Changing income risk across the US skill distribution: Evidence from a generalized Kalman filter*

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Abstract

For whom has earnings risk changed and why? To answer these questions, we develop a filtering method which estimates parameters of an income process and recovers persistent and temporary earnings for every individual at every point in time. Our estimation flexibly allows for first and second moments of shocks to depend upon observables as well as spells of zero earnings (i.e. unemployment) and easily integrates into theoretical models. We apply our filter to a unique linkage of 23.5m SSA-CPS records. We first demonstrate that our earnings-based filter successfully captures observable shocks in the SSA-CPS data such as job switching and layoffs. We then show that despite a decline in overall earnings risk since the 1980s, persistent earnings risk has risen for both employed and unemployed workers, while temporary earnings risk declined. Furthermore, the size of persistent earnings losses associated with full year unemployment has increased by 50%. Using geography, education, and occupation information in the SSA-CPS records, we refute hypotheses related to declining employment and wages among routine workers, declining employment prospects for low-skill workers, as well as geographically concentrated increases in risk around the Rust-Belt. Instead, we provide evidence that the rise in persistent earnings risk is a high skill worker phenomenon. Lastly, we find that rising persistent earnings risk while employed (unemployed) leads to welfare losses equivalent to 1.8% (0.7%) of lifetime consumption, and larger persistent earnings losses while unemployed leads to a 3.3% welfare loss.

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1 Introduction

For whom has persistent and temporary earnings risk changed over time, and why? Prior work has shown that persistent earnings shocks are often not well insured (e.g. Blundell, Pistaferri, and Preston (2008)), and hence understanding how and why the dispersion in persistent and temporary shocks (i.e. persistent and temporary risk) have evolved over time is critical for individual welfare and policy design. To answer these questions, we develop a filtering method which estimates parameters of an income process and recovers persistent and temporary earnings for every individual at every point in time. Using our method on a linked sample of Social Security Administration (SSA) and Current Population Survey Annual Social and Economic Supplement (CPS) earnings records we find that since the 1980s persistent earnings risk has increased while temporary earnings risk has declined. Exploiting the demographic information from our CPS sample we find that the increase in persistent earnings risk is a high-skill worker phenomenon. Finally, we find that there have been large welfare declines due to the increase in persistent earnings risk.

This paper makes three contributions. First, we show how the Kalman filter and an Expectation-Maximization (EM) algorithm can be used to estimate the parameters of a flexible, but easily interpretable model of income dynamics. Consistent with much of the income process literature, we write down a low-dimensional representation of individual earnings as the sum of latent persistent and temporary components. Using the EM algorithm, we derive updating equations for the parameters of the income process, which resemble weighted least squares regressions. These closed form updating equations allow the model to easily handle income process parameters that depend on a potentially large number of observables (e.g., age, employment status, education, occupation, and firm characteristics). We show that the parameters from our estimated income process are simple to integrate into quantitative models. Finally, using the Kalman Filter, we are able to recover estimates of persistent and temporary earnings for each individual in every period, which allow us to conduct further explorations of drivers of income dynamics by studying these filtered estimates directly.

An additional benefit of our Kalman filter and EM algorithm approach is that it naturally allows for the inclusion of individuals with very low or zero earnings. Motivated by economic theories of human capital depreciation during unemployment (e.g. Ljungqvist and Sargent (1998) among others), we posit a law of motion for persistent earnings when individuals have zero earnings (with a slight abuse of convention, we use ‘zero earnings’ and ‘unemployment’ in-

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1While we work with the canonical example where persistent earnings follow a persistent AR(1) process throughout, our approach can naturally extend to incorporate additional linear dynamics including individual fixed effects in earnings levels and growth rates as well as moving average components while remaining tractable.
terchangeably). During unemployment spells individuals receive shocks to persistent earnings with a different mean and variance than the shocks received during periods of employment. Despite individuals not having earnings information, the law of motion for persistent earnings is identified via earnings upon re-entry to work.

We estimate our filter on a linked sample of SSA-CPS earnings records from 1982-2016. The estimated parameters reveal that earnings are moderately persistent, and that the unemployed (i.e those with very low or zero earnings) face substantial earnings risk. We estimate that the standard deviation of shocks to persistent earnings to the unemployed is nearly double that of the employed, and that the unemployed face persistent earnings losses of nearly 15% per year of unemployment. Our estimation also yields a panel of persistent and temporary earnings shocks.

We then compare observed CPS events with our recovered individual-level shocks in order to validate our econometric specification of earnings. More specifically, we plot the distribution of estimated shocks to temporary and persistent earnings in response to job loss and job switching in the SSA-CPS data. Consistent with job ladder models of the labor market, we show that job switching and unemployment are associated with greater dispersion in temporary and persistent shocks relative to job stayers. In particular, layoffs associated with recalls (return to same employer) display much less persistent downside risk compared to non-recall (switch employers) layoffs. Likewise, job switchers are associated with much more dispersed persistent earnings shocks relative to job stayers, and we find that characteristics of the destination firms help to explain this variation. Workers who move up the ladder (switch to higher-paying firms) face considerably less persistent downside risk relative to workers who move down the ladder.

Our second contribution is to examine how earnings risk has varied over time and then use our linked survey data to shed light on a potential mechanism. To answer this question, we extend our filter to allow for age- and time-dependent variances of persistent and temporary earnings shocks. We additionally allow for the mean of persistent earnings shocks to vary over time. We document an upward trend in persistent earnings risk since the 1980s. Among the employed, the standard deviation of persistent earnings shocks rose by over 16%. Conversely, there has been an offsetting downward trend in temporary earnings risk over the same time period. In combination, overall earnings risk among the employed has a moderate downward trend (not statistically significant), consistent with recent findings (e.g. Bloom, Guvenen, Pistaferri, Sabelhaus, Salgado, and Song (2017), and McKinney and Abowd (2020) among others).

One unique feature of the way we filter the data is our ability to measure persistent earnings

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2In our baseline estimation, we define an individual to be unemployed if their annual earnings are below the equivalent of working full-time at the real federal minimum wage for 2-quarters of the year (approximately $8k in 2019 dollars).
trends among the unemployed. We find that persistent earnings losses have been accelerating for unemployed individuals. A year of unemployment translates to a 11% decline in persistent earnings in 1985, but by 2013, this rate of loss accelerates by over 50%, reaching -17% per annum. On top of this more rapid negative drift in persistent earnings, the magnitude of persistent earnings risk has increased among the unemployed by 15%.

By linking our administrative earnings database to survey responses in the CPS, we test a number of explanations of rising persistent earnings risk. Motivated by the job polarization literature, we consider three potential hypotheses relating the rise in persistent earnings risk with: (H1) declining employment prospects of low-skill workers, (H2) the decline of the rust belt (e.g. declining union protection, manufacturing employment, etc.), and (H3) reduced employment and wages in routine occupations. Our results provide strong evidence against H1, H2, and H3.3 We begin by documenting that the rise in persistent earnings risk for both the employed and unemployed is particularly pronounced among college educated workers. Likewise, the acceleration of earnings losses among the unemployed is also pronounced among college educated workers. These facts provide strong evidence against theories related to the declining employment prospects of low-skill workers (e.g. H1). We then show that the rise in persistent earnings risk between 1982 and 2016 is pervasive and fairly uniform across the vast majority of U.S. states. In particular, the well-documented deterioration of labor market conditions in the Rust-Belt is not driving the trends we document, allowing us to rule out hypotheses related to declining union protectionism, and the decline of manufacturing (e.g. H2). Lastly, we use the CPS occupation information to show that the rise in persistent earnings risk is uncorrelated with the routine task content of an occupation, which suggests declining labor demand for workers in routine occupations is not driving the trends in persistent earnings risk.

We argue instead that the increase in persistent earnings risk is a high skill worker phenomenon (H4). To test H4, we consider three measures of the degree to which an occupation is high skill: (1) the degree of non-routine cognitive task content as measured in Acemoglu and Autor (2011), (2) average years of completed education, and (3) average (log) earnings.4 All three measures show that workers employed in high skill occupations have faced a larger increase in persistent earnings risk both while employed and unemployed since the 1980s, as well as larger declines in persistent earnings during spells of unemployment.

One potential mechanism for why high skill workers are facing greater persistent risk is

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3 To clarify the interpretation of this result, note that while a substantial literature has documented trends related to H1, H2, and H3 which explain nontrivial differences in average earnings between groups, we do not find that the increase in earnings risk is particularly pronounced within these groups.

4 We measure average years of completed education and average (log) earnings by occupation at the beginning of our sample.
that they face greater exposure to the introduction of new technologies (e.g. Krueger (1993) and Deming and Noray (2020)). New technologies allow workers to increase their output but they also require workers to have new skills to perform their job. Hence, for workers with the sufficient skill to use the new technology their output increases, which increases their wages. For workers who do not have the skills to use the new technology, the demand for their services declines in their original occupation, which lowers their wages. To empirically test this mechanism, we link Burning Glass vacancy data to our SSA-CPS data in order to measure which occupations introduced computer and software usage in the workplace intensively as a proxy for the introduction of new technologies.\footnote{Burning Glass collects detailed information on the skills listed in vacancies. We follow recent work by Hershein and Kahn (2018) and Atalay, Phongthiengtham, Sotelo, and Tannenbaum (2020), which argues that the skill requirements in vacancy posting is informative on the technology of the firm posting the vacancy.} We find that occupations with high computer usage in 2010 were the occupations that saw the largest increases in persistent earnings risk.

Our third contribution in quantitative. We examine the welfare and macroeconomic effects of changing earnings risk over time. We show that our income process can be easily discretized to nest in a Bewley-Huggett-Aiyagari model of labor income risk with incomplete markets. In the model, agents differ by a permanent type which corresponds to their level of education (e.g. less than college graduate, college graduate, etc.) and receive shocks to labor income based upon our estimated income process. We use the model to measure the welfare losses from each component of the income process: (i) persistent and temporary earnings risk while employed, (ii) persistent earnings risk while unemployed, and (iii) downward drift in earnings while unemployed. We find that the increase in persistent earnings risk while employed (unemployed) generates a welfare losses equivalent to 1.8\% (0.7\%) of lifetime consumption. The acceleration of earnings losses while unemployed causes significant welfare losses equivalent to 3.3\% of lifetime consumption. The welfare losses are largest among the most highly educated, as these individuals have seen the largest increase in persistent earnings risk and the greatest acceleration of persistent earnings losses while unemployed.

\textbf{Literature.} This paper contributes to recent work that has examined how earnings risk has changed over time. While Sabelhaus and Song (2010), and Bloom, Guvenen, Pistaferri, Sabelhaus, Salgado, and Song (2017) find that earnings volatility has declined over time, Moffitt (2020) and the papers summarized therein find no long term trend in male earnings volatility over the past 30 years.\footnote{The papers summarized by Moffitt (2020) are part of a coordinated effort to use common sampling structures and definitions across datasets to document changes in male earnings volatility over time. The papers in this series include: Carr, Moffitt, and Wiemers (2020), McKinney and Abowd (2020), Moffitt and Zhang (2020), and Ziliak, Hokayem, and Bollinger (2020).} We in part reconcile these different findings by showing that the value...
of the minimum earnings criteria (the minimum value of earnings for a person to be included in the sample) plays a large role in shaping the evolution of earnings risk over time. In our baseline estimation, we find a minimal to slightly declining trend in earnings volatility, consistent with the work summarized by Moffitt (2020). We then show that if we lower the value of the minimum earnings criterion to the one set in Bloom et al. (2017), we find a significant decline in earnings risk over time. Further, we show that lowering the minimum earning threshold primarily impacts the trend in temporary earnings over time, making it have a larger decline, which generates a larger decline in overall earnings risk.

Relative to these papers, we make several contributions. First, we decompose earnings risk over time into its temporary and persistent components. This decomposition reveals that trends in overall earnings risk can mask heterogeneous trends in persistent and temporary earnings, as temporary earnings risk has declined since the 1980s while persistent risk has increased. Second, we show that earnings losses of the unemployed have accelerated since the 1980s. Third, we allow for arbitrary spells of unemployment (e.g. low/zero earnings) in the analysis, and allow the mean and variance of persistent shocks to differ during unemployment spells. Finally, we measure the welfare effects of changes in earnings risk, and show that despite the flat trend (or declining trend, depending on the minimum cutoff) there are large welfare implications due to the increase in persistent risk.

This paper also contributes to the literature, which has attempted to estimate persistent earnings at the individual level. While not estimating the income process explicitly, there are influential studies which assume that the persistent component of income is well approximated by a moving average of income and other simple moments. This allows the authors to recover, individual-by-individual, a persistent and temporary component of income. An early example is Gottschalk and Moffitt (1994) who measure persistent component of income using 7-year moving averages of earnings while labeling the residual as temporary earnings. More recently, Guvenen, Ozkan, and Song (2014), and Guvenen, Karahan, Ozkan, and Song (2015) use 1-year and 5-year changes in earnings to approximate temporary and persistent earnings risk, respectively. While intuitive these approaches mix temporary and persistent earnings shocks. As we show, changes in earnings over 1 year (or 5 year) horizons contain a mix of both temporary and persistent earnings shocks.

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7Prior work has examined trends in persistent and temporary earnings risk over time. See Moffitt and Zhang (2018) for a summary of this prior work.
8Prior work has incorporated spells of zero earnings using arc-percent changes. However, arc changes require positive earnings in at least one year. Our method allows for more general and arbitrary spells of zero earnings.
9Another approach to identify individual specific shocks is to use event studies, e.g. tax rebates or mass layoff episodes. Recent work by Baker and Yannelis (2015) and Gelman, Kariv, Shapiro, Silverman, and Tadelis (2015) identify temporary shocks from the 2013 government shutdown. Others have studied tax rebates (e.g. Kaplan and Violante (2014)) and mass layoffs (e.g. Saporta-Eksten (2013)) to isolate specific persistent and temporary income shocks. See Commault (2017) for a recent summary.
persistent shocks, which make interpretation difficult without more structure.

Our paper also contributes to the literature that estimates statistical representations of individual earnings.\(^\text{10}\) There are three ways prior studies have recovered income processes: (1) fitting statistical models of the earnings processes using GMM (e.g. Storesletten et al. (2004), Karahan and Ozkan (2013), Guvenen et al. (2014), and Guvenen et al. (2015)), (2) fitting structural models of earnings processes using simulated method of moments (e.g. Blundell et al. (2008), Guvenen and Smith (2014), and Madera (2016)), and (3) using Bayesian methods to estimate individual specific earnings processes (Geweke and Keane (2000), Jensen and Shore (2011), Nakata and Tonetti (2015), Gu and Koenker (2017), and Chatterjee, Morley, and Singh (2021)).\(^\text{11}\) The first two methods do not allow researchers to recover realized shocks person-by-person, while the Bayesian methods, which encompass the filter presented in this paper, do.\(^\text{12}\) The most closely related papers to ours include Nakata and Tonetti (2015) and Chatterjee et al. (2021). Nakata and Tonetti (2015) review the literature on income process estimation and discuss the small sample properties of Bayesian estimation techniques for earnings processes. Chatterjee et al. (2021) use Bayesian methods to estimate pass-through from income shocks to consumption in the PSID. We build on their work by positing a law of motion for persistent earnings among the unemployed, estimating our earnings process using a combination of the Kalman filter and the EM algorithm, and finally, we use our filtered estimates to understand the drivers of persistent and temporary earnings risk in both the cross-section and over time. As a practical contribution to the literature, we lay out our method in simple terms and show that it yields fast and accurate solutions even with many parameters and in extremely large samples (e.g. using Census servers, estimating the parameters and recovering the full panel of persistent and temporary earnings risk for 23.5 million observations takes roughly 3 hours).

Our paper also contributes to recent work which uses EM algorithms to estimate income processes. Arellano and Bonhomme (2016) and Arellano, Blundell, and Bonhomme (2017) also use an EM approach to estimate models of earnings dynamics using fully nonparametric methods (see also Bonhomme and Robin, 2010, for a related approach). Since we focus on a model of first and second moments only, our approach is more restrictive in some ways than theirs in terms of providing information about higher moments. However, this additional restriction allows us to use closed form expressions, rather than numerical approximations, to compute posterior distributions of latent state variables and confers two substantial advantages. First,

\(^{10}\)See Meghir and Pistaferri (2011) for a detailed discussion of the literature.

\(^{11}\)See also Browning et al. (2010) who estimate rich parametric models of individual income process heterogeneity using simulated minimum distance methods. Meghir and Pistaferri (2004) estimate models of persistent and transitory variances via GMM which also allow for ARCH effects.

\(^{12}\)For detailed discussion of prior work, we refer readers to Gu and Koenker (2017).
our approach is quite fast to implement, even on very large datasets, and does not require specification of additional hyperparameters. Second, since our updating formulas resemble GLS regressions, we can quite easily incorporate information from many sources of observed data (i.e., allow for fairly high dimensional parameter vectors) without suffering from the curse of dimensionality. This allows us to incorporate non-linearities that depend on observable events, such as periods of zero-unemployment or job-switching etc. In other words, our approach can generate non-linearities and still yield very attractive scaling properties by imposing additional structure.

More broadly, by allowing the parameters of our state space representation to be fairly arbitrary linear combinations of functions of observed data, our framework also offers a practical approach to studying the relationship between income risk and other firm/industry/regional-level outcomes. For instance, it is straightforward to extend our approach to allow for firm-level shocks to pass through to workers’ expected level of permanent income (e.g., Guiso et al., 2005; Lamadon et al., 2019; Chan et al., 2020) and income risk (Kogan et al., 2020). Likewise, consistent with our analysis in Section 4.4, one could consider the role of various shocks to local labor markets (e.g. Autor, Dorn, Hanson, and Song (2014)) in affecting workers’ persistent and transitory income dynamics.

2 Empirical framework

In this section, we describe our econometric framework to model income. We then discuss how our framework can be estimated to recover estimates of persistent and temporary earnings at the individual level as well as the parameters that govern the income process.

2.1 Basic setup

We begin with a panel dataset of income, $Y_{i,t}$, where $i \in \{1, \ldots, N\}$ indexes individuals and $t \in \{1, \ldots, T\}$ indexes years. We are interested in understanding the evolution of earnings net of predictable life-cycle components. Let $f(X_{i,t})$ be a function, which characterizes how observable components (e.g. age) influence earnings. Then define residual log earnings, denoted

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13The Guiso et al. (2005) approach to estimating the pass through of a permanent shock to value-added sidesteps the need to estimate a state-space model by employing a novel GMM/instrumental variable approach. However, these approaches utilize moment conditions which are specified in terms of functions of log income, which requires dropping or making other choices to handle years with zero earnings. Our approach offers an alternative which enables the researcher to incorporate these workers (who potentially experience some of the largest shocks) into the analysis.

14For ease of notation, we assume here that the panel is balanced, but the extension to an unbalanced panel setting is immediate.
finally, let \( y_{i,t} = \log(Y_{i,t}) - f(X_{i,t}) \). In the remainder of the paper, we focus on the factors that influence changes in residual log earnings \( y \), which we hereafter refer to as income.

The income process we define below depends upon whether or not an individual is employed in a given year. Let \( l_{i,t} = [l_{E,i,t} \ l_{U,i,t}]' \) be a vector which identifies an individual’s labor market status. Element \( l_{E,i,t} \) is an indicator variable that equals one when individual \( i \) is employed in year \( t \), and zero otherwise. Likewise, \( l_{U,i,t} \) equals one when individual \( i \) is unemployed, and zero otherwise. We define an individual to be employed (unemployed) when they have labor income above (below) a minimum earnings criteria \( \bar{y} \), i.e. \( y_{i,t} > \bar{y} \) \((y_{i,t} \leq \bar{y})\).

For employed individuals, we model the process for income \( y_{i,t} \) as the sum of persistent and temporary earnings. The persistent and temporary components of income \( y_{i,t} \) are not observed. Let \( z_{i,t} \) denote the unobserved persistent component of income and let \( \omega_{i,t} \) denote the temporary shock. When an individual is unemployed we set her income \( (y_{i,t}) \) to missing. Temporary shocks to an individual’s earnings \( (\omega_{i,t}) \) are drawn from a normal distribution with mean zero, and variance \( R(l_{i,t}; X_{i,t}) \), where the variance depends upon the individual’s labor market status as well as other observable variables \( X_{i,t} \). For simplicity, we will begin in Section 3.2 by abstracting away from other observables and assuming that \( R(l_{i,t}; X_{i,t}) = R_E \) for the employed, and \( R(l_{U,i,t}; X_{i,t}) = R_{U} \) for the unemployed. Since \( R_{U} \) is arbitrary and does not inform any parameter estimates, we normalize \( R_{U} = 1 \). An individual’s earnings evolves according to:

\[
y_{i,t} = \begin{cases} 
  z_{i,t} + \omega_{i,t} & \text{if } l_{E,i,t} = 1 \\
  & \text{if } l_{E,i,t} = 0
\end{cases}
\]

\[
V(\omega_{i,t}) = R(l_{E,i,t}; X_{it})
\]

We next discuss the law of motion for persistent earnings. We model the process for persistent earnings \( z_{i,t} \) as an autoregressive process subject to normally distributed innovations. Let \( F(X_{i,t+1}) \) denote the persistence of \( z_{i,t} \). To start, in Section 3.2, we abstract from allowing the persistence to depend upon other observables and assume that \( F(X_{i,t+1}) = F \). Let \( B(l_{i,t}; X_{i,t}) \), denote the drift of an individual’s persistent income in period \( t \). As before, we will also allow for other observables to affect \( B(\cdot) \) but will initially abstract from this dependence and discuss this extension later. We begin by assuming that \( B(l_{i,t}; X_{i,t}) = B_E \) when an individual is employed \( (l_{E,i,t} = 1, l_{U,i,t} = 0) \), and \( B(l_{i,t}; X_{i,t}) = B_{U} \) when an individual is unemployed \( (l_{E,i,t} = 0, l_{U,i,t} = 1) \).\(^{15}\) Therefore, persistent income continues to drift when unemployed. Finally, let \( \nu_{i,t} \) be an i.i.d shock to persistent income. The draw of \( \nu_{i,t} \) is from a normal distribution

\(^{15}\) This process for persistent earnings, where the drift differs by employment status, resembles a Ljungqvist and Sargent (1998) style human capital process.
with mean zero, and variance \( Q(l_{i,t}; X_{i,t}) \), where the variance depends upon the individual’s labor market status and potentially other observables. To start, we assume \( Q(l_{i,t}; X_{i,t}) = Q_E \) for the employed, and \( Q(l_{i,t}; X_{i,t}) = Q_U \) for the unemployed. Since income is set to missing during unemployment, contemporaneous observations of income during unemployment contain no information about \( z_{i,t} \). However, the mean and standard deviation of income at re-employment will inform the parameters governing the law of motion for persistent earnings \( z_{i,t} \) during unemployment. An individual’s persistent income evolves according to the following equation:

\[
\begin{align*}
    z_{i,t+1} &= F(X_{i,t+1}) z_{i,t} + B(l_{i,t+1}; X_{i,t+1}) + v_{i,t+1} \\
    \mathbb{V}(v_{i,t+1}) &= Q(l_{i,t+1}; X_{i,t}),
\end{align*}
\]

When an agent enters the labor market they draw their initial value of persistent earnings, denoted \( z_{i,0} \). We begin by assuming that initial permanent income is drawn from a distribution with mean zero and variance \( u_{z,i0} \). An agent’s initial persistent earnings reflects factors that impact earnings prior to labor market entry, e.g. education, prior experience, etc. In our more general setup, \( z_{i,0} \) may also have first and second moments which depend on a vector of observables \( X_{i0} \).

### 2.2 Recovering individual persistent and temporary earnings and shocks

In this section, we discuss how we can use the income process outlined in equations 1 and 2 to uncover estimates of persistent and temporary earnings at the individual level. For simplicity, we suppress any dependence on \( X_{i,t} \) in this subsection and discuss these extensions in the following section.

To recover estimates of persistent and temporary earnings at the individual level, we recast the income process from Section 2 into state-space form and use the Kalman Filter to recover current and lagged persistent earnings, which are modeled as the state vector. Let \( \zeta_{i,t} \) denote an individual \( i \)'s unobserved state in period \( t \), which is given by:

\[
\zeta_{i,t} = \begin{bmatrix} z_{i,t} \\
                        z_{i,t-1} \end{bmatrix}
\]

where \( z_{i,t} \) is persistent earnings of individual \( i \) in period \( t \). Based off of the law of motion in equation 2, the state vector \( \zeta_{i,t} \) evolves according to the following law of motion, which is
referred to as the state equation,

\[
\begin{bmatrix}
\zeta_{i,t} \\
\end{bmatrix} = \begin{bmatrix}
z_{i,t} \\
 z_{i,t-1}
\end{bmatrix} = \begin{bmatrix}
B(l_{i,t}) & F \\
0 & 1
\end{bmatrix}\begin{bmatrix}
\zeta_{i,t-1} \\

\end{bmatrix} + \begin{bmatrix}
\nu_{i,t} \\
0
\end{bmatrix}
\]

(3)

Using the definition of the state vector \( \zeta_{i,t} \) and the income process specified in equation 1, labor income evolves according to the following equation while employed, which is referred to as the measurement equation,

\[
y_{i,t} = H(l_{i,t})\zeta_{i,t} + I_{E,i,t} \omega_{i,t}
\]

(4)

where \( H(l_{i,t}) = \begin{bmatrix}I_{E,i,t} & 0\end{bmatrix} \) governs the relationship between the state vector \( \zeta_{i,t} \) and earnings \( y_{i,t} \) among individuals who are employed \( (l_{E,i,t} = 1) \).\(^{16}\)

With the income process modeled in state space form (equations 3 and 4) we can use the Kalman Smoother to recover an estimate of mean persistent earnings each period \( \{\hat{z}_{i,t}\}_{t=1}^{T} \) given the observed data. We refer to our estimated mean persistent earnings \( \hat{z}_{i,t} \) as simply persistent earnings, and we define the differenced persistent earnings \( \hat{z}_{i,t} - F\hat{z}_{i,t-1} \) as the persistent earnings shock. Our estimated temporary earnings shock is given by \( \hat{\omega}_{i,t} = y_{i,t} - \hat{z}_{i,t} \). When we recover subpopulation earnings volatility from our estimates of \( \hat{z}_{i,t} \), we adjust for uncertainty of the mean persistent shock \( \hat{z}_{i,t} \) using the law of total variance. In Appendix B we present the further details of our Kalman Filtering and EM estimation algorithms.

2.3 Incorporating additional observable variables and estimation

The specification in Section 2 allows for a rich model of earnings dynamics and is easily estimated via our approach while preserving tractability. For example, in Section 4.4 we are able to tractably estimate measures of persistent and temporary risk over time by geography, occupation, and education. In this section, we briefly describe how the use of the EM algorithm (Dempster, Laird, and Rubin (1977)) makes these extensions feasible, though we continue to relegate the majority of details to Appendix B.

Our flexible time-varying parameter model can incorporate a considerable amount of heterogeneity by allowing the specific parameters of the income process: drift of persistent shock \( B \), variance of the persistent shock \( Q \), persistence coefficient \( F \), variance of the temporary shock

\(^{16}\)When an agent is unemployed \( (l_{E,i,t} = 0) \), the value of the observation \( y_{i,t} \) provides no additional signal about latent earnings other than what can be inferred from other observables, so the Kalman filter will not directly use \( y_{i,t} \) to update its guess about \( z_{i,t} \).
(when the worker is employed), as well as the mean and variance of the initial condition $z_{i0}$ to depend on linear combinations of observable variables (e.g. time, age, education, occupation, etc.). More concretely, we assume that these parameters satisfy a linear-in-parameters structure: i.e., $B(l_{i,t}; X_{i,t}) = g(l_{i,t}; X_{i,t})' \Lambda_B$, where $g(\cdot)$ is a known function mapping the observed data to a set of basis functions and $\Lambda_B$ is an unknown set of parameters. In our simple example in the previous section, $g(l_{i,t}; X_{i,t}) = l_{i,t}$. Via such a specification, we have the ability to characterize rich interactions between observed variables and the dynamics of latent persistent income. We make an analogous assumption for the logarithm of the relevant variances.\textsuperscript{17}

As is shown in Appendix C, updating these parameters turns out to be straightforward, even in high dimensional settings. The EM algorithm oscillates between Expectation and Maximization steps.\textsuperscript{18} In the E step, conditional on an initial guess of the parameters, we can compute a set of sufficient statistics which are required to compute the expected value of the complete data log-likelihood function after integrating out uncertainty about the unobserved state variables (in this case, the history of persistent income). Then, given these sufficient statistics, the M step maximizes the expected value of the likelihood function, which yields an updated guess of the parameter vector. The parameters $F$ and $B$ are updated using a formula which resembles a weighted least squares regression. Then, conditional on mean parameters and a set of sufficient statistics computed from the Kalman filter, the objective function maximized to estimate variance parameters $Q$ and $R$ is strictly convex in the unknown parameters. As such, we can quickly maximize the objective function even with large numbers of unknown parameters.

**A comment on distributional assumptions** Before concluding the layout of the method, we briefly discuss the role of distributional assumptions in our method. To derive the likelihood function associated with the parameters governing our income process, we assume that the shocks to temporary and persistent earnings (for both the employed and unemployed) are normally distributed. As is well known, the likelihood function of the normal distribution yields a least-squares objective function, and, under correct specification, the true conditional expectation minimizes mean squared error. This feature of least squares implies that, under some simple regularity conditions, regardless of the distribution of shocks for temporary and persis-

\textsuperscript{17}We elect to make the log variance a function of observables, e.g. occupation, time, etc. Assuming that the logarithm of variances rather than variances themselves are affine in observables has the advantage of guaranteeing that variances are positive (and guarantees a well-defined log-likelihood). Further, Schmidt and Zhu (2021) show that such a multiplicative separability assumption for variances is implied by a location-scale version of the generalization of differences in differences proposed by Athey and Imbens (2006). See Appendix C for details.

\textsuperscript{18}Dempster et al. (1977) prove that the EM algorithm is guaranteed to increase the likelihood function at each iteration for general MLE problems. While, in principle, poor choices of starting values could lead to convergence to a local maximum, we have found our results to be generally quite insensitive to these choices in our applications and convergence to be quite rapid.
tent earnings, given the linear structure of persistent earnings in equation 2, the Kalman filter produces a consistent estimate of the parameters of our state space model (see, e.g., Hamilton (1994)). Thus, we need not assume shocks are normally distributed for our quasi-maximum likelihood procedure to yield consistent estimates of income process parameters.

Additionally, recent work has emphasized that log income changes are non-Gaussian, and exhibit negative skewness as well as excess kurtosis (e.g. Guvenen et al. (2015)). While the shocks to temporary and persistent earnings in our income process are normally distributed, our income process can produce skewness and kurtosis in log earnings changes by incorporating unemployment spells as well as making the shocks functions of other observables. By conditioning on these observables, we naturally estimate mixture distributions; therefore, integrating out these observables would yield non-Gaussian shock distributions even if shocks were Gaussian conditional on $l_{it}$ and $X_{i,t}$.

**Simple examples: conditioning on job switching/firm characteristics** In most administrative earnings datasets, it is often possible to identify whether or not the identity of a worker’s main employer has changed. Moreover, there is substantial evidence that first and especially second moments of income growth rates differ for job switchers versus job stayers. Since these switching indicators are always available in these datasets, it is easy to allow income process parameters to depend on whether or not workers switch employers, e.g., by having an additional dummy capturing whether an employed worker has recently switched jobs. We could also allow for $B$ and $Q$ to differ for the first versus subsequent years in a nonemployment spell. Likewise, a number of variables can often be computed at the employer or industry level (such as size, average pay per worker, employment/payroll growth rates) which could allow for a richer characterization of income dynamics. In this sense, our approach can be quite flexible.

### 3 Drivers of earnings risk

In this section, we first discuss our source of linked survey and administrative earnings records. Second, we present the benchmark parameter estimates of our income process. Third, we examine the observable labor market events associated with persistent and temporary income shocks identified by our filter.
3.1 Data

We estimate the parameters governing the income process using annual labor earnings from administrative earnings records, which have been linked to survey information. Our source of administrative earnings records is the Social Security Administration’s Detailed Earnings Records (DER). The DER is a database of job-level W-2 earnings from 1978 to 2016. We supplement the DER with survey responses from the Annual Social and Economic Supplement (ASEC) to the Current Population Survey (CPS), which asks a series of questions about labor income, receipt of transfer income, occupation, industry, as well as detailed demographic information. Using scrambled social security numbers (called Protected Identification Keys) the Census Bureau links individuals from the CPS ASEC to their earnings information in the DER.¹⁹

Our sample includes individuals who were in the ASEC in the years 1973, 1991, 1994, 1996-2016. Earnings records from the DER are included in all years that the individual is observed, and not only for the years for which an individual is in the CPS. We use earnings from the DER from 1982 through 2016, due to concerns about data quality prior to 1982.²⁰ For the majority of individuals in our sample, we have 2 years of detailed information on demographics, income (labor and non-labor income), and labor market information (e.g. weeks and hours worked, occupation, etc.), and a full time series of an individual’s labor income over their career from the DER.²¹

To study earnings dynamics we focus on a sample of individuals with a minimum degree of labor force attachment. To be in our estimation sample we require: (1) an individual satisfy a minimum earnings requirement in at least 5 (non-consecutive) years, (2) satisfy the minimum earnings criterion in at least 50% of years (inclusive) between the first and last year they satisfy the minimum earnings criterion, (3) be between the ages of 25 and 60, and (4) enter the sample by 2010. For conditions (1) and (2), we impose a minimum earnings criteria equal to the the real federal minimum wage for 40 hours a week for 26 weeks (on average, approximately $7,900 in 2019 dollars). These criteria allow for extremely long spells of zero earnings, potentially equal to half of the individual’s filtered panel of earnings. Condition (4) is included so that entrants to the sample in the final year are not selected towards individuals with the strongest labor force attachment (i.e. individuals with earnings above the minimum threshold for 5 consecutive years). Finally, to focus on labor market risk for workers, we additionally remove individuals

¹⁹See Wagner and Layne (2014) for more information on the assignment of PIKs to survey and administrative data. Note also that we use the CPS, March CPS, and CPS ASEC interchangeably to denote the CPS ASEC going forward.


²¹In our estimation, we use an individual’s sampling weight from the ASEC.
from the sample who have self-employment income that exceeds 50% of their total income (labor income plus self-employment income) in at least 5 years. This results in a sample of 1,157,000 individuals.\(^{22}\)

We use earnings information from the DER to study income risk. Our measure of income is the sum of Box 1 (total wages, tips, and bonuses) and Box 12 (earnings deferred to a 401(k) type account) earnings across all jobs the individual held during the year. We report earnings in 2019 dollars, where earnings are deflated by the CPI price index. To remove the impact of outliers, we winsorize real earnings at the 99.9th percentile in each year. Table 1 provides summary statistics for the individuals in our sample.

A benefit of our sample is that in addition to a panel of earnings, we have additional information on individuals (e.g. education level, occupation, etc.) from their CPS responses that is not typically found in administrative earnings records for the US. The information on education and occupation allows us to create finely partitioned groups to examine for whom earnings risk has changed over time in a manner that is not possible with U.S. administrative data sources alone.\(^{23}\)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Annual Earnings</td>
<td>$55,650</td>
</tr>
<tr>
<td>Age</td>
<td>40.78</td>
</tr>
<tr>
<td>Share Unemployed</td>
<td>6.9%</td>
</tr>
<tr>
<td>Share Less than College Degree</td>
<td>62.3%</td>
</tr>
<tr>
<td>Share College Degree Plus</td>
<td>29.4%</td>
</tr>
<tr>
<td>Share Education Not Reported</td>
<td>8.3%</td>
</tr>
<tr>
<td>Share Male</td>
<td>52.1%</td>
</tr>
<tr>
<td>Observations</td>
<td>23,500,000</td>
</tr>
<tr>
<td>Individuals</td>
<td>1,157,000</td>
</tr>
</tbody>
</table>

Note: Sample selection criteria in Section 3.1. Real Annual earnings is measured in 2019 dollars. The variable “share unemployed” is the share of individuals whose average earnings in a given year do not satisfy the minimum earnings criteria.


\(^{22}\)Due to Census Bureau disclosure rules, the number of individual is rounded to the nearest thousand.

\(^{23}\)Note that we do not use an individuals reported education if they are less than 25 when in the ASEC to avoid mis-classifying individuals who have not yet completed their education.
3.2 Estimated income process

In this section we discuss the estimated parameters governing the income process presented in Section 2. As a first step, we remove the predictable life-cycle component of log earnings as in Guvenen et al. (2014) (see Appendix A.1 for details). When removing the predictable life-cycle component of log earnings, we only use earnings that satisfy our minimum earnings requirement. Our income process allows for the type of shock an individual receives to depend upon their employment status (i.e. employed or unemployed). For the filtering exercise, we classify individuals with annual earnings below the minimum earnings requirement as unemployed, and individuals with annual earnings above the minimum earnings requirement as employed. To recover the parameters of the income process we use the EM algorithm presented in Appendix C.4.

Table 2 presents the parameter estimates. The parameter estimates reveal that persistent earnings shocks are moderately persistent ($F = 0.94$). Existing estimates of persistence of persistent earnings in mixture models range from .953 to .999 (e.g. Guvenen et al. (2014)). However, we make two departures from the literature, which make comparison difficult. First, the mean (e.g. drift) of the shock to persistent earnings depends upon an individuals employment status. Second, the prior literature drops observations with zero earnings (e.g. earnings below a minimum earnings criteria). Since our approach produces an estimate of persistent earnings even when an individual has zero labor earnings, these observations are taken into account when we estimate $F$. The estimation reveals unemployed individuals experience persistent earnings shocks with both a different mean (captured by the drift) and standard deviation compared to employed individuals. In particular, when an individual is unemployed, shocks to persistent earnings have a standard deviation that is nearly double the size of the standard deviation of shocks for employed individuals ($Q^{1/2}_U = .4171$, $Q^{1/2}_E = .2261$). Additionally, an unemployed individual’s persistent earnings decline on average by nearly 15% per year, while an employed individual’s persistent earnings increase by .4% per year. Hence, the estimation reveals that unemployed individuals experience shocks to persistent earnings from a distribution with a significantly lower mean and much greater amount of dispersion relative to employed individuals.

An additional benefit of our estimation procedure is that we obtain estimates of persistent earnings at the individual level in each period. In the next subsection, we examine how our measures of temporary and persistent earnings shocks align with observable shocks in the SSA-CPS data.
Table 2: Parameter estimates

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistence of Perm. Earnings</td>
<td>$F$</td>
<td>0.9401</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0002)</td>
</tr>
<tr>
<td>Std. Dev. of Shocks to Perm. Earnings (Emp.)</td>
<td>$Q_{E}^{1/2}$</td>
<td>0.2261</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0002)</td>
</tr>
<tr>
<td>Std. Dev. of Shocks to Perm. Earnings (Unemp.)</td>
<td>$Q_{U}^{1/2}$</td>
<td>0.4171</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0008)</td>
</tr>
<tr>
<td>Std. Dev. of Shocks to Temp. Earnings</td>
<td>$R^{1/2}$</td>
<td>0.1604</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0002)</td>
</tr>
<tr>
<td>Drift of Perm Earnings (Emp.)</td>
<td>$B_{E}$</td>
<td>0.0038</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0001)</td>
</tr>
<tr>
<td>Drift of Perm Earnings (Unemp.)</td>
<td>$B_{U}$</td>
<td>-0.1472</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0006)</td>
</tr>
<tr>
<td>Std. Dev. of Initial Draw of Perm Earnings</td>
<td>$z_{0}^{1/2}$</td>
<td>0.7002</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0008)</td>
</tr>
</tbody>
</table>

Note: Table presents parameter estimates from estimating income process in Section 2. Bootstrapped SE in parenthesis.


3.3 What causes persistent and temporary income shocks?

In this section, we demonstrate how our filtered estimates of temporary and persistent earnings shocks align with observable shocks in our sample and allow for examining the distribution of shocks that agents face. In the exercises below we present heatmaps of the distribution of persistent and temporary shocks around observable labor market events. These heatmaps plot the mass of individuals with a given combination of persistent and temporary shocks. In the subsections below we first examine how the distribution of shocks changes around job switching as well as by the type of job switch, and then examine how the distribution of shocks evolves around layoffs.

3.3.1 Job Switching

We first plot heatmaps of the shocks to persistent and temporary earnings for individuals who remain at the same primary employer (EIN) across two consecutive years (Panel (a) of Figure 1) and for individuals who switch their primary employer (Panel (b) of Figure 1). An individual’s primary employer in a given year is the defined as the EIN where the individual had the largest share of earnings in that year.

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persistent earnings. These shocks that individuals face likely reflect changes in hours, weeks worked, as well as promotions and raises, etc. Conversely, Panel (b) shows that among job switchers, the mass of individuals moves out of the middle of the distribution towards the corners of the heatmap where agents are subjected to large persistent and temporary shocks (either both positive or negative). To facilitate comparison, Panel (c) of Figure 1 plots the difference in shares by persistent and temporary shock, and demonstrates how job switching is associated with larger shocks (both positive and negative) to persistent and temporary earnings. Among non-switchers roughly 2.5% have the most extreme earnings outcomes (lowest persistent and temporary or highest persistent and temporary shocks). Among switchers, approximately 10 percent have the most extreme earnings outcomes, representing a 4 fold increase.

We further split job switchers by the type of job switch that they undergo. Using data from the Longitudinal Business Database (LBD), we measure average earnings per employer. We separate job switchers into those who move to an employer with average earnings per worker that are 25% lower (higher) than their previous employer. Panel (d) (panel (e)) of Figure 1 shows that when an individual moves to a lower (higher) paying employer, they become more likely to experience a large negative (positive) shock to persistent earnings. To facilitate comparison, panel (f) of Figure 1 plots the difference in shares by type of job switch, and demonstrates that moving to a higher paying firm is associated with positive shocks, especially to persistent earnings. We find that the mass of individuals with a persistent earnings loss (i.e. all mass left of the origin) is 33% higher among those who switch to firms that pay 25% less rather than 25% more.

The results of this section demonstrate that the estimates of temporary and persistent shocks align with observable labor market events. In particular, the estimates of this section align with job ladder models of the labor market where job switching is associated with larger shocks to temporary and persistent earnings relative to remaining at the same employer. Further, the direction of the job switch (e.g. moving to a higher or lower paying employer) aligns with the notion of climbing up and falling down the job ladder.

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25 See Jarmin and Miranda (2002) for details on the construction of the LBD.
Figure 1: Shocks to temporary and persistent earnings around job switching

(a) Non Job Switchers
(b) Job Switchers
(c) Difference: Job Switch
Share Non Switch Less Share Switch
(d) Job Switch & Move To Lower Paying Firm
(e) Job Switch & Move To Higher Paying Firm
(f) Difference: Job Switch Higher/Lower Paying Firm
Share Switch Higher Paying Less Share Switch Lower Paying

Note: Figure plots a heatmap of temporary and persistent shocks by observable labor market event. Higher (lower) paying firms are identified by moving to an employer with average earnings that are 25% above (below) an individuals current employer. The heatmaps in panels (a), (b), (d) and (e) all use the same scale that is presented in panel (a).
3.3.2 Layoff

In this section we examine how layoffs impact shocks to temporary and persistent earnings. While many papers have studied the average response of earnings to layoffs, we examine the heterogeneous behavior of temporary and persistent earnings following layoff. We document substantial heterogeneity in earnings following layoff and how it correlates with observable features of the layoff.

We identify layoffs using an individual’s CPS responses. In particular, we define an individual to have been laid off in year $t$ if they report having positive weeks on layoff in year $t$, and report zero weeks on layoff in year $t - 1$. We impose the requirement that an individual have zero weeks on layoff in year $t - 1$ so that we are able to accurately measure the inflow of individuals into unemployment. In Panel (a) of Figure 2, we plot the heatmap of persistent and temporary earnings shocks in year $t$ for individuals we identify as laid off in the CPS. The figure shows that there is a large mass of individuals in the bottom left hand corner of the heatmap, which indicates that a sizeable mass of laid off individuals have large negative persistent and temporary shocks. Roughly 29.6% of laid off individuals have negative persistent losses which exceed their temporary losses. Interestingly, there is also a large mass of individuals with small shocks, and even some individuals with positive shocks. We next further decompose the source of this heterogeneity in shocks around layoff.

We further decompose layoffs into individuals that are recalled to their pre-layoff employer and individuals who are not recalled. We identify an individual as having been recalled if their primary employer in the year before layoff is the same employer as their primary employer in the year after layoff.\(^{26}\) Individuals with a different primary employer in the year after layoff relative to the year before layoff are classified as non-recall. Panel (b) of Figure 2 plots the heatmap of persistent and temporary shocks among re-called individuals, while panel (c) of Figure 2 plots the heatmap for non-recalled individuals and Panel (d) illustrates the difference. Comparing panels (b) and (c) shows that individuals who are recalled to their prior employer exhibit much smaller negative shocks to temporary and persistent earnings relative to individuals who are not recalled after layoff, and are more likely to have a positive shock. Among those who are recalled, 19.9% have negative persistent losses which exceed their temporary losses. In contrast, among those who are not recalled, their earnings losses are much more persistent: 33.8% have negative persistent losses which exceed their temporary losses.

\(^{26}\)As in section 3.3.1, we define an individual’s primary employer in a given year as the EIN where they have the largest labor earnings.
Figure 2: Shocks to temporary and persistent earnings around layoff

(a)

(b)

Layoffs

Layoffs: Recalls

(c)

(d)

Note: Figure plots a heatmap of temporary and persistent shocks around layoff. Layoffs are identified using the CPS. An individual is defined to recalled if their primary employer in the year after layoff is the same employer as the year before layoff.

The results of this section showed that our estimates of temporary and persistent earnings align with observable shocks that individuals face in the labor market. As the filter is unaware of the shocks individuals face, we view these results as a validation of the method. We next utilize this method to examine how persistent and temporary earnings risk has changed over time and why.

4 The changing nature of earnings risk

Having validated our filter, we now turn to our main exercise which is to measure time trends in persistent and temporary earnings risk. Exploiting the rich set of demographic, geographic, and occupation information in the linked SSA-CPS data, we then characterize for whom earnings risk has changed.

To measure time trends in earnings risk, we incorporate the following features into our income process: (1) the standard deviation of shocks to temporary and persistent earnings are a function of year fixed effects and a quadratic in age which vary separately for both the employed and unemployed, (2) the standard deviation of initial draws of persistent earnings are also a function of year fixed effects and a quadratic in age, (3) the drift in persistent earnings is also a function of year fixed effects which vary separately for both the employed and unemployed. We include the age quadratic in order to control for changes in the age composition of the sample over time. We relegate a full exposition of this augmented income process to Appendix D. While we use data from 1982 to 2016 to estimate our model, the year fixed effects at the start and end of the sample are not well identified. For this reason, we bin together the first and last three years into a single year fixed effect at the start and end of our sample (e.g., 1982-1984, and 2014-2016.). In the graphs that follow, we omit these grouped year fixed effects, and we present the individual year fixed effects that cover the period 1985-2013.

Our filter also produces estimates of earnings risk over the life-cycle, complementing the work of Karahan and Ozkan (2013) and Blundell, Graber, and Mogstad (2015). Since life-cycle earnings are not the focus of this paper, we relegate these results to Figure 14 in Appendix G.2.

**Estimation Results.** We first illustrate earnings risk among the employed, ignoring the decomposition between persistent and temporary shocks. Figure 3 shows that earnings risk, as measured by the standard deviation of residual log earnings changes, exhibits a mild down-

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27Our results are robust to changing the number of years that are included in the “grouped” fixed effects at the start and end of the sample. The results are also robust to including the 3-year grouped fixed effects at the start and end of the sample.
ward trend. However, we show below that simply looking at trends of earnings changes masks offsetting trends in persistent and temporary earnings risk, as well a major acceleration of persistent earnings losses among the unemployed.

Figure 3: Earnings risk over time

Note: Figure presents the standard deviation of residualized log earnings over time (black, solid line) and a linear trend line (red, dashed line).

We next decompose changes in earnings risk into its persistent and temporary components. Figure 4 illustrates how persistent and temporary earnings risk have changed over time. Panel (a) shows that the standard deviation of persistent earnings among the employed has risen by 2 percentage points (or 9%) from 0.25 to 0.27 between 1985 and 2013. This moderate rise partially reflects the countercyclical nature of persistent earnings risk. The correlation of persistent earnings risk among the employed and real GDP growth is -0.40. The standard deviation of persistent earnings peaks in 2007 at 0.29 but then drops after the Great Recession to 0.26 in 2010. We fit a linear trend line to the plotted path of persistent earnings risk, and we find that there is positive, statistically significant increase in persistent earnings risk despite the recent cyclical variation.

28 In Section 4.1, we discuss how our estimate of earnings risk over time relates to the existing literature.
29 Note that in Figure 4 we present the standard deviation of shocks to income over time, holding the age component fixed at the value for individuals who are 25.
At the same time as the increase in persistent earnings risk, there has been a decline in temporary earnings risk. Panel (b) of Figure 4 plots the standard deviation of temporary earnings among the employed. Temporary earnings risk has a weak negative correlation with real GDP growth of -0.20. When we fit a trend line to temporary earnings between 1985 and 2013, we find a moderate downward trend in temporary earnings among the employed (statistically significant at the 5% level).

An additional feature of our income process is that it estimates the risk faced by unemployed individuals. Panel (c) of Figure 4 shows that the standard deviation of persistent earnings among the unemployed is much larger than that of the employed (roughly 40% larger during the sample period), and has risen by more than 15% between 1985 and 2013. How do we identify persistent earnings risk among the unemployed? The intuition is that persistent earnings risk among the unemployed is identified from the variance of earnings upon re-entry along with the subsequent path of earnings.

Panel (d) of Figure 4 plots the standard deviation of initial persistent earnings. We find no significant time trend. However, the time series exhibits a strong pro-cyclical pattern, with initial persistent earnings being compressed during downturns. We find a positive correlation between initial persistent earnings dispersion and real GDP growth of 0.50.

Panel (e) of Figure 4 plots the drift of persistent earnings among the employed. We find a significant downward time trend, and there is larger downward drift in earnings during recessions. We find a strong positive correlation between the drift of persistent earnings among the employed and real GDP growth of 0.76.

Panel (f) of Figure 4 plots the drift of persistent earnings among the unemployed. Between 1985 and 2013, the downward drift of persistent earnings among the unemployed fell by 6 percentage points (i.e. it became 57% more negative). We find a significant negative time trend in the series. What identifies this downward drift among the unemployed is the ratio of prior earnings to re-entry earnings. Our estimates imply that persistent earnings of the unemployed have begun to deteriorate nearly 50% faster between 1985 and 2013. We find a weak positive correlation of the unemployed’s drift with real GDP growth of 0.27.

In summary, our analysis of the time series yields four facts: (1) the standard deviation of persistent shocks while employed has increased; (2) the standard deviation of temporary shocks has declined; (3) the standard deviation of persistent shocks while unemployed has increased; (4) the decline in persistent earnings while unemployed has increased. We next discuss how our estimates of earnings risk align with the previous literature.
Figure 4: Persistent and temporary earnings risk over time

(a) Std. dev. persistent earnings (Emp.)
(b) Std. dev. temporary earnings
(c) Std. dev. persistent earnings (Unemp.)
(d) Std. dev. initial draw persistent earnings
(e) Drift persistent earnings (Emp.)
(f) Drift persistent earnings (Unemp.)

Note: Figure presents parameter estimates of the shocks to earnings over time (black, solid line) along with a linear trend line (red, dashed line). Dashed gray lines denote a 95% confidence interval.
4.1 Relation to prior measures of earnings risk over time.

In this section, we briefly compare our estimates of earnings risk to existing estimates in the literature. Recently there has been a debate on the extent to which earnings volatility has changed over time, with some papers finding a significant decline in earnings risk over time (e.g. Bloom et al. (2017)) and others reporting no trend (e.g. Moffitt (2020) and papers summarized therein). We find that the value of the minimum earnings cutoff significantly influences the trend in the standard deviation of residual log earnings changes (hereafter earnings risk), however, it does not impact the interpretation of the trends in persistent and temporary earnings risk over time.

In this section, we lower the minimum earnings cutoff to the equivalent of working part-time (20 hours per week) at the real federal minimum wage for one quarter, which corresponds to approximately $2k per year in 2019 dollars.\footnote{In our baseline estimation, the minimum earnings criteria is the equivalent of working full-time at the real federal minimum wage for two quarters.} This value of the minimum earnings criterion follows from Bloom et al. (2017). In this extension, we maintain our prior sample requirements that an individual must have earnings above the alternative minimum earnings criteria in at least 5 (non-consecutive) years and 50% of the years between the first and last year that the individual satisfied the minimum criteria.

We first examine how the lower minimum earnings cutoff impacts earnings risk over time by plotting the standard deviation of residual log earnings changes by year. Panel (a) Figure 5 plots the standard deviation of residual log earnings change by year using our baseline minimum earnings cutoff (black, solid line) along with the standard deviation of residual log earnings changes using the lower threshold (black, solid line with black circles).\footnote{For ease of interpretation, we have normalized the values in 1985 to be equal to 100. In Appendix G.3 we plot the time series of the level of the standard deviation of log earnings changes with the lower minimum earnings cutoff. With the lower minimum earnings cutoff, the level of earnings risk is substantially higher (approximately 40%) than in the baseline.} With the lower minimum earnings cutoff, there is a significant trend decline in earnings risk over time. In particular, with the lower minimum earnings cutoff the standard deviation of residual earnings changes decreases by 10% percent over the sample period, compared to a 5% percent change with the baseline minimum earnings cutoff.

We next examine how the trends in persistent and temporary earnings risk are impacted by the minimum earnings cutoff.\footnote{In Appendix G.3, we present all parameter estimates from the estimation with the lower minimum earnings criteria.} Panel (b) of Figure 5 plots the standard deviation of temporary and persistent shocks over time with the baseline minimum earnings cutoff (blue dashed and red dashed-dotted lines, respectively) along with the corresponding time series with the lower minimum earnings criteria (blue dashed and red dashed-dotted lines with blue and red circles, respectively).
Figure 5: Earnings risk over time by minimum earnings cutoff

(a) Residual log earnings changes
(b) Persistent and temporary earnings risk

Note: Figure examine how the trends in earnings risk have changed over time with the baseline minimum earnings cutoff and a lower minimum earnings cutoff. The baseline (lower) minimum earnings cutoff is set to the equivalent of working 40 (20) hours per week for 2 quarter (1 quarter) at the real federal minimum wage.


respectively). First, we observe a significantly larger decline in temporary earnings risk with the lower minimum earnings threshold compared to the baseline estimate. In particular, the standard deviation of temporary earnings shocks declines by nearly 20% over the sample period with the lower minimum earnings threshold compared to 10% in the baseline. Second, the trends for persistent earnings risk over time are largely unchanged by the minimum earnings cutoff. Under both estimations there is an increase in persistent earnings risk of nearly 10%.

Putting the results of this section together, we find that the value of the minimum earnings criteria plays a large role in the trend in earnings risk through its impact on temporary earnings. With a lower minimum earnings threshold, there is a larger decline in temporary earnings risk over time, which generates a decline in overall earnings risk. However, across both estimations we find a common rise in persistent earnings risk. In the next section, we further decompose the trend in earnings risk into its temporary and persistent components, and discuss why trends in temporary earnings risk play a large share in shaping the trend in overall earnings risk.

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4.2 Decomposing changes in earnings risk over time

In this section, we use the structure of the income process to further decompose changes in earnings risk over time into persistent and temporary components. Given the income process from Section 2, the variance of residual log income changes can be written as a function of persistent and temporary shocks as well as the variance of permanent earnings using the following formula,

\[ \text{var}(y_t - y_{t-1}) = (F - 1)^2 \text{var}(z_{t-1}) + Q_{E,t} + R_t + R_{t-1} \]  

(5)

Using equation 5, we next perform a counterfactual exercise of estimating the standard deviation of residual log earnings changes assuming that there was no change in either persistent or temporary income shocks since 1985.\(^{33}\) Panel (a) of Figure 6 plots the standard deviation of residual log income changes (black, solid line) along with a counterfactual estimate of the standard deviation of residual log income changes where persistent earnings shocks \((Q_{E,t})\) have been fixed at their 1985 values (red, dashed line). For ease of interpretation we have normalized the timeseries to 100 in the year 1985. The figure shows that without the increase in persistent earnings risk, the standard deviation of residual log income changes would have declined by 7.5 percent, slightly more than the nearly 5 percent decline observed in the data.

We next examine the counterfactual path when temporary earnings risk is held fixed. Panel (b) of Figure 6 plots the counterfactual estimate of the standard deviation of residual log income changes where temporary earnings shocks \((R_t \text{ and } R_{t-1})\) are fixed at their 1985 values (blue, dashed line). Without the decline in temporary earnings risk, residual log earnings changes would have increased by over 3 percent during the sample period, significantly higher from the nearly 5 percent decline observed in the data.

The results of this exercise demonstrate that variation in temporary earnings risk plays a larger role than changes in persistent earnings risk in shaping the evolution of overall earnings risk (e.g. the standard deviation of residual log earnings) over time. The rationale for the importance of temporary earnings risk is evident in equation 5, where there are two temporary earnings risk terms but only a single term for persistent earnings risk. Hence, declines in temporary earnings risk hold greater weight in equation 5 and lead to declines in overall earnings risk despite the increasing trend in persistent earnings risk.\(^{34}\) We next examine trends in earn-

\(^{33}\)For this decomposition, we include individuals whose earnings are above the minimum earnings criteria in both year \(t\) and year \(t-1\). We additionally, use the individual level estimates of temporary and persistent risk when performing the decomposition exercise. Additionally note that since \(F = 0.9424\), the first term in equation 5 plays a minor role in changes in the variance of log income.

\(^{34}\)In Appendix G.7, we show how the variance of earnings changes across different horizons can be used to identify the trends in temporary and persistent earnings risk.
ings risk among individuals in the CPS with observable shocks, and then explore the drivers of these time series changes using the labor market, demographic, geographic, and occupation data available in the CPS.

Figure 6: Decomposing changes in earnings risk

(a) Fixing persistent earnings shocks
(b) Fixing temporary earnings shocks

Note: Figure presents the standard deviation of residual log earnings changes over time (black, solid line), along with a counterfactual estimate where persistent earnings shocks are held fixed at their 1985 values (panel (a), red dashed line) and a counterfactual estimate where temporary earnings shocks are held fixed at their 1985 values (panel (b), blue dashed line). The counterfactual estimates are generated using equation 5.


4.3 Earnings risk among CPS unemployed

Our filtered estimates imply a significant worsening of persistent earnings losses during unemployment, and rising persistent earnings risk among both the employed and unemployed. In Section 3.3 we demonstrated that our filter is highly correlated with observable shocks such as job loss. However, one potential concern is that our time series patterns may be attributable to individuals with no observable labor market shocks (i.e. no ‘true’ labor market risk), and thus the time trends reflect misspecification and/or life events that are unrelated to labor market risk. In this section, we validate our finding of rising persistent earnings risk among the unemployed by showing that self-reported CPS job losers exhibit rising persistent earnings risk.

In what follows below we define an individual to be CPS unemployed in year \( t \) if they report being on layoff for at least one week in year \( t \), or report not working in year \( t \). With this defini-
tion, we examine how the standard deviation of shocks changed over time for individuals we classify as CPS unemployed, and how these trends compare to our full sample of individuals.

In Figure 7, we compare the time series of earnings risk among our full sample of individuals to the individuals we classify as CPS unemployed. Panel (a) of Figure 7 compares the time series of the standard deviation of shocks to persistent earnings among individuals in the full sample who we classify as unemployed (black, solid line) to individuals who we classify as CPS unemployed (red, dashed line). The CPS unemployed have a standard deviation of shocks to persistent earnings that is smaller than the full sample of individuals, however, the time series reveals a steadily increasing trend from the early 1990s, similar to the trend we observe in the full sample of unemployed individuals (correlation = 0.887). Hence, under the significantly less stringent definition of CPS unemployment, we see a similar trend increase in the standard deviation of shocks to persistent earnings among the unemployed.

Next, we examine the mean change in persistent earnings among the full sample of unemployed individuals and individuals who report being unemployed in the CPS. Panel (b) of Figure 7 compares the average shock to persistent earnings among individuals in the full sample who we classify as unemployed (black, solid line) to individuals who we classify as CPS unemployed (red, dashed line). The figure shows that those who self-report CPS unemployment have experienced a similar acceleration of persistent earnings losses during unemployment (correlation = 0.794).

In summary, we show that individuals with observable labor market risk in the CPS (i.e. those who self-report positive weeks on layoff) are precisely the individuals who have rising persistent earnings risk implied by our filter. We view this as a demonstration of our filter’s ability to capture economically meaningful measure of labor market risk, not just cross-sectionally, but over time. We next examine for whom earnings risk is changing as a means to better understand the source of the changing nature of persistent earnings risk.

4.4 For whom is earnings volatility changing?

In this section, we test several hypotheses for rising persistent earnings risk. We rule out hypotheses related to declining employment prospects of low skill workers and regional theories of persistent earnings losses, including the decline of the Rust-Belt. We additionally rule out theories based upon routine employment using information on the skill content in occupations (e.g. Acemoglu and Autor (2011)). Instead, we show that the rise in persistent earnings risk is a high skill workers phenomenon, and provide supportive evidence that the rise in persistent earnings is.

35 An individual is defined as full sample unemployed in year $t$ if they have earnings below the minimum earnings cutoff in year $t$. 

29
Figure 7: Comparison to shocks among CPS Unemployed

(a) Std. dev. shock to persistent earnings  (b) Drift to persistent earnings

Note: Figure compares the time series of the standard deviation and mean of filtered shocks to persistent and temporary earnings among individuals in the full estimation sample (black, solid line) and individuals who we classify as CPS unemployed. See Section 4.3 for definitions of CPS Unemployed as well as Full Sample Unemployed.

Risk among high skill workers is due to exposure to the introduction of new technologies. We organize this section around tests of four hypotheses:

**Hypothesis 1:** Low human capital workers have experienced declining employment prospects, which has caused a rise in persistent earnings risk.

**Hypothesis 2:** The declining manufacturing sector along with declining union protection has led to a geographically concentrated rise in persistent earnings risk in the Rust Belt.

**Hypothesis 3:** Declining wages and employment in routine occupations has caused rising persistent earnings risk.

**Hypothesis 4:** High skill workers have experienced the largest increase in persistent earnings risk due to greater exposure to the introduction of new technologies.

To rule out concerns regarding parameter restrictions among these disparate subgroups, we re-estimate our income process for each subgroup (e.g. by education, occupation, gender, state) in all results shown in this section.
4.4.1 Education

We start by testing the first hypothesis that rising persistent earnings risk is being driven by low-human capital workers. We partition individuals into two groups based upon their first recorded level of education in the CPS: those who have less than a college degree, and those who have a college degree or higher. We only use an individual’s reported level of education if they are over the age of 25 at the time of the CPS survey. We then estimate the parameters of our income process separately for each group.\textsuperscript{36}

In Figure 8, we plot the estimated earnings process parameters for the two education groups. Panel (a) shows that employed individuals with less than a college degree (black, solid line) exhibit no trend in persistent earnings risk. On the other hand, there has been a trend increase in persistent earnings risk among employed individuals with a college degree. The trend for college educated workers is very similar to the overall time trend in persistent earnings risk in Figure 4, implying that college educated workers are primarily responsible for rising persistent earnings risk. Panel (b) shows that there is an offsetting reduction in temporary earnings risk among employed college educated workers. Panel (c) plots persistent earnings risk among the unemployed. Unemployed individuals with a college degree face over 50\% greater earnings risk relative to those without a college degree. Moreover, the trend increase in persistent earnings risk among the unemployed is driven almost entirely by college educated workers. Conversely, panels (d) and (e) show that the initial persistent earnings draw and drift while employed do not statistically differ across education groups. Lastly, panel (f) shows that the larger declines in persistent earnings during unemployment are primarily driven by college educated workers.

Taken together, the panels of Figure 8 make a strong argument that individuals with a college degree or higher are driving both (i) the majority of time trends in persistent earnings risk among the employed and unemployed, and (ii) the severe deterioration of persistent earnings losses among the unemployed. We therefore rule out our first hypothesis that rising persistent earnings risk is attributable to low-skill, non-college educated workers.

We further explore differences across genders. In Figure 17 in Appendix G.4, we show that the trends in earnings risk documented above have occurred for both men and women. Therefore, our results are unlikely to be driven by declining employment prospects of low-skill men (e.g. Binder and Bound (2019)).

\textsuperscript{36}Note that we residualize earnings separately for each group.
Figure 8: Changes in earnings risk over time by education

(a) Std. dev. persistent earnings (Emp.)
(b) Std. dev. temporary earnings
(c) Std. dev. persistent earnings (Unemp.)
(d) Std. dev. initial draw persistent earnings
(e) Drift persistent earnings (Emp.)
(f) Drift persistent earnings (Unemp.)

Note: Figure presents parameter estimates of the shocks to earnings over time by level of the education as reported in the CPS. The black solid line denotes individuals with less than a college degree, and the red dashed line represents individuals with a college degree or higher.

4.4.2 Geography

Next, we explore whether persistent earnings risk varies geographically. Our analysis tests theories of regional persistent earnings risk, such as declining manufacturing employment and weakened unions in the Rust-Belt. Since individuals that initially live in hard-hit communities may move states, we split individuals across states using the earliest self-reported state of residence in the CPS survey. We separately re-estimate our income process across states in order to alleviate concerns regarding parameter restrictions. Since our time period is 1985 to 2013 which includes 29 years of observations (a prime number), there is no way to evenly divide the time period. To resolve this issue, we split the time period into 7-year windows (1985-1991, 1992-1998, 1999-2005, and 2006-2013), with the extra year going in the final time interval. We show results that compare the first time window 1985-1991 to the last time window, 2006-2013. Our results are not sensitive to this choice of time windows.

Figure 9 plots the change in our estimated earnings process parameters by state between 1985-1991 (x-axis) to 2006-2013 (y-axis), along with a 45-degree line (red dashed line). Each circle represents a state, with the size of the circle corresponding to the relative population of the state in our sample. Panel (a) plots the change in persistent earnings risk, combined among both the employed and unemployed. In the vast majority of states, we estimate increasing earnings risk over time. Panel (b) plots the change in the drift of persistent earnings among the unemployed. There is considerably more variation across states, but fewer than half of the states had slower persistent earnings deterioration among the unemployed between 2006-2013 relative to 1985-1991. These findings suggest that regional factors are unlikely to explain rising persistent earnings risk. All states followed a fairly similar path of rising persistent earnings risk between 1985 and 2013, with no clear outliers emerging in our analysis. Thus, state-level theories of rising persistent earnings risk such as declining manufacturing employment and/or declining rates of unionization in the Rust-Belt are unlikely to rationalize our results.37

4.4.3 Occupations

In this section, we test our third and fourth hypotheses that rising persistent earnings risk is related to the skill-content of occupations. Using an individual’s earliest reported occupation in the CPS, we classify individuals into one of 334 time-consistent occupation codes developed by Autor and Dorn (2013).38 We then separately estimate the parameters of our income process

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37 In Appendix G.5 we show that changes in persistent earnings risk are largely uncorrelated with changes in state level union coverage and manufacturing employment.

38 We thank Bryan Seegmiller for creating the mapping from CPS occupation codes to the Autor and Dorn (2013) occupation codes.
Figure 9: Changes in persistent earnings risk by U.S. state

(a) Shocks to persistent earnings (comb.) over time

(b) Drift of persistent earnings (unemp.) over time

Note: Figure presents parameter estimates from estimating the income process by state. The x-axis shows the parameter estimate for the 1985-1991 time period, while the y-axis shows the parameter estimate for the 2006-2013 time period. Panel (a) compares the shock to persistent earnings (combined for employed and unemployed) over time, while panel (b) shows the drift of persistent earnings while unemployed over time.


In Figure 10, we plot how the volatility of earnings has changed between 1985-1991 (x-axis) and 2006-2013 (y-axis) for each occupation, along with a 45-degree line (red dashed line). Each circle represents an occupation, with the size of the circle representing the relative employment share of the occupation in our sample. Panel (a) shows that the vast majority of occupations experienced rising persistent earnings risk. However, there remains considerable variation across occupations. Similarly, panel (b) shows that persistent earnings losses among the unemployed accelerated in the majority of occupations, but a significant share of occupations exhibited slower persistent earnings deterioration.

We next exploit variation across occupations to test the third and fourth hypotheses that rising persistent earnings risk is related to the skill-content of occupations. Let $X_o$ be a measure of the skill-content of occupation $o$ (e.g. measure of routine task content, non-routine cognitive task content, etc.). Let $\Delta Y_{o,j} = Y_{o,j} - Y_{o,(1985-1991)}$ denote the change in parameter $Y$ (e.g. the standard deviations of shocks to persistent earnings among employed etc.) for occupation $o$ between time period $j$ and 1985 – 1991.\(^{39}\) Let $\gamma_j$ denote a set of year window fixed effects. The

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specification we use is of the form:

$$\Delta Y_{o,j} = \alpha + \eta X_o + \gamma_j + \epsilon_{o,j}$$  \hspace{1cm} (6)$$

The parameter of interest is $\eta$ which reports if occupations with a greater amount of task $X$ experienced a larger increase in the parameter $Y$ over the sample period. Hence, if $\eta > 0$ then an occupation with a greater amount of skill content $X$ experienced a larger change in earnings risk over the sample period.

Routine Occupations. We first split occupations by their degree of routine skill content as measured by Acemoglu and Autor (2011) using O*NET data.\footnote{Acemoglu and Autor (2011) provide a measure of the routine manual as well as routine cognitive task content of an occupation. We combine their estimates into a single measure of the routine task content of an occupation by averaging the two measures. As in Acemoglu and Autor (2011) we normalize the index to be mean zero and have unit variance. Results with routine manual and routine cognitive skills as the main independent variable are available upon request.} Table 3 presents the results of estimation equation 6 where the independent variable is the measure of the routine task content of an occupation. Column (1) shows that higher routine skill content in an occupation is not correlated with the change in the standard deviation of shocks to persistent earnings while
Table 3: Routine Occupations and Changes in Earnings Risk

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<tbody>
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<td>Routine</td>
<td>-0.000800</td>
<td>-0.00165</td>
<td>-0.000611</td>
<td>0.00314***</td>
<td>0.000919*</td>
<td>0.00418**</td>
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<td>(0.000975)</td>
<td>(0.00225)</td>
<td>(0.000981)</td>
<td>(0.00109)</td>
<td>(0.000506)</td>
<td>(0.00172)</td>
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Year Fixed Effects: Yes, Yes, Yes, Yes, Yes, Yes
R-Sq.: 0.124, 0.054, 0.123, 0.101, 0.212, 0.109
No. Obs. (Occ.): 1000, 1000, 1000, 1000, 1000, 1000

Notes: Table presents results from estimating equation 6 where the independent variable is the measure of routine task content of an occupation from Acemoglu and Autor (2011). The measure of routine task content is normalized to have mean zero and unit variance. Clustered SE in parenthesis, where the clustering is performed at the occupation level. ***p < 0.01, **p < 0.05, *p < 0.1. Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

employed. Similarly, column (2) shows that routine occupations have not experienced greater dispersion in shocks to persistent earnings while unemployed. Finally, column (6) shows that occupations with greater routine task content have experienced smaller declines in persistent earnings during unemployment spells over time relative to non-routine occupations. Putting these results together, we conclude that workers in routine occupations are not driving the increase in persistent earnings risk over time.

High Skill Workers. In this section, we examine the hypothesis that the rise in persistent earnings risk has occurred among high skill workers. We test this high skill workers hypotheses by using three measures of the skill intensity of an occupation. We first split occupation by their degree of “Non-Routine Cognitive Analysis” skills (henceforth, non-routine cognitive skills) as measured by Acemoglu and Autor (2011) using O*NET data.\(^{41}\) We additionally measure the degree to which an occupation is high-skill using its mean years of completed education and mean (log) earnings.\(^{42}\) To ease the comparison across measures, we normalize each measure to have mean zero and unit standard deviation.

\(^{41}\)This measure is created from the O*NET tasks measures on the importance of: (1) Analyzing data/information, (2) Thinking creatively, and (3) Interpreting information for others. The index is constructed to be mean zero and have unit variance. The measure is derived from O*NET vintage 14.0, which contains data collected between 2002-2009.

\(^{42}\)We measure average years of completed education and average (log) earnings in the years 1985-1991.
Figure 11: Changes in earnings risk by non-routine cognitive analysis skills

(a) Chg. std. dev. persistent earnings (Emp.)
(b) Chg. std. dev. temporary earnings
(c) Chg. std. dev. persistent earnings (Unemp.)
(d) Chg. std. dev. persistent earnings (Comb.)
(e) Chg. drift persistent earnings (Emp.)
(f) Chg. drift persistent earnings (Unemp.)

Note: Figure presents a graphical representation of the regression in equation 6, where the measure of task content is non-routine cognitive analytic skills as measured in Acemoglu and Autor (2011).
Figure 11 presents a graphical representation of the regression in equation 6 where the independent variable is the non-routine cognitive skill content of an occupation. Panel (a) plots the change in persistent earnings risk among the employed between 1985-1991 and 2006-2013 on the y-axis, and the workers’ ventile of cognitive skill requirements on the x-axis. Over the time interval, the standard deviation of persistent earnings risk among those in the bottom ventile of cognitive skill requirements increased by 0.005 between 1985-1991 and 2006-2013. For those in the top ventile, their persistent earning risk increased by 0.035 over the same time period, a factor of 7 larger than the bottom ventile. Panel (b) shows that there has been an offsetting decline in temporary earnings among those with the highest cognitive skill requirements. Panel (c) illustrates a strong correlation between cognitive skill requirements and the standard deviation of persistent earnings among the unemployed. Finally, Panel (f) shows that the acceleration of persistent earnings losses among the unemployed is particularly pronounced among those with the highest cognitive skill requirements. Between 1985-1991 and 2006-2013, those in the bottom ventile of cognitive skills had no change in their persistent earnings drift while unemployed. However, persistent earnings among those in the top ventile of cognitive skills deteriorated 6 percentage points per annum faster in the 2006-2013 time period.

Table 4 presents regression results where the independent variable is the non-routine cognitive skill content of an occupation. Consistent with the graphical evidence, occupations with greater non-routine cognitive skill content have experienced: (1) a larger increase in the standard deviation of persistent earnings shocks while employed and unemployed as well as (2) a larger decline in persistent earnings while unemployed. In Appendix G.6 we show that we obtain similar results when using years of completed education as well as average (log) earnings as our measure of the high skill content of an occupation. Putting these results together, we conclude that the increase in persistent earnings risk since the 1980s has been a high skill workers phenomenon.
Table 4: Non-routine cognitive skills and changes in earnings risk

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<td>Non-Routine Cognitive.</td>
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<td>0.0175***</td>
<td>0.00574***</td>
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<td>(0.00412)</td>
<td>(0.00166)</td>
<td>(0.00189)</td>
<td>(0.00109)</td>
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Year FE Yes Yes Yes Yes Yes Yes
R-Sq. 0.186 0.097 0.180 0.095 0.238 0.194
No. Obs. (Occ.) 1000 1000 1000 1000 1000 1000

Note: Table presents results from estimating equation 6 where the independent variable is Non-Routine Cognitive Analysis as measured in Acemoglu and Autor (2011). The variable Non-Routine Cognitive Analysis is constructed to have mean zero and unit variance. Clustered standard errors in parenthesis where the clustering is performed at the occupation level. ***p < 0.01, **p < 0.05, *p < 0.1.


Table 5: Computer skills and changes in earnings risk

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<tbody>
<tr>
<td>Computer Skills</td>
<td>0.00685***</td>
<td>0.00945**</td>
<td>0.00717***</td>
<td>0.00425***</td>
<td>0.00154</td>
<td>-0.0145***</td>
</tr>
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<td></td>
<td>(0.00137)</td>
<td>(0.00382)</td>
<td>(0.00132)</td>
<td>(0.00140)</td>
<td>(0.00118)</td>
<td>(0.00296)</td>
</tr>
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Year Fixed Effects Yes Yes Yes Yes Yes Yes
R-Sq. 0.222 0.068 0.229 0.094 0.219 0.176
No. Obs. (Occ.) 1000 1000 1000 1000 1000 1000

Note: Table presents results from estimating equation 6 where the independent variable is computer skills as measured in Braxton and Taska (2020). Computer skills are measured for each occupation in 2010, and have been normalized to have mean zero and unit variance. Clustered standard errors in parenthesis where the clustering is performed at the occupation level. ***p < 0.01, **p < 0.05, *p < 0.1.

Finally, we examine a potential mechanism for why high skill workers experience a larger increase in persistent earnings risk. Previous work has shown that high skill workers are more exposed to the introduction of new technologies (e.g. Krueger (1993) and Deming and Noray (2020)).\footnote{Krueger (1993) provides evidence that the computer revolution of the workplace was more pronounced among high skill workers. Recently, Deming and Noray (2020) show that there are greater changes in the skill requirements (a proxy for technologies used by firms) of jobs over time for workers with technology intensive college majors, e.g. science, technology, and business.} New technologies allow workers to increase their output but they also require workers to have new skills to perform their job. Hence, for workers with the sufficient skill to use the new technology their output increases, which increases their wages. For workers who do not have the skills to use the new technology, the demand for their services declines in their original occupation (or the worker has to move to another occupation where their skills are still employable), which lowers the wages of the worker.\footnote{Consistent with this mechanism Braxton and Taska (2020) shows that workers displaced from occupations that have experienced a greater increase in computer and software requirements suffer larger earnings losses. Additionally, Kogan, Papanikolaou, Schmidt, and Song (2020) find that within industry increases in the rate of innovation are associated with substantial increases in earnings risk, particularly for high income workers.}

We test this mechanism by exploiting variation in the introduction of new technologies across occupations. In particular, we measure changes in persistent earnings risk among occupations that adopted greater computer and software skill requirements by 2010. Since computers were not prevalent in the workplace during the 1980s, individuals in these occupations faced a greater degree of new technology introduction.\footnote{From Card and DiNardo (2002), "... many observers date the beginning of the computer revolution to the introduction of the IBM-PC in 1981. This was followed by the IBM-XT (the first PC with built-in disk storage) in 1982, and the IBM-AT in 1984."} We measure computer usage in an occupation using detailed skill requirements from online vacancies collected by Burning Glass Technologies.\footnote{We follow recent papers by Hershbein and Kahn (2018) and Atalay et al. (2020), which argue that the skill requirements in vacancies are informative on the technology of the firm posting the vacancy.} As in Hershbein and Kahn (2018) and Braxton and Taska (2020) we measure the degree of computer usage in an occupation by measuring the share of vacancies that list a computer or software requirement.\footnote{See Hershbein and Kahn (2018) and Braxton and Taska (2020) for more details on the Burning Glass database.} To facilitate comparison to our other measures of occupation skill content, we normalize the measure of computer skills to have mean zero and standard deviation equal to one.

Table 5 presents the results of estimating equation 6 where the independent variable is computer skills in 2010. We find that computer usage is a strong predictor of the increase in persistent earnings risk. In particular, Table 5 shows that occupations with a greater amount of computer skill requirements have experienced: (1) a larger increase in persistent earnings risk while employed (Column (1)), (2) a larger increase in persistent earnings risk while unemployed.
(Column (2)), and (3) a greater accelerating of persistent earnings declines while unemployed (Column (6)).

4.5 Taking stock: earnings risk over time

In this section we have shown that since the 1980s there has been a steady increase in persistent earnings risk and a trend decline in temporary earnings risk. Additionally, the average size of persistent earnings losses while unemployed have increased by nearly 50%. We documented that this increase in persistent earnings risk has been a high skill workers phenomenon. In the next section, we quantify the welfare implications of the observed changes in earnings risk over time.

5 Welfare effects of changing earnings risk

In this section, we use a finite life-cycle Bewley-Huggett-Aiyagari style model to examine the welfare effects of changes to earnings volatility between 1985 and 2013.

5.1 Steady State Model

In this section, we introduce a steady state version of a finite life cycle Bewley-Huggett-Aiyagari style model. We assume there are $T \geq 2$ overlapping generations of agents, and let $t \in \{1, ..., T\}$ denote the age of an agent. Agents exit the model exogenously at age $T$, and there is no retirement. Let $i$ denote an agent’s type, and in the estimation that follows, we assume types map to educational attainment. An individual’s type is fixed indefinitely, and let $\pi_i \in [0, 1]$ denote the share of agents that are type $i$.

Agents are heterogeneous along several dimensions. Let $e \in \{E, U\}$ denote the employment status of an agent, where $e = E$ denotes employed, and $e = U$ denotes unemployed. Let $b \in \mathbb{R}$ denote the net asset position of an agent. When $b > 0$ the agent has savings, and when $b < 0$ the agent is borrowing. The agent’s asset choice is constrained by a borrowing limit $b$. Agents save and borrow at the risk free rate, denoted $r_f$. Let $z \in \mathbb{R}$ denote an agent’s persistent earnings. Let $\epsilon \in \mathbb{R}$ denote an agent’s temporary shock to earnings.

At the start of each period, the agent observes their employment status, as well as the shocks to persistent and temporary earnings. Let $\delta_i(z, e) \in [0, 1]$ denote the probability that an agent becomes unemployed. The probability that an agent becomes unemployed depends upon their persistent earnings and employment status from the prior period. In Section 5.2 we discuss how
we estimate the function $\delta_i(z, e)$ using the filtered estimates of persistent earnings $\hat{z}_{i,t-1}$ and realizations of earnings that fall below the minimum earnings threshold (e.g. unemployment).

Let $w_{i,t}(z, e, \epsilon)$ be a function that maps an individual’s (i) type, (ii) age, (iii) persistent earnings, (iv) temporary shock, and (v) employment status into a wage. We define the wage $w_{i,t}(z, e, \epsilon)$ such that

$$w_{i,t}(z, e, \epsilon) = \begin{cases} 
\exp(\kappa_{i,t} + z + \epsilon) & \text{if } e = E \\
\gamma \exp(\kappa_{i,t}) & \text{if } e = U 
\end{cases}$$

where $\kappa_{i,t}$ is a deterministic age profile of log earnings. $\gamma \in [0, 1]$ can be thought of as a replacement rate of persistent income for the unemployed. Wages are subject to labor income taxation. Let $\tilde{w}_{i,t}(z, e, \epsilon)$ denote the after tax income for an age $t$ agent with persistent earnings $z$, temporary shock $\epsilon$ and employment status $e$. We model taxes as Heathcote, Storesletten, and Violante (2017), where after tax income is given by

$$\tilde{w}_{i,t}(z, e, \epsilon) = \lambda w_{i,t}(z, e, \epsilon)^{1-\alpha}.$$ 

The parameter $\alpha > 0$, governs the degree of tax progressivity.

Finally, when an agent enters into the labor market they start as an employed agent, and draw their persistent earnings from a normal distribution with mean zero and variance $u_{z0,i}$. We summarize the choice problem of agents in the paragraph below by writing out the value function of agents.

**Value Functions.** We next define the value function for agents in the model. We write the value function for agents after the shocks to employment status as well as shocks to temporary and persistent earnings have been realized. Let $V_{i,t}(b, z, \epsilon, e)$ denote the value of being an age $t$, type $i$ agent with employment status $e$, persistent earnings $z$ and temporary earnings $e$.\(^{48}\) The agent makes a consumption savings decision in the current period, taking into account the set of potential shocks to income in the next period. The value function for an age $t$ agent is given by,

$$V_{i,t}(b, z, \epsilon, e) = \max_{c, b' \geq b} u(c) + \beta \mathbb{E}_{z', \epsilon', e'} \left[ V_{i,t+1}(b', z', \epsilon', e') \right] \quad \forall t \leq T$$

$$V_{i,T+1}(b, z, \epsilon, e) = 0$$

subject to the budget constraint,

\(^{48}\)Note for unemployed the value of temporary earnings $\epsilon$ is irrelevant.
\[ c + b' \leq b(1 + r_f) + \tilde{\omega}_{i,t}(z,e) \]

the law of motion for employment status,

\[
e' = \begin{cases} 
E & \text{w. prob } 1 - \delta_i(z,e) \\
U & \text{w. prob } \delta_i(z,e)
\end{cases}
\]

and the law of motion for persistent earnings,

\[
z' = \begin{cases} 
F_i z + v_{E,i,t+1} & \text{w. prob } 1 - \delta_i(z,e) \\
F_i z + v_{U,i,t+1} & \text{w. prob } \delta_i(z,e)
\end{cases}
\]

where \( v_{e,i,t+1} \sim N(B_{e,i}, Q_{e,i,t+1}) \). The mean of the shock depends upon an individual's employment status (employed vs. unemployed), and the variance to the shock depends upon the agent's employment status and age.

Finally, the law of motion for temporary earnings is given by,

\[
e' = \begin{cases} 
\epsilon_{i,t+1} & \text{w. prob } 1 - \delta_i(z,e) \\
0 & \text{w. prob } \delta_i(z,e)
\end{cases}
\]

where \( \epsilon_{i,t+1} \sim N(0, R_{i,t+1}) \). The variance of the shock to temporary earnings depends upon the agent's age.

### 5.2 Estimation

We next discuss the estimation of the model.\(^{49}\) Some parameters are assigned using estimates from the literature, while others are calibrated to be consistent with the U.S. labor market in 1985.

**Demographics and preferences.** To align with the sample in Section 3, agents enter the model at age 25 (\( t = 1 \)), and work until age 60 (\( T = 36 \)). When agents enter the model they begin with zero assets and are employed. Upon entering the model, agents learn their type, and types are fixed indefinitely. There are three types of agents in the economy: (1) agents without a college

\(^{49}\)In Appendix E, we discuss how we solve the model numerically.
degree, (2) agents with a college degree or higher, and (3) agents with unknown education.\footnote{In the data, we only classify an individuals level of education if they are over the age of 25 when in the CPS. The third group is comprised of individuals who were in the CPS before the age of 25, for whom we do not classify their level of education. We include them in the model to align aggregate statistics in the model economy with our empirical sample.} We set the shares of each type ($\pi_i$) to be consistent with each group’s share in the sample in 1985. The first row of Panel (b) in Table 6 presents the share of agents by type in the initial steady state.

Agents receive utility from consumption, with preferences given by

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}.$$ 

We set the risk aversion parameter to a standard value, $\sigma = 2$. Agents discount the future at rate $\beta = 0.977$. The parameter $\beta$ is calibrated to match the median ratio of net worth to income. In the SCF, we measure this ratio to be 1.82.

**Income process.** Agents receive wages that are a function of their age, persistent earnings, and temporary earnings. The fixed age component of earnings is estimated as part of the residualization process in Section 4.4.1. Panel (a) of Figure 12 plots the deterministic path of earnings that is used in the model.

When an agent is unemployed they receive (pre-tax and transfers) a fraction $\gamma \in [0, 1]$ of their persistent earnings. The parameter $\gamma$ can be thought as the replacement rate of unemployment insurance. We set $\gamma = 0.4$ as in Shimer (2005).

We next discuss the estimation of the stochastic process that governs how earnings evolve in the model.

**Shocks to income.** The model’s initial steady state is consistent with the 1985 values of the income process estimated in Section 4.4.1, where the income process is estimated separately for individuals by their level of education. The income process allows for the standard deviation of shocks to temporary and persistent income to be a function of age and time. In Panels (b) –(d) of Figure 12, we plot the shocks to persistent earnings while employed and unemployed as well as shocks to temporary earnings over the life cycle that are fed into the model. The figures show that agents of all types face: (1) a U-shaped age profile of persistent earnings risk while employed (Panel (b)), (2) decreasing age profile of temporary earnings risk (Panel (c)), and (3) an increasing profile of persistent earnings risk while unemployed (Panel (d)). The income process additionally allows for the drift of persistent earnings to depend upon
employment status and type. The third and fourth rows of Panel (b) in Table 6 present the drift parameters while employed (row 3) and unemployed (row 4). For each type of agent, there is a modest average increase in persistent earnings while employed, and a large average decline in persistent earnings while unemployed.

**Probability of unemployment.** We next discuss the estimation of the unemployment probability function $\delta_i(z,e)$. We use a functional form that allows an agent’s unemployment probability to flexibly depend upon their prior persistent earnings as well as employment status. The specification we use is of the form:

$$
\delta_i(z,e) = \begin{cases} 
I\{z \geq 0\} \left[ \sum_{k=0}^{2} \alpha_{i,k,E}^+ z^k \right] + I\{z < 0\} \left[ \sum_{k=0}^{2} \alpha_{i,k,E}^- z^k \right] & e = E \\
\alpha_{i,U} & e = U
\end{cases}
$$

(7)

The functional form in equation 7 allows for the unemployment probability of the employed to be a quadratic function of prior persistent earnings ($z$) estimated separately for positive prior persistent earnings or negative prior persistent earnings. We define the unemployment probability for the unemployed to be a constant for each education group.\(^{51}\) We estimate equation 7 separately for each education group using our filtered estimates of persistent earnings and individual realizations of being below the minimum earnings criteria.\(^{52}\) Panel (e) of Figure 12 presents the implied probabilities of becoming unemployed by type and prior persistent earnings for individuals who are employed in the prior period. The figure shows that individuals with lower prior persistent earnings face significantly higher probabilities of becoming unemployed. Additionally, college graduates face a significantly lower probability of becoming unemployed relative to non-college graduates. The second row of Panel (b) in Table 6 contains the parameter estimates that govern the probability of remaining unemployed by education level ($\alpha_{i,U}$).

**Taxes.** We model taxes as in Heathcote et al. (2017). We set the tax progressivity ($\alpha$) parameters as in Heathcote et al. (2017) to be equal to 0.181. In addition to financing the UI system, we model the government as having exogenous expenditures $G$ that are equal to share $g \in [0, 1]$ of

---

\(^{51}\)When we estimated the quadratic function in equation 7 for the unemployed, we obtained results that were consistent with simply using a constant function by education.

\(^{52}\)In the data we use the lag of an individuals estimated persistent earnings ($\hat{z}_{i,t-1}^k$), and the realization of an individuals earnings above or below the minimum earnings cutoff to estimate equation 7. To maximize statistical power, we estimate equation 7 on all sample years, but include year fixed effects in the estimation to obtain a set of parameters for our initial steady state.
before tax labor income. Using NIPA data on personal income and government consumption expenditure and investment, we set $g = 0.264$. We set the level parameter ($\lambda$) so that government revenue from taxes is equated to government spending on transfers and the exogenous government spending. Panel (f) of Figure 12 presents the implied tax function in the model economy. Agents with pre-tax incomes below approximately $10K receive transfers from the government, while individuals with pre-tax incomes greater than $10K pay labor income taxes.

**Asset Markets.** Agents are able to save and borrow using a risk free asset, with a risk-free rate of 4%. We set the borrowing limit $b$ to the “natural borrowing limit,” which requires that individuals exit the model with zero debt. Setting the borrowing limit at the natural borrowing limit represents an upper bound to the extent that agents can use borrowing to smooth shocks to income.

Table 6 and Figure 12 present the parameters that govern model economy. In the next section, we conduct the welfare experiment of adjusting labor income risk as documented in Section 4.

**Table 6: Parameters**

| Panel A: Parameters fixed across types |
|-----------------|-----------------|-----------------|
| Variable        | Value           | Description     |
| $\beta$         | 0.977           | Discount factor |
| $r_f$           | 4%              | Risk-free interest rate |
| $\sigma$        | 2               | Coefficient of relative risk-aversion |
| $\alpha$        | 0.181           | Progressivity of tax function |
| $\gamma$        | 0.4             | Replacement Rate UI |
| $g$             | 0.264           | Ratio of government expenditure to pre-tax income |
| $T$             | 36              | Number of years in labor market |

| Panel B: Income parameters by type |
|-----------------|-----------------|-----------------|
| Variable        | Type 1           | Type 2           | Type 3           | Description                                |
| $\pi_i$         | 0.687            | 0.287            | 0.026            | Shares of agents                           |
| $F_i$           | 0.925            | 0.937            | 0.930            | Persistence                                |
| $\delta_{i,U}$  | 0.389            | 0.413            | 0.379            | Unemp. prob., unemp. in prior year          |
| $B_{E,i}$       | 0.020            | 0.022            | 0.016            | Drift while employed                        |
| $B_{U,i}$       | $-0.123$         | $-0.110$         | $-0.142$         | Drift while unemployed                       |

**Note:** Table presents model parameters. Panel (A) shows parameters that do not depend upon type, and Panel (B) shows parameters of the income process that vary by type. In Panel (B), Type 1 refers to agents who have less than a college degree, Type 2 refers to individuals with a college degree or higher, and Type 3 refers to individuals for whom we do not classify a level of education.
Figure 12: Parameters governing income process

Note: Figure presents parameters that govern the income process for agents in the model that are a function of age and type (Panels (a)-(e)). Panel (f) presents the tax function for the model economy. Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.
5.3 Welfare implications of changing earnings risk

In Section 4, we documented five facts about the changing nature of earnings risk in the U.S. over the past 30 years. We documented: (1) the standard deviation of persistent shocks while employed has increased; (2) the standard deviation of temporary shocks has declined; (3) the standard deviation of persistent shocks while unemployed has increased; (4) the decline in persistent earnings while unemployed has increased. In this section, we use the calibrated model to assess the welfare implications of these changes in earnings risk across steady states of the model by updating each of these components of income risk to their 2013 value.

Changes in standard deviation of shocks. We start by discussing the welfare effects of changes in the standard deviation of shocks (e.g. $Q_E$, $Q_U$, and $R$). In the experiments where we adjust the standard deviation of shocks, to highlight the role of increased risk we adjust the age earnings profiles for each type of agent so that mean earnings are equated between the counterfactual economy and the baseline economy.

We first increase the standard deviation of shocks to persistent earnings of the employed ($Q_E$) from its 1985 values to its 2013 values. Column (2) of Table 7 presents the predictions of the calibrated model from increasing the standard deviation of shocks to persistent earnings for the employed. Across all agents in the economy, the standard deviation of shocks to persistent earnings while employed increases from 0.21 to 0.225. This increase in persistent earnings risk causes an average welfare decline of over 1.75% of lifetime consumption. These welfare losses occur in part as agents accumulate greater precautionary savings to insure against future income risk. From the increase in persistent earnings risk, the median asset to income ratio in the economy increases by over 5 percent from 1.82 to 1.91. Additionally, the welfare loss is heterogeneous across types (e.g. levels of education). Agents with a college degree or higher (type 2) experience the largest welfare losses of nearly $-3.3\%$ of lifetime consumption, while individuals with less than a college degree have earnings losses of $-1.1\%$ of lifetime consumption.

In Column (3) of Table 7 we examine the welfare effects of increases in the standard deviation of shocks to persistent earnings among the unemployed. Increasing the standard deviation of shocks to persistent earnings among the unemployed generates an average welfare loss of $-0.7\%$ of lifetime consumption. The welfare losses of increased persistent earnings risk are smaller because only a fraction of the population is unemployed. However, the greater risk associated with becoming unemployed creates a motive for accumulating more precautionary savings. The results of columns (2) and (3) shows that increased persistent earnings risk has generated a substantial welfare decline. Conversely, in Column (4), we decrease shocks to temporary earnings, and we find that there is only a small welfare gain of 0.03% of lifetime
Finally, we introduce all three changes in earnings risk into the simulated economy, shown in Column (5). Increased persistent earnings risk for both the employed and unemployed, along with the decline in the standard deviation of shocks to temporary earnings, generates an average welfare decline of 2.4%. Since increased persistent earnings risk is concentrated among the college educated (Section 4.4.1), we find the largest welfare losses among these agents, with a welfare loss of nearly $-4.9\%$ of lifetime consumption. Therefore, rising persistent earnings risk has generated substantial welfare losses which are not mitigated by declining temporary earnings risk. We next examine how changes in the means of persistent earnings shocks impact welfare.

**Changes in mean of shocks.** In this section, we examine the welfare implications of changes in the mean (e.g. drifts) of shocks to persistent earnings among the unemployed. We decrease the drift of persistent earnings to the unemployed ($B_U$) from its 1985 value to its 2013 value. Averaging across all agents in the economy, persistent earnings drift of the unemployed decreases from $-0.121$ to $-0.187$. Column (6) of Table 7 shows that accelerating persistent earnings losses among the unemployed causes a welfare loss of 3.3% of lifetime consumption. These welfare losses occur in part due to an increase in precautionary savings, as the median assets to income ratio increases by approximately 10% from 1.82 to 2.

In summary, changes in both the standard deviation and mean of persistent earnings shocks over the past 30 years have generated significant welfare losses across all education groups. But rising persistent earnings risk for college educated workers and faster persistent earnings losses among the unemployed are disproportionately responsible for these welfare losses.
Table 7: Welfare experiment: changes in earnings risk

<table>
<thead>
<tr>
<th></th>
<th>1 Baseline</th>
<th>2 $Q_E$</th>
<th>3 $Q_U$</th>
<th>4 $R$</th>
<th>5 $Q_E, Q_U, R$</th>
<th>6 $B_U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Welfare</td>
<td>−1.773%</td>
<td>−0.691%</td>
<td>0.033%</td>
<td>−2.401%</td>
<td>−3.33%</td>
<td></td>
</tr>
<tr>
<td>Mean Welfare Type 1 (Less than College)</td>
<td>−1.116%</td>
<td>−0.266%</td>
<td>0.018%</td>
<td>−1.353%</td>
<td>−2.837%</td>
<td></td>
</tr>
<tr>
<td>Mean Welfare Type 2 (College Grad Plus)</td>
<td>−3.262%</td>
<td>−1.745%</td>
<td>0.059%</td>
<td>−4.895%</td>
<td>−4.615%</td>
<td></td>
</tr>
<tr>
<td>Mean Welfare Type 3 (Other)</td>
<td>−3.009%</td>
<td>−0.474%</td>
<td>0.171%</td>
<td>−3.069%</td>
<td>−2.459%</td>
<td></td>
</tr>
<tr>
<td>Std. Dev. Perm (Emp.)</td>
<td>0.21</td>
<td>0.225</td>
<td>0.21</td>
<td>0.21</td>
<td>0.225</td>
<td>0.21</td>
</tr>
<tr>
<td>Std. Dev. Perm (Unemp.)</td>
<td>0.407</td>
<td>0.407</td>
<td>0.463</td>
<td>0.407</td>
<td>0.463</td>
<td>0.407</td>
</tr>
<tr>
<td>Std. Dev. Temp.</td>
<td>0.161</td>
<td>0.161</td>
<td>0.161</td>
<td>0.149</td>
<td>0.149</td>
<td>0.161</td>
</tr>
<tr>
<td>Drift Perm. Employed</td>
<td>0.020</td>
<td>0.020</td>
<td>0.020</td>
<td>0.020</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>Drift Perm. Unemployed</td>
<td>-0.122</td>
<td>-0.121</td>
<td>-0.121</td>
<td>-0.122</td>
<td>-0.121</td>
<td>-0.187</td>
</tr>
<tr>
<td>Avg. Earnings Type 1 (Less than College)</td>
<td>39800</td>
<td>39800</td>
<td>39800</td>
<td>39800</td>
<td>39800</td>
<td>39100</td>
</tr>
<tr>
<td>Avg. Earnings Type 2 (College Grad Plus)</td>
<td>71300</td>
<td>71300</td>
<td>71300</td>
<td>71300</td>
<td>71300</td>
<td>68700</td>
</tr>
<tr>
<td>Avg. Earnings Type 3 (Other)</td>
<td>49100</td>
<td>49100</td>
<td>49100</td>
<td>49100</td>
<td>49100</td>
<td>48100</td>
</tr>
<tr>
<td>Assets/Income</td>
<td>1.816</td>
<td>1.912</td>
<td>1.868</td>
<td>1.811</td>
<td>1.955</td>
<td>2.005</td>
</tr>
</tbody>
</table>

Note: Table presents the results of the steady state welfare experiment. Column (1) presents the baseline estimation of the model where the parameters of the income process are set at their 1985 values. Column (2) updates the standard deviation of shocks to persistent earnings among the employed ($Q_E$) to its 2013 values, holding all other parameters fixed. Column (3) updates the standard deviation of shocks to persistent earnings among the unemployed ($Q_U$) to its 2013 value, holding all other parameters fixed. Column (4) updates the standard deviation of shocks to temporary earnings among the unemployed ($R$) to its 2013 value, holding all other parameters fixed. Column (5) updates the parameters $Q_E$, $Q_U$ and $Q_R$ to their 2013 values. Column (6) updates the drift of persistent earnings among the unemployed ($B_U$) to its 2013 values. Type 1 corresponds to agents without a college degree, type 2 corresponds to a college degree or higher, and type 3 corresponds to agents who we do not classify with a level of education. In Appendix F.1 we discuss our measure of welfare.

6 Conclusion

How have temporary and persistent earnings risk changed over time and why? By answering these questions our paper makes several contributions. First, we present a method for estimating temporary and persistent earnings at the individual level as well as the parameters of the income process. We flexibly allow for arbitrary spells of zero earnings, allowing us to include estimates of persistent earnings risk for individuals who would otherwise be omitted from arc percent calculations. We provide a simple persistent-temporary income process that allows for zeros, and thus incorporates skewness, and can easily be integrated into heterogeneous agent models. As a practical contribution, our method of estimating persistent and temporary earnings risk on 23.5 million records can be completed in 3 hours on the Census servers.

Second, we estimate our income process on administrative earnings records that have been linked to survey responses from the CPS ASEC to examine how and why persistent and temporary earnings risk have changed over time. Our analysis of the time series yields four facts: (1) the standard deviation of persistent shocks while employed has increased; (2) the standard deviation of temporary shocks has declined; (3) the standard deviation of persistent shocks while unemployed has increased; (4) the decline in persistent earnings while unemployed has increased. Of particular note, between 1985 and 2013, the downward drift of persistent earnings among the unemployed fell by 6 percentage points (i.e. it became 57% more negative).

We then evaluate various hypotheses for the rise in persistent earnings risk between 1985 and 2013. We rule out hypotheses related to declining employment prospects of low skill workers and regional theories of persistent earnings losses, including the decline of the Rust-Belt. We additionally rule out theories related to declining employment and wages in routine occupations using information on the skill content of occupations (e.g. Acemoglu and Autor (2011)). Instead, we show that the increase in persistent earnings risk is a high skill worker phenomenon.

Our final contribution is to examine the welfare and macroeconomic effects of changing earnings risk over time. We show that the parameter estimates from our filtering method are easily incorporated into a Bewley-Huggett-Aiyagari style model of incomplete markets. While all sources of rising persistent earnings risk generate welfare losses, we find that the accelerating of persistent earnings losses while unemployed generated the largest losses.
References


Chan, M., S. Salgado, and M. Xu (2020). Heterogeneous passthrough from tfp to wages. Available at SSRN 3538503.


A Data and Estimation Details

A.1 Residualizing Earnings

We remove the common age component of earnings (residualizing) as in Guvenen et al. (2014). Using all earnings observations from the base sample, we run a pooled regression of earnings on age and cohort dummies without a constant. This regression recovers the age profile of log earnings. We then scale the age dummies so as to match the average log earnings of 25-year-olds used in the regression. We subtract the age dummies off of the earnings to recover residualized earnings.

B Kalman Filter

In this Appendix, we present the Kalman Filter, which we use to recover estimates of temporary and persistent earnings at the individual level. For now assume that the parameters which govern the income process are all known. In Section C we discuss how we use an EM algorithm to recover the parameters that govern the income process.

For now assume that $F, Q_E, Q_U, B_E, B_U, H_E, H_U, R_E$ and $R_U$ are all known. Starting with estimates $\hat{\zeta}_{i,1|0}$ and $M_{i,1|0}$, which we will define below, we obtain our desired time series for $\zeta_{i,t}$ as follows:

1. Estimate the “Kalman Gain”:

   \[
   K_{i,t} = M_{i,t|t-1}H'(l_{i,t}) \left[ H(l_{i,t})M_{i,t|t-1}H'(l_{i,t}) + R(l_{i,t}) \right]^{-1}
   \]

2. Update the state vector:

   \[
   \hat{\zeta}_{i,t|t} = \hat{\zeta}_{i,t|t-1} + K_{i,t} \left( y_{i,t} - H(l_t)\hat{\zeta}_{i,t|t-1} \right)
   \]

   \[
   \hat{\zeta}_{i,t+1|t} = \begin{bmatrix} F \\ 1 \end{bmatrix} \hat{\zeta}_{i,t|t} + \begin{bmatrix} B(l_{i,t}) \\ 0 \end{bmatrix}
   \]

3. Update the MSE matrix:

   \[
   M_{i,t|t} = M_{i,t|t-1} - K_{i,t}H(l_t)M_{i,t|t-1}
   \]

   \[
   M_{i,t+1|t} = \hat{F}M_{i,t|t}\hat{F}' + Q(l_{t+1})\hat{e}_1\hat{e}_1'
   \]
where $e_t^2 = [1, 0]'$. Repeat steps 1-3 for $t = 2, \ldots, T$, and for each individual $i \in \{1, \ldots, N\}$.

**Setting initial value.** To run the Kalman Filter we need an initial estimate of the mean of the state-vector and the variance-covariance matrix. We set the initial mean of the state vector to zero. The initial variance of the state vector is given by:

$$M_{i,1|0} = \text{var}[\zeta_0|X] = \begin{bmatrix} \text{var}(z_{i,0}) & 0 \\ 0 & 0 \end{bmatrix}$$

where:

$$\text{var}(z_{i,0}) = \text{var}(u_{z,i})$$

**Smoothed Kalman Filter.** Hamilton (1994) comments that when the value of the state vector is of interest in its own right, as in our application, we can improve the inference about the historical values of the state vector took in the middle of the sample by using the smoothed filter. The goal of this section will be to take the sequences we found from the Kalman filter above and estimate the smoothed sequence, which we denote $\{\{\hat{\zeta}_{i,t|T}\}_{t=1}^T\}_{i=1}^N$.

The steps for the smoothed Kalman filter are:

1. Run the Kalman Filter as presented above storing the sequences $\{M_{i,t|t-1}\}_{t=1}^T$ and $\{M_{i,t|t}\}_{t=1}^T$ as well as $\{\hat{\zeta}_{i,t|t-1}\}_{t=1}^T$ and $\{\hat{\zeta}_{i,t|t}\}_{t=1}^T$.
   
   (a) Notice that we will not use the estimates $\hat{\zeta}_{i,T+1|T}$ and $M_{i,T+1|T}$.

2. Store the element $\hat{\zeta}_{i,T|T}$ from $\{\hat{\zeta}_{i,t|t}\}_{t=1}^T$.

3. Calculate the sequence of smoothed estimations $\{\hat{\zeta}_{i,t|T}\}_{t=1}^{T-1}$ in reverse order by iterating on:

$$\hat{\zeta}_{i,t|T} = \hat{\zeta}_{i,t|t} + I_{i,t}(\hat{\zeta}_{i,t+1|T} - \hat{\zeta}_{i,t+1|t})$$

for $t = T - 1, T - 2, \ldots, 1$, where $I_{i,t} = M_{i,t|t}P_{i,t+1|t}^{-1}$.

4. Update the sequence of MSE by iterating on:

$$M_{i,t|T} = M_{i,t|t} + I_{i,t}(M_{i,t+1|T} - M_{i,t+1|t})J'_{i,t}$$
C EM Algorithm

In this Appendix, we outline the EM algorithm we use to estimate the parameters of the income process presented in Section 2.

The EM algorithm is an iterative algorithm to update the parameters that govern the income process. To start the algorithm we make an initial guess of the parameters of the income process, and using these parameters create an estimate of the state vector using the Kalman Filter presented in Appendix B. The next step in the EM algorithm is to use the estimates of the state vector along with the data to update the estimates of the parameters. The parameters are updated using a series of equations that we derive below. The algorithm then repeats by using the new parameters to update the estimate of the state vector, and then using the estimated state vector and data to update the parameters. This process continues until the log likelihood has been maximized. In the subsections below we derive the equations that will allow for closed form updating of the parameters. Finally, we provide a detailed description of the EM algorithm.

C.1 Log Likelihood

The EM algorithm uses a set of closed form updating equations to uncover parameters which allow the log likelihood function to be maximized. To derive these formulas we start with the full-information log likelihood, which is the likelihood function if the state-variables are observed. For an individual \( i \), the full information log likelihood appears as:

\[
LL_i(\{y_{i,t}\}^T_{t=0}, \{l_{i,t}\}^T_{t=0}, \{z_{i,t}\}^T_{t=0} | \theta_0) = -\frac{T + 1}{2} \log(2\pi) \\
- \frac{1}{2} \log(var(u_z)) - \frac{1}{2} \frac{(z_{i0})^2}{\text{var}(u_z)} \\
- \frac{1}{2} \sum_{t=1}^T \log(Q(l_{i,t})) - \frac{1}{2} \sum_{t=1}^T \frac{(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2}{Q_t(l_{i,t})} \\
- \frac{1}{2} \sum_{t=1}^T \log(R(l_{i,t})) - \frac{1}{2} \sum_{t=1}^T \frac{(l_{E,i,t})(y_{i,t} - z_{i,t})^2}{R_{l,t}(l_{i,t})}
\]

C.2 Updating Means

In this section, we derive the expressions that are used to update the mean parameters of income process (e.g. the persistence of persistent earnings, and drifts of persistent earnings when
employed/unemployed). Before deriving the formulas we present a series of useful expressions that will ease the derivations of the updating equations. Additionally, note the following notation. Define \( E[z_{il} | x_{il}, y_{it}, l_{it}] = \mu_{il|T} \), that is the expected value of individual \( i \)'s persistent earnings in period \( t \) (given the data) is denoted by \( \mu_{il|T} \), which corresponds to the output of the smoothed Kalman filter. Define \( \Sigma_{i0|T}(1,1) \) to be the estimated variance of initial persistent earnings. Define \( \Sigma_{i|T}(1,2) \) to be the estimated covariance between \( z_{it|T} \) and \( z_{it-1|T} \).

### C.2.1 Useful Expressions

In this section, we derive a series of useful expressions that will aid in the derivation of the updating equations in the following subsections.

First, we show that \( E[z_{i0}^2 | x_{it}, y_{it}, l_{it}] = \Sigma_{i0|T}(1,1) + \mu_{i|T}^2 \)

\[
E \left[ z_{i0}^2 \right| y_{it}, x_{it}, l_{it} \right] = E \left[ (z_{i0} - \mu_{i0|T} + \mu_{i|T})^2 \right| y_{it}, x_{it}, l_{it} \right] \\
= E \left[ (z_{i0} - \mu_{i0|T})^2 + \mu_{i0|T}^2 + (z_{i0} - \mu_{i0|T}) \mu_{i|T} \right| y_{it}, x_{it}, l_{it} \right] \\
= E \left[ (z_{i0} - \mu_{i0|T})^2 + \mu_{i0|T} \right| y_{it}, x_{it}, l_{it} \right] \\
= \Sigma_{i0|T}(1,1) + \mu_{i|T}^2 \tag{8}
\]

where in the third equality we used the fact that \( E \left[ z_{i0} - \mu_{i0|T} \right| y_{it}, x_{it}, l_{it} \right] = 0 \).

Next, we show that \( E_T [z_{it} z_{i,t-1} | \{y_{it}, x_{it}, l_{it} \}] = \Sigma_{it|T}(1,2) + \mu_{i|T} \mu_{it-1|T} \)

\[
E_T \left[ z_{it} z_{i,t-1} \right| y_{it}, x_{it}, l_{it} \right] = E_T \left[ (z_{it} - \mu_{it|T} + \mu_{i|T}) \left( z_{it-1} - \mu_{it-1|T} + \mu_{it-1|T} \right) \right| y_{it}, x_{it}, l_{it} \right] \\
= E_T \left[ (z_{it} - \mu_{it|T}) \left( z_{it-1} - \mu_{it-1|T} \right) + \mu_{it|T} \mu_{it-1|T} \right| y_{it}, x_{it}, l_{it} \right] \\
= \Sigma_{it|T}(1,2) + \mu_{i|T} \mu_{it-1|T} \tag{9}
\]

where in the second equality we have used the fact that \( E_i \left[ (z_{it} - \mu_{it|T}) \right| y_{it}, x_{it}, l_{it} \right] = 0 \) and \( E_i \left[ (z_{it-1} - \mu_{it-1|T}) \right| y_{it}, x_{it}, l_{it} \right] = 0 \).

---

\(^{53}\)Note this covariance term is the (1,2) element of the matrix \( M_{i,t|T} \).
C.2.2 Updating $F, B_E, B_U$

In this section, we derive the expression we will use to update the parameters $\{F, B_E, B_U\}$. The relevant part of the log likelihood for updating the parameters $\{F, B_E, B_U\}$ is given by:

$$\frac{1}{Q_t(l_{i,t})} \sum_{t=1}^{T} (z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2$$

The expected value can be written as:

$$\frac{1}{Q_t(l_{i,t})} E_T \left[ (z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2 \mid \{y_{it}, x_{it}, l_{it}\} \right]$$

Completing the square we obtain the following expression:

$$\frac{1}{Q_t(l_{i,t})} E_T \left[ z_{i,t}^2 - z_{i,t}Fz_{i,t-1} - z_{i,t}B(l_{i,t}) + F^2z_{i,t-1}^2 - Fz_{i,t-1}Fz_{i,t-1} + Fz_{i,t-1}B(l_{i,t}) + B(l_{i,t})^2 - B(l_{i,t})z_{i,t} + FB(l_{i,t})z_{i,t-1} \mid \{y_{it}, x_{it}, l_{it}\} \right]$$

Combining terms we have:

$$\frac{1}{Q_t(l_{i,t})} E_T \left[ z_{i,t}^2 - 2Fz_{i,t}z_{i,t-1} - 2z_{i,t}B(l_{i,t}) + F^2z_{i,t-1}^2 + 2Fz_{i,t-1}B(l_{i,t}) + B(l_{i,t})^2 \mid \{y_{it}, x_{it}, l_{it}\} \right]$$

(10)

We will next use expressions from Section C.2.1 to simplify equation 10. First using equation 8 (adjusted for period $t$, and period $t + 1$), we have:

$$\frac{1}{Q_t(l_{i,t})} \left( \Sigma_{it|T}(1, 1) + \mu_{it|T}^2 + F^2 \left[ \Sigma_{it-1|T}(1, 1) + \mu_{it-1|T}^2 \right] \right) + \frac{1}{Q_t(l_{i,t})} E_T \left[ -2Fz_{i,t}z_{i,t-1} - 2z_{i,t}B(l_{i,t}) + 2Fz_{i,t-1}B(l_{i,t}) + B(l_{i,t})^2 \mid \{y_{it}, x_{it}, l_{it}\} \right]$$

Next using equation 9, we have:
\[
\frac{1}{Q_t(l_{i,t})} \left( \Sigma_{it|T}(1, 1) + \mu_{it|T}^2 + F^2 \left[ \Sigma_{it-1|T}(1, 1) + \mu_{it-1|T}^2 \right] \right)
\]
\[
\frac{1}{Q_t(l_{i,t})} \left( -2F \left[ \Sigma_{it|T}(1, 2) + \mu_{it|T}\mu_{it-1|T} \right] \right)
\]
\[
+ \frac{1}{Q_t(l_{i,t})} E_T \left[ -2z_{i,t}B(l_{i,t}) + 2Fz_{i,t-1}B(l_{i,t}) + B(l_{i,t})^2 \right] \{y_{it}, x_{it}, l_{it}\}
\]

Then taking the expectation over the remaining terms we have:

\[
\frac{1}{Q_t(l_{i,t})} \left( \Sigma_{it|T}(1, 1) + \mu_{it|T}^2 + F^2 \left[ \Sigma_{it-1|T}(1, 1) + \mu_{it-1|T}^2 \right] \right)
\]
\[
\frac{1}{Q_t(l_{i,t})} \left( -2F \left[ \Sigma_{it|T}(1, 2) + \mu_{it|T}\mu_{it-1|T} \right] \right)
\]
\[
+ \frac{1}{Q_t(l_{i,t})} \left[ \left( -2\mu_{it|T}B(l_{i,t}) + 2F\mu_{it-1|T}B(l_{i,t}) + B(l_{i,t})^2 \right) \right]
\]

We want to optimize equation 11 with respect to \(F, B_E\) and \(B_U\). For ease of exposition, we drop the terms in equation 11 that do not include \(F, B_E\) and \(B_U\), which returns:

\[
\frac{1}{Q_t(l_{i,t})} \left( F^2 \left[ \Sigma_{it-1|T}(1, 1) + \mu_{it-1|T}^2 \right] \right)
\]
\[
\frac{1}{Q_t(l_{i,t})} \left( -2F \left[ \Sigma_{it|T}(1, 2) + \mu_{it|T}\mu_{it-1|T} \right] \right)
\]
\[
+ \frac{1}{Q_t(l_{i,t})} \left[ \left( -2\mu_{it|T}B(l_{i,t}) + 2F\mu_{it-1|T}B(l_{i,t}) + B(l_{i,t})^2 \right) \right]
\]

The expression in 12 gives the expected contribution to the likelihood for individual \(i\) in period \(t\). We want to maximize equation the likelihood across all individuals and time periods. To perform this optimization it will be convenient to define the following vectors and matrices. Define:

\[
X_C \equiv \begin{bmatrix}
\mu_{10|T} & l_{E,1,1} & l_{U,1,1} \\
\vdots & \vdots & \vdots \\
\mu_{1T-1|T} & l_{E,1,T} & l_{U,1,T} \\
\vdots & \vdots & \vdots \\
\mu_{NT-1|T} & l_{E,N,T} & l_{U,N,T}
\end{bmatrix} \quad C \equiv \begin{bmatrix}
F \\
B_E \\
B_U
\end{bmatrix} \quad Y_C \equiv \begin{bmatrix}
\mu_{11|T} \\
\vdots \\
\mu_{NT|T}
\end{bmatrix}
\]
Observe the following:

- $C'X'C = \sum_{i=1}^{N} \sum_{t=1}^{T} \left( F\mu_{it-1|T} + B_E l_{Eit} + B_U l_{Uit} \right)^2$.
- $C'X'C = \sum_{i=1}^{N} \sum_{t=1}^{T} \left( F^2\mu_{it-1|T} + 2FB_E\mu_{it-1|T}l_{Eit} + 2FBU\mu_{it-1|T}l_{Uit} + (B_E l_{Eit})^2 + (B_U l_{Uit})^2 \right)$.
- $C'X'C = \sum_{i=1}^{N} \sum_{t=1}^{T} \left( F^2\mu_{it-1|T} + 2FB_E\mu_{it-1|T}l_{Eit} + 2FBU\mu_{it-1|T}l_{Uit} + B(l_{it})^2 \right)$, where we have used the $B(l_{it})$ notation from above.

- $Y_CX = F\sum_{t=1}^{N} \sum_{i=1}^{T} \mu_{it}|T\mu_{it-1|T} + B_E \sum_{i=1}^{N} \sum_{t=1}^{T} \mu_{it}|Tl_{Eit} + B_U \sum_{i=1}^{N} \sum_{t=1}^{T} \mu_{it}|Tl_{Uit}$.

- $Y_C'X_C = F\sum_{i=1}^{N} \sum_{t=1}^{T} \mu_{it}|T(l_{it})$, where we have used the $B(l_{it})$ notation from above.

To complete writing the sum of the log likelihood across individuals it will be convenient to define the following vectors:

$$\tilde{\sigma}_{t-1}(1,1) \equiv \begin{bmatrix} \Sigma_{10|T}(1,1) \\ \Sigma_{11|T}(1,1) \\ \vdots \\ \Sigma_{NT-1|T}(1,1) \end{bmatrix}$$

$$\tilde{\sigma}_{t}(1,2) \equiv \begin{bmatrix} \Sigma_{11|T}(1,1) \\ \Sigma_{12|T}(1,2) \\ \vdots \\ \Sigma_{NT|T}(1,2) \end{bmatrix}$$

$$e_1^T \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$e_{NT} \equiv \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Observe the following:

- $\sum_{i=1}^{N} \sum_{t=1}^{T} \left( F^2\Sigma_{it-1|T}(1,1) \right) = C'e_1^3 e_1' C\tilde{\sigma}_{t-1}(1,1) e_{NT}$

- $\sum_{i=1}^{N} \sum_{t=1}^{T} \left( F\Sigma_{it|T}(1,2) \right) = e_{NT} e_1 C'\tilde{\sigma}_{t}(1,2) e_1^3 C$

Using 12 and the definitions above we have the following:

$$\sum_{i=1}^{N} \sum_{t=1}^{T} E_T \left[ (z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2 \right] = C'X'C - 2Y_C'X_C - 2e_{NT}^T \tilde{\sigma}_{t}(1,2) e_1^3 C' + C'e_1^3 e_1' C\tilde{\sigma}_{t-1}(1,1) e_{NT}$$

Taking the FOC with respect to $C$ returns:

$$0 = 2C'X'C - 2Y_C'X_C - 2e_{NT}^T \tilde{\sigma}_{t}(1,2) e_1^3 C' + 2C'e_1^3 e_1' \tilde{\sigma}_{t-1}(1,1) e_{NT}$$

(13)
Rearranging equation 13 returns:

\[ C' \left[ X_C'X_C + e_1^3 e_1'^3 \sigma_{t-1}^2 (1,1)e^{NT} \right] = Y_C'X_C + e^{NT} \tilde{\sigma}_t(1,2)e_1'^3 \] (14)

Taking the transpose of both sides of equation 14 returns:

\[ \left[ X_C'X_C + e_1^3 e_1'^3 \sigma_{t-1}^2 (1,1)e^{NT} \right] C = X_C'Y_C + e_1^3 \tilde{\sigma}_t(1,2)e^{NT} \] (15)

where we have exploited the fact that the matrices on the LHS of equation 14 are symmetric. Equation 15 gives us a closed form equation for updating the parameters \( \{ F, B_E, B_U \} \).

**Intuition**  Equation 15 shows that the parameters \( C = [F, B_E, B_U] \) are updated by using a GLS style regression equation. The persistence of persistent earnings \( (F) \) is updated by regressing lagged persistent earnings (the first column of \( X_C \)) onto current persistent earnings \( (Y_C) \), and is then adjusted to take into account the covariance of persistent earnings with its lag as well as the variance of lagged persistent earnings. The drift of persistent earnings when employed is updated by regressing a dummy variable for being employed (the second column of \( X_C \)) onto current persistent earnings \( (Y_C) \). Similarly, the drift of persistent earnings when unemployed is updated by regressing a dummy variable for being unemployed (the third column of \( X_C \)) onto current persistent earnings \( (Y_C) \). The GLS regression formula in equation 15 shows that these parameters are identified by running regressions that are informative about the evolution of persistent earnings over time, as well as during employment and unemployment spells.

C.3 Updating Variances

In this Appendix, we derive the expressions that will be used to update the variance parameters.

C.3.1 Shocks to persistent Earnings When Employed and Unemployed \( (Q_E \text{ and } Q_U) \)

In this section we discuss how we update the variance of persistent earnings for the employed and unemployed. We can write the variance of persistent earnings as:

\[ Q(l_{ii}) = \exp(l_{ii}'\phi_Q) \]

where \( \phi_Q = [\phi_{Q,E}, \phi_{Q,U}] \). The relevant part of the negative log likelihood which depends on \( \phi_Q \) is:
\[ \Theta(\phi_Q; \beta, \omega) \equiv \sum_{i=1}^{N} \sum_{t=1}^{T} \log(Q(l_{it})) + \sum_{i=1}^{N} \sum_{t=1}^{T} \frac{(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2}{Q(l_{it})} \]  

(16)

To arrive at an updating formula for the variance of persistent earnings, we will take the conditional expectation using the posterior distribution of the latent states given all of the missing data, and then take FOC with respect to \( \phi_Q \). Taking the conditional expectation using the posterior distribution of latent states (given all of the missing data) returns:

\[ \Theta(\phi_Q; \beta, F) \equiv \sum_{i=1}^{N} \sum_{t=1}^{T} \log(Q(l_{it})) + \sum_{i=1}^{N} \sum_{t=1}^{T} \frac{E[(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2|z_{i,t}, y_{i,t}, l_{it}]}{Q(l_{it})} \]  

(17)

Observe that this function is convex in \( \phi_Q \). Therefore, if we take a second order approximation of the objective, we get the following:

\[ \Theta(\phi_Q; \beta, F) - \Theta(\phi_Q,0; \beta, \omega) \equiv (\phi_Q - \phi_Q,0)' \nabla \Theta + \frac{1}{2}(\phi_Q - \phi_Q,0)' \nabla^2 \Theta(\phi_Q - \phi_Q,0), \]

where the Jacobian matrix is defined as

\[ \nabla \Theta(\phi_Q,0; \beta, F) \equiv \sum_{i=1}^{N} \sum_{t=1}^{T} \left[ 1 - \frac{E[(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2|z_{i,t}, y_{i,t}, l_{it}]}{Q(l_{it})} \right] l_{i,t} \]

and the Hessian matrix \( \nabla^2 \Theta \) is defined as

\[ \nabla^2 \Theta(\phi_Q,0; \beta, F) \equiv \sum_{i=1}^{N} \sum_{t=1}^{T} \left[ \frac{E[(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2|z_{i,t}, y_{i,t}, l_{it}]}{Q(l_{it})} \right] l_{i,t} l_{i,t}' \]

Taking first order conditions, we get the familiar expressions for Newton’s method:

\[ \nabla^2 \Theta \phi_Q = \nabla^2 \Theta \phi_{Q,0} - \nabla \Theta \iff \phi_Q = \phi_{Q,0} - \left[ \nabla^2 \Theta \right]^{-1} \nabla \Theta, \]

(18)

which gives us a simple way of updating \( \phi_Q \).
Implementation  Note that we can write the conditional expectation term as:

$$E \left[(z_{it} - Fz_{it-1} - B(l_{it}))^2 | x_{it}, y_{it}, l_{it}\right] = E \left[(z_{it} - Fz_{it-1} - B(l_{it})) | x_{it}, y_{it}, l_{it}\right]^2$$

where in the final equality we are using the fact that we are conditioning on $l_{it}$. Then using the definition of $A_{it}$, we have:

$$var(z_{it} - Fz_{it-1} - B(l_{it}) | x_{it}, y_{it}, l_{it}) = var(A_{it} - B(l_{it}) | x_{it}, y_{it}, l_{it})$$

$$= var(A_{it} | x_{it}, y_{it}, l_{it}) + var(B(l_{it}) | x_{it}, y_{it}, l_{it})$$

$$- 2cov(A_{it}, B(l_{it}) | x_{it}, y_{it}, l_{it})$$

$$= var(A_{it} | x_{it}, y_{it}, l_{it})$$

where in the final equality we are using the fact that we are conditioning on $l_{it}$. Then using the definition of $A_{it}$, we have:

$$var(z_{it} - Fz_{it-1} - B(l_{it}) | x_{it}, y_{it}, l_{it}) = var(z_{it} | x_{it}, y_{it}, l_{it}) + F^2 var(z_{it-1} | x_{it}, y_{it}, l_{it})$$

$$- 2Fcov(z_{it}, z_{it-1} | x_{it}, y_{it}, l_{it})$$

Combining equations 19 and 20, we have the following expression for the conditional expectations terms.

$$E \left[(z_{it} - Fz_{it-1} - B(l_{it}))^2 | x_{it}, y_{it}, l_{it}\right] = E \left[(z_{it} - Fz_{it-1} - B(l_{it})) | x_{it}, y_{it}, l_{it}\right]^2$$

$$+ var(z_{it} | x_{it}, y_{it}, l_{it}) + F^2 var(z_{it-1} | x_{it}, y_{it}, l_{it})$$

$$- 2Fcov(z_{it}, z_{it-1} | x_{it}, y_{it}, l_{it})$$

C.3.2 Updated Variance of Temporary Earnings (R)

In this section we discuss how we update the variance of temporary earnings. We can write the variance of persistent earnings as:

$$R(l_{E,i,t}) = \exp(l_{E,i,t} \phi_R)$$

where $l_{E,i,t}$ is a dummy variable denoting whether an individual $i$ is employed in period $t$. 

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The relevant part of the negative log likelihood which depends on $\phi_R$ is:

$$\Theta(\phi_R) \equiv \sum_{i=1}^{N} \sum_{t=1}^{T} \log(R(l_{E,i,t})) + \sum_{i=1}^{N} \sum_{t=1}^{T} \frac{H(l_{i,t}) (y_{i,t} - z_{i,t})^2}{R_{i,t}(l_{E,i,t})}$$

(21)

Taking the conditional expectation using the posterior distribution of latent states (given all of the missing data) returns:

$$\Theta(\phi_R) \equiv \sum_{i=1}^{N} \sum_{t=1}^{T} \log(R(l_{E,i,t})) + \sum_{i=1}^{N} \sum_{t=1}^{T} H(l_{i,t}) \mathbb{E}\left[\frac{(y_{i,t} - z_{i,t})^2}{R_{i,t}(l_{E,i,t})} \big| x_{i,t}, y_{i,t}, l_{i,t}\right]$$

(22)

Similar to above, observe that this function is convex in $\phi_R$. Therefore, if we take a second order approximation of the objective, we get the following:

$$\Theta(\phi_R) - \Theta(\phi_{R,0}) \equiv (\phi_R - \phi_{R,0})' \nabla \Theta + \frac{1}{2} (\phi_R - \phi_{R,0})' \nabla^2 \Theta (\phi_R - \phi_{R,0}),$$

where the Jacobian matrix is defined as

$$\nabla \Theta(\phi_{R,0}) \equiv \sum_{i=1}^{N} \sum_{t=1}^{T} \left[1 - \frac{E\left[ (y_{i,t} - z_{i,t})^2 \big| x_{i,t}, y_{i,t}, l_{i,t}\right]}{R(l_{E,i,t})} \right] l_{E,i,t},$$

and the Hessian matrix $\nabla^2 \Theta$ is defined as

$$\nabla^2 \Theta(\phi_{R,0}) \equiv \sum_{i=1}^{N} \sum_{t=1}^{T} \left[ \frac{E\left[ (y_{i,t} - z_{i,t})^2 \big| x_{i,t}, y_{i,t}, l_{i,t}\right]}{R(l_{E,i,t})} \right] l_{E,i,t} l_{E,i,t}' \right].$$

Taking first order conditions, we get the familiar expressions for Newton’s method:

$$\nabla^2 \Theta \phi_R = \nabla^2 \Theta \phi_{R,0} - \nabla \Theta \quad \iff \quad \phi_R = \phi_{R,0} - \left[ \nabla^2 \Theta \right]^{-1} \nabla \Theta,$$

(23)

**Implementation**  
Note that we can write the conditional expectations term as:

$$E\left[ (y_{i,t} - z_{i,t})^2 \big| x_{i,t}, y_{i,t}, l_{i,t}\right] = E\left[ y_{i,t} - z_{i,t} \big| x_{i,t}, y_{i,t}, l_{i,t}\right]^2 + \text{var}(y_{i,t} - z_{i,t} \big| x_{i,t}, y_{i,t}, l_{i,t})$$

Since we condition on $y_{i,t}$, the conditional variance term can be written as:

$$\text{var}(y_{i,t} - z_{i,t} \big| x_{i,t}, y_{i,t}, l_{i,t}) = \text{var}(z_{i,t} \big| x_{i,t}, y_{i,t}, l_{i,t})$$
Then we have that:

$$E \left[ (y_{i,t} - z_{it})^2 | x_{it}, y_{it}, l_{it} \right] = E \left[ y_{i,t} - z_{it} | x_{it}, y_{it}, l_{it} \right]^2 + \text{var}(z_{it})$$

### C.3.3 Updating Variance of Initial persistent Earnings Draw ($\text{var}(u_z)$)

In this section we discuss how we update the variance of the initial draw of persistent earnings. We can write the variance of initial persistent earnings as:

$$\text{var}(u_z) = \exp(l_{E,i,t}^0 \phi_{u_z})$$

where $l_{E,i,t}^0$ is a dummy variable that is equal to 1 if individual $i$ is employed $E$ for the first time in the sample in period $t$.

The relevant part of the negative log likelihood which depends on $\phi_{u_z}$ is:

$$\Theta(\phi_{u_z}) \equiv \sum_{i=1}^{N} \log(\text{var}(u_z)) + \sum_{i=1}^{N} \frac{(z_{i0})^2}{\text{var}(u_z)}$$

Taking the conditional expectation using the posterior distribution of latent states (given all of the missing data) returns:

$$\Theta(\phi_{u_z}) \equiv \sum_{i=1}^{N} \log(\text{var}(u_z)) + \sum_{i=1}^{N} \frac{E \left[ (z_{i0})^2 | x_{it}, y_{it}, l_{it} \right]}{\text{var}(u_z)}$$

Similar to above, observe that this function is convex in $\phi_{u_z}$. Therefore, if we take a second order approximation of the objective, we get the following:

$$\Theta(\phi_{u_z}) - \Theta(\phi_{u_z,0}) \equiv (\phi_{u_z} - \phi_{u_z,0})' \nabla \Theta + \frac{1}{2} (\phi_{u_z} - \phi_{u_z,0})' \nabla^2 \Theta (\phi_{u_z} - \phi_{u_z,0}),$$

where the Jacobian matrix is defined as

$$\nabla \Theta(\phi_{u_z,0}) \equiv \sum_{i=1}^{N} \left[ 1 - \frac{E \left[ (z_{i0})^2 | x_{it}, y_{it}, l_{it} \right]}{\text{var}(u_z)} \right] l_{E,i,t}^0$$

and the Hessian matrix $\nabla^2 \Theta$ is defined as

$$\nabla^2 \Theta(\phi_{u_z,0}) \equiv \sum_{i=1}^{N} \left[ \frac{E \left[ (z_{i0})^2 | x_{it}, y_{it}, l_{it} \right]}{\text{var}(u_z)} \right] l_{E,i,t}^0 l_{E,i,t}'$$

Taking first order conditions, we get the familiar expressions for Newton’s method:
\[ \nabla^2 \Theta \phi_{u_z} = \nabla^2 \Theta \phi_{z,0} - \nabla \Theta \quad \iff \quad \phi_{u_z} = \phi_{z,0} - \left( \nabla^2 \Theta \right)^{-1} \nabla \Theta, \quad (24) \]

Implementation  
Note that we can write the conditional expectations term as:

\[ E \left[ (z_{i0})^2 | x_{it}, y_{it}, I_{it} \right] = E [z_{i0} | x_{it}, y_{it}, I_{it}]^2 + \text{var} [z_{i0} | x_{it}, y_{it}, I_{it}] \]

C.4 Algorithm

In this section, we present the EM algorithm we use to recover the estimate of persistent earnings as well as the parameters which govern the income process.

1. Guess an initial set of parameters \( \theta_0 = [F, Q_E, Q_U, B_E, B_{UL}, R]' \).

2. Using the parameter guess \( \theta_0 \) use the Kalman Filter for the state-space system in equations 2 and 1 to obtain an estimate of \( \{z_{i,t}\}_{i=1}^{N_i} \}_{t=0}^{T} \) and estimate the log likelihood.

3. Using the estimate of persistent earnings \( \{z_{i,t}\}_{i=1}^{N_i} \}_{t=0}^{T} \) and observed data \( \{y_{i,t}\}_{i=1}^{N_i} \) and \( \{l_{i,t}\}_{i=1}^{N_i} \) and \( \{x_{i,t}\}_{i=1}^{N_i} \), update the parameter vector as follows:
   
   (a) Update \( F, B_{UL}, B_E \) using equation 15.
   
   (b) Update the shocks to persistent earnings by iterating on equation 18.
   
   (c) Update the shocks to temporary earnings by iterating on equation 23.
   
   (d) Update the initial draw of persistent earnings by solving 24.

4. Repeat steps (2) and (3) until the log likelihood is maximized.

D  Extended Model

In this appendix, we present the income process that we use in Section 4. The income process, we estimate in Section 4 includes:

1. The standard deviation of shocks to temporary and persistent earnings are a function of year fixed effects and a quadratic in age which vary separately for both the employed and unemployed.
2. The standard deviation of initial draws of persistent earnings are also a function of year fixed effects and a quadratic in age.

3. The drift in persistent earnings is also a function of year fixed effects which vary separately for both the employed and unemployed.

**Standard deviation of shocks**  Let $\tilde{Q}_{E,t,j}$ denote the log variance of shocks to the employed in year $t$ for an individual of age $j$.\footnote{Assuming that the logarithm of variances rather than variances themselves are affine in observables has the advantage of guaranteeing that variances are positive (and guarantees a well-defined log-likelihood).} We model $\tilde{Q}_{E,t,j}$ as follows:

\[
\tilde{Q}_{E,t,j} = \tilde{Q}_E + \tilde{Q}_Y^E + (j - 25)\tilde{Q}_{E,1}^A + (j - 25)^2\tilde{Q}_{E,2}^A
\]  

(25)

where $\tilde{Q}_E$ denotes the log variance of the shock to persistent earnings in the initial year of the sample for an age 25 individual. The parameter $\tilde{Q}_Y^E$ denotes the year fixed effect for the log variance of the shock to persistent earnings among the employed in year $t$. The parameters $\tilde{Q}_{E,1}^A$ and $\tilde{Q}_{E,2}^A$ govern the age quadratic for the shock to persistent earnings among the employed. Suppose that there are $T$ periods. Then, the estimation recovers a set of parameters $\{\tilde{Q}_E, \tilde{Q}_{E,1}^Y, ..., \tilde{Q}_{E,T}^Y, \tilde{Q}_{E,1}^A, \tilde{Q}_{E,2}^A\}$, which govern the variance of shocks to persistent earnings among the employed.

We similarly define $\tilde{Q}_{U,t,j}$ as the log variance of shocks to persistent earnings for the unemployed in year $t$ for an individual of age $j$, and proceed as above to estimate a set of parameters $\{\tilde{Q}_U, \tilde{Q}_{U,1}^Y, ..., \tilde{Q}_{U,T}^Y, \tilde{Q}_{U,1}^A, \tilde{Q}_{U,2}^A\}$, which govern the log variance of shocks to persistent earnings among the unemployed. Additionally, we define $\tilde{R}_{t,j}$ as the log variance of temporary shocks in year $t$ for an individual of age $j$, and proceed as above to estimate a set of parameters $\{\tilde{R}, \tilde{R}_{1}^Y, ..., \tilde{R}_{T}^Y, \tilde{R}_{1}^A, \tilde{R}_{2}^A\}$, which govern the log variance of shocks to temporary earnings.

Finally, let $\tilde{z}_{0,t,j}$ denote the log variance of the initial draw of persistent earnings for an individual who enter the estimation sample in period $t$ when they are age $j$. We model $\tilde{z}_{0,t,j}$ as follows:

\[
\tilde{z}_{0,t,j} = \tilde{z}_0 + \tilde{z}_{0,t}^Y + \mathbb{I}\{t = t_0\} \left\{ (j - 25)\tilde{z}_{0,1}^A + (j - 25)^2\tilde{z}_{0,2}^A \right\} + \mathbb{I}\{t > t_0\} \left\{ (j - 25)\tilde{z}_{0,3}^A + (j - 25)^2\tilde{z}_{0,4}^A \right\}
\]  

(26)

where $\tilde{z}_0$ denotes the log variance of the initial draw of persistent earnings in the initial year of the sample for an age 25 individual. The parameter $\tilde{z}_{0,t}^Y$ denotes the year fixed effect for the log variance of the initial draw of persistent earnings in year $t$. The parameters that govern the age quadratic depend upon when the individuals enters the sample. We include this distinction,
because individuals of all ages enter the sample in the first year \((t = t_0)\). When individuals enter the sample after the age of 25 in years after the first year, there is information in their later entry to the labor market, e.g. additional schooling, difficulty finding work etc. The parameters \(z_{0,1}^A\) and \(z_{0,2}^A\) govern the age quadratic for individuals who enter the sample in the first year, while the parameters \(z_{0,3}^A\) and \(z_{0,4}^A\) govern the age quadratic for individuals who enter the sample after the first year. The estimation recovers a set of parameters \(\{z_{0,0}^A, z_{0,1}^Y, ..., z_{0,T}^Y, z_{0,1}^A, z_{0,2}^A, z_{0,3}^A, z_{0,4}^A\}\), which govern the standard deviation of shocks to temporary earnings.

**Drift of persistent shocks**  Let \(B_{E,t}\) denote the drift of the persistent earnings shock to the employed in year \(t\). We model the \(B_{E,t}\) as follows:

\[
B_{E,t} = B_E + B_{E,t}^Y
\]

where \(B_E\) denotes the drift of the shock to persistent earnings in the initial year of the sample. The parameter \(B_{E,t}^Y\) denotes the year fixed effect for the drift to the shock of persistent earnings among the employed in year \(t\). Suppose that there are \(T\) periods. Then, the estimation recovers a set of parameters \(\{B_E, B_{E,1}^Y, ..., B_{E,T}^Y\}\), which govern the drift of the shocks to persistent earnings among the employed.

We similarly define \(B_{U,t}\) as the drift of shocks to persistent earnings for the unemployed in year \(t\), and proceed as above to estimate a set of parameters \(\{B_E, B_{E,1}^Y, ..., B_{E,T}^Y\}\).

**D.1 Additional Extensions.**

In Section 4.4 we allow the parameters of the income process to depend upon: education, gender, geography as well as occupation. To recover these parameters, we use the approach from above and simply allow for each parameter to depend upon the appropriate conditioning variable (e.g. education, gender, geography, occupation, etc.).

**E Model Estimation**

In this Appendix, we discuss how we solve the life-cycle Bewley model. We solve the model using value function iteration on grids. Below we outline the algorithm for solving the model and discuss the process for discretizing income shocks.
E.1 Discretization Process (persistent Earnings)

In this section, we outline our process for discretizing shocks to persistent earnings where agents receive different shocks when employed versus unemployed.

At the start of the period an agent draws whether or not they will be employed for the period. Let $\lambda_U \in [0, 1]$ denote the probability that an agent is classified as unemployed. Recall that the process for persistent earnings is given by:

$$z' = \rho z + \mu_e + \eta_e$$

where $e \in \{E, U\}$ denotes employment status, $\mu_e$ denotes the drift of persistent earnings while in employment status $e$, and $\eta_e$ is the shock to persistent earnings while in employment status $e$. We assume that the drifts to persistent earnings and the variance of the shocks to persistent earnings differ by employment status. That is $\eta_U \sim N(0, \sigma_{\eta,U}^2)$, and $\eta_E \sim N(0, \sigma_{\eta,E}^2)$.

Define a transition matrix for agents classified as employed, denoted $\pi^E$, and a transition matrix for agents classified as unemployed, denoted $\pi^U$. The elements of $\pi^e_{jk}$ defines the probability that an agent with employment status $e$, moves from state $j$ today to state $k$ tomorrow.

Assume for now that we have specified a grid of values for $z$ with $N$ grid points, which are given by $[z_1, z_2, ..., z_N]$. Let the points be evenly spaced, with distance between grid points denoted by $d$.\(^{55}\) The transition probability of going from state $j$ today to state $k$ tomorrow for an individual with employment status $e$ is given by

$$\pi^e_{jk} = P(z_k = z_{k+1} \mid z_j)$$

$$= P(z_k - \frac{d}{2} < \rho z_j + \mu_e + \eta_e < z_k + \frac{d}{2})$$

$$= P(z_k - \frac{d}{2} - \rho z_j - \mu_e < \eta_e < z_k + \frac{d}{2} - \rho z_j - \mu_e)$$

(28)

For an interior point on the grid, the probability in equation 28 is given by:

$$\pi^e_{jk} = F\left(\frac{z_k + \frac{d}{2} - \rho z_j - \mu_e}{\sigma_{\eta,e}}\right) - F\left(\frac{z_k - \frac{d}{2} - \rho z_j - \mu_e}{\sigma_{\eta,e}}\right)$$

where $F(\cdot)$ is the standard normal distribution. For the end points of the grid, define the prob-

\(^{55}\)In practice, we define the endpoints of the grid using $z_N = m \left(\frac{\sigma_{\eta,U}^2}{\sigma_{\eta,U}^2 + \sigma_{\eta,E}^2}\right)^\frac{1}{2}$, setting $m = 3$, and $z_1 = -z_N$. 

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abilities using:

\[ \pi_{j1}^e = F\left(\frac{z_1 + \frac{d}{2} - \rho z_j - \mu e}{\sigma_{\eta,e}}\right) \]
\[ \pi_{jN}^e = F\left(\frac{z_N - \frac{d}{2} - \rho z_j - \mu e}{\sigma_{\eta,e}}\right) \]

E.2 Discretization Process (Temporary Earnings)

To discretize the process for temporary earnings, we use Tauchen’s method with the persistence of the shock set to zero.

F Welfare Calculation

In this section, we describe our process for performing the welfare calculation. We first discuss the welfare calculation for the steady state experiments, and then discuss the welfare calculation for the transition path experiment.

F.1 Steady State Welfare Calculation

Let \( \{c_j^l\}_{t=1}^T \) be the consumption policy function for an individual \( j \) over their lifetime with baseline income risk. Let \( \{\tilde{c}_j^l\}_{t=1}^T \) be the consumption policy function for an individual \( j \) under an alternative amount of income risk. We will perform welfare calculations by estimating the share of lifetime consumption an individual would be willing to forgo (or must receive) to leave the baseline economy and move to an economy with an alternative amount of income risk. Formally, we estimate the scaling factor for consumption \( \lambda_j \) that makes individual \( j \) indifferent between living under either amount of income risk:

\[
\sum_{t=1}^T \beta_t \left( \left( \lambda_j c_j^l \right)^{1-\sigma} - 1 \right) = \sum_{t=1}^T \beta_t \left( \left( \tilde{c}_j^l \right)^{1-\sigma} - 1 \right) \quad (29)
\]

Solving equation (29) for \( \lambda_j \) returns:
\[
\lambda_j = \left[ \frac{\sum_{t=1}^{T} \beta_t \left( \left( \hat{c}_j \right)^{1-\sigma} \right)}{\sum_{t=1}^{T} \beta_t \left( \left( \hat{c}_j \right)^{1-\sigma} \right)} \right]^{\frac{1}{1-\sigma}} \tag{30}
\]

We use the model to simulate a large mass of individuals under a series of alternative amounts of labor income risk. Let \( p \) denote an economy with an alternative amount of labor income risk. The utilitarian welfare effect for an alternative economy \( p \), which is denoted \( \text{Welfare}(p) \), is measured as:

\[
\text{Welfare}(p) = \frac{1}{N} \sum_{j=1}^{N} \lambda_{j,p}
\]

### G Additional Results

#### G.1 Unemployment Probability

In this Appendix, we present results on the probability that an agent has earnings below the minimum earnings threshold (e.g. is classified as unemployed). The income process that we define in Section 2 is agnostic on the process for which individuals transition between having earnings above and below the minimum earnings threshold. We then use our estimates of filtered persistent earnings to examine the probability of being unemployed.

Let \( \delta(z,e) \in [0,1] \) denote the probability that an agent becomes unemployed. The probability that an agent becomes unemployed depends upon their persistent earnings and employment status from the prior period. In particular, we model the probability that an individual becomes unemployed using the following functional form:

\[
\delta_i(z,e) = \begin{cases} 
\mathbb{I}\{z \geq 0\} \left[ \sum_{k=0}^{2} \alpha_k^+ z \right] + \mathbb{I}\{z < 0\} \left[ \sum_{k=0}^{2} \alpha_k^- z \right] & e = E \\
\alpha_U & e = U
\end{cases}
\tag{31}
\]

The functional form in equation 31 allows for the unemployment probability of the employed to be a quadratic function of prior persistent earnings (\( \hat{z}_{i,t-1} \)) estimated separately for positive prior persistent earnings or negative prior persistent earnings. We define the unemployment probability for the unemployed to be a constant.\(^{56}\) We estimate equation 31 using our filtered

---

\(^{56}\)When we estimated the quadratic function in equation 31 for the unemployed, we obtained results that were consistent with simply using a constant function by education.
estimates of persistent earnings and individuals realizations of being below the minimum earnings criteria. Table [X] presents the results from estimating equation 31.

To gauge the ability of the functional form in equation 31 to capture the dynamics of becoming unemployed in the data, we assign individuals to ventiles based upon their prior persistent earnings and measure the share of individuals by ventile who become unemployed in the next year. Figure 13 compare the observed share of unemployed individuals by ventile of prior persistent earnings (black, solid line) to the predicted value based upon the results of estimating equation 31 (red, dashed line). The figure shows that the functional form in equation 31 is able to accurately capture the dynamics of who becomes unemployed as a function of their prior persistent earnings.

G.2 Earnings risk over the life cycle

Figure 14 shows parameter estimates of the shocks to earnings by age.57

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57Note that in Figure [X] we present the standard deviation of shocks to income by age, holding the year component fixed at the initial value.
Figure 14: Earnings risk over the life cycle

(a) Std. Dev. of Shock to Permanent Earnings (Emp.)

(b) Std. Dev. of Shock to Permanent Earnings (Unemp.)

(c) Std. Dev. of Shock to Temporary Earnings

(d) Std. Dev. of Initial Draw of Perm. Earnings

Note: Figure presents estimates for the standard deviation of: (1) the shock to persistent earnings among the employed (Panel (a)), (2) the shock to persistent earnings among the unemployed (Panel (b)), (3) the shock to temporary earnings (Panel (c)), and (4) the initial draw of persistent earnings, as a function of age based upon the estimation from Section 4. Dashed line represent a 95% confidence interval.

G.3 Changes in earnings risk with lower minimum earnings cutoff

In this Appendix, we present the results from estimating the income process from Section 2 when the minimum earnings criteria is set to the equivalent of working part-time (20 hours per week) for 1-quarter at the real federal minimum wage, which corresponds to approximately $2k in 2019 dollars. Figure 15 plots the standard deviation of residual log earnings changes by year. The figure shows that with the lower minimum earnings threshold there is a significant decline in earnings risk over the sample period.

Figure 15: Earnings risk over time

![Figure 15: Earnings risk over time](image)

Note: Figure presents the standard deviation of residualized log earnings over time (black, solid line) and a linear trend line (red, dashed line).


Figure 17 plots the parameter estimates from the estimation of the income process in Section 2 with the lower minimum earnings threshold.

G.4 Changes in earnings risk over time by gender

Figure 17 shows parameter estimates of the shocks to earnings by gender over time.
Figure 16: Changes in earnings risk over time with lower minimum earnings cutoff

(a) Std. dev. persistent earnings (Emp.)
(b) Std. dev. persistent earnings (Unemp.)
(c) Std. dev. temporary earnings
(d) Initial draw persistent earnings
(e) Drift persistent earnings (Emp.)
(f) Drift persistent earnings (Unemp.)

Note: Figure presents parameter estimates of the shocks to earnings over time with the lower minimum earnings cutoff.
Figure 17: Changes in earnings risk over time by gender

(a) Std. dev. persistent earnings (Emp.)

(b) Std. dev. persistent earnings (Unemp.)

(c) Std. dev. temporary earnings

(d) Initial draw persistent earnings

(e) Drift persistent earnings (Emp.)

(f) Drift persistent earnings (Unemp.)

Note: Figure presents parameter estimates of the shocks to earnings over time by gender. The black solid line denotes men, and the red dashed line represents women.
G.5 Additional Results: Geographic Variation

In this Appendix, we present additional results where we estimate the income process by state. We formally test the second hypothesis that rising persistent earnings risk is related to the declines in manufacturing employment and union membership. Let $X_s$ be the change in union membership (manufacturing employment) in state $s$ between 1985-1991 and 2006-2013. The share of employed workers that are members of a union is measured in the CPS, while manufacturing employment is based upon Fort and Klimek (2016) industry classifications in the SSA data. Let $\Delta Y_{s,j} = Y_{s,j} - Y_{s,(1985-1991)}$ denote the change in parameter $Y$ (e.g. the standard deviations of shocks to persistent earnings among employed etc.) for state $s$ between time period $j$ and 1985 – 1991.\(^{58}\) Let $\gamma_j$ denote a set of year window fixed effects. The specification we use is of the form:

$$\Delta Y_{s,j} = \alpha + \eta X_s + \gamma_j + \epsilon_{s,j}$$

(32)

The parameter of interest is $\eta$ which reports the correlation between the change in union membership (manufacturing employment) in a state and measures of earnings risk in that state. If $\eta < 0$, then we have evidence that in states with larger declines in union coverage (manufacturing employment) there have been larger increases in earnings risk.

Tables 8 and 9 present the results of estimating equation 32 for changes in union coverage and manufacturing employment, respectively. The tables show that changes in union coverage and manufacturing employment are largely uncorrelated with changes in earnings risk.

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Table 8: Changes in union membership and earnings risk

<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chg. Union Membership</td>
<td>-0.00168</td>
<td>0.00106</td>
<td>-0.00168</td>
<td>-0.00171</td>
<td>0.00250*</td>
<td>0.00480*</td>
</tr>
<tr>
<td></td>
<td>(0.00135)</td>
<td>(0.00317)</td>
<td>(0.00143)</td>
<td>(0.00181)</td>
<td>(0.00125)</td>
<td>(0.00247)</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R-Sq.</td>
<td>0.284</td>
<td>0.391</td>
<td>0.296</td>
<td>0.190</td>
<td>0.330</td>
<td>0.391</td>
</tr>
<tr>
<td>No. Obs (States, rounded)</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

Notes: Table presents results from estimating equation 32 where the independent variable is the change in union membership by state. The change in union membership is measured between 1985-1991 and 2006-2013 for each state, and have been normalized to have mean zero and unit variance. Clustered SE in parenthesis, where the clustering is performed at the state level. ***p < 0.01, **p < 0.05, *p < 0.1.


Table 9: Changes in manufacturing employment and earnings risk

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chg. Manufacturing Share</td>
<td>-0.00348*</td>
<td>0.00598*</td>
<td>-0.00355*</td>
<td>-0.00587**</td>
<td>0.00591***</td>
<td>0.00285</td>
</tr>
<tr>
<td></td>
<td>(0.00175)</td>
<td>(0.00329)</td>
<td>(0.00183)</td>
<td>(0.00254)</td>
<td>(0.00141)</td>
<td>(0.00291)</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R-Sq.</td>
<td>0.315</td>
<td>0.417</td>
<td>0.329</td>
<td>0.271</td>
<td>0.467</td>
<td>0.369</td>
</tr>
<tr>
<td>No. Obs (States, rounded)</td>
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<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

Notes: Table presents results from estimating equation 32 where the independent variable is the change in manufacturing employment share by state. The change in manufacturing employment share is measured between 1985-1991 and 2006-2013 for each state, and have been normalized to have mean zero and unit variance. Clustered SE in parenthesis, where the clustering is performed at the state level. ***p < 0.01, **p < 0.05, *p < 0.1.

G.6 Additional Results: High Skill Occupations

In this Appendix, we present addition results where we estimate the income process by occupation.

**Mean Earnings.** In this section, we split occupations by their mean log earnings for the 1985 – 1991 time period. To ease the interpretation we normalize the statistic to have mean zero and standard deviation equal to one. Table 10 presents the results of estimating equation 6 where the independent variable is mean log earnings in the occupation between 1985 and 1991.

**Years of Education.** In this section, we split occupations by their average years of education in the 1985 – 1991 time period. To ease the interpretation we normalize the statistic to have mean zero and standard deviation equal to one. Table 11 presents the results of estimating equation 6 where the independent variable is the mean years of completed education in the occupation between 1985 and 1991.
Table 10: Mean Earnings and Changes in Earnings Risk

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Log Earnings</td>
<td>0.00588***</td>
<td>0.00981**</td>
<td>0.00592***</td>
<td>-0.000721</td>
<td>-0.000921</td>
<td>-0.0177***</td>
</tr>
<tr>
<td></td>
<td>(0.00118)</td>
<td>(0.00400)</td>
<td>(0.00120)</td>
<td>(0.00212)</td>
<td>(0.00102)</td>
<td>(0.00262)</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R-Sq.</td>
<td>0.196</td>
<td>0.069</td>
<td>0.195</td>
<td>0.064</td>
<td>0.208</td>
<td>0.216</td>
</tr>
<tr>
<td>No. Obs. (Occ.)</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

Notes: Table presents results from estimating equation 6 where the independent variable is mean log earnings in an occupation. Mean log earnings are measured in 1985-1991 for each occupation, and have been normalized to have mean zero and unit variance.


Table 11: Mean Years of Education and Changes in Earnings Risk

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Years Edu.</td>
<td>0.00416**</td>
<td>0.0142***</td>
<td>0.00370**</td>
<td>-0.00812***</td>
<td>-0.00346***</td>
<td>-0.0125***</td>
</tr>
<tr>
<td></td>
<td>(0.00187)</td>
<td>(0.00355)</td>
<td>(0.00184)</td>
<td>(0.00180)</td>
<td>(0.000801)</td>
<td>(0.00307)</td>
</tr>
<tr>
<td>Year Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>R-Sq.</td>
<td>0.159</td>
<td>0.086</td>
<td>0.151</td>
<td>0.174</td>
<td>0.288</td>
<td>0.155</td>
</tr>
<tr>
<td>No. Obs. (Occ.)</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

Notes: Table presents results from estimating equation 6 where the independent variable is mean years of education in an occupation. Mean years of education are measured in 1985-1991 for each occupation, and have been normalized to have mean zero and unit variance.

G.7 Identifying trends in temporary and persistent earnings

In this section, we discuss the intuition for how the trends in temporary and persistent income are identified. We show below that the variance of shocks to persistent and temporary earnings can be identified from the variance in log earnings changes over different horizons. We can then identify trends in temporary and persistent risk by examining how these variances evolve over time. We illustrate below this identification using 1-year and 5-year changes in residual log earnings, however, the full estimation in Section 2 uses the full path of earnings (and hence their changes) for each individual.

For ease of exposition, we abstract from incorporating periods of zero earnings, remove the drift parameters, and assume that persistent earnings following a unit root process. With these assumptions the income process is given by,

\[ y_{i,t} = z_{i,t} + \omega_{i,t} \]
\[ z_{i,t} = z_{i,t-1} + \eta_{i,t} \]

where the temporary earnings shock \((\omega_{i,t})\) has a variance \(R\), and the persistent shock \((\nu_{i,t})\) has variance \(Q\). With the income process specified in this manner, we can write the variance of earnings changes over 1 and 5-year horizons as follows,

\[
\text{var}(y_{i,t} - y_{i,t-1}) = Q + 2R \tag{33}
\]
\[
\text{var}(y_{i,t} - y_{i,t-5}) = 5Q + 2R \tag{34}
\]

Hence, with an estimate of the variance of earnings changes over 1 and 5-year, we can identify the variance of temporary and persistent shocks. Then, by examining these variance at different points in time, we can identify how the variance of temporary and persistent earnings have changed over time. Let \(V_{1,t} = \text{var}(y_{i,t} - y_{i,t-1})\), and \(V_{5,t} = \text{var}(y_{i,t} - y_{i,t-5})\). Then solving the system of equations in 33 and 34, we have,

\[
Q_t = \frac{V_{5,t} - V_{1,t}}{4} \tag{35}
\]
\[
R_t = \frac{5V_{1,t} - V_{5,t}}{4} \tag{36}
\]

In panel (a) of Figure 18, we plot the standard deviation of log earnings changes over a 1-year horizon (black, solid line) and 5-year horizon (red, dashed line). Next, in panel (b) of Figure 18, we plot the standard deviation of shocks to persistent earnings (black, solid line) and temporary earnings (red, dashed line) implied by equations 35 and 36. To ease the illustration
of the trends, we normalize the initial values to be equal to 100. The time series in Panel (b) of Figure 18 show that the path of persistent and temporary earnings implied by the 1 and 5-year variances align with our baseline estimates. In particular, we find an increase in persistent risk and a decline in temporary risk over the sample period.

Figure 18: Identifying trends persistent and temporary risk

(a) Std. residual log earnings changes

(b) Implied persistent and temporary earnings risk

Note: Panel (a) of the figure plots the standard deviation of residual log earnings changes over 1-year (black solid line) and 5-year (red, dashed line) horizons. Panel (b) plots the time series of persistent and temporary earnings implied by equations 35 and 36.