\alpha^p \equiv \kappa + \delta$ and $\alpha^s \equiv \kappa(1 - \bar{z}) + \delta$. The associated steady state values for m_L are given by $m_L^p = \frac{\delta(1-\lambda)}{\delta + \kappa}$ and $m_L^s = \frac{\delta(1-\lambda)}{\delta + \kappa(1-\bar{z})}$. Similar to Section 4, one can show here, too, that the social planner marginally prefers more pooling, that is,

$$\overline{m}_L^* > \overline{m}_L$$

The reason is identical to that in Section 4.

An especially simple welfare result is the comparison of steady state welfares across steady states. Here, the question is whether it is the case that welfare in the pooling steady state exceeds that in the screening steady state. In the context of this model, this is satisfied if

$$(1 - \bar{z} + c\bar{z}) m_L^s + c\bar{z} m_H^* > m_L^p$$

After some algebra, this simplifies to

$$1 + \delta^{-1} \kappa > (1 - \lambda) c^{-1} + \lambda \delta^{-1} \kappa \bar{z}$$

[27] What becomes harder to analyze with a non-constant pool size is slow thawing, since $\theta_t < 1$ affects both type-H and type-L borrowers, i.e. m_H is no longer constant at m_H^*.

which is necessarily the case given our Assumption 1: $c > 1 - \lambda$. Thus, Corollary 3 also carries over to this model.

C Credit bureaus

This appendix provides additional information on credit bureaus around the world. The on-line Appendix D provides the data that underlie this Appendix.

Credit bureaus, as opposed to credit registries, track potential borrowers and provide information about them to potential lenders.[28] When a potential borrower approaches a lender that is a member of a credit bureau, the lender can perform a *credit check* before making a loan, which involves getting a *credit report* from the bureau. Credit reports provide information on potential borrowers including existing credit and payment histories. In addition, many credit bureaus keep track of information about past credit checks and include this information on credit reports. Table 1 describes credit reports for credit bureaus in different countries around the world (underlying data sources are provided in on-line Appendix D).

Table 1: Data captured by credit bureaus

			Credit checks				
	Coverage? (consumers/firms)	Reporting required?	On report for $\leq$ months	Hard check labeled?	Who requested?	Purpose?	Rejections reported?
Advanced Economies							
Australia	Both		60	Yes	Yes	Yes	No
Canada	Both		72	Yes	Yes	No	No
European Union (AnaCredit)	Firms	By law	0	NA	No	No	No
France	Firms	By law	0	NA	No	No	No
Germany	Both	For access	12	Yes	No	No	No
Ireland	Both	By law	60	Yes	Yes	Yes	No
Italy	Both		6	Yes		Yes	No
Japan	Consumers	For access	6	Yes	Yes	Yes	No
Singapore	Both		24	Yes	Yes	Yes	No
South Korea	Both		0	NA	No	No	No
Taiwan	Both		3	Yes	Yes	Yes	No
United Kingdom	Both		24	Yes	Yes	Yes	No
United States	Both	Voluntary	24	Yes	Yes	Yes	No
Emerging Economies							
China	Both	By law	24	Yes	Yes	Yes	No
India	Both		24	Hard only	Yes	Yes	No
Malaysia	Both		12	Hard only	Yes	No	No

Blank cells are missing data.
Note: All information is from consumer credit reports and Bureau FAQs, except for EU and France, see Appendix for sources.

In most countries, a bank that conducts a credit check can generally observe past credit checks and whether the potential borrower subsequently did or did not receive a loan. The

[28]Credit registries are more widespread than credit bureaus, but registries only track the history of outstanding credit and/or loan payments and delinquencies. In our model, and probably in reality, outstanding loans do not assist banks in discriminating among borrowers who have recently been rejected. Credit registries seem to serve the purpose of providing information to assist a bank in setting loan terms, such as loan amount and interest rate based on payments-to-income ratio and/or pre-existing liens on collateral.

information in the bureaus tends to be available only to entities in the bureau's network, although some countries' bureaus sell the information to entities outside the credit market. In some countries like Japan and Germany, bureau members are required to report in exchange for access, but in other countries reporting is voluntary or only required by bureau members (second column of Table 1,). Most credit bureaus, like consumer bureaus in the US, delineate whether a credit check is *hard*, meaning associated with an application for credit, or *soft*, due to account review, marketing, or possibly hiring. Records of credit inquiries stay in credit report from 2 months in Taiwan to 24 months in the U.S. to 60 months in Ireland.

Importantly, however, none of the countries that we investigated have credit bureaus that report whether credit was denied or turned down by the potential borrower (final column of Table 1).[29] Further, credit bureaus generally contain only rudimentary information about the initiator of previous credit checks, such as whether they were banks, mortgage brokers, utilities, etc., and some in some countries, such as South Korea, France, and Germany, even this information is not recorded (fifth column). And only about half of the credit bureaus report the purpose of previous credit checks (sixth column), so that a credit card issuer for example does not know if a previous credit check was associated with an application for a credit card, mortgage, car loan or job.

A past credit check without a subsequent loan does not indicate that a given borrower failed a past lending standard. The borrower may have simply decided not to take the loan (decided not to buy that house or car, or decided to pick a different credit card). Importantly in practice, lenders can evaluate potential borrowers before verifying their information via a credit report and leave no trail of credit checks for rejected borrowers. That is, as is common in mortgage markets for example, banks can fully apply their lending standards on the basis of information reported by the borrower and additional information gathered by the bank, and only verify information with a credit report for borrowers that pass the lending standard. Thus many applications are not recorded.

As noted, our theoretical model suggests that bureaus do not track credit rejections because it is not incentive compatible for banks to report rejections. Statements from credit bureaus are consistent with this reasoning and suggest that credit bureaus are not able to enforce the reporting of soft information that it is not privately optimal to report. First, bureaus state that they want to avoid arbitrating arguments over rejections. Rejection is easy to hide (e.g. just offer unfavorable loan terms) and hard to verify (consistent with our assumptions). Second, bureaus store only verifiable information due to privacy and legal concerns. Credit checks are hard information, rejections are not. Every credit bureau lists data verification and correction measures on their website.

Finally, we re-emphasize that our model may apply even in situations in which credit bureaus accurately track borrowers if lending standards are applied to collateral instead of borrowers. That is, if tight lending standard evaluate and reject on the basis of collateral (e.g. an appraisal of a house), then credit bureaus do not address the externality of tight lending standards because they track borrowers not the assets they wish to fund.

[29] For example, Experian UK states "Here's what our role doesn't involve: - We aren't told which applications are successful or refused. - We don't know why you may have been refused credit."A possible exception is Experian Italy.

We conclude that mature credit markets in legal environments with low-cost enforcement mechanisms may exhibit various mechanisms for mitigating, but maybe not eliminating, the negative externality associated with tight lending standards.