

Online Appendix to

“The Effects of Spending Rules and Asset Allocation on Non-Profit Endowments”

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Abstract

This supplementary online appendix to “The Effects of Spending Rules and Asset Allocation on Non-Profit Endowments” contains four sections. Section A provides supplementary results to the main analyses presented in the paper. It documents endowment growth and spending trajectories for edge cases as well as equal allocation cases for both the SBBI and Common asset classes. It also documents variability of endowment growth and spending trajectories by reporting percentiles of their distribution. Section B documents statistics for the asset returns in both SBBI and Common datasets. It reports means and standard deviations of returns for each asset class, as well as the correlation among asset classes. Section C derives the necessary breakeven return for the endowment to completely offset spending in a given year. These breakeven return bounds are derived for each spending policy studied in the paper. Finally, Section D provides a user guide on how to employ simulation code used in the paper. This code can be adapted for the specific circumstances of any endowment fund.

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A Supplemental Result and Figures

Edge Case Allocation Results

Figures 1-6 show the endowment growth, spending trajectory, and breakeven return for edge case asset allocation based on the SBBI dataset:

- Figure 1. Complete (100%) allocation to U.S. small cap stocks;
- Figure 2. Complete (100%) allocation to U.S. large cap stocks;
- Figure 3. Complete (100%) allocation to U.S. long-term corporate bonds;
- Figure 4. Complete (100%) allocation to U.S. long-term government bonds;
- Figure 5. Complete (100%) allocation to U.S. intermediate government bonds;
- Figure 6. Complete (100%) allocation to U.S. Treasury Bills.

Figure 7 shows the endowment growth, spending trajectory, and breakeven return for an equal allocation to the SBBI asset classes.

Figures 8-15 show the endowment growth, spending trajectory, and breakeven return for edge case asset allocation based on the Common asset classes:

- Figure 8. Complete (100%) allocation to domestic equity;
- Figure 9. Complete (100%) allocation to international equity;
- Figure 10. Complete (100%) allocation to emerging markets equity;
- Figure 11. Complete (100%) allocation to fixed income;
- Figure 12. Complete (100%) allocation to hedge funds;
- Figure 13. Complete (100%) allocation to private equity;
- Figure 14. Complete (100%) allocation to real estate;
- Figure 15. Complete (100%) allocation to real assets.

Figure 16 shows the endowment growth, spending trajectory, and breakeven return for an equal allocation to the Common asset classes.

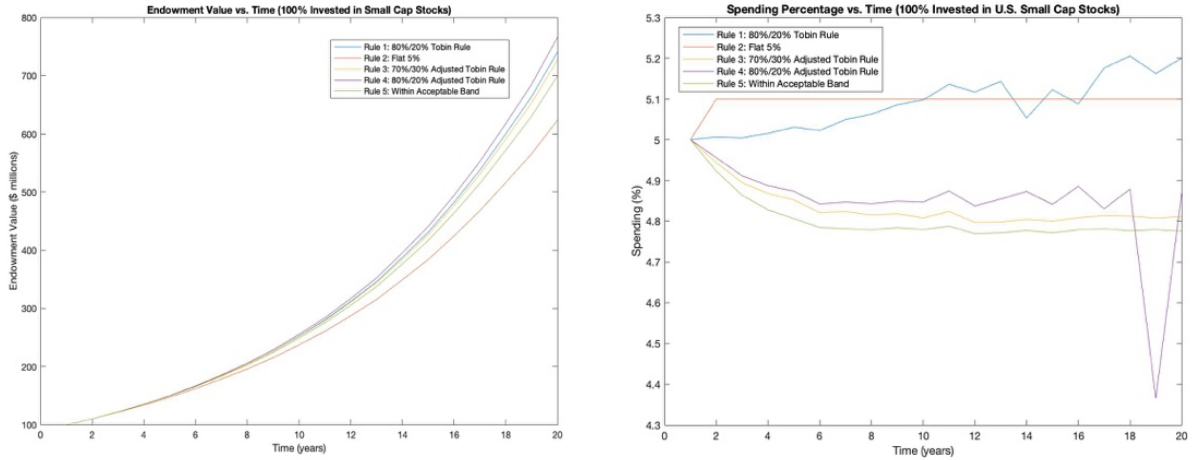
Endowment Value and Spending Percentage Percentiles

Tables 1-4 show the 5th, 25th, 50th, mean, 75th, and 95th percentiles of theoretical endowment and spending growth for the next two decades.

Table 1 shows endowment and spending growth for Harvard University; Table 2 for Yale University; Table 3 for Stanford University; and Table 4 for Massachusetts Institute of Technology.

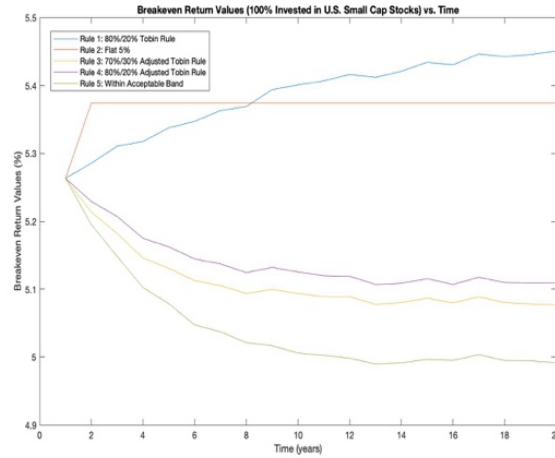
Figure 1 Complete portfolio allocation to U.S. small cap stocks

This figure shows the endowment and spending trajectory for a complete allocation to U.S. small cap stocks. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value

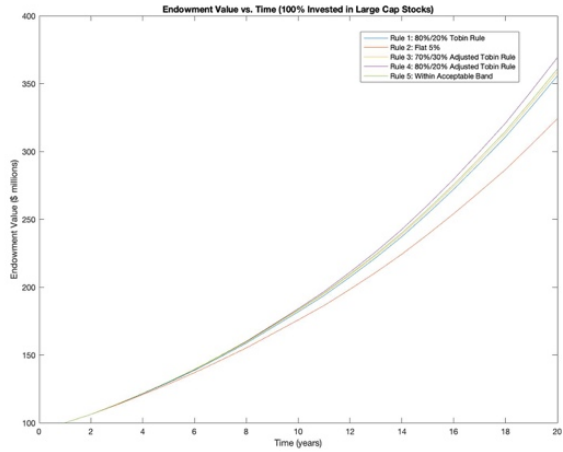
Panel B: Spending Percentage



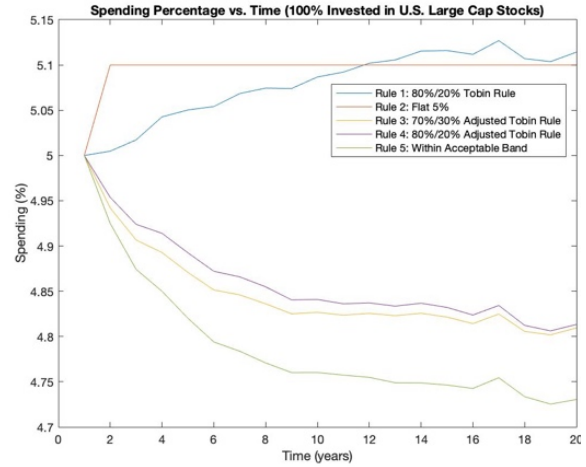
Panel C: Breakeven Return

Figure 2 Complete portfolio allocation to U.S. large cap stocks

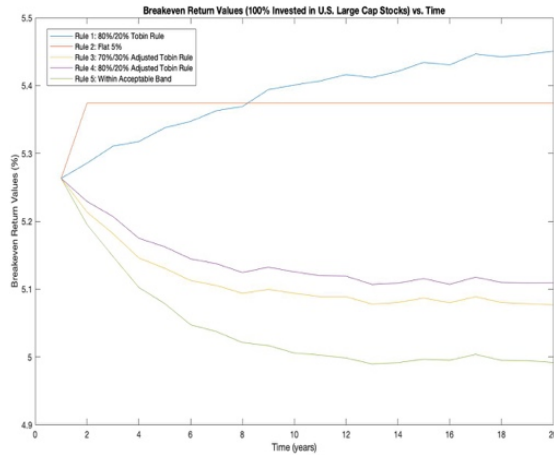
This figure shows the endowment and spending trajectory for a complete allocation to U.S. large cap stocks. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



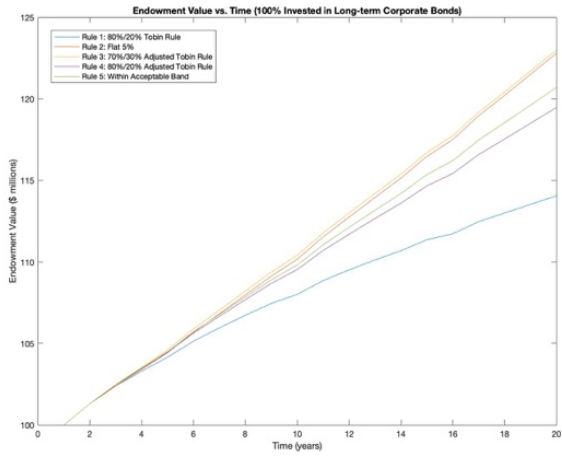
Panel B: Spending Percentage



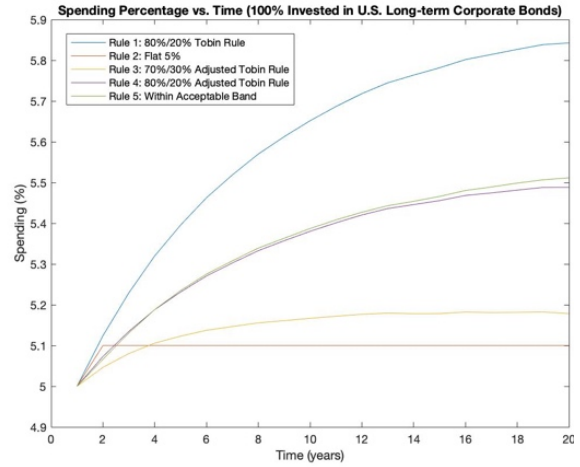
Panel C: Breakeven Return

Figure 3 Complete portfolio allocation to U.S. long-term corporate bonds

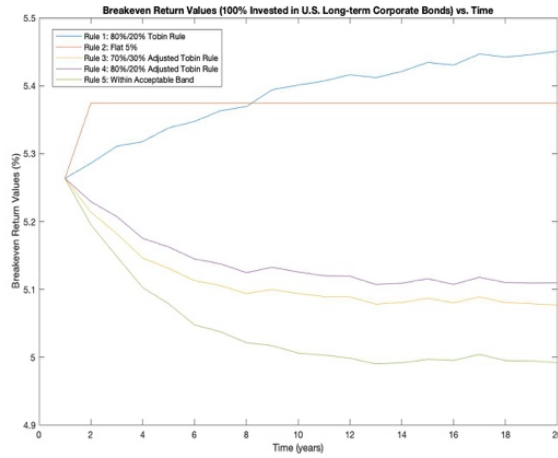
This figure shows the endowment and spending trajectory for a complete allocation to U.S. long-term corporate bonds. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



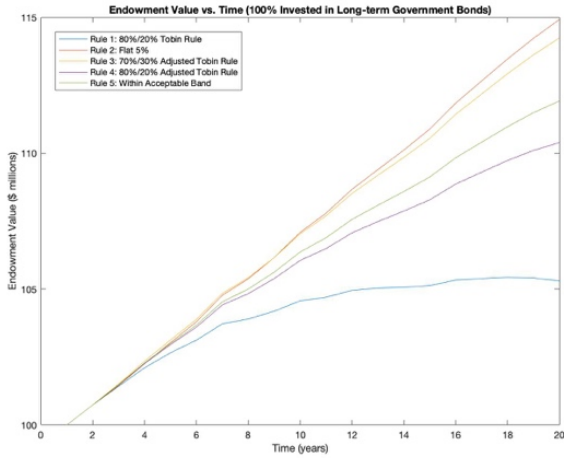
Panel B: Spending Percentage



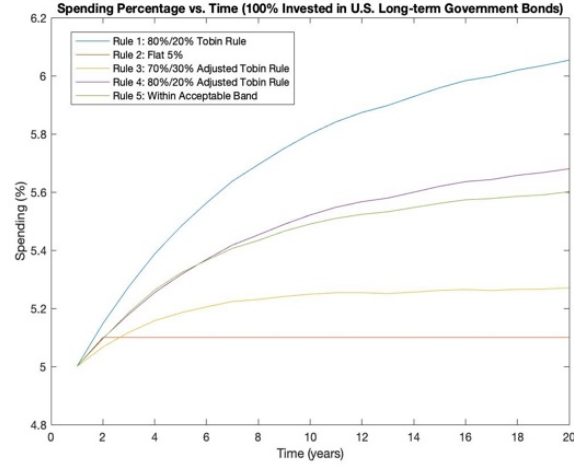
Panel C: Breakeven Return

Figure 4 Complete portfolio allocation to U.S. long-term government bonds

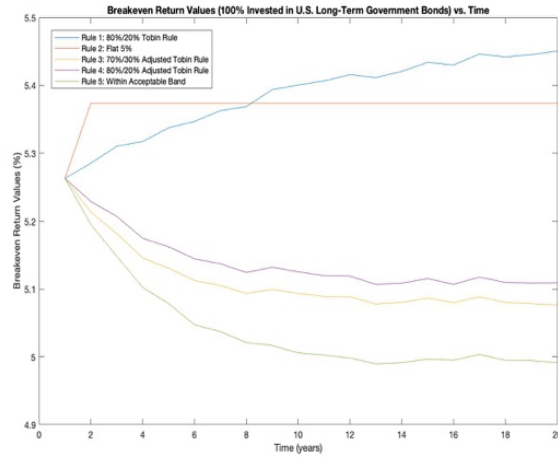
This figure shows the endowment and spending trajectory for a complete allocation to U.S. long-term government bonds. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



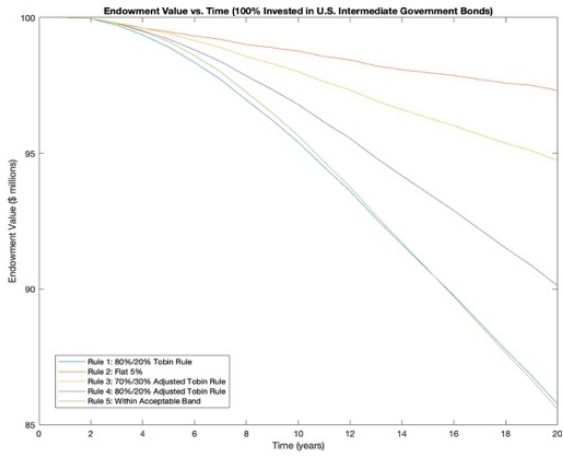
Panel B: Spending Percentage



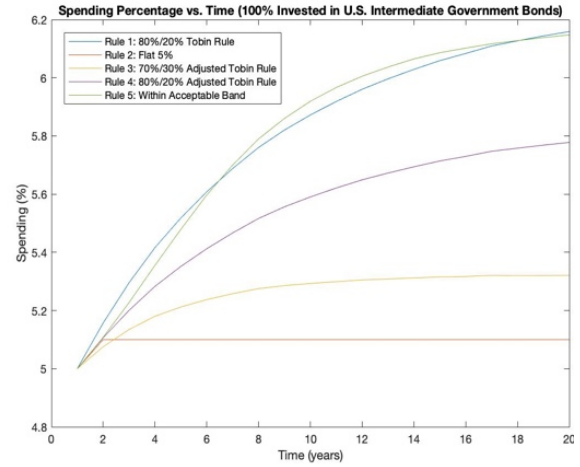
Panel C: Breakeven Return

Figure 5 Complete portfolio allocation to U.S. intermediate government bonds

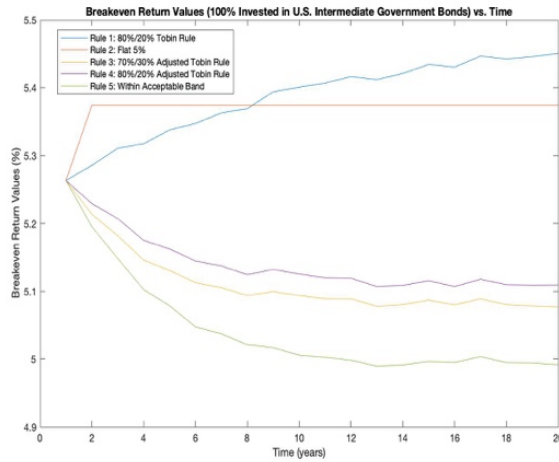
This figure shows the endowment and spending trajectory for a complete allocation to U.S. intermediate government bonds. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



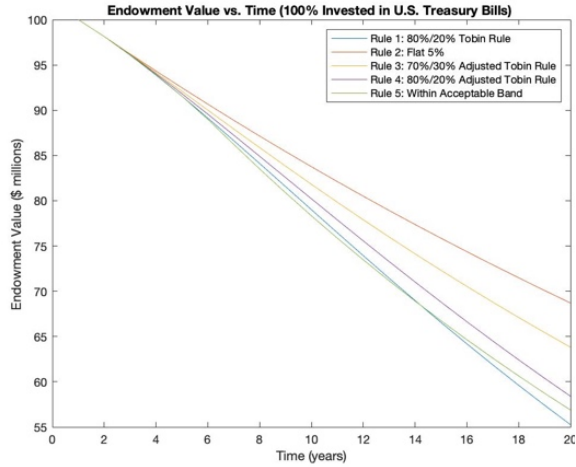
Panel B: Spending Percentage



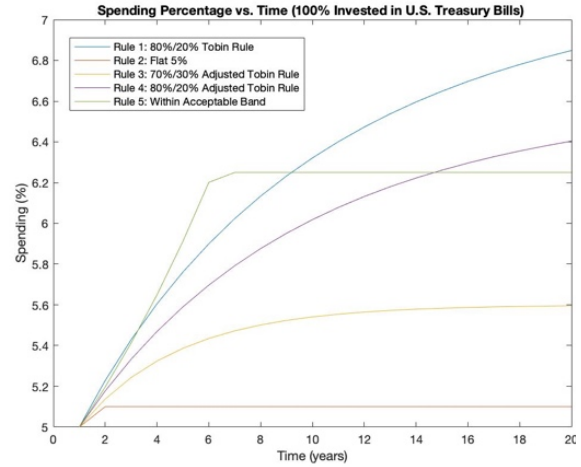
Panel C: Breakeven Return

Figure 6
Complete portfolio allocation to the U.S. Treasury Bills

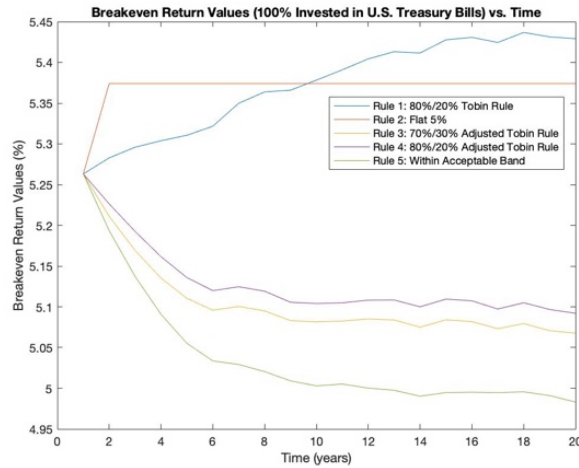
This figure shows the endowment and spending trajectory for a complete allocation to the U.S. Treasury Bills. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



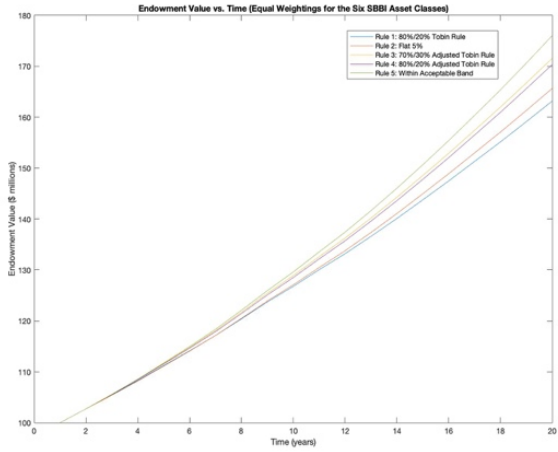
Panel B: Spending Percentage



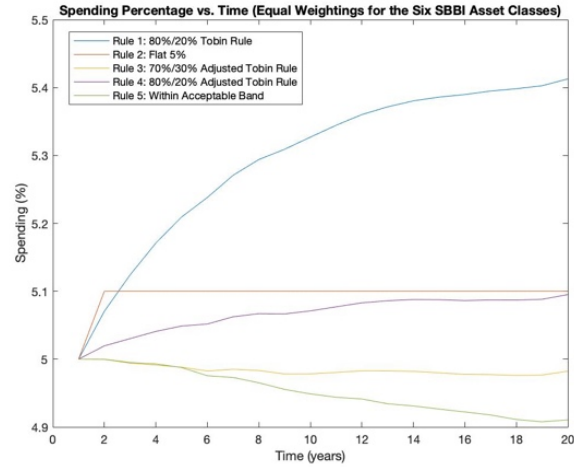
Panel C: Breakeven Return

Figure 7 Equal allocation to SBBI asset classes

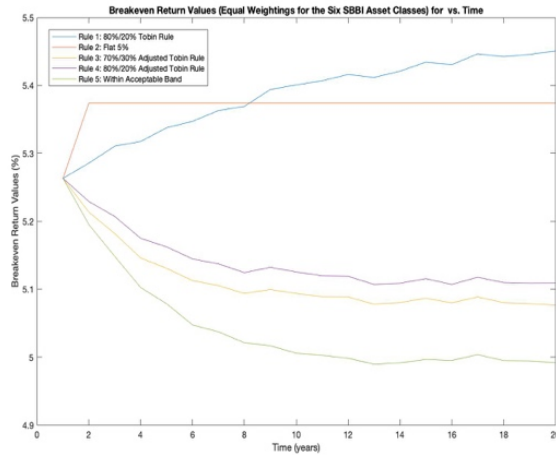
This figure shows the endowment and spending trajectory for an equal allocation to SBBI asset classes. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



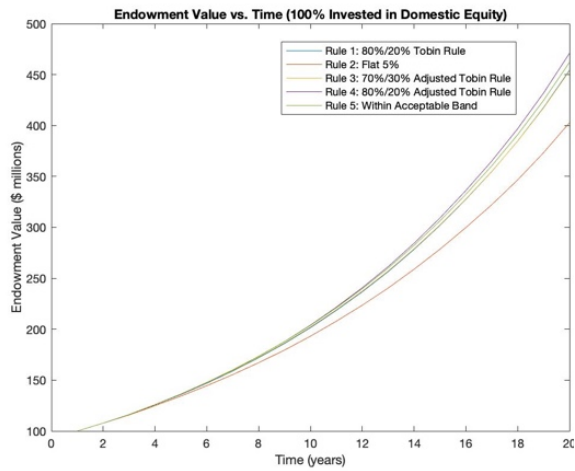
Panel B: Spending Percentage



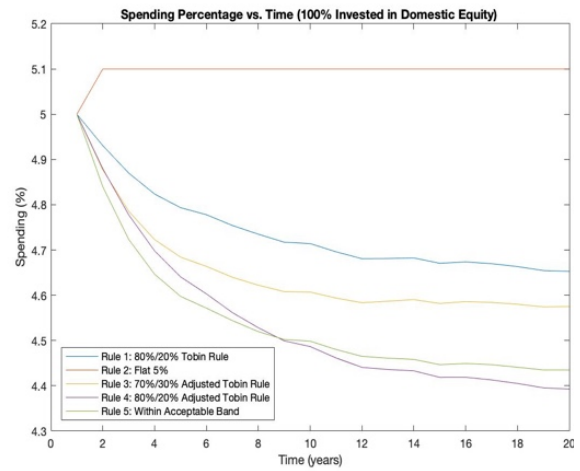
Panel C: Breakeven Return

Figure 8 Complete allocation to domestic equity

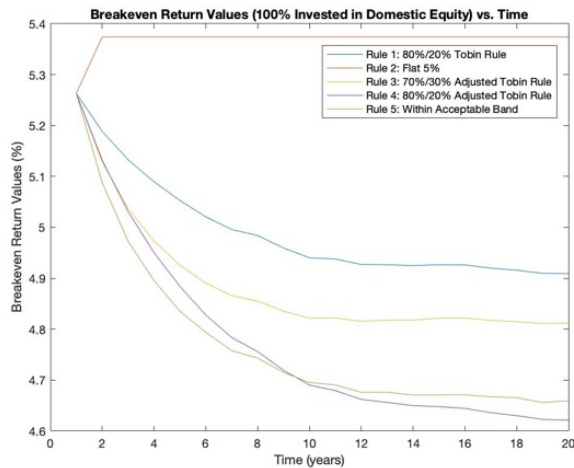
This figure shows the endowment and spending trajectory for a complete allocation to domestic equity. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



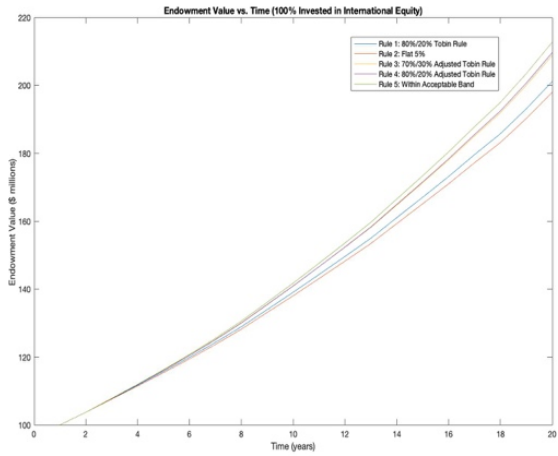
Panel B: Spending Percentage



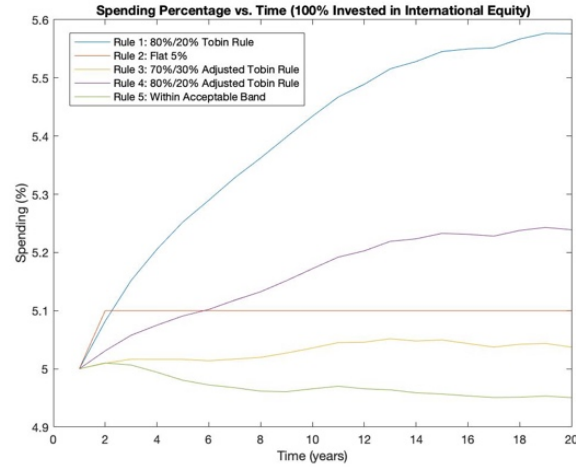
Panel C: Breakeven Return

Figure 9 Complete allocation to international equity

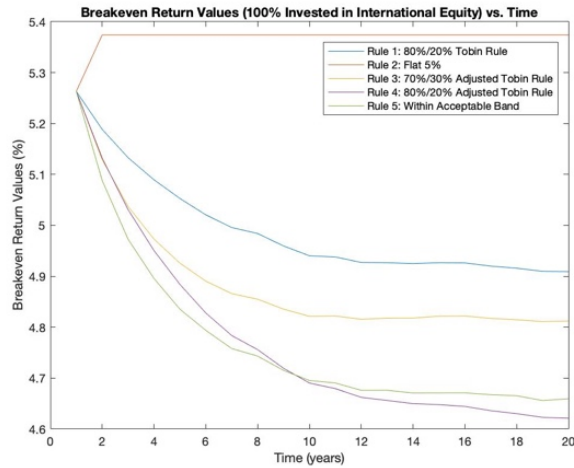
This figure shows the endowment and spending trajectory for a complete allocation to international equity. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



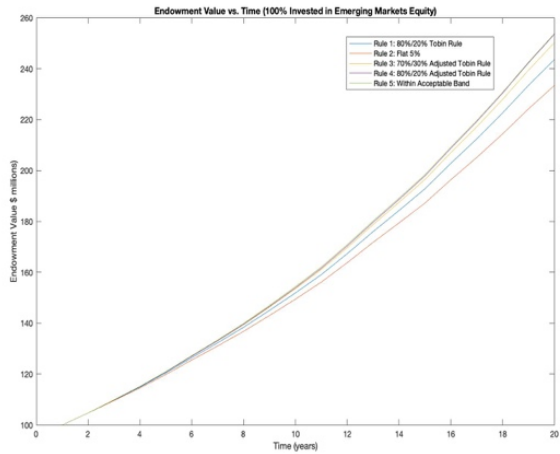
Panel B: Spending Percentage



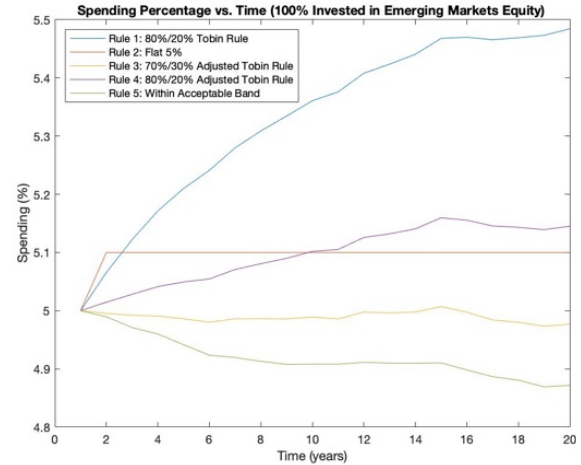
Panel C: Breakeven Return

Figure 10 Complete allocation to emerging markets equity

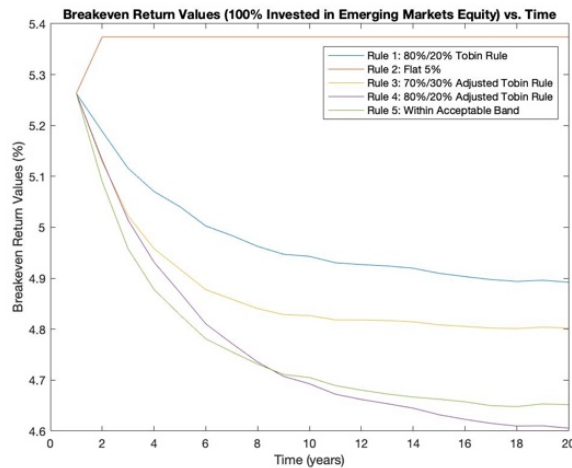
This figure shows the endowment and spending trajectory for a complete allocation to emerging markets equity. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



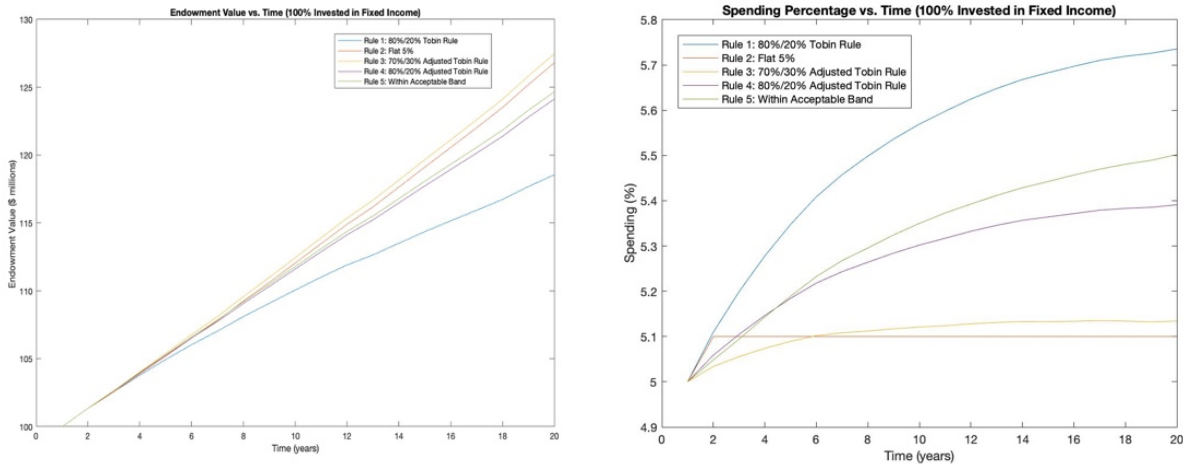
Panel B: Spending Percentage



Panel C: Breakeven Return

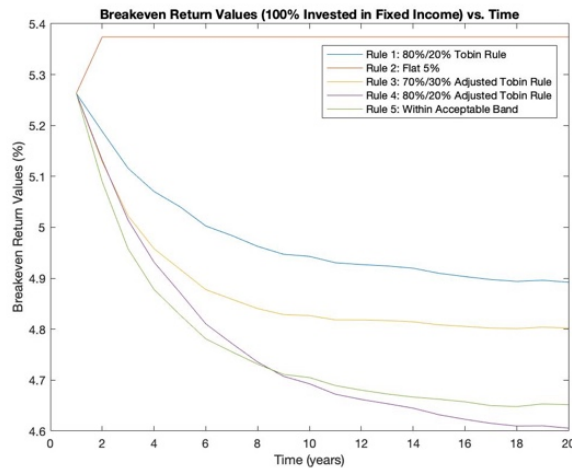
Figure 11 Complete allocation to fixed income

This figure shows the endowment and spending trajectory for a complete allocation to fixed income. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value

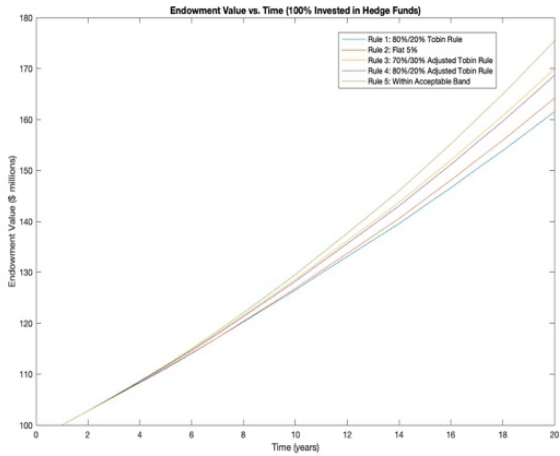
Panel B: Spending Percentage



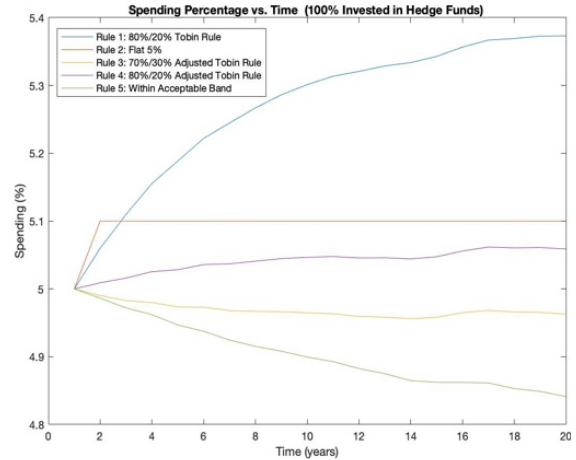
Panel C: Breakeven Return

Figure 12 Complete allocation to hedge funds

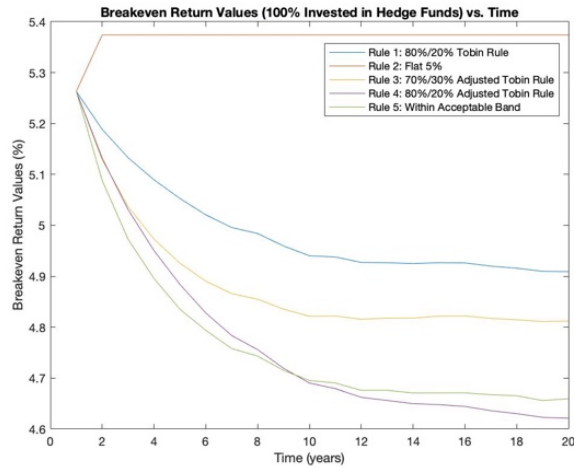
This figure shows the endowment and spending trajectory for a complete allocation to hedge funds. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



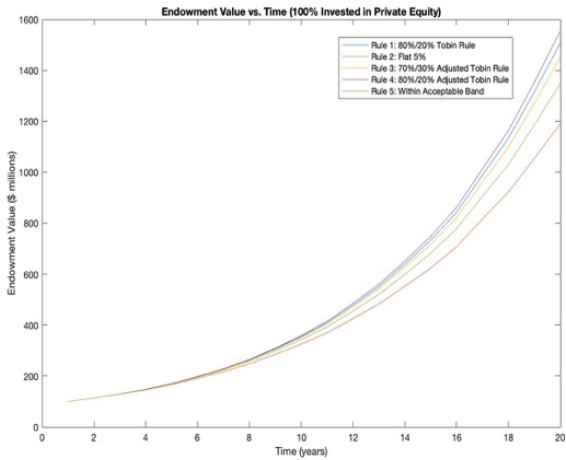
Panel B: Spending Percentage



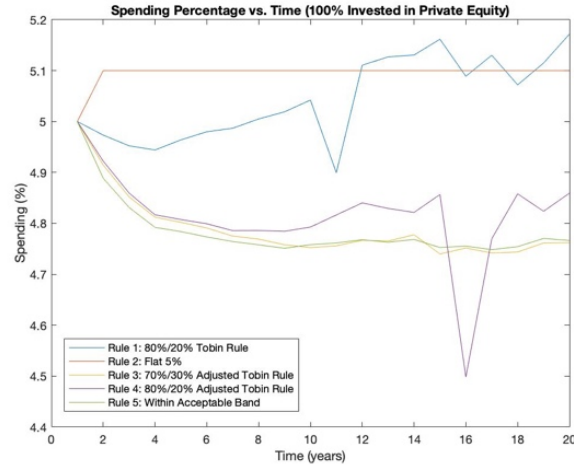
Panel C: Breakeven Return

Figure 13
Complete allocation to private equity

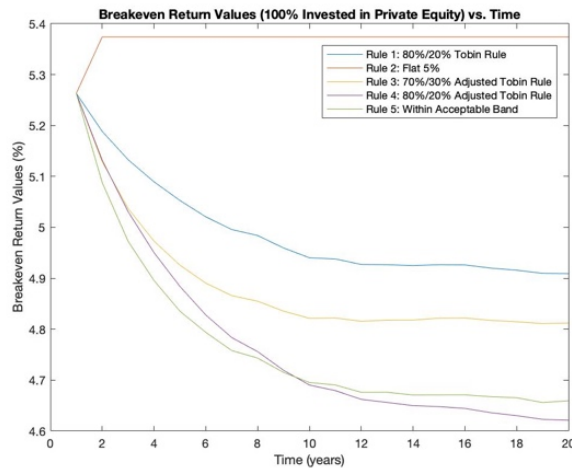
This figure shows the endowment and spending trajectory for a complete allocation to private equity. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



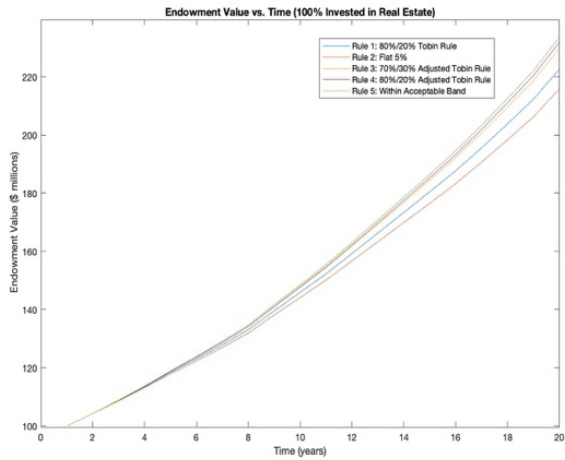
Panel B: Spending Percentage



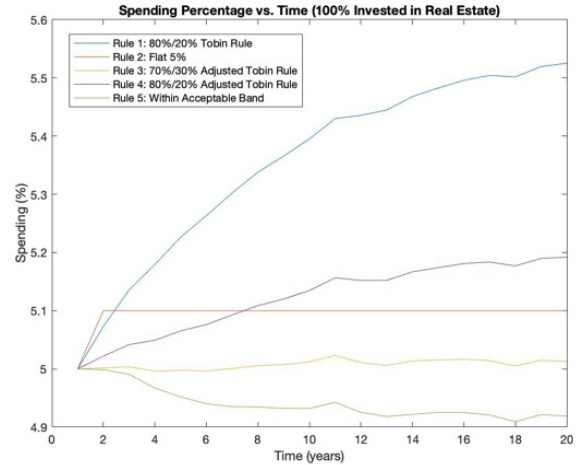
Panel C: Breakeven Return

Figure 14
Complete allocation to real estate

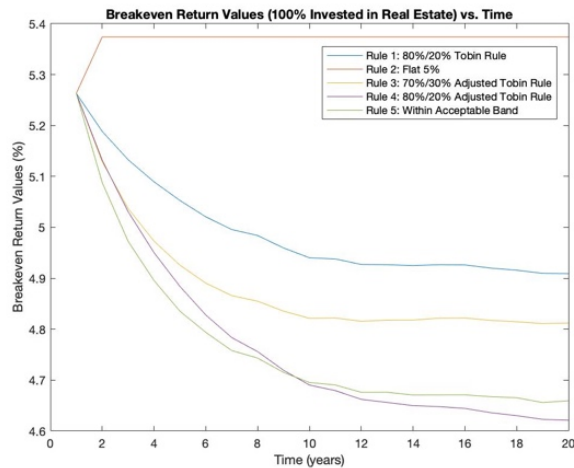
This figure shows the endowment and spending trajectory for a complete allocation to real estate. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



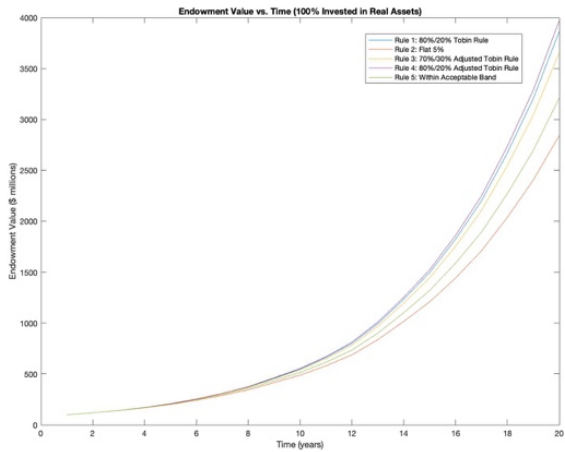
Panel B: Spending Percentage



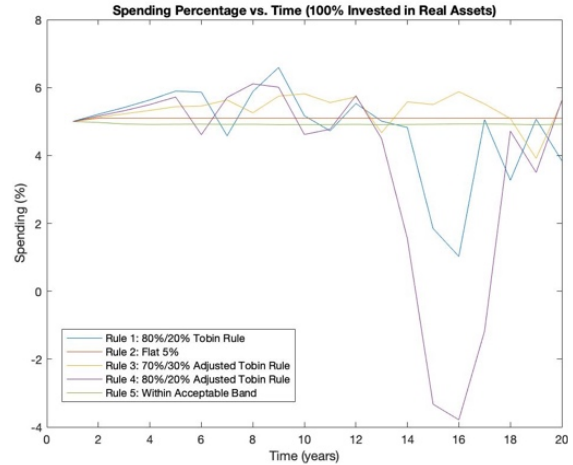
Panel C: Breakeven Return

Figure 15
Complete allocation to real assets

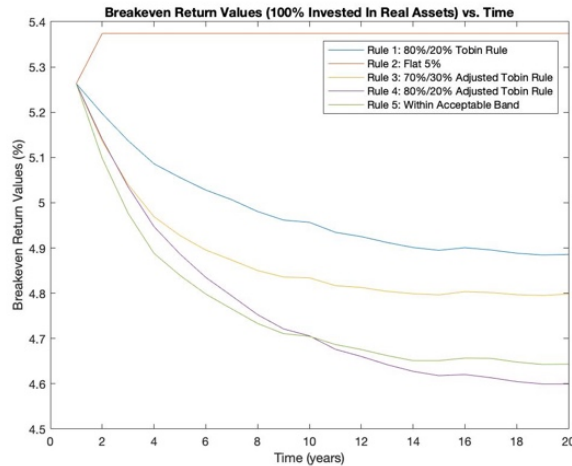
This figure shows the endowment and spending trajectory for a complete allocation to real assets. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



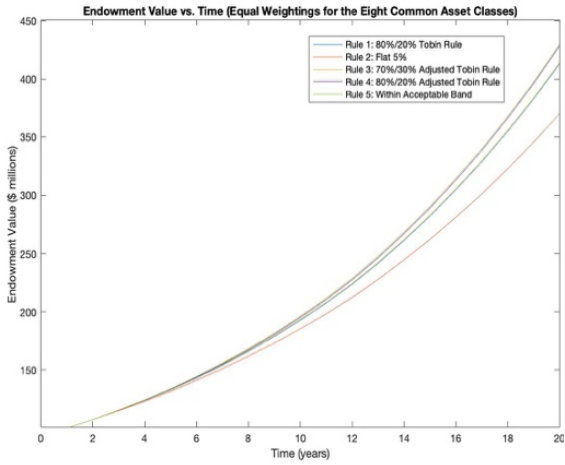
Panel B: Spending Percentage



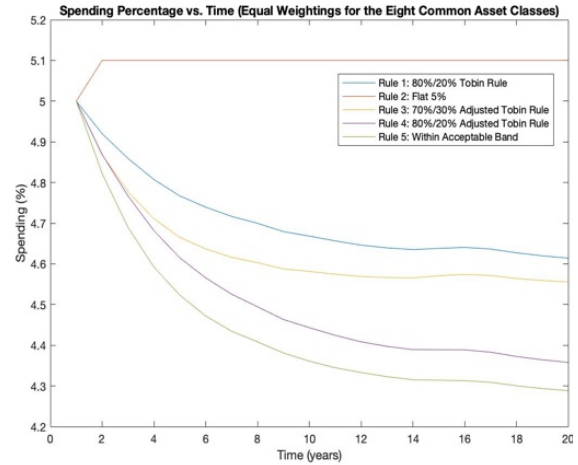
Panel C: Breakeven Return

Figure 16
Equal allocation to the Common asset classes

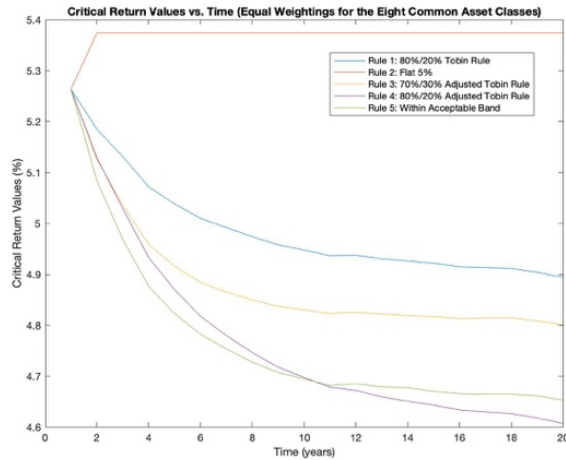
This figure shows the endowment and spending trajectory for an equal allocation to the Common asset classes. Panel A shows predicted endowment value, Panel B shows predicted spending percentage, and Panel C shows breakeven return values over time under the five spending rules.



Panel A: Endowment Value



Panel B: Spending Percentage



Panel C: Breakeven Return

B Asset Return Data

This section shows statistics for the return data used in the paper. Table 5 shows means and standard deviations for the SBBI asset classes. Table 6 shows means and standard deviations for the Common asset classes. Table 7 shows correlations of returns within SBBI asset classes, and Table 8 shows correlations of returns within Common asset classes.

Table 5
Summary statistics for returns on the SBBI asset classes

| Statistic | Ibbotson SBBI US Large Stock TR USD (%Total Return) | U.S. Small Stk TR (%Total Return) | U.S. LT Corp TR (%Total Return) | U.S. LT Gvt TR (%Total Return) | U.S. IT Gvt TR (%Total Return) | U.S. 30 Day TBill TR (%Total Return) |
|------------------------|---|-----------------------------------|---------------------------------|--------------------------------|--------------------------------|--------------------------------------|
| Monthly Mean | 1.0% | 1.2% | 0.5% | 0.5% | 0.4% | 0.3% |
| Monthly STD | 5.4% | 8.1% | 2.2% | 2.5% | 1.2% | 0.3% |
| Annualized Mean | 12.1% | 16.0% | 6.5% | 6.1% | 5.2% | 3.3% |
| Annualized STD | 18.7% | 28.2% | 7.5% | 8.5% | 4.3% | 0.9% |

Table 6
Summary statistics for returns on the Common asset classes

| Statistic | Russell 3000 | MSCI World IMI Index (USD) | MSCI ACWI EM Investable Market Index (IMI) | Dow Jones Corporate Bond Index | Barclays Hedge Fund Index | Private Equity (BX, CG, KKR, APO) | S&P Global REITs | S&P Real Assets (RA) Index |
|------------------------|--------------|----------------------------|--|--------------------------------|---------------------------|-----------------------------------|------------------|----------------------------|
| Monthly Mean | 1.1% | 0.7% | 0.8% | 0.5% | 0.7% | 1.5% | 0.8% | 1.9% |
| Monthly STD | 4.5% | 4.4% | 4.9% | 1.8% | 2.1% | 10.4% | 4.7% | 16.4% |
| Annualized Mean | 13.4% | 9.2% | 10.2% | 6.7% | 8.2% | 20.0% | 9.7% | 25.3% |
| Annualized STD | 15.4% | 15.2% | 17.0% | 6.1% | 7.1% | 36.0% | 16.1% | 56.8% |

Table 7
Return correlation between asset classes in the SBBI dataset

| Correlation | Ibbotson SBBI US Large Stock TR USD (%Total Return) | U.S. Small Stk TR (%Total Return) | U.S. LT Corp TR (%Total Return) | U.S. LT Gvt TR (%Total Return) | U.S. IT Gvt TR (%Total Return) | U.S. 30 Day TBill TR (%Total Return) |
|--|--|--|--|---------------------------------------|---------------------------------------|---|
| Ibbotson SBBI US Large Stock TR USD (%Total Return) | 100.0% | 83.2% | 18.3% | 6.2% | 5.0% | 0.0% |
| U.S. Small Stk TR (%Total Return) | 83.2% | 100.0% | 13.1% | 0.0% | -0.3% | 0.0% |
| U.S. LT Corp TR (%Total Return) | 18.3% | 13.1% | 100.0% | 82.4% | 70.7% | 0.0% |
| U.S. LT Gvt TR (%Total Return) | 6.2% | 0.0% | 82.4% | 100.0% | 82.1% | 0.0% |
| U.S. IT Gvt TR (%Total Return) | 5.0% | -0.3% | 70.7% | 82.1% | 100.0% | 0.0% |
| U.S. 30 Day TBill TR (%Total Return) | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 100.0% |

Table 8
Return correlation between Common asset classes

| Correlation | Russell 3000 | MSCI World IMI Index (USD) | MSCI ACWI EM Investable Market Index (IMI) | Dow Jones Corporate Bond Index | Barclays Hedge Fund Index | Private Equity (BX, CG, KKR, APO) | S&P Global REITs | S&P Real Assets (RA) Index |
|---|---------------------|-----------------------------------|---|---------------------------------------|----------------------------------|--|-----------------------------|---------------------------------------|
| Russell 3000 | 100.0% | 95.8% | 87.6% | 31.7% | 6.3% | 75.6% | 11.9% | -10.5% |
| MSCI World IMI Index (USD) | 95.8% | 100.0% | 91.6% | 37.8% | 3.4% | 79.0% | 12.0% | -9.8% |
| MSCI ACWI EM Investable Market Index (IMI) | 87.6% | 91.6% | 100.0% | 39.4% | 1.7% | 78.5% | -5.5% | -3.6% |
| Dow Jones Corporate Bond Index | 31.7% | 37.8% | 39.4% | 100.0% | -2.5% | 32.6% | 8.0% | 6.7% |
| Barclays Hedge Fund Index | 6.3% | 3.4% | 1.7% | -2.5% | 100.0% | 5.8% | 5.5% | 7.7% |
| Private Equity (BX, CG, KKR, APO) | 75.6% | 79.0% | 78.5% | 32.6% | 5.8% | 100.0% | -13.3% | 1.6% |
| S&P Global REITs | 11.9% | 12.0% | -5.5% | 8.0% | 5.5% | -13.3% | 100.0% | 6.3% |
| S&P Real Assets (RA) Index | -10.5% | -9.8% | -3.6% | 6.7% | 7.7% | 1.6% | 6.3% | 100.0% |

C Breakeven Return Value Derivations for each Spending Rule

We take $s_{i,t}$ to be the amount the endowment spent, under rule i , at time t . $W_{i,t}$ is the total value of the endowment under rule i , at time t . $b_{i,t}$ is the necessary breakeven return for the endowment to completely offset spending for that year under rule i and at time t .

SR 1: 80/20 Tobin Rule

$$\begin{aligned} s_{1,t} &= 0.8 * s_{1,t-1} + 0.2 * (0.0525 * W_{1,t-1}) \\ W_{1,t+1} &= b_{1,t} * (W_{1,t} - s_{1,t}) > W_{1,t} \\ b_{1,t} * W_{1,t} - b_{1,t} * (0.8 * s_{1,t-1} + 0.2 * (0.0525 * W_{1,t-1})) &> W_{1,t} \\ b_{1,t} * (W_{1,t} - 0.8 * s_{1,t-1} - 0.0105 * W_{1,t-1}) &> W_{1,t} \\ b_{1,t} &> \frac{W_{1,t}}{W_{1,t} - 0.8 * s_{1,t-1} - 0.0105 * W_{1,t-1}} - 1 \end{aligned}$$

SR 2: Flat 5%

$$\begin{aligned} s_{2,t} &= 0.05 * W_{2,t-1} \\ W_{2,t+1} &= b_{2,t} * (W_{2,t} - s_{2,t}) > W_{2,t} \\ b_{2,t} * W_{2,t} - b_{2,t} * (0.05 * W_{2,t-1}) &> W_{2,t} \\ b_{2,t} * (W_{2,t} - 0.05 * W_{2,t-1}) &> W_{2,t} \\ b_{2,t} &> \frac{W_{2,t}}{W_{2,t} - 0.05 * W_{2,t-1}} - 1 \end{aligned}$$

SR 3: 70/30 Adjusted Tobin Rule

$$\begin{aligned} s_{3,t} &= 0.7 * s_{3,t-1} + 0.3 * (0.05 * W_{3,t-1}) \\ W_{3,t+1} &= b_{3,t} * (W_{3,t} - s_{3,t}) > W_{3,t} \\ b_{3,t} * W_{3,t} - b_{3,t} * (0.7 * s_{3,t-1} + 0.3 * (0.05 * W_{3,t-1})) &> W_{3,t} \\ b_{3,t} * (W_{3,t} - 0.7 * s_{3,t-1} - 0.015 * W_{3,t-1}) &> W_{3,t} \end{aligned}$$

$$b_{3,t} > \frac{W_{3,t}}{W_{3,t} - 0.7 * s_{3,t-1} - 0.015 * W_{3,t-1}} - 1$$

SR 4: 80/20 Adjusted Tobin Rule

$$s_{4,t} = 0.8 * s_{4,t-1} + 0.2 * (0.051 * W_{4,t-1})$$

$$W_{4,t+1} = b_{4,t} * (W_{4,t} - s_{4,t}) > W_{4,t}$$

$$b_{4,t} * W_{4,t} - b_{4,t} * (0.8 * s_{4,t-1} + 0.2 * (0.051 * W_{4,t-1})) > W_{4,t}$$

$$b_{4,t} * (W_{4,t} - 0.8 * s_{4,t-1} - 0.0102 * W_{4,t-1}) > W_{4,t}$$

$$b_{4,t} > \frac{W_{4,t}}{W_{4,t} - 0.8 * s_{4,t-1} - 0.0102 * W_{4,t-1}} - 1$$

SR 5: Within Acceptable Band

$$0.04 * W_{5,t-1} < s_{5,t} < 0.0625 * W_{5,t-1}$$

$$W_{5,t+1} = b_{5,t} * (W_{5,t} - s_{5,t}) > W_{5,t}$$

$$b_{5,t} * (W_{5,t} - 0.04 * W_{5,t-1}) > W_{5,t}$$

$$b_{5,t} > \frac{W_{5,t}}{W_{5,t} - 0.04 * W_{5,t-1}} - 1$$

$$b_{5,t} * (W_{5,t} - 0.0625 * W_{5,t-1}) > W_{5,t}$$

$$b_{5,t} > \frac{W_{5,t}}{W_{5,t} - 0.0625 * W_{5,t-1}} - 1$$

D How to Use the Simulation Code

Matlab Toolboxes

The simulation code is run in Matlab. Try to have the most up to date version of Matlab when running the simulations. The code requires downloading two toolboxes from Mathworks that are add-ons to the basic Matlab download: “Statistics and Machine Learning Toolbox” and “Financial Toolbox.”

Asset Class Data from Excel

Using the “xlsread” function, import Excel spreadsheets containing annual returns data for different asset classes. Please perform any data processing ahead of the import and remove any blank values as this may impact the calculation of correlation matrices. Then, determine which subset, if any, of these asset classes should be used for the simulations. Please ensure the same years are aligned in the same rows on the spreadsheet. Examples of how to choose a subset of asset classes is in the Matlab code.

Time Horizon

Input the number of years that the simulations should project out to. For example, the current input in the code is 20 which means the simulation projects out 20 years of endowment values with different spending policies.

Number of Spending Rules

Input the number of spending rules the simulations should run through. There are currently five in the code, but one can choose fewer to analyze.

Initial Endowment Value and Spending Percentage

Input the initial endowment value in USD millions and initial spending percentage. Simulations are based off of these initial inputs to model how endowment value and spending percentage change over the desired time horizon or over different asset class weightings.

Number of Monte Carlo simulations

Input the desired of Monte Carlo simulations, which will determine how many trials of random returns are run to determine endowment value and spending percentages. Monte Carlo simulations rely on repeated random sampling, so a higher number of trials will yield a more “random” sample of asset class returns and better simulate future market returns.

Inflation Percentage

Input the desired percentage for inflation based on historical data and investor judgement. Currently, the inflation percentage is set at 2%.

Asset Class Weighting Allocation

Input sets of asset class weightings to test. A weight must be assigned to each asset class imported from the Excel spreadsheet, even if the weight is 0. The weights are stored in an array and can be changed, but they must add up to 1, and the sum is checked by the “sumWeightings” variable. Input these weightings into the “weightings” variable in the order in which they are listed in the Excel spreadsheet.

Putting it Together: How the simulations work

The simulations occur in the “Main Endowment Function” section. With the inputs of initial endowment value and spending percentage, we use the lognormally distributed historical return data and covariance matrix to generate random returns data for the future out to the desired time horizon. We use the “mvnrnd” function to choose random return vectors from a multivariate normal distribution. Then, we exponentiate these returns to bring them back to a lognormal distribution.

The investment contributions from each asset class are summed for each year to get total endowment value for that year. This “main endowment function” repeated over the inputted number of Monte Carlo simulations.