Twisting ARMs:
Who Won and Who Lost from the LIBOR to SOFR Cramdown

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Abstract:

Approximately $800 billion of adjustable-rate mortgages (ARMs) indexed to LIBOR remained outstanding when LIBOR was published for the last time on June 30, 2023. For the many ARMs that lacked fallback language specifying an alternative index, the Federal Reserve mandated replacing LIBOR with the corresponding Term Secured Overnight Funding Rate (Term SOFR) plus a fixed spread. The conversion rule created the possibility of significant wealth transfers between ARM borrowers and lenders because the mandated spreads neglected to account for the difference between rate spreads in the ARM market and in the LIBOR market. To evaluate the size and incidence of realized transfers, we develop a valuation framework that takes the historical spread between LIBOR-indexed and Treasury-indexed ARMs as the best estimator of the neutral spread adjustment, and that projects future cash flows for the affected universe of ARMs using auxiliary models of prepayment, default, and term structure dynamics. We estimate that mortgage lenders benefited significantly at the expense of borrowers, with a total present value transfer of $247 million as of the conversion date. The size of individual transfers varied with mortgage and borrower characteristics. Future unintended wealth transfers could be made less likely by routinizing the inclusion of appropriate fallback language in floating rate contracts offered to households, and with greater transparency on the part of regulators about the redistributive consequences of their policy choices.

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(version from John Ross Wilson's dissertation)

1.1 Introduction

On June 30, 2023, all tenors of the LIBOR index were published for the final time, marking the end of an era in global financial markets. According to Alternative Reference Rates Committee (2021) as of that date, approximately \$74 trillion USD in outstanding financial contracts remained indexed to LIBOR. While most of the contracts contained fallback provisions specifying a replacement index in the event of LIBOR's discontinuation, most of the approximately \$800 billion of LIBOR-indexed consumer retail mortgages that were projected by ARRC (2021) to remain open past the conversion date lacked a specified alternative.

For contracts that were governed by U.S. law and that lacked explicit fallback provisions, the Federal Reserve was authorized to establish a standardized conversion rule. To facilitate the transition away from LIBOR, the Alternative Reference Rates Committee (ARRC)—a group convened by the Federal Reserve—recommended Term SOFR as the preferred replacement benchmark. Specifically, for LIBOR-indexed contracts without fallback provisions, the ARRC mandated conversion to Term SOFR, with the term corresponding to the LIBOR tenor in the original contract. The Secured Overnight Financing Rate (SOFR), published daily, reflects the cost of borrowing overnight using U.S. Treasury securities as collateral. Term SOFR rates are derived from the pricing of SOFR futures contracts, which have payoffs dependent on realizations of compounded overnight SOFR rates prior to contract expiration[1].

To compensate for the typical spread between LIBOR and Term SOFR at popular tenors, the Federal Reserve mandated a schedule of spread adjustments that it based on median

[1]See CME Group Benchmark Administration Limited (2025) for an in-depth explanation on the pricing of the multiple tenors of the Term SOFR index given only the one- and three-month SOFR futures contracts available for trade. This document is based on the methodology described in Heitfield and Park (2019) and is intended to create "forward-looking term reference rates that are conceptually similar to the term LIBOR rates commonly used in loan contracts."

historical spreads between LIBOR and a proxy for Term SOFR (see Appendix 1.A for more detail on that calculation). An adjustment based on historical LIBOR spreads would be predictive of a value-preserving conversion rate for a contract that has LIBOR-like risk and liquidity characteristics.

However, for ARMs whose valuations depend primarily on the pricing of mortgage default and prepayment risk and on liquidity in the mortgage market, the value-preserving spread may bear little resemblance to the historical LIBOR spread. Inferring a value-neutral spread adjustment for ARMs directly by statistical inference is not possible because historically ARMs were not linked to compounded overnight rates. Fortunately, a significant number of ARMs were issued that were quoted at spreads to Treasury rates rather than to LIBOR rates. Theoretically, we show that the spread between LIBOR-linked and Treasury-linked ARMs should be similar to the spread between LIBOR-linked and SOFR-linked ARMs. We find that the historical spread between LIBOR-linked and Treasury-linked ARMs is quite stable, and use it as the best available proxy for the value-preserving spread adjustment for ARMs.

To assess whether the ARRC's prescribed spread adjustments appropriately preserve values for open LIBOR-indexed mortgages, we develop a comprehensive model for ARMs that incorporates both the behavior of the underlying reference rate and borrower-driven factors such as prepayment and default. By pricing the remaining pool of LIBOR-linked ARMs under the Term SOFR plus constant adjustment framework and comparing the resulting value across a suite of alternative constant spreads, we estimate bounds on the value transfer that occurred on the conversion date. To our knowledge, this is the first paper to calculate the value transfer that occurred from the transition.

Estimates of the value transfer depend on several important model parameters. In our preferred specification, we document a value transfer of around 0.51% of the outstanding balance of the loans subject to the conversion, totaling $256 million. We find that this effect is larger for loans that already floated prior to the conversion, loans where the borrowers have lower credit scores, and younger loans. We conclude that the value transfer is sensitive to the value used for a fair spread, with the total transfer ranging from $256 million to $380 million given a reasonable range in fair spread alternatives.

This paper focuses on the value transferred in dollars from borrowers to lenders in the relatively small ARM segment of the mortgage market after the discontinuation of LIBOR. Cooperman et al. (2025) document additional real effects from a shrinking credit supply due to the inability of banks to hedge their risk without LIBOR-indexed lending. Jermann (2024) document the inability of Term SOFR to hedge against credit risk by running a counterfactual analysis which estimates that LIBOR-indexed loans would have accrued less interest (1-2% of notional amount of loans) had they been indexed to a compounded SOFR during the financial crisis. This paper complements those analyses by documenting another channel by which the replacement of LIBOR with Term SOFR impacted cash flows for existing contracts.

There exists some research on drivers of the difference between LIBOR and alternative indices. For example, Skov and Skovmand (2023) decompose the LIBOR-OIS spread into a credit risk component and a funding-liquidity component, showing that the jump in the spread at the onset of COVID-19 is primarily due to credit risk. The presence of these additional risk premiums are the core of the challenge inherent in replacing the index for open LIBOR-issued securities.

The rest of the chapter is organized as follows. In Section 1.2 we provide a theoretical foundation for the imputation of a neutral spread and our mortgage pricing methodology. We highlight the challenge of pricing contracts which are indexed to a rate with risk premia which do not reflect the underlying risk of the contracts. Section 1.3 describes our model for mortgage cash flows and valuation, including the dynamics driving the yield curve and Term SOFR rates. We describe the data used in our analysis in Section 1.4, highlighting interesting features of the pool of open LIBOR-indexed ARMs subject to the LIBOR-Term SOFR conversion. Key parameters of our valuation models are calibrated in Section 1.5. In Section 1.6 we present estimates for the value transfer given our baseline model calibration as well as under a variety of alternative assumptions as a robustness exercise. Finally, Section 1.7 concludes.

1.2 Background and Pricing Considerations

In this section we document the time-varying spread between LIBOR and corresponding tenors of Term SOFR. We argue that a fair spread adjustment must preserve the value of the contracts, rather than simply reflecting differences in the levels between the index being replaced and its replacement. We show that the spread imposed by the Federal Reserve did not consider mortgage valuation. Finally, we explain why the difference in the fair spread adjustment induced a value transfer on the day of the conversion.

1.2.1 LIBOR Versus Term SOFR

Term SOFR rates differ in important ways from the corresponding LIBOR rates that they were used to replace. Because it is based on overnight risk-free rates, Term SOFR rates lack the term premia and a bank credit risk premia present in LIBOR rates. The absence of those premiums causes Term SOFR rates to be systematically lower than LIBOR rates.

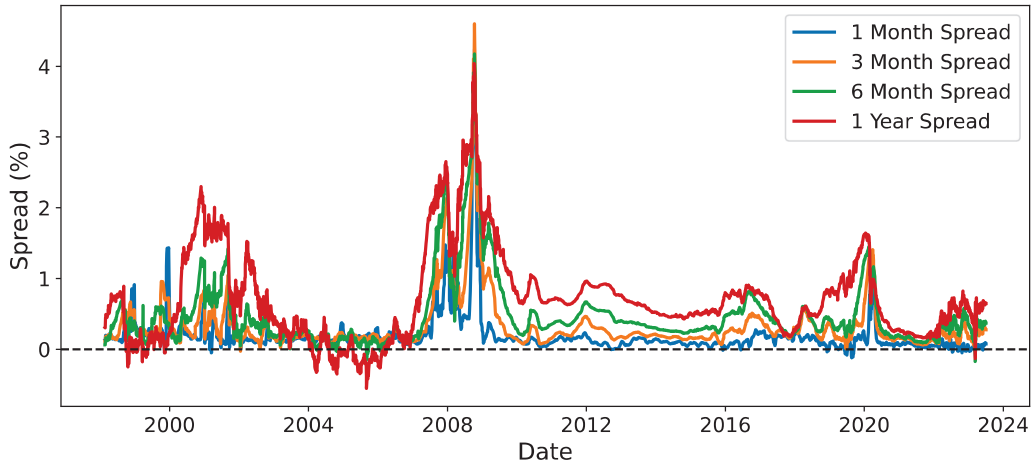
Figure 1.1A illustrates the spread between LIBOR and a proxy for Term SOFR for different tenors, highlighting the magnitude and time-variation of the missing premia. The spreads, which vary over time and with financial market conditions, are wider for longer maturities. In Figure 1.1B, it is apparent that LIBOR is the more volatile series. While some of this is attributable to the rolling average nature of Term SOFR, the volatility of LIBOR is also driven by the volatility of the premiums included in LIBOR but not reflected in SOFR[2].

1.2.2 Determining a Fair Conversion Spread

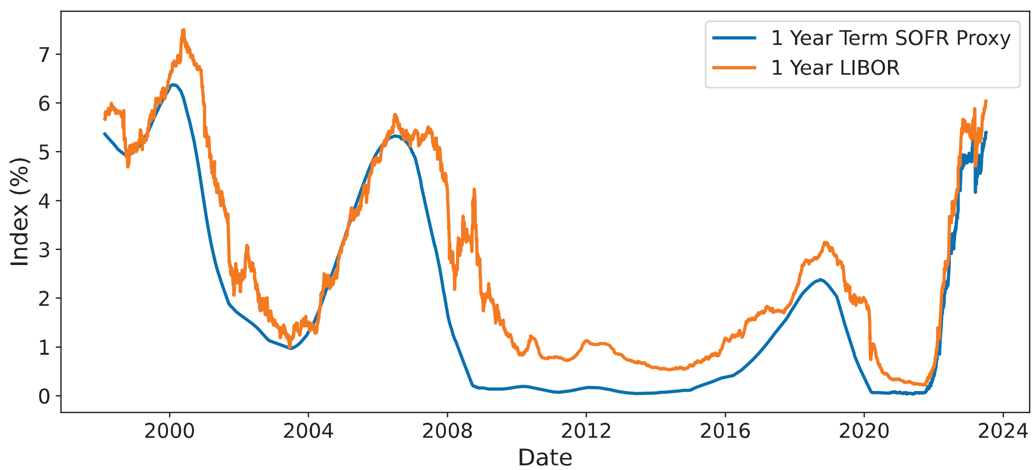
We define the fair or neutral conversion rate spread adjustment, x^*, as the constant that equates the value of an existing LIBOR-linked mortgage, had LIBOR continued, with its value when the index is switched to Term SOFR plus x^*. In other words, x^* is the adjustment spread that is expected to leave borrowers and lenders no better and no worse off than had LIBOR continued. Note that secondary market mortgage values often differ significantly from their outstanding principal because of post-origination changes in perceived credit quality

[2]Term SOFR data was not published before April 21, 2021. Term SOFR data for dates since September 18, 2020 has since been published. The Term SOFR series shown in the graph for dates prior to September 18, 2020 relies on proxies for Term SOFR described in the notes to Figure 1.1.

Figure 1.1: LIBOR and Term SOFR



(A) The spread between LIBOR tenors and the corresponding tenors of a proxy for Term SOFR over time.



(B) One year LIBOR and proxy one year Term SOFR over time.

Note: LIBOR data come from JP Morgan Markets. Our measure for Term SOFR is based on historical Term SOFR data from JP Morgan Markets beginning September 18, 2020 and proxies for Term SOFR prior to that date. The date t proxy for Term SOFR is the forward-looking N-day average of proxies for SOFR. SOFR proxies are as follows: 1) SOFR beginning on its publication date, April 2, 2018; 2) a Fed-published historical proxy for SOFR between August 22, 2014 and March 31, 2018; and 3) a measure of overnight repurchase rates provided by surveying primary dealers in the overnight Treasury general collateral repo market. For a detailed discussion of historical proxies for SOFR, see Bowman (2019).

and market conditions. Hence, an auction mechanism that identified the spread that marked mortgages to market would not reveal x^*.

It is challenging to empirically estimate x^* for several reasons. Clearly, even if there were a liquid secondary market for ARMs, the counterfactual value of a mortgage had LIBOR continued is unobservable. Secondary market prices around the conversion date reflect the effects of the chosen conversion rule on value. While one could build a model of hypothetical future LIBOR and SOFR dynamics and project future mortgage cash flows in each case, the results would be highly sensitive to the interest rate model and to supporting assumptions about discount rates.

In this paper we identify a value-preserving spread x^* using mortgage data. In Section 1.5.3, we leverage ARMs with similar characteristics to those which were subject to the LIBOR conversion but which were indexed to Treasury rates instead of LIBOR. We show that these mortgages have similar initial interest rates but that the Treasury-indexed ARMs have a higher spread than those indexed to LIBOR. This additional spread represents the market's perception of a fair constant adjustment to equate the value of loans indexed to a Treasury rate with the value of ARMs indexed to LIBOR. In Appendix 1.B we show that Treasury rates are approximately equal to Term SOFR in expectation—this allows us to assert that a x^* based on Treasury-indexed ARMs is value-preserving for LIBOR-indexed ARMs converted to Term SOFR.

1.2.3 The Federal Reserve's Conversion Spread

The Federal Reserve imposed a constant spread adjustment equal to the median spread between each tenor of LIBOR and a proxy for each corresponding tenor of Term SOFR over a five-year window. Appendix 1.A details this calculation. This method is a reasonable one to capture the median level of the index rate; however, it does not necessarily preserve mortgage valuation.

This approach is not likely to capture a value-preserving spread x^*, as the valuation of Term-SOFR indexed ARMs may be different from the valuation of LIBOR-indexed ARMs. One possible reason for this difference is that the risk premiums included in LIBOR allowed lenders to hedge their own risk; Term SOFR plus a constant does not achieve the same effect. In addition, by replacing a floating index with another index composed of a variable piece and a fixed piece, the conversion led ARMs to behave more similarly to FRMs than when the entire index was a floating rate. This shift in the importance of the floating rate plausibly impacts mortgage valuation. Our methodology for the computation of x^* described in Section 1.2.2 and implemented in Section 1.5.3 appropriately accounts for differences in how the market prices ARMs indexed to Term SOFR and ARMs indexed to LIBOR.

1.2.4 Value Transfers Under The Federal Reserve's Conversion Spread

Having established that the Fed-imposed constant spread adjustment x likely does not equal the value-preserving constant spread adjustment x^*, we now show why this inequality constitutes a value transfer as of the conversion rate. As a simplified motivating example, consider a balloon, interest-only (IO) loan indexed to Term SOFR plus a constant, with principal P and with T annual payments remaining. Loan A has the value-preserving constant x^* as its constant, and Loan B uses the Fed-imposed constant x. Assume that $x \neq x^*$. Table 1.1 shows the cash flows for these two loans, as well as the difference.

Table 1.1: Cash Flows for Hypothetical Balloon IO Mortgages

	$t = 1$	$t = 2$	$\cdots$	$t = T$
Loan A	$P \times (\text{Term SOFR}(0) + x^*)$	$P \times (\text{Term SOFR}(1) + x^*)$	$\cdots$	$P \times (1 + \text{Term SOFR}(T-1) + x^*)$
Loan B	$P \times (\text{Term SOFR}(0) + x)$	$P \times (\text{Term SOFR}(1) + x)$	$\cdots$	$P \times (1 + \text{Term SOFR}(T-1) + x)$
Difference	$P \times (x^* - x)$	$P \times (x^* - x)$	$\cdots$	$P \times (x^* - x)$

For this hypothetical loan, the value transfer that occurred due to the difference between x and x^* is the present value of the difference in the cash flows. Absent prepayment and default risk, the difference in cash flows between Loan A and Loan B is risk-free and constant (it does not depend on any index). Therefore the value transfer is the sum of each of the cash flows in the final column in Table 1.1 discounted at the risk-free rate. If the value of the difference in values of Loan A and Loan B is positive, that represents a benefit to borrowers as the payments are worth less than they would be under a fair value. This decreased payment burden comes at the expense of lenders. Conversely, if the difference has a negative present value, there was a value transfer in favor of lenders at the cost of borrowers.

Our analysis generalizes that simple example to compare the value of the loans under a fair value spread x^* to the value under the Fed-imposed spread x. However, there is additional complexity for the loans we consider relative to the simplifying assumptions made here. The loans in our analysis are subject to amortization as well as prepayment and default risk. Prepayment and default decrease the estimated value transfer, because they shorten the window during which the difference in cash flows exists.

In addition, amortization of the principal for loans in our analysis introduces non-linearity which prevents the separation ex-ante of a fixed and floating interest component as in the difference row of Table 1.1. This causes the difference in cash flows to not only depend upon the magnitude of the difference between x and x^*, but also on the realized path of Term SOFR.

Another distinction between this hypothetical example and the non-balloon, non-IO ARMs subject to the LIBOR-SOFR conversion is the impact of interest rate caps and floors. A binding interest rate cap or floor attenuates the difference in the total interest rate for loans indexed to Term SOFR plus x relative to Term SOFR plus x^*. This effect on valuation will be exaggerated if the caps and floors bind early in the remaining life of the loans, when the discounting is smallest.

The presence of prepayment and default risk as well as the presence of a floating interest rate index in the difference between cash flows under x and under x^* complicate the determination of the correct discount rate. The floating interest rate index contains risk premiums which do not match the risk inherent in an ARM. This mismatch between the risk premiums included in the index and the risk inherent in the contract presents a unique challenge to valuation.

Contracts with payments that are indexed to a rate which exactly reflects the market price of the risk embedded in the contract will trade at par. For example, LIBOR reflected the cost of unsecured interbank funding. Accordingly, a one-year unsecured interbank loan originated at date t that pays the then-current value of one-year LIBOR at maturity would be worth its face value. However, for a risk-free loan indexed to LIBOR, the risk-free rate would not be the correct discount rate. LIBOR has a negative beta due to the credit risk premium, so this risk-free loan indexed to LIBOR would have a discount rate less than the risk-free rate. In Appendix 1.B we show that for the floating component of mortgage cash flows, the appropriate discount rate is Term SOFR plus a premium for mortgage prepayment and default risk. The discounting of the cash flows in our analysis is detailed in Section 1.3.3.

1.3 Methodology

In order to determine the size of the value transfer, we compare the value of the converted ARMs under the spread imposed by the Fed with their value under a value-preserving spread x^*. We estimate a range of plausible values for x^* in Section 1.5.3.

The valuation of ARMs requires several key components. We must first establish the process for projected Term SOFR values. Given an interest rate path, a detailed accounting of the rules of ARM rates, including any caps and floors, determines the size of a regularly-scheduled payment for a current loan. Prepayment and default risk must also be modeled. Finally, total projected cash flows must be discounted to determine their value on the date of the conversion. The difference in the discounted present value of these simulated cash flows under the Fed-imposed spread and a fair spread represents the value transferred through the LIBOR replacement.

1.3.1 Interest Rates

In this section we explain how the risk free yield curve is simulated and how Term SOFR is derived from simulated paths of the yield curve. We then discuss how simulated Term SOFR values translate into the mortgage note rate. We elaborate on the separation of the mortgage note rate into a fixed and a floating component, an important distinction when discounting mortgage cash flows.

Yield Curve Model

The interest rate process is central to our analysis, as it directly determines the size of mortgage payments (see Section 1.3.3) and serves as a driver of prepayment and default behavior among ARM holders (see Section 1.3.2). We use a simple extension of the Nelson and Siegel (1987) model for the yield curve. If $y_t(\tau)$ is the yield for maturity τ at date t, the baseline for our term structure model is

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1 - e^{-\tau/\lambda_t}}{\tau/\lambda_t}\right) + \beta_{3,t}\left(\frac{1 - e^{-\tau/\lambda_t}}{\tau/\lambda_t} - e^{-\tau/\lambda_t}\right) \tag{1.1}$$

This yield curve specification is a powerful representation in that it can capture a wide variety of yield curves. Diebold and Li (2006) demonstrate the usefulness of this specification in applications where the yield curve must be forecast, especially over long horizons. The first factor, $\beta_{1,t}$, can be interpreted as a level factor, because $\lim_{\tau\to\infty} y_t(\tau) = \beta_{1,t}$. The second factor can be interpreted as a slope factor. This follows from the fact that $\lim_{\tau\to 0} y_t(\tau) = \beta_{1,t} + \beta_{2,t}$; thus, $\beta_{2,t}$ allows for a slope between the yields of very short and very long horizons.

The third factor in this representation, $\beta_{3,t}$, can be considered a curvature or “hump" parameter. Note that the limit of the third factor loading is 0 as τ approaches 0 or infinity and is positive in between. The parameter λ_t controls the maturity at which the third factor loading is maximized and is thus the maturity at which the yield curve has a hump shape. $\beta_{3,t}$ determines the size of this hump.

This specification differs from many other interest rate models such as Cox, Ingersoll, Ross, et al. (1985) in that it directly models the shape of the yield curve, rather than deriving the yield curve from first principals. It was developed to flexibly capture many yield curve shapes that other models are ill-equipped to fit. Using Equation 1.1, we can fit the Nelson-Siegel model to the bootstrapped yield curve on any given day to estimate the parameters $\hat{\beta}_{i,t}$ and λ_t.

Specifying factor dynamics allows us to use the Nelson-Siegel model to forecast the yield curve given a starting set of parameters. We will use simulations of the yield curve to derive our simulated path for Term SOFR values. Let

$$X_0 = \begin{bmatrix} \hat{\beta}_{1,0} \\ \hat{\beta}_{2,0} \\ \hat{\beta}_{3,0} \\ \hat{\lambda}_0 \end{bmatrix}$$

be the set of parameters to minimize the MSE between the Nelson-Siegel modeled yield curve and the bootstrapped yield curve on the conversion date. We assume that the evolution of the vector or model parameters X follows a mean-reverting process, such that

$$X_{t+1} = \delta\overline{X} + (1-\delta)X_t + \sigma\varepsilon_{t+1} \tag{1.2}$$

The calibration of parameters δ, $\overline{X}$, and σ is discussed in Section 1.5. Occasionally, the simulated yield curve factors are such that the yield curve slopes downward over the first few months, something which is not seen in the data. We modify this standard Nelson-Siegel model by adding a factor which decreases the likelihood of negatively sloped yield curves over the first three months. The factor $g(\tau)$ has the following specification:

$$g(\tau) = \begin{cases} -0.0025 + \frac{\tau}{100} & \tau \leq 0.25 \\ 0 & \tau > 0.25 \end{cases}$$

This functional form decreases predicted instantaneous yields by 25 basis points and linearly converges to zero at three months maturity. Our final specification of the yield curve is given by Equation 1.3:

$$\hat{y}_t(\tau) = \hat{\beta}_{1,t} + \hat{\beta}_{2,t}\left(\frac{1-e^{-\tau/\hat{\lambda}_t}}{\tau/\hat{\lambda}_t}\right) + \hat{\beta}_{3,t}\left(\frac{1-e^{-\tau/\hat{\lambda}_t}}{\tau/\hat{\lambda}_t} - e^{-\tau/\hat{\lambda}_t}\right) + g(\tau) \tag{1.3}$$

Term SOFR Simulation

While the Nelson-Siegel model allows us to forecast the yield curve, the rate for floating mortgages is determined by the appropriate tenor of Term SOFR rather than a point on the yield curve. Term SOFR is based on SOFR futures contracts whose payoff depends on realization of compounded overnight interest rates over the relevant period. Term SOFR therefore corresponds to the expected average of overnight rates over the appropriate maturity. In our analysis we simulate the yield curve on a daily basis and define SOFR to be the overnight rate. Each tenor of Term SOFR on date t is then estimated as the compounded overnight rate for the appropriate number of days following date t.

Let $r_t^{\text{ON}} = \hat{y}_t(\frac{1}{360})$ be the simulated overnight rate as of date t. We parameterize the Nelson-Siegel model such that r_t^{ON} is an APR. Then given that Term SOFR is quoted in a simple interest convention, the Term SOFR rate at date t for τ-day maturity is given by

$$s_t^\tau = \left(\prod_{s=0}^{\tau-1} \left(1 + \frac{r_s^{\text{ON}}}{360} \right) - 1 \right) \frac{360}{\tau} \tag{1.4}$$

This definition of Term SOFR implicitly assumes perfect foresight along each simulation path, as Term SOFR always exactly corresponds with the subsequent realization of the simulated path of overnight SOFR. Having described the derivation of Term SOFR, we now outline the projection of mortgage cash flows along each simulated Term SOFR path.

Mortgage Note Rates

We write the interest rate on loan i and date t as $m_{i,t} = r_{i,t} + y_i$, where y_i is the total fixed portion of the interest rate and $r_{i,t}$ is the relevant Term SOFR index value for loan i as of date t. ARMs include a lookback period which specifies how many days before a floating ARM's next interest rate reset date the relevant index is referenced. Due to this lookback period, the gap between interest rate resets for floating loans, and the possibility of mortgage note rate caps and floors, it is likely that $r_{i,t} \neq r_t$, where r_t is the current 12-month Term SOFR rate at date t.

For mortgages making a regular monthly payment, this payment will consist of a portion which goes to paying down principal and a portion made of interest on the current balance, according to the regular amortization schedule of the loan. The interest component can be further divided into two components based on the fixed and floating portions of the overall mortgage note rate. In our notation, the fixed rate component y_i includes the mortgage-specific margin as well as the fixed constant adjustment to the index. $r_{i,t}$ is the value of 12-month Term SOFR when loan i last reset its rate, subject to any caps and floors.

Features in ARM contracts such as infrequent rate resets and lookback periods complicate the division of interest rates into fixed and floating components. For loans that are still on the fixed period there is no floating component to the interest rate, so we set $r_{i,t} = 0$ and y_i equal to the total note rate during the fixed period. As described in Section 1.4.1, occasionally a life or period interest rate cap will bind, causing the mortgage rate at reset to be something other than the relevant index value plus the constants. In those cases, we assign $r_{i,t} = m_{i,t} - y_i$, where $m_{i,t}$ is the mortgage note rate equal to the binding threshold of the relevant cap or floor. We therefore assign differences in the total mortgage note rate due to binding caps and floors to differences in the floating portion of the rate, while keeping the fixed portion of the rate constant.

There exist some loans which are floating on the conversion date but which have not had a rate reset since the conversion date. For example, a loan which resets every 12 months and which reset in June 2023 will not reset again until June 2024, meaning that it will not be subject to the LIBOR-SOFR conversion until that date. Other loans exist which reset soon after the conversion date but which have a long enough lookback period such that the new rate following the conversion date is based on a value of LIBOR prior to the conversion. From the perspective of modeling the value transfer, the cash flows given the fair spread adjustment x^* and the Fed-imposed adjustment x are identical until the first reset which references Term SOFR instead of LIBOR. Therefore, for floating loans which have not yet reset to a Term SOFR rate, we let $r_{i,t} = 0$ and y_i. When the loan observes its first post-conversion rate reset which references Term SOFR instead of LIBOR, we revert to the usual characterization of y_i and $r_{i,t}$.

1.3.2 Prepayment and Default

In addition to the mortgage note rate, prepayment and default play a key role in determining the magnitude of mortgage cash flows. To capture prepayment and default along each simulated path, we follow Fuster and Willen (2017) and use a hazard model for prepayment and default given by Equation 1.5 for each $n \in \{\text{prepayment}, \text{default}\}$.

$$h^n(t|X_{i,t-1}) = h_0^n(t) f^n(X_{i,t-1}\beta^n) \tag{1.5}$$

The specification and calibration of the hazard model in Equation 1.5 is discussed in Section 1.5.1. Along a simulation path, we our calibrated hazard model to determine probabilities of prepayment and default, which are used to simulate loan prepayment and default behavior. At each given time step, we draw a random number $x_{i,t}$ for each loan from the standard uniform distribution over $[0, 1]$. The outcome for a loan in date $t + 1$ is then as follows:

$$\begin{cases} x_{i,t} \leq h^{\text{prepay}}(t+1|X_{i,t}) & \text{Loan } i \text{ prepays} \\ x_{i,t} \geq 1 - h^{\text{default}}(t+1|X_{i,t}) & \text{Loan } i \text{ defaults} \\ x_{i,t} \in \left(h^{\text{prepay}}(t+1|X_{i,t}), 1 - h^{\text{default}}(t+1|X_{i,t})\right) & \text{Loan } i \text{ makes a regular payment} \end{cases}$$

When comparing cash flows across alternative interest rate spread specifications, we want to compare the value transferred while holding prepayment and default behavior constant. In our specification this is only relevant for default as our model for prepayment will not change with alternative values of a constant spread. The loan's LTV is impacted by the spread through alternative amortization schedules implied by different interest rates. In our simulation we track cash flows assuming that the default probabilities are given by our alternative spread adjustment x^* estimated in Section 1.5.3. When comparing results across a grid of feasible values for x^*, we use the same monthly draws of $x_{i,t}$ to simulate prepayment and default for each alternative fair spread adjustment within a given simulation.

1.3.3 Modeling Cash Flows

Given the mortgage note rate $m_{i,t} = r_{i,t} + y_i$, we now discuss how mortgage cash flows are determined, and how they are divided into fixed and floating portions. Along each simulation path, we start by determining the value of a fully-amortizing payment in case the loan does not prepay or default. Let $B_{i,t}$ be the ex-payment balance of loan i on date t. Suppose loan i has τ_i months remaining until maturity. While mortgage rates are quoted on an APR basis, assume that $r_{i,t}$ and y_i are expressed as the mortgage rate on a monthly basis instead. According to standard mortgage amortization rules, a full payment $P_{i,t+1}$ is given by

$$P_{i,t+1} = \frac{B_{i,t}(r_{i,t} + y_i)}{1 - (1 + r_{i,t} + y_i)^{-\tau_i}} \tag{1.6}$$

At each date t along a simulated path there exist known probabilities for loan i's prepayment and default in date $t+1$ given by $\Pr(\text{prepay}_{i,t+1}) = p_1(X_{i,t})$ and $\Pr(\text{default}_{i,t+1}) = p_2(X_{i,t})$. Section 1.3.2 describes the determination of the functions p_1 and p_2. Given these probabilities, random draws are used to determine if the loan prepays or defaults at date t along a given simulation path.

The date $t+1$ cash flow and balance of the loan will depend on whether the loan prepays or defaults. From t to $t+1$, the loan will accrue interest and then reduce balance from either prepayment, default, or the regularly-amortizing payment. Let $\mathbb{1}_d$ indicate default, $\mathbb{1}_p$ indicate prepayment, and $\mathbb{1}_r = 1 - \mathbb{1}_d - \mathbb{1}_p$ be the indicator for no default nor prepayment between date t and date $t+1$. Then given the date t ex-payment balance $B_{i,t}$,

$$\begin{aligned} B_{i,t+1} &= \mathbb{1}_r \left[B_{i,t}(1 + r_{i,t} + y_i) - P_{t+1}\right] + \mathbb{1}_d 0 + \mathbb{1}_p 0 \\ &= B_{i,t} - \underbrace{\mathbb{1}_r \left[P_{t+1} - B_{i,t}(r_{i,t} + y_i)\right]}_{\text{scheduled principal reduction}} - \underbrace{\mathbb{1}_p B_{i,t}}_{\text{prepayment}} - \underbrace{\mathbb{1}_d B_{i,t}}_{\text{default}} \end{aligned} \tag{1.7}$$

In our analysis, we assume that there is no partial prepayment and that default is instantaneous, such that there is no delinquency period. Therefore, Equation 1.7 assumes that prepayment and default both result in setting the loan balance to zero. The scheduled principal reduction is only realized in case there is no prepayment or default.

We can then express the cash flow at date t+1 in each case: prepayment, default, or the regularly amortizing payment. In case of prepayment, we assume that the loan accrues interest between date t and date $t+1$, at which point the total balance is paid off. The cash flow from prepayment is given by $B_t(1 + r_{i,t} + y_i)$. In case of default, we assume that a constant portion of the date t balance is recovered; let γ be the recovery rate. Putting it all together, we have

$$CF_{t+1} = \mathbb{1}_r P_{t+1} + \mathbb{1}_d \gamma B_t + \mathbb{1}_p B_t(1 + r_{i,t} + y_i) \tag{1.8}$$

For loans that are fixed or have not yet reset since the conversion (i.e., loans where $r_{i,t} = 0$), we use Equation 1.8 to model the total monthly cash flow for a given loan along a simulation path. For loans that are floating and have had a rate reset since the conversion date (i.e., loans where $r_{i,t} \neq 0$), we can expand the cash flow expression to divide it into a component based on the floating rate and another component that contains the remainder of the cash flow.

$$\begin{aligned}
CF_{t+1} &= \mathbb{1}_r P_{t+1} + \mathbb{1}_d \gamma B_t + \mathbb{1}_p B_t(1 + r_{i,t} + y_i) \\
&= B_t(r_{i,t} + y_i) + \mathbb{1}_r \left[P_{t+1} - B_t(r_{i,t} + y_i)\right] + \mathbb{1}_d B_t(\gamma - r_{i,t} - y_i) + \mathbb{1}_p B_t \\
&= B_t(1 + r_{i,t}) - B_t + B_t y_i + \mathbb{1}_r \left[P_{t+1} - B_t(r_{i,t} + y_i)\right] + \mathbb{1}_d B_t(\gamma - r_{i,t} - y_i) + \mathbb{1}_p B_t
\end{aligned} \tag{1.9}$$

The algebra between the first and third lines in Equation 1.9 allows us to write a component of loan i's date $t+1$ cash flow that is determined solely by $r_{i,t}$. We will treat this component separately from the remaining terms in discounting these cash flows back to the conversion date.

Equation 1.9 demonstrates that as of date t, a Term SOFR-indexed loan can be thought of as a portfolio of two one-period loans: one which pays the Term SOFR rate, and the other which pays the remainder of the monthly payment accounting for prepayment and default cash flows. The portion of the cash flow indexed to Term SOFR is the floating component of interest payments. In Appendix 1.B we showed that under the assumption of no prepayment or default, the appropriate one-period discount rate for this floating cash flow is Term SOFR. We will account for these hazards using a premium for prepayment and default which we will call μ. The other component of the cash flow which is not strictly indexed to Term SOFR we will discount at the risk-free rate as well as the premium for prepayment and default risk.

Let $f^0_{t,t+1}$ be the date 0 implied futures rate for risk-free cash flows between date t and $t+1$. We will use the futures rates to discount future cash flows along each simulation period-by-period in order to capture their projected value as of date 0. Recall that r_t is the date t value of 12-month Term SOFR. For floating loans that have reset to a Term SOFR index, we can write the date t value of the date $t+1$ cash flows expressed in Equation 1.9.

$$V_t(CF_{t+1}) = \frac{B_t(1 + r_{i,t})}{(1 + r_t)(1 + \mu)} + \frac{B_t y - B_t + \mathbb{1}_r \left[P_{i,t+1} - B_t(r_{i,t} + y)\right] + \mathbb{1}_d B_t(\gamma - r_{i,t} - y) + \mathbb{1}_p B_t}{(1 + f^0_{t,t+1})(1 + \mu)} \tag{1.10}$$

The first term represents the floating component of the cash flow, discounted by Term SOFR and adjusting for the risk of prepayment and default. The second term is the fixed component of the cash flow, discounted by the risk free rate and adjusted for the risk of prepayment and default.

For loans which are fixed or have not reset to Term SOFR since the conversion date, recall that we set $r_{i,t} = 0$ and that the fixed component of the interest rate y_i is set equal to the total interest rate. Therefore the date $t+1$ cash flow given in Equation 1.8 has no floating component, and the value in date t is simply given by

$$V_t(CF_{t+1}) = \frac{\mathbb{1}_r P_{t+1} + \mathbb{1}_d \gamma B_t + \mathbb{1}_p B_t(1 + r_{i,t} + y_i)}{(1 + f^0_{t,t+1})(1 + \mu)} \tag{1.11}$$

This formulation allows for the term premium inherent in $f^0_{t,t+1}$ as well as the premium for prepayment and default risk. For each of these one-period discounted cash flows expressed in Equations 1.10 and 1.11, we iterate and discount this projected value back to date 0 using the risk free rate as well as $1+\mu$. This results in a date 0 value of the date $t+1$ cash flows for mortgage i given by

$$V_0(CF_{t+1}) = \frac{V_t(CF_{t+1})}{\prod_{s=0}^{t-1}\left(f^0_{s,s+1}(1+\mu)\right)} = \frac{V_t(CF_{t+1})}{r^f_{0,t}(1+\mu)^t}$$

where $r^f_{0,t}$ is the risk free rate between date 0 and date t.

The total value of the mortgage at date 0 (the conversion date) is then the sum of the present value of its forecasted cash flows, averaged across multiple simulations. The difference between this value under the value-preserving constant adjustment x^* and the Fed-imposed constant adjustment x constitutes a value transfer due to the LIBOR-Term SOFR conversion.

1.4 Data Description

For mortgage data we use Black Knight's McDash dataset. McDash contains mortgage origination data such as LTV, loan size, and 3-digit ZIP geography. Borrower characteristics such as DTI and FICO are also included. For ARMs, there is information detailing the fixed rate period, the frequency of loan resets, and any caps and floors for the loan. The relevant index is also encoded, however, the tenor of the index is not. For example, the data indicates that a loan is indexed to LIBOR but not 12-month LIBOR. As such, we assume that the tenor of the index corresponds to the reset frequency, so that a LIBOR-indexed loan resetting every three months is indexed to 3-month LIBOR[3].

McDash also reports monthly performance data, including the unpaid balance and the current interest rate of the loan each month. Our sample consists of first-lien 30-year LIBOR-indexed ARMs that remained open as of June 30, 2023, the date LIBOR was discontinued. We also impose that the loans reset every 12 months and are therefore likely to have been indexed to one-year LIBOR. This assumption allows us to compare the margin for these loans with the margin on Treasury-indexed ARMs, for which the one-year rate is the most common index. We also impose that the mortgages be prime mortgages[4] for a primary residence, that they are conventional loans without private mortgage insurance, that they are not balloon mortgages, and that loan LTV as of origination does not exceed 100%. We do not include interest-only (IO) loans in our main specification. This set of restrictions ensures that we observe a homogeneous population of loans. Based on the current criteria, our final dataset includes 99,986 mortgages, with a total principal balance of $50.5 billion as of the conversion date.

Table 1.2 shows descriptive statistics for open loans as of the conversion date. Of the loans included in this sample, only 39% are floating. This helps to explain the relatively low average interest rate as of the conversion date. Spread at origination is determined by subtracting the average Fannie Freddie rate during the month of mortgage origination from

[3]This assumption appears to hold in the data by inspection when comparing the reported interest rate of a loan to the corresponding tenor of LIBOR plus the loan-specific margin.

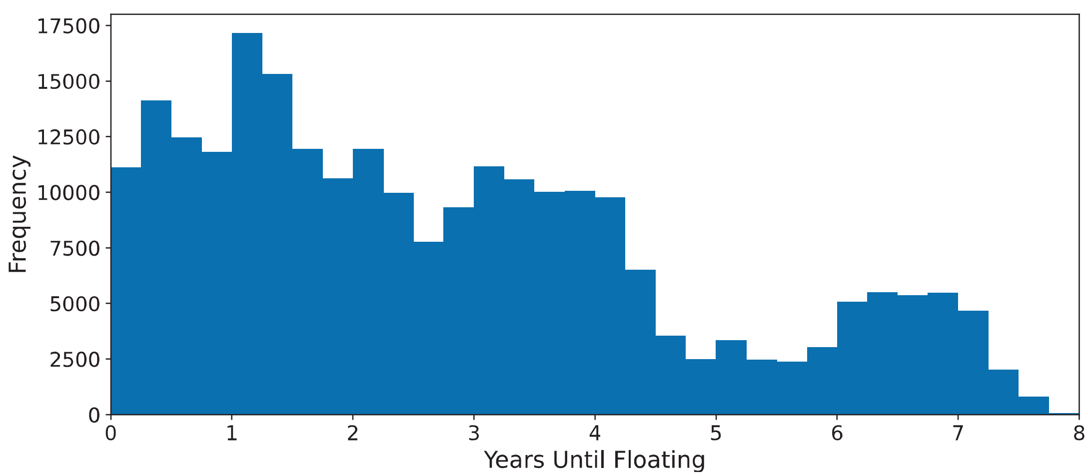
[4]Subprime loans fell in popularity following the 2008 recession and observed high rates of prepayment and default; as such, very few LIBOR-indexed ARMs remained open as of June 30, 2023.

Table 1.2: Main Sample Descriptive Statistics

	Mean	Std Dev	Min	Median	Max
Original Interest Rate (%)	3.30	0.80	1.00	3.10	10.20
Interest Rate on Conversion (%)	3.90	1.30	1.40	3.50	8.60
ARM Margin (%)	2.20	0.20	0	2.20	5.10
Original Loan Amount ($1,000s)	632.33	607.96	15.00	485.00	30,000.00
Unpaid Balance at Conversion ($1,000s)	505.34	544.08	1.00	355.00	27,181.00
Original LTV (%)	64.30	17.30	1.50	69.60	100
Loan Age At Conversion (Years)	7.54	4.87	0.92	6.17	25.17

the original loan rate. It is unsurprising that the mean spread at origination is negative. This is because most mortgages included in the calculation of the Fannie Freddie are FRMs which typically have a higher interest rate than ARMs.

Figure 1.2: Time Until Floating for Fixed LIBOR Loans



Note: 248 loans with more than eight years until floating are not shown to enhance readability.

For the 61% of loans that are not yet floating as of the conversion date, there is significant heterogeneity in the time until floating. Figure 1.2 shows a histogram of the time to float in years. These loans are significantly weighted towards floating soon: 42.2% of the loans which have not floated yet will float within three years if not prepaid or defaulted before then. This is important for our analysis because loans that are not floating will not be impacted by the transition from LIBOR to Term SOFR until they do float. We expect the impact of the transition to be smaller for loans that are far from floating.

Figure 1.2 also shows that there is a mass of loans with four or seven years until floating. These correspond to ARMs originated during the surge in refinancing activity around the COVID-19 pandemic and which are three years matured by the LIBOR conversion date. Chapter 2 shows that over 90% of ARMs originated during that time were ARMs with a seven- or ten-year fixed rate period.

The duration of the fixed period is encoded in months by McDash and often does not equal a multiple of twelve months, suggesting that the McDash encodings may be based on differing fixed rate starting points. To categorize the loans into discrete year buckets for descriptive statistics, we flag loans as having a fixed rate period of less than 24 months, equal to 24 months, between 25 and 36 months, between 37 and 60 months, between 61 and 84 months, and greater than 84 months. We will call these 1/1, 2/1, 3/1, 5/1, 7/1, and 10/1 loans, respectively[5].

Table 1.3: Summary Statistics by Loan Fixed Period

Fixed Period	Count	Original Loan Amount ($K)	Principal ($K)	Loan Age (Years)	Margin (%)	LTV (%)
1 Year	1,245	438.48 (384.84)	333.14 (326.50)	9.82 (6.24)	2.32 (0.10)	70.03 (17.73)
2 Years	1	113.00 (—)	60.00 (—)	19.42 (—)	3.25 (—)	97.72 (—)
3 Years	1,626	353.30 (562.55)	219.39 (494.60)	17.02 (4.04)	2.25 (0.49)	69.04 (15.88)
5 Years	21,467	401.08 (489.78)	278.88 (425.93)	11.41 (5.82)	2.22 (0.27)	64.50 (17.75)
7 Years	47,708	678.75 (582.52)	546.65 (519.95)	6.52 (3.70)	2.25 (0.10)	63.80 (17.25)
10 Years	27,939	755.65 (684.00)	633.11 (612.10)	5.65 (3.25)	2.25 (0.06)	64.37 (16.90)

Note: Means are displayed with standard deviations below in parenthesis.

Table 1.3 shows summary statistics for loans within each category of fixed rate period length. Note that there is only one loan with a fixed rate period of exactly 24 months. Chapter 2 shows that this loan type was popular prior to 2008 and that its rate of default is significantly higher than for other kinds of ARMs. As discussed in Section 1.5.1, 3.6% of loans in our training sample are 2/1 ARMs and are treated separately in the logit estimation. As such, we maintain this categorization here despite the sparse representation of that category in loans as of the conversion date.

We find substantial heterogeneity in loan characteristics both within and between loan categories. For example, loan size differs by length of fixed period, with longer fixed periods tending to correspond with larger loans. An exception to this observation is the 1/1 loans, which are larger and have higher margins. The 5/1, 7/1, and 10/1 loans are younger, reflecting a shift away from shorter fixed rate periods in new mortgage origination.

1.4.1 Rate Caps and Floors

Adjustable-rate mortgages typically include both per-period and lifetime constraints on interest rate adjustments. Per-period rate caps and floors limit the extent to which the interest rate on a mortgage can change from one reset period to the next. For example, a

[5]To get a sense for the importance of this assumption, note that 99.90% of loans have fixed rate periods of exactly 12, 24, 36, 60, 84, or 120 months or one month less than those benchmarks, suggesting that very few loans could potentially be incorrectly sorted by this mechanism.

loan with a current interest rate of 5% and a per-period cap of 2.5 percentage points would be restricted to a maximum rate of 7.5% at the time of reset, even if the index rate plus the margin would otherwise imply a higher rate. Lifetime caps and floors, in contrast, define absolute upper and lower bounds on the interest rate over the life of the loan, ensuring that large fluctuations in the underlying index do not drive the mortgage rate beyond these predefined thresholds.

Table 1.4: Summary of Loan Caps and Floors

	Life Cap	Life Floor	Period Cap	Period Floor
Fraction of Loans with This Restriction	99%	100%	100%	77%
Most Common Value	5%	2.25%	5%	2%
Fraction of Loans with Most Common Value	88%	74%	49%	15%

Table 1.4 contains descriptive statistics for caps and floors in the data. We find that most loans have both lifetime caps and floors, as well as period caps. Period floors are far less common and are more widely distributed than other caps and floors, with the most common period floor only representing 15% of the sample. Note that the life caps and floors are represented as relative to the original interest rate, so a 5% cap imposes a maximum interest rate of 5% above the starting rate rather than an absolute cap of 5%. Period caps and floors are relative to the current rate on a given mortgage.

McDash appears to include both absolute lifetime caps as well as lifetime caps relative to the starting rate. If the reported lifetime cap is exactly 5% larger than the original interest rate, we assume that that loan has a lifetime cap relative to the initial rate of 5%. We do so because of bunching at 5% in the distribution of the reported lifetime cap minus the original interest rate. Additional controls for data quality include multiplying the reported cap or floor by the appropriate power of 10 for some loans with reported caps and floors less than 10 basis points, and dropping period rate floors less than 2% because for these loans, the period rate floor is equal to the original interest rate minus the margin for the loan and is not credible.

1.4.2 Timing of Rate Resets

In addition to interest rate caps and floors, the timing of rate resets introduces further complexity in modeling adjustable-rate mortgage payments. ARM contracts typically specify both the specific day of the month on which the new interest rate takes effect and a "lookback period," which determines how far in advance the index rate is observed for the reset. For instance, a loan scheduled to reset on February 15 with a 45-day lookback period would reference the index rate from January 1 when determining its new interest rate.

The McDash dataset does not provide information on the exact reset date within each month for individual loans, preventing differentiation between mortgages that reset at the beginning versus the end of the month. Given this limitation, we assume that all resets occur simultaneously at the beginning of each month. Consequently, we downsample our projected Term SOFR values to a monthly frequency. Additionally, we approximate the lookback effect by assuming that loans with a lookback period of 30 days or less reference the index rate from the preceding month and those with a lookback period of 31 to 60 days use the index rate from two months prior. In our data, 76% of loans are assigned a two-month lookback and 24% are assigned a one-month lookback.

1.5 Model Calibration

In this section we describe the calibration of the model used to project cash flows. This includes the estimation of the logit models for prepayment and default described in Section 1.3.2 and the calibration of the dynamics for the Nelson-Siegel model for the yield curve. We also describe the method we use to identify a constant spread over Term SOFR which is consistent with market valuation. Finally, we derive an estimate of the premium for the risk of prepayment and default, μ.

1.5.1 Logit Hazard Model

To estimate the logit model for prepayment and default hazard rates described in Section 1.3.2, we use McDash mortgage performance data. Rather than select a set of mortgages and estimate the model on the resulting panel of data, we select a time window and fit the logit model to all open loans during that window subject to the same restrictions described in Section 1.4.

Namely, we require loans to be LIBOR-indexed, first-lien 30-year ARMs. In addition, we require the mortgages to be conventional without private mortgage insurance, to not be balloon or IO loans, and to have LTV less than or equal to 100%. Unlike in our main analysis, we do not exclude loans which reset at frequencies other than one year. This bolsters the amount of loans included in our analysis.

The baseline hazard $h_0^n(t)$ for prepayment and default in Equation 1.5 is the loan age, which we specify as a polynomial in loan age for the first seven years, then an indicator for loans aged eight through 15 years, followed by another indicator for loans older than 15 years. This flexible specification permits granular variation in baseline hazard for younger loans, where the data is rich, and acknowledges the noise introduced by older loans where there are fewer observations.

We treat as default an event where a loan becomes 90 or more days delinquent[6] using the Mortgage Bankers Association definition of delinquency, similar to the approach used in Fuster and Willen (2017). Loans are classified as prepaid when the balance is set to 0 prior to loan maturity. In case of prepayment or default, a loan is censored from the sample. Loans are also treated as censored if they are transferred out of the data feed and performance becomes unavailable.

To avoid overfitting the hazard model, we exclude the interest rate and the current spread from the logit models for prepayment and default. The downward-sloping yield curve on the conversion date was such that the rate on floating loans was at a historic high, as was the spread between the current rate and the 10-year rate. While the current rate and the current spread terms may be natural to understand as economic drivers of prepayment and default behavior, logit models are unable to capture their dynamics at unusually high levels. As such, they are not included in our specification.

[6] Results are similar when using alternative thresholds for mortgage delinquency.

The window we select for our analysis includes monthly observations for all loans which were not prepaid or defaulted at any point between January 2015 and December 2019. This window is selected to represent a time in which there is some variability in the interest rate yet no salient financial crisis likely to have driven abnormal aggregate prepayment or default behavior. We allow loans which originate during this window as well as loans which originated before the sample window and have not prepaid or defaulted by January 2015 to be included in our sample in order to capture the prepayment and default behavior across the entire lifetime of a loan.

Table 1.5: ARM Characteristics Within the Logit Data

Fixed Rate Period Length			**Reset Frequency Once Floating**		
Value	Freq.	%	Value	Freq.	%
1 Year	1,299,667	6.83	1	159,019	0.84
2 Years	681,628	3.58	6	1,094,465	5.75
3 Years	498,860	2.62	12	17,783,496	93.42
5 Years	5,567,181	29.24			
7 Years	7,896,302	41.48			
10 Years	3,093,342	16.25			
Total	**19,036,980**	**100.00**	**Total**	**19,036,980**	**100.00**

Table 1.5 describes the distribution of the length of the fixed rate period as well as the frequency of interest rate resets once floating for the ARMs in our analysis. We find that the vast majority of loans in our analysis are loans which reset every 12 months, with only 6.58% of loans resetting at other frequencies. There is a large mass of loans with a fixed rate period of exactly 24 months, suggesting that these loans merit different treatment in our logit model. As such, we assign different indicators to loans within each of the fixed rate period categories in Table 1.5 in our logit estimation.

Table 1.6: Descriptive Statistics for Logit Sample

	Mean	Std Dev	Min	Median	Max
Original Interest Rate (%)	3.64	1.37	1.00	3.25	15.89
ARM Margin (%)	2.41	0.80	0.02	2.25	9.99
Original Loan Amount ($1,000s)	477.86	443.99	10.00	368.00	45,000.00
Original LTV (%)	66.53	16.81	0.58	71.82	100.00
Spread At Origination (%)	-0.86	1.09	-6.28	-0.80	10.70

The loan characteristics from the pool of loans used to estimate logit are similar to the characteristics of the loans impacted by the LIBOR-SOFR cramdown described in Section 1.4. Table 1.6 displays additional descriptive statistics for the loans used to estimate the logit models for prepayment and default. By comparing these numbers with those contained in Table 1.2, we see that the loans used to estimate the logit have similar LTV, starting interest rate, origination spread, and margin to those from our main pool of ARMs. The average loan size at origination is larger for our main sample, but we attribute this to the increased house price for loans which originated after this window.

Table 1.7: Logit Regression Estimates for Prepayment and Default

Variable	Prepayment	Default
2 Year Fixed Period	-0.724^{***}	0.899^{***}
	(0.0230)	(0.0392)
3 Year Fixed Period	-0.00621	0.756^{***}
	(0.0153)	(0.0392)
5 Year Fixed Period	0.168^{***}	0.469^{***}
	(0.00918)	(0.0338)
7 Year Fixed Period	-0.128^{***}	0.392^{***}
	(0.00953)	(0.0391)
10 Year Fixed Period	-0.407^{***}	0.285^{***}
	(0.0106)	(0.0505)
Spread At Origination	-3.520^{***}	8.289^{***}
	(0.340)	(0.575)
(Spread At Origination)^2	-101.0^{***}	-29.54^{*}
	(11.88)	(14.02)
(FICO at Origination) /100	0.180^{***}	-0.495^{***}
	(0.00491)	(0.00824)
log Loan Size	0.103^{***}	0.00464
	(0.00280)	(0.0115)
Origination LTV	-0.00895	1.370^{***}
	(0.0641)	(0.377)
(Origination LTV)^2	0.136^{*}	-1.564^{***}
	(0.0548)	(0.265)
Origination LTV = 80%	-0.0154^{**}	-0.0237
	(0.00587)	(0.0172)
Full Documentation	-0.0417^{***}	-0.430^{***}
	(0.00581)	(0.0235)
No Documentation	-0.0828^{***}	-0.0196
	(0.00870)	(0.0255)
Cash Out Refi	-0.0700^{***}	0.209^{***}
	(0.00682)	(0.0301)
Non-Cash Out Refi	-0.0711^{***}	-0.0222
	(0.00448)	(0.0153)
Condo	-0.0441^{***}	-0.259^{***}
	(0.00488)	(0.0219)
(Loan Age ≤ 7) $\times$ Loan Age	0.0996^{***}	0.877^{***}
	(0.00951)	(0.111)
(Loan Age ≤ 7) $\times$ Loan Age^2	0.0347^{***}	-0.120^{***}
	(0.00290)	(0.0299)
(Loan Age ≤ 7) $\times$ Loan Age^3	-0.00412^{***}	0.00928^{***}
	(0.000251)	(0.00233)
Loan Age $\in (7, 15]$	0.431^{***}	4.238^{***}
	(0.0108)	(0.121)
Loan Age > 15	0.398^{***}	4.632^{***}
	(0.0402)	(0.137)
Current LTV		3.282^{***}
		(0.165)
Current LTV Squared		-1.010^{***}
		(0.113)
Constant	-7.255^{***}	-8.552^{***}
	(0.0561)	(0.229)
Observations	19,036,980	19,036,980
Pseudo R^2	0.184	0.014

Figure 1.3: Prepayment and Default Hazard Rates in Logit Training Data

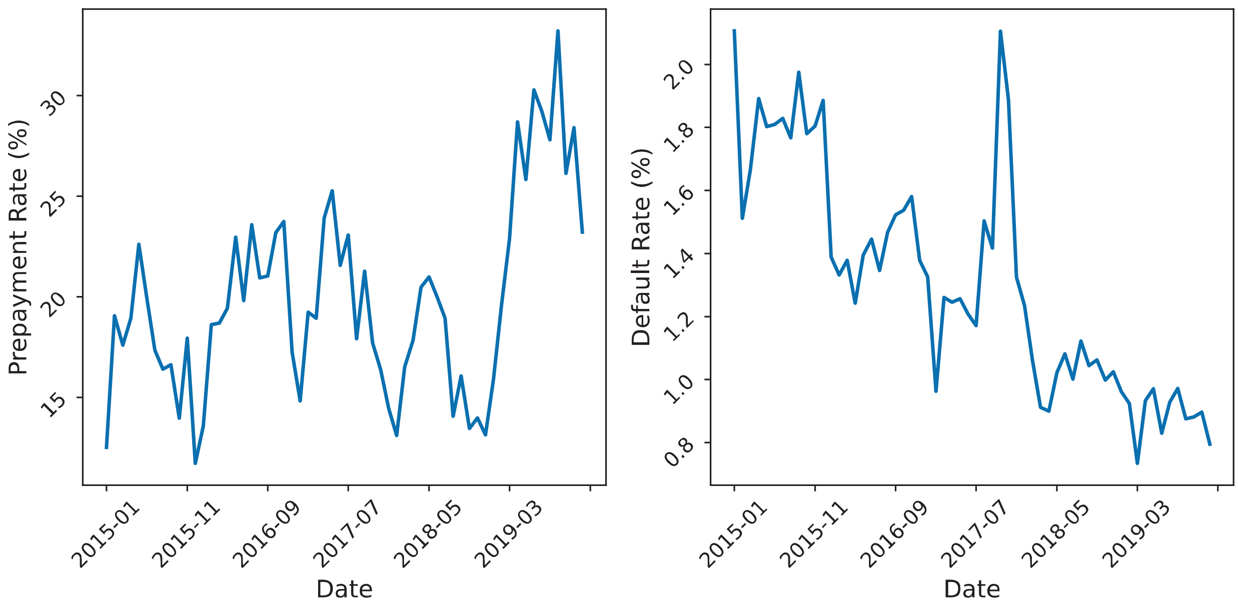


Table 1.7 shows the estimated logit coefficients. The baseline loan is a 1/1 ARM with incomplete documentation which is not for a condo and not a refinancing loan. We include an indicator for the LTV at origination being equal to 80%. This addresses the concern that first-lien mortgages with LTV equal to 80% may be likely to take a second mortgage to target the desired amount of debt without being required to pay for private mortgage insurance. We find that when this constraint binds, a mortgage is less likely to prepay, which is consistent with the hypothesis that when LTV is bound at 80%, the borrower is likely to have a second lien on the mortgage. We also find that we are able to capture a phenomenon similar to the well-documented burnout for mortgage prepayment (for example, see Richard and Roll (1989)) in that very mature loans are less likely to prepay than younger loans, despite the reduced principal remaining on the loan.

We find that our model for prepayment is able to describe a larger proportion of the observed prepayment behavior than the model of default can for default behavior. There may be several reasons for this finding. One possible explanation is that we have omitted or imprecisely measured variables which are important predictors of default. For example, debt to income ratios as reported by McDash only include the debt burden of the mortgage and not of other debt obligations. In addition, the measure we use for LTV does not include other liens on the property[7]. While these imprecisely measured variables appear in the models for both prepayment and default, we would expect the measures of the debt burden to matter more for models of prepayment than default.

Figure 1.3 displays the aggregate prepayment and default behavior for all loans in our sample throughout the window of observation. This highlights another possible reason for the lower pseudo R^2 in our model for default relative to prepayment. While throughout the window the rate of prepayment is between 11.7% and 33.2% of all open loans per year, the

[7]McDash does report a combined LTV field, but the field is very sparsely populated.

rate of default is only between 0.7% and 2.1% per year. King and Zeng (2001) demonstrate that low rates of the outcome of interest in logistic regressions can result in a bias of the estimated coefficients. In case of default, we set the recovery rate γ to 70%.

We add a loan's current estimated LTV to the logit specification for default because of its important role in predicting default rates. Homeowners with a high equity stake are highly unlikely to default on a loan because of the forfeited equity that would result.

To estimate the current LTV for a loan we use the loan's current unpaid balance divided by an estimate for the current value of a home. Without regular transactions we cannot observe the market value of a property, so we estimate the value using the annual FHFA All-Transactions House Price Index at the three-digit ZIP level. We used the annual data set instead of the quarterly for its increased historical coverage. The house price indices are then linearly smoothed throughout each year and matched to the McDash monthly performance data. Given the house price index, we can estimate

$$\frac{L_t}{V_t} = \frac{L_0}{V_0} \times \frac{V_0}{V_t} \times L_t \times \frac{1}{L_0} \tag{1.12}$$

where $\frac{L_0}{V_0}$ is LTV at origination, $\frac{V_0}{V_t}$ is determined using the house price index, L_t is the current unpaid balance, and L_0 is the original loan amount. For our simulation exercise, we project house prices to grow at the mean house price growth rate over the last five years in the loans' three-digit ZIP area.

Figure 1.4: Forecast Prepayment and Default Rates

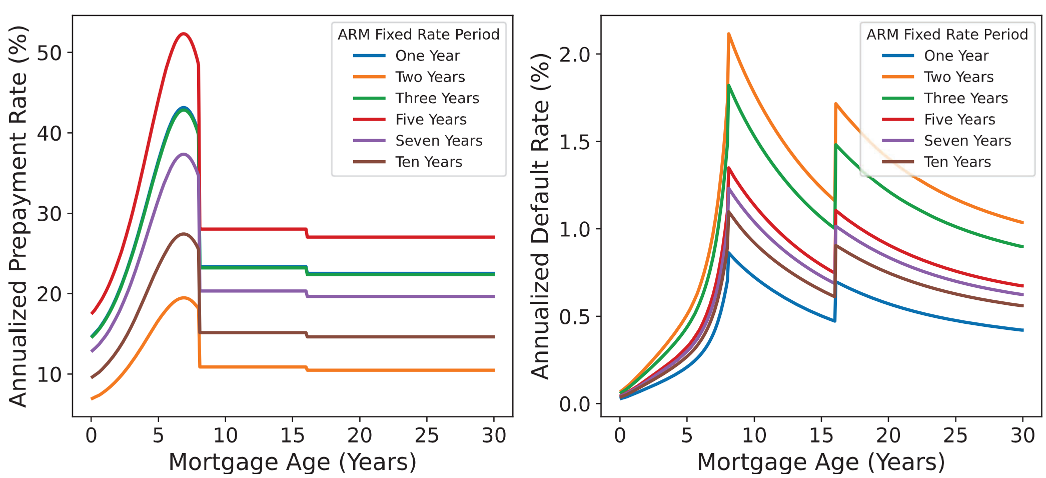


Figure 1.4 shows the forecast annualized hazard rates for prepayment and default for representative loans with characteristics set to the median values of the characteristics of the loans converted from LIBOR to Term SOFR. Prepayment and default rates have clear spikes where the age crosses indicator thresholds included in the logit specification. LTV decreases over time for these loans due to amortization and projected house price growth, which drives the downward-sloping rates between the jumps in the default rate. There is clear heterogeneity in the hazard rate for loans of different fixed rate periods, with the five,

seven, and ten year fixed period ARMs having the lowest default rates. Most of the loans in our main sample have fixed period lengths of five, seven, or ten years, suggesting that default is not a crucial driver of our results.

1.5.2 Nelson-Siegel Factor Dynamics

Calibration of the dynamic Nelson-Siegel model we use to simulate the evolution of the yield curve dynamics requires specifying an initial point to capture the yield curve on the day of the conversion and parameterization of the factor dynamics described in Equation 1.2.

We estimate the starting point X_0 by minimizing the mean squared error between the Nelson-Siegel implied yield curve and a bootstrapped treasury yield curve on the date of the conversion[8]. This exercise results in

$$X_0 = \begin{bmatrix} 0.034 \\ 0.018 \\ 0.033 \\ 0.512 \end{bmatrix}$$

For the factor dynamics, we target a few key properties. The first is that mean reversion is not too fast, so that the model predicts reasonably slow changes in the yield curve. We also target a long-run rate of around 3.5% and a short rate of 2.5%, allowing the yield curve to be upward sloping. For the factor controlling the curvature, we set parameters to that there is occasionally a hump-shaped yield curve in the long-run distribution of simulated yield curves, and that the hump generally falls between one and five year maturities. Assuming the time step in Equation 1.2 is monthly[9], we set

$$\delta = \begin{bmatrix} 0.05 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ \overline{X} = \begin{bmatrix} 0.035 \\ -0.01 \\ 0.015 \\ 1.67 \end{bmatrix}, \text{ and } \sigma = \begin{bmatrix} 0.001 & 0 & 0 & 0 \\ 0 & 0.002 & 0 & 0 \\ 0 & 0 & 0.005 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}$$

where $\varepsilon_{t+1} \sim N(0,1)$ is a multivariate distribution. This dynamic specification allows us to simulate the yield curve's evolution following the conversion date.

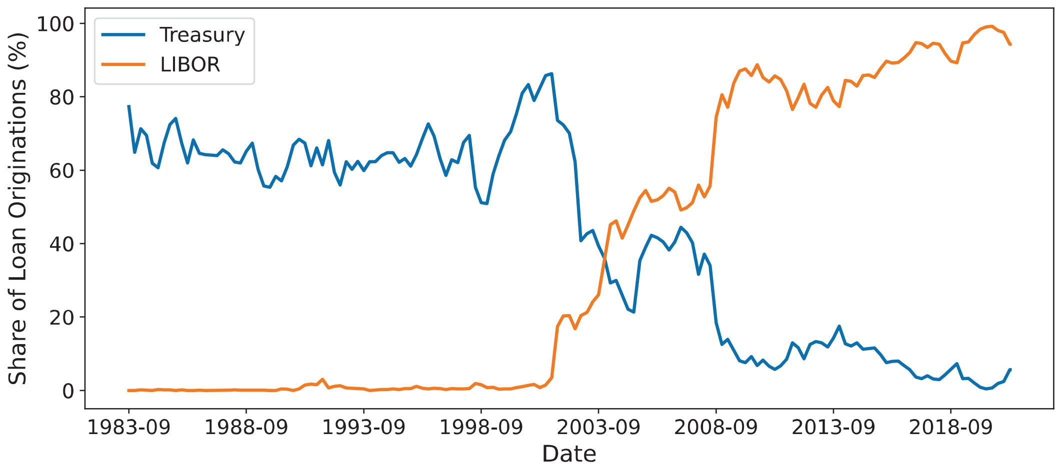
1.5.3 Fair Spread Candidates

The constant spread adjustment x imposed by the Fed and reported in Table 1.12 is based on the median spread between LIBOR and proxies for Term SOFR over a five-year window. For

[8]Specifically, we minimize the sum of squared errors between monthly observations for the first five years of the model-implied yield curve and a yield curve bootstrapped using T-bill data for short maturities and constant maturity treasury data for longer maturities. By focusing on the first five years, we emphasize goodness of fit for the shorter maturities matching mortgage duration. We bootstrap the yield curve by assuming that discount factors are log-linear in the maturity.

[9]The parameters are displayed on a monthly frequency to anhance readability. However, to simulate Term SOFR following the methodology in Equation 1.4, we must simulate daily instances of the yield curve. Assuming 30 days per month, we can easily map the monthly parameters into daily parameters using the identities $\delta_{\text{day}} = 1-(1-\delta_{\text{month}})^{1/30}$ and $\sigma_{\text{day}} = \sigma_{\text{month}}\sqrt{\frac{1-(1-\delta_{\text{day}})^2}{1-(1-\delta_{\text{day}})^{60}}}$. This mapping preserves the conditional distribution of X_{t+1} given X_t whether $t+1$ represents a single monthly draw or 30 daily draws.

Figure 1.5: ARM Treasury and LIBOR Index Share Over Time



contracts indexed to one-year LIBOR, the spread adjustment $x = 71.513$ basis points. This methodology does not necessarily capture the fixed spread that a market applies to newly issued Term SOFR contracts in comparison with LIBOR contracts.

The ideal thought experiment is to observe two identical mortgages issued to two identical borrowers. One borrower is assigned a mortgage indexed to Term SOFR; the other is issues a mortgage indexed to LIBOR. To the extent that the market cares about the level difference and different risk sensitivities of Term SOFR and LIBOR, there will be a difference in the margin applied to each mortgage. Specifically, because LIBOR incorporates more risk premiums than Term SOFR, a larger margin would be applied to the Term SOFR-indexed ARM than the LIBOR-indexed ARM. The size of the difference in margins represents the premium required by the market for a mortgage being indexed to Term SOFR versus LIBOR.

In principle we could approximate this thought experiment by taking a pool of LIBOR and Term SOFR-indexed ARMs and regressing the ARM margin on loan characteristics and an indicator for a Term SOFR index. However, this analysis is complicated by the fact that new ARMs are not indexed to Term SOFR. According to a Freddie Mac fact sheet,[10] GSEs only purchase new SOFR-indexed loans if they are indexed to the backward-looking 30-day average SOFR rate published daily by the Federal Reserve Bank of New York.

To get around this limitation, we use Treasury-indexed ARMs instead of Term SOFR-indexed ARMs as a baseline to determine a fair constant spread. Treasury-indexed ARMs are a well-suited substitute for Term SOFR-indexed ARMs because they share the same pricing property described in Appendix 1.B. Namely, the floating component of their cash flows will also price at par absent prepayment and default because otherwise, there is an arbitrage opportunity available through investing in risk-free securities.

Figure 1.5 shows the proportion of new ARMs indexed to the one-year tenors of LIBOR and the Treasury rate which fit the restrictions described in Section 1.4. Our method requires that both LIBOR-indexed and Treasury-indexed loans are issued concurrently. We find that

[10] https://sf.freddiemac.com/working-with-us/origination-underwriting/mortgage-products/sofr-indexed-arms

this is the case, especially starting after the year 2000. In using historical data to estimate the spread between Treasury- and LIBOR-indexed ARMs, we impose that there are at least 50 LIBOR-indexed ARMs and 50 Treasury-indexed ARMs originated during each included quarter. Our sample window includes loans issued between July 2001 and March 2021.

$$\text{ARM Margin}_i = \alpha + \beta X_i + \gamma_t(i) + \delta \mathbb{1}[\text{Treasury-Indexed ARM}(i)] + \varepsilon_i \tag{1.13}$$

We use Equation 1.13 to isolate the different spread applied to Treasury-indexed ARMs versus LIBOR-indexed ARMs. We control for mortgage characteristics such as log of loan size, credit score at origination, LTV at origination, and fixed effects for the quarter of origination. δ represents the mean spread over time between Treasury- and LIBOR-indexed ARMs after controlling for loan characteristics and origination quarter. To estimate the equation, we take the full set of ARMs in McDash including ARMs not indexed to LIBOR but applying the other restrictions applied to our main sample of LIBOR loans. To address outliers, we impose that the ARM margin be non-negative and less than or equal to 5%.

Table 1.8: Estimates for Treasury-Indexed Margin and Initial Rate Premiums

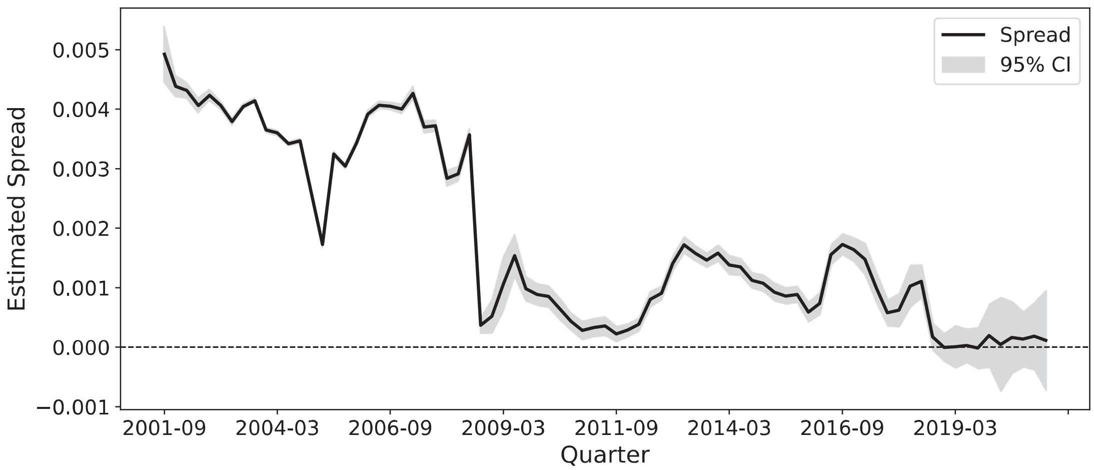
	ARM Margin	**Initial Interest Rate**
Treasury-Indexed Indicator	$3.09 \times 10^{-3***}$	$2.46 \times 10^{-4***}$
	(5.20×10^{-6})	(1.18×10^{-5})
Original LTV	$2.68 \times 10^{-5*}$	$2.61 \times 10^{-3***}$
	(1.42×10^{-5})	(3.21×10^{-5})
Log Loan Size	$-7.74 \times 10^{-5***}$	$-6.83 \times 10^{-4***}$
	(3.50×10^{-6})	(8.00×10^{-6})
Original Credit Score	$-5.54 \times 10^{-7***}$	$-1.94 \times 10^{-5***}$
	(4.85×10^{-8})	(1.10×10^{-7})
Constant	$2.53 \times 10^{-2***}$	$8.33 \times 10^{-2***}$
	(7.30×10^{-5})	(1.65×10^{-4})
Origination Quarter FE	✓	✓
R^2	0.2140	0.6528
Observations	2,051,525	2,051,496

Robust standard errors in parentheses
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Column 1 in Table 1.8 displays the regression estimates for Equation 1.13. We find that our point estimate for the average additional spread applied to Treasury-indexed loans relative to LIBOR-indexed loans is insensitive to the inclusion of origination quarter fixed effects.

This analysis relies on a few key assumptions. The first is that assignment to a mortgage index is quasi-random, so that loans indexed to LIBOR and loans indexed to the Treasury rate have similar risk. Gupta and Hansman (2021) argue for quasi-random index assignment, showing that most of the variation between ARM indices comes from variation in the lender, with different lenders specializing in one index. Bucks and Pence (2008) use survey data to show that most ARM borrowers do not know their mortgage terms, which supports our random-assignment assumption, as ARM borrowers are unaware which index pertains to their loan.

Figure 1.6: Premium for Treasury-Indexed ARMs Over Time.



A second assumption is that the difference in pricing between LIBOR- and Treasury-indexed ARMs is only in the index. It is possible that lenders assign the same margin to ARMs with different indices but adjusting for level difference in the index thorough differences in the introductory rate during the fixed rate period. To test this assumption, we regress the initial interest rate on mortgage characteristics and an indicator for Treasury-indexed ARMs, as in Equation 1.13 with the left-hand side being replaced by the initial interest rate. Column 2 of Table 1.8 presents regression results indicating a 2.46 basis point premium on the initial rate for Treasury-indexed ARMs relative to LIBOR-indexed ARMs. We conclude that while statistically significant, this premium is not economically significant and our assumption that the only pricing adjustment is through the index is valid.

An additional assumption is that the difference in interest rate margin applied to LIBOR-versus Treasury-indexed ARMs at origination is the same as the difference in margin which would be applied should the loan refinance partway through its life. We also assume that there is a time-invariant spread between the margins on loans indexed to each of these indices. We relax this assumption to estimate the variation in the spread over time.

$$\text{ARM Margin}_i = \alpha + \beta X_i + \gamma_t(i) + \delta_t(i)\mathbb{1}[\text{Treasury-Indexed ARM}(i)] + \varepsilon_i \tag{1.14}$$

Figure 1.6 plots our estimates for $\delta_t(i)$ in Equation 1.14. We find substantial variation in the premium required for Treasury-indexed ARMs over time. We find a large decrease in the spread at the end of 2008, during the recession. We also find that there is no significant spread since 2018, shortly after the end of support for LIBOR was announced in 2017. In our main specification we rely on the 30 basis point constant spread adjustment estimated in the left-hand panel of Table 1.8. In Section 1.6.3, we estimate the value transfer for alternative values of a fair spread adjustment.

1.5.4 Mortgage Risk Premium

In order to determine a range of appropriate values for μ used in Equation 1.10, we can simulate cash flows from representative mortgages. The fair μ for each representative loan is then one that sets the loan's value at origination to 102% of par, using the valuation method described in Section 1.3. Fuster, Lo, and Willen (2024) document that loans price at different fractions of par across time. We pick 102% as our baseline and examine the sensitivity of our results to alternative estimates for μ.

To construct a representative loan on a given month, we must make choices about its characteristics. For each month, we construct several loans which vary by loan size and the duration of the fixed period. We allow loan sizes of $250,000, $500,000, $750,000, and $1,000,000. We also vary the fixed-rate period such that each loan is either a 3/1, 5/1, 7/1, or 10/1 ARM. The representative loans are indexed to one-year Term SOFR plus a 30 basis point constant spread to closely reflect the estimated market value of the loans in our sample post-conversion.

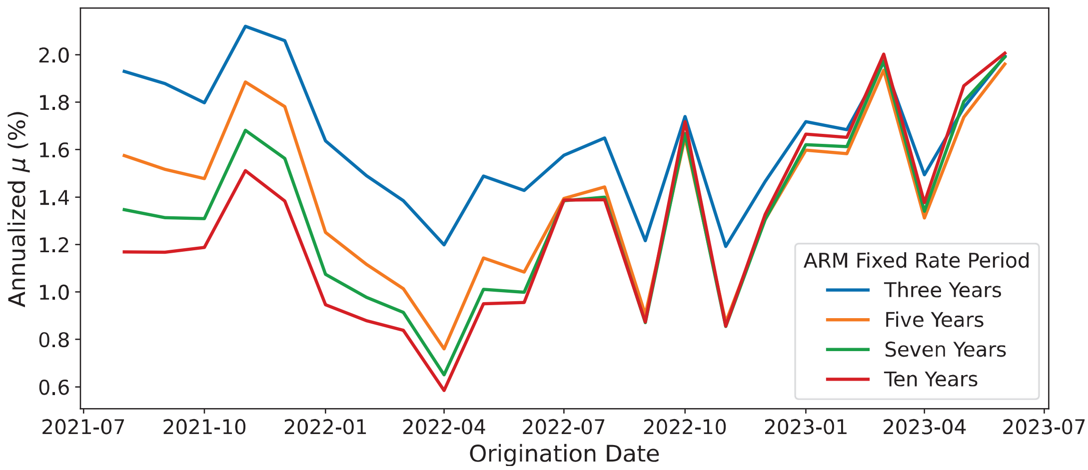
For each of these loans we set the rest of their characteristics at some common values. LTV at origination is set to 80% and the margin is set to 2.25%. We need to make an assumption about the house price growth rate, as we don't assign the representative loan to a specific ZIP code. However, we want a house price growth rate comparable with what we expect as of the conversion date; as such, we set the house price growth rate for each representative loan to the median of the average house price growth rates for the open loans on the conversion date across the five years leading up to conversion. The starting interest rate for each loan is set using the average new rate for ARMs originated that given month.

We must also specify parameters used in the Nelson-Siegel model to model cash flows for these representative loans. We re-fit the Nelson-Seigel vector of parameters X_0 given the origination date for these representative loans, and simulate the yield curve using the same dynamics in Equation 1.2. Because we will price a range of representative loans across a variety of origination dates, we will be able to assess within-pool heterogeneity of μ on a given date, as well as the stability of μ for a specific representative loan if it originated in different months.

Figure 1.7 shows the results of this exercise. The estimated mortgage risk premium varies both within periods and across time. Within periods, most of the variation comes from the length of the fixed-rate period: 10/1 ARMs correspond to the lowest values of μ, while 3/1 ARMs correspond to higher values. This pattern is consistent with the decreasing risk profiles associated with longer fixed-rate periods, as seen in Table 1.7 and discussed in Chapter 2. In June 2023—the month of the LIBOR–SOFR conversion—our annualized estimates of μ have relatively little variation across loan class, ranging from 1.96% to 2.03%. In contrast with the length of the fixed rate period, the size of the ARM has negligible impact on the estimate for the mortgage risk premium μ.

Across time, the variation is substantial, but there is strong correlation between the estimated mortgage risk premiums for different classes of ARMs. This variation over time implies that we cannot identify a narrow range of μ values for which mortgages consistently price near 102% of par. In Section 1.6.2, we evaluate the value transfer across a range of μ values, with our baseline results based on the average transfer using each estimated μ on the conversion date.

Figure 1.7: Estimates for Mortgage Risk Premium μ



Note: μ is quoted on an annual basis. Results are based on 100,000 cash flow simulations for each loan on each origination date.

1.6 Estimated Value Transfer

In this section we discuss our estimates for the value transfer implied by our baseline specification. We then test the sensitivity of our findings to alternative calibrations. Specifically, we consider the value transfer implied by different mortgage risk premiums μ and alternative fair constant spread adjustments. Our methodology for simulating cash flows allows for prepayment and default, which options have their own value that we do not explicitly price. We evaluate the importance of the loan caps and floors by testing the value transfer when there are no caps or floors, and we test the value transfer implied by our model when there is no chance of prepayment or default. Finally, we discuss extensions of our estimates to other LIBOR-indexed loans which were converted when LIBOR was discontinued.

1.6.1 Baseline Results

Our baseline results are based on the assumption that a fair constant spread adjustment is 30 basis points and that μ is based on our estimate from representative loans originated on the conversion date. The value transfer we estimate is the value of the mortgage cash flows under the Fed-imposed constant spread adjustment of 71.513 basis points minus the value of the remaining cash flows under our estimated 30 basis point alternative constant spread adjustment.

The first row of Table 1.9 shows our estimates for the total value transferred due to the cramdown in our baseline specification. The total estimated value transfer under our baseline specification is \$248.73 million, which is 0.49% of the total balance of the pool. Loans which are fixed as of the conversion date experience the smallest value transfers. This is because the alternative constant spread adjustment only applies when a loan is floating, so

Table 1.9: Value Transfer Estimates

Specification	Aggregate Value Transfer		Individual Value Transfer	
	Total	% of Balance	Floating	Fixed
Baseline	\$248.73 M	0.49%	0.94% (0.23%)	0.48% (0.31%)
Alternative Specifications				
20 Basis Point Spread	\$309.09 M	0.61%	1.17% (0.28%)	0.60% (0.39%)
10 Basis Point Spread	\$369.61 M	0.73%	1.40% (0.33%)	0.72% (0.46%)
No Caps/Floors	\$274.13 M	0.54%	0.99% (0.20%)	0.53% (0.32%)
No Hazards	\$1.28 B	2.54%	2.61% (0.66%)	2.58% (0.58%)

Note: Standard deviations are displayed below means in parentheses.

the discounting applied to the cash flow projections makes the difference smaller for floating loans. In addition, loans which are not yet floating have a high likelihood of prepayment prior to floating. A loan which prepays prior to floating to a Term SOFR rate will not undergo a value transfer.

To better understand the drivers of different sizes of value transfer, we partition the pool of loans into two groups based on whether the loan is fixed or floating as of the conversion date. Within each of these two groups, we compare the means and standard deviations of the value transfer between loans within the highest and lowest quartiles of age, margin, credit score, and LTV.

Table 1.10: Value Transfer by Loan Characteristic Quartile

	Fixed Loans		Floating Loans	
Variable	Lower Quartile	Upper Quartile	Lower Quartile	Upper Quartile
Loan Age	0.224	0.888	0.897	0.774
	(0.083)	*(0.230)*	*(0.230)*	*(0.121)*
ARM Margin	0.479	0.481	0.947	0.945
	(0.312)	*(0.313)*	*(0.228)*	*(0.227)*
Original Credit Score	0.592	0.421	0.936	0.945
	(0.330)	*(0.294)*	*(0.227)*	*(0.228)*
Conversion LTV	0.629	0.284	0.965	0.908
	(0.346)	*(0.180)*	*(0.244)*	*(0.215)*

Note: Means displayed with standard deviation below in parentheses. All numbers are reported as percentages of principal. Based on 100,000 simulations using the baseline specification.

Table 1.10 shows the results of this analysis. A few notable relationships stand out. For fixed loans, older loans have a larger value transfer. This is because they are closer to floating and thus the difference in cash flows between the Fed-imposed spread and our alternative 30 basis point spread is discounted less. However, for floating loans, older loans have smaller value transfers. This is because there are more remaining periods with a difference in cash flows before loan maturity.

Credit score is an important predictor of the value transfer for fixed loans, but not for floating loans. This may be due to the lower chance of prepayment from loans with a lower credit score, as found in Table 1.7. Higher values for LTV at origination values correlate with higher value transfers. Higher LTVs correspond newer loans which have longer remaining lives. The ARM margin does not predict significantly different value transfers for either fixed or floating loans.

The fact that LTV correlates strongly with age suggests that the results in Table 1.10 are driven by correlations with important drivers of heterogeneity in the loan size. To address this concern and control for the relationships between alternative predictors of the value transfer, we regress the value transferred as a fraction of unpaid balance on the conversion date relative to the 30 basis point baseline on mortgage characteristics as of the origination date. Table 1.11 contains the results of this regression.

Contrary to the descriptive exercise of Table 1.10, we find that the margin has a significant impact on the size of the transfer. Loans with a large ARM margin experience smaller transfers, all else equal. This may be due to differences in credit quality which are not captured in the credit score but are reflected in the margin. Loans with a larger balance as of the conversion date have smaller value transfers, due to the increased prepayment rate of these loans described is Table 1.7. The value transfer is regressive, due to the increased value transfer for smaller loans for borrowers with lower credit scores.

1.6.2 Alternative Mortgage Risk Premium

In Section 1.5.4, we calibrated μ, the mortgage risk premium used in discounting, by pricing a set of representative loans at 102% of par. We found heterogeneity in the estimated value for μ both across time and across alternative representative loans. In our baseline estimates, we report the average value transfer as the average value transfer implied by the range of estimated values for μ across alternative representative loans issued in the month of the conversion.

Figure 1.8 demonstrates the total value transferred across alternative values for μ. The range of the values for μ considered spans the minimum and maximum values estimated across all loans and all time periods discussed in Section 1.5.4. We find there is a small, nearly-linear sensitivity to μ in the size of the value transfer, with a 1% increase in μ leading to a decrease of $17.83 million (0.04% of the balance of the pool) in the size of the value transfer.

1.6.3 Alternative Constant Spread Adjustments

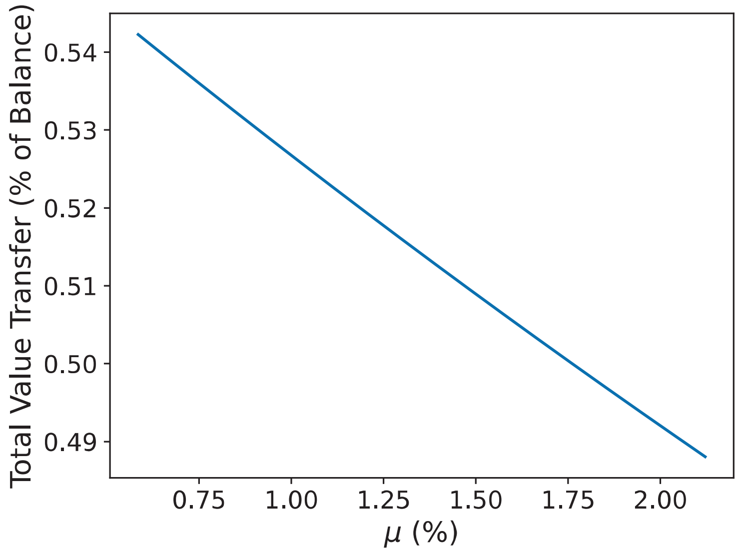
In Section 1.5.3 we discuss our reasoning for basing our estimates of the realized value transfer on the difference in the present value of the loan cash flows when the constant spread adjustment is 71.513 basis points, as imposed by the Federal Reserve, and when the constant spread adjustment is 30 basis points, the historical premium imposed in the ARM margins of Treasury-indexed ARMs relative to LIBOR-indexed ARMs. Figure 1.6 shows that this premium varies over time. Following the 2008 recession, the premium applied

Table 1.11: Baseline Value Transfer Regression

	Fixed Loans	Floating Loans
Original Credit Score (/100)	$-1.27 \times 10^{-3***}$	$-5.25 \times 10^{-4***}$
	(1.25×10^{-5})	(1.23×10^{-5})
Loan Age (Years)	$6.92 \times 10^{-3***}$	$-1.42 \times 10^{-4***}$
	(1.68×10^{-3})	(1.57×10^{-6})
ARM Margin (%)	$-2.06 \times 10^{-4**}$	$6.22 \times 10^{-5**}$
	(8.64×10^{-5})	(2.48×10^{-5})
2 Year Fixed Period	—	5.67×10^{-4}
		(1.20×10^{-3})
3 Year Fixed Period	—	$-5.06 \times 10^{-4***}$
		(4.66×10^{-5})
5 Year Fixed Period	$-1.42 \times 10^{-2***}$	$-8.60 \times 10^{-4***}$
	(3.37×10^{-3})	(3.58×10^{-5})
7 Year Fixed Period	$-2.50 \times 10^{-2***}$	$2.08 \times 10^{-3***}$
	(6.72×10^{-3})	(3.56×10^{-5})
10 Year Fixed Period	$-4.22 \times 10^{-2***}$	$4.30 \times 10^{-3***}$
	(1.18×10^{-2})	(4.04×10^{-5})
LTV as of Conversion Date (%)	$-1.08 \times 10^{-5***}$	$-1.57 \times 10^{-5***}$
	(3.09×10^{-7})	(5.53×10^{-7})
Balance as of Conversion Date ($1,000)	$-7.27 \times 10^{-7***}$	$-1.83 \times 10^{-6***}$
	(7.32×10^{-9})	(2.27×10^{-8})
Time to Floating (Years)	$5.35 \times 10^{-3***}$	—
	(1.68×10^{-3})	
Constant	-3.19×10^{-3}	$1.50 \times 10^{-2***}$
	(4.99×10^{-3})	(1.23×10^{-4})
Observations	60,430	39,556
R-squared	0.8933	0.7201

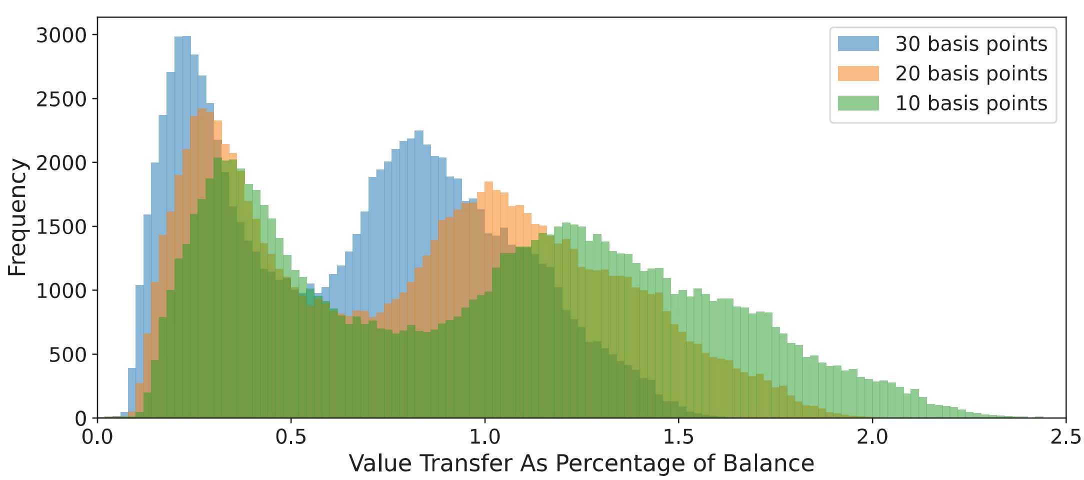
Robust standard errors in parentheses
$^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$

Figure 1.8: Value Transfer for Alternative Mortgage Risk Premium Estimates



to Treasury-indexed ARMs relative to LIBOR-indexed ARMs fell to between zero and 20 basis points. Here, we consider the sensitivity of our model to alternative values for the fair constant spread adjustment.

Figure 1.9: Distribution of Value Transfers for Alternative Fair Spread Adjustments



Note: Result based on 100,000 simulations.

In Figure 1.9 we display estimates of the value transfer given alternative constant spread adjustments of 10, 20, and 30 basis points. Summary statistics for the value transfer under these alternatives are also shows in Table 1.9. In each case the estimated value transfer is the difference between the present value of the cash flows under the 71.513 basis point spread and the relevant alternative spread. Smaller alternative constant spread adjustments correspond with larger value transfers due to the larger difference between the Fed-imposed spread and the alternative constant spread. This effect is larger for loans with higher sensitivity to the interest rate, leading to larger differences in the value transfer under alterative spread adjustments for loans that experience a large value transfer in the baseline specification. The total estimated value transfer due to the cramdown is estimated at $369.61 million when the alternative spread is 10 basis points, $309.09 million when the alternative spread is 20 basis points, and $248.73 million when the alternative spread is 30 basis points.

1.6.4 Caps and Floors Sensitivity

As discussed in Section 1.4.1, the encoding of caps and floors in our data requires some assumptions to apply those constraints to our simulation. It is possible that we have incorrectly applied caps and floors in our data, and that this has a large impact on our estimates for the value transfer. The caps and floors impact our analysis in two ways. The first is the size of the cash flow when the cap or floor on the interest rate is binding. The second is that when the caps or floors bind, we discount the entire cash flow using Term SOFR rates instead of discounting the fixed portion of the interest payment using the implied forward rates. In this section we estimate the value transfer absent any caps or floors on the loans in our sample at assess their impact on this analysis.

The second row of Figure 1.10 displays the distribution of estimated value transfers for fixed and floating ARMs under the assumption that loans have no caps or floors. This distribution closely resembles that of the baseline specification, though it is slightly shifted to the right. Table 1.9 quantifies this shift, showing that the estimated value transfer is approximately 10% larger in the absence of caps and floors.

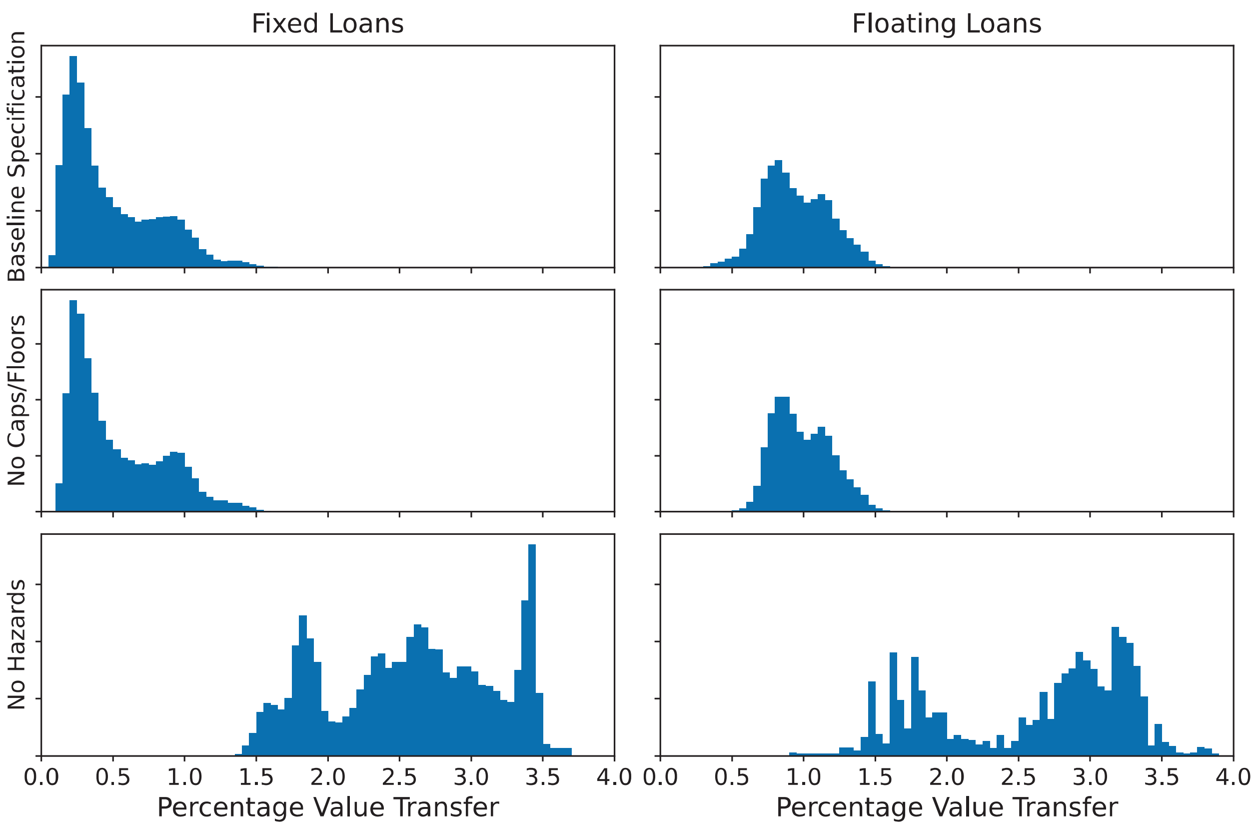
This increase arises because a substantial share of loans in the sample encounter binding period rate caps at their first post-conversion reset. For example, one-year LIBOR rose from 3.62% on June 30, 2022, to 6.04% on June 30, 2023. Since 45.25% of the loans carry a 2% period rate cap, many were constrained during the first year after conversion. When the cap binds, cash flows are identical across alternative constant spread adjustments. Because this binding occurs at the earliest resets—when differences in cash flows are least attenuated by discounting—the effect amplifies the estimated value transfer in the capped specification.

1.6.5 Alternative Specification of Prepayment and Default

In our simulations of the cash flows, the average aggregate probabilities for prepayment and default for the entire loan pool change over time. Figure 1.3 shows the annualized probabilities of prepayment and default given our logit specification for a representative loan. When simulating the loan pool, the aggregate prepayment and default risk decrease over time because the riskiest loans prepay or default early in the simulation and because the logit model predicts less hazard risk as loans age. This effect results in the highest prepayment and default rates early in the life of a loan, when the cash flows have the biggest impact on the value transfer due to discounting.

By comparing the value of the cash flows under alternative constant spread adjustments to Term SOFR while simulating prepayment and default, we may be understating the size of the value transfer. Our model does not explicitly price the value of the prepayment option, which

Figure 1.10: Distribution of Value Transfer Under Alternative Specifications



Note: Results based on 100,000 simulations.

can change with the current note rate of the loan (see Agarwal, Driscoll, and Laibson (2013) for example). The note rate is in turn impacted by the constant spread. To address this concern, we simulate the value of the loans under our estimated fair spread adjustment and the Fed-imposed spread adjustment without prepayment or default risk. The resulting value transfer can be interpreted as the present value of the change in the total debt burden for households which cannot prepay and do not default. This analysis also serves as a robustness check for our logit hazard model.

Figure 1.10 shows the distribution of the value transfer for all loans assuming there is no prepayment or default. Table 1.9 documents descriptive statistics of our estimated value transfer under this specification. Absent prepayment and default, the total value transfer is estimated at $1.28 billion, which is 2.54% of the balance of the loans in our pool. The large increase in the value transfer for fixed loans relative to the baseline scenario is due to the inability of loans to refinance or default prior to floating. There is no fixed ARM in this specification which does not experience a change in value due to the LIBOR conversion.

1.6.6 Extension

In our primary analysis we focus on prime, first-lien mortgages for primary residence and resetting every 12 months. We also require the loans to be conventional without PMI and that the loans are not IO or balloon. This subset of loans permits us to accurately model prepayment and default behavior by estimating a logit model on a similar subset of loans. However, the total balance of the pool of loans we consider in McDash is only $50.5 billion. This is in stark contrast with the estimate of $800 billion in open, LIBOR-linked loans provided in Alternative Reference Rates Committee (2021).

We found that the value transfer totaled $248.73 million, which is 0.49% of the principal. If we apply that 0.49% estimate to the total ARRC-estimated value of LIBOR loans as of the conversion date ($800 billion), we find that the value transfer can be as large as $3.94 billion. Without prepayment or default, the estimated value transfer in our sample was $1.28 billion, which is 2.54% of the sample balance. Applying the 2.54% to the $800 billion estimate, we find that the value transfer could have been as large as $20.30 billion.

There are several reasons why the application of our estimated value transfer as a fraction of principal does not apply to loans outside of our sample. For subprime loans, we would expect the value transfer to depend on the forecasted prepayment and default behavior, which will not be accurately captured in our logit model described in Section 1.5.1. IO loans will likely have a larger value transfer than the loans in our sample. IO loans do not amortize during the IO portion of the loan. Therefore the difference in cash flows under different constant spread adjustments is larger over time than it is for loans which do amortize. We provide these numbers as indications for how big the value transfer may have been rather than precise estimates of how large it was.

1.7 Conclusion

LIBOR was last published on June 30, 2023. After that date, open LIBOR-indexed contracts needed a suitable replacement for LIBOR. The Federal Reserve bore the responsibility to select that replacement for a wide variety of different contracts, including derivatives, business loans, securitizations, and ARMs. For ARMs lacking fallback language which specified a replacement index, the Federal Reserve imposed that Term SOFR would be the new index. This change had economic consequences because Term SOFR lacks many risk premiums which are included in LIBOR. To address the level difference between Term SOFR and LIBOR, the Federal Reserve added a constant to the Term SOFR index used as a replacement.

We find that the Federal Reserve's imposed constant (71.513 basis points), based on historical spreads between LIBOR and Term SOFR, exceeded the historical premium for mortgages indexed to an index similar to Term SOFR. We estimate this historical premium to be 30 basis points. The 41.513 basis point difference in the spread results in an extra \$248.73 million in estimated cash flows from the mortgages. This represents an increase in the value of the mortgages, at the expense of borrowers. This effect is largest among ARMs which were floating as of June 30, 2023, because they are immediately subject to the too-high constant spread adjustment that was imposed. We also find that the value transfer is larger for loans with low credit scores, perhaps because they are unable to obtain refinancing to prepay the loan. Indeed, in an extension where there is no prepayment or default, we find that the total value transfer is estimated at \$1.28 billion, representing a 2.54% increase in the debt burden for borrowers who are unable to prepay and do not default.

Our estimates for the size of the value transfer are sensitive to several assumptions we made, including the risk premium due to prepayment and default risk which is used to discount cash flows, as well as the premium which the market assigns to ARMs indexed to a rate which does not incorporate the same risk premiums that are reflected in LIBOR. This sensitivity highlights the importance of using market prices to determine a value-preserving spread, suggesting that the historical median spread used by the Federal Reserve for ARMs does not preserve mortgage valuation.

These results point to broader concerns—ARMs are only a small subset of the estimated open LIBOR-indexed financial contracts. This chapter emphasizes the importance of using market-based (rather than historically-based) data in applications of eminent domain, or any case of a repricing by fiat. It also highlights the importance of robust fallback language in financial contracts, in case a relevant index is deemed untrustworthy.

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Appendices to Chapter 1

1.A The Federal Reserve's LIBOR-SOFR Spread Adjustment

The ARRC determined its recommended spread adjustment by first constructing a historical proxy for SOFR and Term SOFR using RP rates. Proxies were necessary because official SOFR data was only published starting in April 2018, and Term SOFR was not introduced until July 2021. Using actual and proxy SOFR data, the Federal Reserve compounded overnight rates to approximate Term SOFR for various maturities. The ARRC then calculated the five-year median spread between the proxy Term SOFR and the corresponding LIBOR tenor over the period March 5, 2016 – March 5, 2021. The results, shown in Table 1.12, are the basis for the recommended adjustments following a one-year phase-in[11]. For example, a contract originally indexed to 12-month LIBOR was converted to one-year Term SOFR plus 71.513 basis points.

Table 1.12: ARRC-Recommended Term SOFR Constant Spread Adjustments

Tenor	**Spread Adjustment (Basis Points)**
Overnight	0.644
One Week	3.839
One Month	11.448
Two Month	18.456
Three Month	26.161
Six Month	42.826
One Year	71.513

[11]The constant spread adjustment started as the average realized spread for 10 days prior to the conversion day and converged to the spreads in Table 1.12 linearly over the first year. Our simulation code reflects this one-year adjustment.

1.B No-Arbitrage Proofs

In this section we prove through no-arbitrage arguments that Term SOFR rates are approximately equal to Treasury rates of matching maturities. This justifies our use of the spread between the margin on Treasury- and LIBOR-indexed ARMs as a fair spread adjustment for LIBOR-indexed ARMs with the index changed to Term SOFR. We then make a similar no-arbitrage argument that absent prepayment and default risk, the appropriate discount rate for the floating component of the interest on a Term-SOFR indexed ARM is Term SOFR.

Term SOFR Rates Approximately Equal Treasury Rates

Suppose at date 0 an investor wishes to lock in rates for a risk-free investment of principal P at date X with maturity T. There are two contracts available to do so: a Treasury forward contract and a SOFR futures contract.

For the Treasury forward contract, suppose the forward rate is $F^T(0, X, T)$. In that case, at date X the investor can invest P and receive a risk-free cash flow of $P(1 + F^T(0, X, T))$ at date T. Similarly, if the SOFR futures rate is $F^S(0, X, T)$, at date X the investor can invest P and receive a cash flow of $P(1 + F^S(0, X, T))$ at date T.

We now show that as of date X, the return on each of these investments is approximately equal. Consider borrowing P in the overnight RP market at date X to finance the long position in either the Treasury forward or the SOFR future. Continue rolling this debt forward at overnight RP rates until date T. The cost of financing is

$$\prod_{i=X}^{T}(1 + \mathrm{RP}_i),$$

where RP_i is the overnight RP rate (SOFR) on date i. By definition, the payoff from the SOFR futures contract exactly covers this financing cost at maturity T. In addition, because this overnight compounding is approximately risk-free, the payoff from the Treasury forward also covers the position. Thus,

$$F^T(0, X, T) \approx F^S(0, X, T) = \prod_{i=X}^{T}(1 + \mathrm{RP}_i) - 1.$$

There are a couple of differences between Treasury rates and compounded overnight RP rates which prevent total equality of Treasury forward rates and SOFR futures rates. Treasuries are more liquid (which influences pricing), and holding a Treasury to maturity avoids the interest rate risk inherent in rolling overnight RP rates forward. However, there would be arbitrage if rolling RP rates systematically differed from the return on Treasury bills. Because Term SOFR is based on SOFR futures, by setting $X = 0$ we conclude that Term SOFR is approximately equal to Treasury rates.

Discount Rate for Riskless Contracts Indexed to Term SOFR

Loans in our analysis have a fixed and a floating component to the interest rate. The fixed component is due to the margin and the constant spread adjustment; the floating component comes from the Term SOFR index. Each component is subject to prepayment and default risk; but the component indexed to Term SOFR carries additional interest rate risk. We establish here that this component prices at par absent prepayment and default risk. In Section 1.3.3 we address the how to correct for the premium we assign to prepayment and default risk which is then calibrated in Section 1.5.4.

Consider a one-period loan with no risk of prepayment or default, with a floating rate indexed to Term SOFR. Specifically, assume that at date $t = 1$ the loan pays $P(1 + \text{Term SOFR}(0))$. To determine the value of this contract at $t = 0$, suppose that the value is $P - D$. There is now possible arbitrage if $D \neq 0$. Lend $P - D$ worth of debt at $t = 0$ using this contract. To cover the cost of this investment, borrow $P - D$ in an overnight repurchase agreement, and keep rolling forward the debt until $t = 1$.

From the Term SOFR-indexed loan, the expected cash flow will be $P(1 + \text{Term SOFR}(0))$. The cost of the debt, having rolled overnight RPs forward until $t = 1$, is the product of the overnight RP rates. By definition, the date 0 expected cost of the debt would be $-(P - D)\text{Term SOFR}(0)$. Therefore, the expectation of the profit from this strategy is $D\text{Term SOFR}(0)$.

This is not a true arbitrage because the net cash flow is positive only in expectation, rather than with certainty. Unexpected fluctuations in the overnight RP rate can cause this to be a losing portfolio. The positive expected cash flow compensates for interest rate risk. We assume there is no interest rate risk and therefore conclude that a cash flow indexed to Term SOFR will price at par through one-period discounting. Over short time periods, such as month-to-month with the ARMs, the assumption of no interest rate risk is reasonable. Therefore, the appropriate discount rate for a date $t + 1$ floating cash flow determined by Term SOFR(0) and without prepayment or default risk is Term SOFR(0). This result is applied to price the floating component of cash flows in Section 1.3.3, where we also account for prepayment and default risk.