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Leveraging Predictive Analytics to Control and Coordinate Operations, Asset Loading and Maintenance

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Abstract

This paper aims to advance decision-making in power systems by proposing an integrated framework that combines sensor data analytics and optimization. Our modeling framework consists of two components: (1) a predictive analytics methodology that uses real-time sensor data to predict future degradation and remaining lifetime of generators as a function of the loading conditions, and (2) a mixed integer optimization model that transforms these predictions into cost-optimal maintenance and operational decisions. We model the key balance between meeting demand with very high confidence and at the same time prolonging the lifetime of generation assets. To do so, we encapsulate stochastic loading-dependent predictive models for asset condition within our optimization model. The methodology is validated and evaluated using IEEE 118-bus system that has been augmented using real-world sensor-based vibration signals from rotating machinery to emulate physical degradation of generators. Our experiments suggest that the proposed framework provides considerable improvements over conventional methods in terms of cost and reliability.

Index Terms

Loading-dependent degradation models, generation maintenance, condition based maintenance, asset reliability and sustainability, mixed integer optimization

I. Introduction

The modern grid creates a highly dynamic operational environment for capital intensive generation assets. Critical assets used in power plants that were originally designed to operate at steady base loads are today required to adjust their loading schedules and profiles to compensate for the dynamic grid conditions. These loading profiles play a critical role in how long power generation assets can operate before requiring maintenance. In general, assets that operate under harsh loading conditions are bound

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to fail faster than similar units operating under milder conditions. For instance, production decisions can have significant impact on the useful life of capital-intensive generation assets. As a result, accounting for asset loading conditions and their effects on asset degradation is very important for generating cost-effective maintenance schedules that do not disrupt daily power network operations. It also has significant implications on cost and reliability. It is, therefore, key to consider the unit commitment (UC), asset loading, and generation maintenance problems, simultaneously.

Generation maintenance and unit commitment are fundamental optimization problems in power systems analysis. Generation maintenance identifies the optimal time to conduct repairs and maintenance for generation assets. Given a maintenance schedule, UC problem determines the commitment and dispatch decisions that define which generation assets should be committed to producing electric power and how much each asset should produce. Generation maintenance is largely based on time-based schedules [1]–[4]. Time-based maintenance policies often recommend repairs on a periodic (calendar-based) schedule regardless of the operating conditions or the degradation state of the asset. As a result, many power plants are faced with one of the following extreme scenarios; they either experience significant unexpected failures, or employ a conservative schedule that drives frequent unnecessary maintenance events that usually impact asset availability and increase the likelihood of human errors. With many generation assets operating beyond their design life, the limitations of time-based policies are becoming increasingly apparent.

Today, advances in sensor technology and wireless communication are playing a vital role in enabling what many refer to as the Industrial Internet of Things (IIoT). Remote "condition monitoring" of physical and performance degradation of long-term capital-intensive plant assets (e.g. turbines, generators, boilers, etc.) is one of the most important IIoT applications in the energy sector. The goal of condition monitoring is to reduce the risk of unexpected failures by providing advance warning of any impending faults. Numerous examples of condition monitoring of generation assets have been presented in the literature [5]–[7]. The majority of the literature in this domain rests on the premise of utilizing sensor data to detect equipment faults by studying deviations from baselines data signatures that represent normal operating conditions. *Predictive degradation modeling*, however, extends the value of condition monitoring applications to the prediction of asset remaining operational life. This is accomplished by identifying and studying characteristic trends in condition monitoring data, especially ones that are correlated with the severity of physical degradation. When modeled properly, degradation-based sensor signals (degradation signals) can be used to predict remaining lifetime.

Remaining lifetime predictions are a fundamental component of proactive maintenance (aka. predictive or condition-based maintenance). Proactive maintenance policies leverage life predictions to optimize maintenance and repair schedules. Unlike time-based maintenance, proactive maintenance considers the state-of-health of assets and generates significant improvements in asset reliability and availability. According to [8], cost savings generated by implementing proactive maintenance can sometimes exceed 50%. Yet, the majority of the UC literature does not consider proactive maintenance policies. Most UC models employ basic constraints to capture the maintenance of critical assets [2], [3], [9], [10]. A commonly used constraint is one where an asset is required to undergo at least one maintenance event within a specified time period, say every year [1]–[3], [9]–[13]. Some other approaches consider additional maintenance dependencies, such as priorities, exclusions, and separations between consecutive maintenances [1], [10], [12]. In a recent work [14], [15], the authors proposed a joint optimization model that integrates predictive degradation modeling and UC. In [14], the authors present a mixed-integer programming model for generation maintenance scheduling that utilizes real-time condition monitoring data to schedule maintenance of power generation assets across the network. In the second part of their work [15], the authors extended their framework to account for the impact of maintenance on network operations by coordinating generator maintenance schedules with UC and and dispatch decisions.

However, one of the key limitations of current maintenance policies, time-based and proactive alike, is that the prevailing environmental and/or operational conditions are assumed to be constant. In other words, the future operating and loading conditions in which the assets will operate are not considered when scheduling maintenance and UC decisions. Accounting for future asset loading conditions is very important for generating cost-effective maintenance schedules. By controlling the loading conditions, asset maintenance can be delayed (or expedited) without increasing the risk of unexpected failure. By capturing the interaction between operational decisions and asset loading profiles, system operators can exercise significant control over the rate of degradation experienced by various critical plant assets. This paper presents a methodology that enhances our understanding of the role of asset loading conditions in jointly optimizing generation maintenance schedules and UC decisions. Specifically, the paper provides a modeling framework that i) uses sensor data to predict the failure times of critical generation assets *under different loading profiles*, and ii) jointly optimizes maintenance and UC by using asset loading to control commitment, dispatching, and generation maintenance decisions. Unique to our approach, is the integration of asset loading to operations and maintenance scheduling in power systems. The main contributions of the paper can be summarized as follows:

1) We formulate a MIP that jointly optimizes generation maintenance and UC in the context of asset loading. Specifically, our model formulation allows us to better understand and characterize how dispatch decisions impact the loading conditions, and the remaining lifetimes of power generation assets. This information is then used to optimize maintenance decisions accordingly.

2) We consider a two-step stochastic degradation modeling framework that predicts and updates, in real-time, statistical distributions for the remaining lifetime of power generation assets. First, we develop a Bayesian updating procedure that uses condition monitoring data to improve the accuracy of the predictive degradation models. Second, we derive a closed-form expression that utilizes the updated degradation models to compute posterior remaining life distributions of the generation assets operating in time-varying loading conditions. This degradation framework provides a one-to-one mapping between current and future asset loading conditions, and their effects on the remaining life distributions.
3) We construct an extensive experimental platform that uses real-world condition monitoring data from machines subjected to different loading conditions. We utilize the sensor data and loading decisions from the optimization model to mimic degradation of generation assets in dynamic environments. Key maintenance, operational and cost performance metrics are evaluated within this framework.

Experiments on the IEEE 118-bus system indicate that the proposed method provides significant advantages over existing approaches in terms of the effective use of equipment lifetime, asset reliability, and total cost. The remainder of the paper proceeds as follows. Section II presents the predictive degradation model. Section III introduces a joint optimization model that determines optimal scheduling decisions by taking into account the effects of generator loading on maintenance and operations. In Section IV, we present the experimental framework and the results of our numerical studies. Conclusions are provided in Section V.

II. Predictive Degradation Modeling

We model the degradation signal as a continuous-time continuous-state stochastic process with a combination of fixed and random parameters. Fixed parameters are used to capture deterministic degradation attributes that are common across a population of identical generation assets. Random parameters are assumed to follow a statistical distribution, and capture unit-to-unit variability among individual generation assets. Degradation process also depends on the future loading conditions. If we are modeling the degradation signal at time t, we need complete information on the future loading conditions until time t, namely $\{\gamma(s) : 0 \leq s \leq t\}$. Our key underlying assumption is that changes in loading conditions manifest themselves in the rate at which the degradation signal increases/decreases and the signal-to-noise ratio – high loading generate noisier degradation signals. Formally, the degradation signal can be expressed as follows:

$$D_i(t, \gamma) = \theta_i + \int_0^t r_i\left(\gamma(s)\right) ds + \int_0^t v_i(\gamma(s)) dW(s). \tag{1}$$

where $D_i(t,\gamma)$ is the amplitude of the degradation signal of generation asset i at time t. θ_i denotes the initial amplitude of the degradation signal, which follows a normal distribution $\pi_i(\theta_i) \sim N(\mu_0, \sigma_0^2)$. $r_i\left(\gamma(t)\right)$ and $v_i\left(\gamma(t)\right)$ are the functions associated with rate and diffusion of degradation signal, respectively. These functions can be further decomposed as $r_i\left(\gamma(t)\right) = \beta_i \cdot \Psi(\gamma(t))$ and $v_i\left(\gamma(t)\right) = [\sigma^2 \cdot \Psi(\gamma(t))]^{1/2}$, where the $\Psi : \mathbb{R}_{\geq 0}$ provides a mapping between the loading condition $\gamma(t)$ and the resulting multipliers for rate and diffusion. β_i denotes the nominal rate of degradation for generation asset i, which follows a normal distribution $\pi_i(\beta_i) \sim N(\mu_1, \sigma_1^2)$. $\{W(t) : t > 0\}$ is a standard Brownian process that captures signal noise. Related yet distinct degradation models in literature can be found in [16], [17].

We define the remaining life R_{i,t_0} for a new generation asset i, as the first time that the degradation function $\{D_i(t,\gamma) : t > 0\}$ reaches a failure threshold Λ_i. More specifically, $R_{i,t_0} = \inf\{t > 0 : D_i(t,\gamma) \geq \Lambda_i\}$.

A. *Updating the Degradation Model using Real-Time Sensor Data*

Ideally, a general predictive degradation model is defined for each asset family. This model is assumed to capture variability in the degradation rates of the asset population using the random model parameters. Asset-specific sensor data is then used to derive updated instances of the degradation model based on the unique degradation characteristics of each asset. This is significant because even identical generation assets operating under the same loading conditions degrade differently due to differences in manufacturing tolerances and other processing and material homogeneities. The model is also updated based on the current and future loading conditions experienced by the asset. This updating process allows us to account for added variability resulting from load changes.

The updating process is performed using a Bayesian framework. We begin by defining the likelihood function of an observed degradation signal. Without loss of generality, we consider a partial degradation signal $D(t_1,\gamma),\ldots,D(t_k,\gamma)$ at times $t_1,\ldots t_k$, and define the signal increment $d_j = D(t_j,\gamma) - D(t_{j-1},\gamma)$. Given θ_i and β_i, $d_1,\ldots,d_k$ are independent and identically distributed normal random variables with mean $\beta_i \int_{t_{j-1}}^{t_j} \Psi(\gamma(s))ds$ and variance $\sigma^2 \int_{t_{j-1}}^{t_j} \Psi(\gamma(s))ds$. The likelihood function of $d_1,\ldots,d_k$ can be expressed as follows:

$$
\begin{aligned}
f(d_1,\ldots,d_k|\theta_i,\beta_i) = & \prod_{j=1}^{k} \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) \\
& \cdot \exp\left[-\frac{(d_1-\theta_i-\beta_i\xi_1)^2}{2\sigma^2\xi_1} - \sum_{j=2}^{k}\left(\frac{(d_j-\beta_i\xi_j)^2}{2\sigma^2\xi_j}\right)\right],
\end{aligned} \tag{2}
$$

where $\xi_j = \int_{t_{j-1}}^{t_j} \Psi(\gamma(s))ds$. The above likelihood function along with the prior distributions, $\pi_i(\theta_i)$ and $\pi_i(\beta_i)$, are used to derive the posterior distribution of θ_i and β_i as outlined in Proposition 1.

Proposition 1. *Given the observed data $d_1, \ldots, d_k$, the posterior distribution of the degradation parameters (θ_i, β_i) follows a bivariate normal distribution with mean $(\mu_{\theta_i}, \mu_{\beta_i})$, variance $(\sigma^2_{\theta_i}, \sigma^2_{\beta_i})$ and correlation coefficient ρ_i*

$$\begin{aligned}
\mu_{\theta_i} &= \frac{\left(d_1\sigma_0^2 + \mu_0\sigma^2\xi_1\right)\left(\sigma_1^2\Upsilon_k + \sigma^2\right) - \sigma_0^2\xi_1^{\mu}\left(\sigma_1^2 D_k + \mu_1\sigma^2\right)}{\left(\sigma_0^2 + \sigma^2\xi_1\right)\left(\sigma_1^2\Upsilon_k + \sigma^2\right) - \sigma_0^2\sigma_1^2\xi_1}, \\
\sigma^2_{\theta_i} &= \frac{\sigma^2\sigma_0^2\xi_1\left(\sigma_1^2\Upsilon_k + \sigma^2\right)}{\left(\sigma_0^2 + \sigma^2\xi_1\right)\left(\sigma_1^2\Upsilon_k + \sigma^2\right) - \sigma_0^2\sigma_1^2\xi_1}, \\
\mu_{\beta_i} &= \frac{\left(\sigma_1^2 D_k + \mu_1\sigma^2\right)\left(\sigma_0^2 + \sigma^2\xi_1^{\mu}\right) - \sigma_1^2\left(d_1\sigma_0^2 + \mu_0\sigma^2\xi_1\right)}{\left(\sigma_0^2 + \sigma^2\xi_1\right)\left(\sigma_1^2\Upsilon_k + \sigma^2\right) - \sigma_0^2\sigma_1^2\xi_1}, \\
\sigma^2_{\beta_i} &= \frac{\sigma^2\sigma_1^2\left(\sigma_0^2 + \sigma^2\xi_1^{\mu}\right)}{\left(\sigma_0^2 + \sigma^2\xi_1\right)\left(\sigma_1^2\Upsilon_k + \sigma^2\right) - \sigma_0^2\sigma_1^2\xi_1}, \\
\rho_i &= -\frac{\sigma_0\sigma_1\sqrt{\xi_1}}{\sqrt{\left(\sigma_0^2 + \sigma^2\xi_1\right)\left(\sigma_1^2\Upsilon_k + \sigma^2\right)}},
\end{aligned}$$

where $\Upsilon_k = \sum_{j=1}^{k} \xi_j$ and $D_k = D(t_k) = \sum_{j=1}^{k} d_j$.

Proof: See Appendix A. ■

Updated posterior distribution of the degradation parameters is key to revising our predictions on remaining life of generation assets.

B. Predicting the Remaining Life Distribution using Bayesian Updating

We leverage on the findings from [18] and [19] to provide a closed form expression for remaining life that conditions on the loading conditions and the posterior mean of the degradation parameters:

Lemma 1. *Given the degradation function defined in* (1) *with an associated continuous loading condition function $\gamma(s)$; the distribution of the remaining life at the time of observation t^ℓ, is $P(R_{i,t^\ell} \leq t|\zeta_i, \alpha_i, \boldsymbol{\gamma}_i^t) = IG(\tau(t)|\zeta_i, \alpha_i)$, $t > 0$, where $IG(x|a, b)$ defines the CDF of an inverse Gaussian distribution with shape and mean parameters a and b:*

$$\begin{aligned}
P(R_{i,t^\ell} \leq t|\mu_\beta, \boldsymbol{\gamma}_i^t) &= P\left(\tau(R_{i,t^\ell}) \leq \tau(t) = \int_{s=1}^{t} \Psi_i(\gamma(s))ds\right) \\
&= IG(\tau_i(t)|\zeta_i, \alpha_i) = \Phi\left(\sqrt{\frac{\alpha_i}{\tau(t)}}\exp\left(\frac{\tau(t)}{\zeta_i} - 1\right)\right) \\
&\qquad + \exp\left(\frac{2\alpha_i}{\zeta_i}\right)\Phi\left(-\sqrt{\frac{\alpha_i}{\tau(t)}}\exp\left(\frac{\tau(t)}{\zeta_i} + 1\right)\right),
\end{aligned} \tag{3}$$

where $\zeta_i = \frac{\Lambda_i - d_i(t_o)}{\mu_\beta}$, $\alpha_i = \frac{(\Lambda_i - d_i(t_o))^2}{\sigma^2}$, and $\tau(t) = \int_{s=1}^{t} \Psi_i(\gamma(s))ds$.

The main strategy behind Lemma 1 is to obtain an appropriate time-transformation to project the degradation function to an alternative domain, where an equivalent degradation function would have constant rate and diffusion terms. The mapping between the actual time t, and the transformed time $\tau(t)$ depends on the loading condition $\{\gamma(s) : 0 \leq s \leq t\}$. However, the projected degradation function is independent of the loading conditions. In fact, it reduces to a Brownian Motion with positive drift, whose remaining life can be obtained by the inverse Gaussian distribution.

As a motivating example, we subject a generation asset to three different loading conditions. In the first case, generation asset operates under harsh condition in the first week, and returns to nominal condition in the second week, i.e. $\{\Psi_i(\gamma(t)) = 2 : 0 \leq t < 1, \Psi_i(\gamma(t)) = 1 : 1 \leq t \leq 2\}$. In the second case, we reverse the ordering of the loading conditions. The third case operates the generation asset under nominal loading for three weeks, i.e. $\{\Psi_i(\gamma(t)) = 1 : 0 \leq t < 3\}$. The failure probabilities in all three cases would be identical - i.e. sum of loading conditions is 3 in all cases. This example demonstrates that the remaining life predictions are order-invariant (i.e. Cases 1 & 2), and duration-invariant (i.e. Cases 1 & 3). The only information needed to predict the distribution of the remaining life is the transformed time $\tau(t) = \int_0^t \Psi(\gamma(s))ds$.

C. Updating the Dynamic Maintenance Cost Function

Our proposed dynamic maintenance cost function models the tradeoff between the cost of premature maintenance vs. the cost of unexpected failures. The basis of this function is proposed by [14], [15], [20]. Unique to our approach is the incorporation of the loading conditions. Instead of defining the dynamic maintenance cost as a function of the loading conditions, we rederive this function in the time transformed domain. The resulting function considers the transformed time $\tau(t) = \int_{s=t_o}^t \Psi_i(\gamma(s))ds$, without explicitly modeling the complete information on the future loading conditions starting from the time of observation t^ℓ, namely $\{\Psi_i(\gamma(s)) : t^\ell \leq s \leq t\}$. The proposed dynamic maintenance cost function can be represented as follows:

$$C^{d,i}_{t_i^\ell,\tau(t)} = \frac{c_i^p P(R_i' > \tau(t)) + c_i^f P(R_i' \leq \tau(t))}{\int_0^{\tau(t)} P(R_i' > z)dz + \tau(t^\ell)}, \tag{4}$$

where $C^{d,i}_{t_i^\ell,\tau(t)}$ is the cost rate associated with conducting maintenance of generation asset i at transformed time $\tau(t)$. The term $\tau(t^\ell)$ is the transformed time of observation, R_i' is the remaining life in the transformed time domain, c_i^p and c_i^f are the costs of planned maintenance and failure replacement, respectively.

Dynamic maintenance cost function is continuously revised and updated as new condition monitoring data becomes available. The probability $P(R_i' > \tau(t))$ within the dynamic maintenance cost function, captures both the impact of loading conditions, and the updated remaining life predictions that are evaluated using expression (3).

III. Adaptive Optimization Model

In this section, we present the Load Dependent Adaptive Predictive Maintenance (LDAPM) model. LDAPM is an integrated model that simultaneously determines optimal generation maintenance and operations scheduling for a fleet of generation assets by explicitly characterizing the interactions between generation loading and asset degradation. Generation maintenance involves using degradation-based sensor data measured from the generation assets to determine the time of maintenance within a planning horizon. The maintenance problem is subject to several constraints that include labor capacity, minimum duration between successive maintenances, and dependencies between generator maintenances. Operations scheduling involves solving the UC problem for a fleet of generation assets. Unlike conventional UC models, we consider operations schedules that also determine the loading conditions of the generation assets and how these conditions interact with maintenance scheduling decisions.

A. Key Variables

We first introduce the key variables for maintenance $\boldsymbol{\nu}, \boldsymbol{z}$ and loading $\boldsymbol{\gamma}$. The binary variables $\boldsymbol{\nu}$, $\boldsymbol{z}$ correspond to the actual, and the transformed time of maintenance, respectively. $\nu_{t,i} = 1$ indicates that maintenance of generation asset i is scheduled at time t. Similarly, $z_{t,i,1} = 1$ indicates that the maintenance of generation asset i occurs at t in the transformed time domain. Finally, $\boldsymbol{\gamma}$ denotes the loading condition, where $\gamma_{t,i,l} = 1$ means that the loading on generation asset i at time t is at least l. We elucidate these variables using a simple example that considers maintenance and operations scheduling of a single generation asset. For ease of exposition, we let $\boldsymbol{\nu}_{.,i} = \{\nu_{1,i}, \ldots, \nu_{|\mathcal{T}|,i}\}$. We use a similar convention for the variables $\boldsymbol{z}$, and $\boldsymbol{\gamma}$. Consider the following schedule:

$$\begin{aligned}
&\text{Maintenance - \textit{Actual Time}:} && \boldsymbol{\nu}_{.,i} = [0,0,0,0,1,0,0],\\
&\text{Maintenance - \textit{Transformed Time}:} && \boldsymbol{z}_{.,i} = [0,0,0,0,0,1,0],\\
&\text{Loading Conditions - \textit{Level 1}:} && \boldsymbol{\gamma}_{.,i,1} = [1,1,1,1,0,0,0],\\
&\text{Loading Conditions - \textit{Level 2}:} && \boldsymbol{\gamma}_{.,i,2} = [0,0,1,1,0,0,0].
\end{aligned}$$

In this schedule, generation asset i experiences a maintenance at time 5 as shown by the variable $\boldsymbol{\nu}$. Loading is indicated by the variable $\boldsymbol{\gamma}$. In the example, generator i is subjected to nominal loadings during times 1 and 2. Therefore, $\gamma_{1,i,1} = \gamma_{2,i,1} = 1$. At time periods 3 and 4, the generation asset is subjected to harsh loading, thus both the first level and the second level loading variables are 1. More specifically, $\gamma_{3,i,1} + \gamma_{3,i,2} = 2$ and $\gamma_{4,i,1} + \gamma_{4,i,2} = 2$. All loading variables are zero during time 5 since there is an ongoing maintenance.

To evaluate the transformed time of maintenance, we need the actual time of maintenance $\boldsymbol{\nu}_{.,i}$, and the loading decisions $\boldsymbol{\gamma}$. By summing over the loading levels until the time of maintenance, we obtain the transformed time of maintenance. For this example, the transformed time of maintenance is $\sum_{l=1}^{2}\sum_{t=1}^{5}\gamma_{t,i,l} = 6$, which is indicated by the variable $\boldsymbol{z}_{.,i}$.

B. Objective Function

Our objective is to minimize the total cost of maintenance and operations:

$$\begin{aligned}\min \quad & \xi_m \sum_{i\in\mathcal{G}} \sum_{t\in\Theta} C^{d,i}_{t^\ell_i, t-T^R_i} \cdot z_{t,i} - \sum_{i\in\mathcal{G}} P^R_i \left(\sum_{t\in\mathcal{T}} t\cdot \nu_{t,i,1}\right) \\ & + \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{G}}\sum_{s\in\mathcal{S}} \left(V^t_i \cdot x^t_{s,i} + P^t_{U,i}\cdot \pi^{U,t}_{s,i} + P^t_{D,i}\cdot \pi^{D,t}_{s,i} + B^t_i \cdot y^t_{s,i}\right) \\ & + \sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}} \left(\sum_{p\in\mathcal{D}} \left(P_{DC}\cdot \psi^{DC,t}_{s,p}\right) + \sum_{r\in\mathcal{R}} \left(P_{TL}\cdot \psi^{TL,t}_{s,r}\right)\right),\end{aligned}$$

where ξ_m is the maintenance criticality coefficient. We note that the first expression of the first line identifies the sensor-driven dynamic maintenance cost, captured through the cost parameter $C^{d,i}_{t^\ell_i, t-T^R_i}$. The calculation of this parameter, and the development of the sensor-driven approach is presented in detail in §II-C. The second expression evaluates the reward for operating the generators for longer time periods before scheduling them for maintenance, where P^R_i is an incentive for extending generation asset's useful life.

The second and the third lines of the objective function provides the operational cost due to commitment & dispatch, and demand curtailment & line capacity penalty, respectively. For generation asset i, maintenance period t, and subperiod s; the terms $y^t_{s,i}$, $x^t_{s,i}$, $\pi^{U,t}_{s,i}$, $\pi^{D,t}_{s,i}$, indicate the continuous variable for generation dispatch, and the binary variables for commitment, start-up, and shut-down, respectively. These variables have associated costs B^t_i, V^t_i, $P^t_{U,i}$, and $P^t_{D,i}$. Demand curtailment for demand location p during period t and subperiod s, is denoted by $\psi^{DC,t}_{s,p}$, with corresponding cost P_{DC}. Likewise, $\psi^{TL,t}_{s,r}$ denotes the transmission line overload for line r during period t and subperiod s, with associated cost P_{TL}. Lastly, $\mathcal{G}, \mathcal{T}, \Theta, \mathcal{S}, \mathcal{D}$, and $\mathcal{R}$ correspond to the sets of generators, maintenance time periods (i.e. weeks), transformed time periods, subperiods (i.e. hours), demand points, and transmission lines, respectively.

C. Constraints for Modeling Maintenance Actions

In the next set of constraints, we establish basic rules for the time of maintenance indicated by variable $\boldsymbol{\nu}$. Constraint (5) selects a maintenance start time for each generation asset i. In constraint (6), we enforce that a unit maintenance cannot be started if there is an ongoing maintenance.

$$\sum_{t\in\mathcal{T}} \nu_{t,i} = 1, \qquad \forall i \in \mathcal{G}, \tag{5}$$

$$\sum_{t\in\mathcal{T}} t\cdot \nu_{t,i,1} \geq T^R_i + 1, \qquad \forall i \in \mathcal{G}, \tag{6}$$

where T^R_i is the remaining time for an ongoing maintenance of generation asset i. Finally we also impose maintenance priorities, exclusions and separations. We present these constraints in compact form as $\boldsymbol{H\nu} \leq \boldsymbol{p}$ (see [1], [10], [12]).

Next, we shift our focus to the time of maintenance in the transformed time domain. In Constraint (7), we ensure that a certain time is selected for the transformed time of maintenance. We also assure that the transformed time of maintenance is before a predefined threshold ζ_i. This threshold can be set to a time period when the generation asset failure probability exceeds a certain threshold.

$$\sum_{t \in \Theta} z_{t,i} = 1, \qquad \forall i \in \mathcal{G}, \tag{7}$$

$$\sum_{t \in \Theta} t \cdot z_{t,i} \leq \zeta_i, \qquad \forall i \in \mathcal{G}. \tag{8}$$

Lastly, we focus on the labor resources. Constraint (9) ensure that the number of ongoing maintenances at time t does not exceed the labor capacity Y_t.

$$\sum_{i \in \mathcal{G}} \sum_{e=0}^{T_i^M - 1} \nu_{t-e,i} \leq Y_t, \qquad \forall t \in \mathcal{T}, \tag{9}$$

where T_i^M is the duration of maintenance for generation asset i.

D. Constraints for Modeling Unit Commitment

Constraint (10) stipulates that if a generation asset i is experiencing an ongoing maintenance at the start of the planning horizon. the associated commitment variables should be set to zero.

$$x_{s,i}^t = 0, \quad \forall i \in \mathcal{G}, \forall s \in \mathcal{S}, \forall t \in \{1, \ldots, T_i^R\}. \tag{10}$$

We couple the maintenance decision variable $\boldsymbol{\nu}$ with generator commitment variable $\boldsymbol{x}$. Constraint (11) ensures that if a unit is under maintenance during maintenance epoch i, it cannot be committed in any of the days within that epoch. To verify that unit i is not under maintenance at time t, we check that a maintenance activity on generator i did not start during any of the following maintenance periods $\{t - T_i^M + 1, \ldots, t\}$.

$$x_{s,i}^t \leq 1 - \sum_{e=0}^{T_i^M - 1} \nu_{t-e,i}, \qquad \forall i \in \mathcal{G}, \forall t \in \mathcal{T}, \forall s \in \mathcal{S}. \tag{11}$$

We next model the conventional UC constraints such as minimum up/down, start-up/shut-down, energy balance, transmission limit and ramping, minimum and maximum dispatch levels for each generator based on the commitment status (see the Appendix of [21] for the detailed formulation). In its compact form, we represent this set of constraints as follows:

$$\boldsymbol{F}\boldsymbol{x} + \boldsymbol{G}\boldsymbol{y} \leq \boldsymbol{\ell}, \tag{12}$$

where $\boldsymbol{x}$ denotes the binary variables of UC such as commitment, start-up, and shut-down variables, and $\boldsymbol{y}$ denotes the remaining dispatch, demand curtailment, and line slack variables.

E. Constraints for Modeling Load-Dependent Degradation

In this section, we capture the relationship between operations, loading, and maintenance. The following set of constraints embed the load-dependent predictive degradation model discussed in Section II into our optimization framework. First, we evaluate the actual and the transformed times associated with maintenances - i.e. the variables $\boldsymbol{\nu}$ and $\boldsymbol{z}$ provide the actual maintenance time t, and the corresponding transformed time $\tau(t)$ as defined in (3), respectively. This relationship depends on the variable $\boldsymbol{\gamma}$ as it contains the information $\boldsymbol{\gamma}_i^t$ in (3). Second, we determine the generator loading $\boldsymbol{\gamma}$ given the operational decisions. By coupling loading profiles with the maintenance and UC decisions of the adaptive optimization framework, the following set of constraints enables the schedulers to control the degradation of the generation assets.

1) Capturing the transformed time of maintenance: Our first objective is to couple the loading and maintenance decisions with the corresponding transformed time of maintenance.

Constraint (13) coordinates the transformed time variable $\boldsymbol{z}$ with the loading variable $\boldsymbol{\gamma}$. More specifically, it provides a mapping between the loading conditions at each time period t, with the transformed time when the preventive maintenance is scheduled. $\sum_{\ell=0}^{L} Q_{\ell,i}\gamma_{t,i,\ell}$ gives impact of the loading condition at time t, $\Psi(\gamma(t))$. By summing $\sum_{\ell=0}^{L} Q_{\ell,i}\gamma_{t,i,\ell} = \Psi(\gamma(t))$ over time periods until the first maintenance, we can get the transformed time until the first preventive maintenance, given by $\sum_{t\in\mathcal{T}} t\, z_{t,i,1}$. To account for non-integer solutions, we take a conservative approach and round off the total loading to the upper integer value.

$$\sum_{t\in\mathcal{T}} t\, z_{t,i} - 1 \le \sum_{t\in\mathcal{T}}\sum_{\ell\in\mathcal{L}} Q_{\ell,i}\gamma_{e,i,\ell} \le \sum_{t\in\mathcal{T}} t\, z_{t,i}, \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{G}. \tag{13}$$

We next ensure some logical constraints on the loading variables. In (14), we enforce that generation i cannot have experience any loading (thus remains offline) at time t, if there is an ongoing maintenance:

$$\gamma_{t,i}^{o} + \gamma_{t,i,\ell} \le 1 - \sum_{e=0}^{T_i^M - 1} \nu_{t-e,i}, \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}, \tag{14}$$

where $\gamma_{t,i}^{o}$ is a loading variable of generation asset i for time periods after its last maintenance.

In (15), we ensure that if the loading of the generator at time t is ℓ, then the $\boldsymbol{\gamma}$ variables for the ℓ^{th} level and all the levels before ℓ gets the value 1, or more specifically $\gamma_{t,i,\ell'} = 1$ for all $\ell' \le \ell$:

$$\gamma_{t,i,\ell} \le \gamma_{t,i,\ell-1}, \qquad \forall l \in \mathcal{L}/\{0\}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}. \tag{15}$$

Lastly, we ensure that $\gamma^o_{t,i}$ is zero for all time periods before maintenance (16), and the loading variables $\gamma_{t,i,\ell}$ cannot be 1 for any time period t that is after the scheduled maintenance (17):

$$\gamma^o_{t,i} = \sum_{e=1}^{t-T^M_i+1} \nu_{e,i}, \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{G}, \tag{16}$$

$$\gamma_{t,i,\ell} \leq \sum_{e=t}^{H} \nu_{e,i}, \qquad \forall l \in \mathcal{L}, \forall t \in \mathcal{T}, \forall i \in \mathcal{G}. \tag{17}$$

2) Capturing the interaction between operational decisions and loading: Our next challenge is to establish the relationship between the operational decisions, and the loading imposed onto the generators. We study the case, where the load severity depends on the dispatch level of the generation asset. To do so, we define the loading condition of a generation asset as a function of the average production within a maintenance period.

$$\frac{\sum_{s\in\mathcal{S}} y^t_{s,i}}{|\mathcal{S}|} \geq \left[\Gamma^L_0 \cdot \gamma_{t,i,0} + \sum_{l\in\mathcal{L}/0} \left((\Gamma^L_l - \Gamma^L_{l-1})\gamma_{t,i,l}\right)\right], \tag{18a}$$

$$\frac{\sum_{s\in\mathcal{S}} y^t_{s,i}}{|\mathcal{S}|} \leq \left[\sum_{l\in\mathcal{L}} \left((\Gamma^L_{l+1} - \Gamma^L_l)\gamma_{t,i,l}\right) + p^{max}_i \cdot \gamma^o_{t,i}\right], \tag{18b}$$

$$\forall t \in \mathcal{T}, \forall i \in \mathcal{G},$$

where Γ^L_l is the average load level to reach to degradation regime l. Constraint (18) considers two cases. If the maintenance for generator i is scheduled before time t, then $\gamma_{t,i,l} = 0$ for all $l \in \mathcal{L}$, and $\gamma^o_{i,l} = 1$ (due to Constraints (16),(17)) makes the inequality redundant. It only ensures that the average dispatch is between 0 and p^{max}_i, which is already imposed through (12). If the time t is before the last maintenance, it stipulates that the average production should be between Γ^L_l and Γ^L_{l+1} if the current loading level is l - i.e. $\gamma_{t,i,l'} = 1$, $\forall l' \leq l$ and $\gamma_{t,i,l'} = 0$, $\forall l' > l$ due to (15). Note that the right hand size in (18a), and (18b) construct the sum for the lower bound Γ^L_l, and upper bound Γ^L_{l+1}, respectively.

IV. Experiments

In this section, we present an extensive experiments to highlight the performance of LDAPM. In our study, we schedule the maintenance and operations of 54 generators using the IEEE 118-Bus case. We provide a benchmark analysis that compares the proposed model against the models in literature that do not use sensor information.

To evaluate the performance of different scheduling models, we develop an experimental framework that incorporates real-time condition monitoring data and a dynamic loading environment. The framework uses a rolling horizon model that is composed of two modules: optimization module, and the execution module. In the optimization module, we use the dynamic sensor-updated cost functions to obtain the optimal maintenance and operations decisions. In the execution module, we model the chain of events that occur during a freeze period. To do so, we first evaluate the loading conditions on each generator using the results of LDAPM. More specifically, the optimal decision for the variable γ is used to model the rate and signal-to-noise ratio of the degradation signals for each generation asset. We then determine whether an unexpected failure or a successful maintenance have occurred during any time point within the freeze period. If a preventive maintenance is experienced, the generator is taken offline for 3 weeks. Otherwise, if the generator fails unexpectedly before the time of its scheduled maintenance, it remains offline for the duration of 6 weeks.

For every time period within the planning horizon, we solve a UC model with the available generators (those that are not undergoing a preventive or corrective maintenance). The solution of this problem provides the operational cost. We also evaluate the maintenance cost by finding the number of preventive and corrective maintenances and multiplying those instances by the cost of preventive maintenance c_i^p and corrective maintenance c_i^f, respectively. In all our experiments, these costs are fixed across generators, where $c_i^f = 4 \cdot c_i^p = \$800,000$. In our framework, the maintenance decisions are weekly, and the unit commitment decisions are hourly. Planning horizon for every problem is 80 weeks, and the maintenance and operations scheduling is updated every $\tau_R = 8$ weeks. The experiments are solved using Gurobi. In every case study, we execute the implementation for a period of 48 weeks using a rolling horizon simulation. Age of the generators at the start of the experiments are obtained by running the generators for the duration of a warming period.

A. Comparative Study on LDAPM

In this section we present a comparative study to illustrate the advantages of using LDAPM. We consider the scenario where increasing the average production from a generator, also increases its loading (i.e. the case considered through Constraint (18)). In order to make a fair comparison, we perform benchmark analysis for LDAPM against two conventional methods in literature, namely the periodic model (PM), and the reliability based model (RBM). These approaches rely on population estimates (without condition monitoring data), and are not adaptive to the loading conditions. For the PM case, we enforce a constraint to ensure the preventive maintenance takes place at a specific age range for every generator, with the objective of minimizing total operational cost. We look at the overall demand and the available generator capacities to adjust the optimal period. We therefore devise a smarter periodic policy that is not extremely conservative. For the RBM case, we define the dynamic maintenance cost function using a Weibull distribution. Weibull estimates are derived using the failure times from a rotating machinery application subjected to an approximate average loading environment. We also condition on the time of survival to estimate the RLD and the associated maintenance costs. This distribution provides the best estimate for RLD without using sensor data (see [22]). These benchmark models do not control generator loading, however the resulting loading decisions drive the way we emulate degradation in the execution module.

We use a congested IEEE 118-Bus system to better illustrate the dependency between maintenance, loading and operational decisions. Table I-III presents the reliability and cost metrics for the three policies considered in the first study. In this section, we consider 3 scenarios that differ in terms of the number of loading levels. The first scenario considers a constant loading environment. In other words, we assume that a generator degrades in harsh environment whenever it remains operational. We do not allow control of the loading levels (i.e. there is only one loading level), thus the advantages of LDAPM in this case are purely due to integration of the improved remaining life predictions into maintenance and operations scheduling. It can be observed that LDAPM improves both maintenance and operational metrics compared to the benchmark models.

In the second scenario, we incorporate two loading levels. In the zero loading case, we let $\Psi(.) = 0$, whereas in the severe loading case we accelerate degradation to the harsh environment, or more specifically we set $\Psi(.) = 2$. In other words, a generator remains in zero loading environment when it halts production. Otherwise, it operates under harsh loading environment. This case provides a more dynamic environment that allows the schedulers to control the loading levels to some extent. For this case, the advantages of LDAPM is twofolds. First, LDAPM leverages on the condition monitoring data to have an accurate estimation on the RLD of the generation assets. Second, LDAPM captures the interaction between the operational decisions and degradation, which allows the operators to control the loading conditions while scheduling maintenance. Evidently, LDAPM provides a maintenance schedule that decreases the number of preventive maintenances (by $\%30.43$ and $\%31.91$ for PM and RBM, respectively), and unexpected failures (by $\%58.82$ and $\%50.00$ for PM and RBM, respectively), while also ensuring a significant reduction in the mean loading level as well (by $\%8.21$ and $\%4.38$ for PM and RBM, respectively).

In addition to improving maintenance metrics and associated costs, LDAPM also minimizes the impact of maintenance onto operations. Generation assets age slower in LDAPM, because the model typically lowers the average loading unless there is a significant advantage in using the full capacity of the generators. LDAPM captures the dependency of load and sensor information into its life prediction, therefore incurs less unexpected failures while executing a more liberal maintenance policy. Lastly, LDAPM has significantly more flexibility for delaying the optimal maintenance time of the generator. Thus, it can control the production level and minimize the risk of multiple failures occurring simultaneously. This flexibility significantly improves the operational costs. We observe that LDAPM provides $\%14.73$ and $\%30.99$ savings compared to the operational costs of PM and RBM. A similar trend is apparent in

TABLE I: Benchmark Analysis: # Loading Condition $L = 1$

	Periodic	**RBM**	**LDAPM**
# Preventive	40	42	55
# Failures	29	24	7
# Total Outages	69	66	62
Mean Loading	2.00	2.00	2.00
Maintenance Cost	$ 31.2M	$27.6 M	$16.6 M
Operations Cost	$ 126.7 M	$160.7 M	$ 121.2 M
Total Cost	$ 157.9M	$188.3 M	$ 137.8 M

TABLE II: Benchmark Analysis: # Loading Conditions $L = 2$

	Periodic	**RBM**	**LDAPM**
# Preventive	46	47	32
# Failures	17	14	7
# Total Outages	63	61	39
Mean Loading	1.61	1.51	1.44
Maintenance Cost	$22.8 M	$20.6 M	$12.0 M
Operations Cost	$138.2 M	$170.8 M	$ 117.9 M
Total Cost	$161.0 M	$191.4 M	$ 129.9 M

TABLE III: Benchmark Analysis: # Loading Conditions $L = 3$

	Periodic	**RBM**	**LDAPM**
# Preventive	50	51	21
# Failures	6	8	4
# Total Outages	56	59	25
Mean Loading	1.06	1.02	0.92
Maintenance Cost	$14.8 M	$16.6 M	$7.4 M
Operations Cost	$147.1 M	$178.9 M	$ 107.7 M
Total Cost	$161.9 M	$195.5 M	$ 115.4 M

terms of the total cost as well.

To further illustrate the advantages of our approach, we consider a more interesting scenario, where the number of loading levels increases to 3. The first level $\ell = 0$ covers the loading environment where a generator does not produce any power (turned off) during the entire week. In this case, we assume that the generator does not experience any degradation during that week. If its average dispatch is positive and below 70% of its maximum capacity p_i^{max}, then the generator operates in nominal loading environment. Otherwise, the generator is subjected to harsh loading. The loading case $\ell = 1$ is the nominal case, where the associated $\Psi(.) = 1$, whereas in the severe loading case, like in the previous study, we accelerate degradation by a factor of two, or more specifically we set $\Psi(.) = 2$.

Similar conclusions can be made for this study. However, we note that the advantages of our model becomes more pronounced in this case. As the number of loading levels increase, so does the ability of LDAPM to finetune the control of the loading conditions. In other words, detailed modeling of the degradation and loading dependency allows LDAPM to provide further improvements over the benchmarks. We observe that LDAPM decreases the number of unexpected failures, outages, as well as the costs associated with maintenance (decreasing the cost of maintenance by 50.00% and 55.42% compared to PM and RBM, respectively) and operations (this time reducing by 26.79% and 39.79% compared to PM and RBM, respectively). The mean loading of LDAPM in this scenario was reduced more significantly (incurring a decrease in mean loading by 13.22% and 9.69% compared to PM and RBM, respectively).

V. Conclusion

In this paper, we developed a unified framework that combines i) load dependent predictive degradation models for generation assets, and ii) a joint MIP that simultaneously models maintenance, loading and UC decisions. Novel to our framework is the ability to incorporate load dependent degradation within a large scale UC and generation maintenance model. Our experiments on IEEE 118-Bus case show that our proposed policy can provide significant savings in both maintenance and operations cost, while ensuring a reliable electricity system, and an effective use of asset lifetime. More specifically, our policy reduces the costs associated with maintenance by $> 50\%$, UC by $> 14.73\%$, while also maintaining a favorable loading profile. We also show that the proposed method fully adapts to changes in the way we model the degradation process (i.e. number of loading levels as in §IV-A).

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Appendix A
Proof of Proposition 1

Proof: Given the prior distributions $\pi_i(\theta_i), \pi_i(\beta_i)$, we can obtain the posterior distribution $\upsilon(\theta_i, \beta_i)$ as follows:

$$
\begin{aligned}
&P(\theta_i, \beta_i | d_1, \ldots, d_k) \propto f(d_1, \ldots, d_k | \theta_i, \beta_i) \pi_i(\theta_i) \pi_i(\beta_i) \\
&\propto \exp\left[-\frac{(d_1 - \theta_i - \beta_i \xi_1)^2}{2\sigma^2 \xi_1} - \sum_{j=2}^{k} \left(\frac{(d_j - \beta_i \xi_j)^2}{2\sigma^2 \xi_j} \right) \right] \cdot \exp\left[\frac{-(\theta_i - \mu_0)^2}{2\sigma_0^2} \right] \exp\left[\frac{-(\beta_i - \mu_1)^2}{2\sigma_1^2} \right] \\
&\propto \exp\left[-\frac{1}{2}\theta_i^2 \left(\frac{\sigma_0^2 + \sigma^2 \xi_1}{\sigma^2 \sigma_0^2 \xi_1} \right) - \frac{1}{2}\beta_i^2 \left(\frac{\sigma_1^2 \Upsilon_k + \sigma^2}{\sigma^2 \sigma_1^2} \right) - \frac{\theta_i \beta_i}{\sigma^2} \right] \cdot \exp\left[\theta_i \left(\frac{d_1 \sigma_0^2 + \mu_0 \sigma^2 \xi_1}{\sigma^2 \sigma_0^2 \xi_1} \right) + \beta_i \left(\frac{\sigma_1^2 D_k + \mu_1 \sigma^2}{\sigma^2 \sigma_1^2} \right) \right] \\
&\propto \exp\left[-\frac{1}{2}\theta_i^2 \left(\frac{1}{\sigma_{\theta_i}^2 (1 - \rho_i^2)} \right) - \frac{1}{2}\beta^2 \left(\frac{1}{\sigma_{\beta_i}^2 (1 - \rho_i^2)} \right) \right] \cdot \exp\left[\theta_i \beta_i \frac{-\rho_i}{\sigma\theta_i \sigma\beta_i (1 - \rho_i^2)} \right] \\
&\quad \cdot \exp\left[\theta_i \left(\frac{\mu_{\theta_i}}{\sigma_{\theta_i}^2 (1 - \rho_i^2)} - \frac{\mu_{\beta_i} \rho}{\sigma_{\theta_i} \sigma_{\beta_i} (1 - \rho^2)} \right) \right] \cdot \exp\left[\beta_i \left(\frac{\mu_{\beta_i}}{\sigma_{\beta_i}^2 (1 - \rho_i^2)} - \frac{\mu_{\theta_i} \rho}{\sigma_{\theta_i} \sigma_{\beta_i} (1 - \rho^2)} \right) \right],
\end{aligned}
$$

which follows a bivariate normal distribution, with the parameters defined in the proposition. ■